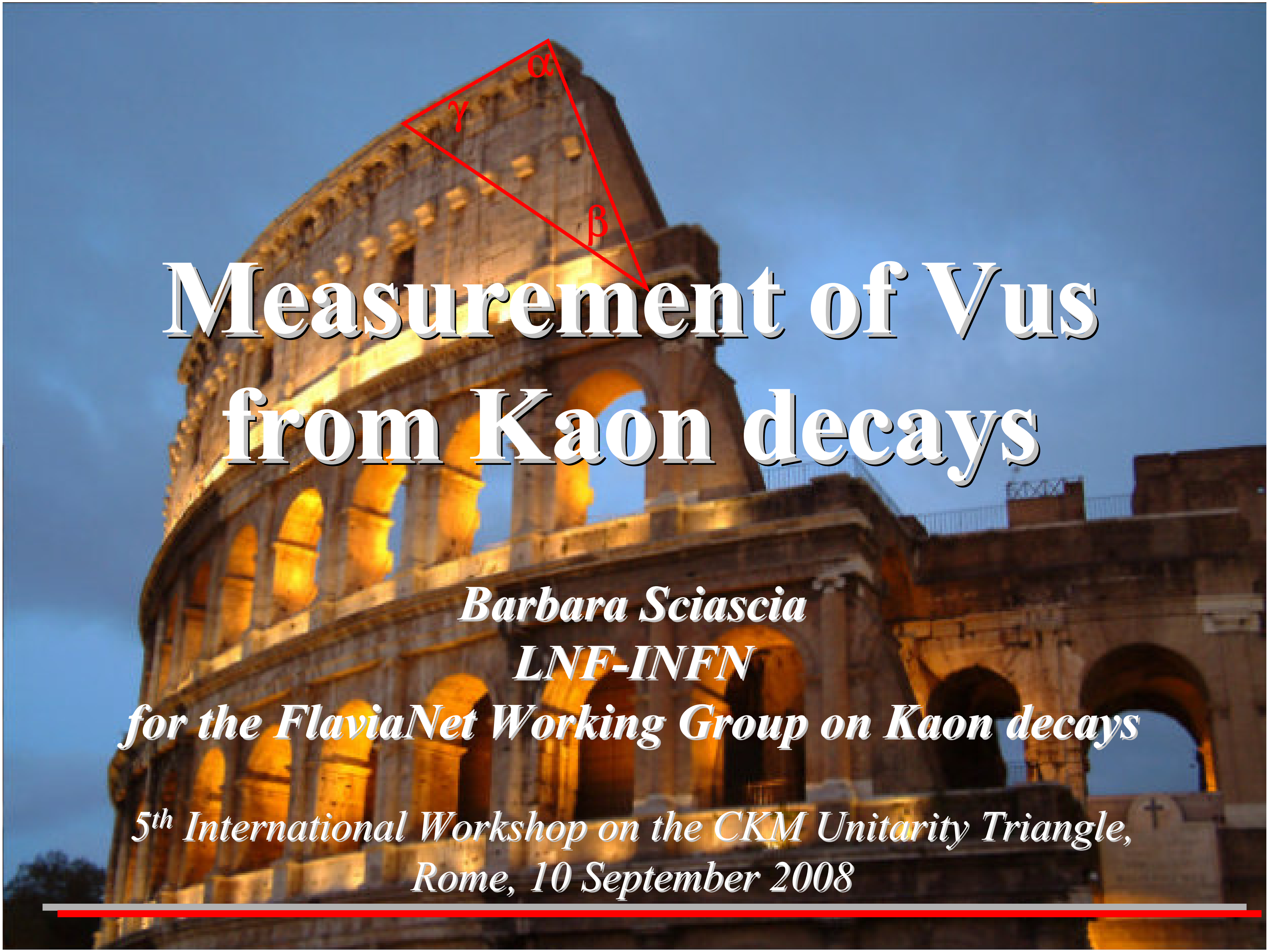

Measurement of Vus from Kaon decays

Barbara Sciascia
LNF-INFN
for the FlaviaNet Working Group on Kaon decays

5th International Workshop on the CKM Unitarity Triangle,
Rome, 10 September 2008

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The FlaviaNet Kaon working group

- **The FlaviaNet Kaon WG (www.lnf.infn.it/wg/vus/)**. Recent kaon physics results come from many experimental (BNL-E865, ISTRA+, KLOE, KTeV, NA48) and theoretical (Lattice, χ_{PT},). The main purpose of this working group is to perform precision tests of the Standard Model and to determine with high accuracy fundamental couplings (such as V_{us}) using all existing (published and/or preliminary) data on kaon decays, taking correlations into account.

- WG note: ***Precision tests of the Standard Model with leptonic and semileptonic kaon decays***, arXiv:0801.1817 [hep-ph] 11 Jan 2008.

Summary of the talk

Global fits and averages:

- K_L, K_S, and $K^{\pm}$, dominant BRs and lifetime.
- Parameterization of the $K \rightarrow \pi$ interaction (form factor)

Physics results:

- $|V_{us}| \times f_+(0)$
- Test of lepton universality with $K_{\ell 3}$.
- $|V_{us}|/|V_{ud}| \times f_K/f_\pi$.
- Theoretical estimations of $f_+(0)$ and f_K/f_π.
- V_{us} and V_{ud} determinations.
- Experimental evaluation of $f_+(0)$ and f_K/f_π.
- Bounds on helicity suppressed amplitudes.
- The special role of $BR(K^{\pm}e2)/BR(K^{\pm}\mu 2)$

see T.Spadaro talk on "Kℓ2 review"

Update results of the WG note.

K_L leading branching ratios and τ_L

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18 input measurements:

5 KTeV ratios
NA48 K_{e3} /2t and $\Gamma(3\pi^0)$
4 KLOE BRs
KLOE, **NA48** $\pi^+\pi^-/K_{l3}$
KLOE, **NA48** $\gamma\gamma/3\pi^0$
PDG ETAFIT for $\pi^+\pi^-/\pi^0\pi^0$
KLOE τ_L from $3\pi^0$
Vosburgh '72 τ_L

Parameter	Value	S
$BR(K_{e3})$	$0.4056(7)$	1.1
$BR(K_{\mu3})$	$0.2705(7)$	1.1
$BR(3\pi^0)$	$0.1951(9)$	1.2
$BR(\pi^+\pi^-\pi^0)$	$0.1254(6)$	1.1
$BR(\pi^+\pi^-)$	$1.997(7) \times 10^{-3}$	1.1
$BR(2\pi^0)$	$8.64(4) \times 10^{-4}$	1.3
$BR(\gamma\gamma)$	$5.47(4) \times 10^{-4}$	1.1
τ_L	$51.17(20)$ ns	1.1

8 free parameters, 1 constraint: $\Sigma BR=1$

Main differences wrt PDG08:
- For KLOE and KTeV, use values obtained before applying constraints.
- Make use of preliminary $BR(3\pi^0)$ from NA48
- Fit parameter $BR(\pi^+\pi^-)$ is understood to be inclusive of the DE component.

Evolution of the average BR values

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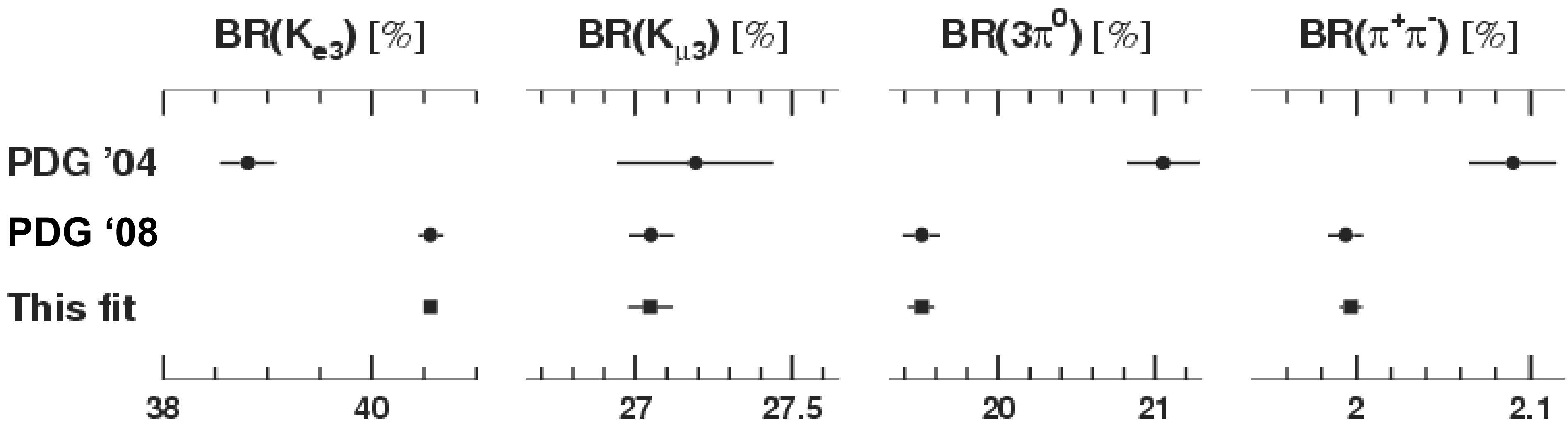
This fit χ^2/ndf = 20.2/11 (4.3%)

PDG08 fit: main BR's coincide even if χ^2/ndf = 35.7/17 (0.5%)

Differences wrt PDG08:

- contrast between KLOE BR($3\pi^0$) and other inputs involving BR($2\pi^0$) and BR($3\pi^0$)
- **treatment of the correlated KLOE and KTeV inputs**: more uniform scale factor in this fit and significantly smaller uncertainty for BR(Ke3)
- treatment of $K_L 2\pi(\gamma)$ decays (IB vs DE): dramatic worsen of fit quality.



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K_S leading branching ratios and τ_S

4 input measurements:

KLOE BR(Ke3)/BR($\pi\pi$)
KLOE BR($\pi\pi$)/BR($\pi^0\pi^0$)
Universal lepton coupling
NA48 BR(Ke3)
τ_S: non CPT-constrained fit value, dominated by 2002 NA48 and 2003 KTeV measurements

4 free parameters: $K_S\pi\pi$, $K_S\pi^0\pi^0$, K_Se3, $K_S\mu3$, 1 constraint: ΣBR=1

- **KLOE meas. completely determine the leading BR values**.
- NA48 Ke3 input improve the BR(Ke3) accuracy of about 10%.
- BR(K_Se3)/BR(K_Le3) from NA48 not included (need of a K_L and K_S combined fit)
- Combined K_S–K_L fit would be useful in properly account for preliminary NA48 $\Gamma(K_L\to3\pi^0)$ and PDG ETAFIT, used in the K_L fit.

$K^{\pm}$ leading branching ratios and $\tau_{\pm}$

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25 input measurements:

5 older τ values in PDG

KLOE τ

KLOE BR($K\mu2$)

KLOE *Ke3*, *Kμ3*, and *Kπ2* BRs

ISTRA+ $K_{e3}/\pi\pi^0$

NA48/2 $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$

BNL-E865 K_{e3}/Kdal

3 old $\pi\pi^0/\mu\nu$

2 old *Ke3*/2 body

3 *Kμ3/Ke3* (2 old)

2 old + 1 KLOE results on 3π

7 free parameters,
1 constraint: ΣBR=1

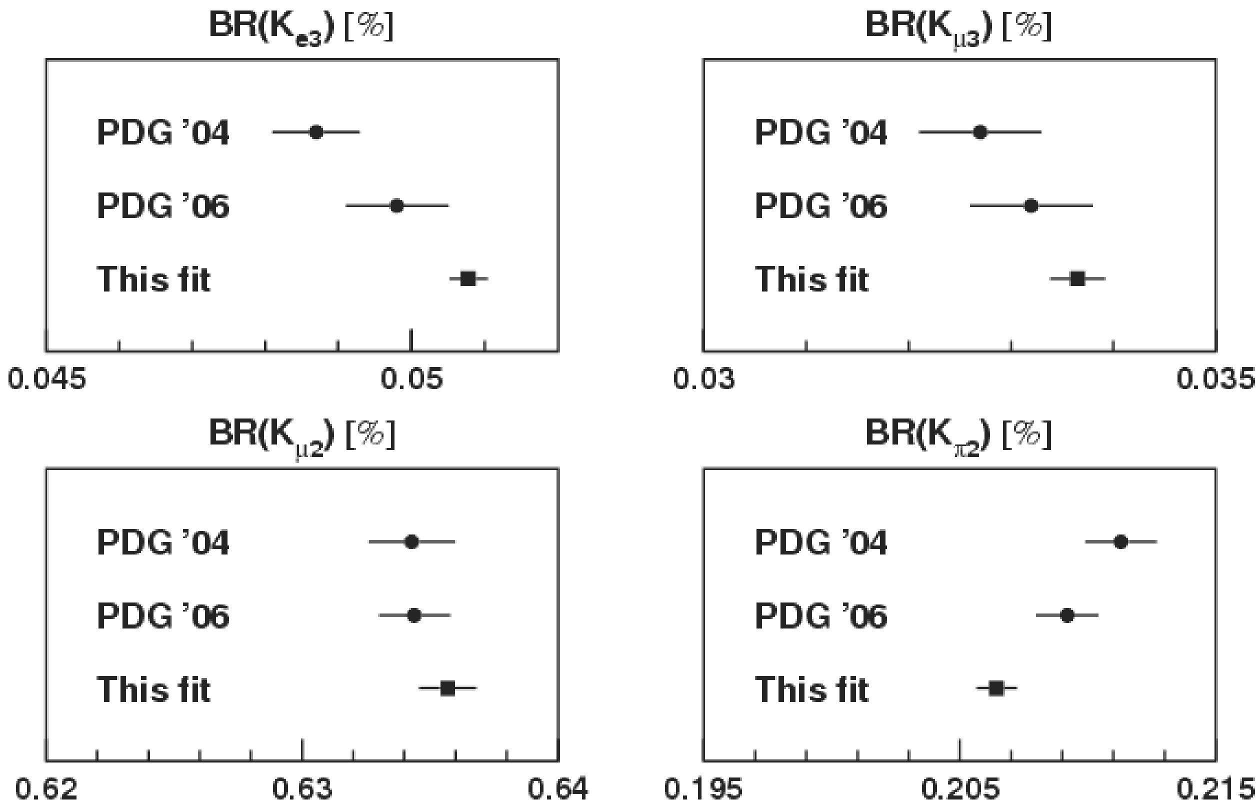
Parameter	Value	S
BR(Kμ2)	63.57(10)%	1.1
BR(Kπ2)	20.64(7)%	1.1
BR(Kπππ)	5.593(32)%	1.1
BR(Ke3)	5.078(25)%	1.2
BR(Kμ3)	3.365(26)%	1.7
BR($K\pi\pi^0\pi^0$)	1.750(26)%	1.1
τ	12.379(21) ns	1.9

Don't use the 6 BR meas. from Chiang;
- no implementation of radiative corrections
- 6 BR constrained to sum to unit.
- the correlation matrix not available.

Try to discard many other old meas.:
- no recent meas. involving BR(πππ)
- fit instable if only recent are used.

Evolution of the average BR values

- This fit **χ^2/ndf = 42.6/19 (0.15%)**; PDG08 fit: χ^2/ndf = 51.7/24 (0.08%)
- include new results
- **some conflict among newer meas. involving BR(Ke3)**: the pulls are +1.04, -0.26, -0.73, and -2.13, for NA48, BNL-E865, ISTRA+, and KLOE respectively.



- Evolution of the BR($K_{\ell3}$) and for the important normalization channels.

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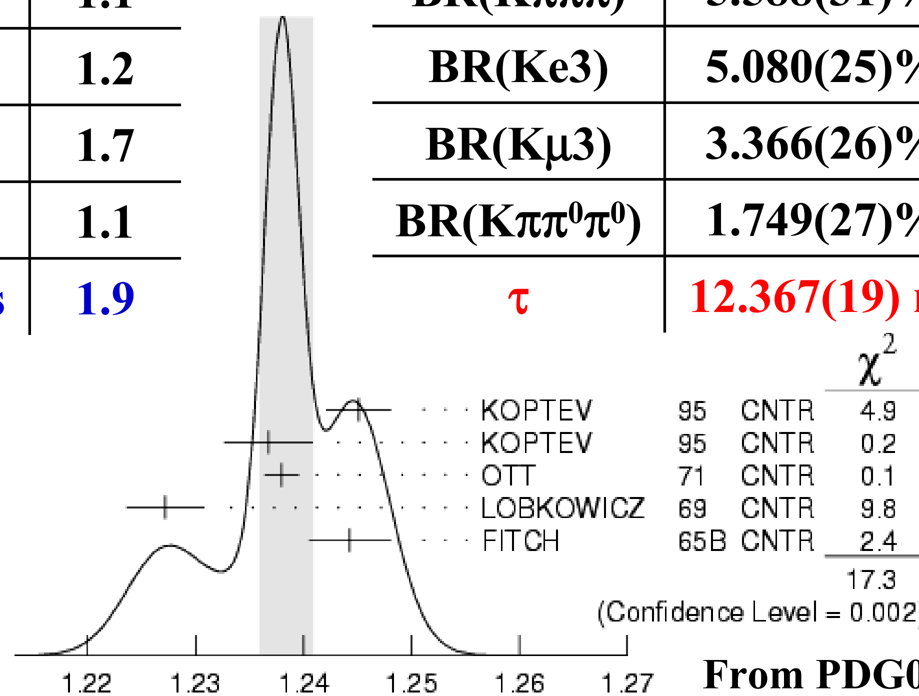
A note on $K^{\pm}$ lifetime measurements

• If 5 older $\tau_{\pm}$ measurements replaced by PDG06 avg (with S=2.1), **χ^2/ ndf = 24.8/15 (5.3%)** with **no significant changes to central values or errors, but $\tau_{\pm}$ itself (0.6σ)**.

Parameter	Value	S
BR($K\mu2$)	63.57(10)%	1.1
BR($K\pi2$)	20.64(7)%	1.1
BR($K\pi\pi\pi$)	5.593(32)%	1.1
BR(Ke3)	5.078(25)%	1.2
BR($K\mu3$)	3.365(26)%	1.7
BR($K\pi\pi^0\pi^0$)	1.750(26)%	1.1
τ	12.379(21) ns	1.9

Parameter	Value	S
BR($K\mu2$)	63.57(10)%	1.1
BR($K\pi2$)	20.65(7)%	1.1
BR($K\pi\pi\pi$)	5.588(31)%	1.0
BR(Ke3)	5.080(25)%	1.2
BR($K\mu3$)	3.366(26)%	1.7
BR($K\pi\pi^0\pi^0$)	1.749(27)%	1.2
τ	12.367(19) ns	1.0



From PDG06

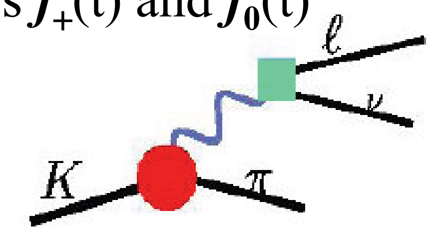
• Need of a new list of accepted $\tau_{\pm}$ measurements (deeper analysis of single papers).

Parameterization of $K_{\ell 3}$ form factors

• Hadronic K→π matrix element is described by two form factors $f_+(t)$ and $f_0(t)$ defined by:

$$\langle \pi^-(k)|\bar{s}\gamma^\mu u|K^0(p)\rangle = (p+k)^\mu f_+(t) + (p-k)^\mu f_-(t)$$

$$f_-(t) = \frac{m_K^2 - m_\pi^2}{t}\left(f_0(t) - f_+(t)\right)$$



• Experimental or theoretical inputs to define t-dependence of $f_{+,0}(t)$.

• $f_-(t)$ term negligible for K_{e3}.

• Taylor expansion:

$$\tilde{f}_{+,0}(t) \equiv \frac{f_{+,0}(t)}{f_+(0)} = 1 + \lambda'_{+,0}\,\frac{t}{m_\pi^2} + \frac{1}{2}\,\lambda''_{+,0}\left(\frac{t}{m_\pi^2}\right)^2 + \ \ldots .$$

• Obtain λ′ ,λ″, from fit to data distributions (more accurate than theor. predictions).

• λ′ and λ″ are **strongly correlated: –95% for $f_+(t)$, and –99.96% for $f_0(t)$**.

One parameter parameterizations:

• Pole parameterization
 (what vector/scalar state should be used?)

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

• Dispersive approach plus Kπ scattering data for both $f_+(t)$ and $f_0(t)$

Vector form factor

Quadratic expansion:

- Measurements from ISTRA+, KLOE, KTeV, NA48 with K_Le3 and K^-e3 decays.
- **Good fit quality: χ^2/ndf=5.3/6(51%) for all data**; χ^2/ndf=4.7/4(32%) for K_L only
- The significance of the quadratic term is 4.2σ from all data and 3.5σ from K_L only.
- Using all data or K_L only changes the space phase integrals I^0_{e3} and $I^{\pm}_{e3}$ by 0.07% .
- Errors on I_{e3} are significantly smaller when K^- data are included.

A **pole parameterization** is in good agreement with present data:

$$\tilde{f}_+(t) = M_V^2/(M_V^2 - t), \text{ with } M_V \sim 892 \text{ MeV} \qquad \lambda' = (m_{\pi+}/M)^2;\ \lambda'' = 2\lambda'^2$$

- KLOE, KTeV, NA48 quote value for M_V for pole fit to K_Le3 data (χ^2/ndf=1.8/2)
- The values for λ_+' and λ_+'' from pole expansion are in agreement with quadratic fit results.
- Using quadratic averages or pole fit results changes I^0_{e3} by 0.03% .

Improvements: **dispersive parameterization** for $f_+(t)$, with analytical and unitarity properties and a correct threshold behavior, (e.g.Passemar arXiv:0708.1235[hep-ph]) Dispersive results for λ_+ and λ_0 are in agreement with pole parameterization.

Vector and scalar form factor from $K_{\mu 3}$

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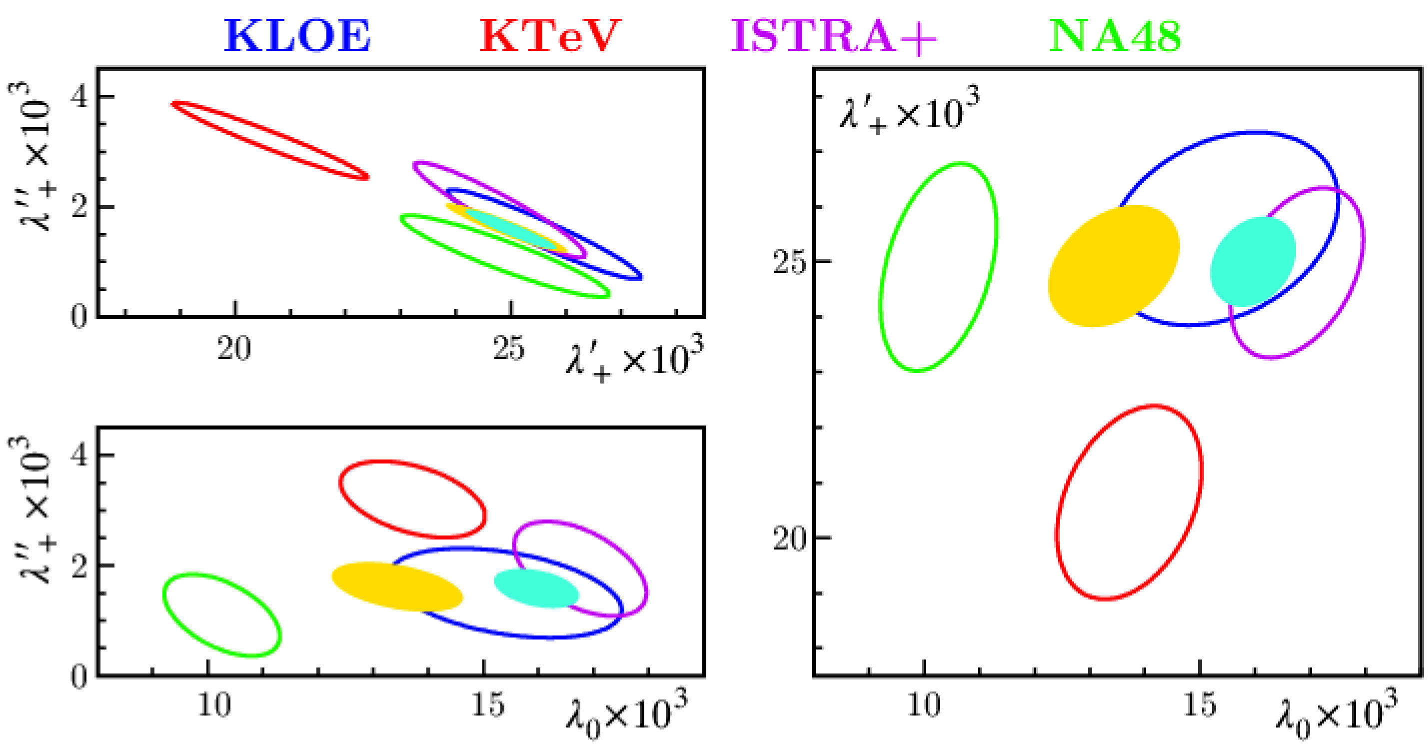
- λ_+', λ_+'' and λ_0 measured for Kμ3 from ISTRA+, KLOE, KTeV, and NA48.
- **NA48 result lies out of correlation directions in the [λ_+', λ_+'', λ_0] space**.
- Fit probability varies from 1×10^{-6} (with NA48) to 22.3% (without NA48).



1σ contour for all the experimental results.

- Neglecting a quadratic term in the param. of scalar FF implies: $\lambda_0' \rightarrow \lambda_0' + 3.5\lambda_0''$
- Because of correlation, is not possible measure λ_0'' at any plausible level of stat.

Vector and scalar form factor from $K_{\ell 3}$

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- Slope parameters λ_+', λ_+'' and λ_0 from ISTRA+, KLOE, KTeV, and NA48.

	K_L and K^-	K_L only
Measurements	16	11
χ^2/ndf	54/13 (7×10^{-7})	33/8 (8×10^{-5})
$\lambda'_+ \times 10^3$	24.9 ± 1.1 ($S = 1.4$)	24.0 ± 1.5 ($S = 1.5$)
$\lambda''_+ \times 10^3$	1.6 ± 0.5 ($S = 1.3$)	2.0 ± 0.6 ($S = 1.6$)
$\lambda_0 \times 10^3$	13.4 ± 1.2 ($S = 1.9$)	11.7 ± 1.2 ($S = 1.7$)
$\rho(\lambda'_+, \lambda''_+)$	-0.94	-0.97
$\rho(\lambda'_+, \lambda_0)$	$+0.33$	$+0.72$
$\rho(\lambda''_+, \lambda_0)$	-0.44	-0.70
$I(K^0_{e3})$	0.15457(29)	0.1544(4)
$I(K^\pm_{e3})$	0.15892(30)	0.1587(4)
$I(K^0_{\mu3})$	0.10212(31)	0.1016(4)
$I(K^\pm_{\mu3})$	0.10507(32)	0.1046(4)
$\rho(I_{e3}, I_{\mu3})$	$+0.63$	$+0.89$

Averages of quadratic fit results for Ke3 and Kμ3 slopes.

Space integral used for the $|V_{us}|f_+(0)$ determination

- Adding Kμ3 data to the fit doesn't cause significant changes to I^0_{e3} and $I^\pm_{e3}$.
- The significance of the quadratic term λ_+'' is strong (3.6σ from to fit to all data).
- **NA48: Δ[I(Kμ3)] = 0.6%, but Ke3+Kμ3 average gives Δ[$V_{us}f_+(0)$] = –0.08%**.

Determination of $|V_{us}| \times f_+(0)$

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{+,0}) (1+\delta^K_{SU(2)}+\delta^{K\ell}_{em})^2$$

with $K = K^+, K^0; \ell = e, \mu$ and $C_K^2 = 1/2$ for K^+, 1 for K^0

Inputs from theory:

S_{EW} Universal short distance EW correction (1.0232)

$\delta^K_{SU(2)}$ Form factor correction for strong SU(2) breaking

$\delta^{K\ell}_{em}$ Long distance EM effects

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element at zero momentum transfer (t=0)

Inputs from experiment:

$\Gamma(K_{l3(\gamma)})$ **Branching ratios** with well determined treatment of radiative decays; **lifetimes**

$I_{K\ell}(\lambda)$ Phase space integral: λ's parameterize form factor dependence on t :

K_{e3}: only λ_+

$K_{\mu3}$: need λ_+ and λ_0

CT

SU(2) and *em* corrections

	$\delta^{K}_{SU(2)}$(%)	$\delta^{K\ell}_{em}$(%)
K^0e3	0	+0.50(11)
$K^0\mu3$	0	+0.70(11)
K^+e3	+2.36(22)	+0.05(13)
$K^+\mu3$	+2.36(22)	+0.01(12)

	K^0e3	$K^0\mu3$	K^+e3	$K^+\mu3$
K^0e3	1	0.69	0.08	–0.15
$K^0\mu3$		1	–0.15	0.08
K^+e3			1	0.76
$K^+\mu3$				1

values used to extract $|V_{us}|f_+(0)$

- New values for $\delta^{K\ell}_{em}$ from ChPT $O(e^2p^2)$ (Cirigliano, Giannotti, and Neufeld, 0807.4507[hep-ph]: preliminary results of this work should be considered obsolete).
- Detailed error matrix assuming 100% fractional error on LECs and 20% on $\delta_{i=1,2,4}$. δ_{em} for full phase space: all measurements assumed fully inclusive.
- **Different estimates of δ_{em} agree within the quoted errors**.
- Available correlation matrix between different δ_{em} corrections.

- New value for $\delta^{K}_{SU(2)}$=2.9(4)% (Kastner and Neufeld: 0805.2222[hep-ph]) not used (larger error).

Determination of $|V_{us}| \times f_+(0)$

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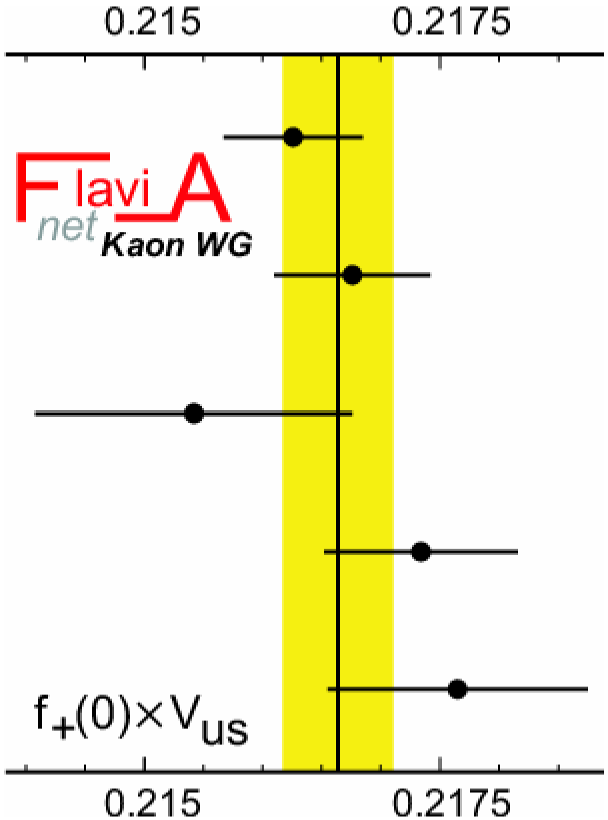
$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{+,0}) (1+\delta^K_{SU(2)}+\delta^{K\ell}_{em})^2$$



		% err	Approx. contr. to % err from: BR	τ	δ	$I_{K\ell}$
K_Le3	**0.2164(6)**	**0.26**	0.09	**0.19**	0.11	0.09
$K_L\mu3$	**0.2170(6)**	**0.29**	0.10	**0.18**	0.11	0.15
K_Se3	**0.2156(13)**	**0.62**	**0.60**	0.03	0.11	0.09
$K^\pm e3$	**0.2174(8)**	**0.38**	**0.26**	0.13	**0.25**	0.09
$K^\pm\mu3$	**0.2177(11)**	**0.51**	**0.40**	0.13	**0.25**	0.15

Average: $|V_{us}| f_+(0) = 0.2167(5)$ $\chi^2/\text{ndf} = 2.83/4$ (59%)

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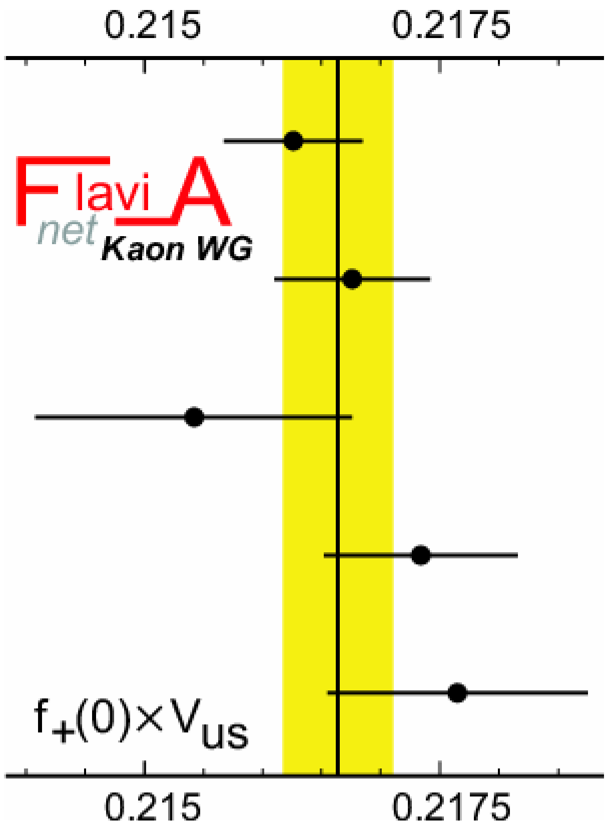
Determination of $|V_{us}| \times f_+(0)$



$K_L e3$	**0.2164(6)**
$K_L \mu 3$	**0.2170(6)**
$K_S e3$	**0.2156(13)**
$K^{\pm} e3$	**0.2174(8)**
$K^{\pm} \mu 3$	**0.2177(11)**

- Evaluate $V_{us} f_+(0)$ by charge:

  $K_{L,S}$ = 0.2165(5),

  $K^{\pm}$ = 0.2174(8)

- Average: 0.2167(5), **χ^2/ndf = 1.04/1** (31% probability).

- Comparing values obtained for K_L and $K^{\pm}$ (**without $\delta^K_{SU(2)}$ correction**) allows the empirical evaluation of SU(2) breaking correction : **2.81(38)%**.
To be compared with χ_{PT} prediction **2.36(22)%** (Kastner and Neufeld: **2.9(4)%**).

Test of Lepton Universality from $K\ell3$

Test of Lepton Flavor Universality: comparing Ke3 and Kμ3 modes constraints possible anomalous LF dependence in the leading weak vector current. Evaluate $R_{K\mu3/Ke3}$:

$$\frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})} = \left(\frac{G_F^\mu}{G_F^e}\right)^2 \frac{I_K^\mu}{I_K^e} \frac{(1+\delta_K^\mu)^2}{(1+\delta_K^e)^2}$$

Compare experimental results with SM prevision:

$$\mathbf{r}_{\mu\mathbf{e}} = \frac{(R_{K\mu3/Ke3})_{\text{obs}}}{(R_{K\mu3/Ke3})_{\text{SM}}} = \frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})} \frac{I_K^e}{I_K^\mu} \frac{(1+\delta_K^e)^2}{(1+\delta_K^\mu)^2} = \frac{\mathbf{G}_\mathbf{F}^\mu}{\mathbf{G}_\mathbf{F}^\mathbf{e}}$$

Using FlaviaNet results get accuracy ~0.5%,

K_L $r_{\mu e} = 1.0055(47)$

$K^\pm$ $r_{\mu e} = 1.0031(78)$

Average $r_{\mu e} = 1.0050(44)$

Comparable with other determinations:

- τ decays: $(r_{\mu e})_\tau = 1.0005(41)$ (PDG08)
- π decays: $(r_{\mu e})_\pi = 1.0042(33)$

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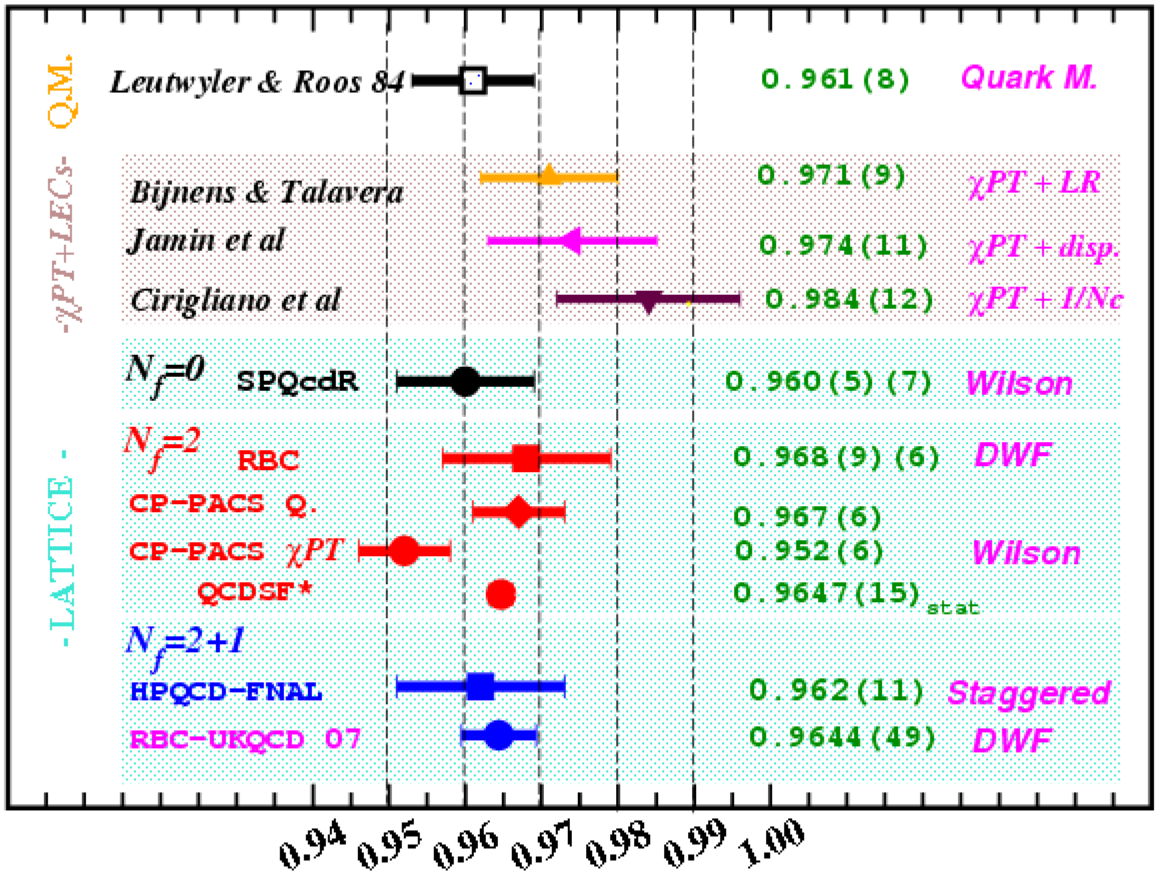
Theoretical estimate of $f_+(0)$

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{+,0}) (1+\delta^K_{SU(2)}+\delta^{K\ell}_{em})^2$$

Leutwyler & Roos estimate still widely used:
$f_+(0) = 0.961(8)$.

Lattice evaluations generally agree well with this value; use RBC-UKQCD07 value:
$f_+(0) = 0.9644(49)$ (0.5% accuracy, also syst. err.).



V_{us}/V_{ud} determination from BR($K_{\mu 2}$)

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$$\frac{\Gamma(K_{\mu2(\gamma)})}{\Gamma(\pi_{\mu2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K}{f_\pi} \times \frac{M_K(1-m_\mu^2/M_K^2)^2}{m_\pi(1-m_\mu^2/m_\pi^2)^2} \times (1+\alpha(C_K-C_\pi))$$

Inputs from experiment:

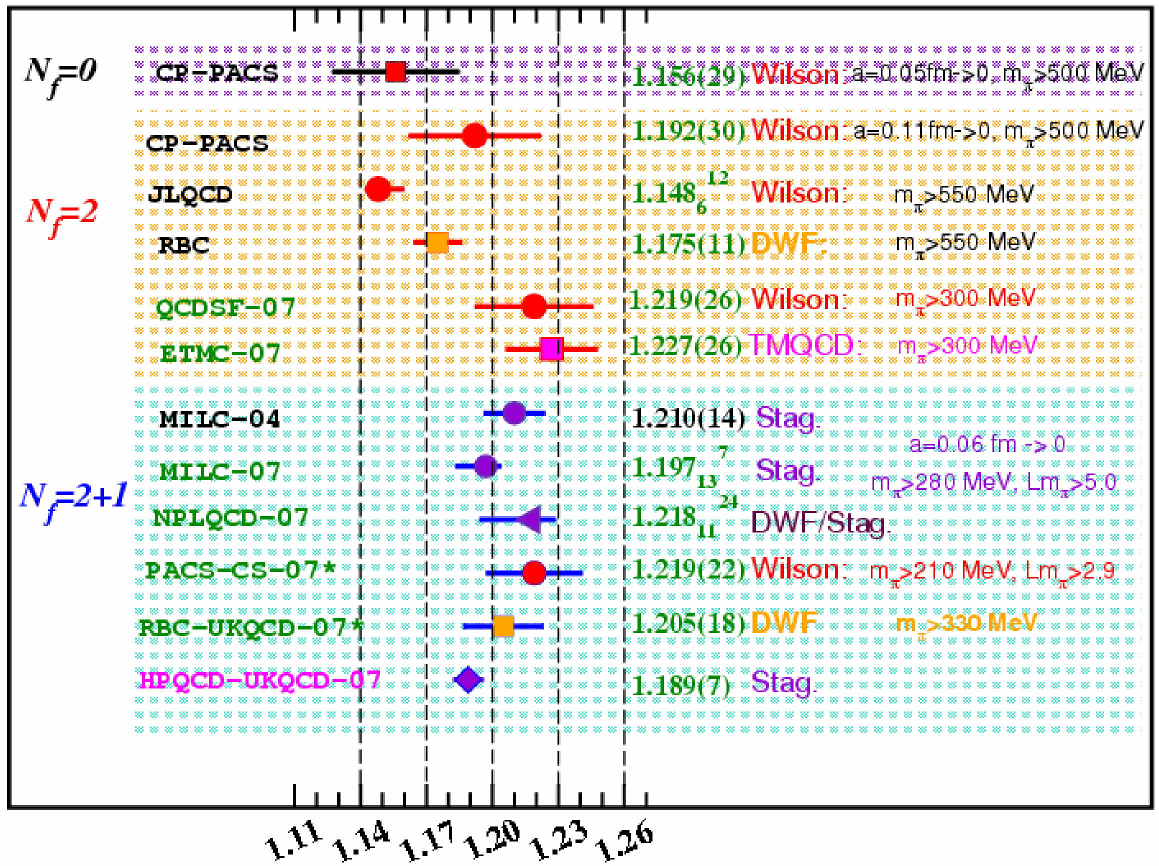
$\Gamma(\pi,K_{l2(\gamma)})$ **BR** with well determined treatment of radiative decays; **lifetimes**

Inputs from theory:

$C_{K,\pi}$ rad. inclusive EW corr.

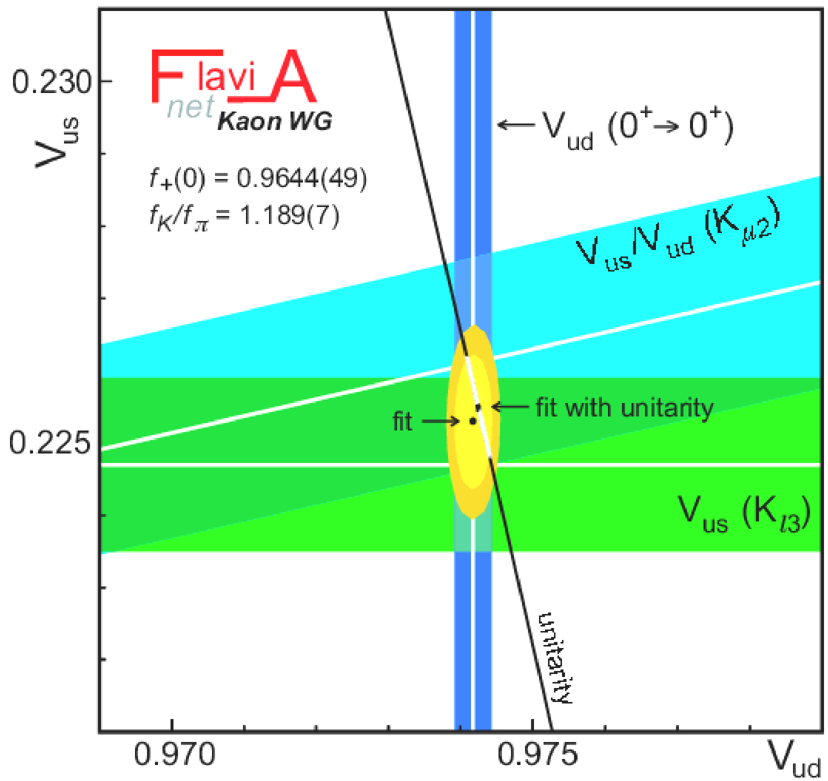
f_K/f_π Not protected by the Ademollo-Gatto theorem: only lattice.

- Lattice calculation of f_K/f_π and radiative corrections benefit of cancellations.
- Use HPQCD-UKQCD07 value: f_K/f_π **= 1.189(7)**.



V_{ud}, V_{us} and V_{us}/V_{ud}

K13: $|V_{us}| f_+(0) = 0.2167(5)$ and $f_+(0) = 0.964(5)$, obtain $|V_{us}| = \mathbf{0.2247(12)}$
K12: $|V_{us}|/|V_{ud}| f_K/f_\pi = 0.2760(6)$ and $f_K/f_\pi = 1.189(7)$, obtain $|V_{us}|/|V_{ud}| = \mathbf{0.2322(15)}$
V_{ud} from nuclear β decay: $V_{ud} = \mathbf{0.97425(23)}$ [Towner, CKM08]



Fit (no CKM unitarity constraint):

$\mathbf{V_{ud} = 0.97425(23); V_{us} = 0.2254(9)}$
$\chi^2/\text{ndf} = 0.60/1$ (44%)

- Unitarity: $\mathbf{1 - V_{ud}^2 - V_{us}^2 = 0.00003(60)}$

- The test on the unitarity of CKM can be also interpreted as a **test of the universality of lepton and quark gauge coupling**:

$$G_{\rm CKM} \equiv G_\mu \left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right]^{1/2} = (1.1662 \pm 0.0004) \times 10^{-5}\ \text{GeV}^{-2}$$

$$G_\mu = (1.166371 \pm 0.000007) \times 10^{-5}\ \text{GeV}^{-2}$$

Fit (with CKM unitarity constraint): $\mathbf{V_{us} = 0.2254(7)}$ $\chi^2/\text{ndf} = 0.60/2$ (74%)

A test of lattice calculations

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Scalar form factor $f_0(t) = \tilde{f}_0(t)\, f_+(0)$ extrapolation at **Callan-Treiman** point:

$$\tilde{f}_0(\Delta_{K\pi}) = \frac{f_K}{f_\pi}\frac{1}{f(0)} + \Delta_{CT}, \quad \Delta_{CT} \simeq -3.4 \times 10^{-3}$$

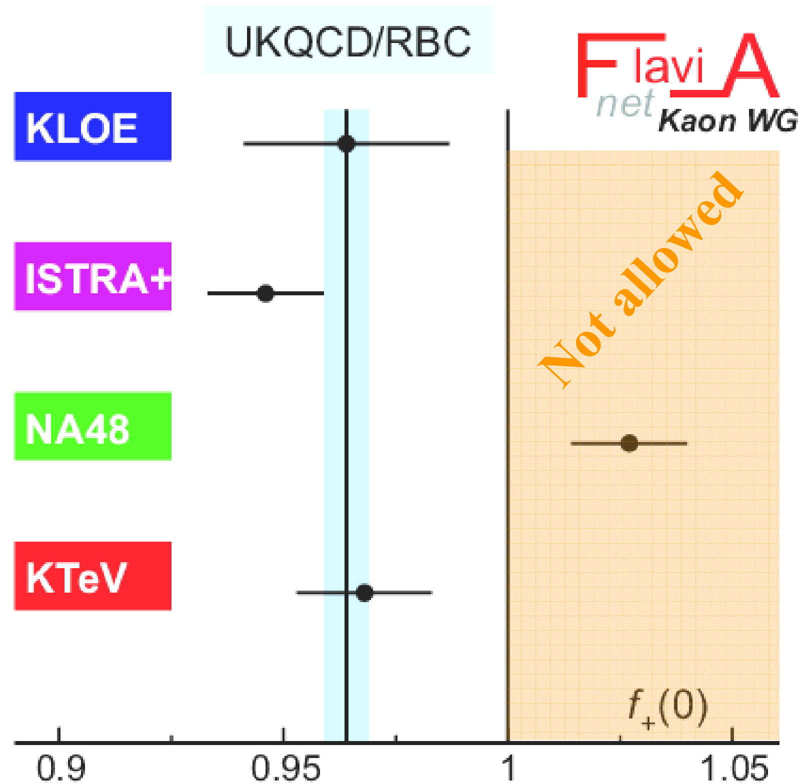
- links $f_+(0)$ and f_K/f_π with λ_0 measured in Kμ3 decays.

$\tilde{f}_0(\Delta_{K\pi})$ is evaluated fitting $K_L\mu3$ with a dispersive parameterization

$$\tilde{f}_0(t) = \exp\left(\frac{t}{\Delta_{K\pi}} \log(C - G(t))\right)$$

G(t) from Kπ scattering data.
To fit we use a 3rd order expansion

From CT, using f_K/f_π=1.189(7) [HPQCD-UKQCD07] obtain: $f_+(0)$=0.964(23) in agreement with RBC/UKQCD07 value: $f_+(0)$ = 0.9644(49).



FlaviA net Kaon WG — Experimental evaluation of $f_+(0)$ and f_K/f_π

$$\frac{\Gamma(K_{\mu2(\gamma)})}{\Gamma(\pi_{\mu2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K}{f_\pi} \times \frac{M_K(1-m_\mu^2/M_K^2)^2}{m_\pi(1-m_\mu^2/m_\pi^2)^2} \times (1+\alpha(C_K-C_\pi))$$

C: constant, including corrections.

$$\frac{\Gamma(K_{\mu2(\gamma)})}{\Gamma(\pi_{\mu2(\gamma)})} = \frac{|V_{us} f_+(0)|^2}{|V_{ud}|^2} \times \frac{1}{f_+(0)} \frac{f_K}{f_\pi} \times C$$

From decay rates

From Kl3

From nucl. β-decay

Straight calculation from Kμ2/πμ2 relation:

- **$f_K/f_\pi/f_+(0) = 1.2409(46)$**
- **using $f_+(0) = 0.9644(49)$ obtains $f_K/f_\pi = 1.1967(75)$**
- **using $f_K/f_\pi = 1.189(7)$ obtains $f+(0) = 0.9582(67)$**

~1σ agreement between experimental and theoretical evaluations

FlaviA net Kaon WG — Experimental evaluation of $f_+(0)$ and f_K/f_π

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us} f_+(0)|^2}{|V_{ud}|^2} \times \frac{1}{f_+(0)} \frac{f_K}{f_\pi} \times C$$

From decay rates (→ $\Gamma(K_{\mu 2(\gamma)})/\Gamma(\pi_{\mu 2(\gamma)})$)

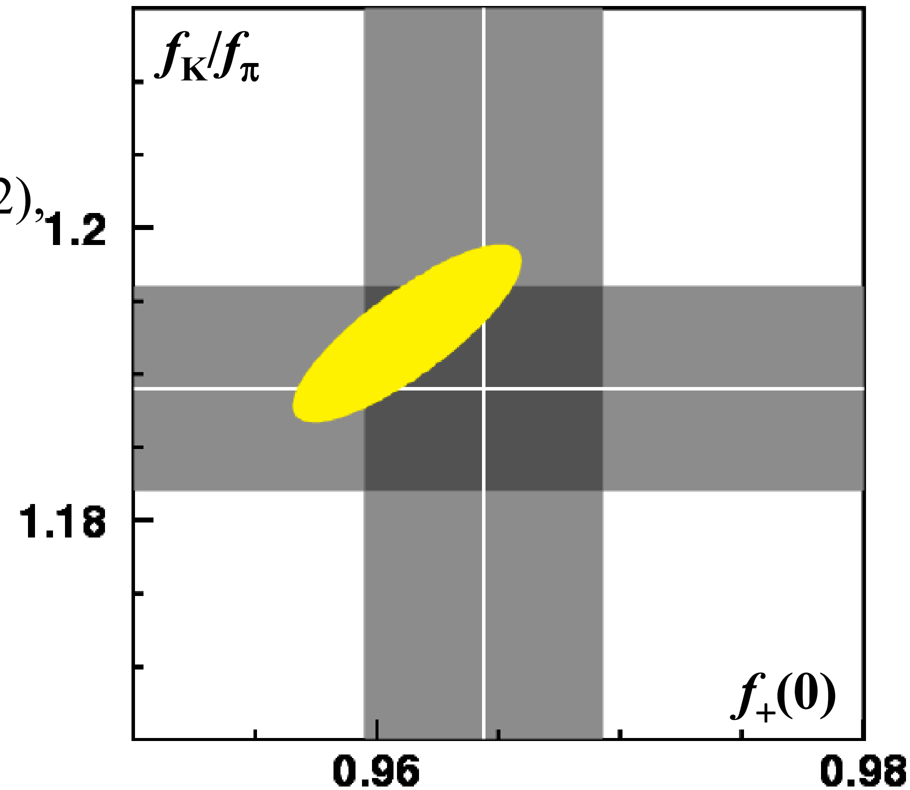
From Kl3 (→ $|V_{us} f_+(0)|^2$)

From nucl. β-decay (→ $|V_{ud}|^2$)

- Obtain f_K/f_π and $1/f_+(0)$ values from a fit (M.Moulson for the FlaviaNet Kaon WG):
  - 3 inputs: V_{ud}=0.97425(23), $V_{us}f_+(0)|$=0.21673(46), and C=0.076197(322),
  - 1 constraint: Γ(Kμ2)/Γ(πμ2) relation
  - obtain:

  f_K/f_π = 1.1928(61) and **$f_+(0)$ = 0.9612(47)**

  with a correlation of 0.82 (complete correlation matrix available)



- Very good agreement with th. estimations: f_K/f_π = 1.189(7) and $f_+(0)$ = 0.9644(49).

FlaviA net Kaon WG

Experimental evaluation of $f_+(0)$ and f_K/f_π

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us} f_+(0)|^2}{|V_{ud}|^2} \times \frac{1}{f_+(0)} \frac{f_K}{f_\pi} \times C$$

From decay rates

From Kl3

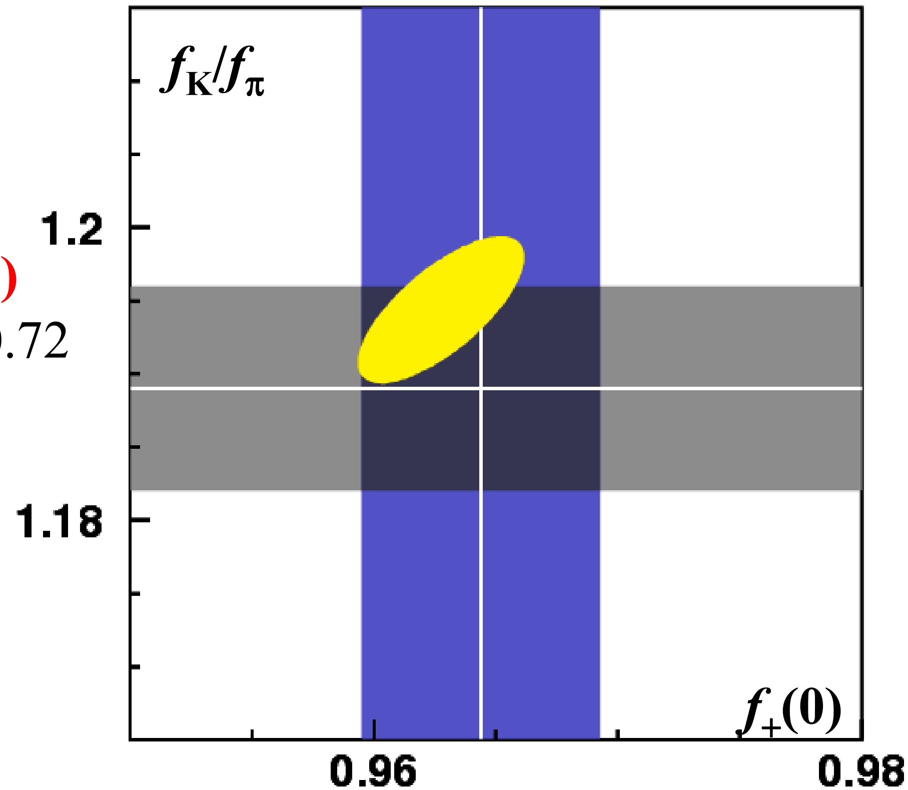
From nucl. β-decay

- Obtain f_K/f_π and $1/f_+(0)$ values from a fit:
  - one more input: $f_+(0) = 0.9644(49)$
  - 1 constraint: $\Gamma(K\mu 2)/\Gamma(\pi\mu 2)$ relation,
  - obtain:

  $f_K/f_\pi = 1.1944(50)$ and **$f_+(0) = 0.9628(34)$**

  with χ^2=0.22/1 (64%) and a correlation of 0.72 (complete correlation matrix available)

- Again very good agreement with th. estimations: $f_K/f_\pi = 1.189(7)$ and $f_+(0) = 0.9644(49)$ (used as fit input).



FlaviA net Kaon WG

Conclusions

- Dominant K_S, K_L, and $K^{\pm}$ BRs, and lifetime known with very good accuracy.
- Dispersive approach for form factors.
- New ***em*** corrections for $K\ell3$, with error matrix.
- Constant improvements from lattice calculations of $f_+(0)$ and f_K/f_π:
  - lack of sys. errors; needed to average different evaluations.
  - Callan-Treiman relation allows checks from measurements;
  - experimental determination of $f_+(0)$ and f_K/f_π from $\Gamma(K\mu2)/\Gamma(\pi\mu2)$ relation.

- $|V_{us}|\, f_+(0)$ at 0.2% level.
- Test of LU with Kl3 decays with 0.5% accuracy.
- $|V_{us}|$ measured with 0.4 accuracy (with $f_+(0)$= 0.9644(49))
  - Dominant contribution to uncertainty on $|V_{us}|$ still from $f_+(0)$.
  - CKM unitarity test satisfied at 0.3σ level
  - test of lepton-quark universality

- Near future: K_S–K_L combined fit, deeper understanding of $\tau^{\pm}$ and $K_L\pi\pi$ determinations,....

Additional information

Dispersive approach: details

Dispersive approach

New parametrizations based on dispersive relations (doubly-subtracted) and $K\pi$ scattering data: f_+ and f_0 depend only on parameters λ_+ and λ_0

[Passemar-Stern, Jamin-Pich]

$$\lambda_+ = (\,25.7 \pm 0.6\,)\times 10^{-3}$$
$$\lambda_0 = (\,14.0 \pm 2.1\,)\times 10^{-3}$$

$\chi^2/\text{ndf} = 2.6/3 \quad \rho = -0.26$

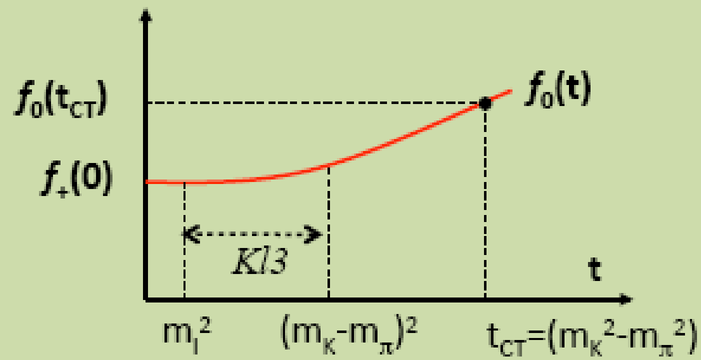
Phase-space integrals change by 0.04% and 0.09% for K_{e3} and $K_{\mu3}$

Test of lattice QCD with Callan-Treiman relation

$$f_0(t_{CT}) = f_K / f_\pi + \Delta_{CT}$$

$f_K / f_\pi = 1.189 \pm 0.007$ (HPQCD/UKQCD)

$f_+(0) = 0.967 \pm 0.025$



Compare with $f_+(0) = 0.9644 \pm 0.0049$ (RBC/UKQCD)

$K_{\mu 2}$: sensitivity to NP

Comparison of V_{us} from $K_{\ell 2}$ (helicity suppressed) and from $K_{\ell 3}$ (helicity allowed)
To reduce theoretical uncertainties study the quantity:

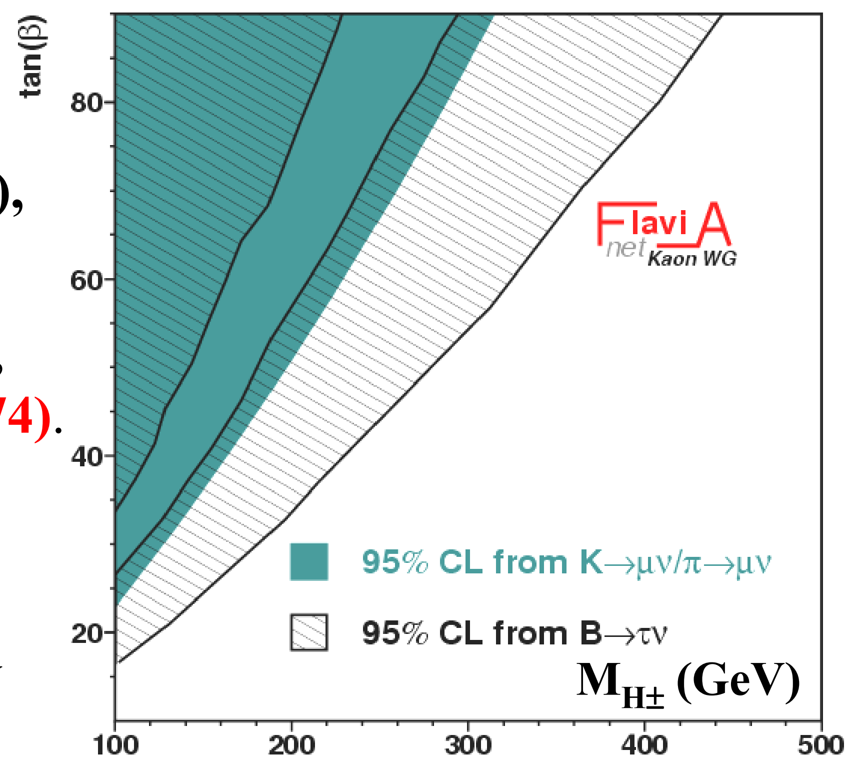
$$R_{l23} = \left| \frac{V_{us}(K_{\ell 2})}{V_{us}(K_{\ell 3})} \times \frac{V_{ud}(0^+ \to 0^+)}{V_{ud}(\pi_{\ell 2})} \right|$$

Within SM $R_{\ell 23} = 1$; NP effects can show as scalar currents due to a charged Higgs:

$$R_{l23} = \left| 1 - \frac{m_{K+}^2}{M_{H+}^2} \left(1 - \frac{m_d}{m_s}\right) \frac{\tan^2\beta}{1 + \epsilon_0 \tan\beta} \right|$$

$R_{\ell 23}$ is accessible via $BR(K_{\mu 2})/BR(\pi_{\mu 2})$, $V_{us}f_+(0)$, and V_{ud}, and $f_K/f_\pi/f_+(0)$ determinations.

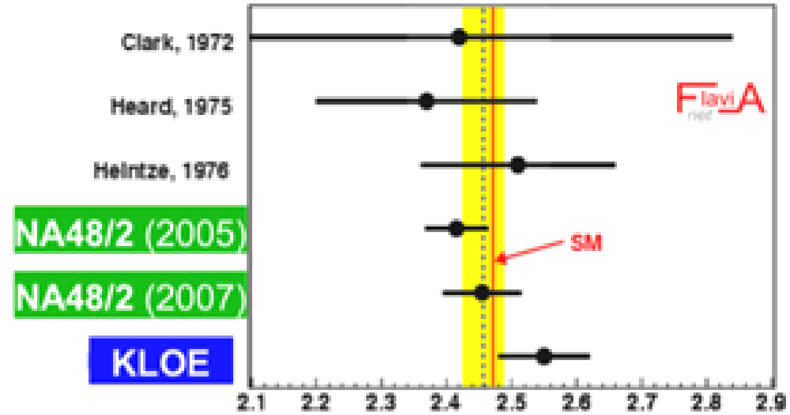
- Using $K^\pm$ fit results, assuming unitarity for $K_{\ell 3}$, and using $f_K/f_\pi/f_+(0)$ from lattice: **$R_{\ell 23} = 1.0028(74)$**.
- Uncertainty dominated by $f_K/f_\pi/f_+(0)$.
- 95% CL excluded region (with $\varepsilon_0 \sim 0.01$).
- In $\tan\beta$-$M_{H\pm}$ plane, $R_{\ell 23}$ **fully cover the region uncovered by $BR(B\to\tau\nu)$**. To be included: new **$B\to\tau\nu$** measurements from Belle and BaBar.



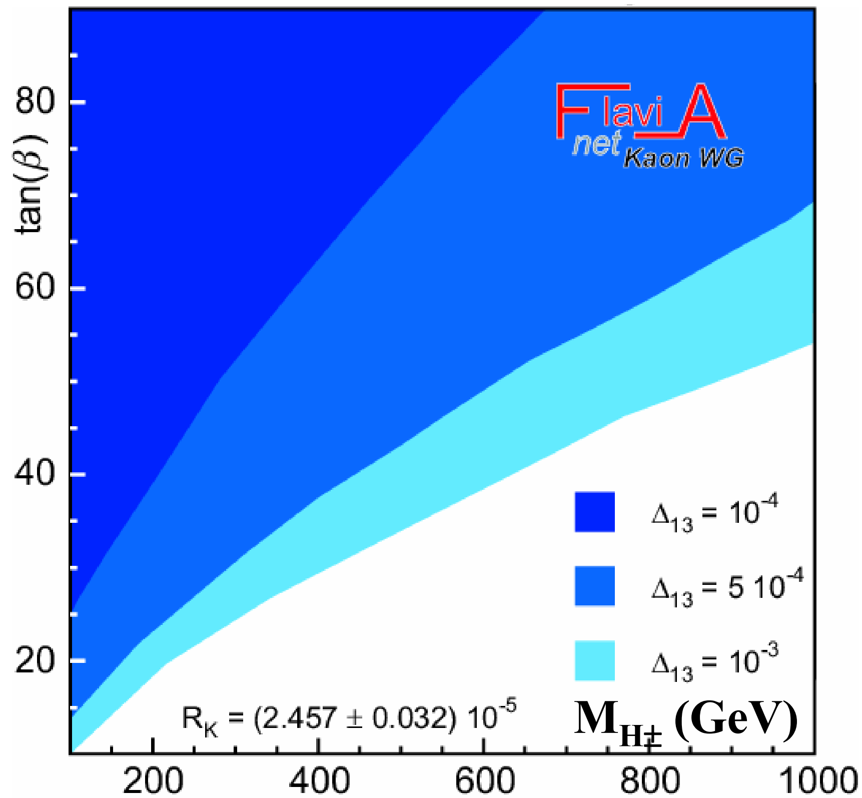
Measurement of $R_K=\Gamma(K_{e2})/\Gamma(K_{\mu 2})$

FlaviA net Kaon WG

- PDG06: 5% precision from 3 old mnts
- 2 preliminary meas. from NA48 (see M.Raggi talk); waiting for new data result.
- 1 preliminary from KLOE see (A.Passeri talk); waiting for final.

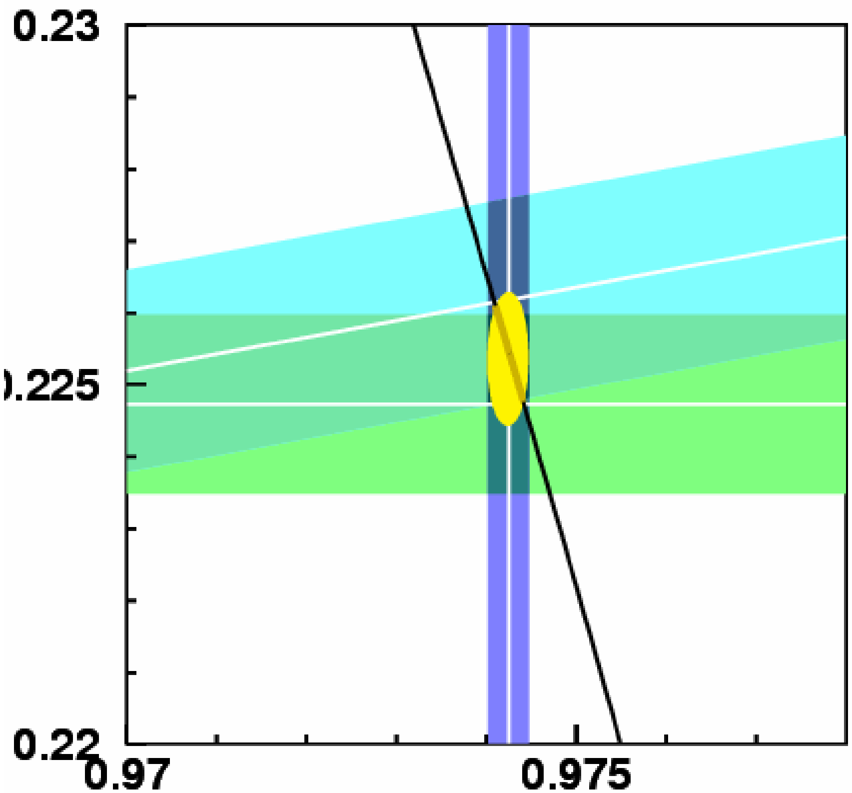


- **New average**: $R_K=2.457(32)\times10^{-5}$.
- Perfect agreement with SM expectations: $R_K^{SM}=2.477(1)\times10^{-5}$.
- In SUSY(MSSM) LFV appear at 1-loop level (effective $H^+\ell\nu_\tau$ Yukawa interaction). For moderately large tanβ values, enhance R_K up to few %.
- The world average gives strong constrains for tanβ and $M_{H\pm}$.
- **95%-CL excluded regions in the tanβ - M_H plane, for $\Delta_{13} = 10^{-4}$, 0.5×10^{-4}, 10^{-3}**.



V_{ud}, V_{us} and V_{us}/V_{ud}

K13: $|V_{us}| f_+(0) = 0.2167(5)$ and $f_+(0) = 0.964(5)$, obtain $\mathbf{|V_{us}| = 0.2247(12)}$
K12: $|V_{us}|/|V_{ud}| f_K/f_\pi = 0.2760(6)$ and $f_K/f_\pi = 1.189(7)$, obtain $\mathbf{|V_{us}|/|V_{ud}|=0.2322(15)}$
V_{ud} from nuclear β decay: $\mathbf{V_{ud} = 0.97425(23)}$ [Towner, CKM08]



Fit (no CKM unitarity constraint):

V_{ud} = 0.97425(23); V_{us} = 0.2254(9)
$\chi^2/\text{ndf} = 0.60/1$ (44%)

- Unitarity: $\mathbf{1 - V_{ud}^2 - V_{us}^2 = 0.00003(60)}$
- The test on the unitarity of CKM can be also interpreted as a **test of the universality of lepton and quark gauge coupling**:

$$G_{\rm CKM} \equiv G_\mu \left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right]^{1/2} = (1.1662 \pm 0.0004) \times 10^{-5}\ \text{GeV}^{-2}$$

$$G_\mu = (1.166371 \pm 0.000007) \times 10^{-5}\ \text{GeV}^{-2}$$

Fit (with CKM unitarity constraint): **V_{us} = 0.2254(7)** $\chi^2/\text{ndf} = 0.60/2$ (74%)

Dispersive parameterization

$$\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$

$$\tilde{f}_+(t) = 1 + \lambda_+ \frac{t}{m^2} + \frac{{\lambda_+}^2 + p_2}{2}\left(\frac{t}{m^2}\right)^2 + \frac{{\lambda_+}^3 + 3p_2\lambda_+ + p_3}{6}\left(\frac{t}{m^2}\right)^3$$

p_n	$\tilde{f}_+(t)$	$\tilde{f}_0(t)$
$p_2 \times 10^4$	5.84 ± 0.93	4.16 ± 0.50
$p_3 \times 10^4$	0.30 ± 0.02	0.27 ± 0.01

Table 1: Constants appearing in the dispersive form of vector and scalar form factors.

$$\tilde{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}(\ln C - G(t))\right]$$

$$\tilde{f}_0(t) = 1 + \lambda_0 \frac{t}{m^2} + \frac{{\lambda_0}^2 + p_2}{2}\left(\frac{t}{m^2}\right)^2 + \frac{{\lambda_0}^3 + 3p_2\lambda_0 + p_3}{6}\left(\frac{t}{m^2}\right)^3$$

With or without NA48 Kμ3 data

	K_L and K^-	K_L only	K_L and K^-	K_L only
Measurements	16	11	13	8
χ^2/ndf	$54/13$ (7×10^{-7})	$33/8$ (8×10^{-5})	$13/9$ (24.9%)	$9/5$ (12.3%)
$\lambda'_+\times10^3$	24.9 ± 1.1 $(S=1.4)$	24.0 ± 1.5 $(S=1.5)$	25.0 ± 0.8	24.5 ± 1.1
$\lambda''_+\times10^3$	1.6 ± 0.5 $(S=1.3)$	2.0 ± 0.6 $(S=1.6)$	1.6 ± 0.4	1.8 ± 0.4
$\lambda_0\times10^3$	13.4 ± 1.2 $(S=1.9)$	11.7 ± 1.2 $(S=1.7)$	16.0 ± 0.8	14.8 ± 1.1
$\rho(\lambda'_+,\lambda''_+)$	-0.94	-0.97	-0.94	-0.95
$\rho(\lambda'_+,\lambda_0)$	$+0.33$	$+0.72$	$+0.26$	$+0.28$
$\rho(\lambda''_+,\lambda_0)$	-0.44	-0.70	-0.37	-0.38
$I(K^0_{e3})$	0.15457(29)	0.1544(4)	0.15459(20)	0.15446(27)
$I(K^\pm_{e3})$	0.15892(30)	0.1587(4)	0.15894(21)	0.15881(28)
$I(K^0_{\mu3})$	0.10212(31)	0.1016(4)	0.10268(20)	0.10236(28)
$I(K^\pm_{\mu3})$	0.10507(32)	0.1046(4)	0.10559(20)	0.10532(29)
$\rho(I_{e3},I_{\mu3})$	$+0.63$	$+0.89$	$+0.59$	$+0.62$

wNA48-w/oNA48:	-0.00002 (0.01%)	-0.00006 (0.04%)	$\Delta I(K^0e3)$
	-0.00002 (0.01%)	-0.00011 (0.07%)	$\Delta I(K^\pm e3)$
	-0.00056 (0.55%)	-0.00076 (0.75%)	$\Delta I(K^0\mu3)$
	-0.00052 (0.49%)	-0.00072 (0.69%)	$\Delta I(K^\pm\mu3)$

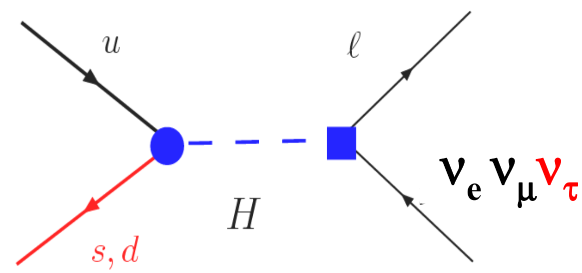
Lepton universality from $K_{e2}/K_{\mu 2}$

SM: no hadronic uncertainties (no f_K) $\rightarrow 0.4\times10^{-3}$

In MSSM, LFV can give up to % deviations [Masiero, Paradisi, Petronzio]

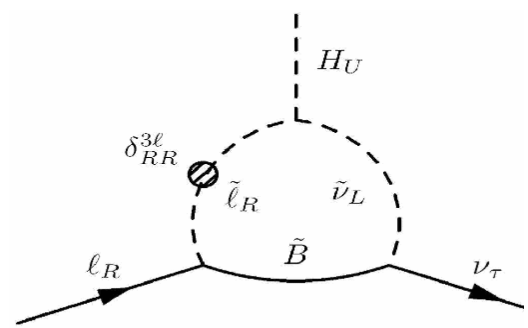
NP dominated by contribution of $e\nu_\tau$

$$R_K \approx \frac{\Gamma(K\rightarrow e\nu_e)+\Gamma(K\rightarrow e\nu_\tau)}{\Gamma(K\rightarrow \mu\nu_\mu)}$$



with effective coupling:

$$eH^{\pm}\nu_\tau \rightarrow \frac{g_2}{\sqrt{2}}\frac{m_\tau}{M_W}\Delta_R^{31}\tan^2\beta$$



$$\mathbf{R_K \approx R_K^{SM}\left[\, 1+ \frac{m_K^4}{m_H^4}\frac{m_\tau^2}{m_e^2}\,|\Delta^R_{31}|^2 \tan^6\beta\right]}$$

1% effect (Δ^R_{31} ~5x10^{-4}, tanβ~40, m_H~500GeV) not unnatural

Present accuracy on R_K @ 6% ; need for precise (<1%) measurements

FlaviA net Kaon WG

Vector form factor from K_{e3}

- Quadratic from ISTRA+, KLOE, KTeV, NA48 with K_L and K^- decays.

	K_L and K^- data 4 measurements $\chi^2/\text{ndf} = 5.3/6$ (51%)	K_L data only 3 measurements $\chi^2/\text{ndf} = 4.7/4$ (32%)
$\lambda'_+ \times 10^3$	25.2 ± 0.9	24.9 ± 1.1
$\lambda''_+ \times 10^3$	1.6 ± 0.4	1.6 ± 0.5
$\rho(\lambda'_+, \lambda+'')$	-0.94	-0.95
$I(K^0_{e3})$	0.15465(24)	0.15456(31)
$I(K^\pm_{e3})$	0.15901(24)	0.15891(32)

- The significance of the quadratic term is 4.2σ from all data and 3.5σ from K_L only.
- Using all data or K_L only changes the space phase integrals I^0_{e3} and $I^\pm_{e3}$ by 0.07% .
- Errors on I_{e3} are significantly smaller when K^- data are included.

Vector form factor from K_{e3}

• KLOE, KTeV, NA48 quote value for M_V for pole fit to $K_L e3$ data.

Experiment	M_V (MeV)	$\langle M_V \rangle = 875 \pm 5$ MeV
KLOE	$870 \pm 6 \pm 7$	$\chi^2/\mathrm{ndf} = 1.8/2$
KTeV	881.03 ± 7.11	$\lambda'_+ \times 10^3 = 25.42(31)$
NA48	859 ± 18	$\lambda''_+ = 2 \times \lambda'^2_+$
		$I(K^0_{e3}) = 0.15470(19)$

• The values for λ_+' and λ_+'' from pole expansion are in agreement with quadratic fit results.

• Using quadratic averages or pole fit results changes I^0_{e3} by 0.03% .

Experimental evaluation of $f_+(0)$ and f_K/f_π

Correlation matrix:

+1.0000	+0.0003	-0.0008	-0.0001
+0.0003	+1.0000	-0.0002	+0.9007
-0.0008	-0.0002	+1.0000	+0.4338
-0.0001	+0.9007	+0.4338	+1.0000
+0.4147	+0.9098	-0.0006	+0.8196

Free fit

Correlation matrix:

+1.0000	+0.0014	-0.0021	+0.0001
+0.0014	+1.0000	-0.2576	+0.8313
-0.0021	-0.2576	+1.0000	+0.3223
+0.0001	+0.8313	+0.3223	+1.0000
+0.5037	+0.8641	-0.2237	+0.7184

Use also $\boldsymbol{f_+}(0)$ value as input

Introduction

M.Moulson – CKM 2006

K_{l3} decays, V_{us}, and CKM uni[illegible]

At present, most precise test of CKM unitarity is from 1[illegible]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \equiv 1 - \Delta$$

$0^+ \to 0^+ \beta$ decays: $2|V_{ud}|\delta V_{ud} = 0.0$[illegible]

$Kl3$ decays: $2|V_{us}|\delta V_{us} < 0.001$[illegible]

→ 2002 (2004 PDG)	**Old $Kl3$ data give Δ = 0.0035(15)** A 2.3σ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \to \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-2006	**Many new measurements from KTeV, KLOE, ISTRA+, NA48** • **BRs, lifetimes, form-factor slopes** • **Much higher statistics** than older measurements • Importance of **radiative corrections** • Proper reporting of **correlations** between measurements
2005 CKM WG1 report	**$|V_{us}|f_+(0)$ = 0.2173(8): unitarity to better than 1σ** Includes many (but not all) of these important developments
This talk	**Update with all recent measurements (even if preliminary)**

V_{us} from kaon decays - M. Moulson for the FlaviaNet Kaon Working Group - CKM 2006 - Nagoya, Japan - 15 Dec 2006

Introduction

Summary and outlook

M.Moulson – CKM 2006

- Several new $K^{\pm}$ measurements! None yet published
- BR/lifetime fits for both K_L and $K^{\pm}$ have χ^2 probability ~ 5%
  For $K^{\pm}$, new measurements in normalization channels could help
- New NA48 $K\mu 3$ form-factor slopes disagree with other data
- Accuracy of $\Delta^{SU(2)}$, Δ^{EM} a significant issue for charged modes
  Some evidence that $\Delta^{SU(2)}$ may be underestimated
- Experimental uncertainty on $|V_{us}|\, f_+(0)$ at 0.2% level
- Dominant contribution to uncertainty on $|V_{us}|$ still from $f_+(0)$
- With $f_+(0)$ = 0.961(8), first-row unitarity test satisfied at ~1σ level

K_{l3} average: $|V_{us}|\, f_+(0)$ = 0.21686(49)

V_{us} from kaon decays - M. Moulson for the FlaviaNet Kaon Working Group - CKM 2006 - Nagoya, Japan - 15 Dec 2006

Introduction

Analysis of leptonic and semileptonic kaon decays data
- provide precise determination of **fundamental SM couplings**;
- set stringent SM tests, almost **free from hadronic uncertainties**;
- discriminate between **different NP scenarios**.

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{\rm ew} \, |V_{us}|^2 f_+(0)^2 \, I_K^\ell(\lambda_{+,0}) \left(1 + \delta_{SU(2)}^K + \delta_{\rm em}^{K\ell}\right)^2$$

$$\frac{\Gamma(K^\pm_{\ell 2(\gamma)})}{\Gamma(\pi^\pm_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right)^2 \times (1 + \delta_{\rm em})$$

- Test **unitarity of the quark mixing matrix** (V_{CKM}):

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \epsilon_{\rm NP} \qquad \epsilon_{\rm NP} \sim M_W^2/\Lambda_{\rm NP}^2$$

→ present precision on V_{us} (dominant source of error) and V_{ub} negligible ($|V_{ub}^2| \sim 10^{-5}$) set bounds on NP well above 1 TeV.

- Comparison of Ke3 and Kμ3 modes, tests the **lepton universality**.