Theoretical Methods

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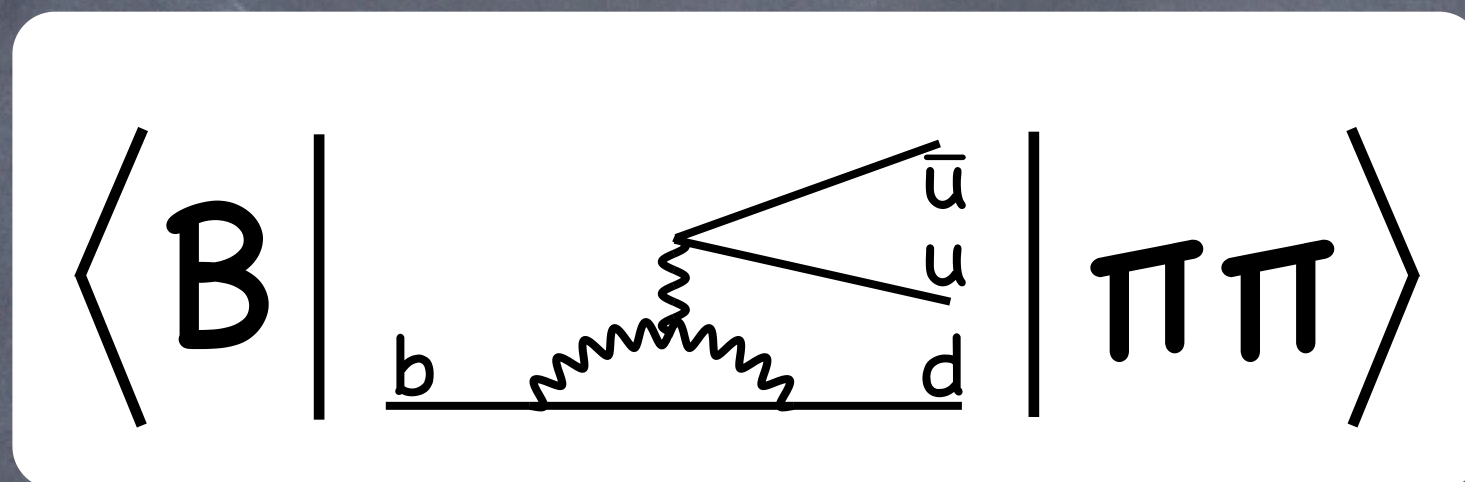
Factorization Theorems

We measure a combination of short distance EW physics and long distance hadronization effects



Factorization theorems separate short distance from long-distance physics

Long distance physics has to be isolated into a few measurable parameters, such that EW physics can be determined from measured data

Outline

- The different scales in non-leptonic B decays
- A quick introduction to SCET
- Factorization theorems from scale separation
- What are the implications the?
- What are the differences between different approaches?

The different scales in the problem

The different scales

m_W, m_Z, ... — Electroweak physics we are after

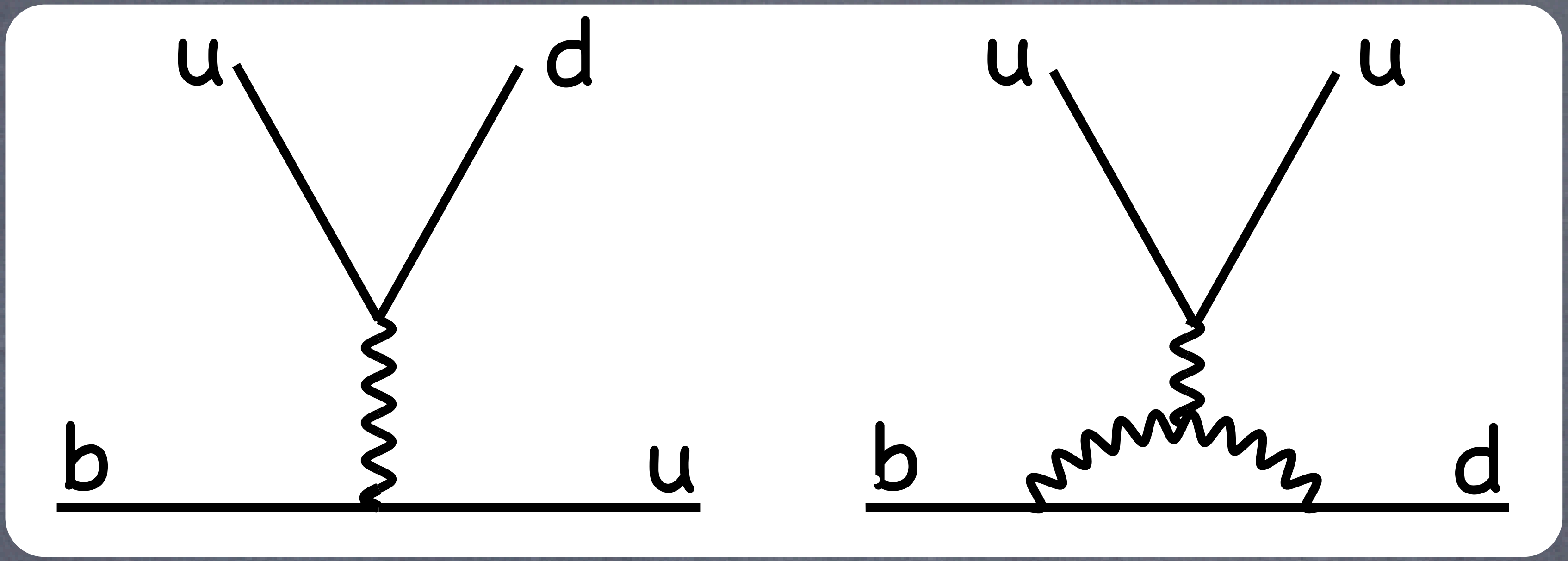
m_b — Short distance, perturbative

$(\Lambda_{QCD}\, m_b)^{1/2}$ — Scale arising dynamically

Λ_{QCD} — Long distance, nonperturbative

Scales m_W, m_Z

Mediates the Flavor processes in the SM



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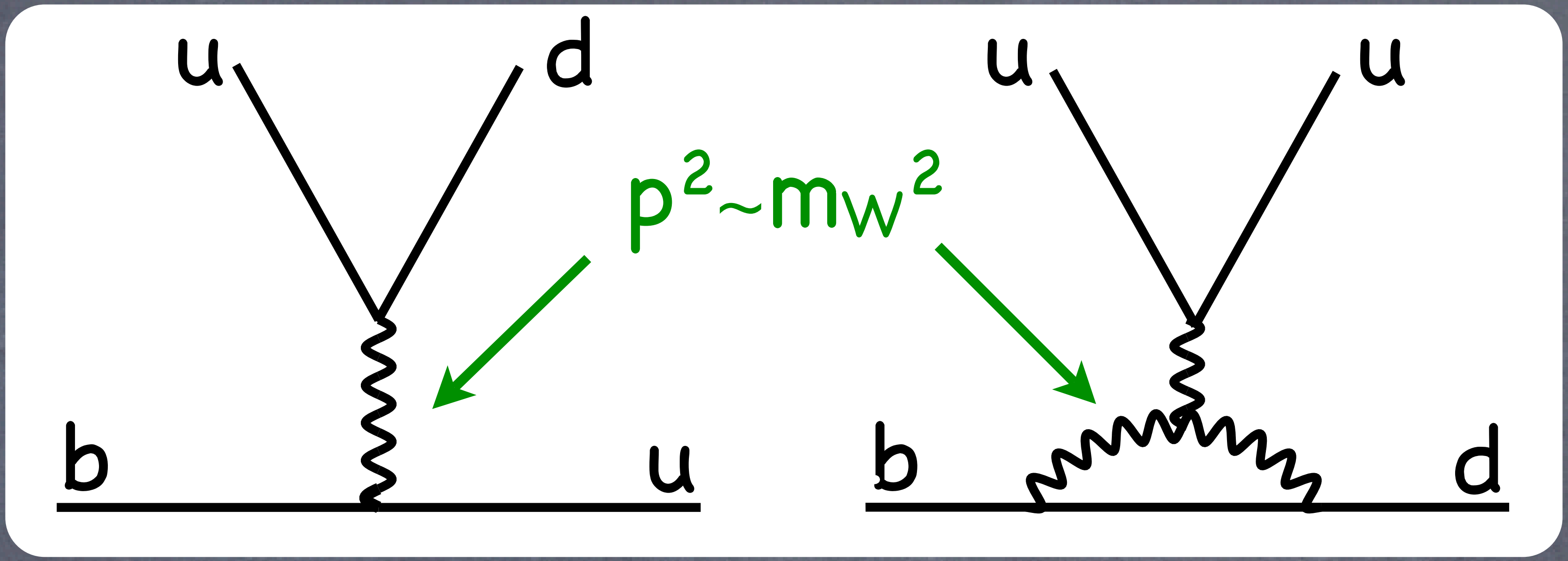

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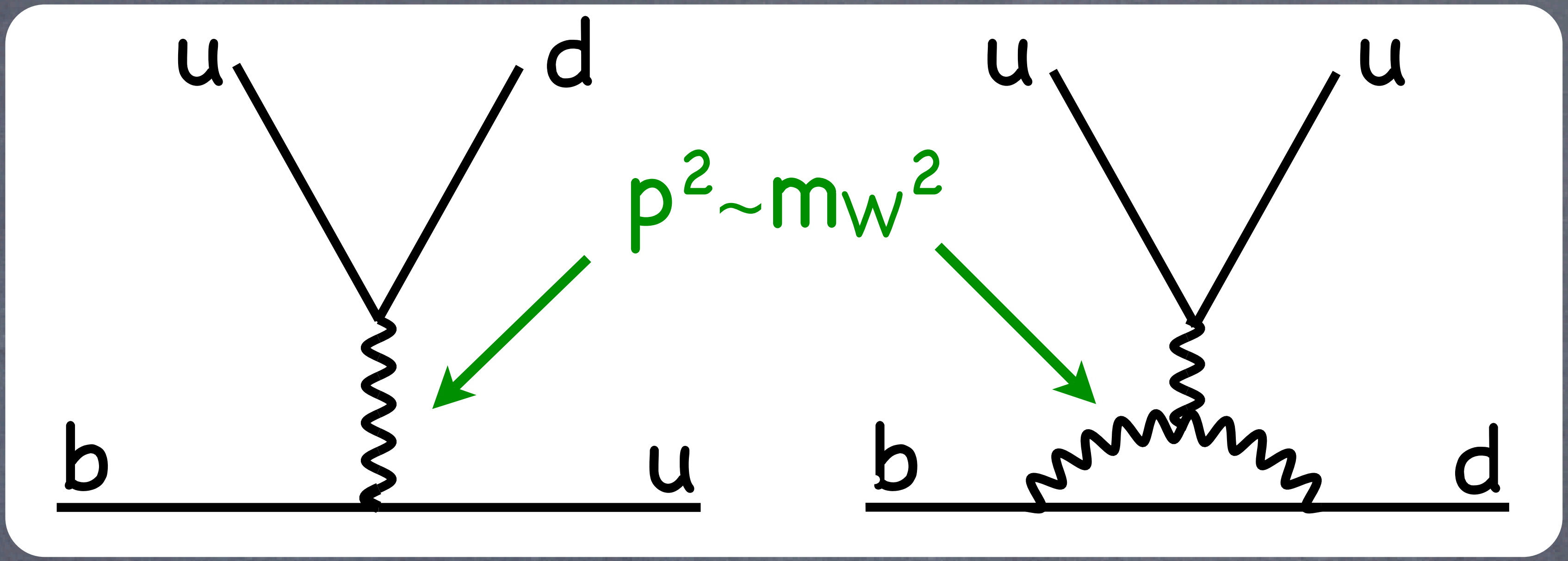
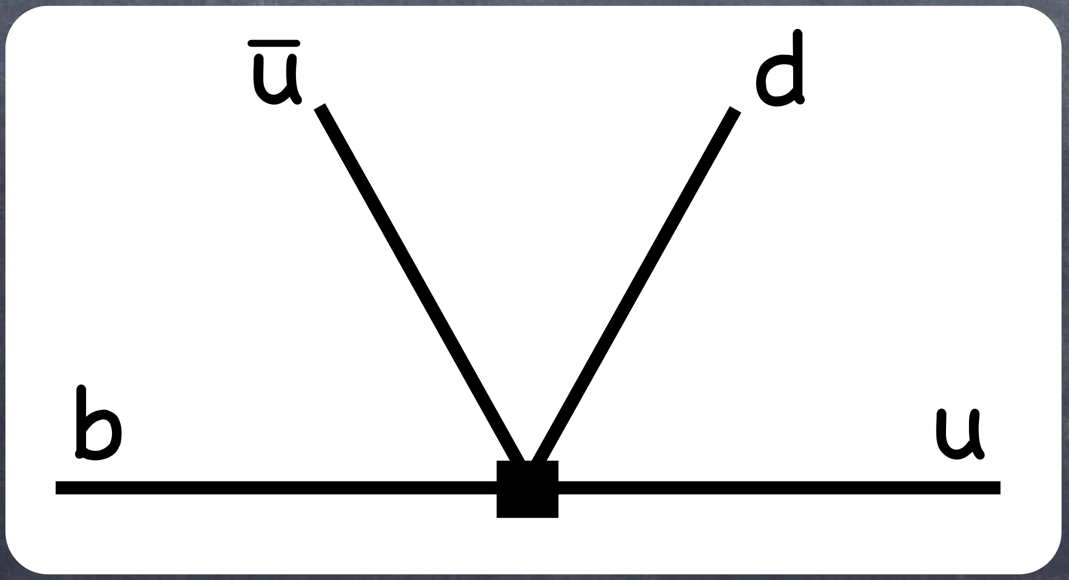
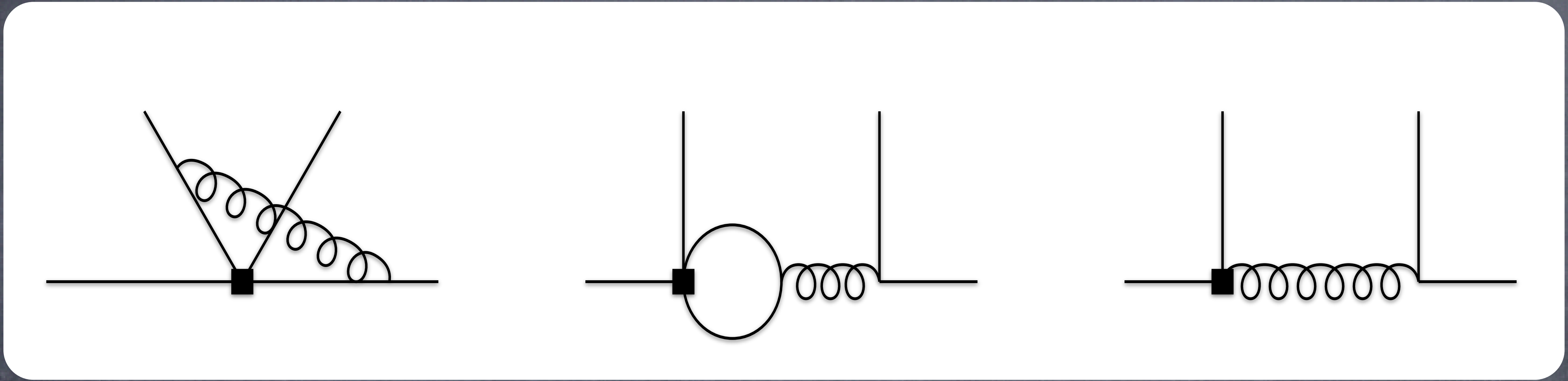




Scale m_b, E_M

Describes short distance QCD effects



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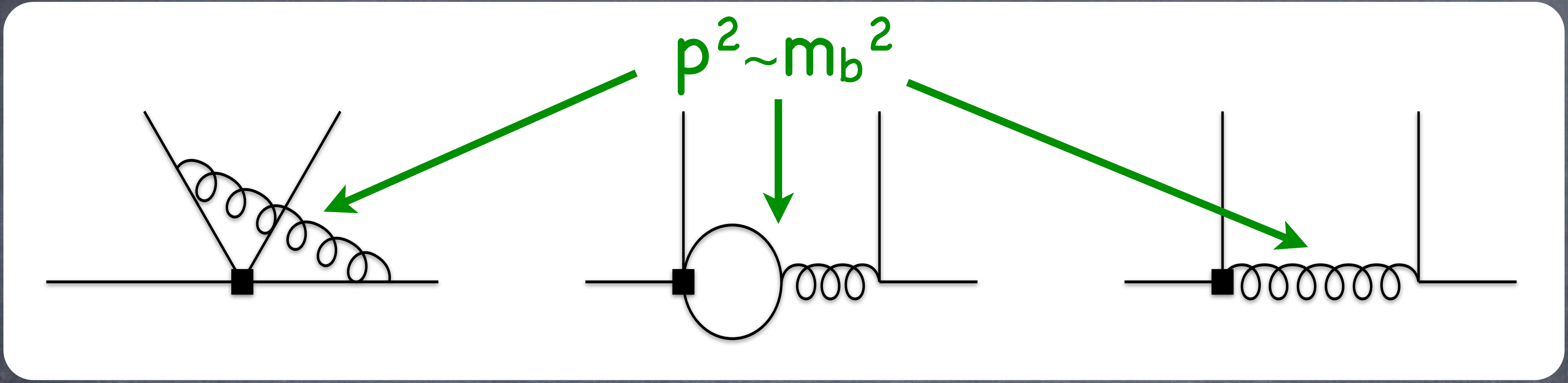

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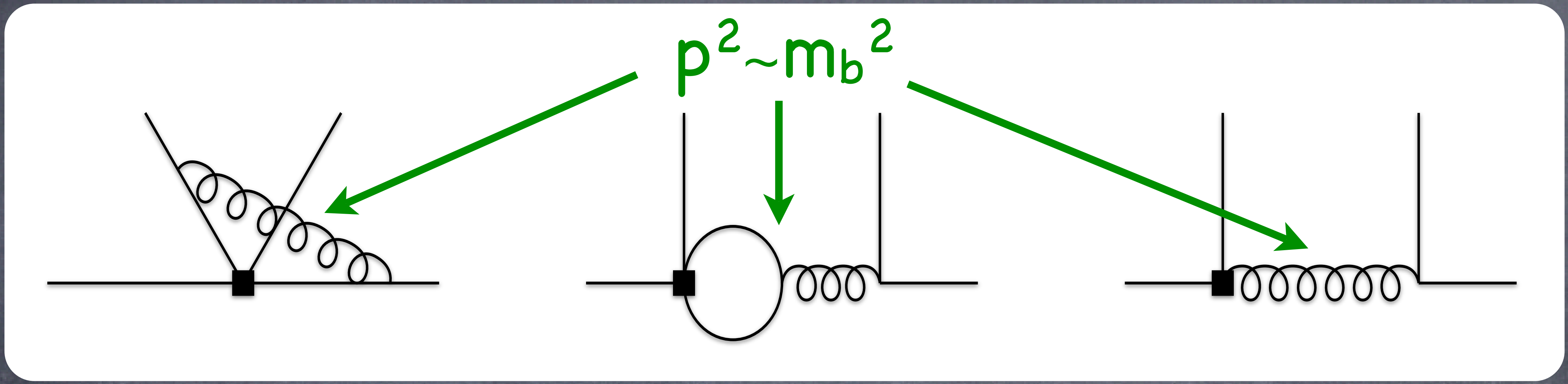
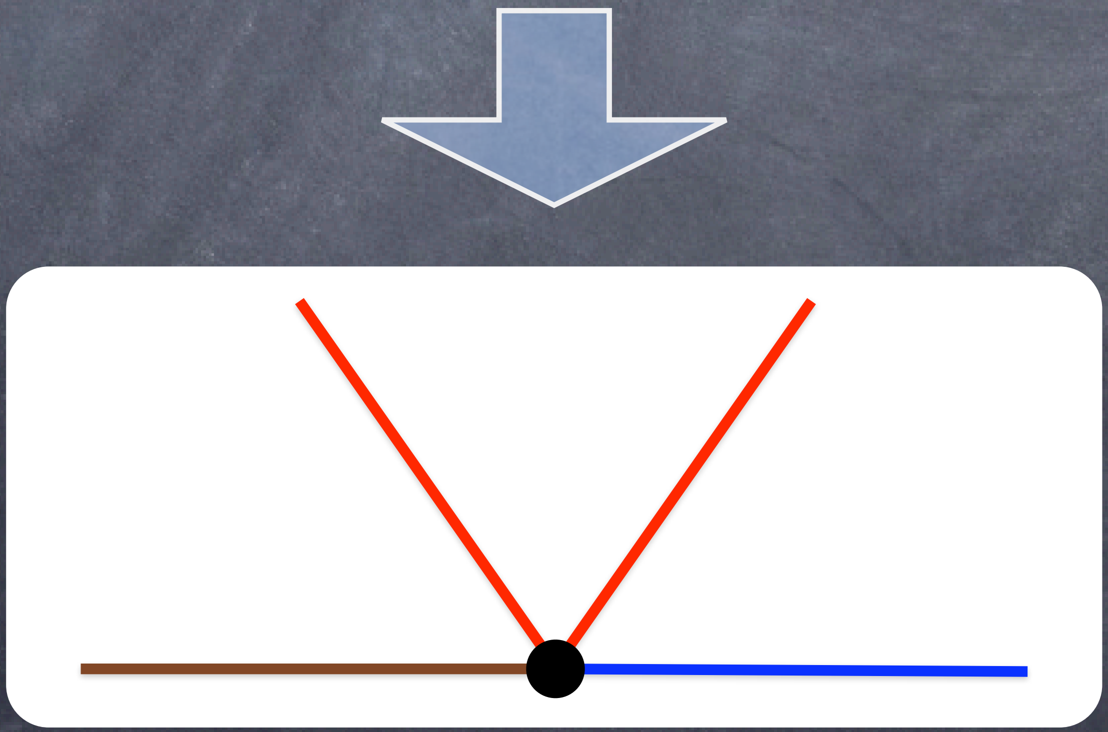
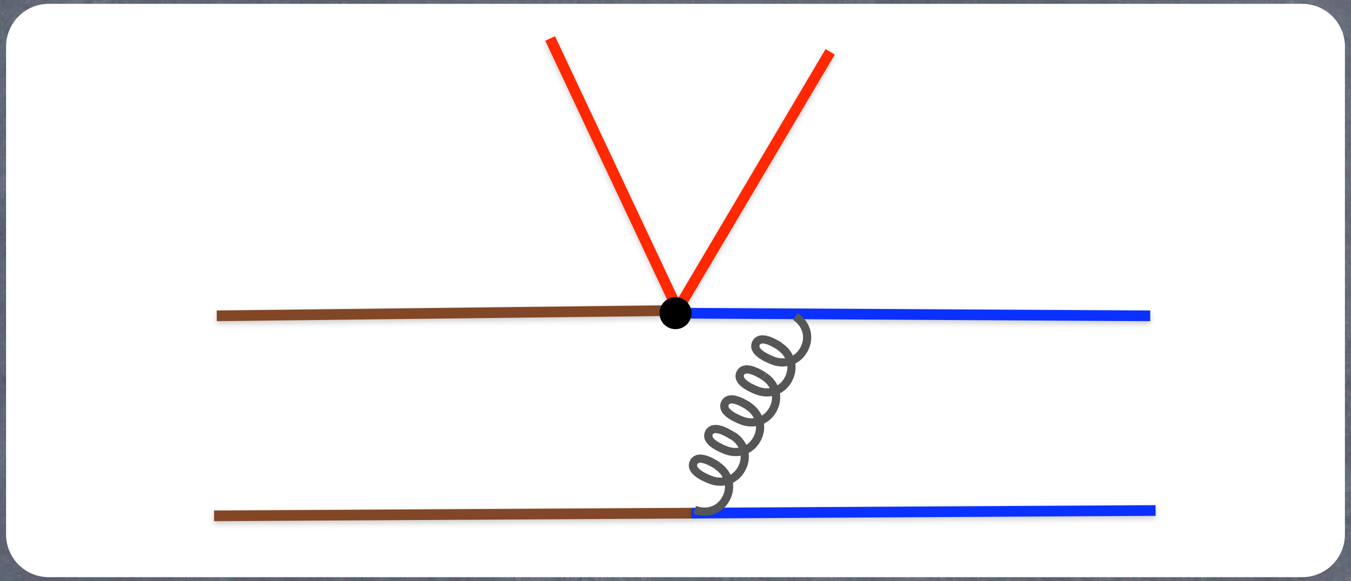




Scale $(\Lambda_{QCD} m_b)^{1/2}$

Arises from turning soft spectator in B meson into collinear spectator in light meson



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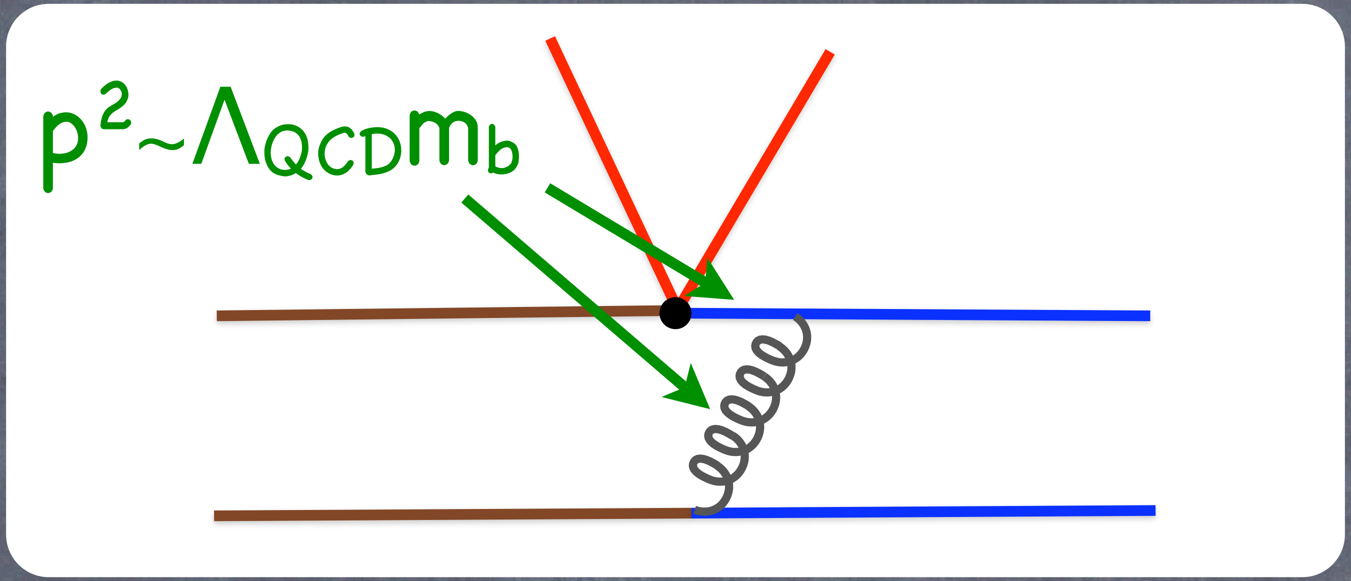

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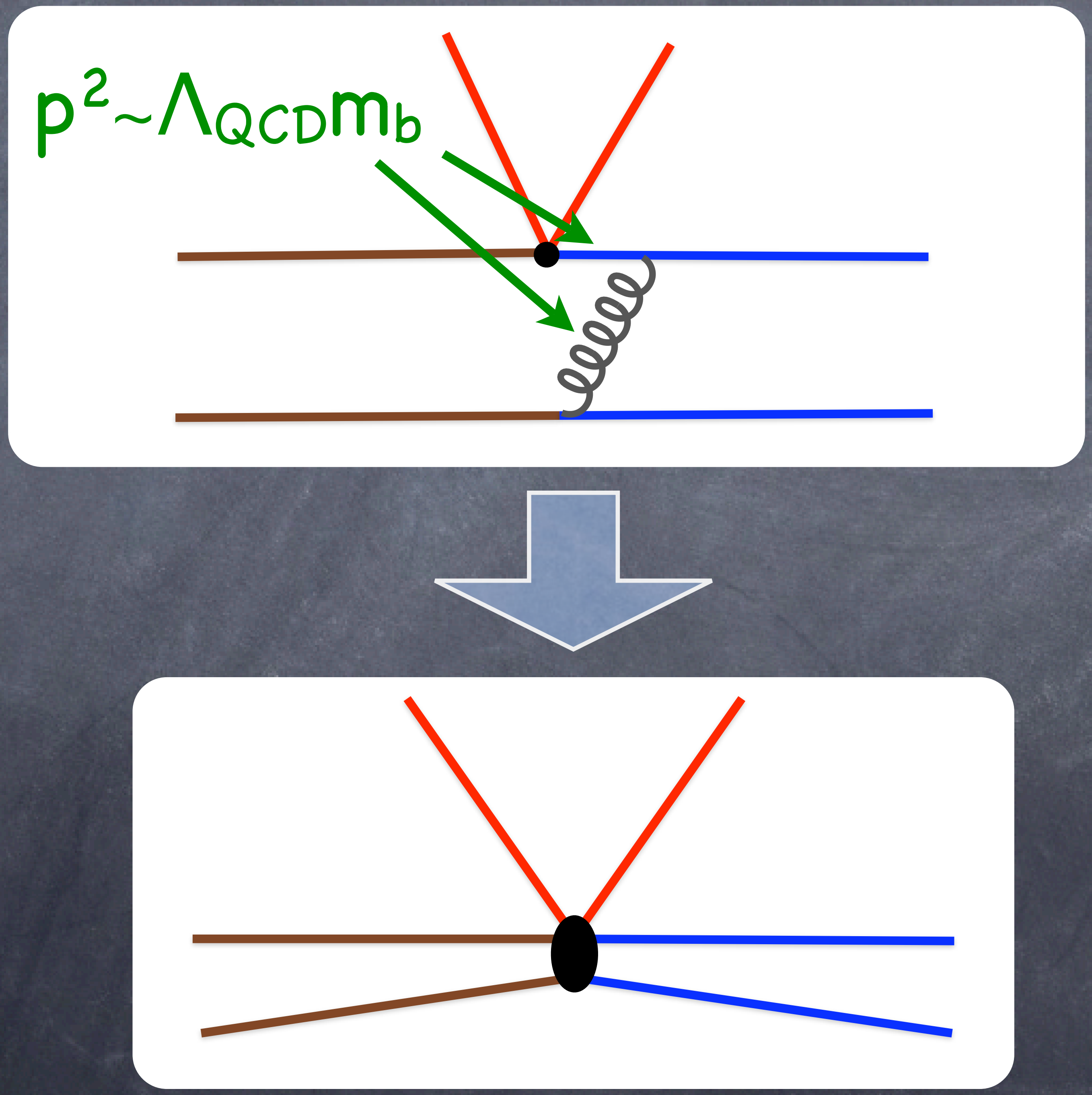
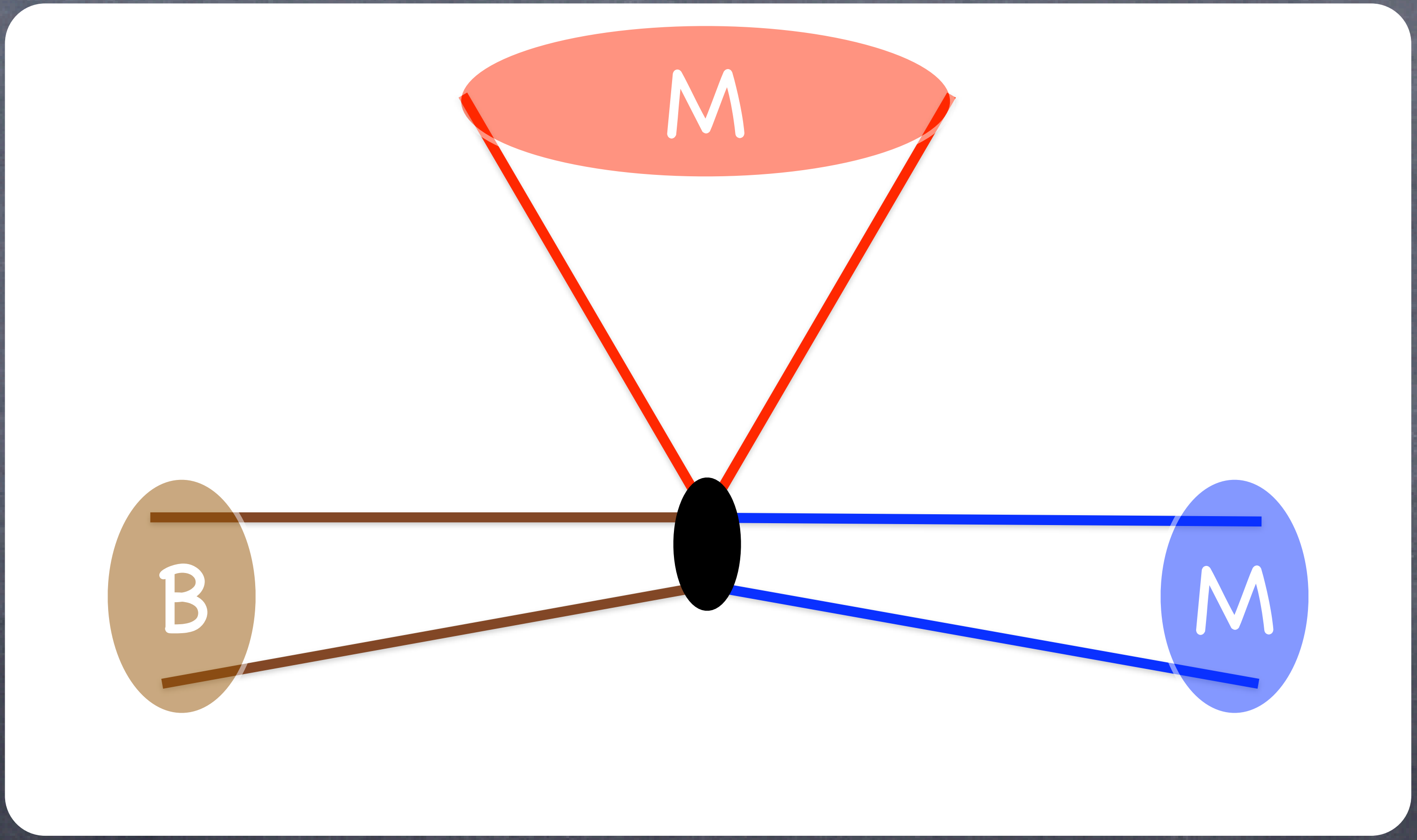


Scale Λ_{QCD}

The final hadronization of the partons into hadrons



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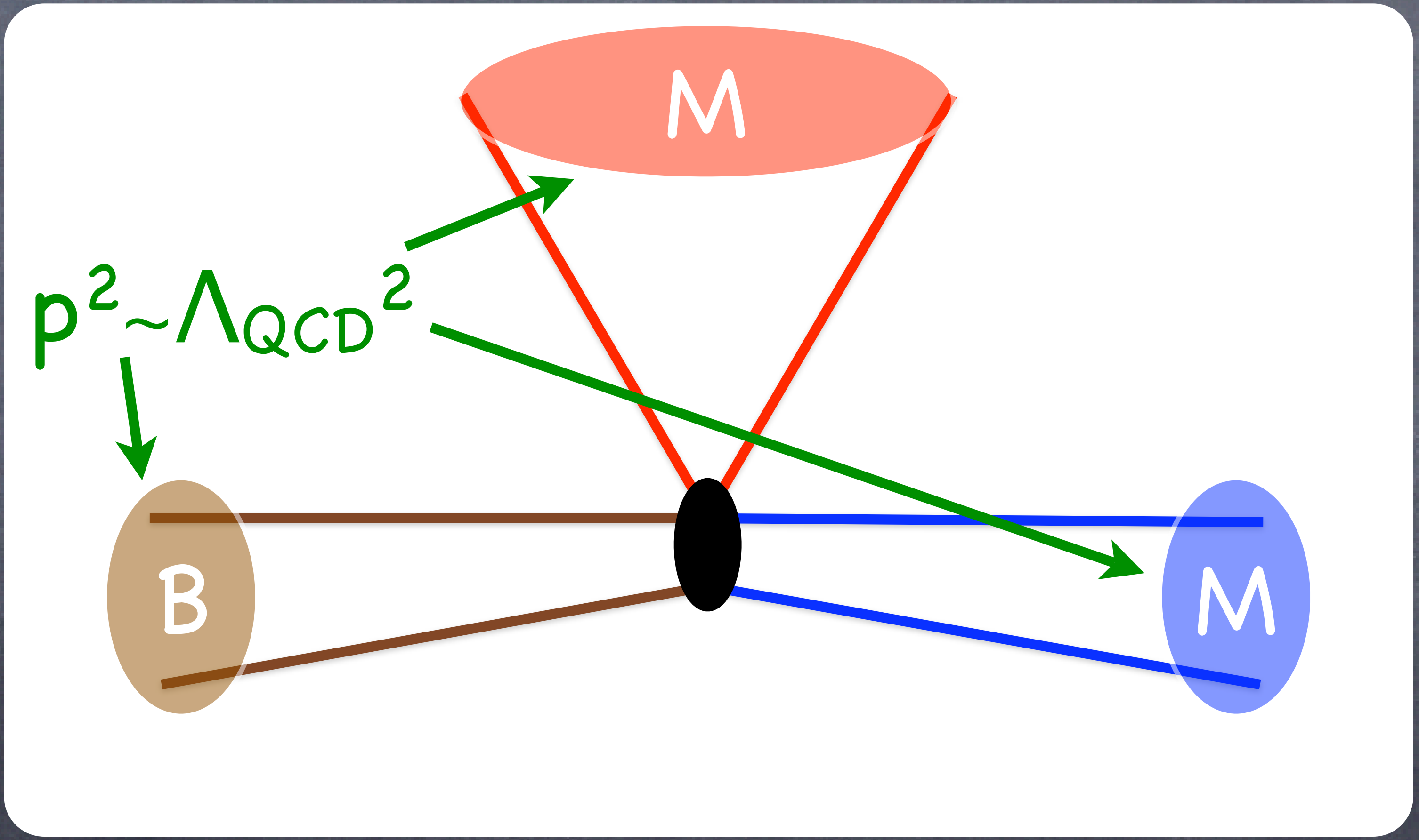

Quick introduction to SCET

Field content of SCET

Differentiate collinear and soft DOF

Type	Momentum	Fields
collinear	$E \gg \Lambda_{QCD}$	χ_n, A_n
soft	$E \sim \Lambda_{QCD}$	q_s, A_s

Need a different collinear field for each direction

Interactions in SCET

Leading order collinear Lagrangian

$$\mathcal{L} = \sum_n \bar{\chi}_n \left[in\cdot D_n + gn\cdot A_s + i\not{D}_n^{\perp} \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^{\perp} \right] \frac{\not{\bar{n}}}{2} \chi_n$$

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Collinear fields

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Collinear fields

Soft gluon

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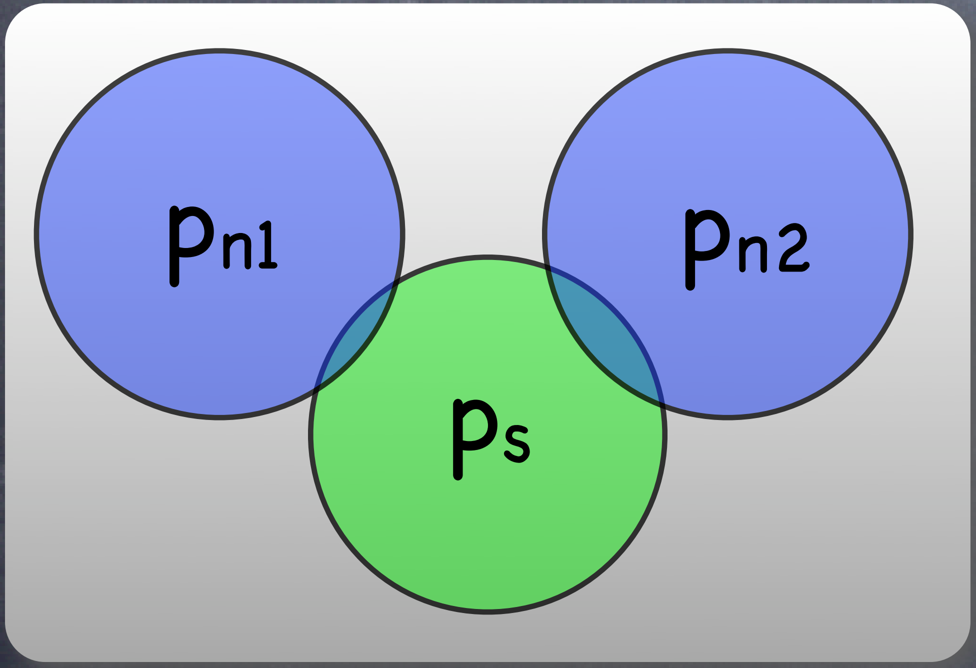
Collinear fields

Soft gluon

- No interactions between collinear fields of different directions
- Interaction between collinear and soft fields only via one single term

Soft/collinear decoupling

$$\mathcal{L} = \sum_n \bar{\chi}_n \left[in\cdot D_n + g n\cdot A_s + i\not{D}_n^{\perp} \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^{\perp} \right] \frac{\bar{\not{n}}}{2} \chi_n$$



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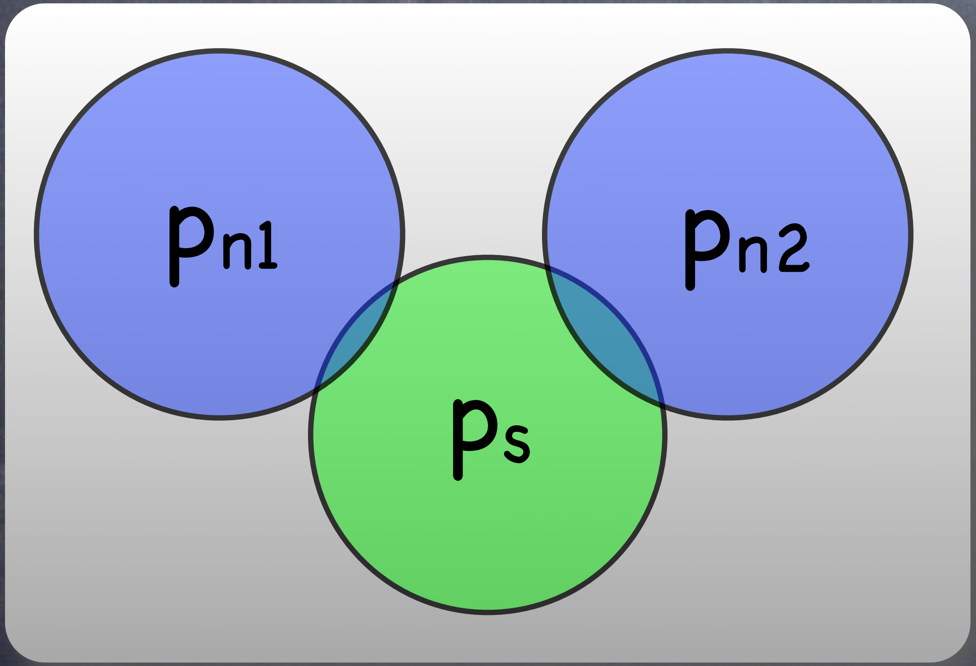

Perform field redefinition

$$\chi_n = Y_n \chi_n^{(0)} \qquad A_n = Y_n A_n^{(0)} Y_n^{\dagger}$$

$$Y_n = \mathrm{P}\exp\left[ig \int_0^{\infty} ds\; n\cdot A_s(ns) \right]$$

$Y_n Y_n^{\dagger} = 1$ $\qquad$ $inDY_n = Y_n in\partial$

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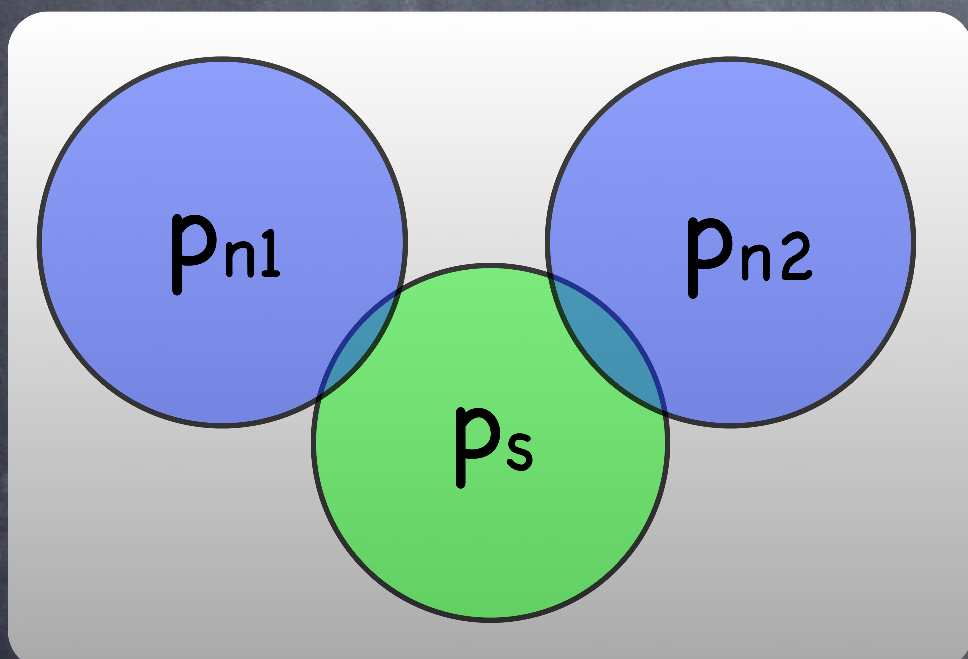

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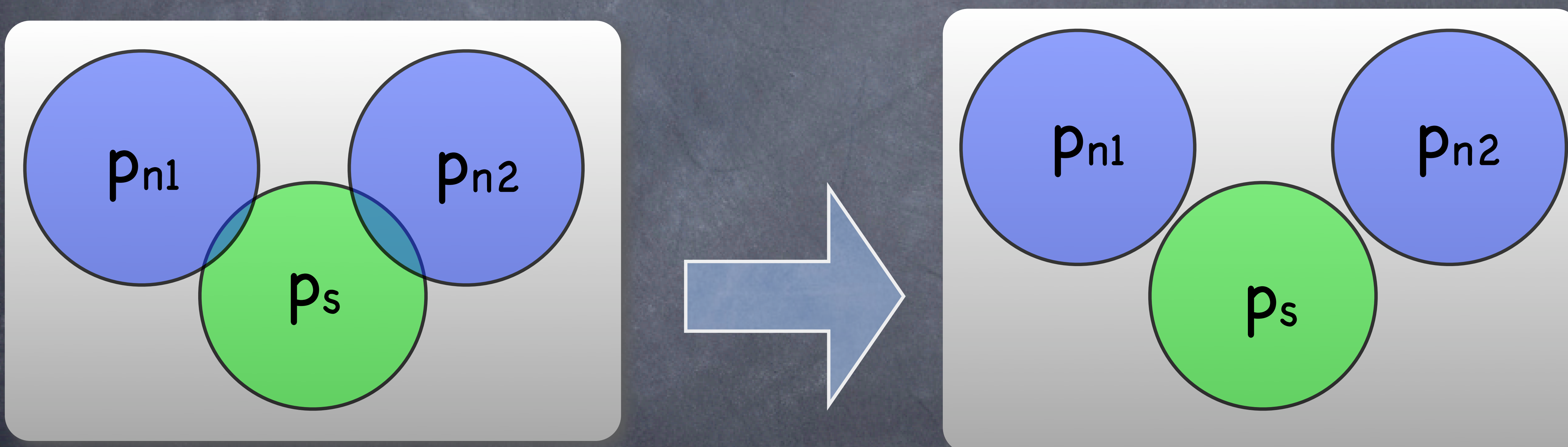

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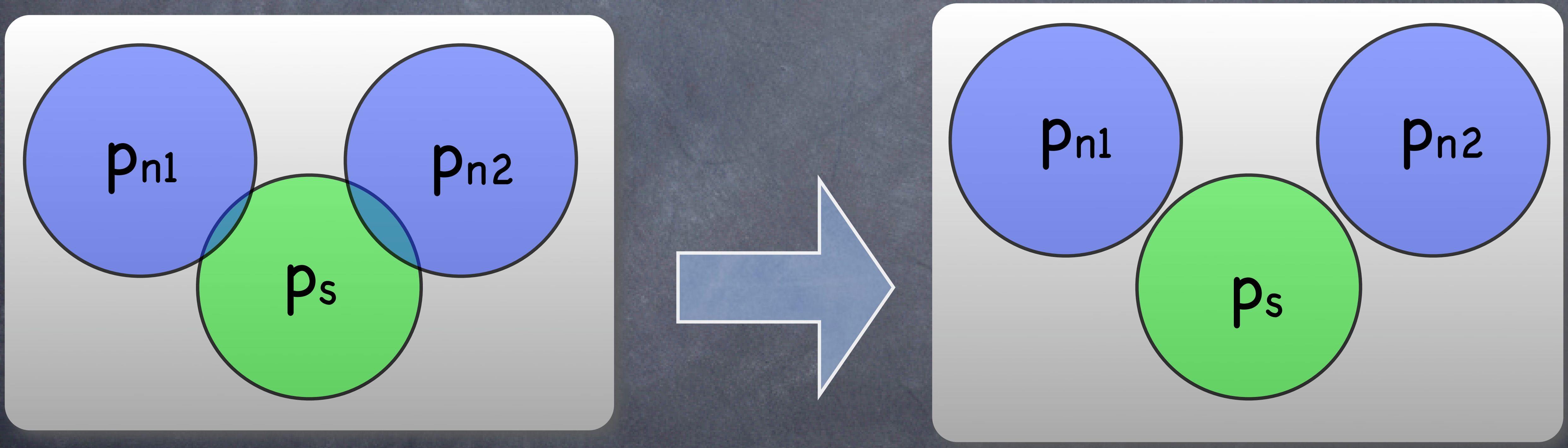
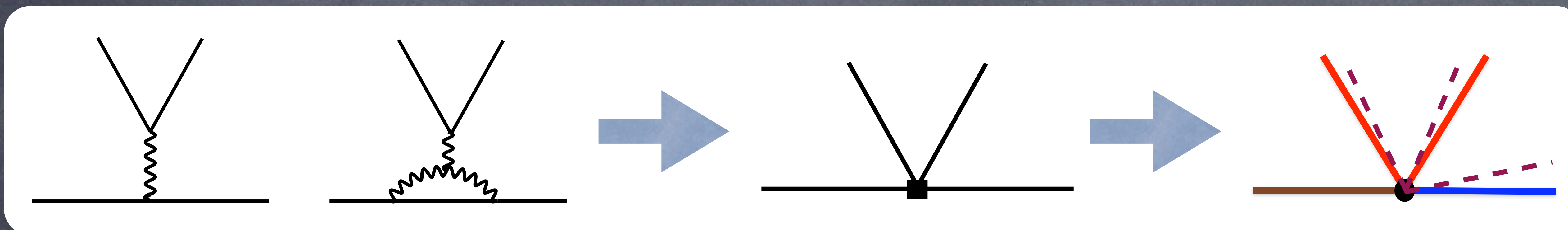


Only true at leading order in Λ_{QCD}/E

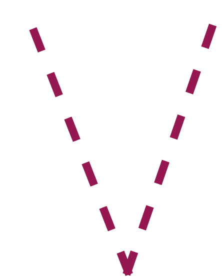
Factorization from scale separation

Matching onto SCET

All approaches remove scales m_W and m_b by matching onto SCET (even though might not use these words)



Soft Wilson lines between particles in same direction cancels



$$= Y^\dagger Y = 1$$

Immediately find separation of one meson from rest of process

The factorization theorem

Uses that at Lagrangian level different fields in SCET completely decouple at leading order in Λ_{QCD}/E

No expansion in $\alpha_s(\sqrt{\Lambda_{QCD} m_b})$

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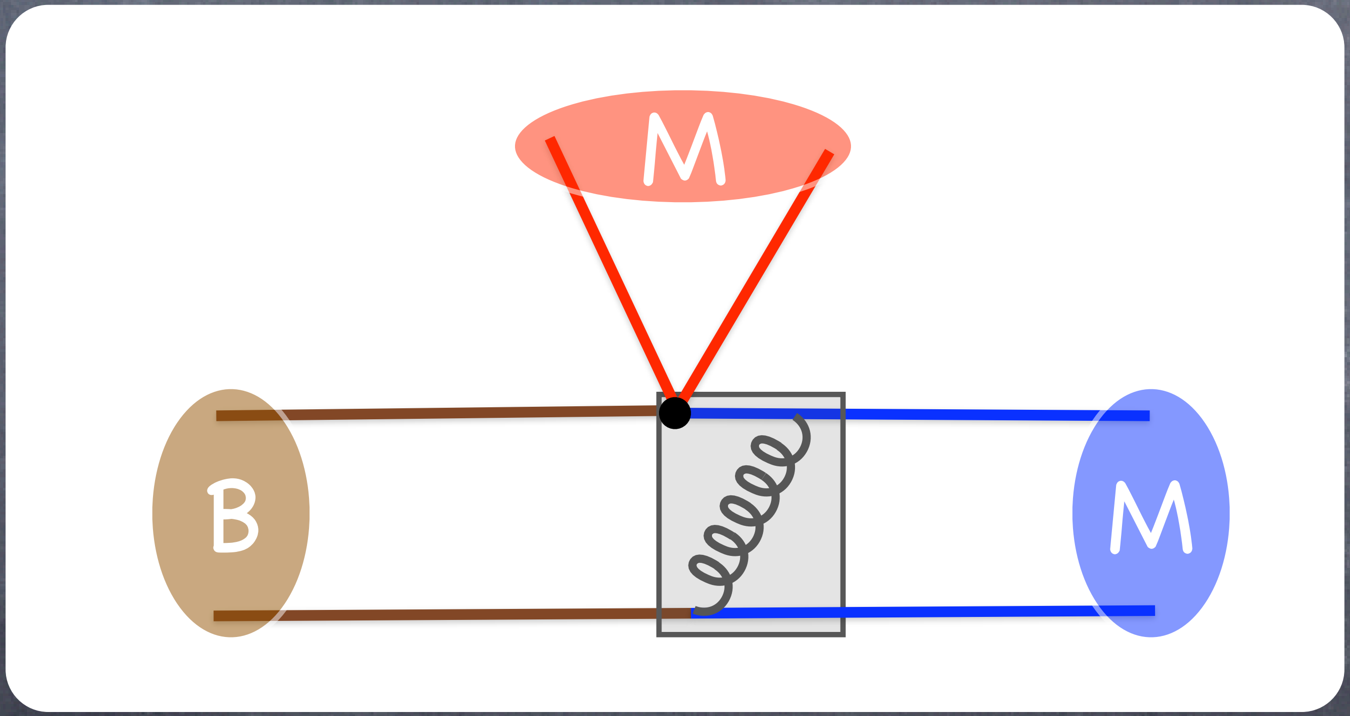

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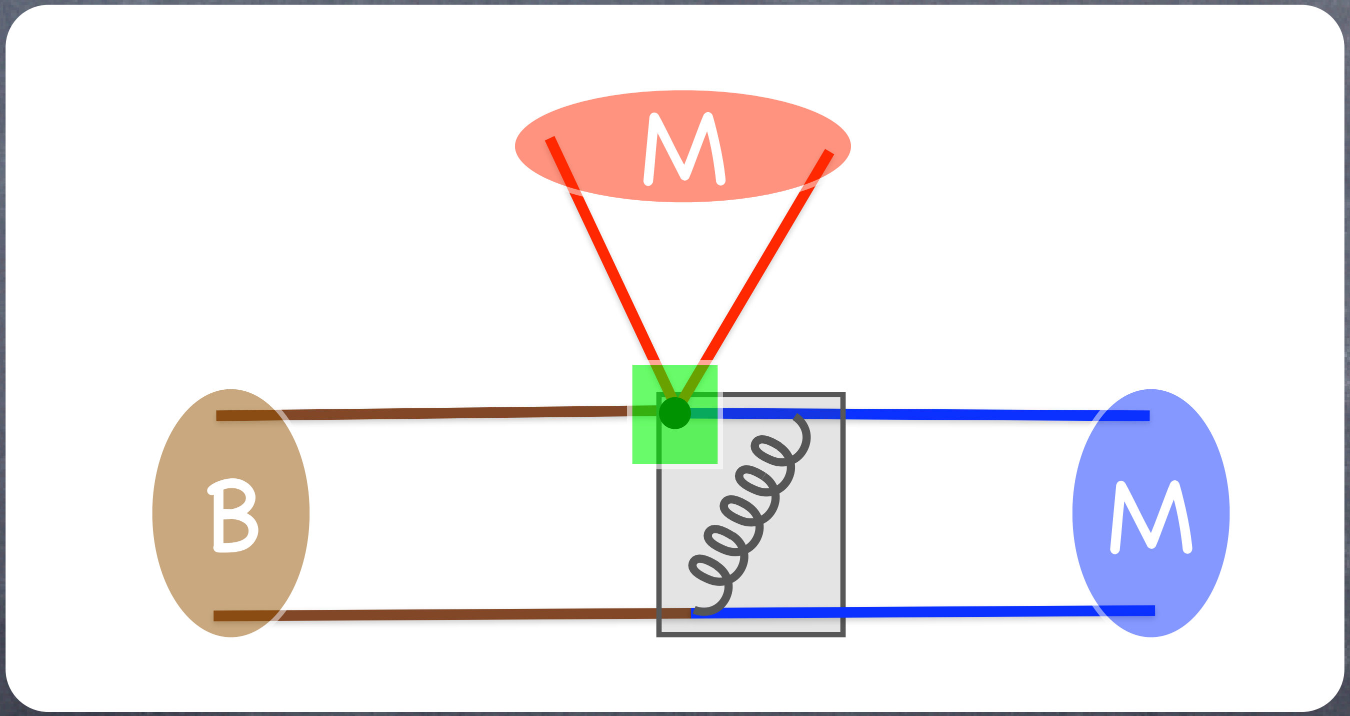

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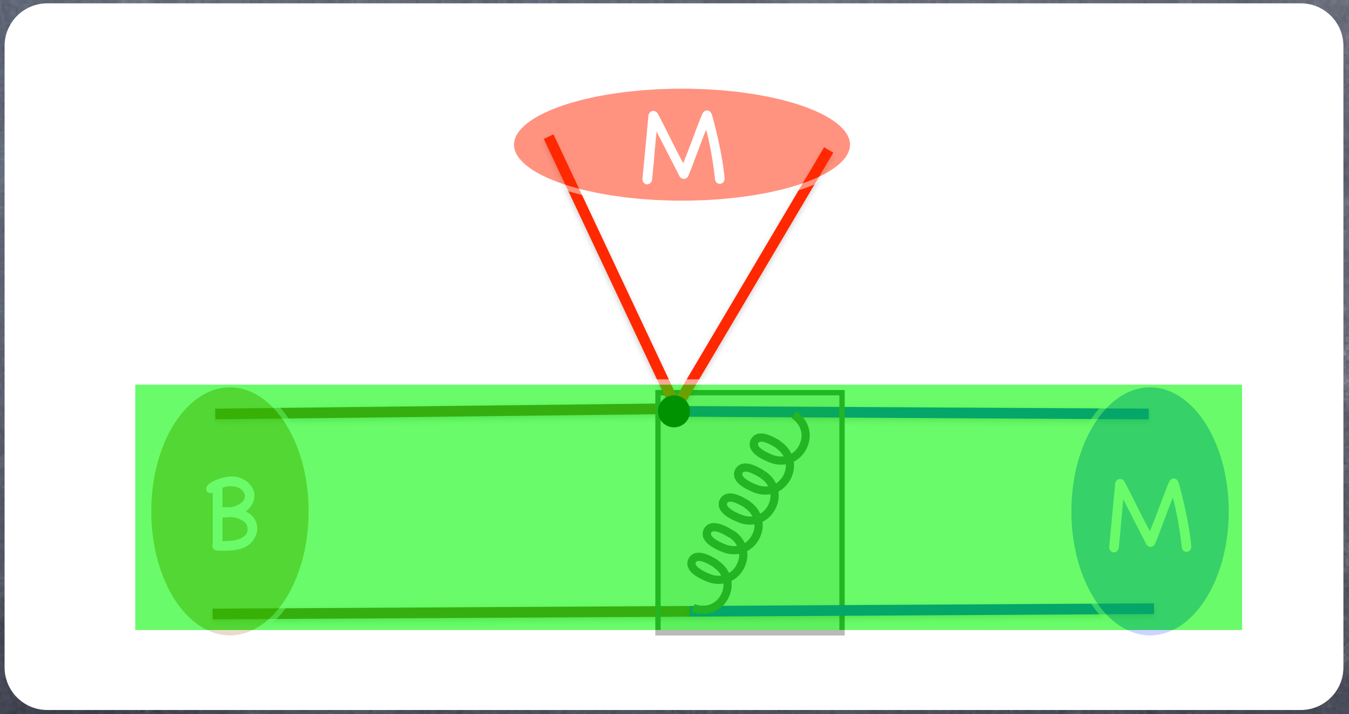

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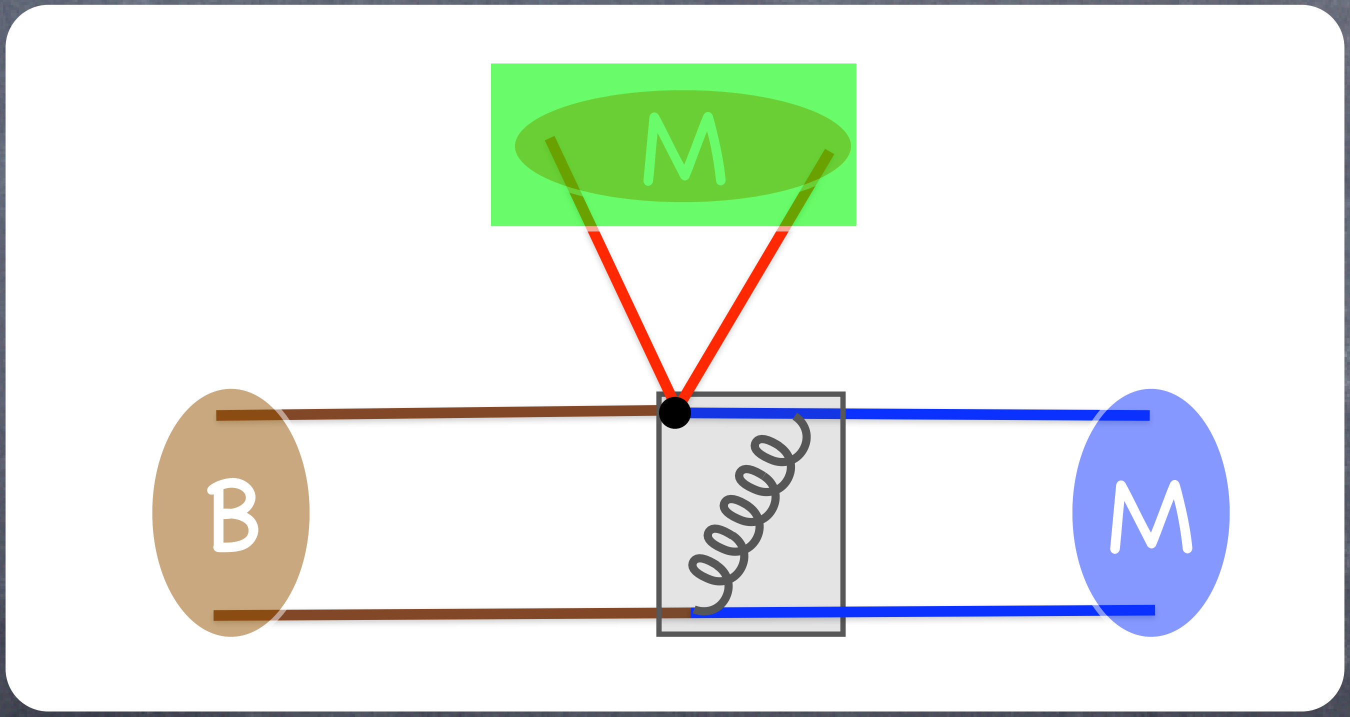

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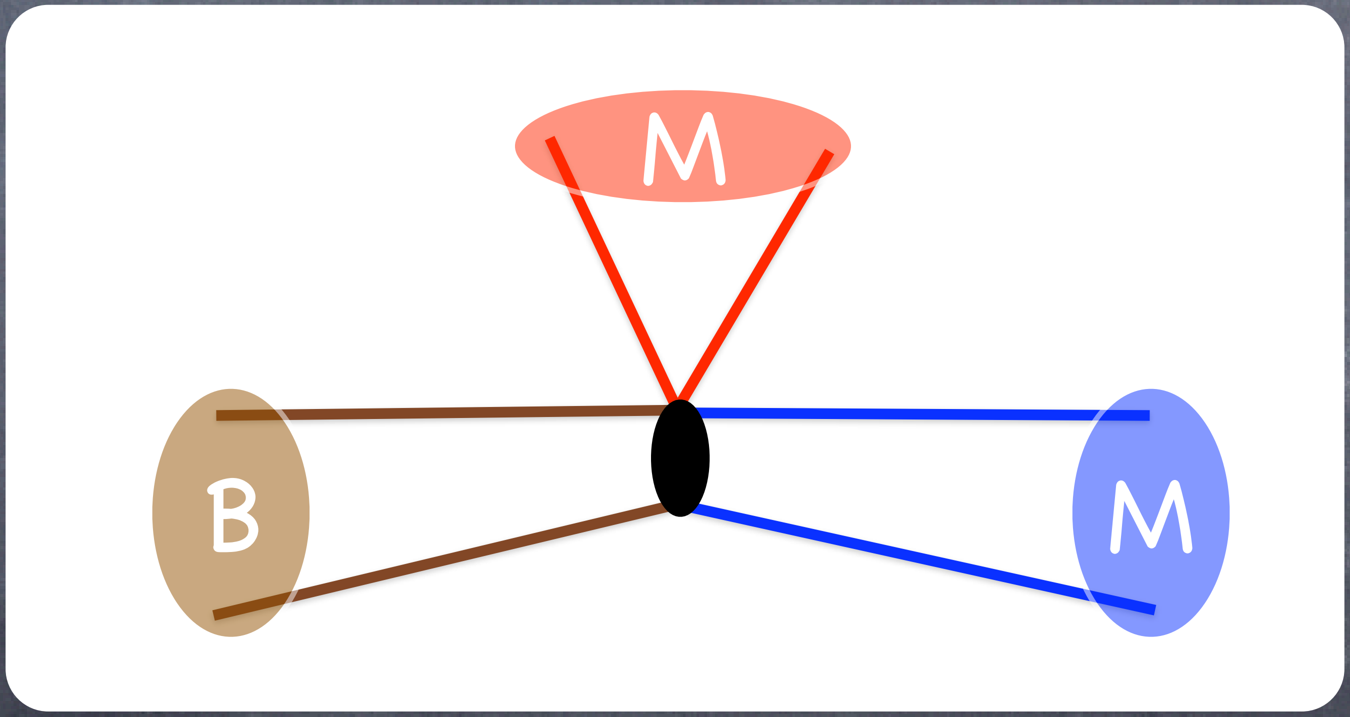

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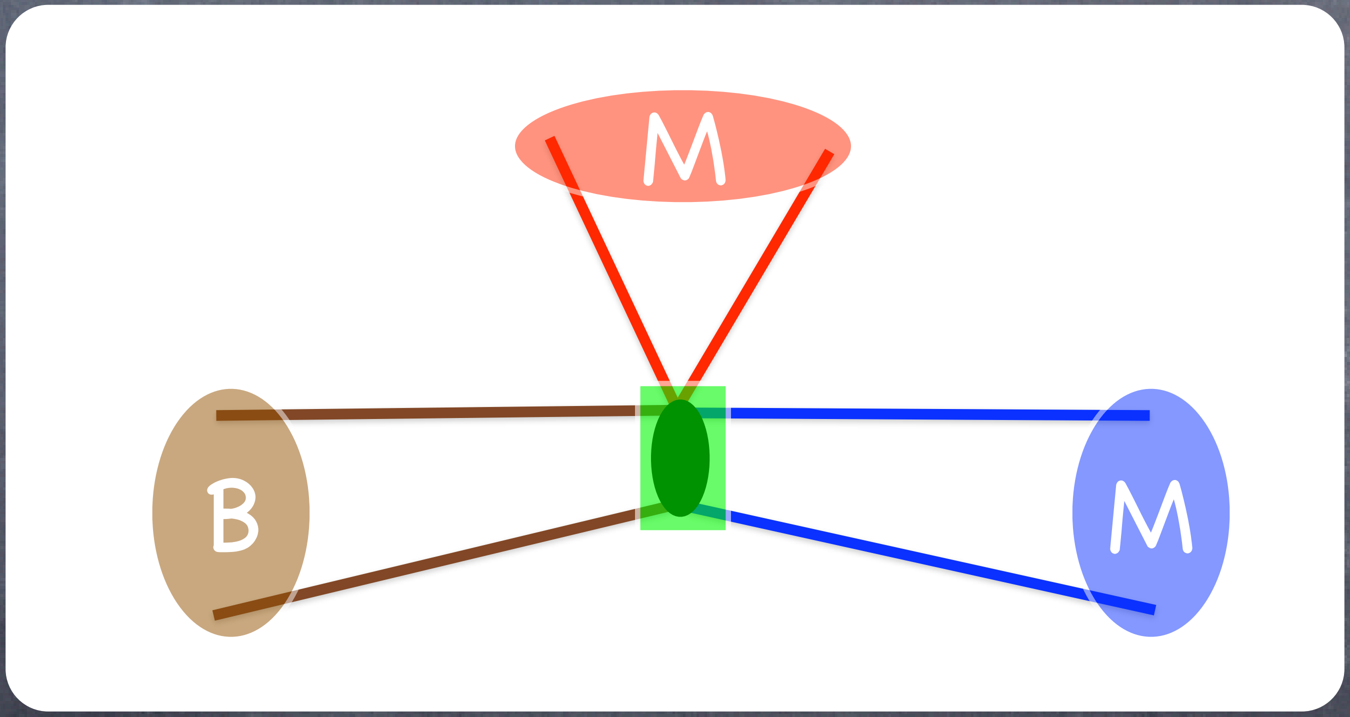

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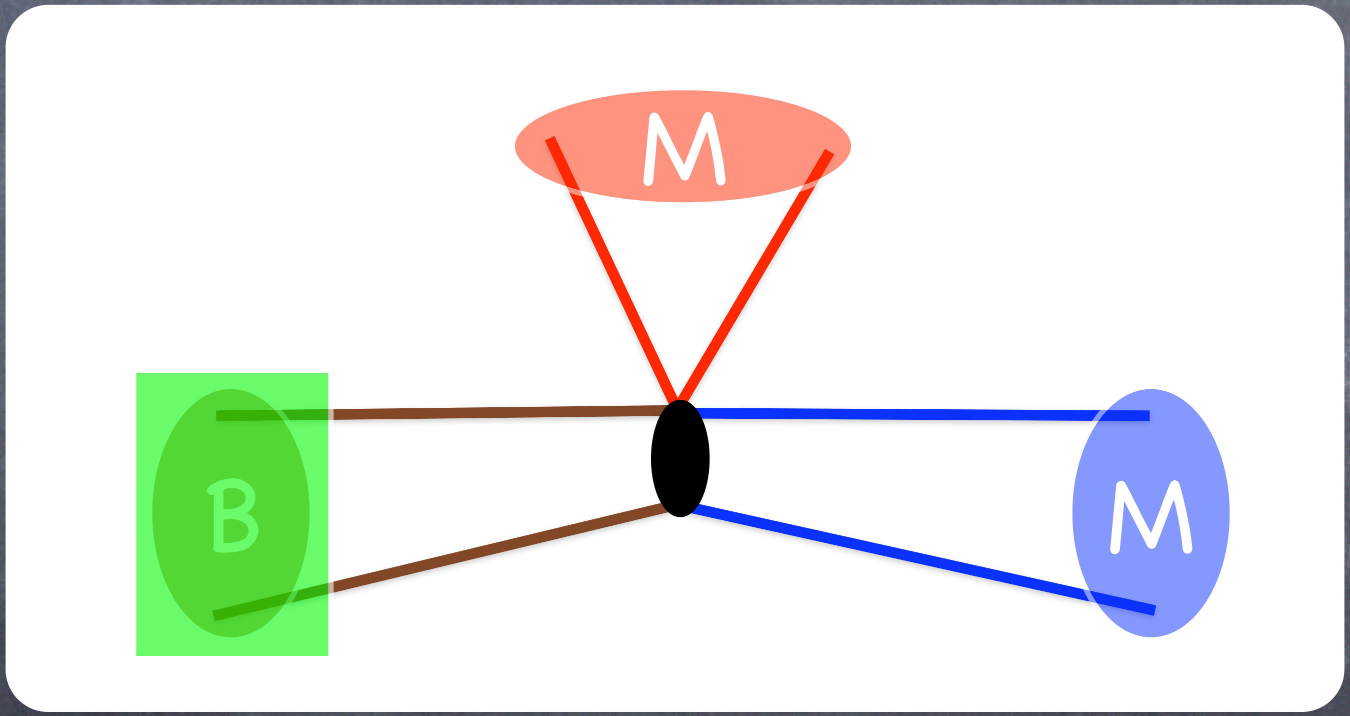

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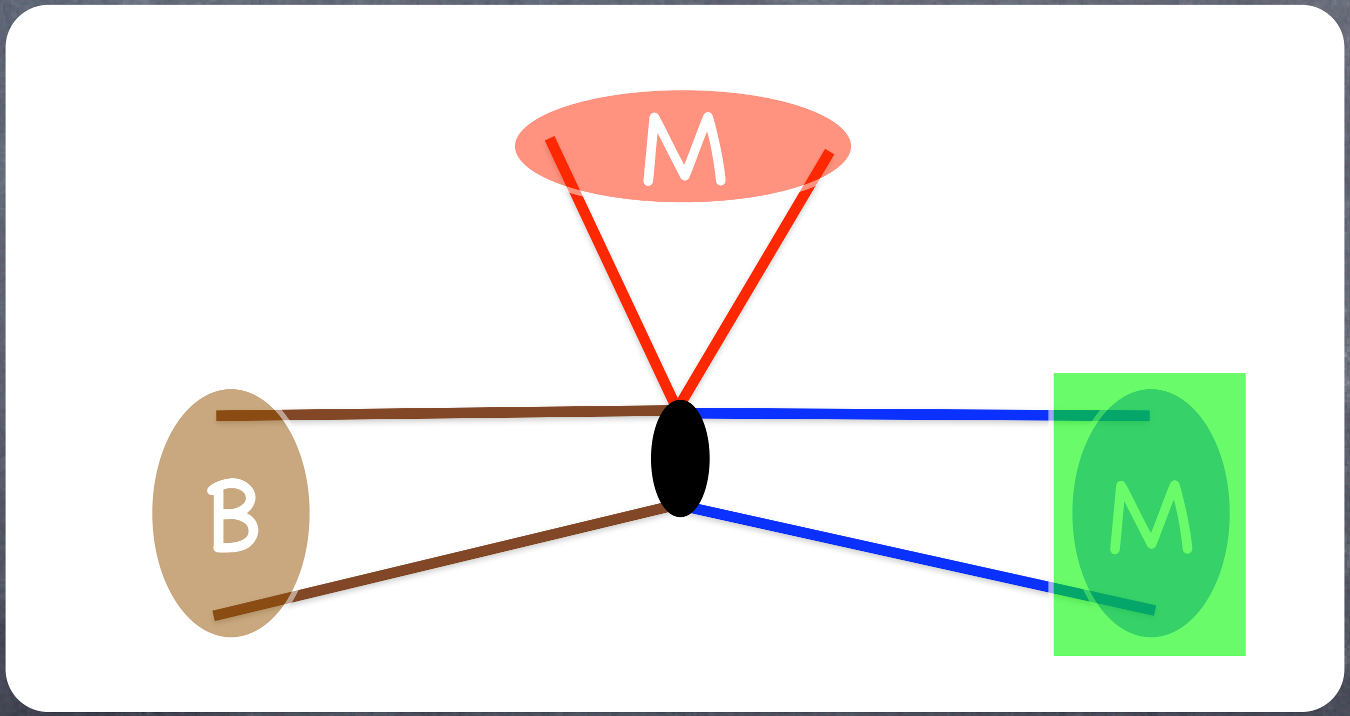

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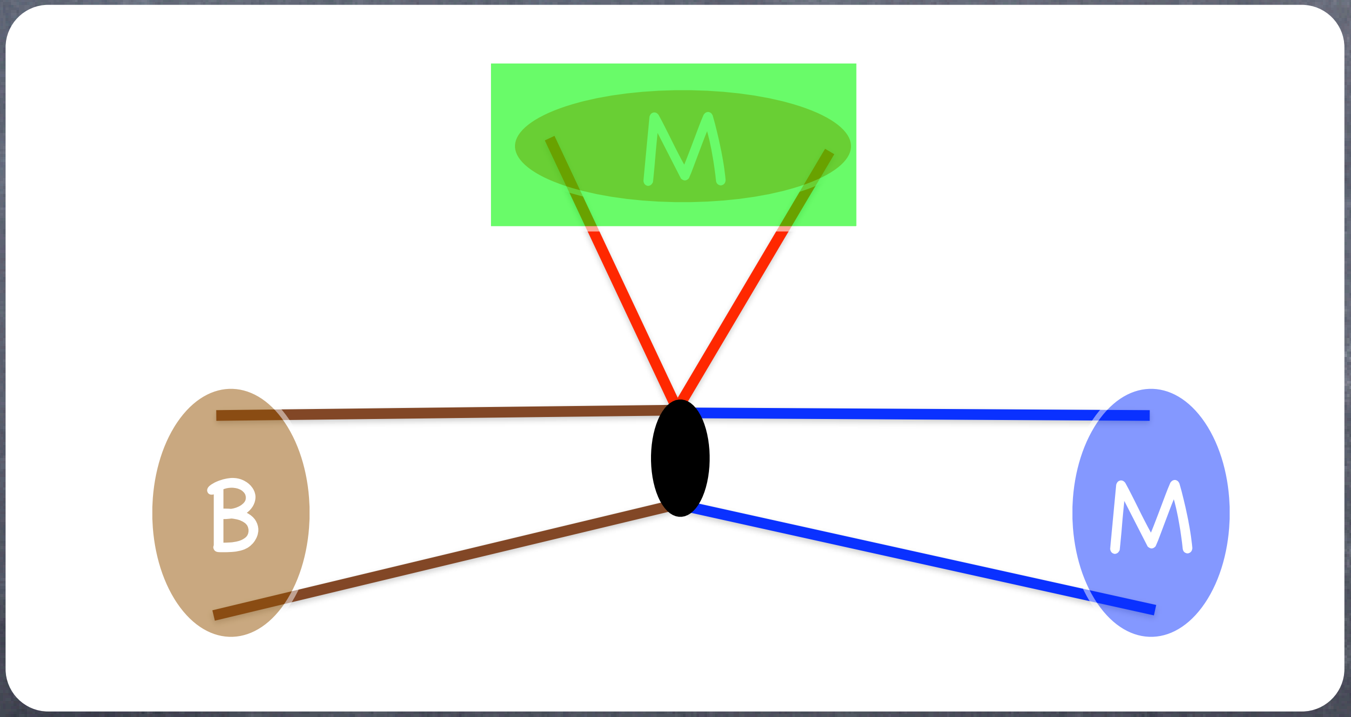

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Do we trust the method?

- SCET agrees with decades of accumulated knowledge on collider physics phenomenology
  - Same effective theory and same manipulations reproduce known factorization theorems in collider physics
  - Solving RG equations of SCET reproduces resumation of known logarithms in DIS, DY and Higgs production
  - SCET at leading order reproduces the parton shower

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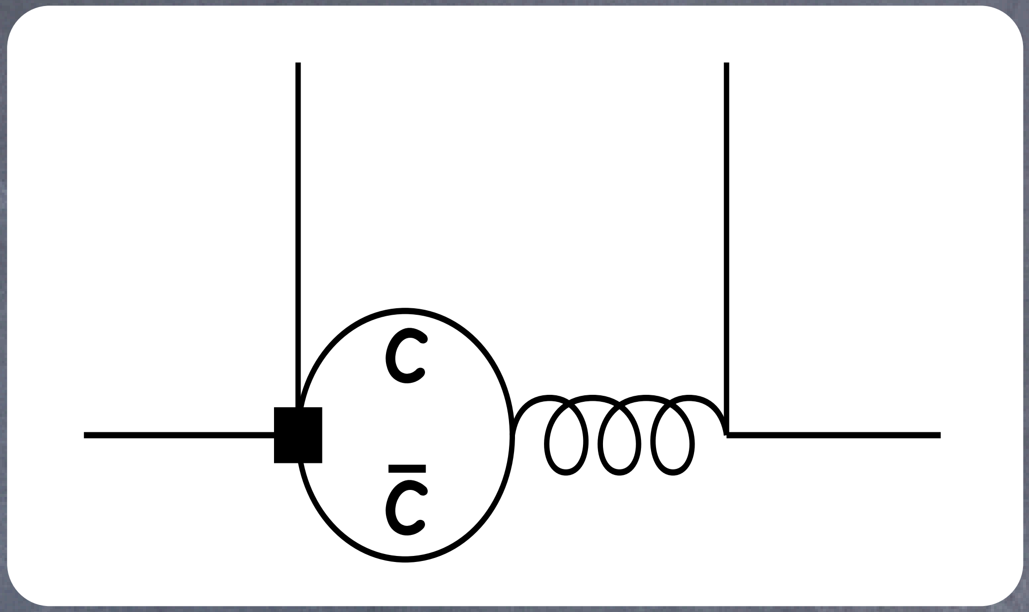
  - SCET at leading order reproduces the parton shower

We do know that SCET reproduces the long distance physics of QCD reliably

Main question is how well leading order works, or how large Λ_{QCD}/E power corrections are

One subtlety...

Intermediate charm loop



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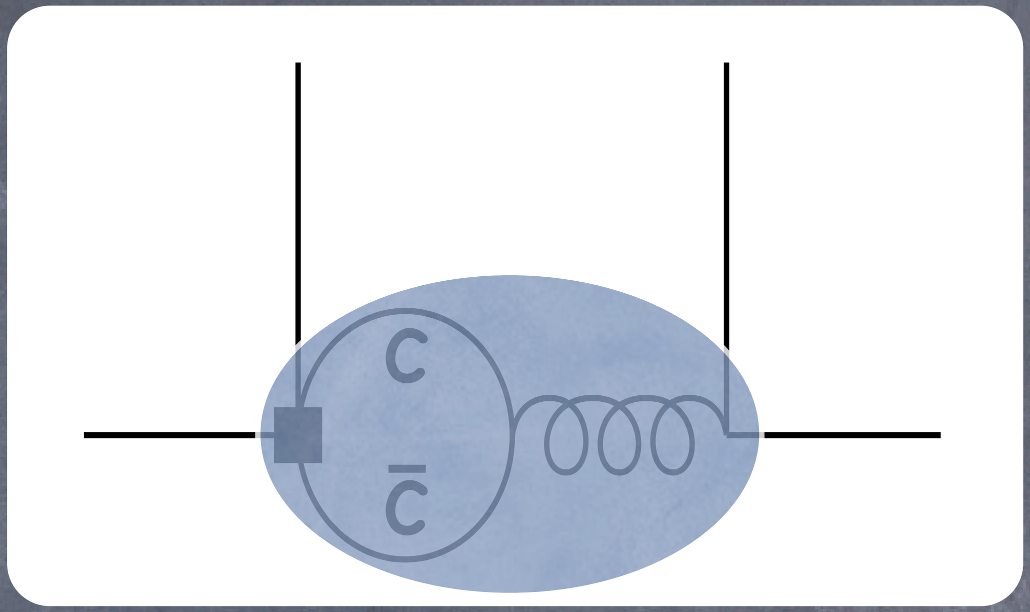

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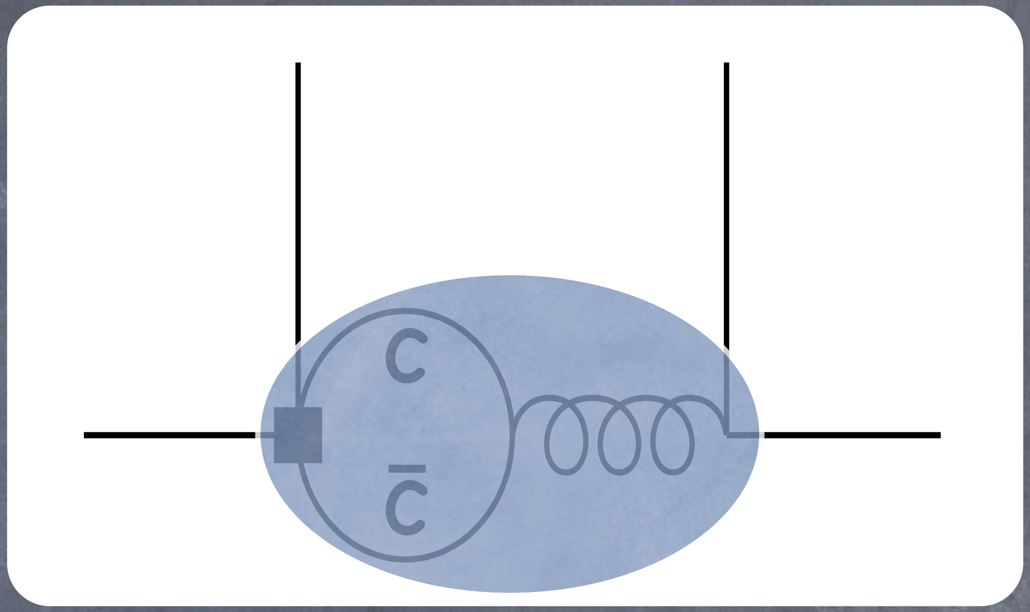
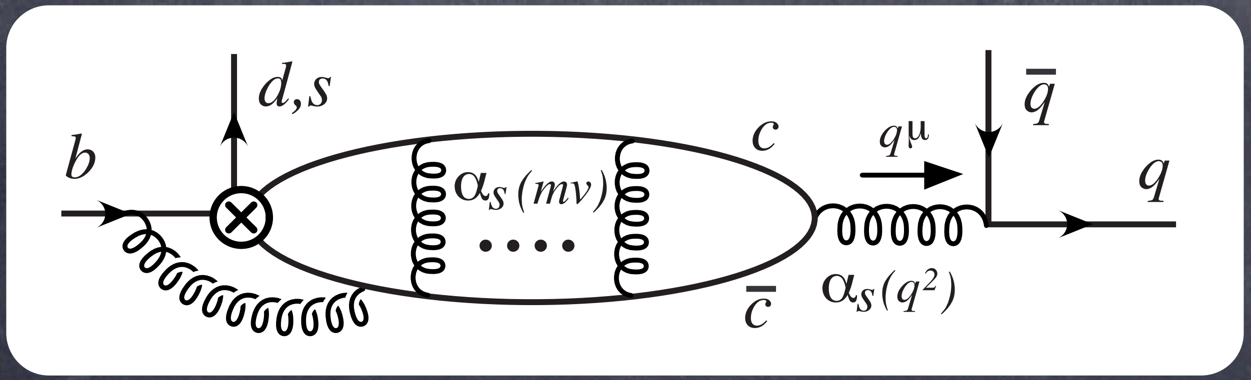


Does this lead to local, perturbative effect, or can cc-pair travel a large distance, creating new non-perturbative effect?



What are the implications?

Strong phases

$$A = H \otimes \zeta_{BM} \otimes \Phi_M + A_{cc}$$

ζ_{BM} and Φ_M both real

$\Rightarrow$ strong phases only from H and A_{cc}

At least if A_{cc} does not contribute, strong phases suppressed by $\alpha_s(m_b)$ or Λ_{QCD}/E

Reduction in hadronic parameters

Even if don't expand in $\alpha_s(\sqrt{\Lambda_{QCD} m_b})$ and don't assume anything for A_{cc}, very few hadronic parameters

<table>
<tr><th></th><th>no expns</th><th>SU(2)</th><th>SU(3)</th><th>SCET +SU(2)</th><th>SCET +SU(3)</th></tr>
<tr><td>ππ</td><td>11</td><td>7/5</td><td rowspan="2">15/13</td><td>4</td><td rowspan="3">4</td></tr>
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One important prediction:

$$A_{CP}^{K^+\pi^0} < A_{CP}^{K^+\pi^-}$$

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Data

$$A_{CP}^{K^+\pi^0} = 0.050 \pm 0.025$$

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Large power corrections or new physics?

Differences between different approaches?

PT at $\alpha_s(1.3\text{GeV})$?

All approaches use PT at hard scale $\mu_h \sim m_b$

Not all use PT at intermediate scale $\mu_h \sim \sqrt{\Lambda_{QCD}E})$

BPRS	QCDF	PQCD
No PT at intermediate scale	PT at intermediate scale	PT at intermediate scale

While using PT does lead to more universality, for non-leptonic B decays get predictive power without

Endpoint divergences

PT at intermediate scale gives rise to divergent convolutions with meson wave functions

How do we regulate these singularities?

BPRS	QCDF	PQCD
No singularities since no PT at intermediate scale	New hadronic parameters for singular convolutions	Singularities regulated with "unphysical" k_T

Charm Loops

As we have seen, cc pair in charm loop can potentially generate new non-perturbative (complex) matrix element, introducing non-perturbative strong phases

Disagreement if PT can be used or not

BPRS	QCDF	PQCD
Treat charm loop as non-perurbative	Use PT for charm loops	Use PT for charm loops

Comparison

	BPRS	QCDF	PQCD
Expansion in $\alpha_s(\mu_i)$?	No	Yes	Yes
Singular convolutions	N/A	New parameters	"Unphyiscal" k_T
Charm Loop?	Non-perturbative	Perturbative	Perturbative
Number of parameters	Most	Middle	Least
How conservative?	Most	Middle	Least

Conclusions

- Need factorization theorems to disentangle EW physics from hadronization effects
- Scale separation using effective theories has given theoretical handle
- Theoretical concept is quite well tested in other processes
- Factorization is established at leading order in Λ_{QCD}/E
- How small are power correction?
- Of different approaches, BPRS most conservative, but has more hadronic parameters than QCDF or PQCD