



Principles and Applications of Betatron Xray generation

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Laser-wakefield acceleration

A back of the envelope approach





Case Study:

*W. Lu et al., PRSTAB 10, 061301 (2007)

100-TW class Ti:Sa laser, 10 Hz

Peak power:

area under a Gaussian with 30 fs FWHM duration:

$$\sigma = \frac{\Delta T}{2\sqrt{2\ln 2}} \to P_0 = \frac{2\sqrt{2\ln 2}E}{\sqrt{2\pi}\Delta T} \approx 0.94 \frac{E}{\Delta T} = 94 \text{ TW}$$

Wavelength = 800 nm Duration = 30 fs FWHM Energy = 3 J

$$\begin{split} & \underline{\mathsf{Matched plasma density:}} \quad \lambda_p = 2\pi c/\omega_p = 2c\Delta T \Longleftrightarrow \omega_p = k_p/c = \pi/\Delta T \\ & \to n_e = \frac{\epsilon_0 m_e \omega_p^2}{e^2} = 3.45 \times 10^{18} \text{ cm}^{-3} \\ & \underline{\mathsf{Peak intensity:}} \text{ matched spot size}^* \quad w_0 = 2\sqrt{a_0}/k_p \\ & a_0 = \frac{e}{c\omega_L m_e} \sqrt{\frac{2I_L}{\epsilon_0 c}} \underbrace{=}_{I_{Gauss} = \frac{2P}{\pi w_0^2}} \underbrace{\frac{e}{w_0 c\omega_L m_e} \sqrt{\frac{4P}{\pi \epsilon_0 c}}_{i=\frac{C}{w_0}} := \frac{C}{w_0}}_{I_{Gauss} = \frac{e}{\pi w_0^2}} \underbrace{\frac{e}{w_0 c\omega_L m_e} \sqrt{\frac{4P}{\pi \epsilon_0 c}}_{i=\frac{C}{w_0}} := \frac{C}{w_0}}_{u_{p,rel} = \frac{\omega_p}{\sqrt{1 + \frac{a_0^2}{4}}}} \\ & \Rightarrow w_0 = \left(\frac{2}{k_p}\sqrt{C}\right)^{2/3} = 12\mu m \Rightarrow d_{FWHM} = 20\mu m \\ & I_0 = \frac{2}{\pi} \frac{P_0}{w_0^2} \underbrace{=}_{real \text{ spot}, P=0.7P_0} 3.2 \times 10^{19} \frac{W}{\text{cm}^2} \Rightarrow a_0 \approx 4 \end{split}$$





Case Study:

 $a_0=4$ leads to highly relativistics electrons, which in turn change the plasma frequency according to:

$$\omega_{p,rel} = \omega_p / \sqrt{\langle \gamma \rangle}$$
 with $\langle \gamma \rangle \approx 1 + a_0^2 / 4$

10

We can now solve the previous equations iteratively by first plugging $a_0=4$ into the expression for λ_0 , and then the respective results.

After a few steps (<10) we converge at the following laser and plasma parameters:

$$n_e = 1.2 \times 10^{18} \text{ cm}^{-3} \iff \lambda_p = 30.6 \mu \text{m} \iff \omega_p = 6.2 \times 10^{13} \text{Hz}$$

 $d_{FWHM} = 27 \mu \text{m} \iff w_0 = 16.1 \mu \text{m} \iff I_L = 1.6 \times 10^{19} \text{ W/cm}^2$

$$a_0 = 2.75$$

100-TW class Ti:Sa laser, 10 Hz

Wavelength = 800 nmDuration = 30 fs FWHM Energy = 3 J

19









$$E_r = \frac{m_e c^2 2\pi}{2e\lambda_p} r = 1.65 \times 10^{11} \text{V/m}|_{r=\lambda_p/2} \simeq 344 \text{MT/m}$$

 E_r causes a radial confinement potential of 1.2 MV. Approx. the same field accelerates electrons.







Other results:

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Sears et al., Phys. Rev. STAB 13, 0928303 (2010) G.H. Welsh et al., MOPE 072, IPAC proceedings 2010 S. Fritzler et al., Phys. Rev. Lett 92, 165006 (2004)



Temporal characterization by coherent TR spectroscopy









LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

UNIVERSITAT

MUNCHEN

MPG

LMU





Dulse

plasma



target

high harmonic sources

filter





Laser-driven Xray sources

(metal jet) bremsstrahlung and line (K- α)-sources

relativistic electron beam +

- undulator = undulator source, FEL
- plasma fields = Betatron source
- laser pulse = Thomson/Compton source





"Wiggly" electron X-ray sources: Ingredients: relativistic electron beam +



plasma fields



laser fields



undulator radiation, FEL 100's eV - keV $\lambda_u \approx 1$ cm $\lambda (K^2)$

$$\lambda_{x-ray} = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

Betatron radiation keV – 10's keV λ_b≈500μm Thomson scattering 10's keV - MeV λ_l≈1μm

 $\lambda_{x-ray} = \frac{\lambda_l}{4\gamma^2} \left(1 + \frac{a_0^2}{2} + \gamma^2 \theta^2 \right)$





Assume:

500 MeV e⁻

 $r_{\beta}=1\mu m$

osiris

UCLA

v2.0

In the plasma fields, electrons can oscillate radially if they start off-axis:

Charge Density



They oscillate with a wavelength

 $\lambda_{\beta} = \lambda_p \sqrt{2\gamma} \approx 1.4 \mathrm{mm}$

that depends on the electrons' instantaneous relativistic factor and emit radiation at

$$\sum_{n \in \mathbb{R}} \lambda_{X-ray}(n) = \frac{\lambda_p}{2n\gamma^2} \left(1 + \frac{K_\beta^2}{2} + \gamma^2 \theta^2 \right) = \frac{8 \text{ nm}}{n}$$

where *n* is the harmonic number, θ the observation angle and K_{β} the betatron strength parameter given by:

$$K_{\beta} = \gamma r_{\beta} \frac{2\pi}{\lambda_{\beta}} = 4.54$$

The number of harmonics is given by the critical harmonic number

$$n_c \approx \frac{3}{4} K_\beta^3 = 70.3$$

leading to a critical energy of 10 keV



...and can be controlled

Popp et al., Phys. Rev., Lett. 105 215001 (2010)

(keV)



(keV)

Electron oscillations have been observed...

Y.Glinec et al., Europhys.. Lett. 81 64001 (2008)

Laser polarization a) 5 🚍 (arb. u. 10-(mm) a) -5 0 × -10^{-10} 115 135 E (MeV) b) 165 10b) 100 🔒 λ.=8 um () Angle [mrad] (arb. 10 -10t= 0.08 0.93 2 10-C 0 Schnell et al., Nat. Comm. 4 2421 (2013) -10^{-10} а nce (mrad) 10d) ay 0 Diver -10-Ę 100 120 3 4 5 6 Photon energy 60 70 80 160 200 60 70 80 100 120 160 200 3 4 5 6 Electron energy (MeV) Electron energy (MeV) Photon energy -10^{-10} (keV) b (keV) (mrad) intensity 50 100 200 ∞ Energy [MeV] 30 av Dive 5 60 70 80 100 120 160 200 70 200 3 4 5 6 Photon energy 60 80 100 120 160 3456 Electron energy (MeV) Electron energy (MeV) Photon energy (keV) С (keV) Divergence (mrad) 3 0 60 70 80 100 120 160 200 3 4 5 6 Photon energy 60 70 80 100 120 160 200 56 4 Electron energy (MeV) Electron energy (MeV) Photon energy





For constant γ_z of the electrons in the plasma (no acceleration), Esarey et al.* give the following expression for the asymptotic on-axis spectrum:

$$\frac{d^2 I(\theta=0)}{d\omega d\Omega} \propto \frac{\omega}{\omega_c} K_{2/3}^2 \left(\frac{\omega}{\omega_c}\right) \text{ with } \omega_c \simeq 3K_\beta \gamma^2 \omega_\beta$$

In reality, the situation is even more complicated:

- Electrons are accelerated during radiation: γ , λ_{β} , λ_{X-ray} , K_{β} , n_{c} are non-constant.
- Distribution of r_{β} and E in electron bunch: Further broadening of spectrum.







Betatron spectrum for flat-top electron spectrum: only max. r_{β} (dashed) vs. $0 < r_{\beta} < r_{\beta,max}$ (solid)



Overall betatron spectrum depends on energy and injection radius distribution of the electron bunch \Rightarrow numerical calculation necessary.





Detection: X-ray CCD as imager and spectrometer (below 30-40 keV)

Imaging mode:



- Requires many photons/pixel for lownoise image
- Spatial resolution given by pixel size and projection factor
- Absorption and phase-contrast imaging possible
- Can be combined with filter-based spectrometers to give spectral information

Single-hit spectroscopy



- Guaranteed less than one photon/pixel (not more than 1% of pixels see signal)
- Single pixel value gives energy for single photon
- Histogram of all pixels gives spectrum
- Spatial resolution given by necessary binning for spectrum statistics





First observation at SLAC: S. Wang et al., Phys. Rev. Lett. 88 135004 (2002)



From LWFA beams: A. Rousse et al., Phys. Rev. Lett. 93 135005 (2004)









betatron beam profile:





Sidky et al, J. Appl. Phys. 97, 124701 (2005)









reconstructed Betatron spectrum











Single hit spectroscopy with low charge bunches







Angular resolved photon energy







Betatron radiation source characteristics



peaks at 5.5 keV

best fit 1.7 μm

assuming a 5-fs pulse duration, this infers a peak brilliance of 2×10^{22} ph/(s²mm²mrad² 0.1% bandwidth)

J. Wenz et al., Nat. Comm. 6 7568 (2015)











Betatron applications: 1. LWFA diagnostic





Poor man's plasma diagnostics:

*S. Corde et al., Rev. Mod. Phys. 85 1 (2013) **F. Albert et al., Phys. Rev. E 77 056402(2008)

Electron injection:

- How large is injection radius?
- Is injection polarization dependent?

→ Knife-edge diffraction directly yields average betatron radius r_{β} . With $r_0 = r_{\beta} \gamma^{1/4} *$ follows injection radius r_0 .

Similar information by analysis of spectrum and spectrum model simulations retrieved by** (not quite so poor man-wise...)

Transverse fields:

- How strong are transverse wakefields?
- What is the average wiggling parameter?

 \rightarrow Beam divergence is K/ γ . From measured beam divergence and measured spectrum (and some modeling) one gets effective K and effective fields. Unisotropy givens information on wiggling (and injection) plane.





Detailed electron trajectory modelling can match the experimental X-ray profiles













S. Corde et al., Plasma Phys. Contr. Fus. 54, 124023 (2012)







Betatron applications: 2. Imaging





Phase-contrast CT allows excellent soft-tissue discrimination

conventional CT



phase contrast CT



Slices through a mouse thorax using equal radiation dose.

(F. Pfeiffer)

Perspectives



Results extremely important for :

Designing future accelerators Compact X ray source (Thomson, Compton, Betatron, or FEL) Applications (chemistry, radiotherapy, medicine, material science, ultrafast phenomena studies, etc...)

First X rays betatron contrast images

S. Fourmaux *et al.*, Opt. Lett. **36**, 13 (2011)

S. Kneip *et al.*, Appl. Phys. Lett. **99**, 093701 (2011)





Courtesy of K. Krushelnick

V. Malka *et al.*, Nature Physics **4** (2008) E. Esarey et al., Rev. Mod. Phys. **81**, 1229 (2009) S. Corde et al., Rev. of Modern Physics **85**, 1 (2013)

HELL Experimental Platform - Detailed Used Requirements Workshop Institute of Physics of the Academy of Science, Praha Czech Republic, January 28 (2014)



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POLYTECHNIQUE

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Are betatron beams any good for applications? \rightarrow Single-shot phase contrast imaging









Tomography: Line projections and Radon transform:



Parametrize each point on ray by a normal unit vector w, distance to rotation center t and longitudinal position s:

$$f\left(\vec{x}\right) = f\left(\vec{\omega}t + s\vec{\omega}^{\perp}\right)$$

Then the Radon transform yields a representation of the object function f in the variables t and w:

$$Rf(t,\omega) = \int_{x\cdot\omega=t} f(x) dx = \int_{-\infty}^{\infty} f(\vec{\omega}t + s\vec{\omega}^{\perp}) ds$$







- Projections are (n-1)-dim. distribution functions representing the line integrals of the n-dim. density distribution along each ray path.
- The set of projections under different angles α constitute a sinogram:







Reconstruction: Inversion of Radon transform:

2 angles

Overlapping backprojections

360 angles





Filtered backprojection formula:





propagation-based phase-contrast imaging



The intensity distribution on the detector is a result of wavefront distortions introduced by phase object. The Transport of Intensity Equation relates sample thickness to measured intensity distribution:



 $\delta_{\rm poly}$ and $\mu_{\rm poly}$ are polychromatic refraction and absorption coefficients, respectively





The transport-of intensity-equation (TIE) relates the edge-enhanced image at the detector (a) to the phase map of the insect (b)

tomographic reconstruction of 2-D projections yields cuts through sample (edge anhancement (a) and phase images (b,c))







3D rendering of the fly (with S. Schleede, F. Pfeiffer et al., TUM)



J. Wenz et al., Nat. Comm. 6 7568 (2015)

- Demostrates suitability for high-resolution imaging (well below 1 mm) for an alloptical source
- Photon energies for human diagnosis require 10J-class laser, long scan times.

Bone Tomography

by courtesy of S.P.D. Mangles





- Trabecular/cancellous bone intricate spongy internal structure
- Efficient distribution of mechanical stress throughout bone volume
- Very high surface area to volume ratio site of intense bone remodelling



by courtesy of S.P.D. Mangles



X-rays produced on Astra Gemini are ideal for imaging these bone samples



by courtesy of S.P.D. Mangles



Tomographic 3D reconstruction of human trabecular bone

Projection at 1 degrees





Sinogram

by courtesy of S.P.D. Mangles



Tomographic 3D reconstruction of human trabecular bone



- Voxel size: 4.8×4.8×4.8 μm
 - Limited by geometric magnification
 - -Resolution $\approx 50 \ \mu m$
- Total scan time 4 hours
 - -@ 10 Hz laser operation this image could be achieved in 3.6 seconds
- Total dose ≃ 40 mGy
 - -potential for in-vivo studies
- Data quality already suitable for studies of osteoporosis

Cole, J. M., Wood et al. Plasma Phys. Contr. Fusion, 58(1), 014008 (2016)







Betatron applications: 3. ultrafast studies

JAI

Imperial College London

Shock Imaging Setup



Wakefield driver: (12.2 +/- 0.3) J, 45 fs, 800 nm. Shock driver: (15.7 +/- 1.0) J, 1.5 ns, 800 nm. Max Intensity ≈ 2x10¹³ Wcm⁻². Material Pressure ≈ 10-20 GPa. Target magnification = 29. Variable betatron probe delay.





Ran FLASH simulations for using experimental drive spatial profile, but at normal incidence rather than 30°.

Simulated 2 ns top hat pulse whereas the experimental pulse was more complicated.

FLASH neglects material strength.

J. Wood, S.P.D. Mangles, Z. Najmudin, private communication





Thomson scattering radiation

K. Khrennikov, J. Wenz + L. Veisz group et al.







Experimental setup



driver: 1.2 J, 28 fs, 4.2×10^{19} W/cm² ($a_0=4.4$) colliding pulse: 0.3 J, 28 fs, 1.8×10^{18} W/cm² ($a_0=0.9$)

Electron beam size at interaction point decreases from 30 μm at 15 MeV to 17 μm at 45 MeV





Hard X-Rays recorded with an intensified camera







Thomson scattering $(a_0=0.9)$

Shock-front injected e-beams: Electron energy (red – averaged) X-ray energy (red – averaged; white – expected (SPECTRA 9.0))







X-ray energy matches expectations from electron energy







Thank you!

Betatron

(Rousse et al., 2004), (Kneip et al., 2010), (Albert et al., 2008), (L. M. Chen et al., 2013), (Wenz et al., 2015), (Cole et al., 2016), (Fourmaux et al., 2011), (Cipiccia et al., 2011), (Schnell et al., 2012)

Thomson/Compton

(Schwoerer, Liesfeld, Schlenvoigt, Amthor, & Sauerbrey, 2006), (Powers et al., 2013), (Sarri et al., 2014), (Ta Phuoc et al., 2012), (S. Chen et al., 2013), (Khrennikov et al., 2015)

Undulator sources

(Anania et al., 2014),(Fuchs et al., 2009),(Schlenvoigt et al., 2007)

image reference:

http://photon-

science.desy.de/research/studentsteaching/sr_and_fel_basics/fel_basics/tdr_spectral_c haracteristics/index_eng.html

http://www.interactions.org/cms/?pid=2100&image_no=DE0057

http://photon-

science.desy.de/sites/site_photonscience/content/e62/e189219/e187240/e187241/e1 87242/infoboxContent187244/f2_eng.pdf

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