

# TeV muon beams photoproduction with Hadron-Photon Colliders based on high efficiency FELs

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Trends in Free Electron Laser Physics, Erice

Conceptual study to generate  
TeV low emittance muon beams  
in highly Lorentz boosted frame  
by colliding TeV protons (LHC/FCC) and  
high brilliance keV photon beams (FEL)

# OUTLINE

- Introduction to Hadron-Photon Collider
- Kinematics of relevant reactions and description of event-generator codes to simulate the secondary beams
- Phase space and luminosity analysis in various scenarios

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- TeV protons keV photons: very asymmetrical collision

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close to  $\gamma$  of protons  $\rightarrow$

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- energy of photons in protons rest frame  
much higher than in laboratory  $\rightarrow$   
maximum efficiency above threshold  
even at keV photon energies

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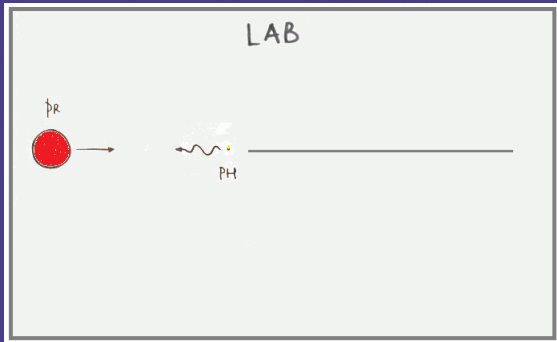
$$p + h\nu \rightarrow p' + e^- e^+$$

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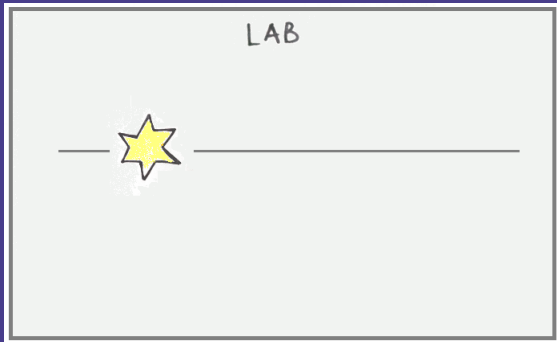
$$p + h\nu \longrightarrow \pi^+ + n$$

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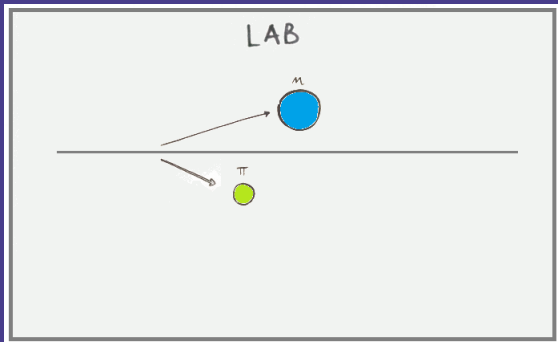
$$p + h\nu \rightarrow \pi^+ + n \text{ AND } \pi^+ \rightarrow \mu^+ + \nu_\mu$$



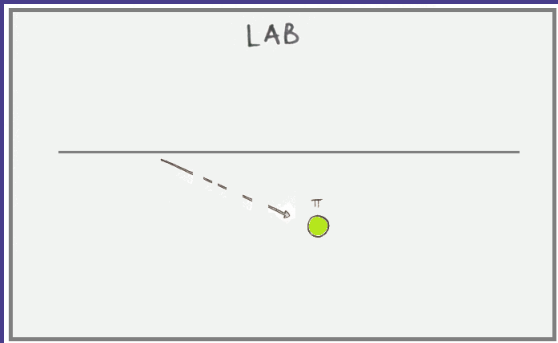
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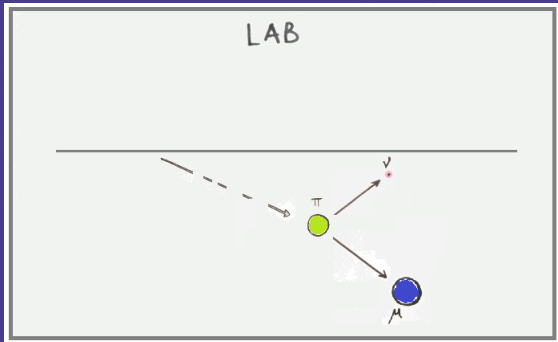
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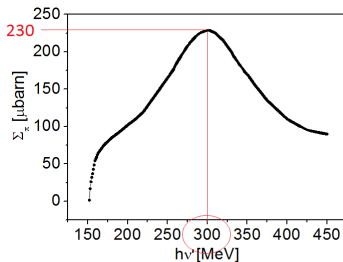






PR source	$E_p$ [TeV]	$N_p$	$\sigma_0$ [ $\mu\text{m}$ ]	PH source	$h\nu$ [keV]	$N_{ph}$
LHC	7	$2 \cdot 10^{11}$	7	FEL	20	$10^{13}$
FCC	50	$10^{11}$	1.6	FEL	3	$10^{14}$

$h\nu$  chosen to maximize Lorentz invariant total cross section of pion photo-production on protons



J. Phys. G: Nucl. Part. Phys. **18** (1992) 449–497.

Dieter Drechsel and Lothar Tiator



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photon energy seen by the proton:

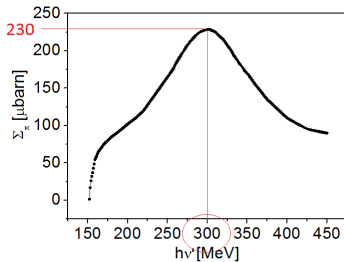
$$h\nu' = h\nu \gamma (1 - \beta \cdot \underline{e}_k)$$

photon  
energy  
in lab

proton  
gamma

photon  
direction

proton  
velocity



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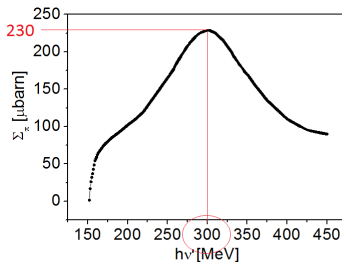


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photon energy seen by the proton:

$$h\nu' \simeq 2h\nu\gamma$$

for head-on collision  
of ultra-relativistic  
proton vs keV photon



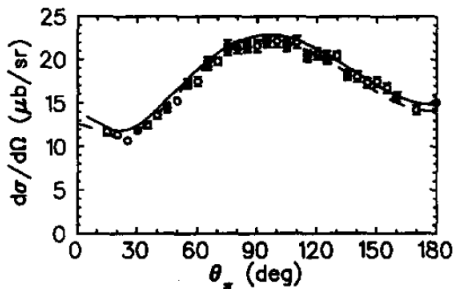
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The probability of the pion to be emitted at a certain angle is proportional to the differential cross section

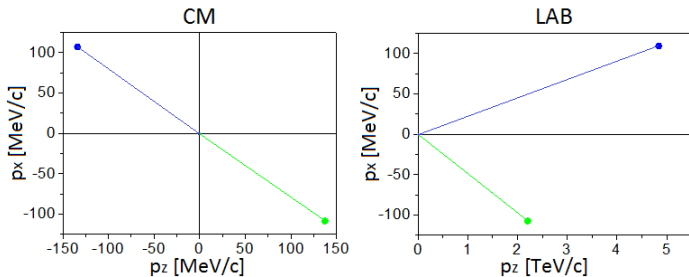
Ex:  $E_p = 7$  TeV,  $h\nu = 20$  keV



D. Drechsel *et al.*, *A unitary isobar model for pion photo- and electroproduction on the proton up to 1 GeV*, Nucl. Phys. A 645 (1999) 145-174



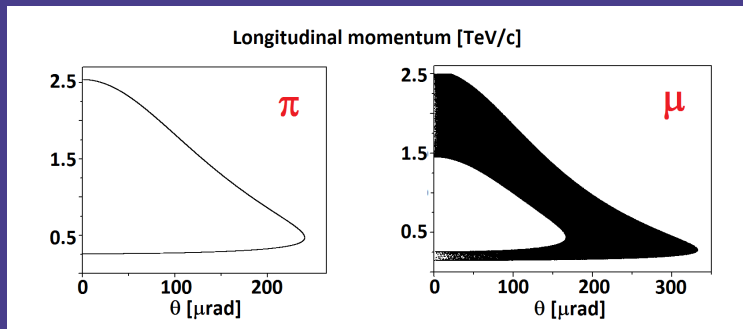
## Lorentz boost along z



$$\begin{cases} p_{x\pi} = p_{\pi}^* \sin \theta^* \cos \phi^* \\ p_{y\pi} = p_{\pi}^* \sin \theta^* \sin \phi^* \\ p_{z\pi} = \gamma_{CM} \left( \sqrt{1 - \frac{1}{\gamma_{CM}^2}} E_{\pi}^* + p_{\pi}^* \cos \theta^* \right) \end{cases}$$

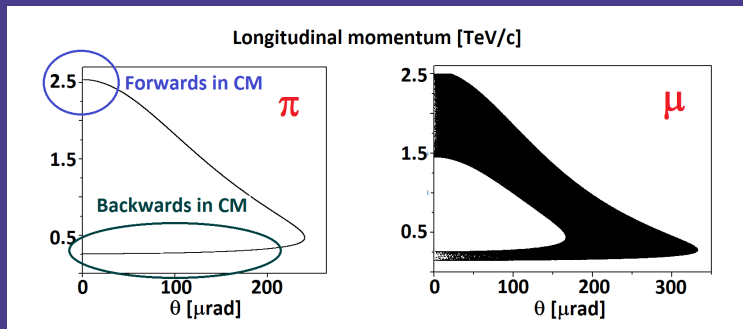
$$p + h\nu \rightarrow \pi^+ + n \text{ AND } \pi^+ \rightarrow \mu^+ + \nu_\mu$$

Event-generator code: results for  $E_p = 7$  TeV and  $h\nu = 20$  keV,  
no proton beam emittance.



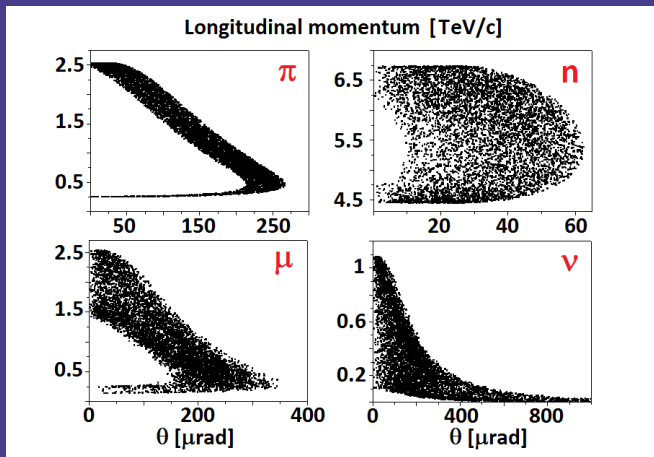
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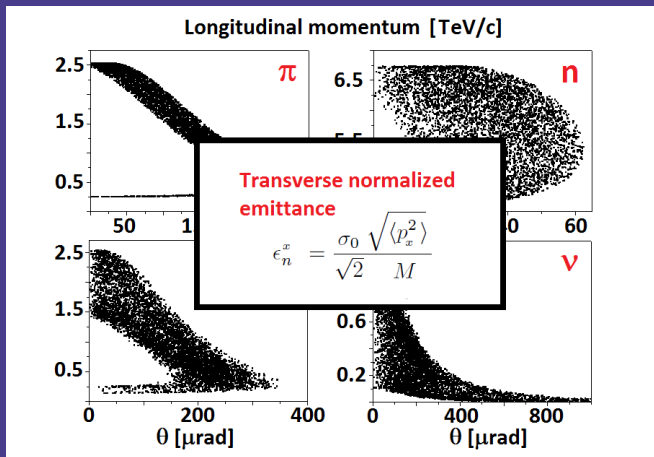
With proton emittance: increase of angular spread and dispersion in momentum for all particles







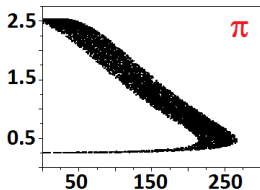
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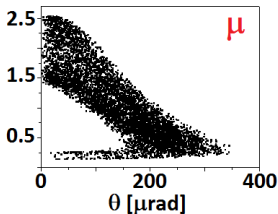


With proton emittance: enlargement of angular spread and dispersion in momentum for all particles

Longitudinal momentum [TeV/c]



$$\leftarrow \epsilon_n^x = 3.88 \text{ mm mrad}$$

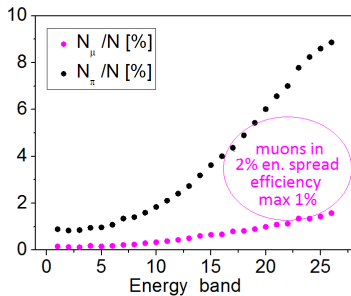
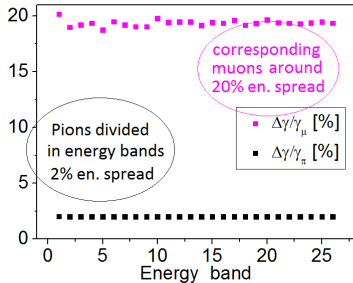


$$\leftarrow \epsilon_n^x = 4.14 \text{ mm mrad}$$



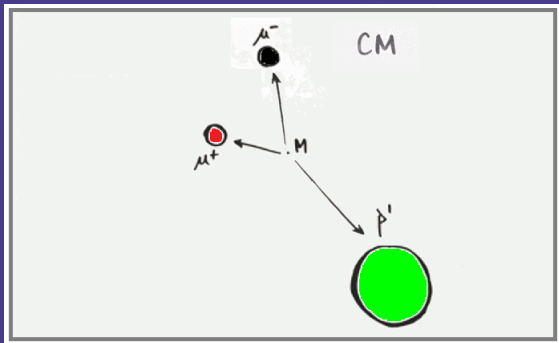
Pions have to run in a storage ring for a time sufficient to their decay into muons and the produced muons have to remain in the ring, i.e. only the muons produced within the energy acceptance of the ring survive.

Ex:  $E_p = 50 \text{ TeV}$ ,  $h\nu = 2.251 \text{ keV}$  and ring 2% en. spread acceptance.

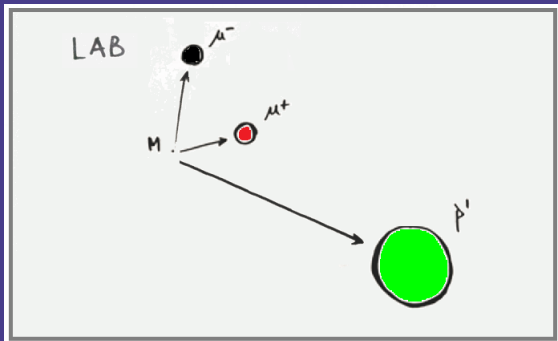


$$p + h\nu \longrightarrow p' + \mu^+ \mu^-$$

$$p + h\nu \rightarrow p' + \mu^- \mu^+$$

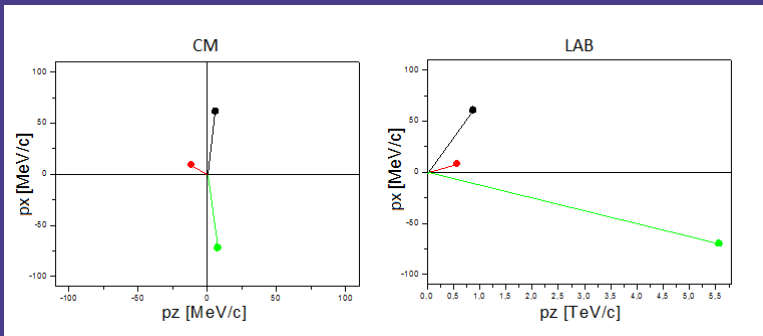


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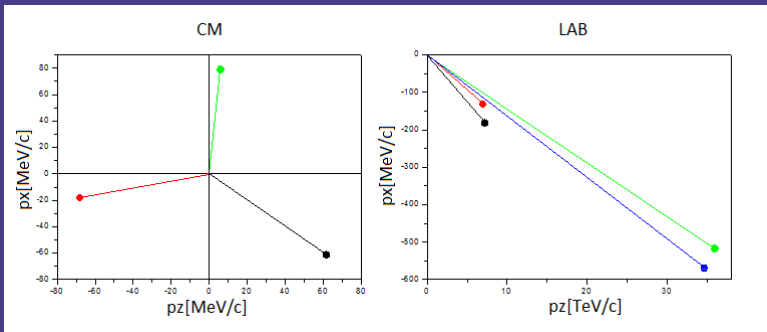


Ex: Lorentz transformation of the momenta of the product particles with boost along the  $\vec{\beta}_{CM} = \beta_{CM}\vec{e}_z$ .



$$p + h\nu \rightarrow p' + \mu^- \mu^+$$

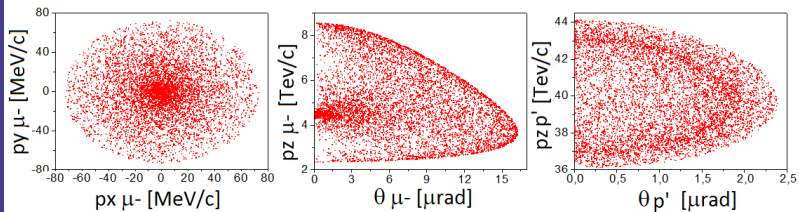
If the emittance of the incident protons is considered  $\Rightarrow$   
Lorentz transformation in the generic direction





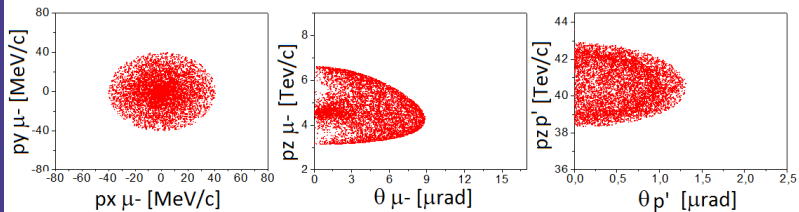
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$E_p = 50 \text{ TeV}$     $h\nu = 2.5 \text{ KeV}$



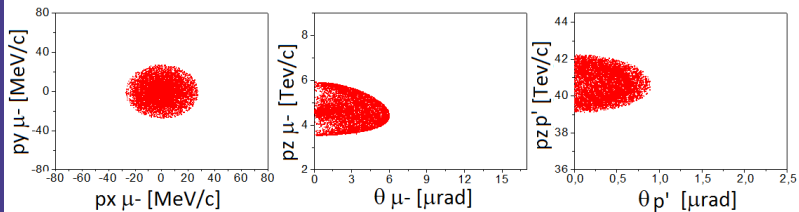
$$p + h\nu \rightarrow p' + \mu^- \mu^+$$

$E_p = 50 \text{ TeV}$     $h\nu = 2.3 \text{ KeV}$



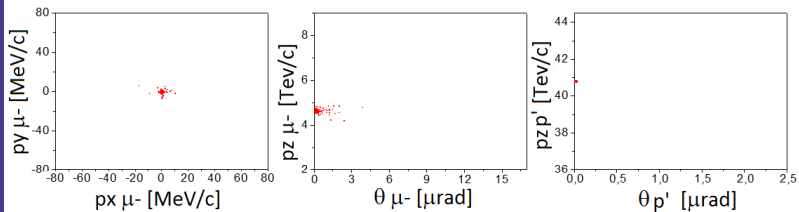
$$p + h\nu \rightarrow p' + \mu^- \mu^+$$

$E_p = 50 \text{ TeV}$     $h\nu = 2.25 \text{ KeV}$



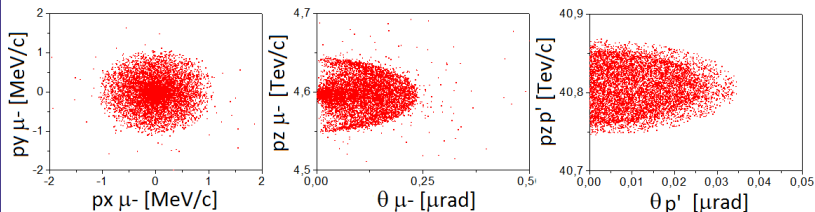
$$p + h\nu \rightarrow p' + \mu^- \mu^+$$

$E_p = 50 \text{ TeV}$     $h\nu = 2.2053 \text{ KeV}$





$E_p = 50 \text{ TeV}$     $h\nu = 2.2053 \text{ KeV}$



Threshold photon energy in proton rest frame for pair production:

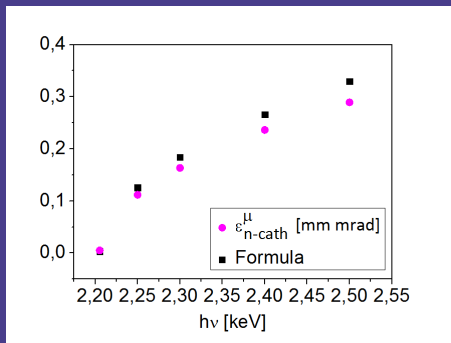
$$h\nu'^{th} = \frac{(2M_\mu + M_p)^2 - M_p^2}{2M_p} = 235 \text{ MeV}$$

$$\text{Ex: } E_p = 50 \text{ TeV} \Rightarrow h\nu = h\nu'^{th} / (2\gamma) = 2.2053 \text{ keV}$$



$E_p = 50$  TeV, no proton beam emittance,  $\Delta_p = 4h\nu\gamma/M_p$

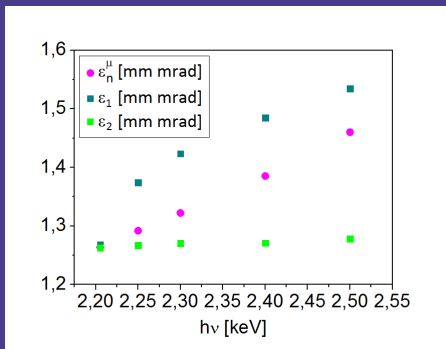
$$\epsilon_{n-cath}^\mu \leq \frac{1}{\sqrt{3}} \frac{\sigma_0}{\sqrt{2}} \sqrt{\frac{M_p^2}{4M_\mu^2} \left( \sqrt{1 + \Delta_p} - 1 \right)^2 - 1}$$





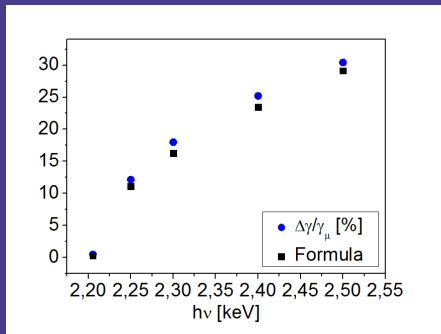
$E_p = 50$  TeV, with incoming proton beam emittance

$$\epsilon_2 = \epsilon_{n-cath}^\mu + \epsilon_n^{p'} < \epsilon_n^\mu < \epsilon_1 = \sqrt{(\epsilon_{n-cath}^\mu)^2 + (\epsilon_n^{p'})^2}$$





$$\frac{\Delta\gamma}{\gamma_\mu} = \frac{1}{\sqrt{3}} \sqrt{\frac{M_p^2}{4M_\mu^2} \left( \sqrt{1 + \Delta_p} - 1 \right)^2 - 1}$$



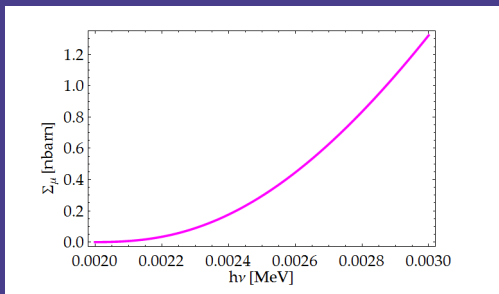




Total cross section close to muon pair threshold: Racah formula

$$\sigma \simeq \alpha Z^2 r_0^2 \frac{2\pi}{3} \left( \frac{k-2}{k} \right)^3 \left( 1 + \frac{\epsilon}{2} \right)$$

where  $k = h\nu' / M_\mu$ ,  $\epsilon = (k-2)/(k+2)$ ,  
 $r_0 = r_e (M_e / M_\mu) = 2.82 \cdot 10^{-15} (0.511 / 105.65)$  m,  $\alpha = 1/137$  and  $Z = 1$ .



$$p + h\nu \rightarrow p' + \mu^- \mu^+$$

IDEA: CLOSE TO THRESHOLD TO MINIMIZE ENERGY SPREAD  
BUT VERY HARD TO FIND THE CORRECT  
DIFFERENTIAL CROSS SECTIONS AT SUCH LOW ENERGIES  
UP TO NOW: FLAT DIFFERENTIAL CROSS SECTION

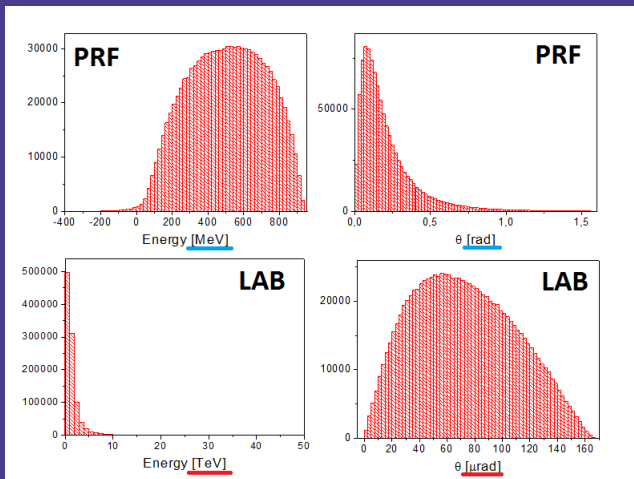
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NEXT IDEA: MOVE FROM THRESHOLD SINCE TOTAL  
CROSS SECTION HIGHER AND THANKS TO  
A NEW CODE BASED ON GEANT4 APPROACH  
(CALCULATIONS IN PRF + LORENTZ TRANS TO LAB)  
TAKE ADVANTAGE OF THE MUONS' ENERGY DISTRIBUTION

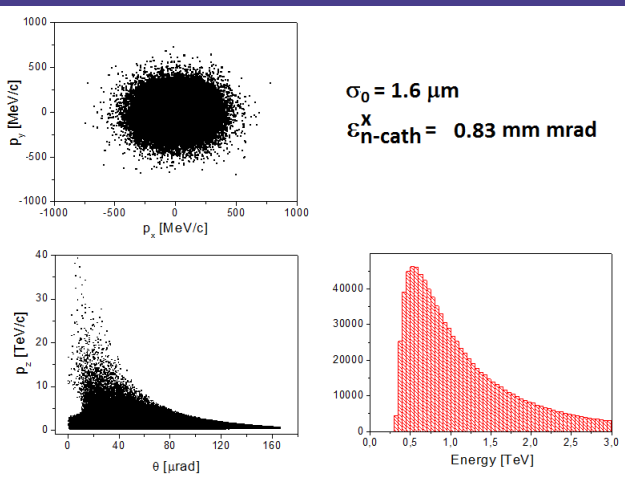


Ex:  $E_p = 50 \text{ TeV}$ ,  $h\nu = 10 \text{ keV}$  ( $h\nu' = 1.066 \text{ GeV}$ )



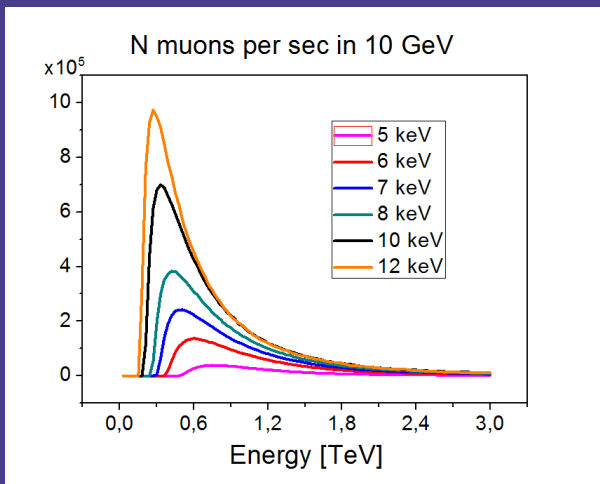
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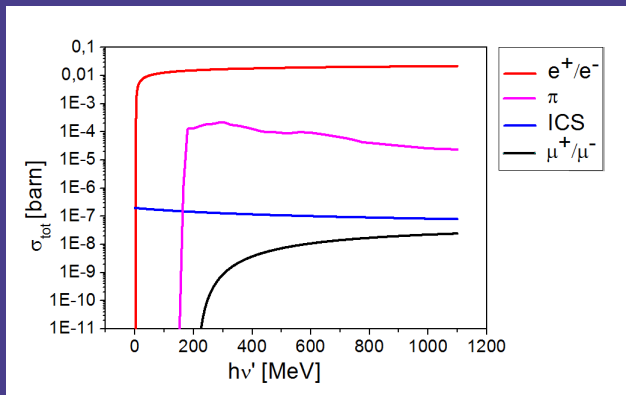
Ex:  $E_p = 50$  TeV and various photon energies



# LUMINOSITY AND FLUX

$$\mathcal{N} = \mathcal{L} \cdot \sigma_{tot}$$

Total cross section of the relevant reactions



# LUMINOSITY AND FLUX

$$\cdot \text{ FCC case: } \mathcal{L} = \frac{N_p N_{phr}}{4\pi\sigma_0^2} = \frac{10^{11} \cdot 10^{14} \cdot 10^7}{4 \cdot \pi \cdot 1.6^2 \cdot 10^{-12}} = 3.1 \cdot 10^{38} \text{ cm}^{-2}\text{s}^{-1}$$



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$E_p$ [TeV]	$h\nu$ [keV]	$\mathcal{N}_\pi$ [s $^{-1}$ ]	$\mathcal{N}_{\mu^-/\mu^+}$ [s $^{-1}$ ]	$\mathcal{N}_{e^-/e^+}$ [s $^{-1}$ ]
50	1.43 ( $\pi^{th}$ )	$1.86 \cdot 10^{10}$	0	$4.5 \cdot 10^{12}$
50	2.2053 ( $\mu^{th}$ )	$3.72 \cdot 10^{10}$	$1.25 \cdot 10^4$	$5 \cdot 10^{12}$
50	3	$6.5 \cdot 10^{10}$	$4 \cdot 10^5$	$5.4 \cdot 10^{12}$
50	5	$2.48 \cdot 10^{10}$	$1.2 \cdot 10^6$	$5.6 \cdot 10^{12}$
50	10	$8.6 \cdot 10^9$	$4.8 \cdot 10^6$	$6.5 \cdot 10^{12}$
50	12	$6.8 \cdot 10^9$	$5.6 \cdot 10^6$	$6.8 \cdot 10^{12}$

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high energy, high collimation, low emittance

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high energy, high collimation, low emittance
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- Direct muon pair production:  
easier to be implemented even if total cross section is low, differential cross section gives a narrow peak + idea to gain a factor  $\sim 50 - 500$  is to use lead ions

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easier to be implemented even if total cross section is low, differential cross section gives a narrow peak + idea to gain a factor  $\sim 50 - 500$  is to use lead ions
- Important to evaluate the impact on the proton beam of electron/positron pair production and ICS: need to implement a simulation taking into account all the reactions at the same time.

# Thank you for your attention!

-  L. Serafini, C. Curatolo and V. Petrillo, *Low emittance pion beams generation from bright photons and relativistic protons*, <http://arxiv.org/pdf/1507.06626.pdf> (2015)
-  C. Curatolo, *PhD Thesis: High brilliance photon pulses interacting with relativistic electron and proton beams* (2016)
-  J. W. Motz, H. A. Olsen and H. W. Koch, *Pair production by photons*, *Rev. Mod. Phys.* 41 (1969)
-  D. Drechsel and L. Tiator, *Threshold pion photoproduction on nucleons*, *Nucl. Part. Phys.* 18 (1992)
-  C. Emma, K. Fang, J. Wu and C. Pellegrini, *High efficiency, multiterawatt x-ray free electron lasers*, *Phys. Rev. AB* 19 (2016)
-  H. Burkhardt, S. R. Kelner and R. P. Kokoulin, *Monte Carlo generator for muon pair production*, *Clic-note-511* (2002)