INFN

Istituto Nazionale di Fisica Nucleare

Laboratori Nazionali di Frascati

Hybrid beam driven PWFA scheme for high brilliance beams acceleration

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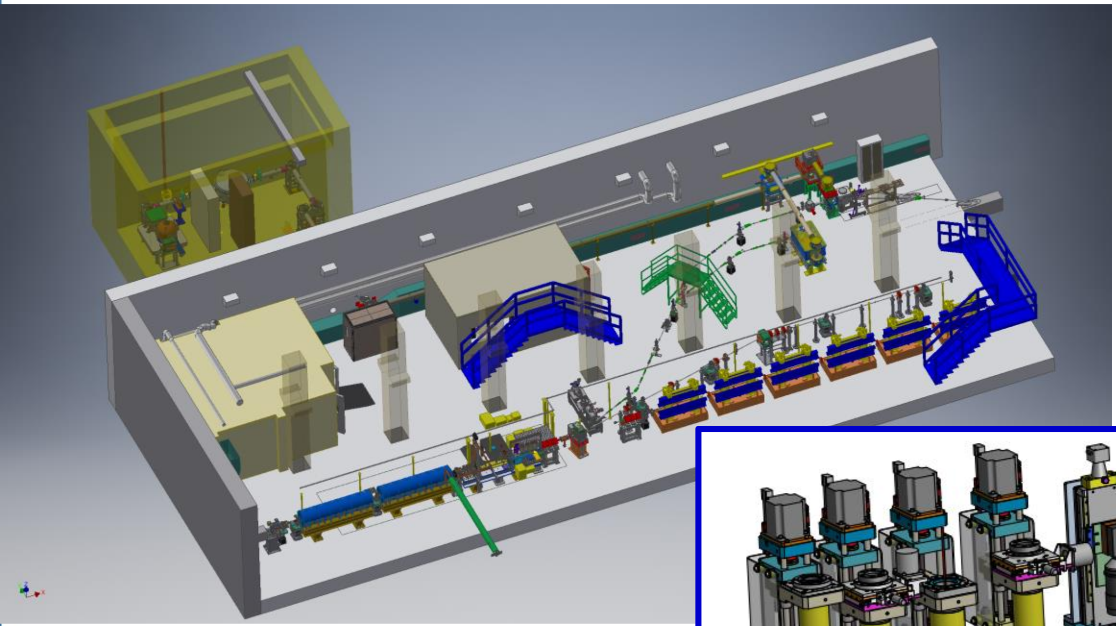
on behalf of SPARC_LAB collaboration

SPARC LAB

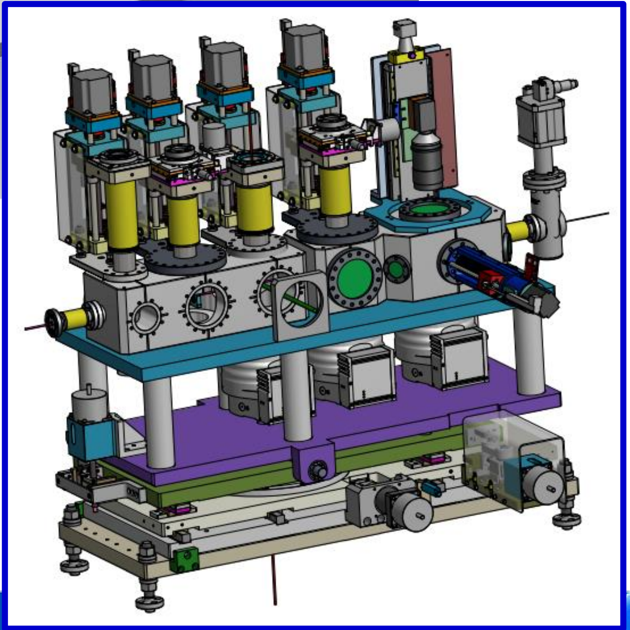
Trends in Free Electron Laser Physics, Erice 2016-05-20

Plasma driven Free Electron Laser @SPARC_LAB?



A plasma driven Free Electron Laser experiment could be performed at SPARC_LAB

- Beam driven plasma accelerating structure experiments scheduled

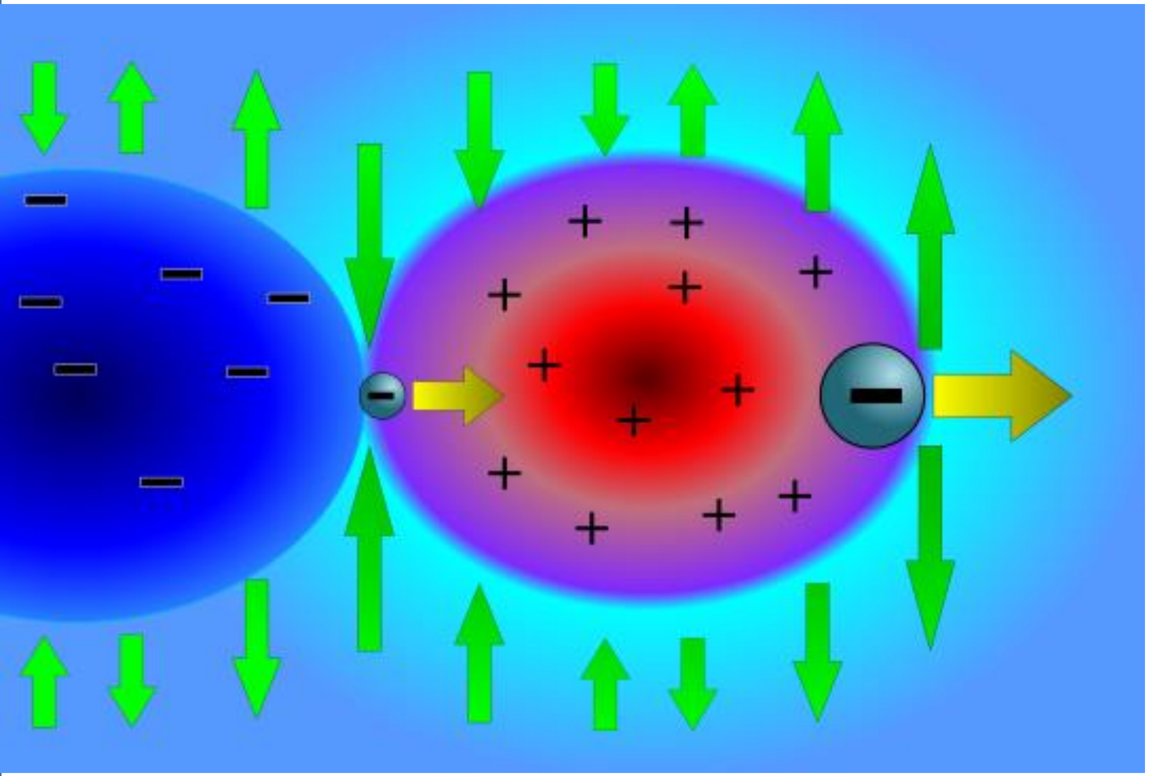


Plasma chamber already installed and under test

Beam driven

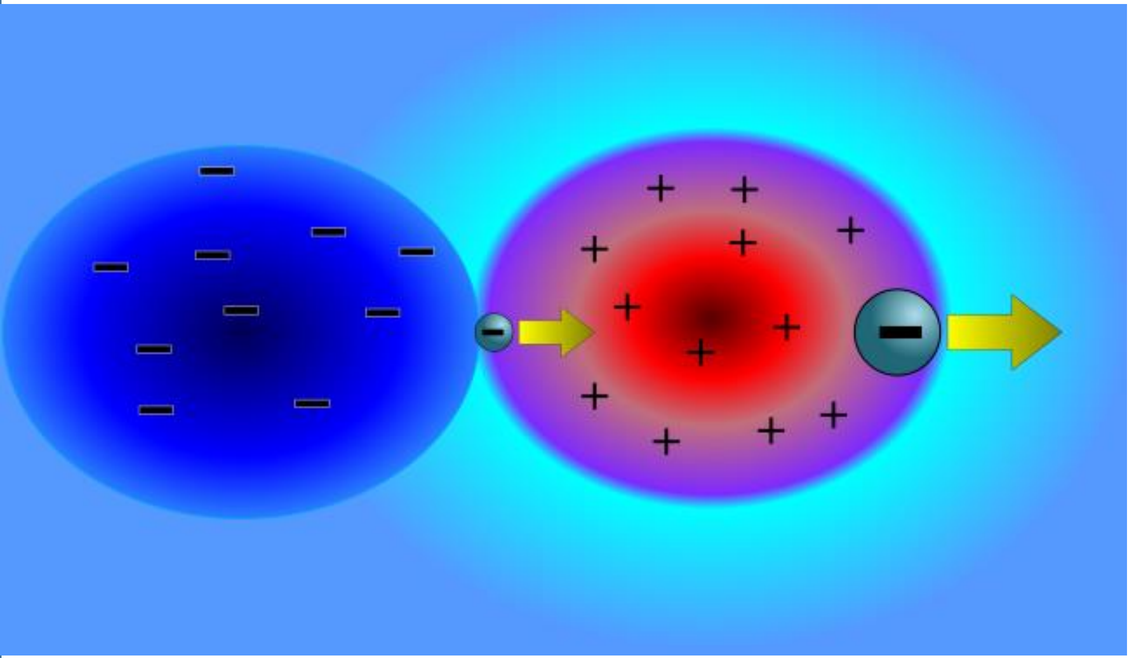
- Beam driven plasma wakefield acceleration scheme
- 1 Driver+1 Witness: driver loosing energy inside plasma, accelerating the witness
- **GOAL: High quality accelerated bunches, low energy spread and emittance at the exit**

Beam Driven Plasma Wakefield Acceleration

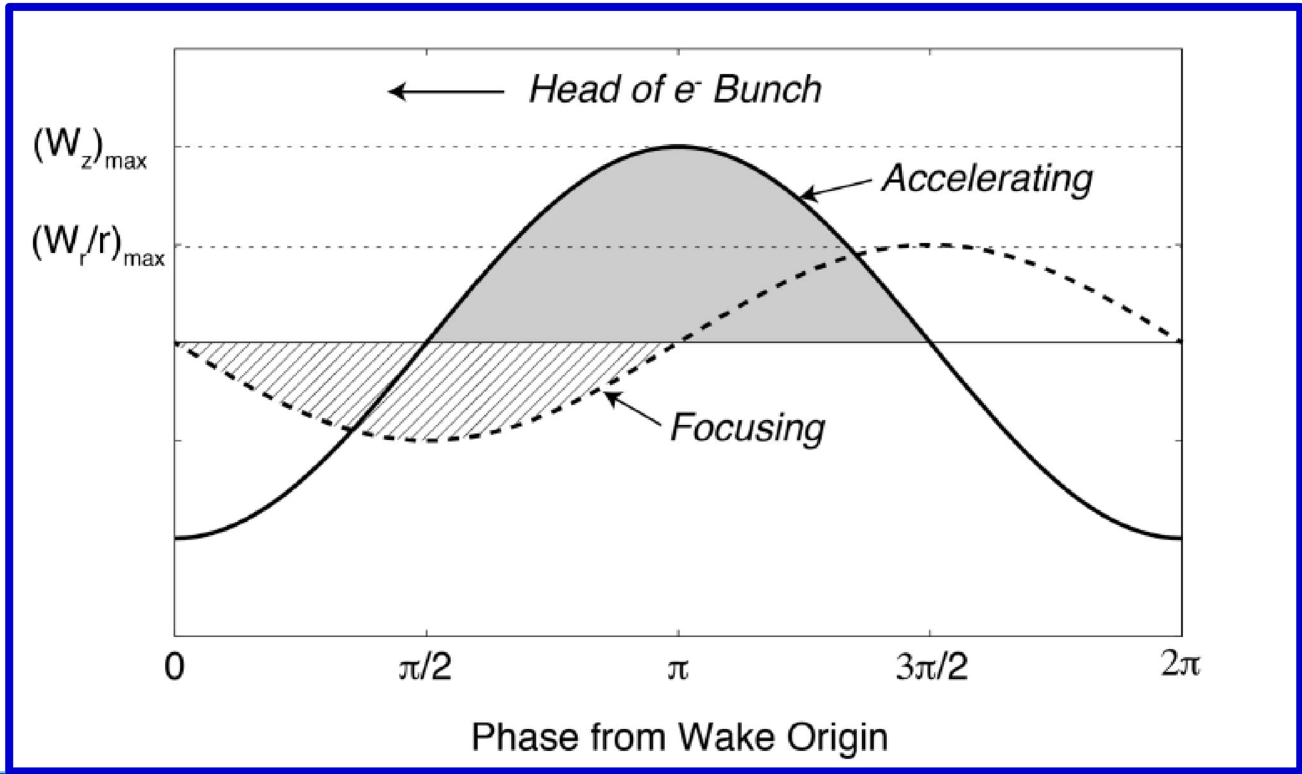


- In plasma wakefield acceleration, a pulse looses energy inside plasma, exciting a quasi-periodic wave that tranfers energy to a beam
- The pulse generating the wave can be both a laser or a beam
- In beam driven scheme, two bunches are injected inside plasma
- The first bunch looses energy and the second one gains energy
- We can roughly regroup the kind of plasma wake excitation in two regimes:
  - Linear regime
  - Blow-out or non-linear regime

Linear regime



- $\alpha = \frac{n_b}{n_0} < 1$
- We are inside the limit of small oscillations
- The plasma behaviour is harmonic



- In the region very far behind the driver, the longitudinal and transverse field have sinusoidal behaviour in the longitudinal coordinate
- The shape of the fields in transverse coordinate depends mostly on the shape of the beam that generates the wakefield

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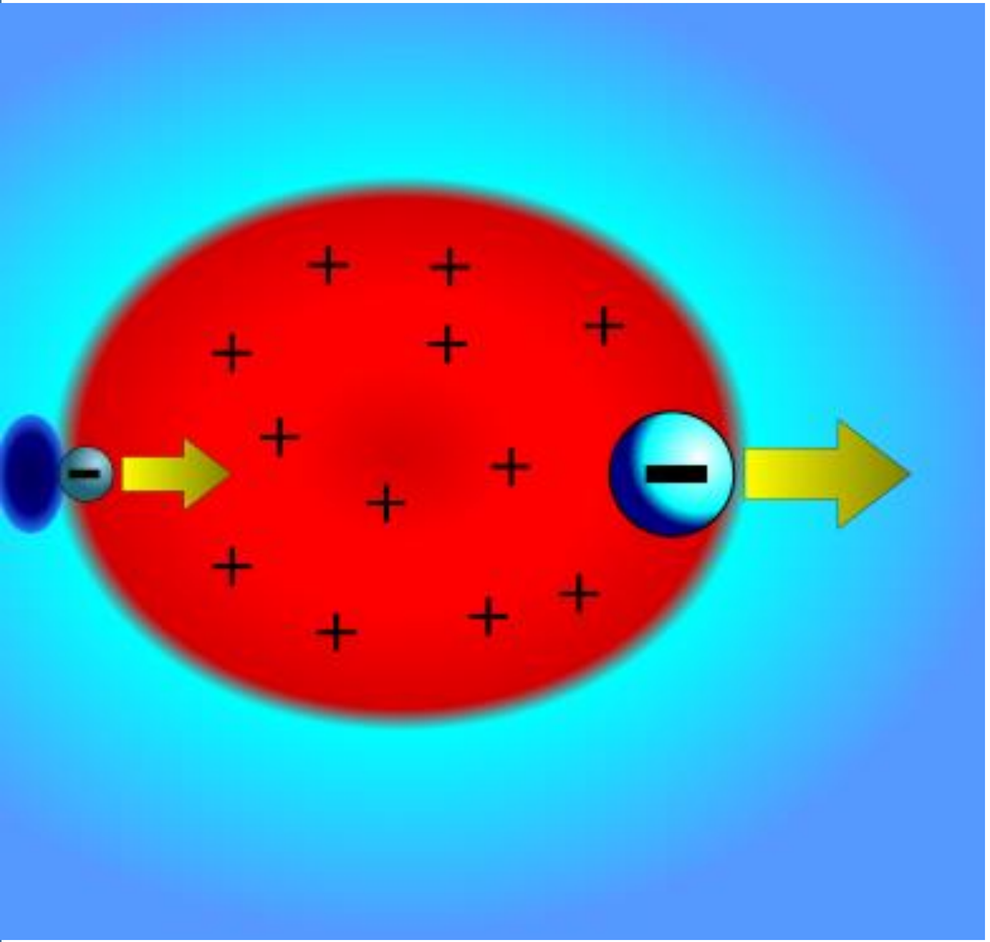
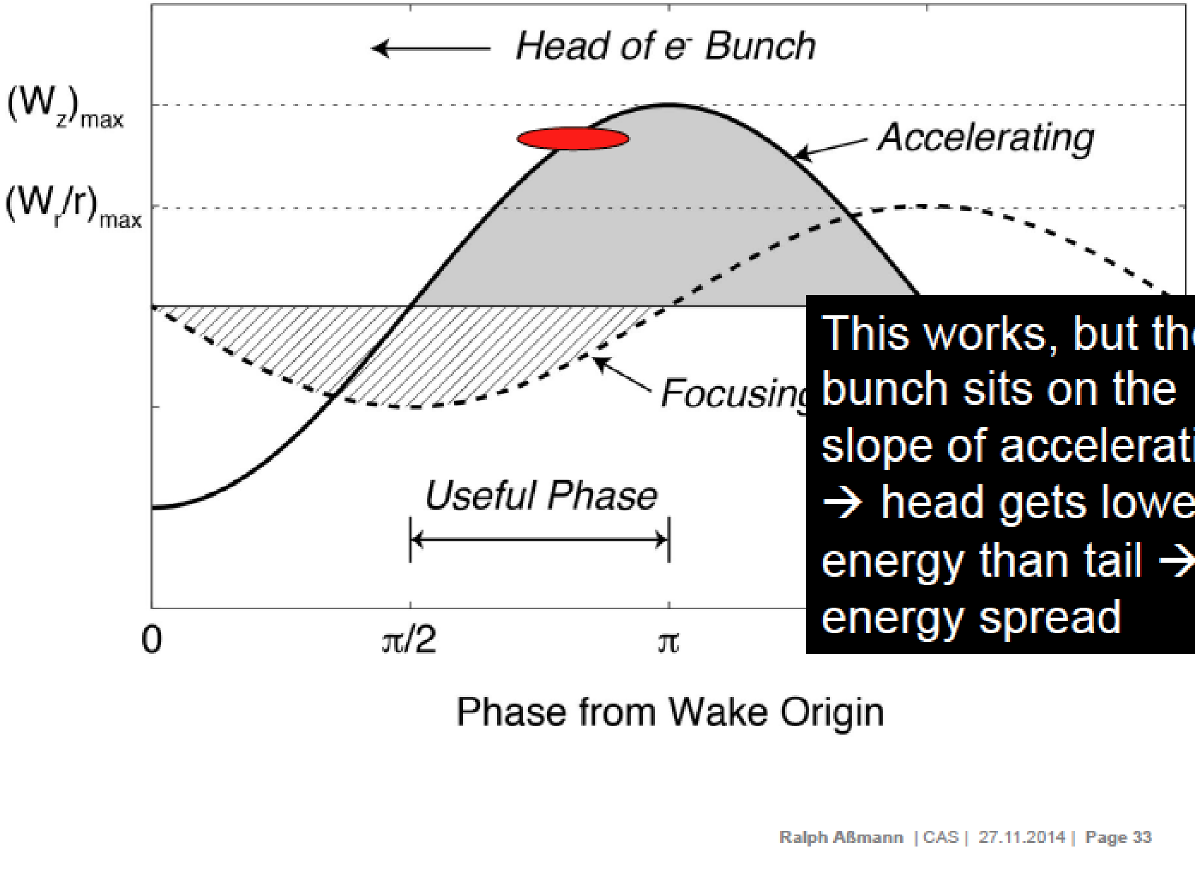
Blow-out regime



- An high density bunch ($\alpha \gg 1$) creates a region inside plasma with total absence of electrons
- Trajectories of electrons are no more similiar to harmonic oscillators
- Trajectory intersections occur and behind the bubble it’s created a region with very high local electron density
  [Lu, Wei, et al. "Nonlinear theory for relativistic plasma wakefields in the blowout regime." *Physical review letters* 96.16 (2006): 165002.]
- Inside the bubble the longitudinal field is a function of longitudinal coordinate and constant over the transverse coordinate
  [**Lotov, K. V.** "Blowout regimes of plasma wakefield acceleration." *Physical Review E* 69.4 (2004): 046405.]
- The transverse fields have a linear dependency on the transverse position

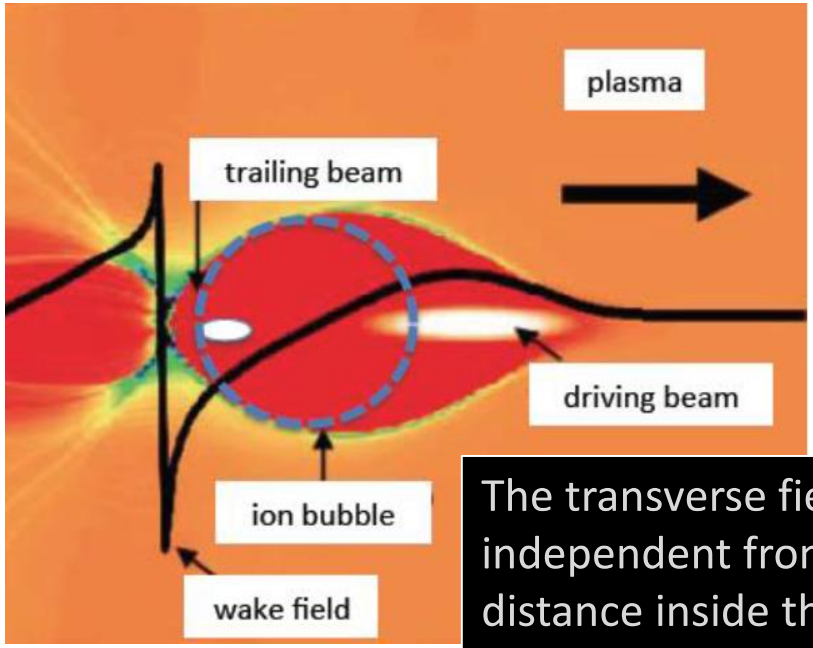
Witness injection

Linear



This works, but the bunch sits on the slope of acceleration → head gets lower energy than tail → energy spread

Non Linear



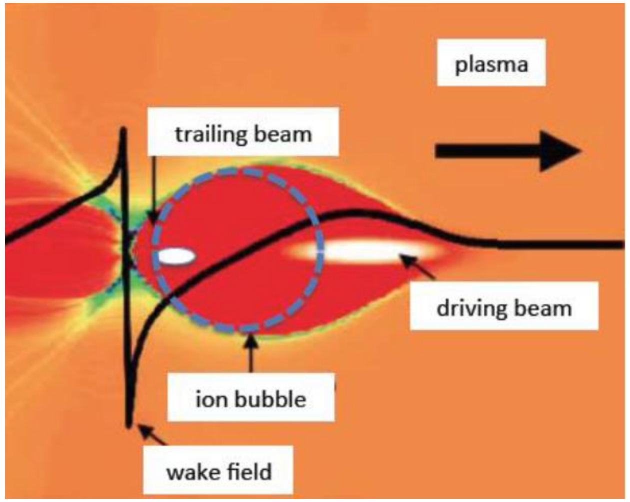
The transverse field is independent from the injection distance inside the bubble → in order to get the maximum energy the injection is placed near to the end of the bubble → head gets lower energy than tail → energy spread

Minimization of energy spread

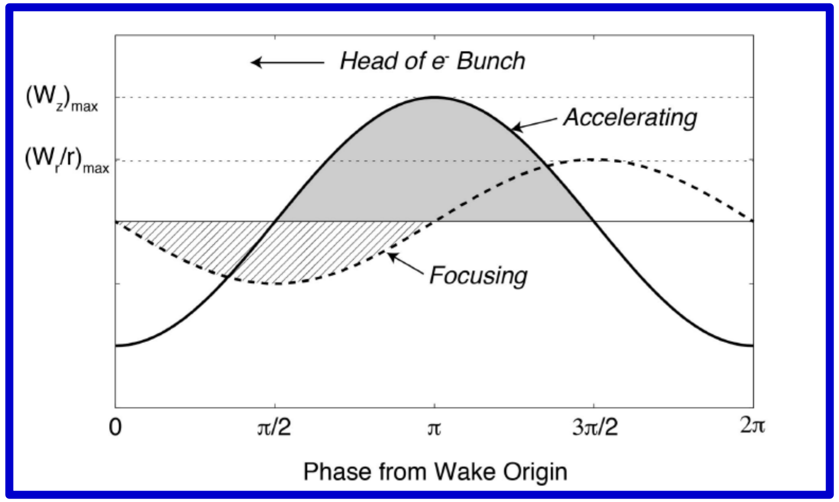
- The energy spread obtained by a longitudinal sinusoidal wake can be written in the form:

$$\sigma_E = 1 - \cos(k_p \sigma_z) + \tan\varphi \sin(k_p \sigma_z)$$

- With a bunch of finite length σ_z, the only simple way to minimize the energy spread is to have the angle $\varphi = 0$ respect the frame where the coordinate ξ corresponds to the peak of the accelerating field



In the bubble regime it is not possible to minimize the energy spread, because of the spike localized at the end of the bubble



In linear regime the injection at the peak is forbidden by the 0 focusing field

Further, in linear regime, the longitudinal field is not constant over the transverse dimension

1-D linear theory

KATSOULEAS, T, et al. "BEAM LOADING IN PLASMA ACCELERATORS" Particle Accelerators, 1987, Vol. 22, pp. 81-99

If we suppose to have a beam with a charge distribution over space that is separable over its longitudinal and transverse components it is possible to write the fields as:

$$E_z = -2qk_p^2 K_0(k_p r)\theta(t - z/c)\cos\omega_p(t - z/c)$$

$$W_\perp = (E_r - B_\theta) = \int dz \frac{\partial W_\parallel}{\partial r} = -2qk_p^2 K_1(k_p r)\theta(t - z/c)\sin\omega_p(t - z/c)$$

$$W_\parallel = E_z(r, \zeta) = Z'(\zeta)R(r)$$

$$W_\perp = Z(\zeta)R'(r)$$

$$Z'(\zeta) = -4\pi\int_\infty^\zeta d\zeta'\rho_\parallel(\zeta')\cos k_p(\zeta - \zeta')$$

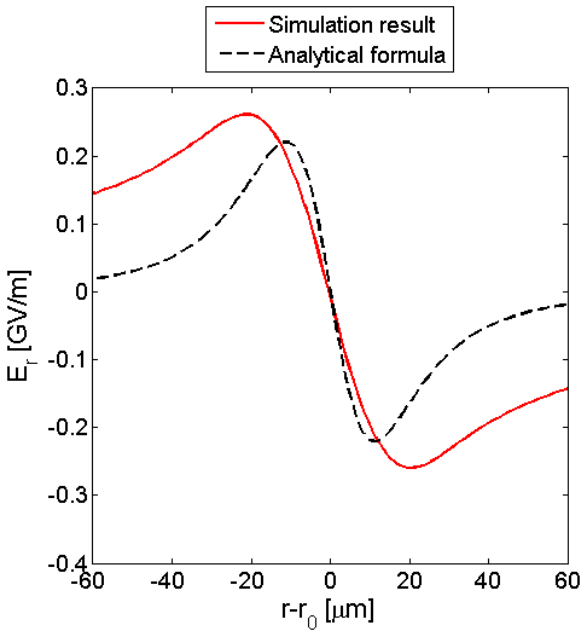
$$R(r) = \frac{k_p^2}{2\pi}\int_0^{2\pi} d\theta\int_0^\infty r'\,dr'\rho_\perp(r', \theta)K_0(k_p|\mathbf{r} - \mathbf{r}'|)$$

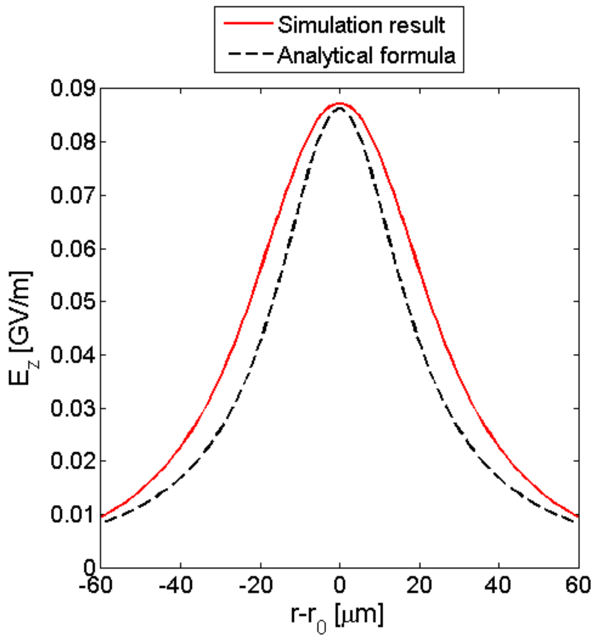
Field scaling laws in linear regime

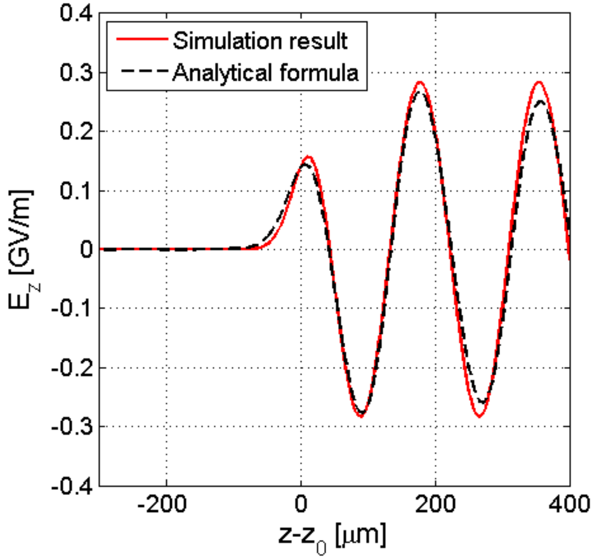
These expressions for the fields were obtained with some numerical considerations over the range $0.05 \le k_p\sigma_r \le 1$

$$E_z(r,\xi) = \frac{\sqrt{\pi}}{2}\alpha\left(\frac{c^2 m_e}{e}\right)k_p^2\sigma_z \frac{0.39(k_p\sigma_r)^2}{(k_p r)^2+0.79(k_p\sigma_r)^{0.59}} Re\left\{e^{ik_p\xi}\left[1-erf\left(\frac{\xi}{\sqrt{2}\sigma_z}+i\frac{k_p\sigma_z}{\sqrt{2}}\right)\right]\right\}$$

$$E_r(r,\xi) - \beta c B_\vartheta(r,\xi) = \sqrt{\pi}\alpha\left(\frac{c^2 m_e}{e}\right)k_p\sigma_z e^{-\frac{k_p^2\sigma_z^2}{2}} \frac{0.39(k_p\sigma_r)^2 k_p^2 r}{\left[(k_p r)^2+0.79(k_p\sigma_r)^{0.59}\right]^2} Im\left\{e^{ik_p\xi}\left[1-erf\left(\frac{\xi}{\sqrt{2}\sigma_z}+i\frac{k_p\sigma_z}{\sqrt{2}}\right)\right]\right\}$$







Transverse contribution to energy spread can be evaluated as

$$\sigma_E = \frac{k_p^{7/5}\sigma_{r_w}^2}{0.79\sigma_{d_w}^{3/5}}$$

Minimization of energy spread: conclusions

$$\sigma_E = 1 - \cos(k_p\sigma_z) + \tan\varphi \sin(k_p\sigma_z)$$

$$\sigma_E = \frac{k_p^{7/5}\sigma_{r_w}^2}{0.79\sigma_{d_w}^{3/5}}$$

The minimum energy spread occurs by injecting a very short bunch ($k_p\sigma_z \approx 0$) injected on crest ($\varphi = 0$) with a minimal spot size ($k_p\sigma_r \approx 0$)

For such a bunch, we require a focusing field and a matching

Transverse matching of witness

$$\sigma_r'' + \frac{\gamma'}{\gamma}\sigma_r' - \frac{1}{\sigma_r}\frac{\langle rF_{r,ext}\rangle}{\beta cp} = \frac{\varepsilon_n^2}{\gamma^2\sigma_r^3}$$

Linear

$$E_r(r,\xi) - \beta c B_\vartheta(r,\xi) = \sqrt{\pi}\alpha\left(\frac{c^2 m_e}{e}\right)k_p\sigma_z e^{-\frac{k_p^2\sigma_z^2}{2}} \times$$
$$\times \frac{0.39(k_p\sigma_r)^2 k_p^2 r}{\left[(k_p r)^2 + 0.79(k_p\sigma_r)^{0.59}\right]^2} \times$$
$$\times Im\left\{e^{ik_p\xi}\left[1 - erf\left(\frac{\xi}{\sqrt{2}\sigma_z} + i\frac{k_p\sigma_z}{\sqrt{2}}\right)\right]\right\}$$

Direct dependence of transverse matching from injection distance (0 in crest)

In both cases fast oscillations of envelope of the driver could lead to strong variations of the fields over witness and increase emittance

Non Linear

Cylindrical ion column

$$E_r = \frac{en_0}{2\varepsilon_0}$$

$$\sigma_r = \sqrt[4]{\frac{2}{\gamma}}\sqrt{\frac{\varepsilon_n}{k_p}}$$

Transverse matching of driver

Linear

$$E_r(r,0) - \beta c B_\vartheta(r,0) = \sqrt{\pi}\alpha\left(\frac{c^2 m_e}{e}\right)k_p\sigma_z e^{-\frac{k_p^2\sigma_z^2}{2}} \times$$
$$\times \frac{0.39(k_p\sigma_r)^2 k_p^2 r}{\left[(k_p r)^2 + 0.79(k_p\sigma_r)^{0.59}\right]^2} erf\left(\frac{k_p\sigma_z}{\sqrt{2}}\right)$$

Taylor expansion

$$\sigma_r = \left(\frac{\varepsilon_n^2}{t\alpha\gamma}\right)^{\frac{5}{24}} k_p^{-\frac{7}{12}}$$

Non Linear

Cylindrical ion column

$$E_r = \frac{e n_0}{2\varepsilon_0}$$

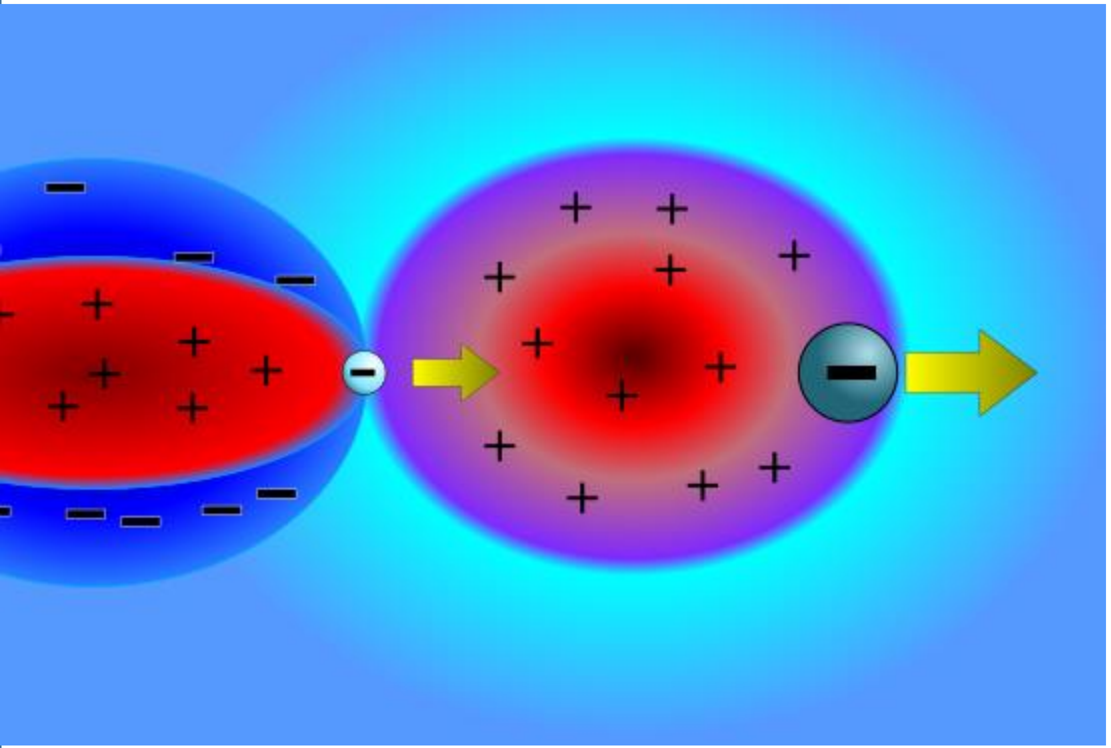
$$\sigma_r = \sqrt[4]{\frac{2}{\gamma}}\sqrt{\frac{\varepsilon_n}{k_p}}$$

Matching condition of a driver in non linear regime are calculated supposing

- Cylindrical simmetry of the problem
- Complete blow-out regime ($\alpha \gg 1$)
- The bubble is completely formed in the very head of the bunch ($k_p\sigma_z \approx 1$)

[**Rosenzweig, J. B., et al.** "Acceleration and focusing of electrons in two-dimensional nonlinear plasma wake fields." *Physical Review A* 44.10 (1991): R6189.]

New concept: hybrid scheme



Within the new hybrid scheme

- The driver generates a linear field
- The witness is:
  - Short ($k_p\sigma_z \ll 1$)
  - Ultra dense ($k_p\sigma_r \ll 1, \alpha \gg 1$)
  - Injected in the region of the crest
  - Mainly focused by the wakefields generated by the witness itself

The longitudinal field crest region

$$Z'(\zeta) = -4\pi \int_{\infty}^{\zeta} d\zeta' \rho_{\|}(\zeta') \cos k_p(\zeta - \zeta')$$

[**Lu, W., et al.** "Limits of linear plasma wakefield theory for electron or positron beams." *Physics of Plasmas (1994-present)* 12.6 (2005): 063101.]

Very far behind the driver ($\xi \ll -\sigma_z$)

$$E_z(\xi,0) = \sqrt{\pi}\alpha\left(\frac{c^2 m_e}{e}\right) k_p^2 \sigma_z e^{-\frac{k_p^2\sigma_z^2}{2}} R(0) \cos k_p\xi$$

$$Z(\xi) = \sqrt{\pi}\alpha\left(\frac{c^2 m_e}{e}\right) k_p \sigma_z e^{-\frac{k_p^2\sigma_z^2}{2}} \sin k_p\xi$$

$$B_\vartheta(\xi,r) \approx 0$$

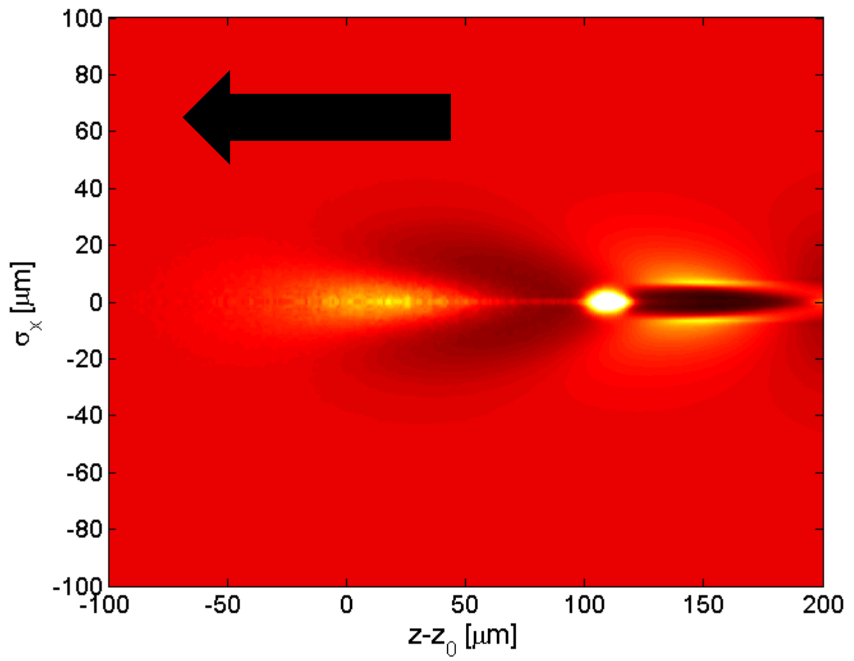
$$E_r(\xi,r) = \sqrt{\pi}\alpha\left(\frac{c^2 m_e}{e}\right) k_p \sigma_z e^{-\frac{k_p^2\sigma_z^2}{2}} R(r) \sin k_p\xi$$

$$\nabla\tilde{E} = \frac{Z(\xi)R'(r)}{r} + Z(\xi)R''(r) + Z''(\xi)R(r)$$

$$\frac{\rho}{\varepsilon_0} = 0$$

$$n_1 = n_0$$

In the crest region the plasma is locally neutral



For the transverse matching at first order, the injection of witness on crest is equivalent to the injection inside neutral plasma

Matching conditions for short ultra dense bunch

One of the hypotesis to calculate the matching of driver was that the bubble is completely formed in the very head of the bunch

Cylindrical ion column

$$E_r = \frac{e n_0}{2\varepsilon_0} \Rightarrow \sigma_r = \sqrt[4]{\frac{2}{\gamma}}\sqrt{\frac{\varepsilon_n}{k_p}}$$

Half density ion column

$$E_r = \frac{e n_0}{4\varepsilon_0} \Rightarrow \sigma_r = \sqrt[4]{\frac{1}{\gamma}}\sqrt{\frac{2\varepsilon_n}{k_p}}$$

New hypotesis

- Short ($k_p\sigma_z \ll 1$)
- Ultra dense ($k_p\sigma_r \ll 1,\ \alpha \gg 1$)
- Ultra relativistic ($\beta \approx 1$)

The local electron plasma density n_1 is:

- Almost constant inside bunch
- 0 behind the bunch
- Equal to n_0 before the bunch

We conclude that in the region inside the bunch, the local plasma density $n_1 = n_0/2$

The matching conditions of witness are completely independent from driver

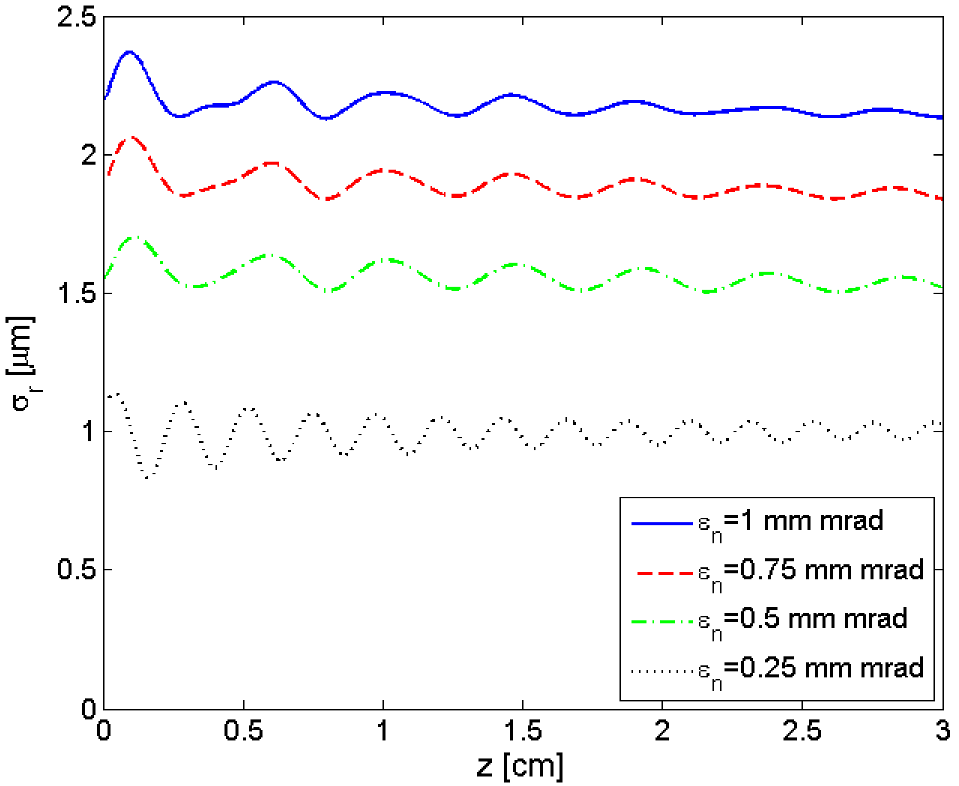
Simulation parameters 1° case

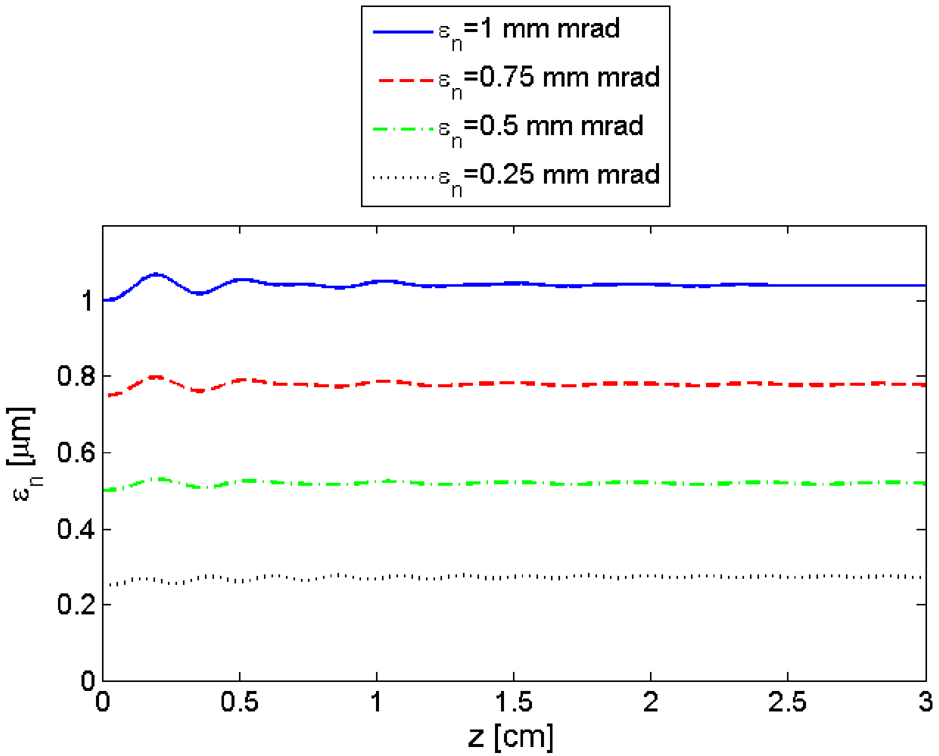
- Plasma density $n_0 = 2 \cdot 10^{16}$

- **Driver:**
  - Energy $\gamma = 240$
  - Charge $Q = 120pC$
  - Length $\sigma_z = \frac{1}{k_p}$
  - $\alpha = 0.5$
  - Emittance $\varepsilon_n = 10\ mm\ mrad$

- **Witness:**
  - Energy $\gamma = 240$
  - Length $\sigma_z = 5.31\mu m$
  - $\alpha = 25$
  - Emittance scan $0.25 \leq \varepsilon_n \leq 1\ mm\ mrad$
  - Spot size $\sigma_r = \sqrt[4]{\frac{1}{\gamma}}\sqrt{\frac{2\varepsilon_n}{k_p}}$

Matching conditions check





The matching condition we used gave a good result in a simulation scan at various emittances

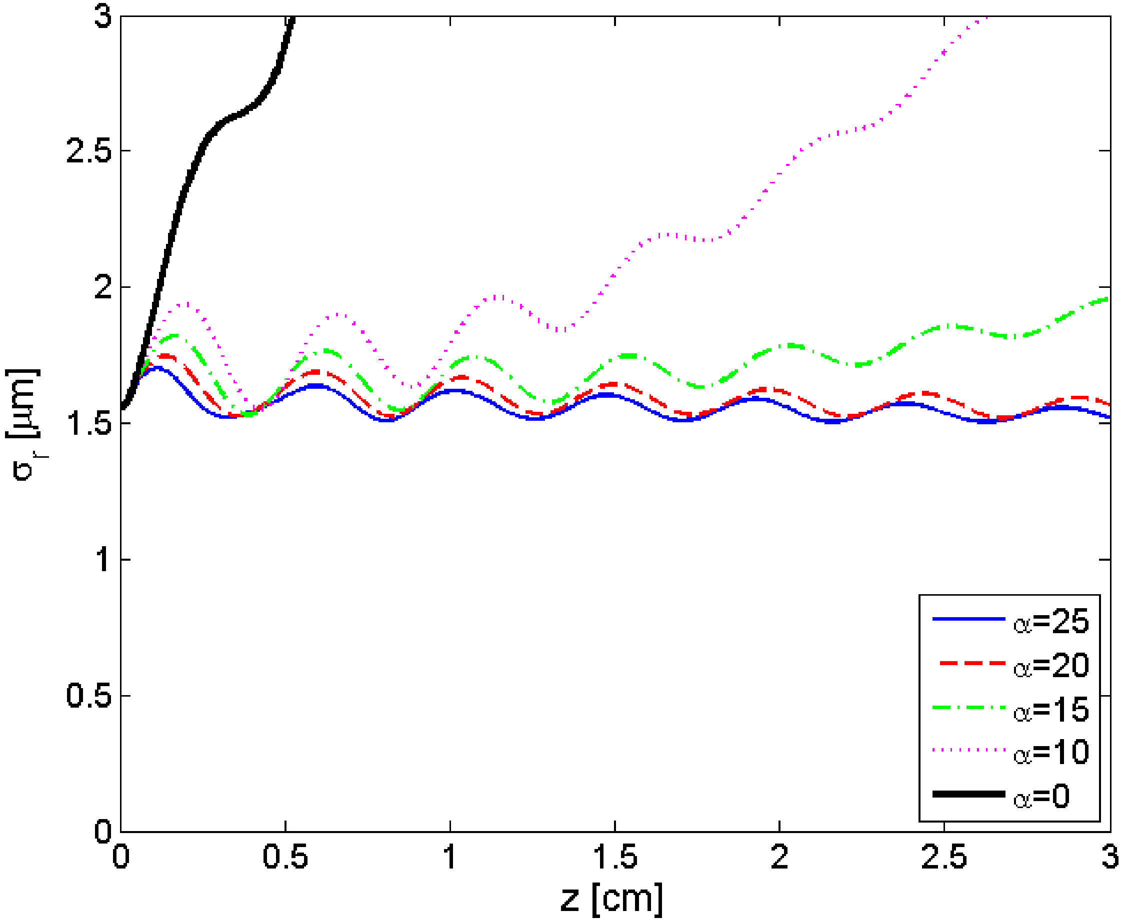
Simulation parameters 2° case

- Plasma density $n_0 = 2 \cdot 10^{16}$

- **Driver:**
  - Energy $\gamma = 240$
  - Charge $Q = 120pC$
  - Length $\sigma_z = \frac{1}{k_p}$
  - $\alpha = 0.5$
  - Emittance $\varepsilon_n = 10\, mm\, mrad$

- **Witness:**
  - Energy $\gamma = 240$
  - Length $\sigma_z = 5.31\mu m$
  - α-scan $0 \leq \alpha \leq 25$ (2° case)
  - $\varepsilon_n = 0.5\, mm\, mrad$ (2° case)
  - Spot size $\sigma_r = \sqrt[4]{\frac{1}{\gamma}}\sqrt{\frac{2\varepsilon_n}{k_p}}$

Stability of the scheme check



Simulation scan at variable α with fixed emittance showed that

- The witness is effectively matched to its own field (no matching at $\alpha = 0$)
- At small α the envelope is not stable, probably due to the fall of ultra dense bunch hypotesis
- At high α the envelope seems to be independent from bunch density, following a non linear behaviour

Simulation parameters 3° case

- Plasma density $n_0 = 2 \cdot 10^{16}$

Driver:

- Energy $\gamma = 200$
- Charge $Q = 120pC$
- Length $\sigma_z = 37.6\mu m$
- Emittance $\varepsilon_n = 10\ mm\ mrad$
- Spot size $\sigma_r = 11.4\mu m$
- $\alpha = 0.5$

Witness:

- Energy $\gamma = 200$
- Charge $Q = 8pC$
- Length $\sigma_z = 5.3\mu m$
- Emittance $\varepsilon_n = 0.3\ mm\ mrad$
- Spot size $\sigma_r = 1.26\mu m$

Preliminary case of study

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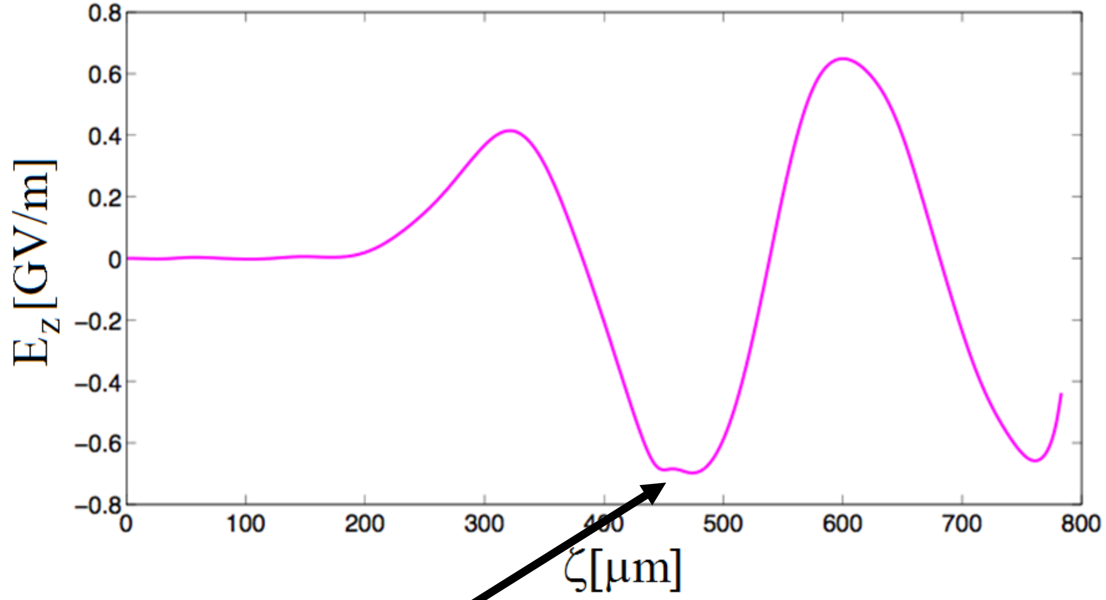
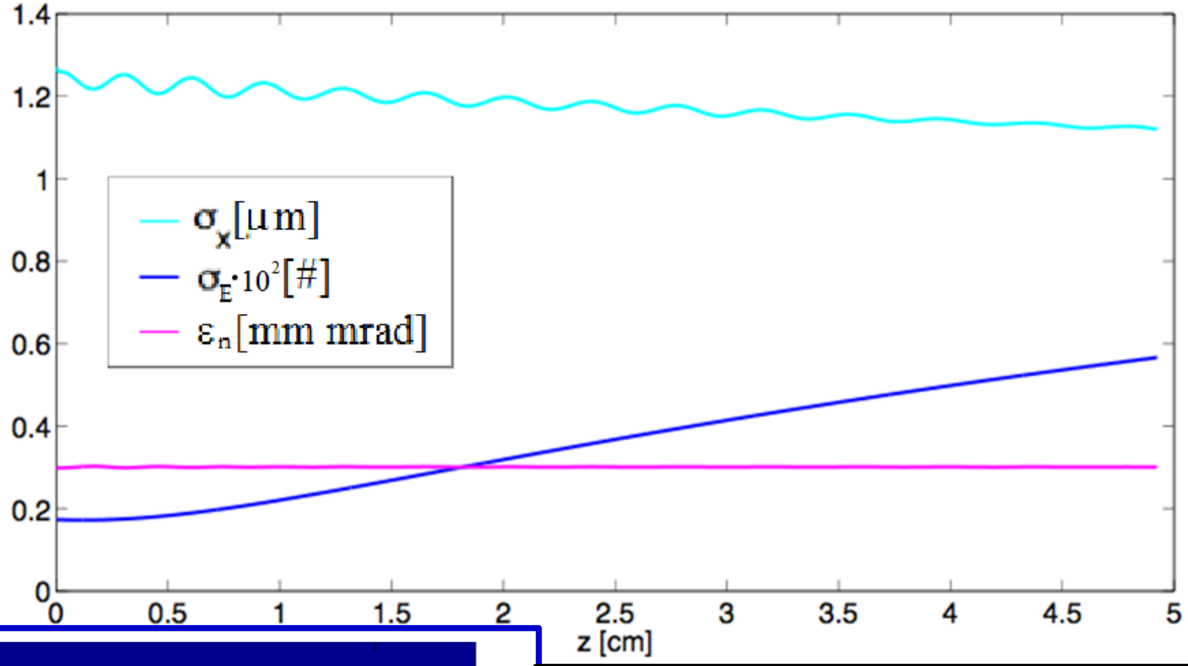
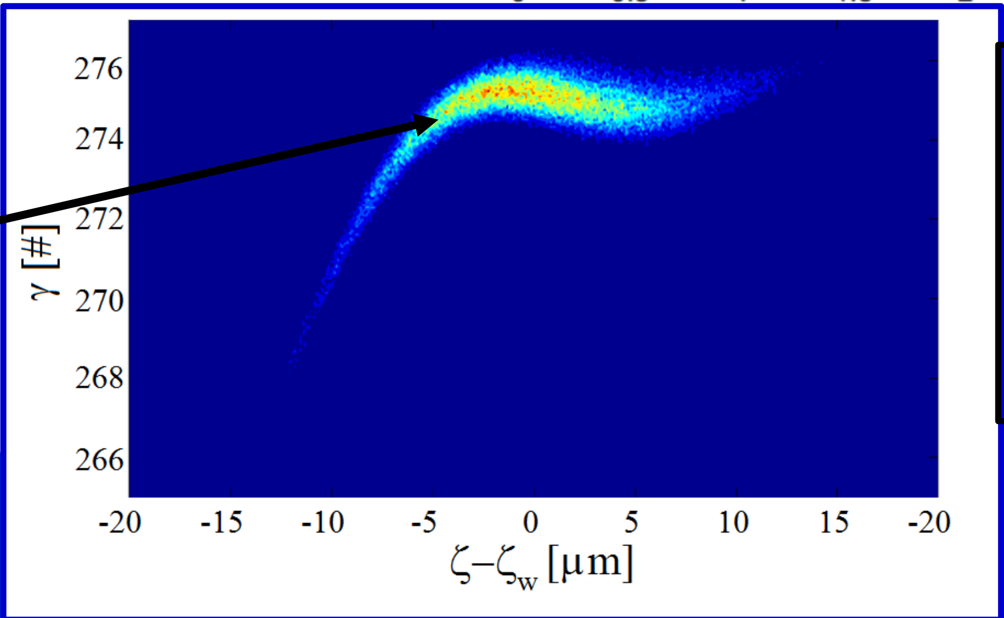




Low energy spread
+
Beam loading optimization



Accelerating gradient of 0.75 GV/m

Energy spread growth below 0.5% over 5 cm!

Conclusions

- **The hybrid scheme advantages:**
  - Injection of witness on a linear crest lets to minimize the energy spread growth
  - The matching condition of witness is performed on the fields generated by witness itself, increasing the stability of the envelope
  - A driver mismatch doesn’t increase the witness emittance, relaxing the very restrictive injection conditions
- **The hybrid scheme disadvantages:**
  - High density witness is required to allow the hybrid scheme to work
  - More tests at longer distance are required to verify the stability of the scheme

Thanks for the attention

“Aut inveniam viam aut faciam” - *"I shall either find a way or make one.“*
Hannibal, Carthaginian general, before crossing the Alps by using elefants