Quantum FEL Theory and its Classical limits

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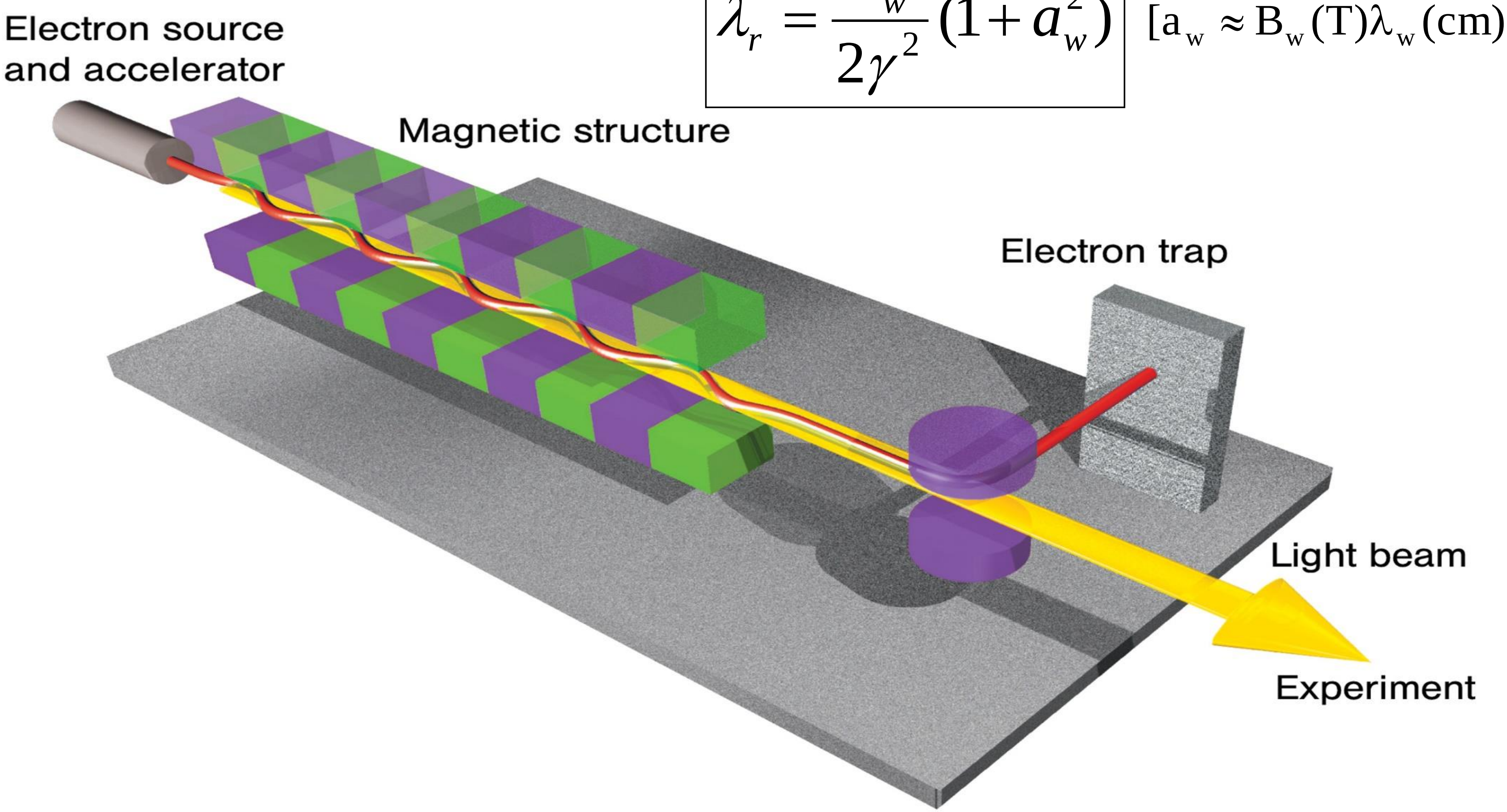
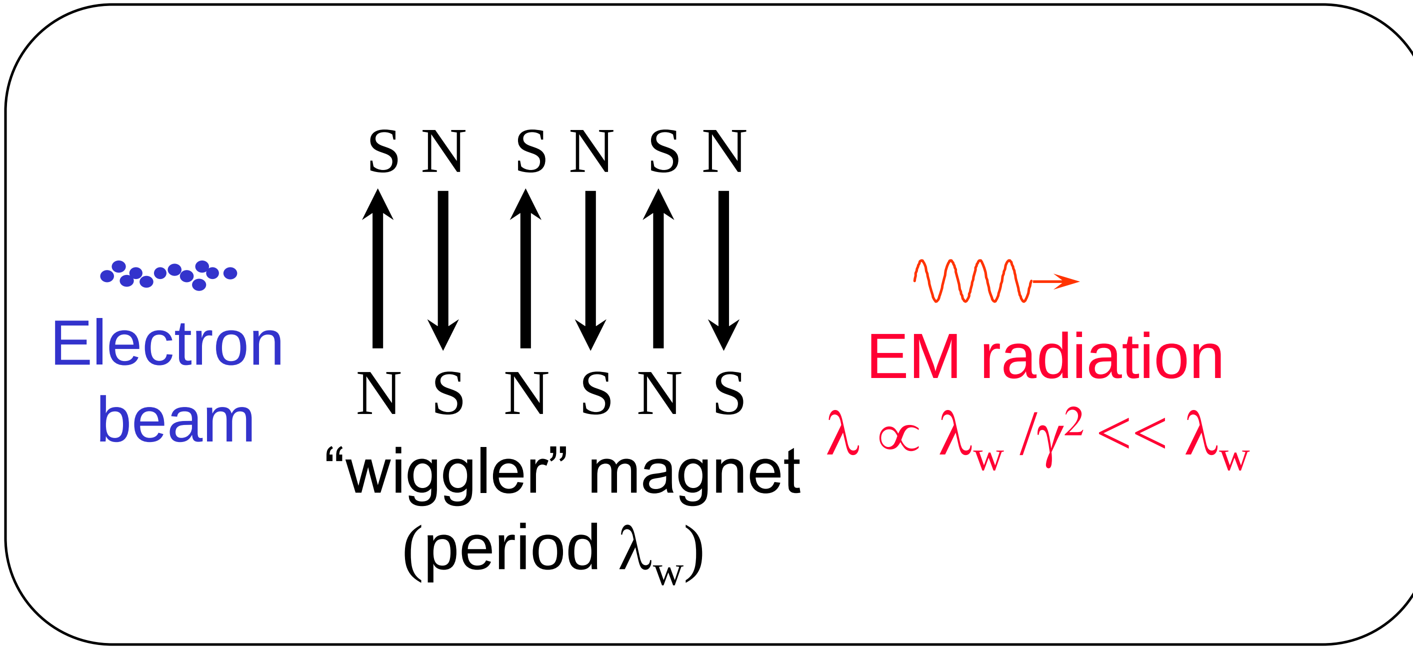
- High Gain FEL basic concepts
- Quantum FEL model and classical limit
- Classical SASE
- Quantum purification of SASE
- Toward a Quantum FEL realization
- The atomic analogue of FEL: the Collective Atomic Recoil Laser (CARL)

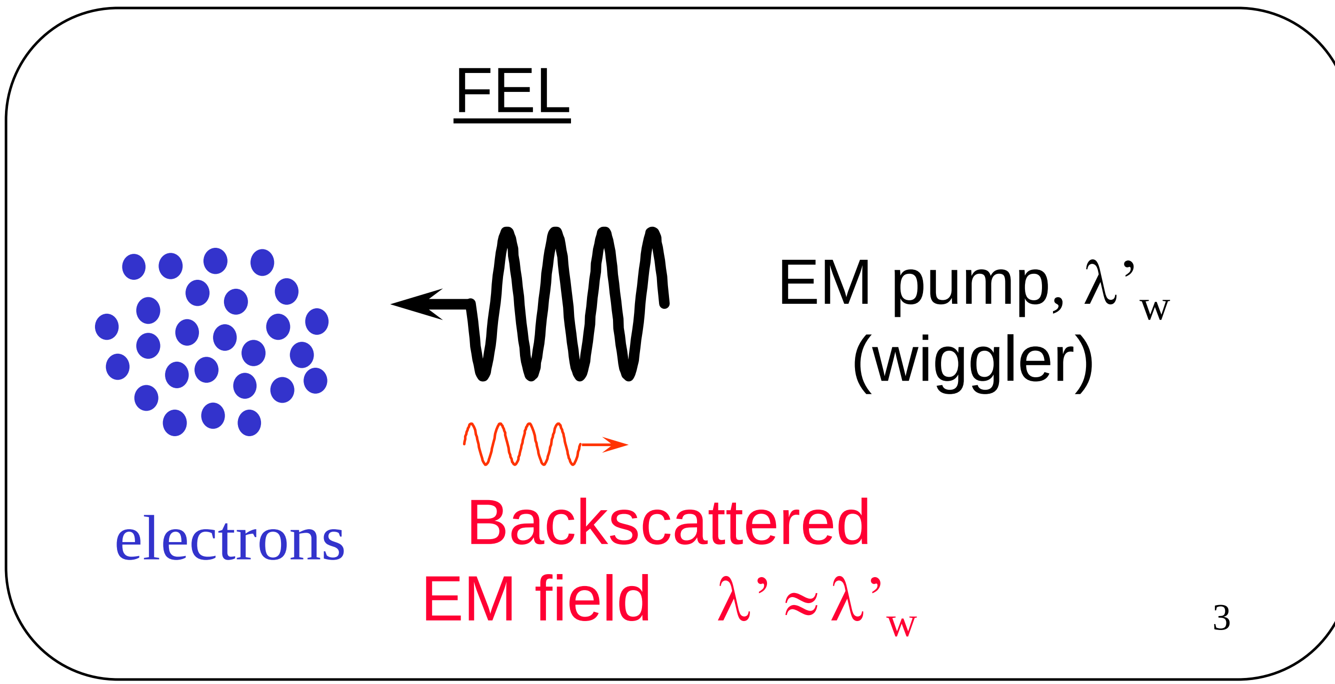
High-gain Free Electron Laser (FEL)

$$\lambda_r = \frac{\lambda_w}{2\gamma^2}(1+a_w^2)$$ $[a_w \approx B_w(T)\lambda_w(cm)]$

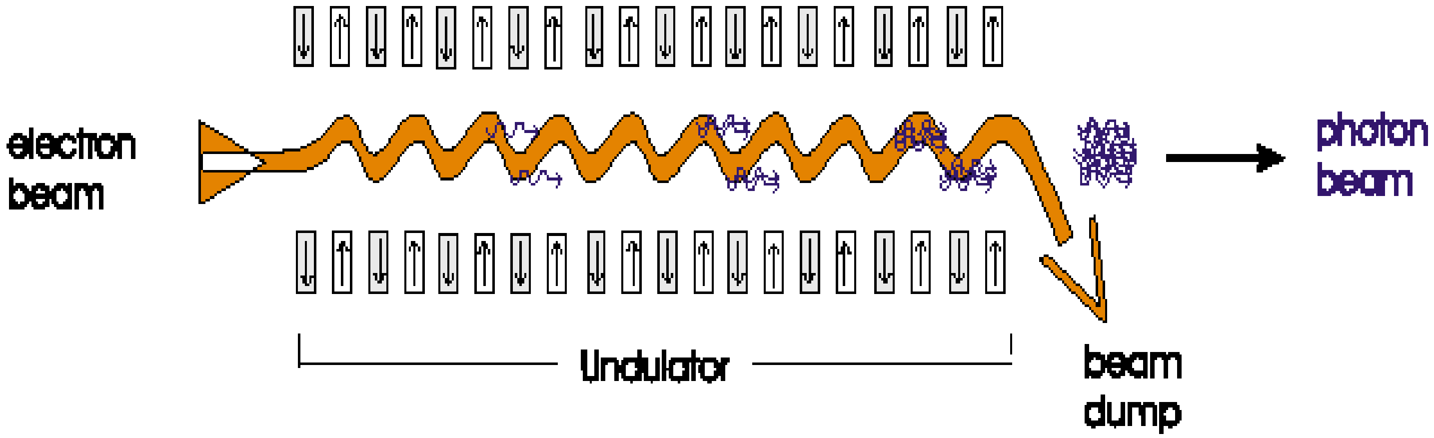


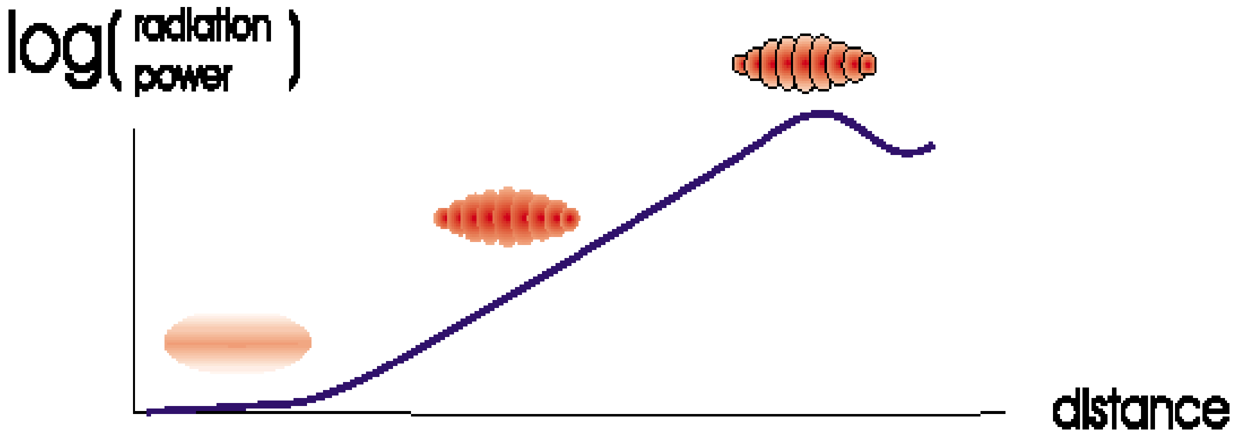
FEL can be understood by transforming to a frame moving with electrons





HIGH-GAIN REGIME





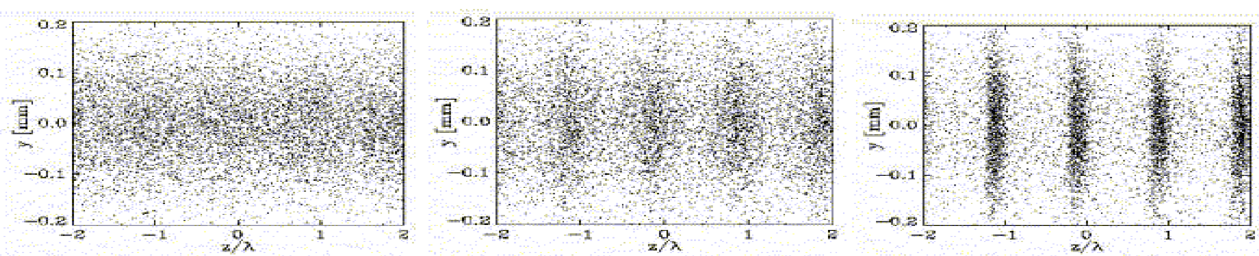
HIGH-GAIN REGIME

- exponential growth of intensity and bunching
- saturation ($P_{rad} \sim \rho \, P_{beam}$) after several gain lengths L_g

$$\left(L_g = \frac{\lambda_w}{4\pi\rho} \right)$$

b~0 → b~0.8

bunching:



$$b = \frac{1}{N} \sum_{j=1}^{N} e^{-ikz_j}$$

wiggler length (several L_g)

basic parameter of the High-Gain FEL

$$\rho = \frac{1}{2\gamma_0}\left(\frac{I}{17kA}\right)^{1/3}\left(\frac{\lambda_w a_w}{2\pi\sigma_b}\right)^{2/3}$$

(typically $\rho \sim 10^{-3} - 10^{-4}$)

- **efficiency ($P_{rad}/P_{beam} \sim \rho$)**
- **gain bandwidth ($\Delta\lambda_r/\lambda_r \sim \rho$)**
- **Saturation length ($L_{sat} \sim \lambda_w/\rho$)**
- **Minimum energy spread** ($\Delta\gamma/\gamma < \rho$)

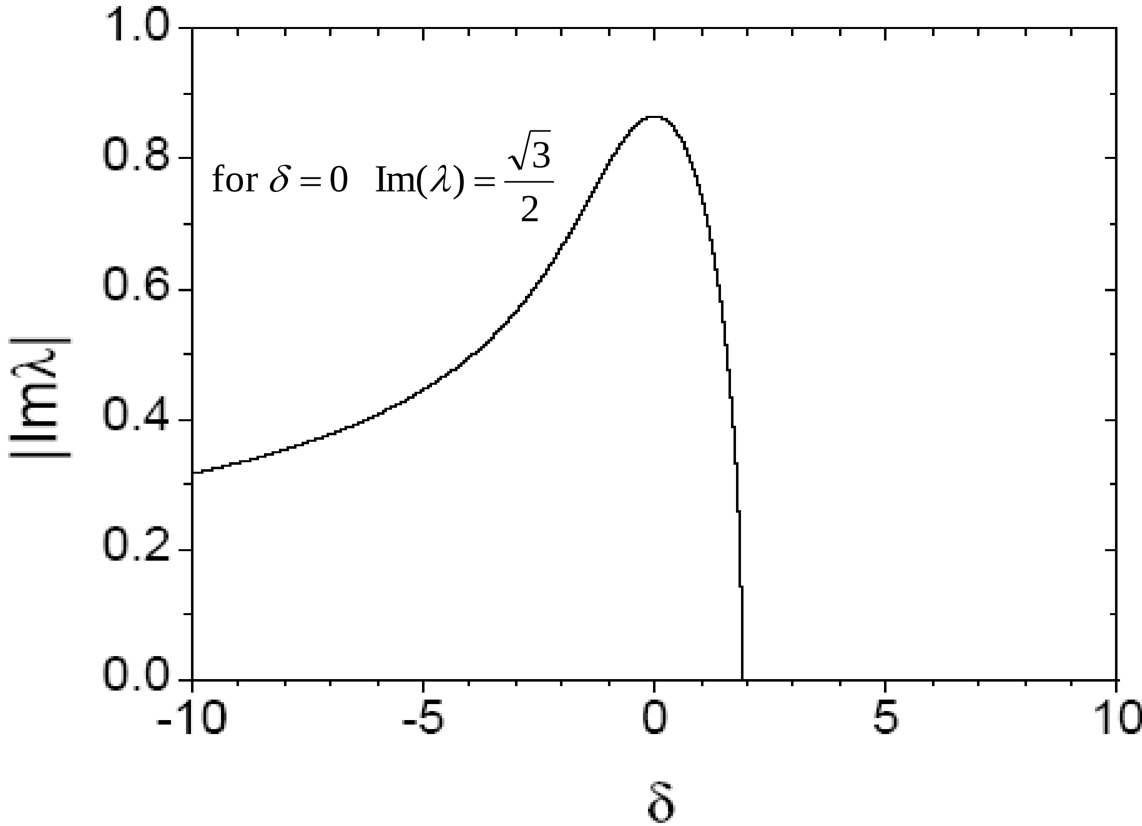
Collective instability

From a linear instability analysis of classical FEL equations:

$$E_{rad}(z) \propto e^{i\lambda(z/L_g)}$$

$$(\lambda-\delta)\lambda^2+1=0$$

runaway solution if $\mathrm{Im}(\lambda)<0$



$$\delta=\frac{\gamma_0-\gamma_R}{\rho\gamma_R}=\frac{\omega_r-\omega}{2\rho\omega_r}$$

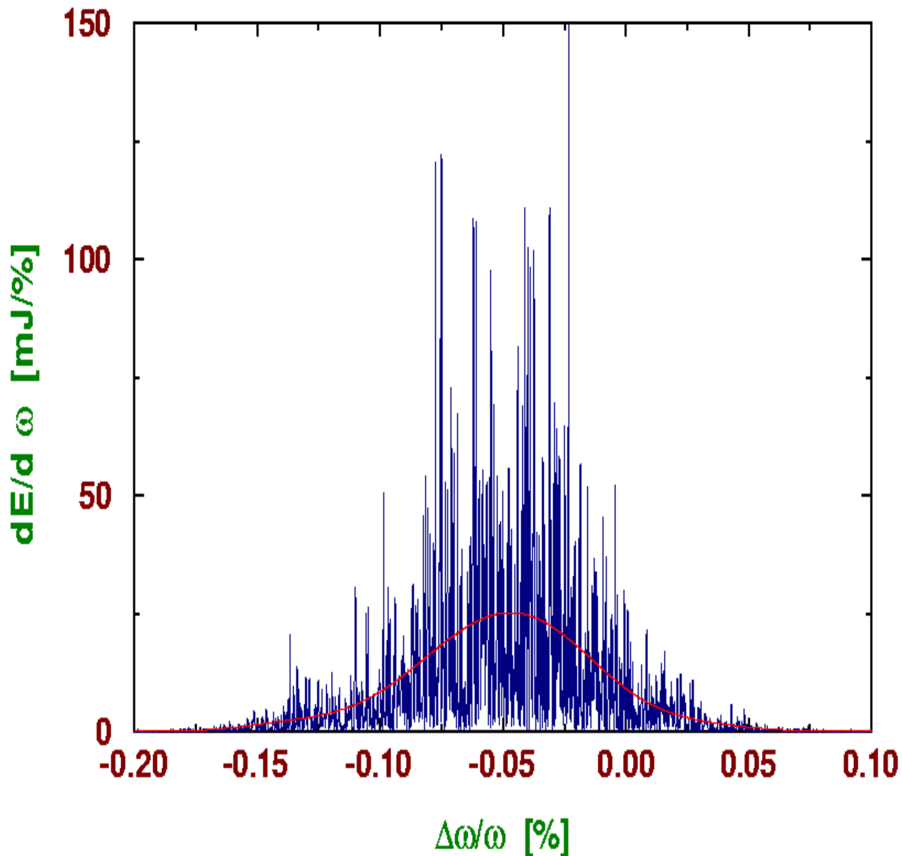
$$\omega=\frac{2\gamma_R^2}{1+a_w^2}ck_w \quad , \quad \omega_r=\frac{2\gamma_0^2}{1+a_w^2}ck_w$$

'CLASSICAL' SASE
(Self Amplified Spontaneous Emission)

exponential amplification of shot noise (see later..)

many random spikes in the GAIN BANDWIDTH

$$\frac{\Delta\omega}{\omega} \approx 2\rho$$



WHY A QUANTUM FEL THEORY?

In a classical theory the electron momentum recoil is a continuous variable

WRONG: if an electron emits n photons

$$\Delta p = n(\hbar k) \Rightarrow \text{QUANTUM THEORY}$$

QUANTUM FEL parameter: $\bar{\rho} = \rho\left(\frac{mc\gamma_0}{\hbar k}\right)$

since $\frac{\Delta\gamma}{\gamma_0} \approx \rho$, then $\Delta p = mc\Delta\gamma \approx (mc\gamma_0)\rho \Rightarrow \frac{\Delta p}{\hbar k} \approx \bar{\rho}$

if $\bar{\rho} >> 1$ classical limit

if $\bar{\rho} < 1$ strong quantum effects

QUANTUM FEL MODEL

Procedure :

Describe N-particle system as a Quantum Mechanical ensemble

Write a Schrödinger equation for macroscopic wavefunction Ψ :

$$i\frac{\partial\Psi(\theta,\bar{z})}{\partial\bar{z}} = H\ \Psi(\theta,\bar{z})$$

$$\theta = (k+k_w)z - ckt \quad , \quad \bar{z} = z/L_g$$

$$H = \frac{p^2}{2\bar{\rho}} - i\bar{\rho}\left(Ae^{i\theta} - c.c.\right) \qquad [\theta, p] = i \qquad p = -i\frac{\partial}{\partial\theta}$$

QUANTUM FEL MODEL

$$i\frac{\partial\Psi(\theta,\bar{z})}{\partial\bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2\Psi(\theta,\bar{z})}{\partial\theta^2} - i\bar{\rho}\left\{A(\bar{z})e^{i\theta} - c.c.\right\}\Psi(\theta,\bar{z})$$

$$\frac{dA(\bar{z})}{d\bar{z}} = \int_0^{2\pi} |\Psi(\theta,\bar{z})|^2 e^{-i\theta} d\theta$$

A: normalized radiation amplitude

G. Preparata, Phys. Rev. A (1988)

CLASSICAL LIMIT for $\bar{\rho} >> 1$

$$\Psi(\theta,\bar{z}) = \sqrt{n(\theta,\bar{z})}\, e^{i\bar{\rho}S(\theta,\bar{z})} \quad , \quad u(\theta,\bar{z}) = \frac{\partial S(\theta,\bar{z})}{\partial\theta}$$

Madelung quantum fluid description

$$\frac{\partial n}{\partial\bar{z}} + \frac{\partial nu}{\partial\theta} = 0$$

$$\frac{\partial u}{\partial\bar{z}} + u\frac{\partial u}{\partial\theta} = -\left(Ae^{i\theta} + c.c.\right) + \frac{1}{2\bar{\rho}^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sqrt{n}}\frac{\partial^2\sqrt{n}}{\partial\theta^2}\right] \rightarrow 0 \quad \text{for } \bar{\rho}\rightarrow\infty$$

$$\frac{dA}{d\bar{z}} = \int_0^{2\pi} n(\theta,\bar{z})e^{-i\theta}$$

Equations for a classical fluid of density **n** an mean velocity **u**

CLASSICAL FEL 1D MODEL

$$\frac{d^2\theta_j}{d\bar{z}^2} = -(Ae^{i\theta_j} + c.c.)$$

$$\frac{dA}{d\bar{z}} = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

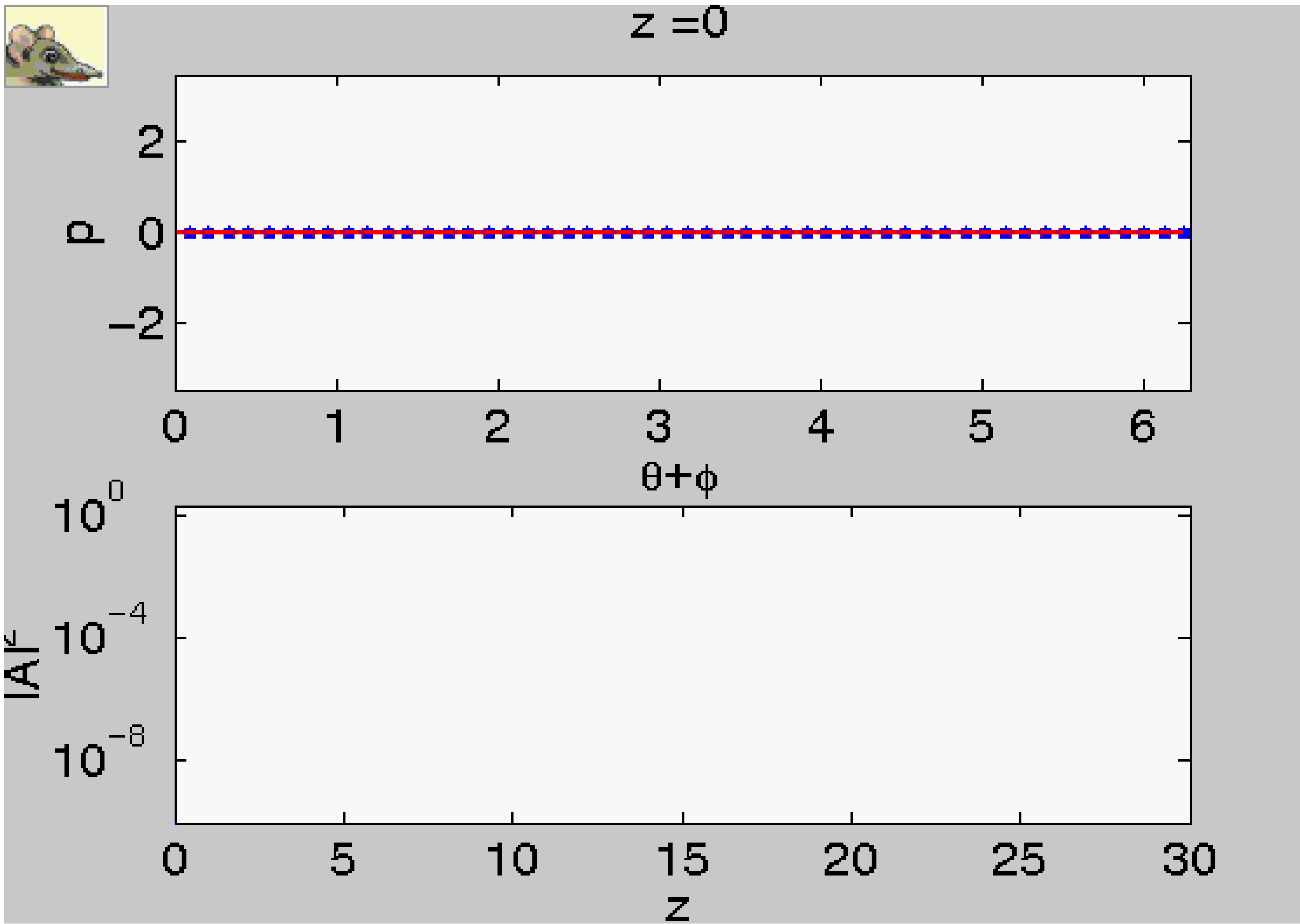
j=1,.., N

$$\rho\,|A|^2 = \frac{P_{rad}}{P_{beam}}$$

R.Bonifacio, C.Pellegrini, L.Narducci, Opt. Comm. (1984)

Classical FEL instability animation

Animation shows evolution phase space (θ,p) of the electrons in the dynamic pendulum potential and the FEL intensity $|A|^2$



$$p = \frac{\gamma - \gamma_0}{\rho \gamma_0}$$

Propagation effects and SUPERRADIANCE

Radiation advance electrons by one wavelength λ_r every undulator period λ_w

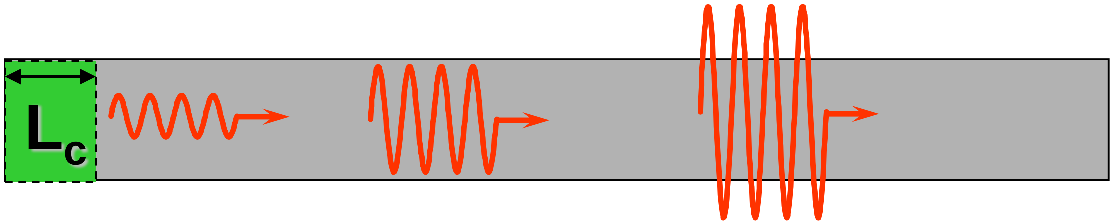
slippage length $(\lambda_r/\lambda_w)L_w = \lambda_r N_w$

electrons in a cooperation length

$$L_c = (\lambda_r/\lambda_w)L_g = \lambda_r/4\pi\rho$$

emit SUPERRADIANTLY (i.e. ~ N^2)

.



CLASSICAL FEL 1D MODEL
with propagation

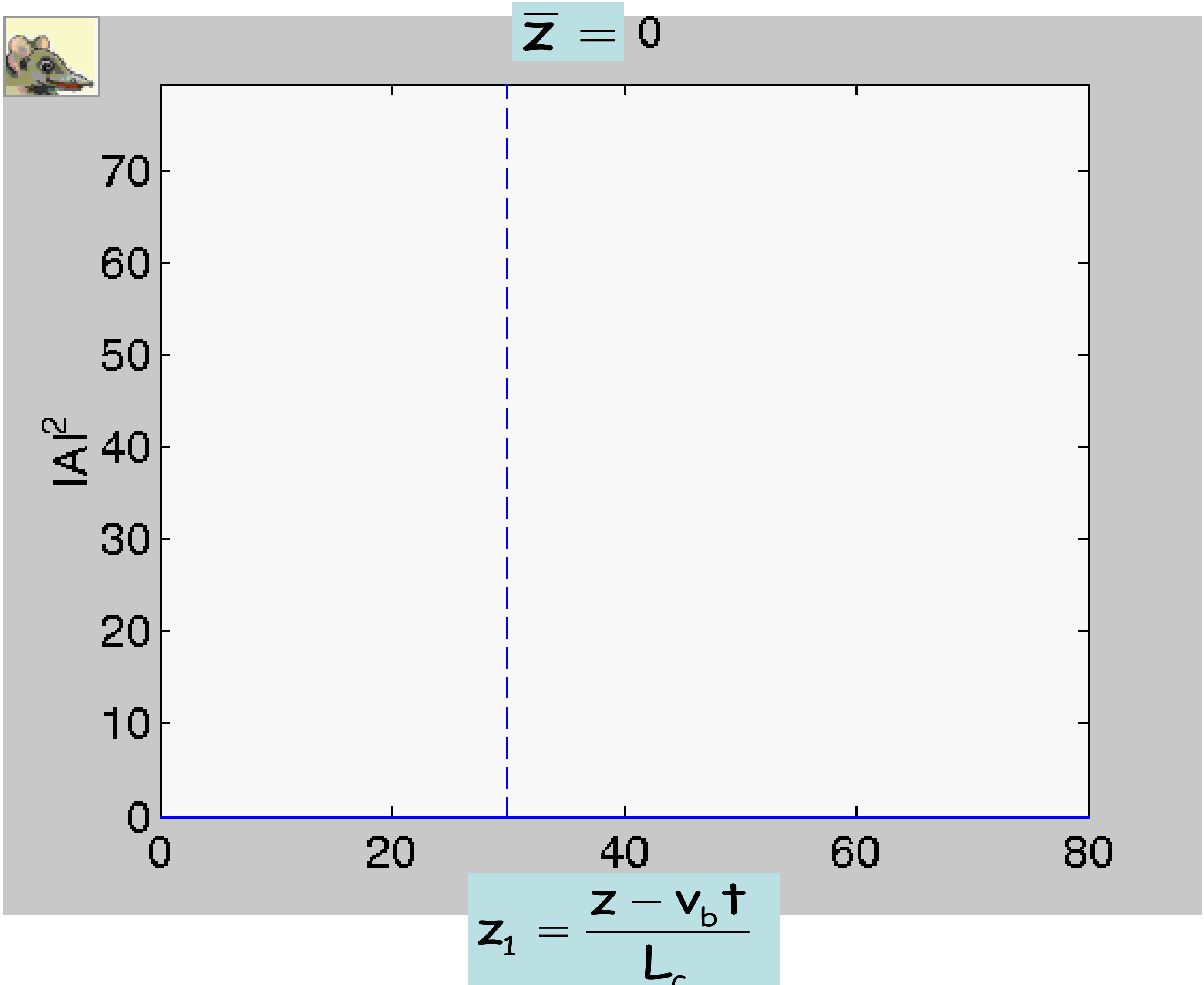
$$\frac{\partial^2 \theta_j}{\partial \bar{z}^2} = -(A e^{i\theta_j} + c.c.)$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

$$\bar{z} = \frac{z}{L_g}, \quad z_1 = \frac{z - v_b t}{L_c}$$

For long beams ($L >> L_c$) → Superradiant Instability

$L_b = 30 L_c$



SASE mode for FELs

Ingredients of **Self Amplified Spontaneous Emission** are:

i) Start up from noise
ii) Superradiance instability

each cooperation length in the e-beam radiates a SR spike which is amplified when it propagates forward on the beam

Most of present and next x-ray FELs (LCLS, XFEL etc..) work in the SASE mode.

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Spectrum, Temporal Structure, and Fluctuations in a High-Gain Free-Electron Laser Starting from Noise

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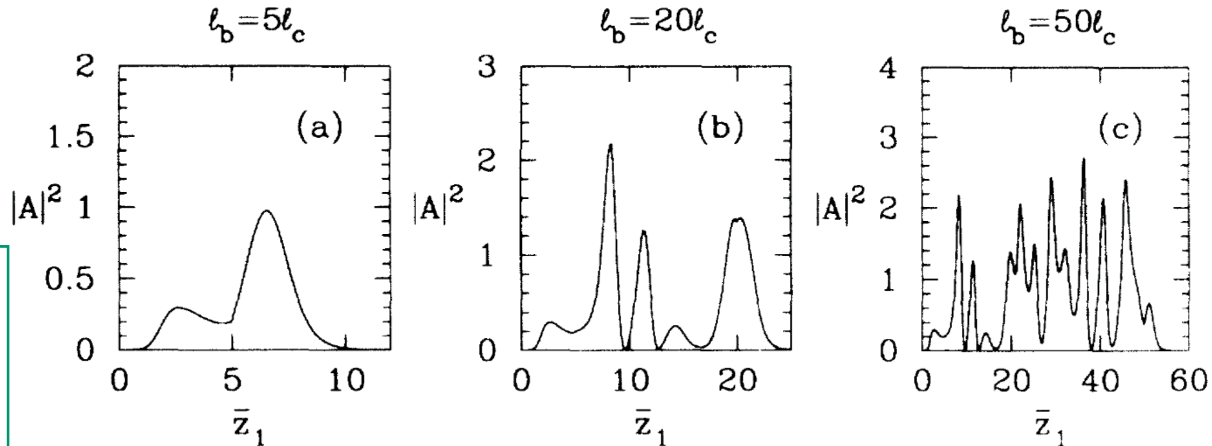
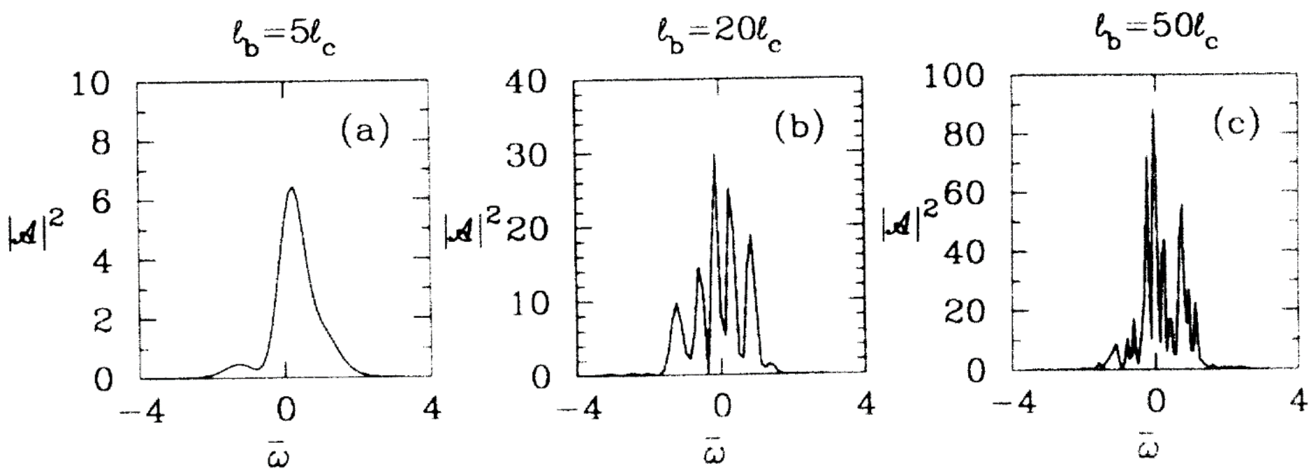
(Received 14 July 1993)

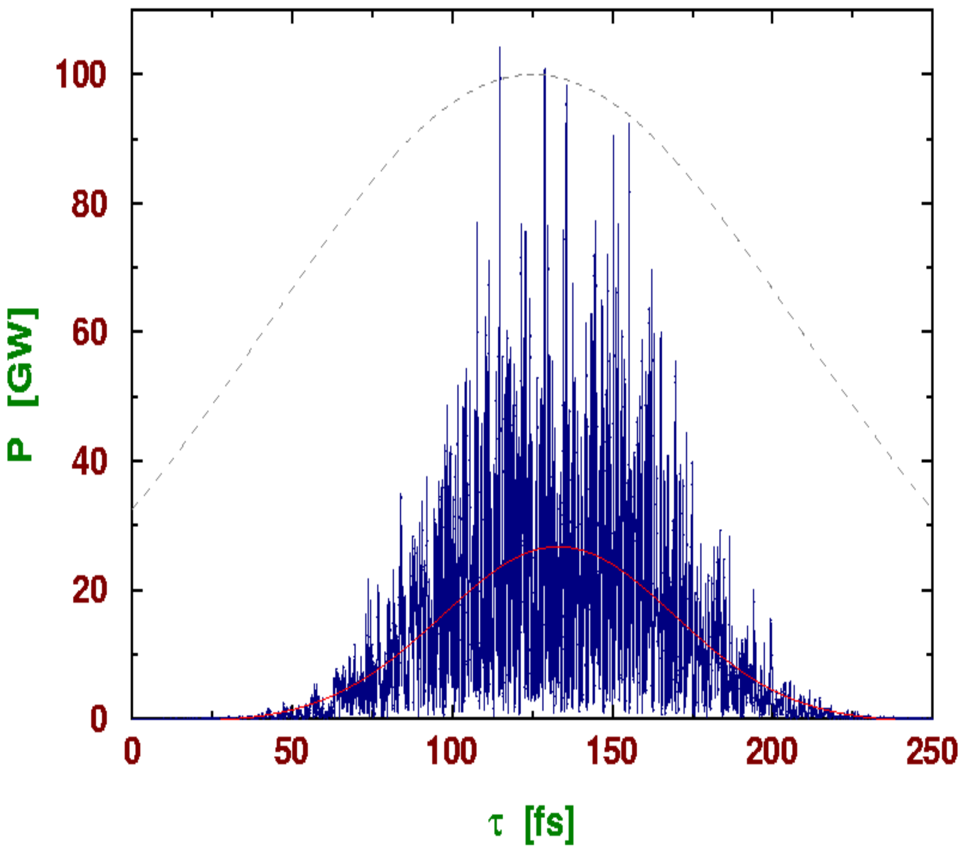
$$N_s = \frac{L_b}{2\pi L_c}$$

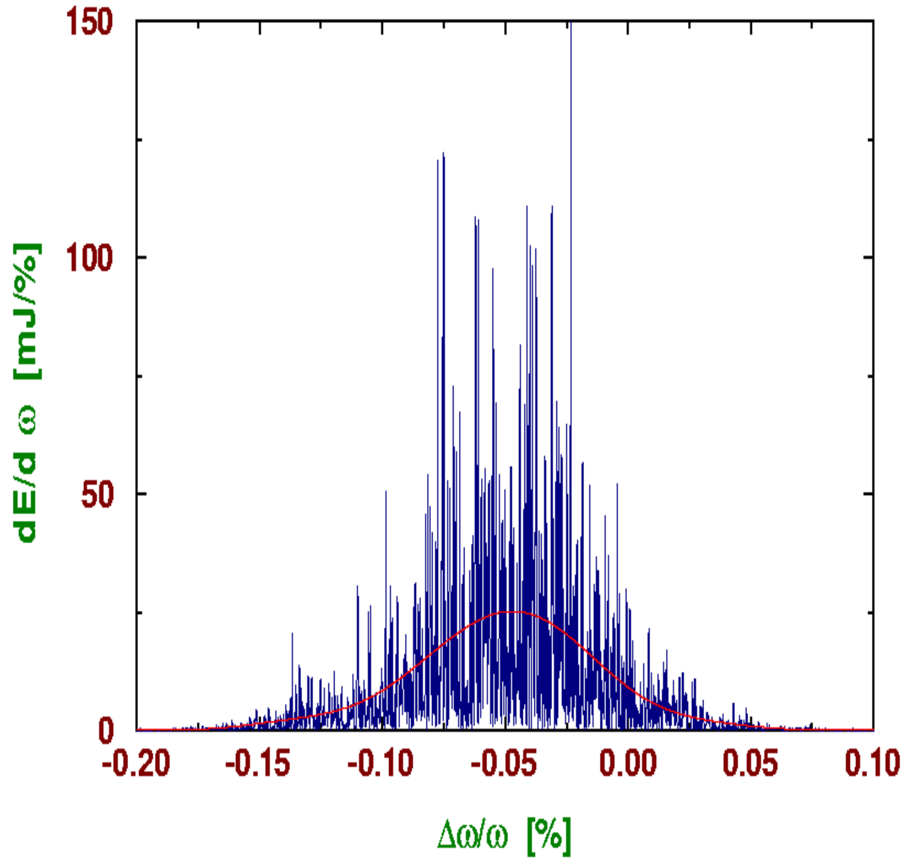




Radiation has poor temporal coherence (many random spikes (~ L_b/L_c))

Broad and noisy spectrum at short wavelengths (x-ray FELs) **(see Huang and Ding talks)**





QUANTUM FEL MODEL with propagation

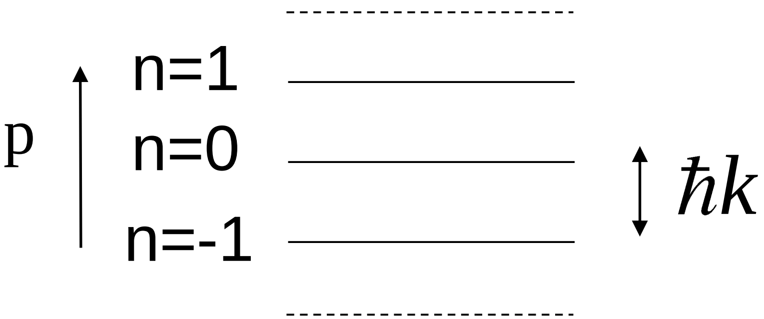
$$i\frac{\partial\Psi(\theta,\bar{z},z_1)}{\partial\bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2\Psi(\theta,\bar{z},z_1)}{\partial\theta^2} - i\bar{\rho}\left\{A(\bar{z},z_1)e^{i\theta} - c.c.\right\}\Psi(\theta,\bar{z},z_1)$$

$$\frac{\partial A(\bar{z},z_1)}{\partial\bar{z}} + \frac{\partial A(\bar{z},z_1)}{\partial z_1} = \int_0^{2\pi}|\Psi(\theta,\bar{z},z_1)|^2 e^{-i\theta}d\theta$$

Quantum FEL model:

electron momentum changes by discrete steps of photon momentum

$p = mc(\gamma - \gamma_0) = n\,(\hbar k)$, $n = 0, \pm 1, ..$



$$\Psi(\theta, \bar{z}, z_1) = \sum_{n=-\infty}^{\infty} c_n(\bar{z}, z_1) e^{in\theta}$$

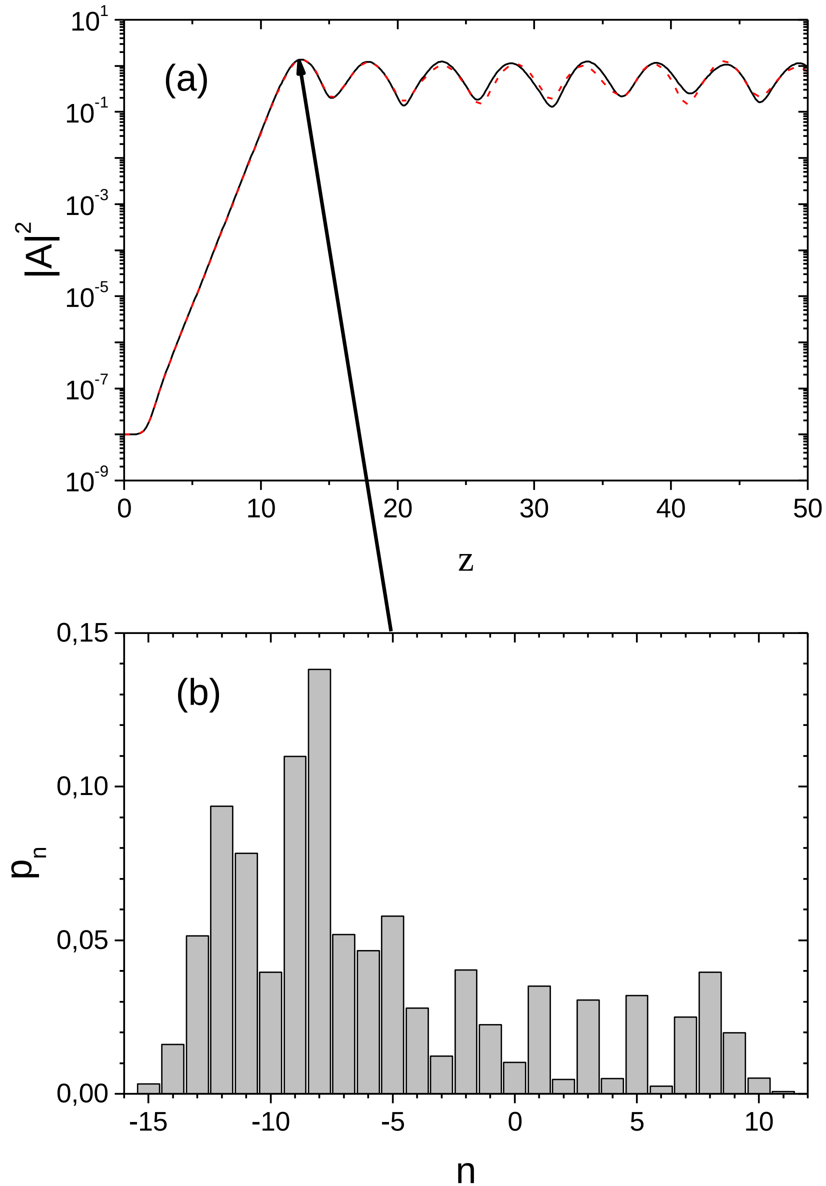
$$\frac{\partial c_n}{\partial \bar{z}} = -\frac{in^2}{2\bar{\rho}} c_n - \bar{\rho}\left(A c_{n-1} - A^* c_{n+1}\right)$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \sum_{n=-\infty}^{\infty} c_n c_{n-1}^*$$

The classical limit is recovered for

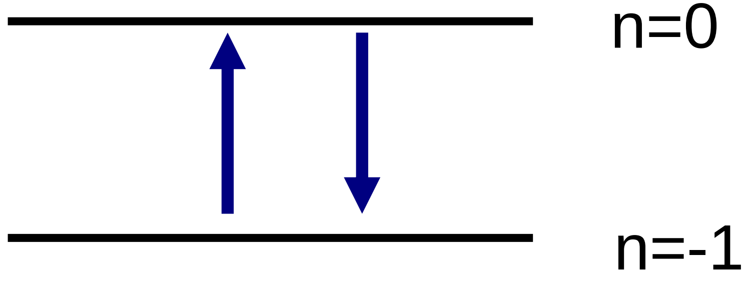
$$\bar{\rho} >> 1$$

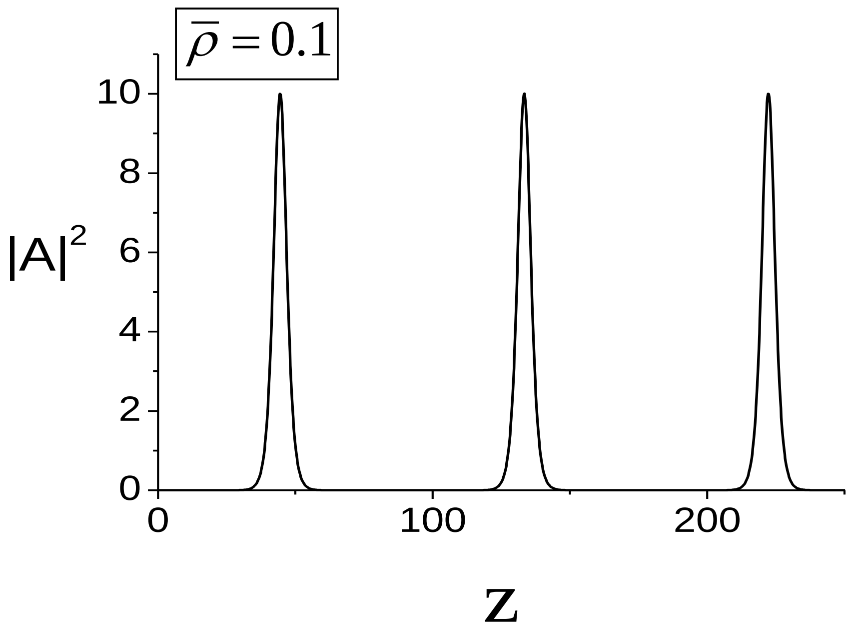
$\bar{\rho}$=10, δ=0, no propagation



Quantum limit for $\bar{\rho} \leq 1$

Only TWO momentum states : p=0 and p= - ħk





2-level system coupled to an optical field,

as in a LASER!

QUANTUM REGIME of FEL occurs when:

$$mc(\delta\gamma) \le \hbar k \quad \Rightarrow \quad \bar{\rho} < 1$$

each electron emits only one photon!

$\Downarrow$

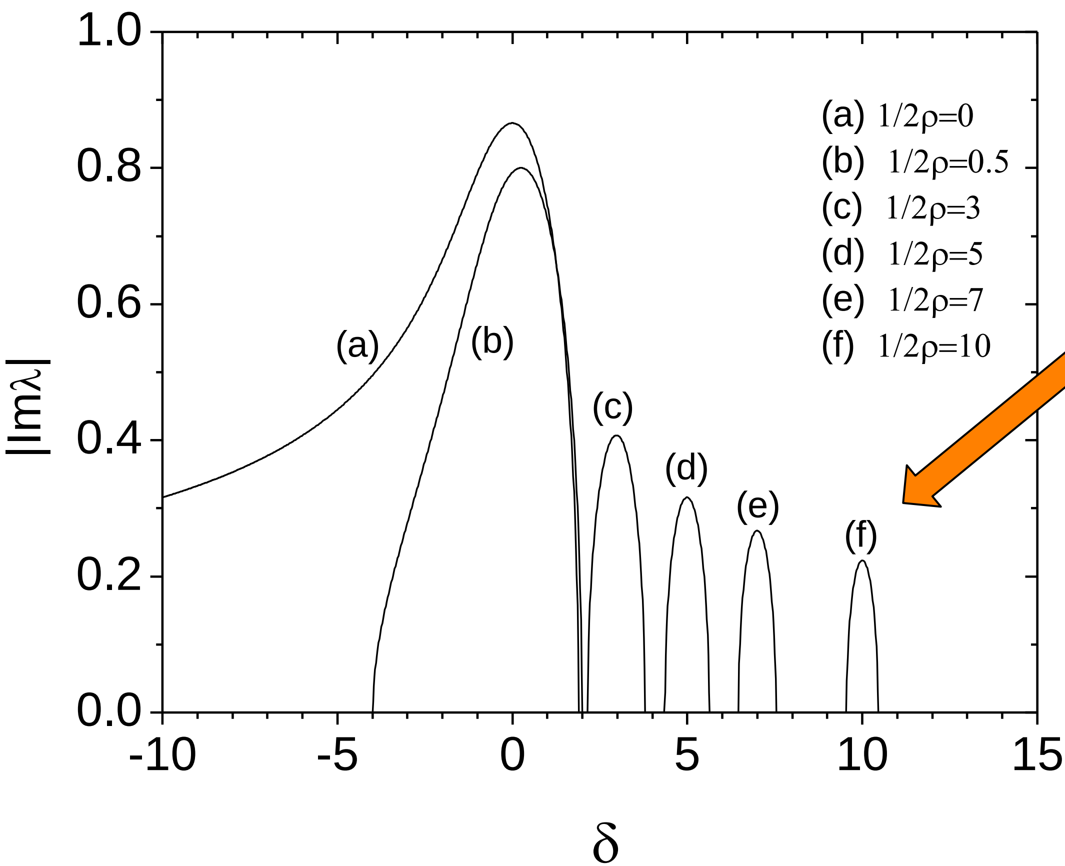
QUANTUM COHERENCE

Quantum FEL behaves like a two-level system (i.e. a 'laser')

Quantum Linear Theory

$$\left(A \propto e^{i\lambda z/L_g}\right)$$

$$(\lambda-\delta)\left(\lambda^2-\frac{1}{4\bar{\rho}^2}\right)+1=0$$



$$\delta=\frac{1}{2\bar{\rho}} \Rightarrow \omega=\omega_s-\omega_{rec}$$

$$\omega_{rec}=\frac{\hbar k^2}{m\gamma_0}$$

$$\frac{\Delta\omega}{\omega}\approx 2\rho\sqrt{\bar{\rho}} << 2\rho$$

QUANTUM-SASE REGIME

- In the quantum regime an FEL behaves as a two-level system
- electrons emit coherent photons similarly as in a LASER
- in the SASE mode the spectrum is intrinsically narrow ('quantum purification')
- the transition between the classical and the quantum regimes depends on a single parameter:

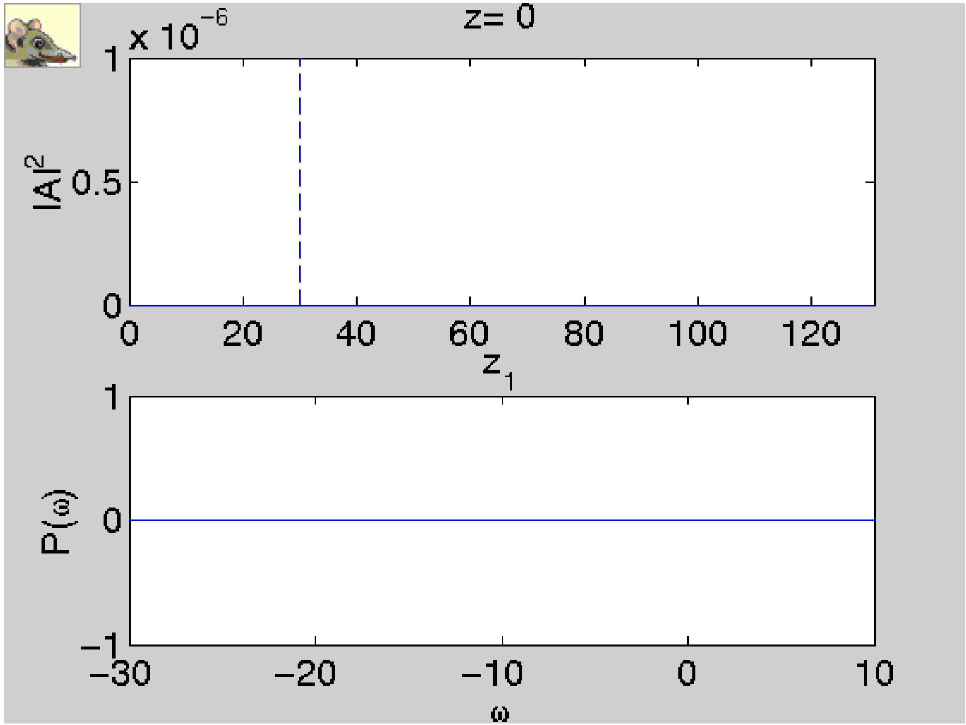
$$\bar{\rho} = \left(\frac{mc\gamma_0}{\hbar k} \right) \rho$$

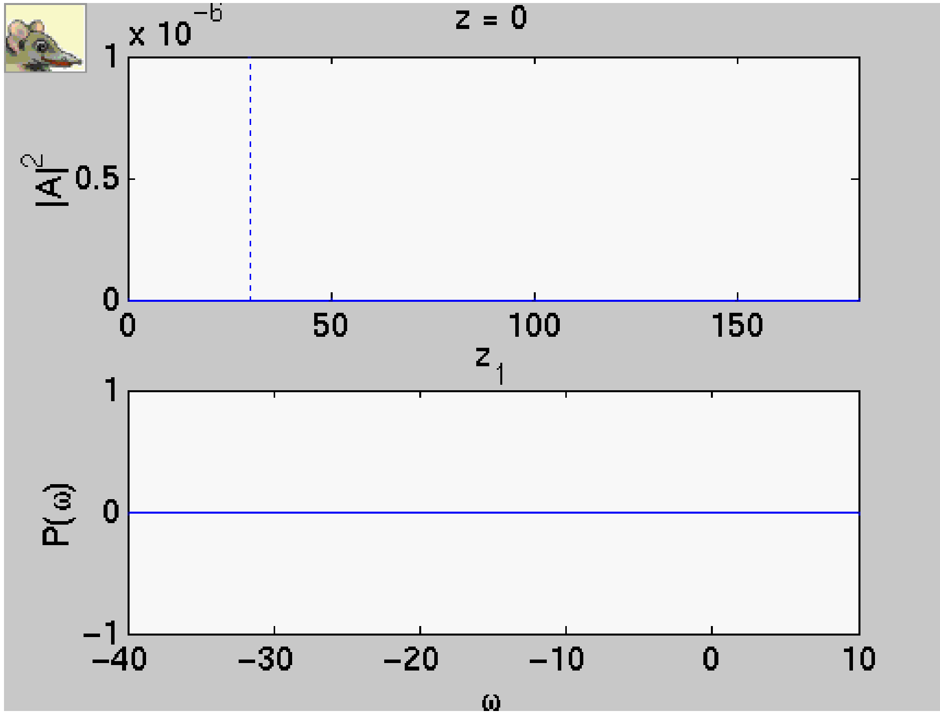
QUANTUM SASE REGIME

$\bar{\rho}$ = photon mean number emitted per electron
$\bar{\rho}$ > 1 classical SASE (incoherent spiking)
$\bar{\rho}$ < 1 quantum SASE (coherent)

$\bar{\rho} = 5$ **(CLASSICAL REGIME)**

$\bar{\rho} = 0.05$ **(QUANTUM REGIME)**



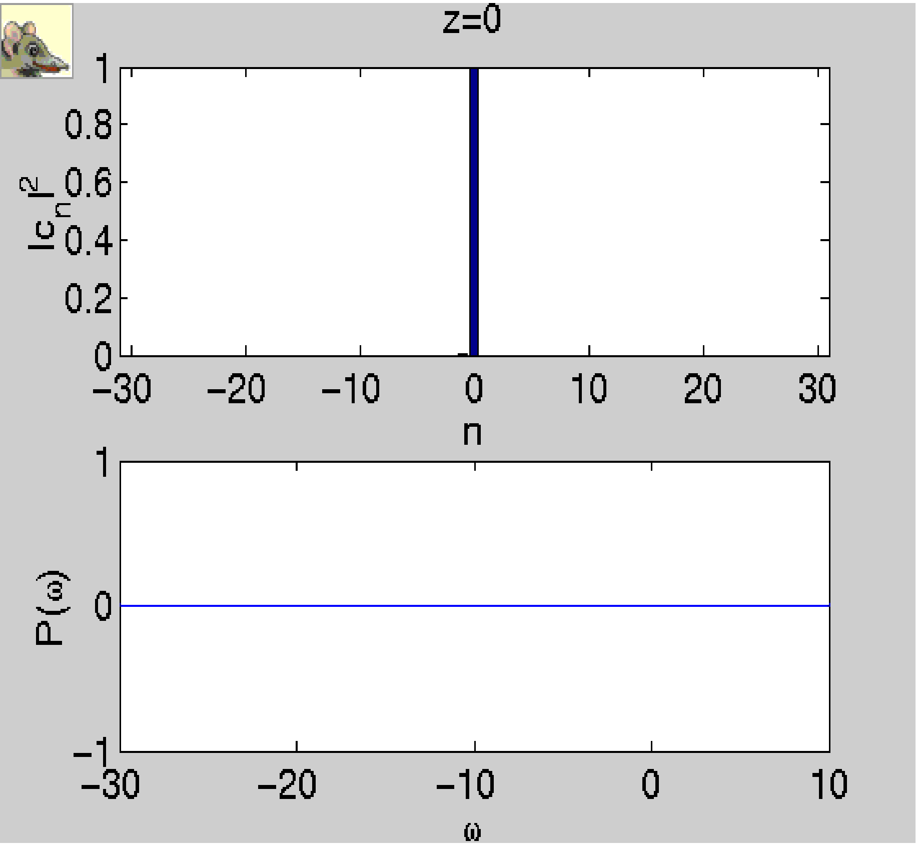


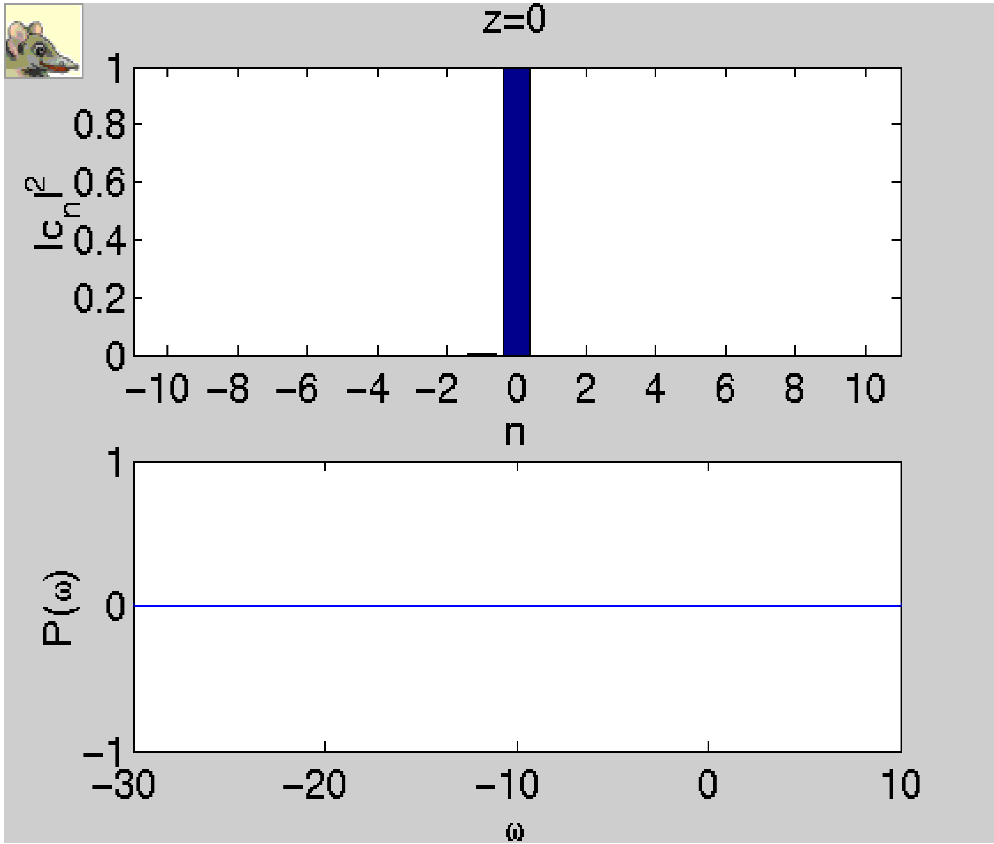
R. Bonifacio, N. Piovella, G.R.M. Robb, NIMA 543 (2005) 645.

momentum distribution for SASE

CLASSICAL REGIME: $\bar{\rho} = 5$

QUANTUM REGIME: $\bar{\rho} = 0.1$



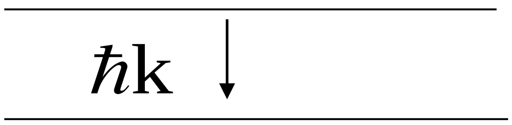


Classical regime:
both n<0 and n>0 occupied

Quantum regime:
sequential SR decay, only n<0

QUANTUM INTERPRETATION of SASE

at each momentum transition a spike is emitted



$$p = (n)\hbar k$$

$$p = (n-1)\,\hbar k$$

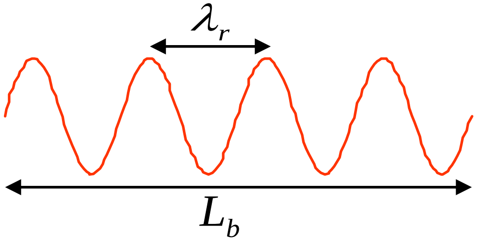
$$n = 0, -1, -2, \ldots$$

for $\bar{\rho} >> 1$ MANY RANDOM SPIKES → **CLASSICAL SASE**

for $\bar{\rho} \leq 1$ A SINGLE SPIKE IS EMITTED! → **QUANTUM SASE**

SPIKE'S WIDTH:

$$\left(\frac{\Delta\omega}{\omega}\right)_{QFEL} \approx \frac{\lambda_r}{L_b}$$



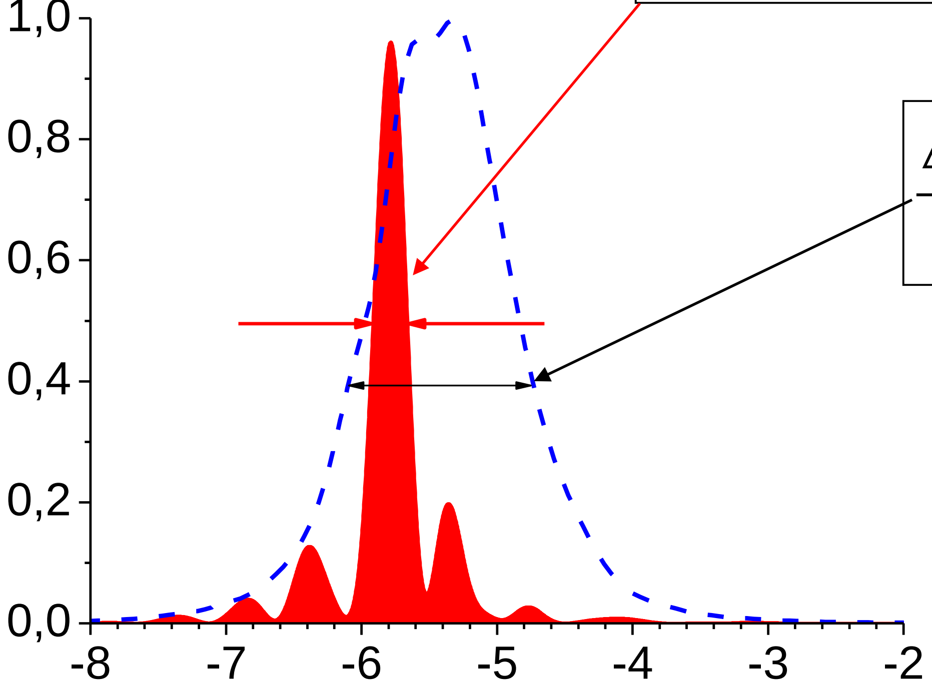
QUANTUM SASE SPECTRUM

$$\frac{\Delta\omega}{\omega} \approx \frac{\lambda}{L_b}$$

(SINGLE SPIKE)

$$\frac{\Delta\omega}{\omega} \approx 2\rho\sqrt{\vec{\rho}}$$

(ENVELOPE)

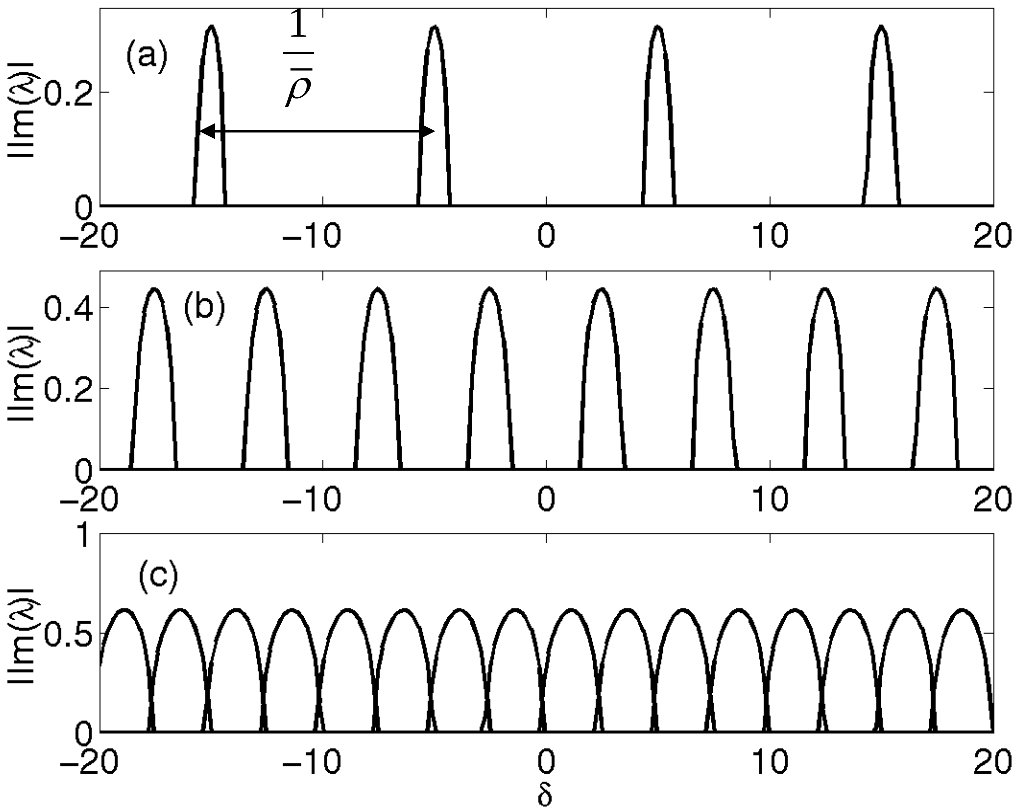


DISCRETE FREQUENCIES AS IN A CAVITY

$\bar{\rho} = 0.1$

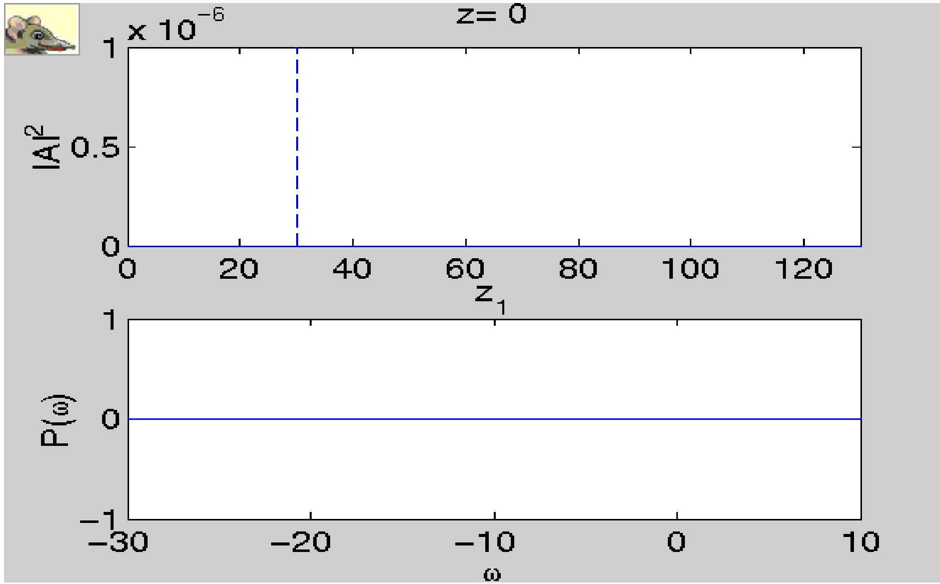
$\bar{\rho} = 0.2$

$\bar{\rho} = 0.4$

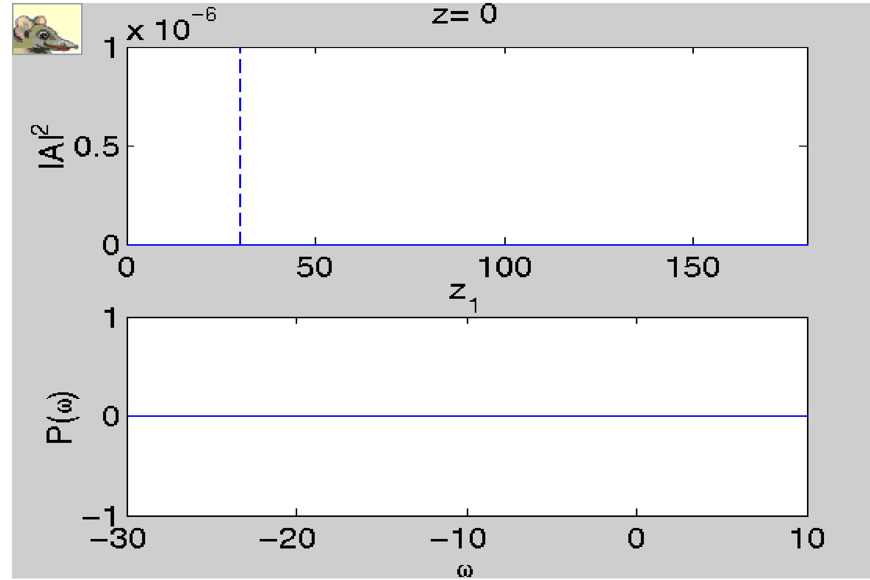


CONTINUOUS LIMIT FOR $\bar{\rho} > 0.4$

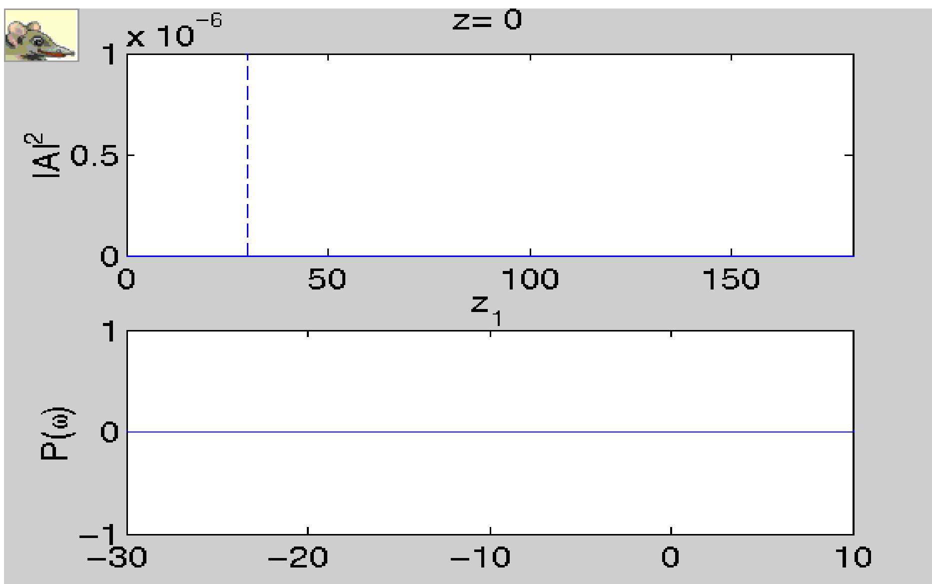
$\bar{\rho} = 0.1 \quad 1/\bar{\rho} = 10$



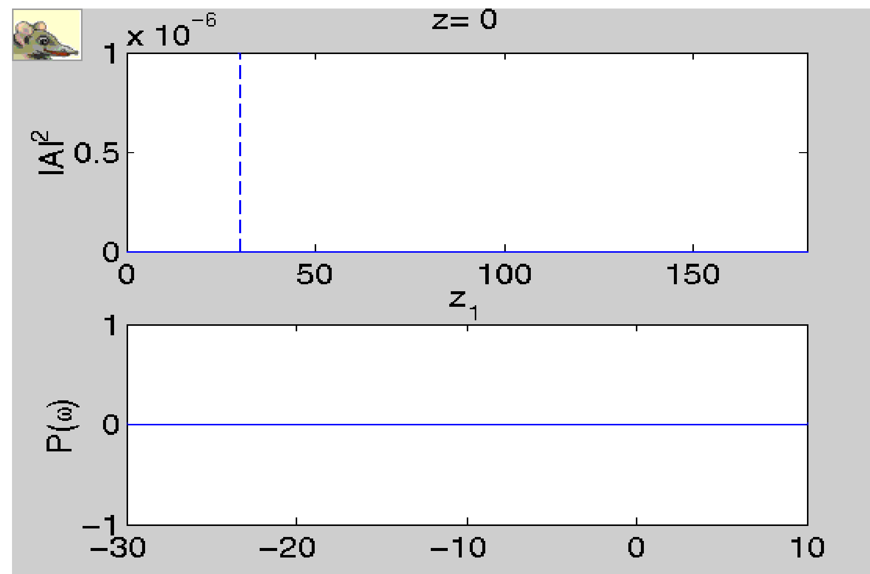
$\bar{\rho} = 0.2 \quad 1/\bar{\rho} = 5$



$\bar{\rho} = 0.3 \quad 1/\bar{\rho} = 3.3$



$\bar{\rho} = 0.4 \quad 1/\bar{\rho} = 2.5$



When a quantum FEL can be realized for x-rays (λ~1 Å) ?

magnetic undulator: $\lambda_r \approx \frac{\lambda_w}{2\gamma^2}(1+a_w^2)$ **λ_w~ 1 cm, E=3.5 GeV**

$$\bar{\rho} < 1 \Rightarrow \rho < 3\cdot 10^{-6} \Rightarrow L_w \approx \frac{\lambda_W}{\rho} \approx 3\text{km!}$$

laser undulator: $\lambda_r \approx \frac{\lambda_L}{4\gamma^2}(1+a_L^2)$ **λ_L~ 1 μm, E=25 MeV**

$$\bar{\rho} < 1 \Rightarrow \rho < 5\cdot 10^{-4} \Rightarrow L_w \approx \frac{\lambda_L}{\rho} \approx 2\text{mm!}$$

needs a laser of high power (TW) and long duration (ps)

$$\left[a_L \approx \sqrt{P_L(TW)}\,\frac{\lambda_L}{R} \right]$$

CONCLUSIONS (I)

We presented a quantum wave model for FELs where the e-beam is described by a coherent macroscopic wavefunction Ψ.

The quantum FEL model:

- allows to define the classical FEL limit;
- predicts that the e-momentum $p_z = mc\gamma$ is discretized in units of the photon recoil $\hbar k$;
- predicts a new quantum regime, with only two momentum states occupied and N coherent emitted photons;
- in the SASE mode operation, a spectral quantum purification occurs, from the classical spiky and broad spectrum to a quantum single-line spectrum.

BREAK

QFEL requires a high quality e-beam

low energy spread: $$\frac{\delta\gamma}{\gamma_0} \leq \rho\sqrt{\bar{\rho}} < \frac{\hbar k}{mc\gamma_0} = \frac{\lambda_c}{\lambda_r\gamma_0} \approx 10^{-4}$$

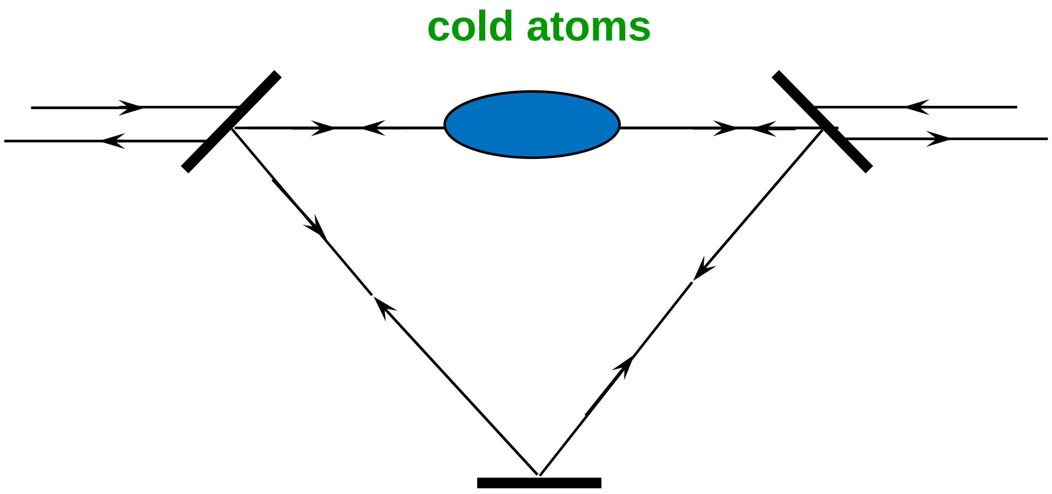
low emittance :

$$\varepsilon_n < \frac{\gamma\lambda_L}{4\pi}\left(\frac{\sigma_{beam}}{\sigma_{laser}}\right)^2$$

$$\varepsilon_n < \frac{\gamma\lambda_r}{2\pi}\sqrt{\frac{Z_r}{L_g}}$$

(geometrical condition) **(inhomogeneous condition)**

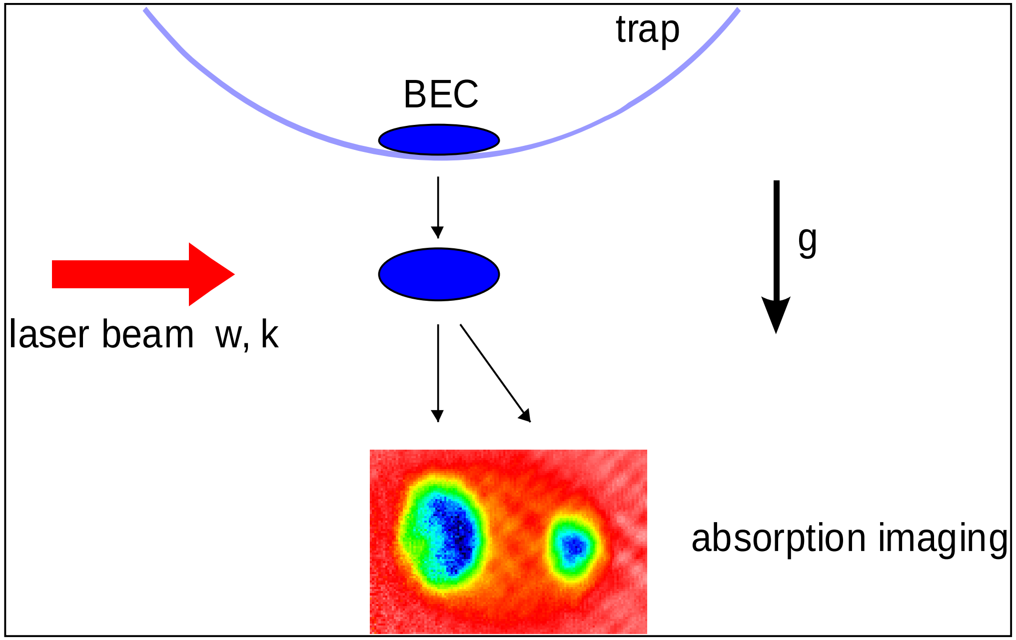
The atomic analogue of FEL: the Collective Atomic Recoil Laser (CARL)



cold atoms driven by a detuned laser emit in a reverse mode growing exponentially and get spatially bunched at the wavelength scale

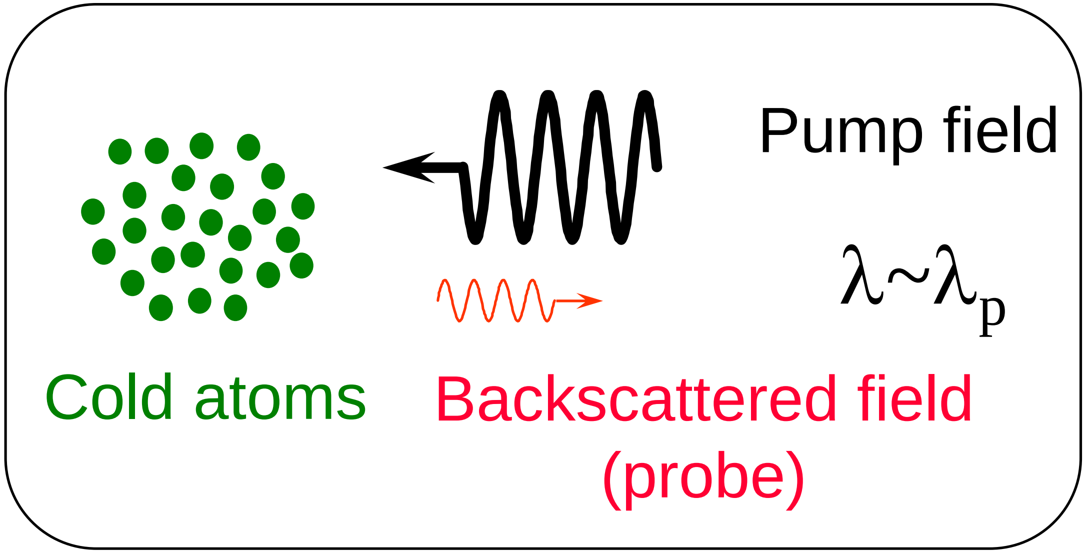
CARL, with cold atoms driven by a pump laser, and FEL are described by the same theoretical model.

The quantum regime of CARL has been already demonstrated with a BEC, with discrete atomic recoil momentum.

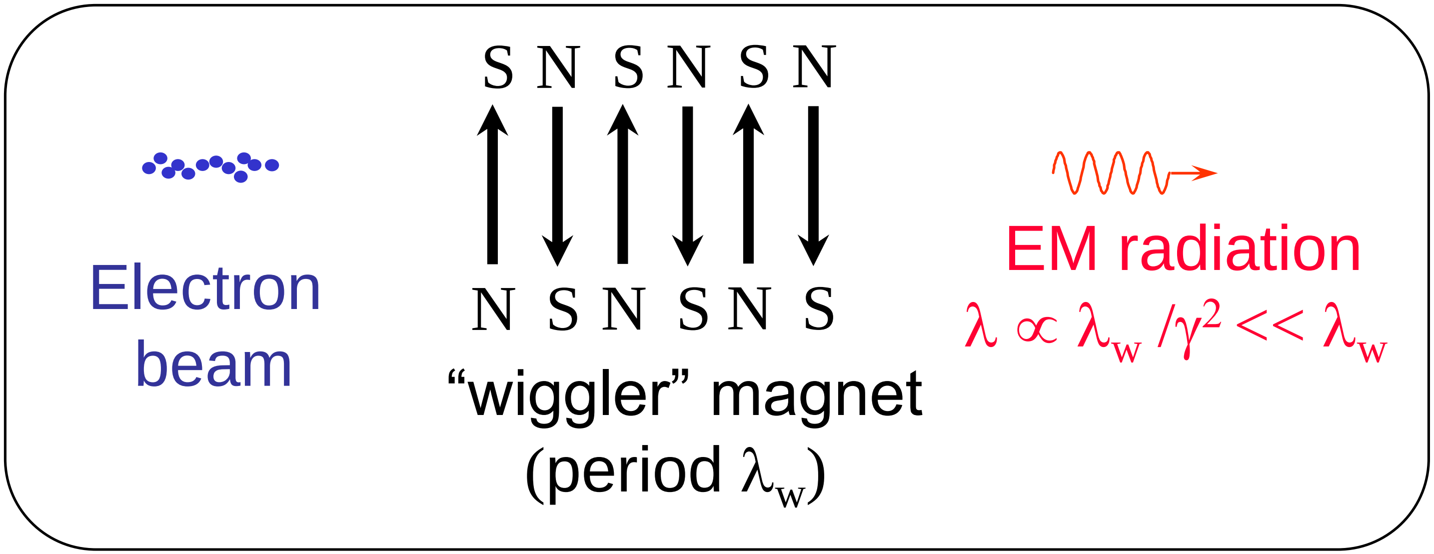


Both FEL and CARL are examples of collective recoil lasing

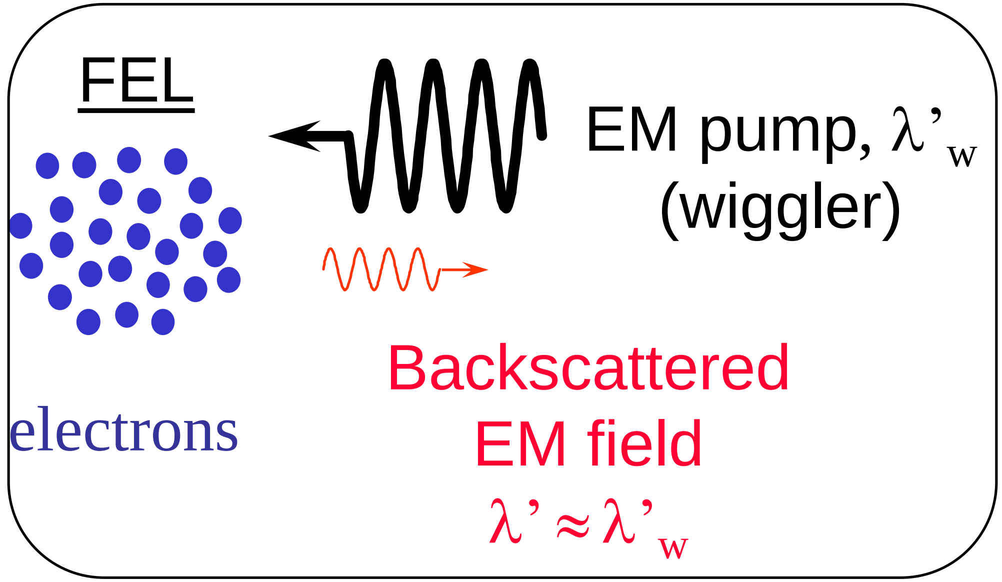
CARL

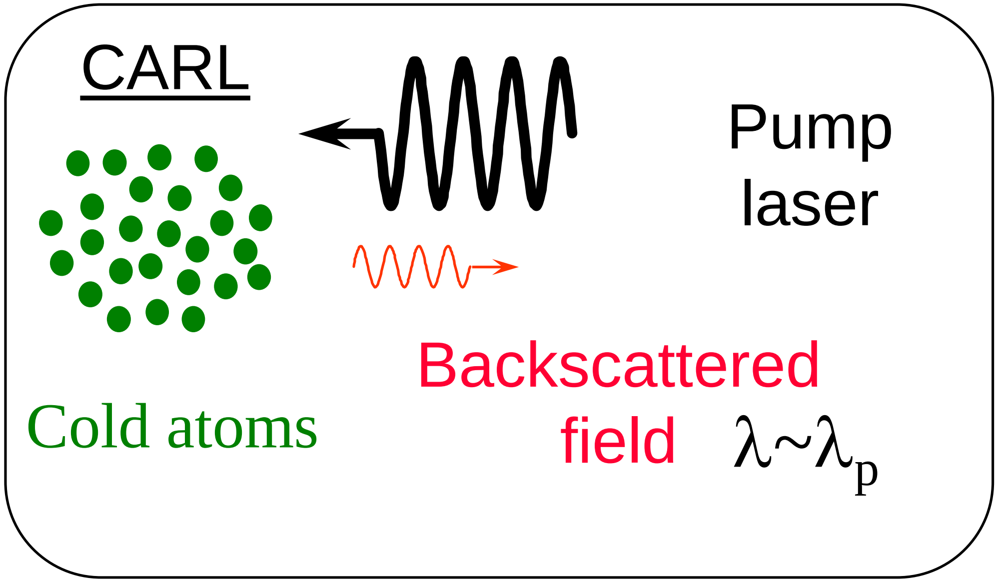


FEL



Connection between CARL and FEL can be seen more easily by transforming to a frame moving with electrons





CARL was first proposed in 1994..

•R. Bonifacio. & L. De Salvo, NIMA 341, 360 (1994)
•R. Bonifacio, L. De Salvo, L.M. Narducci & E.J. D'Angelo PRA 50, 1716 (1994).

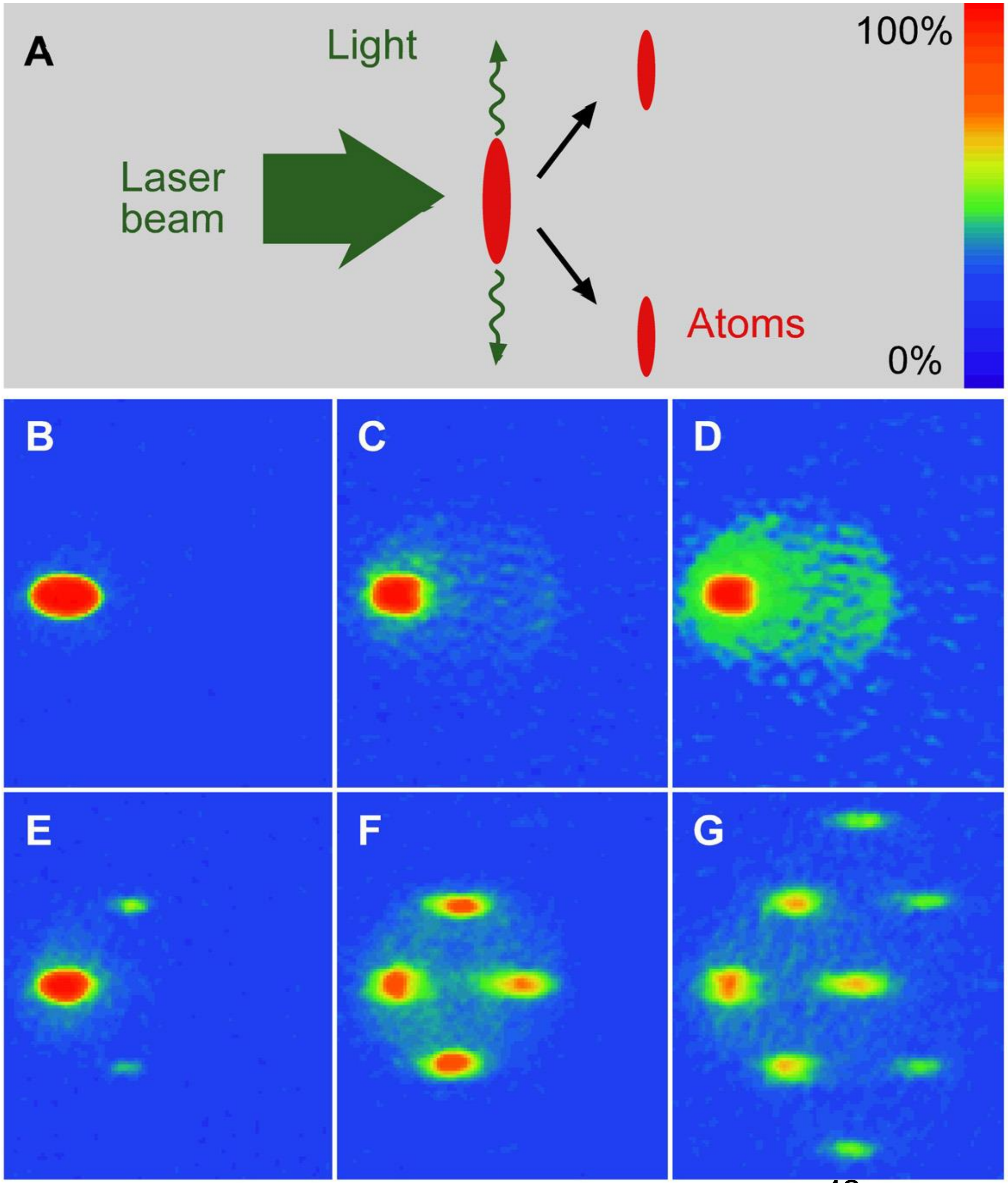
Experiments with BECs without cavity at MIT and LENS:

S. Inouye et al. Science, 285, 571 (1999)
L. Fallani et al. PRA 71, 033612 (2005)

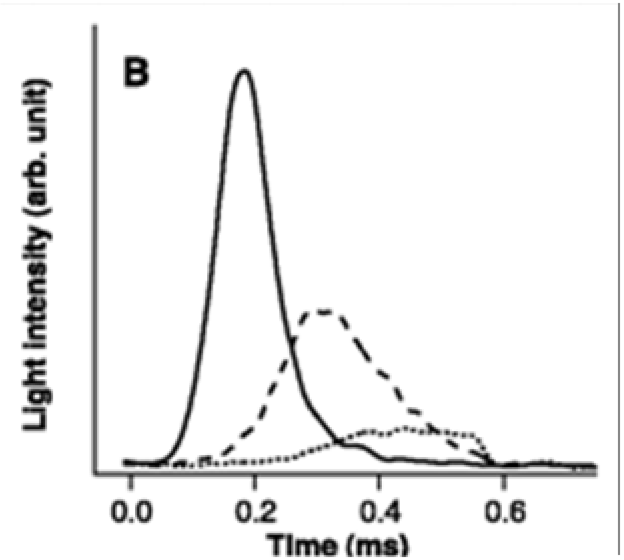
..and experimentally observed at Tűbingen with ring cavity:

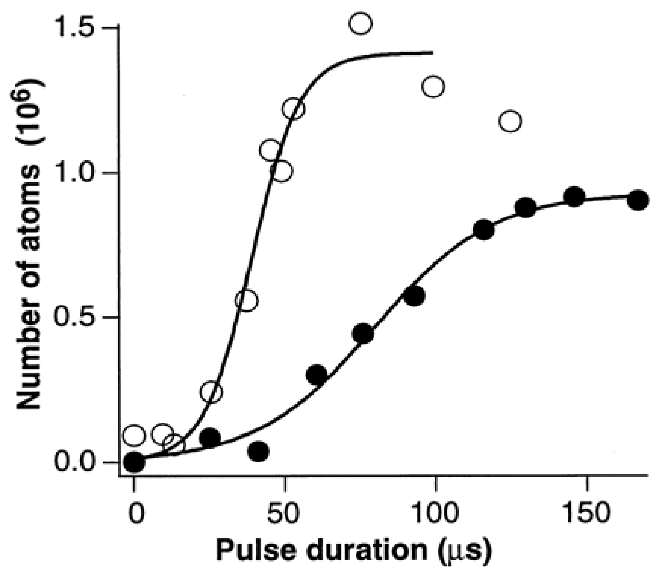
D. Kruse et al., PRL 91, 183601 (2003)
S. Slama et al, PRL 98, 053603 (2007)
S. Bux et al, PRL 106, 203601 (2011)

Superradiant Rayleigh Scattering in a BEC (Ketterle, 1991)



MIT experiment





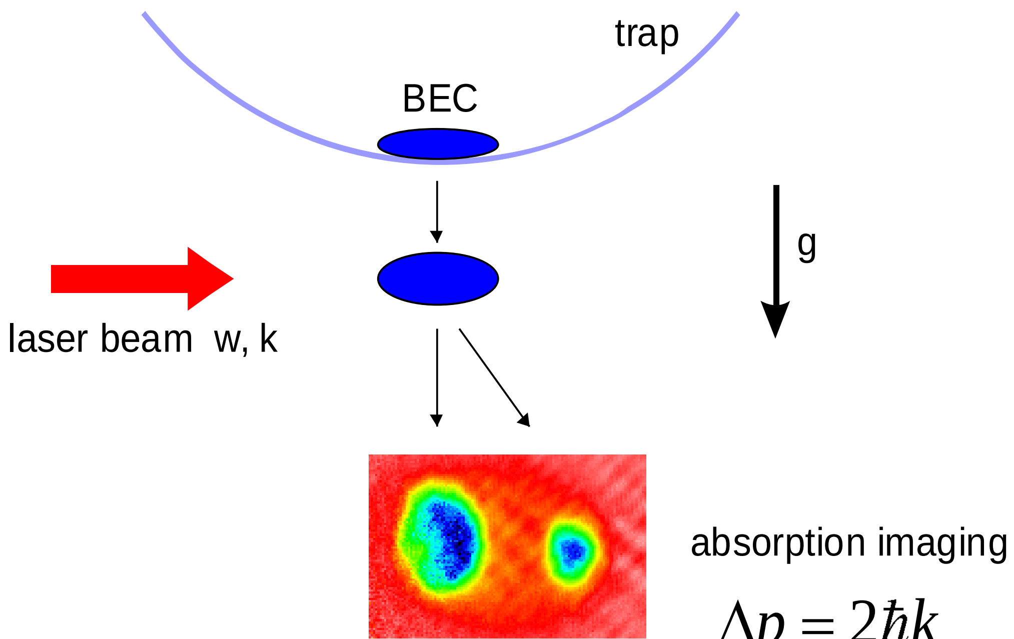
Back scattered intensity for different laser powers: 3.8 2.4 1.4 mW/cm^2 Duration 550 µs

Number of recoiled particles for different laser intensity (25 & 45 mW/cm^2). Total number of atoms $2 \cdot 10^7$

Experimental Evidence of Quantum Dynamics
The LENS Experiment

- Production of an elongated ^{87}Rb BEC in a magnetic trap
- Laser pulse during first expansion of the condensate
- Absorption imaging of the momentum components of the cloud

☺ see Cataliotti's talk on Friday!



$$\Delta p = 2\hbar k$$

Experimental values:

Δ = 13 GHz
w = 750 mm
P = 13 mW

R. Bonifacio, F.S. Cataliotti, M.M. Cola, L. Fallani, C. Fort, N. Piovella, M. Inguscio,

Optics Comm. **233**, 155(2004) and Phys. Rev. A 71, 033612 (2005)

Classical and quantum CARL in a ring cavity at Tűbingen (D)

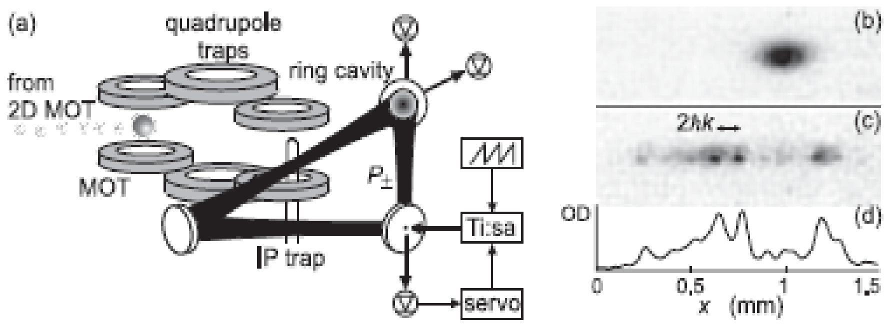


FIG. 1. (a) Schematic view of the experimental setup. A two-dimensional MOT (2D MOT) feeds a MOT in the main chamber. From here the cloud is transferred adiabatically in several intermediate steps into a Ioffe-Pritchard (IP) type magnetic trap overlapping with the ring cavity mode volume. A Ti:sapphire laser resonantly pumps the cavity mode P_+. Both cavity modes $P_\pm$ are observed via the light fields leaking out through one of the cavity mirrors. The atomic cloud can be visualized by absorption imaging. Typical images of a condensate cloud at $T = 0.5T_c$ having and not having interacted with the cavity are shown in (c) and (b), respectively. The images are recorded after 10 ms of free expansion. Curve (d) shows the vertically integrated optical density (OD) of image (c).

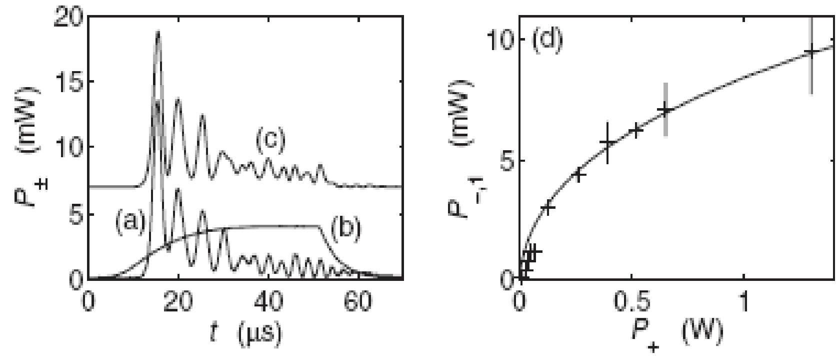
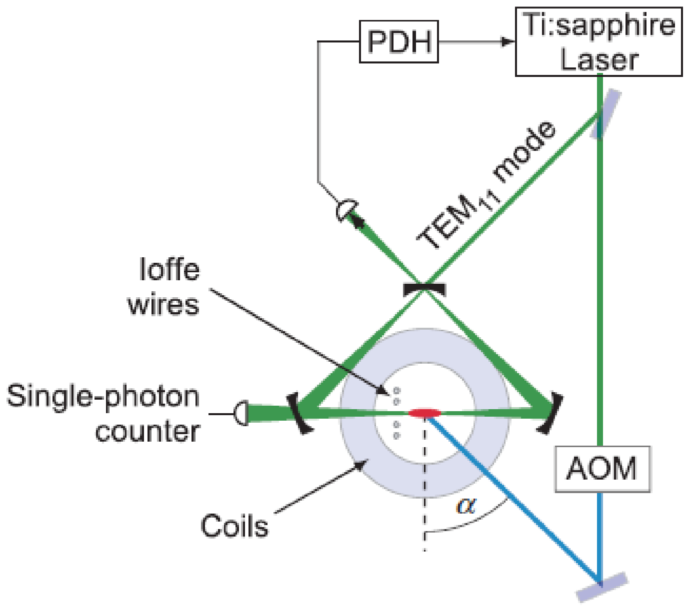


FIG. 2. (a) Measured time evolution of the reverse power P_-. The pump laser power is $P_+ = 4$ W. The cavity is operated at high finesse. The atom number is $N = 1.5 \times 10^6$ and the laser wavelength is $\lambda = 797.3$ nm. Curve (b) marks the time evolution of the recorded pump laser power scaled down by 1000. Curve (c) shows (offset by 7 mW) a numerical simulation of the reverse power using the above parameters (see text). To account for the finite switch-on time of the pump laser power, its experimentally recorded time evolution is plugged into the simulations, where we assume that the pump laser frequency is fixed and resonant to a cavity mode. (d) Measured and calculated (solid line) height $P_{-,1}$ of the first peak as a function of pump power P_+. Here $N = 2.4 \times 10^6$ and $\lambda = 796.1$ nm.

..and 2D QUANTUM CARL (2011)



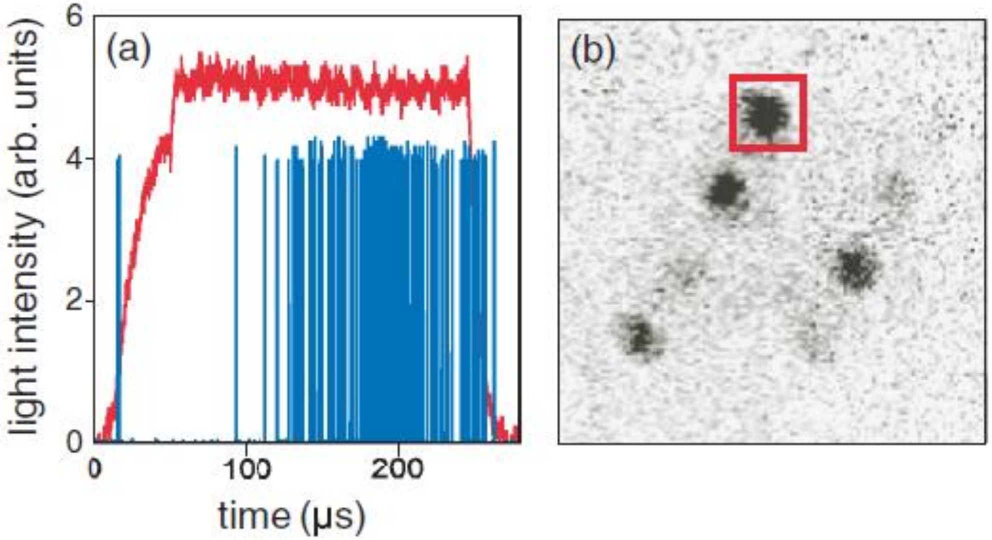
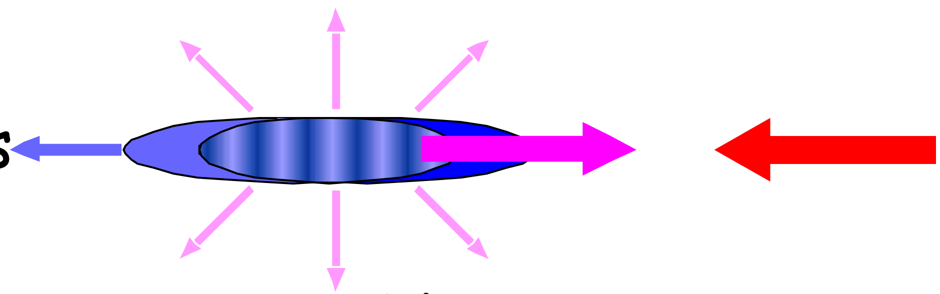


FIG. 2. (Color online) Main signatures of the experiment. (a) A single-photon detector counts the photons which are scattered into the left running cavity mode and leak out through one of the high-reflectivity mirrors. The red (light gray) line indicates the power of the pump beam in arbitrary units. The blue (dark gray) spikes represent single-photon counts. (b) Atomic momentum distribution observed via time-of-flight absorption imaging. The populations in the individual momentum states are obtained by summing the pixel values in a fixed area (red rectangle).

QUANTUM EFFECTS FOR $T < T_{recoil}$

momentum spread $\sigma_p \leq 2\hbar k$ ➡ Requires $T < T_{recoil}$ (a BEC)

$$(k_B T_{rec} = \hbar \omega_{rec})$$

recoiling atoms  pump laser

two possible regimes:

'classical' REGIME:

$\rho >> 1$

$\langle p_z \rangle_{max} \approx \rho (2\hbar k) >> 2\hbar k$

$N_{photon} \approx \rho N >> N$

"quantum' REGIME:

$\rho < 1$

$\langle p_z \rangle = 0 \text{ or } -2\hbar k$

$N_{photon} = N$

CONCLUSIONS (II)

- The quantum discretization of e- momentum p_z has already been observed with BECs (Quantum CARL)
- It can be considered an indirect proof for the Quantum FEL regime.
- May an e-beam be described by a macroscopic wave function Ψ?

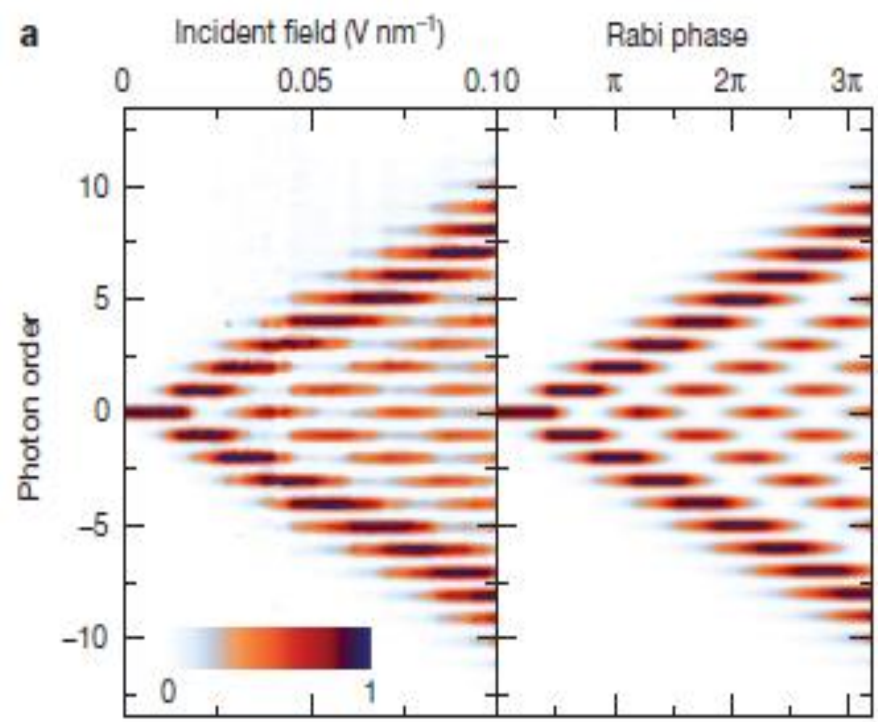
LETTER

doi:10.1038/nature14463

Quantum coherent optical phase modulation in an ultrafast transmission electron microscope

Armin Feist[1], Katharina E. Echternkamp[1], Jakob Schauss[1], Sergey V. Yalunin[1], Sascha Schäfer[1] & Claus Ropers[1]

…
Here we demonstrate the **coherent quantum state manipulation of free electron populations** in an electron microscope beam.
…
Our results reveal the potential of quantum control for the precision structuring of electron densities, with possible applications ranging from ultrafast electron spectroscopy and microscopy to **accelerator science and free-electron lasers**.



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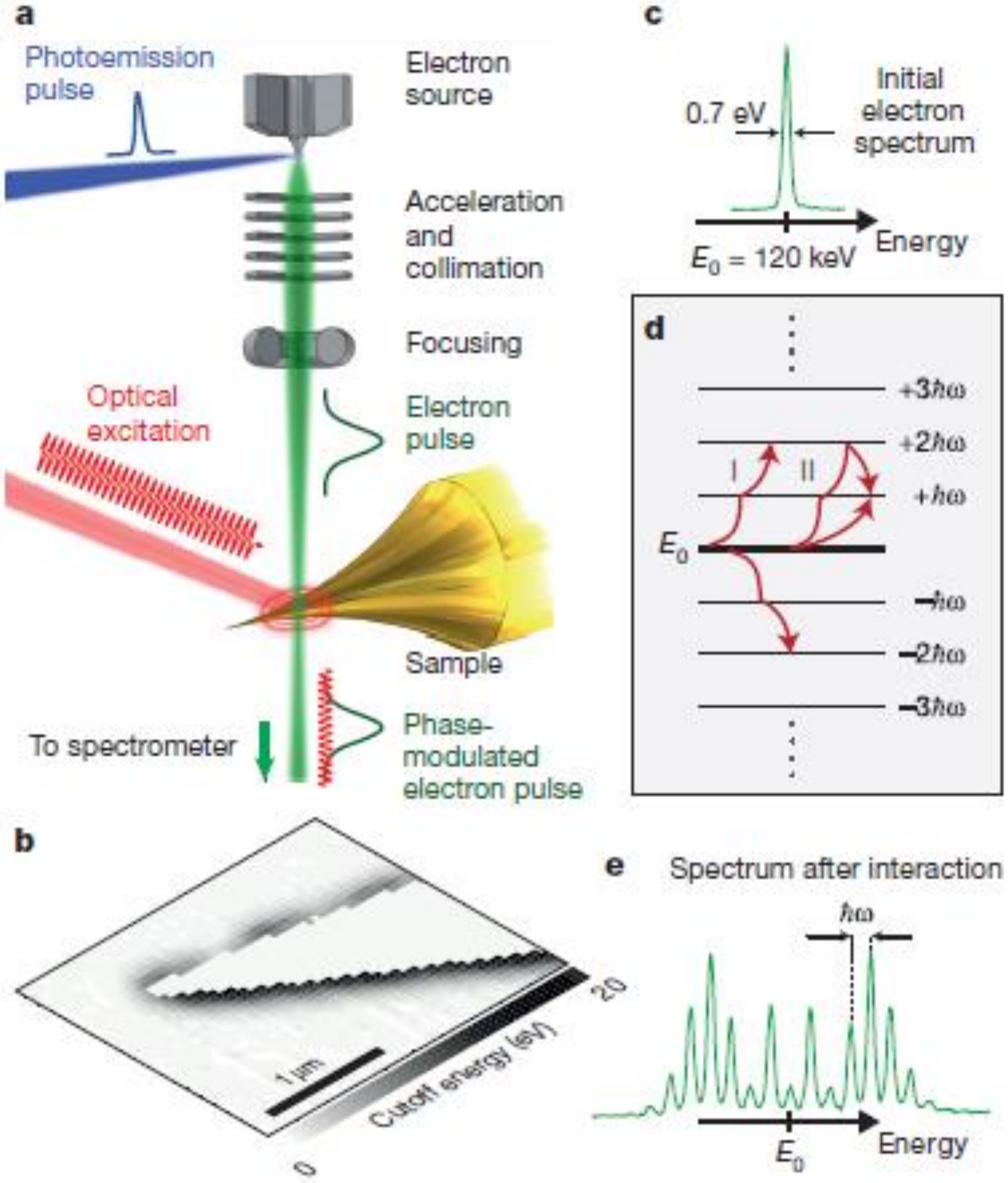


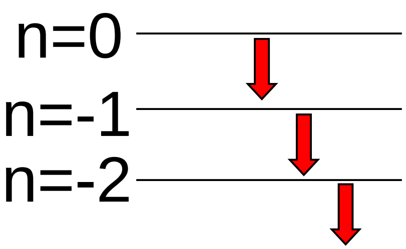
Figure 1 | Schematic and principles of coherent inelastic electron scattering by optical near-fields. **a**, Experimental scheme. Ultrashort electron pulses generated by nanotip photoemission are accelerated and focused to a beam that interacts with the optical near-field of a nanostructure, phase-modulating the electron pulse and exchanging energy in integer multiples of the photon energy. **b**, Raster-scanned image of the energy cutoff in the inelastic electron scattering spectra, representing the local transition amplitude (see text). **c**, Incident kinetic energy spectrum (full-width at half-maximum, 0.7 eV) centred at E_0 = 120 keV. **d**, Energy level diagram of ladder states with spacing $\hbar\omega$ coupled to the initial state at E_0. Arrows indicate sequential multistate population transfer (type I) and interfering quantum paths (type II) leading to multilevel Rabi oscillations. **e**, Example of kinetic energy spectrum after the near-field interaction, exhibiting a spectral comb with multiple sidebands separated by the photon energy and modulated in occupation.

QUANTUM SUPERRADIANT REGIME:

photons escape quickly from the atomic cloud (for large K)..

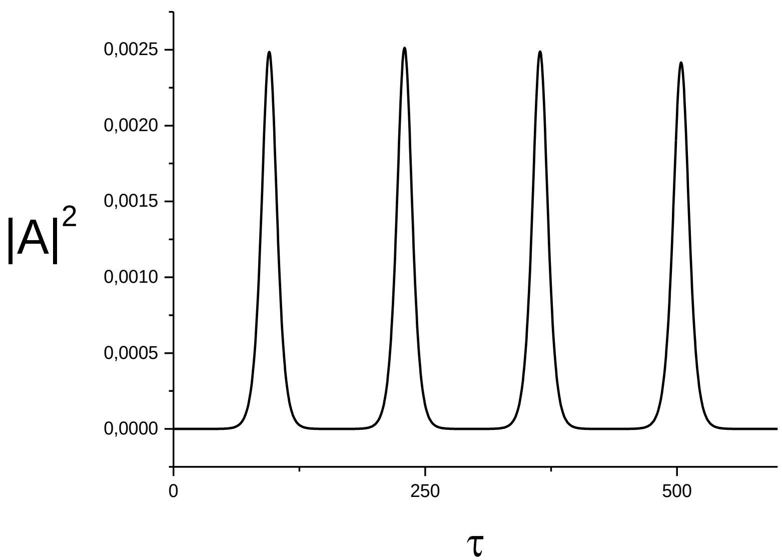
SEQUENTIAL SUPERRADIANT SCATTERING:

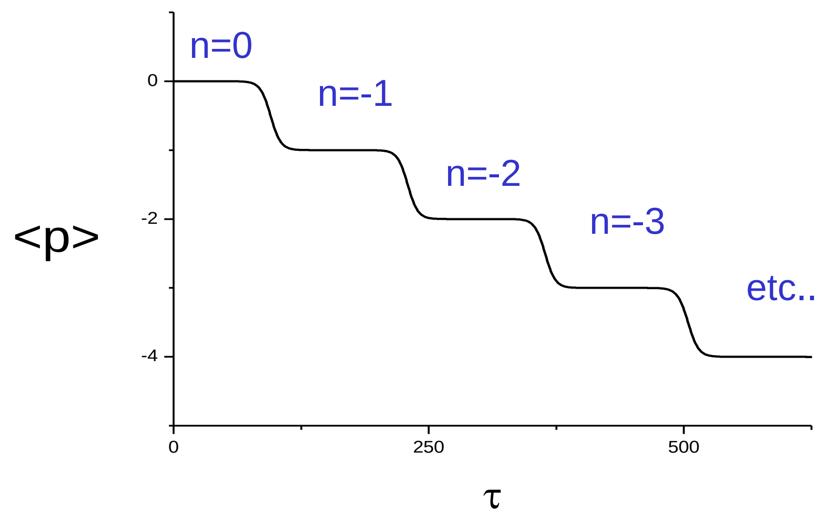
N atoms recoil by **$2\hbar k$**, emitting a SR pulse



$$K_{cav} >> \omega_{rec} > G_{SR}$$

$$G_{SR} = \frac{g_2^2 N}{K_{cav}}$$

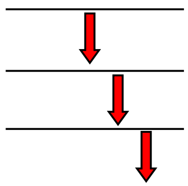




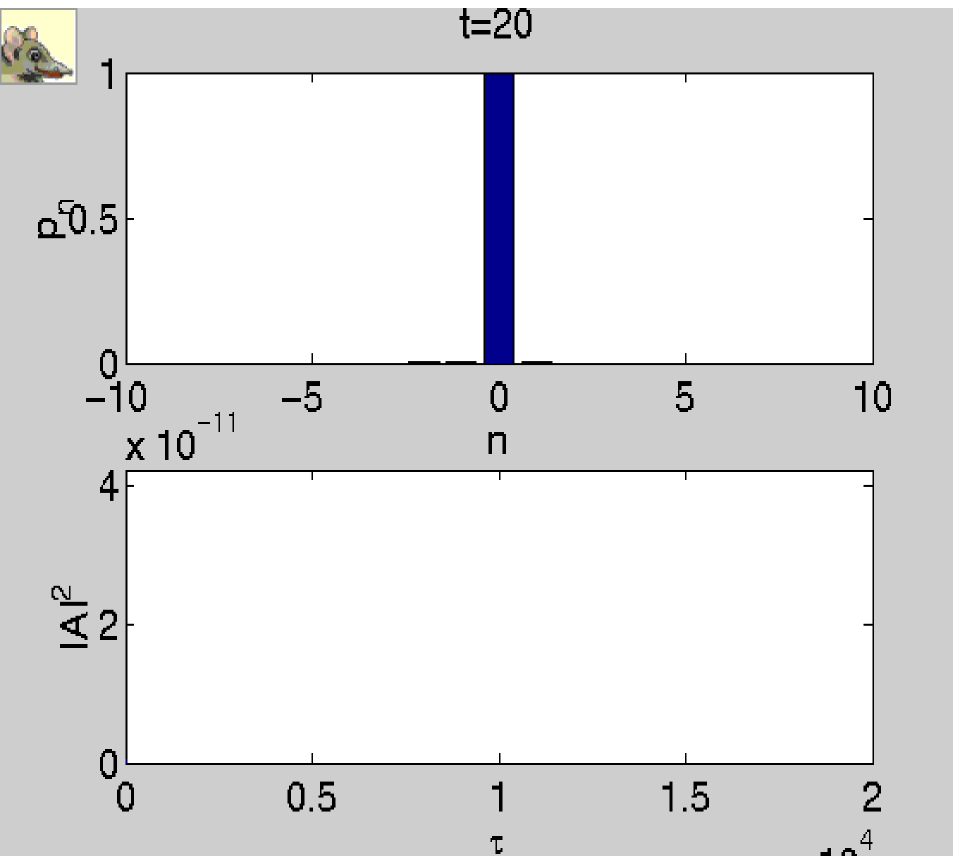
train of sech² pulses

CLASSICAL & QUANTUM SUPERRADIANT CARL:

QUANTUM LIMIT
(sequential SR CARL)



$K_{cav} >> \omega_{rec} > G_{SR}$



CLASSICAL LIMIT
(SR CARL)

$K_{cav} > G_{SR} >> \omega_{rec}$