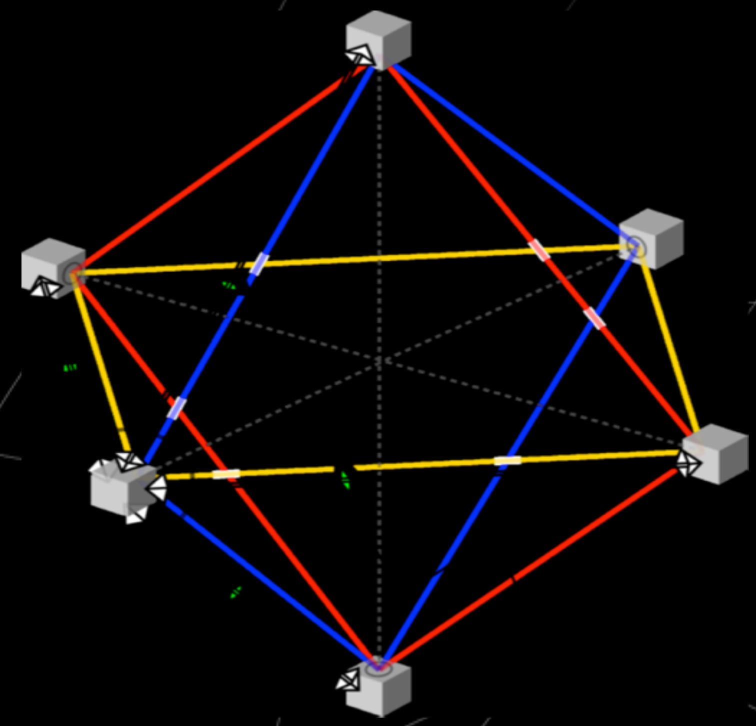
Active Control of the Shape & Pose of optical systems

"How to control the pose and shape of an etherolitic optical system with respect to an absolute reference frame"



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Shape & Pose Definitions

An heterolitic optical system, i.e. a system made by the interaction of several optical elements, can be described as:

- POSE: The joint description of system's position and orientation.
- SHAPE: The geometry of the relative displacement of the optical elements.

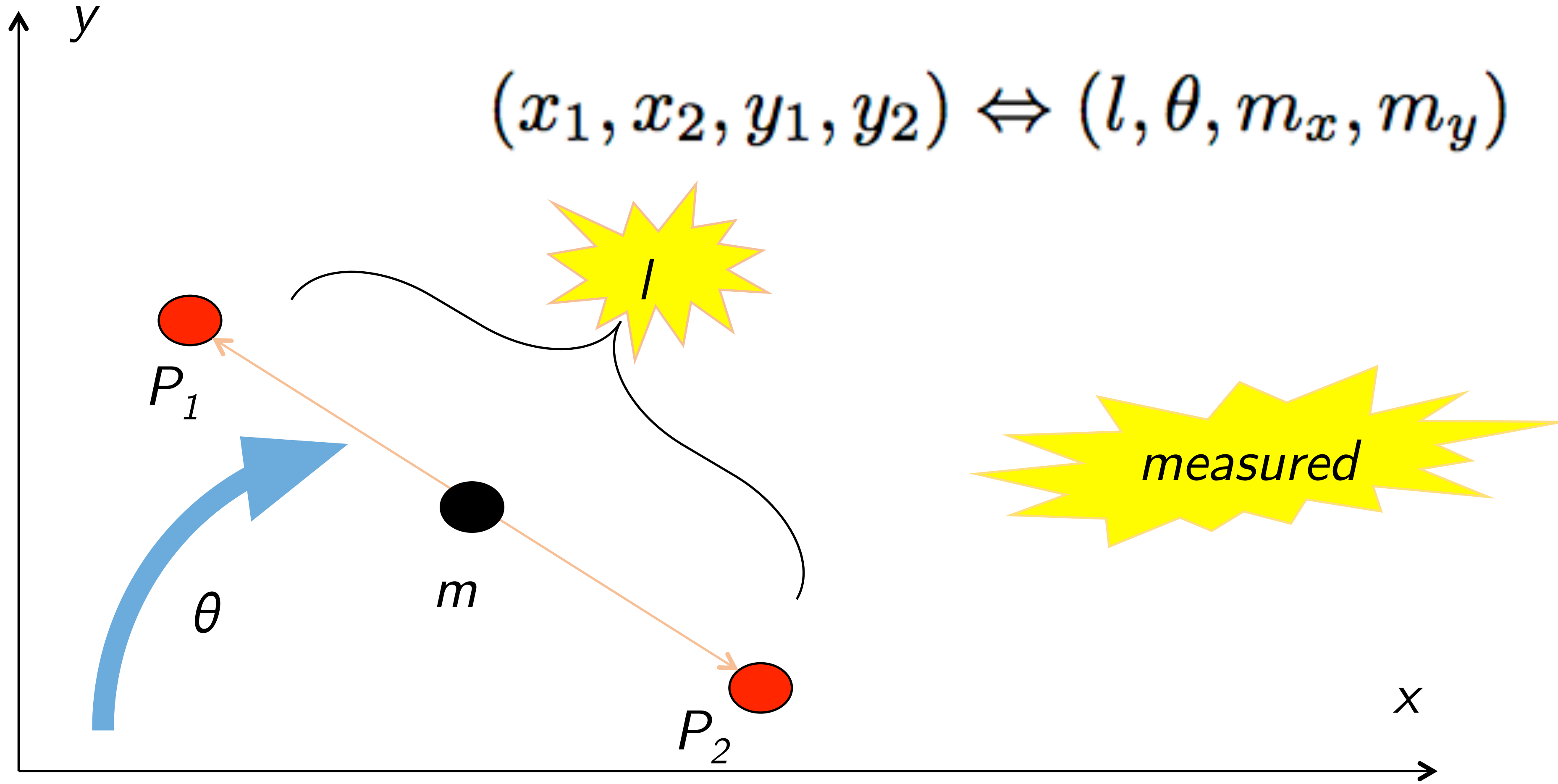
Shape & Pose Assumptions

- Optical elements are represented by tridimensional points.
- The displacement of all optical elements is actuated along fixed directions, thus providing repeatable movements.
- The distances among different system elements are monitored.

2 Optical Elements System, 1/2

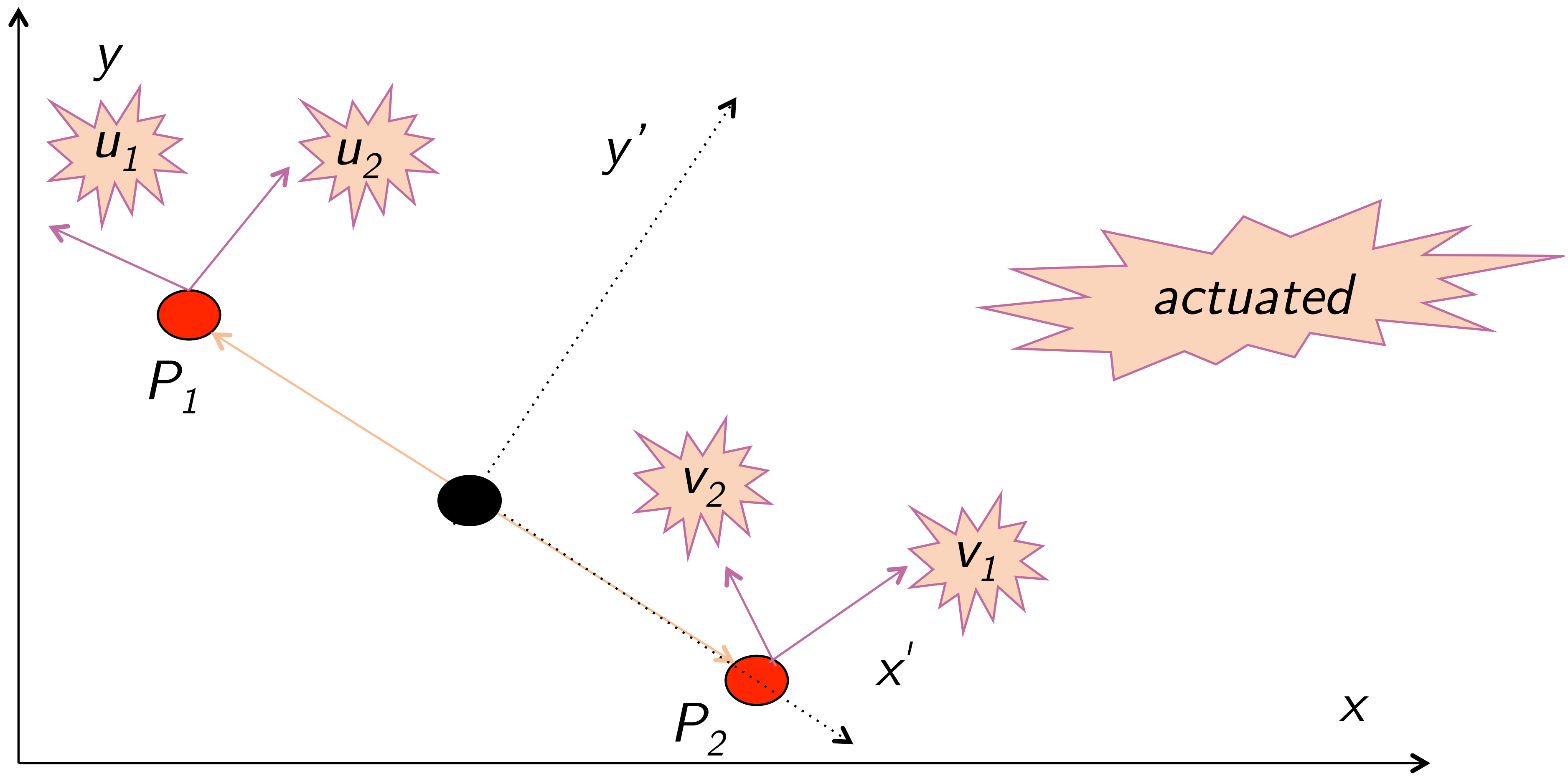
$$P = [\mathbf{p}_1, \mathbf{p}_2], \quad \mathbf{p}_{1,2} \in \mathbb{R}^2, \quad \mathbf{p}_{1,2} = \begin{pmatrix} x_{1,2} \\ y_{1,2} \end{pmatrix}$$

$$(x_1, x_2, y_1, y_2) \Leftrightarrow (l, \theta, m_x, m_y)$$



2 Optical Elements System, 2/2

To account for Shape and Pose change we fix the reference frame. The detection of shape changes is performed with respect to the frame (x′,y′).



Shape/Pose ⟷ Coordinates

$$T(P) = \begin{cases} l & = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ \theta & = \arctan\left(\frac{y_1 - y_2}{x_1 - x_2}\right) \\ \mathbf{m} & = (\mathbf{p}_1 + \mathbf{p}_2)/2 \end{cases}$$

$$T^{-1}(l, \theta, \mathbf{m}) = \begin{cases} \mathbf{p}_1 & = \mathbf{m} + \frac{l}{2}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \\ \mathbf{p}_2 & = \mathbf{m} - \frac{l}{2}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \end{cases}$$

Control Analysis

We impose $$P_{k+1} = \bar{P}$$

With dynamics given by $$P_{k+1} = \bar{P} + P_k - \hat{P}_k = \bar{P} + \delta P_k$$

The system errors in coordinates are, up to the first order

$$\delta P_k = \nabla T^{-1}\left(\hat{l_k}, \hat{\theta}_k, \hat{\mathbf{m}}_k\right)[\delta\theta_k, \delta\mathbf{m}_k]$$

expanding the direct relation T up to the first order we get

$$T\left(P_{k+1}\right) = T\left(\bar{P} + \delta P_k\right) = T\left(\bar{P}\right) + \nabla T\left(\bar{P}\right)[\delta P_k]$$

Control Analysis

The latter equations provide us with:

$$T\left(P_{k+1}\right) - T\left(\bar{P}\right) = \nabla T\left(\bar{P}\right)\left[\nabla T^{-1}\left(\hat{l_k}, \hat{\theta}_k, \hat{\mathbf{m}}_k\right)\left[\delta l_k, \delta\theta_k, \delta\mathbf{m}_k\right]\right]$$

Remark: A shape actuation error at the moment $k+1$

is the effect of a Pose estimation error at the instant k

Conclusions

Thanks for the attention!

The international collaboration

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