

# On the inverse problem for extragalactic cosmic ray nuclei with energies $10^{18}$ to $10^{20}$ eV

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The inverse problem of cosmic ray transport of ultra-high energy cosmic rays is considered. The analysis of Auger data on energy spectrum, energy dependence of mean logarithm of atomic mass number and its variance allows definite conclusions on the shape of the source spectrum in the frameworks of the inverse problem approach. The discussion on regularization procedure for considered ill-posed problem is presented.

## I. INTRODUCTION

The origin of cosmic rays with energies  $E > 10^{18}$  eV is a key problem of cosmic ray astrophysics. The observed suppression of cosmic ray flux at energies above  $\sim 5 \times 10^{19}$  eV seems confirm the presence of the GZK cutoff predicted in [1, 2] although the suppression due to the acceleration limits in cosmic ray sources can not be excluded [3, 4]. The occurrence of the GZK suppression and the high isotropy of the highest energy cosmic rays are indicative of their extragalactic origin. The list of potential sources which could give the observed cosmic ray flux includes active galactic nuclei, gamma-ray bursts, fast spinning newborn pulsars, interacting galaxies, large-scale structure formation shocks and some other objects, see reviews [5–7] and references therein.

The present knowledge about the highest energy cosmic rays was mainly acquired from the High Resolution Fly's Eye Experiment (HiRes), Pierre Auger Observatory (Auger), Telescope Array experiment (TA), and from the Yakutsk complex EAS array, see [5, 8, 9]. The mass composition of these cosmic rays remains uncertain. The interpretation of HiRes and TA data favors predominantly proton composition at energies  $10^{18}$  to  $5 \times 10^{19}$  eV, whereas the Auger data indicate that the cosmic ray composition is becoming heavier with energies changing from predominantly proton at  $10^{18}$  eV to more heavy composition at about  $5 \times 10^{19}$  eV. The mass composition interpretation of the measured quantities depends on the assumed hadronic model of particle interactions which is based on not well determined extrapolation of physics from lower energies.

The energy spectrum in extragalactic sources is commonly determined by the trail-and-error method when one makes the calculations of the expected at the Earth cosmic ray intensity assuming some shape of the source energy spectrum and the source composition. The calculations follow cosmic ray propagation from the source to the observer, e. g. [10]. The standard assumption is that the source spectrum is a power law on magnetic rigidity up to some maximum rigidity.

In our previous work [11] we showed how to inverse the procedure and calculate the source function start-

ing from the observed at the Earth spectrum without ad hoc assumptions about the shape of source spectrum. Simple cases of the source composition that includes protons and Iron nuclei were considered. The more realistic chemical composition including other nuclei is considered in the present work.

## II. SOLUTION OF INVERSE PROBLEM FOR A SYSTEM OF COSMIC-RAY TRANSPORT EQUATIONS

We use the following transport equation for cosmic ray protons and nuclei in the expanding Universe filled with the background electromagnetic radiation (see [12] for detail):

$$\begin{aligned} & -\frac{\partial}{\partial \varepsilon} \left( \varepsilon \left( \frac{H(z)}{(1+z)^3} + \frac{1}{\tau(A, \varepsilon, z)} \right) F(A, \varepsilon, z) \right) \\ & - H(z)(1+z) \frac{\partial}{\partial z} \left( \frac{F(A, \varepsilon, z)}{(1+z)^3} \right) + \nu(A, \varepsilon, z) F(A, \varepsilon, z) \\ & = \sum_{i=1,2,\dots} \nu(A+i \rightarrow A, \varepsilon, z) F(A+i, \varepsilon, z) + q(A, \varepsilon)(1+z)^m \end{aligned} \quad (1)$$

The system of eqs. (1) for all kinds of nuclei with different mass numbers  $A$  from Iron to Hydrogen should be solved simultaneously. The energy per nucleon  $\varepsilon = E/A$  is used here because it is approximately conserved in a process of nuclear photodisintegration,  $F(A, \varepsilon, z)$  is the corresponding cosmic-ray distribution function,  $z$  is the redshift,  $q(A, \varepsilon)$  is the density of cosmic-ray sources at the present epoch  $z = 0$ ,  $m$  characterizes the source evolution (the evolution is absent for  $m = 0$ ),  $\tau(A, \varepsilon, z)$  is the characteristic time of energy loss by the production of  $e^-e^+$  pairs and pions,  $\nu(A, \varepsilon, z)$  is the frequency of nuclear photodisintegration, the sum in the right side of eq. (1) describes the contribution of secondary nuclei produced by the photodisintegration of heavier nuclei,  $H(z) = H_0((1+z)^3 \Omega_m + \Omega_\Lambda)^{1/2}$  is the Hubble parameter in a flat universe with the matter density  $\Omega_m (= 0.3)$  and the  $\Lambda$ -term  $\Omega_\Lambda (= 0.7)$ .

A comprehensive analysis of cosmic ray propagation in the intergalactic space was presented in [13].

The numerical solution of cosmic-ray transport equations follows the finite differences method. The variables are the redshift  $z$  and  $\log(E/A)$ .

Let us introduce solution  $G(A, \varepsilon; A_s, \varepsilon_s)$  of eqs. (1) at  $z = 0$  for a delta-source  $q(A, \varepsilon) = \delta_{AA_s} \delta(\varepsilon - \varepsilon_s)$ . This source function describes the emission of nuclei with mass number  $A_s$  and energy  $\varepsilon_s$  from cosmic ray sources distributed over all  $z$  up to some  $z_{\max}$ . The general solution of eqs. (1) at the observer location  $z = 0$  can now be presented as

$$F(A, \varepsilon, z = 0) = \sum_{A'} \int d\varepsilon' G(A, \varepsilon; A', \varepsilon') q(A', \varepsilon'). \quad (2)$$

The observed all-particle spectrum is determined by the summation over all types of nuclei  $\sum_A F(A, E/A, z = 0)/A$  that is

$$N(E) = \sum_{A, A'} A^{-1} \int d\varepsilon' G(A, E/A; A', \varepsilon') q(A', \varepsilon'). \quad (3)$$

We shall assume below that source spectra of nuclei can be expressed in terms of one function on rigidity:

$$q(A, \varepsilon) = k(A) Q(\varepsilon A/Z) \quad (4)$$

Here  $Q(\varepsilon)$  is the source proton spectrum and coefficients  $k(A)$  determine the source chemical composition.

The set of discrete values of particle energy  $\varepsilon_i$  is defined to solve the transport equation numerically. The grid with constant  $\Delta\varepsilon/\varepsilon$  and with 25 energy bins per decade is used in our calculations. Eq. (3) in the discrete form is

$$N_i = \sum_j S_{ij} Q_j, \quad (5)$$

$$S_{ij} = \sum_{A, A'} \frac{Z(A') k(A')}{A' A} \Delta\varepsilon_j G_{ij}(A, E_i/A; A', \varepsilon_j Z(A')/A'),$$

where the subscript indexes  $i$  and  $j$  denote the corresponding energies  $\varepsilon_i$  and  $\varepsilon_j$ .

The source term  $Q_j$  can be found from this set of linear eqs. (5) if the observed all particle spectrum  $N(E)$  and chemical composition of the source are known. We have already considered the case of protons and iron nuclei in the source [11]. It was found that the solutions of equation (5) have a physical meaning only for a limited range of proton to iron ratio. In addition the solution can be unstable relative small deviations of the left hand side of Eq. (5) so that the inverse problem is ill-posed. We shall used

the following regularization procedure [14] for this set of equations below.

Let introduce the functional  $L$

$$L = \sum_i \left( 1 - \frac{1}{N_i} \sum_j S_{ij} Q_j \right)^2 + \varepsilon_R \sum_j (Q_{j-1} - 2Q_j + Q_{j+1})^2 \quad (6)$$

Here  $\varepsilon_R$  is the regularization parameter. The first term in this equation is simply the sum of squared relative deviations from the observable spectrum  $N(E)$ . For  $\varepsilon_R = 0$  this functional is minimized by solutions of Eqs. (5) and its value equals to zero.

Renormalized set of equations is found from the condition  $\partial L / \partial Q_j = 0$ :

$$\sum_j S_{kj}^R Q_j = N_k^R, \quad N_k^R = \sum_i \frac{1}{N_i} S_{ik}, \quad S_{kj}^R =$$

$$\sum_i \frac{1}{N_i^2} S_{ik} S_{ij} + \varepsilon_R (6\delta_{kj} - 4\delta_{k,j-1} - 4\delta_{k,j+1} + \delta_{k,j-2} + \delta_{k,j+2}), \quad (7)$$

### III. APPROXIMATION OF EXPERIMENTAL DATA

To simplify calculations and damp the spread of data points in the measured at the Earth cosmic ray spectrum, we use its analytical approximations.

The formula

$$J(E) \propto E^{-3.23}, \quad E < 5 \times 10^{18} \text{ eV}; \\ J(E) \propto E^{-2.63} \times [1 + \exp(\log(E/10^{19.63} \text{ eV})/0.15)]^{-1} \times \\ \exp(-(E/(1.5 \times 10^{20} \text{ eV}))^4), \quad E > 5 \times 10^{18} \text{ eV}. \quad (8)$$

is used in our calculations to approximate the Auger data [15]. This formula is similar to the equation suggested by the Auger team but contains  $\exp(-(E/1.5 \times 10^{20} \text{ eV})^4)$  factor of cosmic ray flux suppression at energies  $\gtrsim 1.5 \times 10^{20}$  eV.

### IV. RESULTS

The minimal value  $10^{-3} - 10^{-2}$  of the parameter  $\varepsilon_R$  was adjusted to provide the smooth positive source spectrum  $Q_j$ . We found that this method does not work for any chemical composition. However the range of the chemical composition is strongly extended in comparison with the exact solution of Eq. (5).

The results obtained for light and heavy composition of cosmic ray non-evolutionary sources ( $m = 0$ ) are shown in Figures 1-4. The maximum redshift  $z_{\max} = 3$  was used. The coefficients  $k(A)$  are given in Table I. The light composition corresponds to the

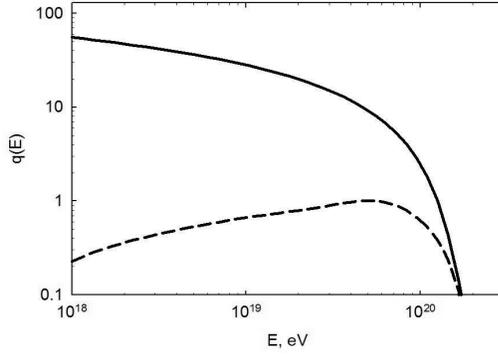


FIG. 1: Calculated source spectra of Iron in arbitrary units obtained for light (solid line) and heavy (dash line) composition.

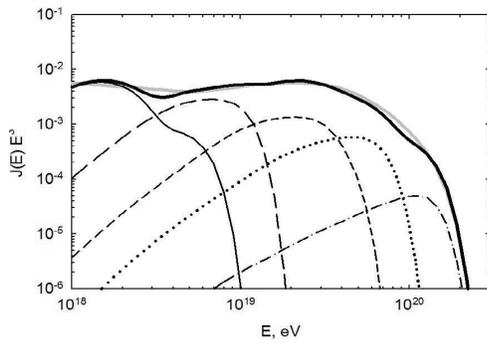


FIG. 2: Calculated spectra of protons (solid), He (dashed), C (short dashed), Si (dotted), Fe (dot-dashed line) for heavy composition of sources. The all particle spectrum (thick solid) and the analytical approximation Auger cosmic ray spectrum (gray solid line) are also shown.

composition of Galactic cosmic rays. We adjust the heavy composition to reproduce the Auger data on energy dependence of the mean logarithm of the atomic mass number  $\langle \ln A \rangle$  calculated in the EPOS-LHC model of particle interactions in the atmosphere [15].

It is evident that our model reproduces the observed all particle spectrum and measured mean logarithm  $\langle \ln A \rangle$ .

## V. DISCUSSION AND CONCLUSION

We showed how one can find average spectrum of extragalactic sources from the cosmic ray spectrum observed at the Earth. This task was formulated as an inverse problem for the system of transport eqs. (1) that describe the propagation of ultra-high energy cosmic rays in the expanding Universe filled with the background electromagnetic radiation.

Mathematically, the inverse problems for transport

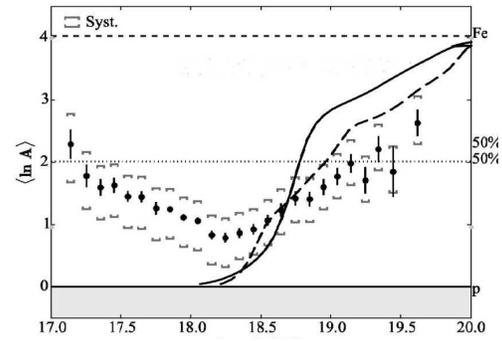


FIG. 3: Calculated value of  $\langle \ln A \rangle$  for light (solid line) and heavy (dashed line) composition together with corresponding Auger data (dots and gray regions which characterizes errors in determination of  $\langle \ln A \rangle$  in the EPOS LHC interaction model).

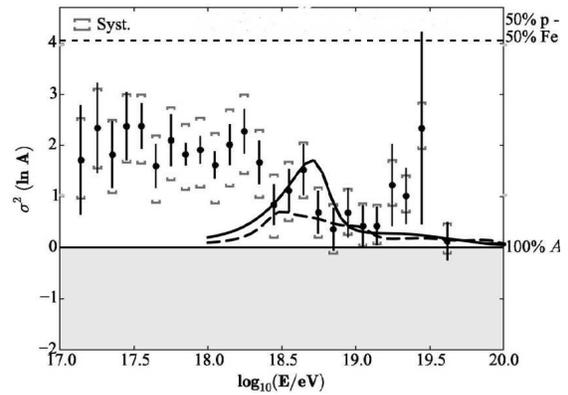


FIG. 4: Calculated variance of  $\langle \ln A \rangle$  for light (solid line) and heavy (dashed line) composition together with corresponding Auger data (dots and gray regions which characterizes errors in determination of variance of  $\langle \ln A \rangle$  in the EPOS LHC interaction model).

equations (1) are ill-posed in the general case that manifests itself in the instability of derived solutions. To avoid this problem we use the regularization procedure (Eqs. 6,7) and perform calculations for a realistic chemical composition. In addition the same spectral function on the rigidity for the source spectra of different nuclei was assumed.

We found that assumption of heavy composition permits to explain the Auger data [15]. The Auger data favor the transition from a proton source composition to the heavier one as the energy is rising. With our heavy source composition, this case is most closely reproduced by the calculations illustrated in figure 2. The obtained source spectra (see figure 1) resemble the results [16, 17] based on the analysis of direct transport problems with a power law source spec-

TABLE I: Coefficients  $k(A)$  describing the chemical composition of sources

	H	He	C	O	Mg	Si	Fe
A	1	4	12	16	24	28	56
light	1	0.2	0.05	0.05	0.015	0.04	0.004
heavy	1	6	0.65	0.2	0.1	0.12	0.015

trum. The maximum energy of accelerated particles  $(3...5)Z \times 10^{18}$  eV is relatively low in this case that alleviates the problem of cosmic ray acceleration. The calculated composition of cosmic rays at the Earth shown in figures 3,4 is also in accordance with the Auger measurements.

The study of inverse transport problem is a useful tool for the investigation of ultra high energy cosmic rays allowing the abandonment of the standard assumption of power law source spectrum with an abrupt cutoff at some maximum magnetic rigidity as it is usually assumed when the direct problem is considered.

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- [1] K. Greisen, *End to the cosmic ray spectrum?*, *Phys. Rev. Lett.* **16** (1966) 748
- [2] G.T. Zatsepin and V.A. Kuzmin, *Upper limit of the spectrum of cosmic rays*, *JETP Lett.* **4** (1966) 78
- [3] D. Allard, *Extragalactic propagation of ultrahigh energy cosmic rays*, *Astropart. Phys.* **39** (2012) 33
- [4] R. Aloisio, V.Berezinsky, P Blasi, *Ultra-high energy cosmic rays: implications of Auger data for source spectra and chemical composition*, eprint *arXiv:1312.7459v1* (2013)
- [5] K. Kotera and A.V. Olinto, *The astrophysics of ultra-high energy cosmic rays*, *Annu. Rev. Astron. Astrophys.* **49** (2011) 119
- [6] M. Lemoine, *Acceleration and propagation of ultrahigh energy cosmic rays*, *JPhCS* **409** (2012) 012007
- [7] P. Blasi, *Origin of very high and ultra high energy cosmic rays* eprint *arXiv:1403.2967v1* 2014
- [8] S.V. Troitsky, *Cosmic particles with energies above  $10^{19}$  eV: a brief review of results*, *Physics Uspekhi* **56** (2013) 304
- [9] A. Watson, *High-energy cosmic rays and the Greisen-Zatsepin-Kuzmin effect*, *Reports on Progress in Physics* **77** (2014) 036901
- [10] D. Allard, *Propagation of extragalactic ultra-high energy cosmic-ray nuclei: implications for the observed spectrum and composition*, eprint *arXiv:0906.3156v1* (2009)
- [11] Ptuskin V.S., Rogovaya S.I., Zirakashvili V.N. *Inverse problem for extragalactic transport of ultra-high energy cosmic rays* *Journal of Cosmology and Astroparticle Physics* 2015. Issue 03. article id. 054.
- [12] V.S. Ptuskin, S.I. Rogovaya and V.N. Zirakashvili, *On ultra-high energy cosmic rays: origin in AGN jets and transport in expanding Universe*, *Adv. Space Res.* **51** (2013) 315
- [13] V.S. Berezinsky, A. Gazizov and S. Grigorieva, *On astrophysical solution to ultrahigh energy cosmic rays*, *Phys. Rev. D* **74** (2006) 3005
- [14] A.N. Tikhonov and V.Y. Arsenin, *Solutions of Ill-Posed Problems*. New York: Winston, 1977
- [15] A. Letessier-Selvon for the Pierre Auger Collaboration, *Highlights from the Pierre Auger Observatory 33rd International Cosmic Ray Conference, Rio de Janeiro, 2013*, in press eprint *arXiv:1310.2118v1* (2013)
- [16] D. Allard, N.G.Busca, G.Decerprit, A.V.Olinto, E.Parizot, *Implications of the cosmic ray spectrum for the mass composition at the highest energies*, *J. Cosmol. Astropart.Phys.* **10** (2008) 33
- [17] R. Aloisio, V.Berezinsky, A.Gazizov, *Ultra high energy cosmic rays: the disappointing model* *Astropart. Phys.* **34** (2011) 620