To visualize the indication of the dipole, an average flux smoothed out at an angular scale $Q$ per solid angle unit can be derived using the joint data set in the following way:

$$f(n_i)Q = \frac{1}{R} \int d^2 \theta \int d^2 \psi f(n, n_0) \bar{w}(n_0) dN(n_0) d\psi,$$

with $f$ the top-hat filter function at the angular scale $Q$. This average flux is displayed using the Mollweide projection in fig. 3, in km$^2$yr$^{-1}$sr$^{-1}$ units. This map is drawn in equatorial coordinates.

To exhibit the dipole structure, the angular window is chosen to be $Q = 60$. The directions of the reconstructed dipole is shown as the blue cross.

The angular power spectrum $C_`$ is a coordinate-independent quantity, defined as the average $|a_m|^2$ as a function of $`$,

$$C_` = \frac{1}{2} \hat{A}_m = \frac{1}{2} \frac{\int \int d^2 \theta \int d^2 \psi f(n, n_0) \bar{w}(n_0) dN(n_0) d\psi}{\int \int d^2 \theta \int d^2 \psi f(n, n_0) \bar{w}(n_0) dN(n_0) d\psi}.$$

In the same way as the multipole coefficients, any significant anisotropy of the angular distribution over scales near $1/`$ radians would be captured in a non-zero power in the mode $`$. Although the exhaustive information of the distribution of arrival directions is encoded in the full set of multipole coefficients, the characterisation of any important overall property of the anisotropy is hard to handle in a summary plot from this set of coefficients. Conversely, the angular power spectrum does provide such a summary plot. In addition, it is possible that for some fixed mode numbers $`, all individual $a_m$ coefficients do not stand above the background noise but meanwhile do so once summed quadratically.

From the set of estimated coefficients $\bar{a}_m$, the measured power spectrum is shown in fig. 4. The gray band stands for the RMS of power around the mean values expected from an isotropic Auger/TA $>10$ EeV.
Cosmic Rays and Propagation Regimes

Origin of ultrahigh energy cosmic rays

Magnetic fields

At low energy Galactic Magnetic Field $B_{\text{GMF}}$ and InterGalactic Magnetic Field $B_{\text{IGMFI}}$ deflect CR particles. UHECRs are very little deflected for $E/Z > 10^{19}$ eV.

Cosmic rays are deflected as:

$\vec{B}_{\text{uG}} \times \vec{r} \approx \text{L}_{\text{pc}}$

$\vec{B}_{\text{rG}}$ is coherent over scales of kpc.

$B_{\text{rms}} = \text{few } \mu\text{G}$

Intergalactic magnetic field?

Only for $E/Z \gg 10^{19}$ eV it is possible to point to the source direction.

Carla MACOLINO hLPHE, EnCNRS Parisi

The Pierre Auger Observatory and Cosmic Ray Physics

Janssen & Farrar, 2012

Harari et al., 1999

Diffusion

Diffusion/Drift

Ballistic?
Anisotropy and CR Observatories

- Challenge: Control of the counting rate at the level of the anisotropy contrast searched for (weather modulations, local-angle dependences of the energy estimators, etc.)
- Of particular interest/difficulty: dipole
- In most cases, searches in right ascension only (be it in several declination bands): not the `real` dipole!

Example of `good` analysis:

Compton-Getting (solar)

Sidereal

Fluctuations (anti-sidereal)

Dipole Observations

- Northern hemisphere: Tibet ASγ, Super-Kamiokande, Milagro, EAS-TOP, MINOS, ARGO-YBJ
- Southern hemisphere: IceCube/IceTop Tibet

≈10^{-3} anisotropy contrast
Dipole Observations

Amplitude increasing up to 10 TeV, and then decreasing

Phase steadily migrating and then suddenly changes/flips
Diffusion and Anisotropy

Propagating in a turbulent magnetic field - resonant mode:

Diffusion approximation: scattering on isotropic diffusion centers each $\lambda$

Angular distribution at the observation point:

$$\Phi_{obs}(\theta, \varphi) = \frac{c}{4\pi \lambda} \int_0^\lambda dr \, \tilde{n}(r, \theta, \varphi).$$

Density to first order:

$$\tilde{n}(r, \theta) \approx \tilde{n}(r \cos \theta, 0) \approx \tilde{n}(0) + r \cos \theta \frac{\partial \tilde{n}(0)}{\partial r}$$

$$\Phi_{obs}(\theta, \varphi) \approx \frac{c}{4\pi} \tilde{n}(0) \left( 1 + \frac{\lambda}{2} \tilde{n}(0) \frac{\partial \tilde{n}(0)}{\partial r} \cos \theta + ... \right)$$

$$\Phi_{obs}(\mathbf{n}) \approx \frac{c}{4\pi} \tilde{n}(0) \left( 1 + \frac{3D \tilde{\nabla} \tilde{n}(0)}{c} \cdot \mathbf{n} + ... \right)$$

$$\Delta = \frac{3D \tilde{\nabla} \tilde{n}(0)}{c} = \frac{3D \tilde{\nabla} n(r_o)}{c}$$

- For each single source, anisotropy $\approx$ dipole
- A dipole is represented by a vector
- Observed dipole = Sum of all individual dipoles
Diffusion and Dipole

- Relevant benchmark scenario: discrete sources stochastically distributed in space and time in the Galaxy [Erlykin & Wolfendale (2006)]

Amato & Blasi (2011): Green function satisfying the boundary conditions from the geometry of the Galaxy

(see also Erlykin & Wolfendale (2006), Ptuskin et al. (2012), Pohl & Eichler (2012), Streshnikova et al. (2013), Kumar & Eichler (2014), Mertsch & Funk (2014))

- Observed dipole dominated by the most `local` source at some energy
- In average, increase with energy (diffus. coeff.), but compensation mechanism due to the summing rule of the dipoles in each realization + changes in mass composition
- Abrupt changes of phase with energy, when the contribution of one source dominates the global vector (`local` source)
Impact of Local Environment

- Anisotropic diffusion induced by the local ordered magnetic field [Ahlers, 1605.06446]
  - Larmor radius much smaller than typical scattering length in local ordered $B$
    - Diffusion tensor $K_{ij} = \frac{\hat{B}_i \hat{B}_j}{3\nu_\parallel} + \frac{\delta_{ij} - \hat{B}_i \hat{B}_j}{3\nu_\perp} + \frac{\epsilon_{ijk} \hat{B}_k}{3\nu_A}$ dominated by the first term
  - Projection of the CR gradient onto the magnetic field direction

- Subtracting CG effect corresponding to the Sun’s motion towards the solar apex
- Equatorial components of the dipole corrected for projection effects

Toy model: impact of Vela SNR on the average anisotropy of all Galactic SNRs:
Beyond the Dipole Observations

- Features observed by Milagro, ARGO, IceCube, HAWC

- Differences between Northern experiments and IceCube: energy dependence, trigger bias against heavy nuclei for IceCube

- Indications for harder spectra in some regions of excess

\( \approx 10^{-4} \) anisotropy contrast
Beyond the Dipole Observations

From Gradient Density to Angular Distribution

Effect of the magnetic turbulence geometry within the last sphere of diffusion [Giacinti & Sigl, 2012, also Ahlers 2014]

- Connect the observed direction $n$ to the flux at the entrance point of the last diffusion sphere

- Liouville: the isotropic flux outside the scattering mean free path remains isotropic
- Higher-order multipoles generated from the initial dipole, conserving the anisotropic fraction of the flux
- Energy-dependent structures at different angular scales
Probing the Magnetic Field Turbulence?

Test particle trajectories integrated in a compressible sub-Alfvénic isothermal MHD turbulence in low gas-to-magnetic pressure value $\beta$ [Lopez-Barquero et al., 2015]

- $\beta=0.2, \, M_A=0.773$
- Controlled parameter in this MHD model of the turbulence: external mean magnetic field
- Reasonable reproduction of the power spectrum
Small Scales and Heliospheric Electric Field

[Drury, 2013]

- e.m. field purely magnetic in the plasma rest frame, but appearing with an induced electric component in a moving frame

- Above TeV energies, little deflections while penetrating the heliosphere

- With heliospheric length scale of ≈100 AU, velocity scale of ≈10^4-10^5 m/s and magnetic field of ≈nT, induced potential shift of ≈100 MV-1 GV

  - For same incoming directions/energy bands, TeV energies shifted at the ≈10^{-4} level
  
  - Small-scale TeV anisotropies as the signatures of the heliospheric electric field structure

Alternative: Magnetic funnelling [Drury & Aharonian 2006, Salvati 2010], Anisotropic turbulence [Malkov et al., 2010], Local source in the heliotail [Lazarian & Desiati, 2010], Dark matter [Harding 2013], etc.
PeV-EeV Anisotropies

- Energy range where the Larmor radius exceeds the largest turbulence scales $\rightarrow$ Effect of the regular field amplified
- The full diffusion tensor matters, the non-diagonal elements inducing drift motions (Ptuskin et al., 1993)
- Direction of the dipole: not necessarily aligned with the dominant source(s):
  \[
  \delta_r = \frac{3}{cN} \left( -D_\perp \frac{\partial N}{\partial r} + D_{A\text{sign}}(B^\phi_{\text{reg}}) \frac{\partial N}{\partial z} \right), \quad \delta_z = \frac{3}{cN} \left( -D_\perp \frac{\partial N}{\partial z} - D_{A\text{sign}}(B^\phi_{\text{reg}}) \frac{\partial N}{\partial r} \right)
  \]
- Amplification/Distortions of the density gradients:

\[\text{Candia et al. (2002)}\]

★ Solutions for stationary states only
PeV-EeV Observations

direction/amplitude of the dipole in right ascension:

- Phases fairly consistent about the RA of the GC
- Possible/expected increase of amplitudes do not compensate the decrease in statistics
  - Much larger exposure needed to probe the amplitudes
EeV and >EeV CRs

- Hard, meal-rich injection, low cutoff ($R_{cut} < 10^{18.7} \text{ V}$)
  
- Mainly due to narrow $X_{\text{max}}$ distributions (little mixing of different masses at the same energy)
  
- NB: Relies on extrapolations of the mass at UHE

Di Matteo (Auger collab.), 2015
EeV and >EeV Large-Scale Anisotropies

- Back-tracking anti-particles with random directions from the Earth to outside the Galaxy [Thielheim & Langhoff 1968, J. Phys A 694]
- Each test particle probes the total luminosity along the path of propagation from each direction as seen from the Earth
- For stationary sources emitting equally in all directions, the time spent in the source region is proportional to the flux detected in that direction

→ EeV protons not from the Galaxy

★ Caveat: Temporal-dependent solutions for discrete sources?
EeV and >EeV Observations

- At EeV energies, two different components with (almost) opposite phases?

- Need for composition-based measurements of anisotropies...

(Auger collab., 2013)
Full-Sky Map > 10 EeV (Auger/Telescope Array)

Equatorial Coordinates - 60° smoothing

Power Spectrum

199% CL isotropy

• Interesting dipole effect (≈3σ) to monitor

Constraints on Extragalactic Scenarios?

- Benchmark-scenario: Dipole at the entrance of the Galaxy
- Back-tracking technique to connect the observed n (random) to the flux outside from the Galaxy
  - Dipole not `destroyed` by the GMF (JF12 model here)
  - Detection of higher orders: probe of the extragalactic CR gradient outside from the Galaxy
UHE Anisotropies (?)

J. Miró - Drowned Sun
Southern hemisphere (Auger, 2015)

- Scan on energy threshold $E$ and circular window radius $\Psi$ to compute the obs/exp number of events
- $4.3\sigma$ for $E>54$ EeV and $\Psi=12^\circ$
- Post-trial p-value: 69%

Cross-correlation with catalogs of extragalactic matter:

<table>
<thead>
<tr>
<th>Objects</th>
<th>$E_{th}$ [EeV]</th>
<th>$\Psi$ [$^\circ$]</th>
<th>$D$ [Mpc]</th>
<th>$f_{min}$ [erg/s]</th>
<th>$\mathcal{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2MRS Galaxies</td>
<td>52</td>
<td>9</td>
<td>90</td>
<td>$1.5\times10^{-3}$</td>
<td>24%</td>
</tr>
<tr>
<td>Swift AGNs</td>
<td>58</td>
<td>1</td>
<td>80</td>
<td>$6\times10^{-5}$</td>
<td>6%</td>
</tr>
<tr>
<td>Radio galaxies</td>
<td>72</td>
<td>4.75</td>
<td>90</td>
<td>$2\times10^{-4}$</td>
<td>8%</td>
</tr>
</tbody>
</table>

No significant indication of anisotropy
Northern hemisphere (Telescope Array, 2016)

- `Hot spot` status above 57 EeV
  - 109 events, 24 within the 20° window (6.88 exp.)
  - Post-trial significance: $P=3.7 \times 10^{-4}$ (3.4σ)
  - Same significance as in 2014 (2 more years of data analysed here)

- Energy spectrum ON/OFF:

- Correlation with LSS?

Expectations from LSS for protons >57EeV smeared through GMF:

- Tension with isotropy?
Multi-Messenger Approach: IceCube/Auger/TA

- Cross-correlations between UHECRs and IceCube neutrinos

- Smallest p-value: IceCube high-energy cascades, angular scale of 22°, post-trial p-value: $5 \times 10^{-4}$ (considering isotropic UHECRs)

- To be continued...
Summary

- Anisotropies up to ≈PeV energies well established
  - Not only dipoles!
  - Important developments for local CR propagation

- Quest of UHECR origin more difficult than expected
  - No small-scale clustering observation, only dipoles seem at reach!
  - Need for composition-based searches
  - Need for (much) larger exposure keeping similar resolutions...