Higher order QED radiative corrections to Bhabha scattering

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Outline

- Bhabha: hierarchy of contributions
- Higher order corrections: how much?
- NLO factorization for exclusive Bhabha
- MC integrator
- 2-loop Soft + Virtual
- 2 Hard photons
- 1 Hard photon $\otimes$ 1-loop Soft + Virtual
- e^+e^- soft + virtual pair corrections
- Outlook

Event selection conditions

Numerical illustrations are given for the simple set of conditions:

- $E_{\rm beam} = 1$ **GeV**
- $\pi/3 < \theta_{+,-} < 2\pi/3$
- $E_{+,-} > 50$ **MeV**
- $\theta_{\rm acoll} > 30$ **mrad**
- **no** $e\gamma$ **recombination: BARE event selection**

Bhabha: hierarchy of contributions

- **At the Born level:** $t-$**channel dominates even for large angles:** $\sigma_{\rm total} = 104.5419$ **nb** $=$ $\sigma_t + \sigma_s + \sigma_{\rm int}$,

  $\sigma_t = 118.43$ **nb,** $\sigma_s = 8.82$ **nb,** $\sigma_{\rm int} = -22.71$ **nb**

- Z**-boson exchange is suppressed by (at least)** s/M_Z^2**:**
  $\sigma_{\gamma+Z} = 104.5363$ **nb,** $\delta_Z = -0.005\%$

- $\mathcal{O}(\alpha)$**:** $\frac{\alpha}{\pi} \approx \frac{1}{400} \sim 0.5\%$**, but we get a few percent** (-4.4%) **due to large logs,** $L = \ln(M^2/m_e^2)$**, where** $M^2 = t,\ s,\ tu/s,\ \ldots$

- **Vacuum polarization:** $\sim \frac{\alpha}{\pi} \sum \ln(M^2/m_i^2)$
  so that we get as much as $+7.7\%$

- $\mathcal{O}(\alpha^2)$**: expand in the large log**
  LLA (LO) $\mathcal{O}(\alpha^2 L^2)$, **NLO** $\mathcal{O}(\alpha^2 L^1)$, **NNLO** $\mathcal{O}(\alpha^2 L^0)$

- $\mathcal{O}(\alpha^n)$, $n \geq 3$: **LLA** $\mathcal{O}(\alpha^n L^n)$; **exponentiation**

Particular NLO contributions: Soft+Virt (2)

$L = \ln \frac{M^2}{m_e^2}$ $\qquad$ $E_{\gamma_i} < \Delta E_{beam}$

A.A. et al., JETP Lett. 2006: $\qquad$ **using $r_0^{(2)}$ from A. Penin**

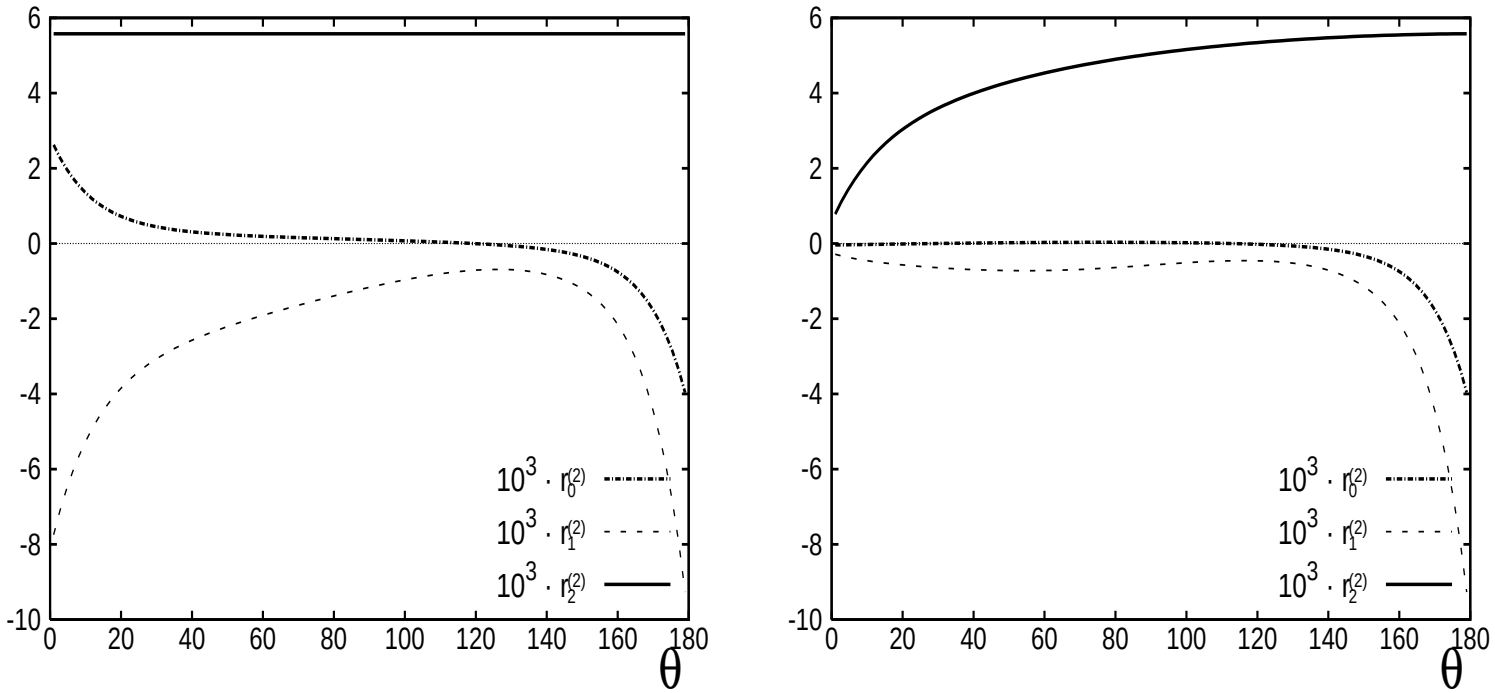

Figure: Soft and virtual second order photonic radiative corrections *versus* the scattering angle in degrees for $\Delta = 1$, $\sqrt{s}$=1 GeV; $M = \sqrt{s}$ on the left side and $M = \sqrt{-t}$ on the right side.

QED factorization theorem

QCD factorization theorem can be adopted for the QED case
e.g. **for Bhabha:**

$$\mathrm{d}\sigma = \sum_{a,b,c,d=e^{\pm},\gamma} \int_{\bar{z}_1}^{1} \mathrm{d}z_1 \int_{\bar{z}_2}^{1} \mathrm{d}z_2 \mathcal{D}^{\mathrm{str}}_{ae^-}(z_1)\mathcal{D}^{\mathrm{str}}_{be^+}(z_2)\Big[\mathrm{d}\sigma^{(0)}_{ab\to cd}(z_1,z_2) + \mathrm{d}\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2)$$
$$+ \mathcal{O}\left(\alpha^2 L^0\right)\Big] \int_{\bar{y}_1}^{1} \frac{\mathrm{d}y_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{\mathrm{d}y_2}{Y_2} \mathcal{D}^{\mathrm{frg}}_{e^-c}\left(\frac{y_1}{Y_1}\right)\mathcal{D}^{\mathrm{frg}}_{e^+d}\left(\frac{y_2}{Y_2}\right),$$

where $\sigma^{(0)}_{a+b\to c+d}$ is the Born-level partonic cross section,
$\bar{\sigma}^{(1)}$ is the $\overline{\mathrm{MS}}$ subtracted $\mathcal{O}(\alpha)$ contribution to it,

$$\mathcal{D}^{\mathrm{str,frg}}_{ee}(z) = \delta(1-z) + \frac{\alpha}{2\pi} d^{(1)}(z,\mu_0,m_e) + \frac{\alpha}{2\pi} LP^{(0)}(z)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{1}{2} L^2 P^{(0)} \otimes P^{(0)}(z) + LP^{(0)} \otimes d^{(1)}(z,\mu_0,m_e)\right.$$
$$\left. + LP^{(1,\gamma)\mathrm{str,frg}}_{ee}(z)\right) + \mathcal{O}\left(\alpha^2 L^0, \alpha^3\right)$$

QED Master Formula Ansatz

Using slicing in the photon energy, we cast the corrected cross section in the form

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \mathrm{d}\sigma^{(1)}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(1)}_{\mathrm{H}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} + \mathrm{d}\sigma^{(3)LO} + \ldots$$

For many observables we need to know the complete kinematics including hard photon angles, which are integrated over in the above QCD-like formula. Let us decompose the $\mathcal{O}\left(\alpha^2 L^{2,1}\right)$ hard γ radiation contribution

$$\mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} = \mathrm{d}\sigma^{(2)}_{\mathrm{HH(coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{HH(s-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(n-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(coll)}}$$

where slicing in the photon emission angle is applied:

- **"coll" means collinear photon(s) with $\vartheta_\gamma < \theta_0 \ll 1$,**
- **"n-coll" means non-collinear photon with $\vartheta_\gamma > \theta_0$,**
- **"HH(s-coll)" means semi-collinear kinematics,**
  i.e. **one collinear photon and one non-collinear**

Particular NLO contributions: H×H, H×(S+V)

Phase space of two hard photon emission (HH) is splitted into three regions:

1. non-collinear: $\theta_{1,2} > \vartheta_0$
   suited for Monte Carlo simulations
2. semi-collinear: $\theta_1 > \vartheta_0$ and $\theta_2 < \vartheta_0$
   in $\mathcal{O}\left(\alpha^2 L\right)$ has factorized form $\mathrm{d}\sigma_{\mathrm{H}}^{(1)} \otimes R_{\mathrm{H}}^{\mathrm{ISR,FSR}}(z)$
3. collinear: $\theta_{1,2} < \vartheta_0$ is described by the HH radiation factor convoluted with the Born

Emission of one hard photon in $\mathcal{O}\left(\alpha^2 L\right)$ can be sliced into two domains:

1. non-collinear: $\theta_\gamma > \vartheta_0$ as a product of two factors $\mathrm{d}\sigma_{\mathrm{H}}^{(1)} \times \delta_{\mathrm{Soft+Virt}}^{\mathrm{LO}}$
2. collinear: $\theta_\gamma < \vartheta_0$ is described by the collinear NLO H radiation factor

QED Collinear Radiation Factors in NLO (1)

A.A., E. Scherbakova, Phys. Lett. B 660 (2008) 37

$$\mathrm{d}\sigma[a(p_1)+b(p_2)\to c(q_1)+d(q_2)+\gamma(k\sim(1-z)p_1)]$$
$$=\mathrm{d}\hat{\sigma}[a(zp_1)+b(p_2)\to c(q_1)+d(q_2)]\otimes R_{\mathrm{H}}^{\mathrm{ISR}}(z)$$

Emission of collinear photons in FSR and ISR with conditions

$$\vartheta_\gamma<\vartheta_0,\qquad \frac{m}{E}\ll\vartheta_0\ll 1,\qquad l_0=\ln\frac{\vartheta_0^2}{4},\qquad \frac{E_\gamma}{E}>\Delta\ll 1$$

In $\mathcal{O}(\alpha)$ the result is well known:

$$R_{\mathrm{H}}^{\mathrm{ISR}}(z)=\frac{\alpha}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{E^2}{m^2}-1+l_0\right)+1-z+\mathcal{O}\left(\frac{m^2}{E^2}\right)+\mathcal{O}\left(\vartheta_0^2\right)\right]$$

QED Collinear Radiation Factors in NLO (2)

Emission of two collinear photons (HH) in the same direction is described by a one-fold integral of results from [A.A. et al., Nucl. Phys. B 483 (1997) 83]:

$$R_{\rm HH}^{\rm ISR}(z) = \left(\frac{\alpha}{2\pi}\right)^2 L\Bigg\{(L+2l_0)\Bigg(\frac{1+z^2}{1-z}(2\ln(1-z)-2\ln\Delta-\ln z)$$
$$+\frac{1+z}{2}\ln z - 1 + z\Bigg) + \frac{1+z^2}{1-z}\Big(\ln^2 z + 2\ln z - 4\ln(1-z) + 4\ln\Delta\Big)$$
$$+(1-z)\Big(2\ln(1-z) - 2\ln\Delta - \ln z + 3\Big) + \frac{1+z}{2}\ln^2 z\Bigg\}$$

FSR factor is restored with help of the Gribov-Lipatov relation generalized for the collinear emission case:

$$R_{\rm HH}^{\rm FSR}(z) = -zR_{\rm HH}^{\rm ISR}\left(\frac{1}{z}\right)\Bigg|_{\ln\Delta\to\ln\Delta-\ln z;\ l_0\to l_0+2\ln z}$$

QED Collinear Radiation Factors in NLO (3)

Emission of one collinear hard photon accompanied by one-loop soft and virtual correction (H(S+V)) is received using the NLO QED splitting functions

$$R^{\rm ISR}_{\rm H(S+V)}(z)\otimes {\rm d}\hat{\sigma}(z) = \delta^{(1)}_{\rm (S+V)} R^{\rm ISR}_{\rm H}(z)\otimes {\rm d}\sigma^{(0)}(z)$$
$$+\left(\frac{\alpha}{2\pi}\right)^2 L\left[2\frac{1+z^2}{1-z}\left({\rm Li}_2\left(1-z\right)-\ln(1-z)\ln z\right)\right.$$
$$\left.-(1+z)\ln^2 z+(1-z)\ln z+z\right]\otimes {\rm d}\sigma^{(0)}(z),$$

$$\delta^{(1)}_{\rm (S+V)} = \frac{{\rm d}\sigma^{(1)}_{\rm Soft}+{\rm d}\sigma^{(1)}_{\rm Virt}}{{\rm d}\sigma^{(0)}},$$

where $\sigma^{(0)}(z)$ is the boosted Born cross section, and $\delta^{(1)}_{\rm (S+V)}$ is the relative $\mathcal{O}\left(\alpha\right)$ Soft + Virtual radiative correction with $E^{\rm Soft}_{\gamma} < \Delta E$. The corresponding FSR factor is received again using the Gribov-Lipatov relation.

Pair Corrections (I)

Leptonic and hadronic pair corrections are important for a number of precision observables. Exclusive treatment here is of ultimate importance. Monte Carlo has to be used for real or for hard pairs, then soft and virtual ones can be treated analytically (semi-analytically for the hadronic case).

Singlet and non-singlet NLO pair contributions in $\mathcal{O}\left(\alpha^2 L\right)$ to inclusive observables can be described within the QCD-like factorization approach.

But if we have a MC for hard pairs, we can extract analytically the soft+virtual part, so that

$$\mathrm{d}\sigma^{(2)}_{\mathrm{pair}} = \mathrm{d}\sigma^{(2)MC}_{\mathrm{H\ pair}} \ + \ \mathrm{d}\sigma^{(0)} \times \delta^{(2)}_{\mathrm{S+V\ pair}}$$

Pair Corrections (II)

For e^+e^- pairs the Soft+Virt part is estimated by the Δ-part of non-singlet pair contribution to electron structure function

$$\delta^{(2)}_{\mathrm{S+V\ pair}} = 4\left(\frac{\alpha}{2\pi}\right)^2 \left\{\frac{1}{3}L^2\left(2\ln\Delta + \frac{3}{2}\right) + L\left(-\frac{20}{9}\ln\Delta - \frac{4\pi^2}{18} - \frac{1}{6}\right)\right\}$$

Note cancellation of $\mathcal{O}\left(\alpha^2 L^3\right)$ terms in the sum of virtual and soft pair contributions

For $E_{\mathrm{pair}} < 50$ MeV, $L = \ln(-t/m_e^2)$ **the effect is -0.16%, or -0.20% in LLA**
(for $E_{\mathrm{pair}} < 100$ MeV it makes -0.11% in NLLA)

$\mathcal{O}\left(\alpha^2\right)$ photonic contributions

LLA: $\mathcal{O}\left(\alpha^2 L_t^2\right)$ **gives** $+0.43\%$

N.B.: Pair corrections $\sim$ $1/3 \times$ $\mathcal{O}\left(\alpha^2\right)$ **photonic ones**

NLLA: $\mathcal{O}\left(\alpha^2 L_t^1\right)$ **gives** -0.05%

NNLLA: using the Soft+Virt (w/o log of soft-hard separator) we estimate it by the order of magnitude: 0.006% (for L_t) and 0.013% (for L_s)

$\mathcal{O}\left(\alpha^n L_t^n\right)$ **with** $n \geq 3$ **gives** $\lesssim -0.01\%$

MC-integrator structure

According to dimension of integration:

- **1d** **Born + Soft + Virt (γ+e^+e^-pair) in** $\mathcal{O}\left(\alpha^0, \alpha^1 L, \alpha^1, \alpha^2 L, \alpha^3 L^3\right)$
- **2d** **1 collinear hard photon in** $\mathcal{O}\left(\alpha^1 L, \alpha^1, \alpha^2 L^2, \alpha^2 L, \alpha^3 L^3\right)$**: 4 cases**
- **3d** **2 collinear hard photons in different directions** $\mathcal{O}\left(\alpha^2 L^2, \alpha^2 L, \alpha^3 L^3\right)$**: 6 cases**
- **4d(a)** **3 collinear hard photons in different directions** $\mathcal{O}\left(\alpha^3 L^3\right)$**: 4 cases**
- **4d(b)** **1 non-collinear hard photon** $\mathcal{O}\left(\alpha^1, \alpha^2 L\right)$
- **5d** **2 hard photons: collinear and non-collinear** $\mathcal{O}\left(\alpha^2 L\right)$ **new for MCJPG**

Residual dependence on Δ and ϑ_0 is at the level 0.03% mainly from $\mathcal{O}\left(\alpha^1\right)$ and $\mathcal{O}\left(\alpha^2 L\right)$ (should be improved)

Approximations

- In $\mathcal{O}\left(\alpha^1\right)$ radiation factor terms of the order $\mathcal{O}\left(m^2/E^2\right)$ and $\mathcal{O}\left(\theta_0^2\right)$ can be restored if required
- Factorization scale dependence is considerable for LO, and rather small for NLO QED
- The collinear cone can be transformed into any other form, *e.g.* into $k_t < k_t^0$ region
- $\mathcal{O}\left(\alpha^2 L^0\right)$ terms in particular kinematical domains can be added in a straightforward manner
- Negatively weighted events within this approach are possible but not numerous (parameters can be adjusted to reduce them)

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- What do we need?