
PHOTOS Monte Carlo,

$$e^+ e^- \rightarrow \gamma^* \rightarrow \pi^+ \pi^- (\gamma)$$

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Brief Introduction to PHOTOS

- PHOTOS is used to simulate effect of QED in decays
- PHOTOS can be combined with other main process, generators since it works on four-momenta, most of kinematical variables are needed for one step of iteration
- In soft and collinear limits, the matrix element can be factorized into the Born matrix element times a photo emission (an **eikonal**) factor
- Similar factorization is done for phase space, too
- Process independent kernel in PHOTOS can be written as Born matrix element times an **eikonal** factor

Brief Introduction to PHOTOS

- QED, scalar QED decaying processes in PHOTOS
 - $Z \ (\gamma^*, H) \rightarrow \mu^+ \mu^- (\gamma)$
 - $B^0 \rightarrow K^+ K^-, \pi^+ \pi^-, K^+ \pi^- (\gamma), B^\pm \rightarrow K^\pm K^0, \pi^\pm \pi^0 (\gamma)$
 - $W \rightarrow l \nu_l (\gamma)$ will be published soon
 - $\gamma^* \rightarrow \pi^- \pi^+ (\gamma)$ will be published soon
- These processes can be simulated using PHOTOS kernel and exact matrix element
- The results were compared process after process. We found PHOTOS kernel is a very good approximation for these processes

Motivation of $\gamma^* \rightarrow \pi^+\pi^-(\gamma)$

- Precise measurement of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-(\gamma)$ is used for parameterization of **photon vacuum polarization**, hence it is useful to improve standard model prediction on
 - Muon anomalous magnetic moment a_μ
 - Electromagnetic coupling constant $\alpha_{em}(s)$This could also give a hint for possible new physics contribution
- Simulating $\gamma^* \rightarrow \pi^+\pi^-(\gamma)$ will be useful for rare semileptonic decays $K^\pm \rightarrow \pi^+\pi^- l^\pm \nu_l(\gamma)$ since $\rho \rightarrow \pi^+\pi^-(\gamma)$ has same spin amplitude structure as $\gamma^* \rightarrow \pi^+\pi^-(\gamma)$

scalar QED kernel in PHOTOS

In PHOTOS, **scalar QED kernel** in case of B decays

$B_0(P) \rightarrow \pi^\pm(q_1)K^\mp(q_2)\gamma(k, \epsilon)$ is

$$|M|_{\text{PHOTOS}}^2 = 4\pi\alpha|M_{\text{Born}}|^2 \left(Q_1 \frac{q_1 \cdot \epsilon}{q_1 \cdot k} - Q_2 \frac{q_2 \cdot \epsilon}{q_2 \cdot k} \right)^2$$

Q_1, Q_2 are the charges of final masons

Since **spin structure** of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$ is different from B decays, $\sum_{\lambda,\epsilon} |M|^2(\gamma^* \rightarrow \pi^+\pi^-\gamma)$ is different from **Scalar QED kernel** in PHOTOS!

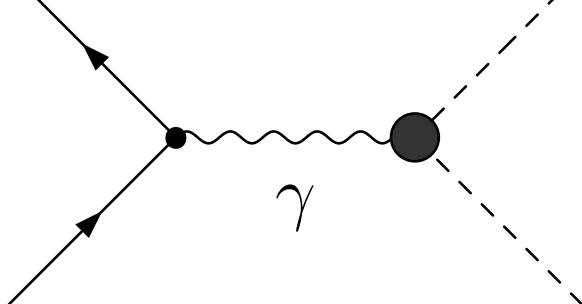
$\sum_{\lambda,\epsilon} |M|^2(\gamma^* \rightarrow \pi^+\pi^-\gamma) = \text{Born-like expression} \times \text{eikonal factor} + \text{remaining part}$

Born-level amplitude

Born-level spin amplitude of

$$e^+(p_1, \lambda_1)e^-(p_2, \lambda_2) \rightarrow \gamma^* \rightarrow \pi^+(q_1)\pi^-(q_2)$$

$$e^+$$



$$M_{born} = V_\mu H_0^\mu$$

$$V_\mu = ie\bar{u}(p_1, \lambda_1)\gamma_\mu v(p_2, \lambda_2)$$

$$H_0^\mu = \frac{eF_{2\pi}(S)}{S}(q_1 - q_2)^\mu$$

$$e^- \quad \pi^-$$

Square of amplitude and sum over spin degrees of freedom

$$\sum_\lambda |M_{Born}|^2(S, T, U) = \frac{8(4\pi\alpha)^2 F_{2\pi}^2(S)}{S^2} (TU - m_\pi^2 S) \propto q^2 \sin \theta_B^2$$

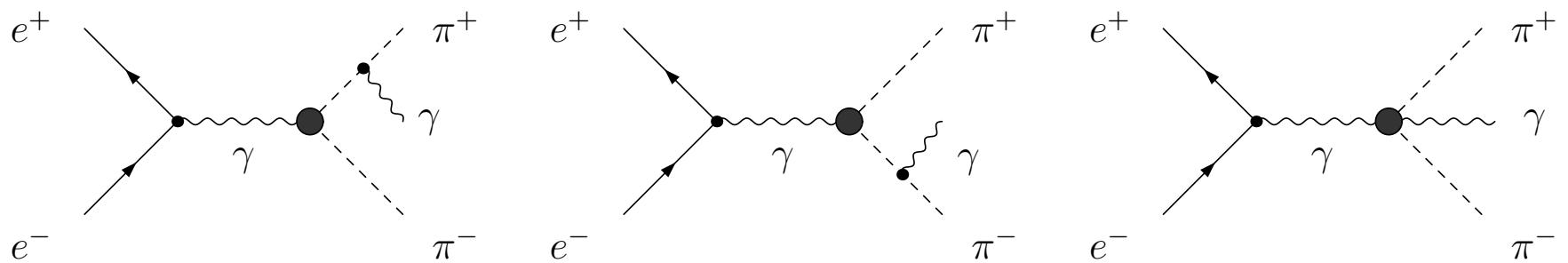
$$S = 2p_1 \cdot p_2, \quad T = 2p_1 \cdot q_1, \quad U = 2p_1 \cdot q_2$$

q is the length of \vec{q}_1 , $\theta_B = \angle p_1 q_1$

Amplitude of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$

Spin amplitude of

$$e^+(p_1, \lambda_1)e^-(p_2, \lambda_2) \rightarrow \gamma^* \rightarrow \pi^+(q_1)\pi^-(q_2)\gamma(k, \epsilon)$$



$$M = V_\mu H^\mu , \quad V_\mu = ie\bar{u}(p_1, \lambda_1)\gamma_\mu v(p_2, \lambda_2)$$

$$H^\mu = \frac{e^2 F_{2\pi}(S)}{S} \left\{ (q_1 + k - q_2)^\mu \frac{q_1 \cdot \epsilon}{q_1 \cdot k} + (q_2 + k - q_1)^\mu \frac{q_2 \cdot \epsilon}{q_2 \cdot k} - 2\epsilon^\mu \right\}$$

In order to obtain high precision of PHOTOS and KKMC Monte Carlo programs, one must analysis exact spin amplitudes and their parts of processes

Amplitude of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$

Rewrite H^μ into two gauge invariant parts $H^\mu = H_I^\mu + H_{II}^\mu$

$$H_I^\mu = \frac{e^2 F_{2\pi}(S)}{S} (\mathbf{q}_1 - \mathbf{q}_2)^\mu \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} - \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right) \rightarrow H_0^\mu \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} - \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right) \text{ for soft limit}$$

$$H_{II}^\mu = \frac{e^2 F_{2\pi}(S)}{S} \left(k^\mu \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} + \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right) - 2\epsilon^\mu \right), \text{ free of soft singularity}$$

One can also rewrite $H^\mu = H_{I'}^\mu + H_{II'}^\mu$

$$H_{I'}^\mu = \frac{e^2 F_{2\pi}(S)}{S} \left((\mathbf{q}_1 - \mathbf{q}_2)^\mu + k^\mu \frac{\mathbf{q}_2 \cdot k - \mathbf{q}_1 \cdot k}{\mathbf{q}_2 \cdot k + \mathbf{q}_1 \cdot k} \right) \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} - \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right)$$

$$H_{I'}^\mu \rightarrow H_0^\mu \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} - \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right) \text{ for soft and collinear limits}$$

$$\begin{aligned} H_{II'}^\mu &= \frac{e^2 F_{2\pi}(S)}{S} \left(k^\mu \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} + \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right) - 2\epsilon^\mu - k^\mu \frac{\mathbf{q}_2 \cdot k - \mathbf{q}_1 \cdot k}{\mathbf{q}_2 \cdot k + \mathbf{q}_1 \cdot k} \left(\frac{\mathbf{q}_1 \cdot \epsilon}{\mathbf{q}_1 \cdot k} - \frac{\mathbf{q}_2 \cdot \epsilon}{\mathbf{q}_2 \cdot k} \right) \right) \\ &= \frac{2e^2 F_{2\pi}(S)}{S} \left(\frac{k^\mu}{\mathbf{q}_2 \cdot k + \mathbf{q}_1 \cdot k} (\mathbf{q}_1 \cdot \epsilon + \mathbf{q}_2 \cdot \epsilon) - \epsilon^{*\mu} \right) \end{aligned}$$

free of soft and collinear and singularities!

This due to the factor $k^\mu \frac{\mathbf{q}_2 \cdot k - \mathbf{q}_1 \cdot k}{\mathbf{q}_2 \cdot k + \mathbf{q}_1 \cdot k}$, similar factor appears in QCD amplitude,

see A. van Hameren and Z. Was, arXiv:0802.2182

Amplitude of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$

For the first separation,

$$\sum_{\lambda,\epsilon} |M|^2 = \sum_{\lambda,\epsilon} |M_I|^2 + \sum_{\lambda,\epsilon} |M_{II}|^2 + 2 \sum_{\lambda,\epsilon} M_I M_{II}^*$$

$$A_I = \sum_{\lambda,\epsilon} |M_I|^2 , A_{II} = \sum_{\lambda,\epsilon} |M_{II}|^2 , A_{III} = 2 \sum_{\lambda,\epsilon} M_I M_{II}^*$$

$$A_I = (4\pi\alpha) \left(\frac{(A+B+C+D)}{4} - (4\pi\alpha)^2 \frac{8F_{2\pi}^2(S)}{S^2} (q_1 \cdot k)(q_2 \cdot k) \right) \sum_{\epsilon} \left(\frac{q_1 \cdot \epsilon}{q_1 \cdot k} - \frac{q_2 \cdot \epsilon}{q_2 \cdot k} \right)^2$$

$$A_{II} = (4\pi\alpha) \left(\frac{(A+B-C-D)}{4} + (4\pi\alpha)^2 \frac{8F_{2\pi}^2(S)}{S^2} (q_1 \cdot k)(q_2 \cdot k) \right) \sum_{\epsilon} \left(\frac{q_1 \cdot \epsilon}{q_1 \cdot k} + \frac{q_2 \cdot \epsilon}{q_2 \cdot k} \right)^2$$

$$- \frac{4\pi\alpha}{q_1 \cdot k} \left(B - \frac{1}{2}(C+D) \right) - \frac{4\pi\alpha}{q_2 \cdot k} \left(A - \frac{1}{2}(C+D) \right) + 8(4\pi\alpha)^3 \frac{F_{2\pi}^2(S)}{S^2} (S' + 2m_{\pi}^2 +$$

$$A_{III} = -(4\pi\alpha) \left(\frac{m_{\pi}^2}{(q_1 \cdot k)^2} - \frac{m_{\pi}^2}{(q_2 \cdot k)^2} \right) \frac{A-B}{2} + 8(4\pi\alpha)^3 \frac{F_{2\pi}^2(S)}{S^2} (S' + 2m_{\pi}^2 - S)$$

$$+ \frac{4\pi\alpha}{q_1 \cdot k} \left(B + \frac{1}{2}(C+D) \right) + \frac{4\pi\alpha}{q_2 \cdot k} \left(A + \frac{1}{2}(C+D) \right)$$

Amplitude of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$

$$A = \sum_{\lambda} |M_{Born}|^2(S, T', U) , B = \sum_{\lambda} |M_{Born}|^2(S, T, U') ,$$

$$C = \sum_{\lambda} |M_{Born}|^2(S, T, U) , D = \sum_{\lambda} |M_{Born}|^2(S, T', U') ,$$

$$E = 32(4\pi\alpha)^3 m_{\pi}^2 \frac{F_{2\pi}^2(S)}{S^2}$$

Mandelstam variables

$$S = 2p_1 \cdot p_2, \quad S' = 2q_1 \cdot q_2,$$

$$T = 2p_1 \cdot q_1, \quad T' = 2p_2 \cdot q_2,$$

$$U = 2p_1 \cdot q_2, \quad U' = 2p_2 \cdot q_1$$

$$\sum_{\lambda, \epsilon} |M|^2 = 4\pi\alpha \left\{ \frac{-m_{\pi}^2}{(q_1 \cdot k)^2} A + \frac{-m_{\pi}^2}{(q_2 \cdot k)^2} B + \frac{S - 2m_{\pi}^2}{2(q_1 \cdot k)(q_2 \cdot k)} (C + D) \right\} + E$$

We keep m_{π} all the time!

Amplitude of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$

For the second separation,

$$\sum_{\lambda,\epsilon} |M|^2 = \sum_{\lambda,\epsilon} |M_{I'}|^2 + \sum_{\lambda,\epsilon} |M_{II'}|^2 + 2 \sum_{\lambda,\epsilon} M_{I'} M_{II'}^*$$

$$A_{I'} = \sum_{\lambda,\epsilon} |M_{I'}|^2 , A_{II'} = \sum_{\lambda,\epsilon} |M_{II'}|^2 , A_{III'} = 2 \sum_{\lambda,\epsilon} M_{I'} M_{II'}^*$$

$$A_{I'} = (4\pi\alpha) \left(\frac{(q_2 \cdot k)^2}{(q_1 \cdot k + q_2 \cdot k)^2} A + \frac{(q_1 \cdot k)^2}{(q_1 \cdot k + q_2 \cdot k)^2} B + \frac{(q_1 \cdot k)(q_2 \cdot k)}{(q_1 \cdot k + q_2 \cdot k)^2} (C + D) \right.$$

$$\left. - (4\pi\alpha)^2 \frac{32F_{2\pi}^2(S)}{S^2} \frac{(q_1 \cdot k)^2(q_2 \cdot k)^2}{(q_1 \cdot k + q_2 \cdot k)^2} \right) \sum_{\epsilon} \left(\frac{q_1 \cdot \epsilon}{q_1 \cdot k} - \frac{q_2 \cdot \epsilon}{q_2 \cdot k} \right)^2$$

$$A_{II'} = \frac{-(4\pi\alpha)S}{(q_1 \cdot k + q_2 \cdot k)^2} (A + B - C - D) + \frac{16(4\pi\alpha)^3 F_{2\pi}^2(S)}{S^2} \frac{(q_1 \cdot k)^2 + (q_2 \cdot k)^2}{(q_1 \cdot k + q_2 \cdot k)^2} S$$

$$A_{III'} = \frac{-4\pi\alpha}{(q_1 \cdot k + q_2 \cdot k)^2} \left\{ \left((2m_{\pi}^2 + S') \frac{q_2 \cdot k}{q_1 \cdot k} - S \right) A + \left((2m_{\pi}^2 + S') \frac{q_1 \cdot k}{q_2 \cdot k} - S \right) B \right.$$

$$\left. - \left(\frac{S}{2} \left(\frac{q_1 \cdot k}{q_2 \cdot k} + \frac{q_2 \cdot k}{q_1 \cdot k} \right) - 2m_{\pi}^2 - S' \right) (C + D) \right\} -$$

$$\frac{32(4\pi\alpha)^3 F_{2\pi}^2(S)}{S^2(q_1 \cdot k + q_2 \cdot k)^2} \left[\frac{S}{2} ((q_1 \cdot k)^2 + (q_2 \cdot k)^2) - (2m_{\pi}^2 + S')(q_1 \cdot k)(q_2 \cdot k) \right]$$

Amplitude of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma$

Normalize lengths of 3-vectors $\vec{q}_1 - \vec{q}_2$ and $\vec{q}_1 - \vec{q}_2 + \vec{k} \frac{q_2 \cdot k - q_1 \cdot k}{q_2 \cdot k + q_1 \cdot k}$ to make them be the same as $|\vec{q}_1 - \vec{q}_2|_{\text{Born}}$

For the first separation,

$$\sum_{\lambda, \epsilon} |M|^2 = A'_I + A_{\text{remain}}, \quad A'_I = A_I \frac{S - 4m_\pi^2}{|\vec{q}_1 - \vec{q}_2|^2}$$

For the second separation,

$$\sum_{\lambda, \epsilon} |M|^2 = A'_{I'} + A'_{\text{remain}}, \quad A'_{I'} = A_{I'} \frac{S - 4m_\pi^2}{|\vec{q}_1 - \vec{q}_2 + \vec{k} \frac{q_2 \cdot k - q_1 \cdot k}{q_2 \cdot k + q_1 \cdot k}|^2}$$

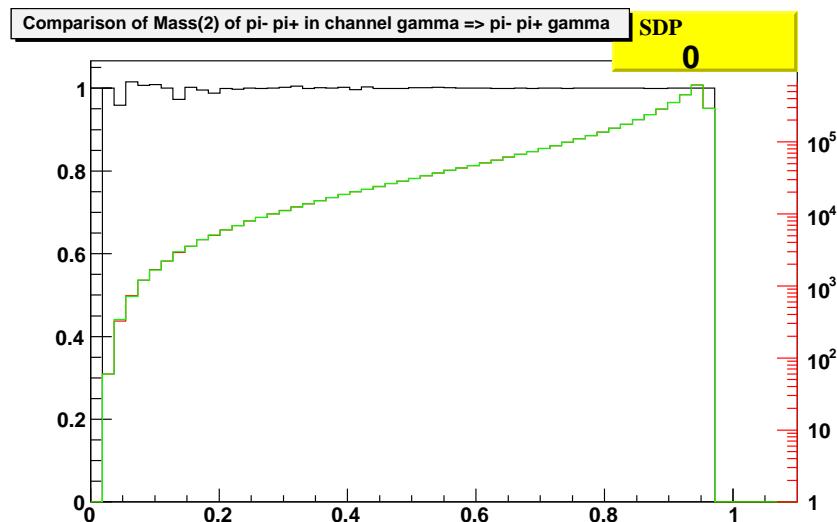
Conclusion:

$$\sum_{\lambda, \epsilon} |M|^2 = 4\pi\alpha \sum_{\lambda} |M_{\text{Born}}|^2 \sum_{\epsilon} \left(\frac{q_1 \cdot \epsilon}{q_1 \cdot k} - \frac{q_2 \cdot \epsilon}{q_2 \cdot k} \right)^2 + \text{remaining part}$$

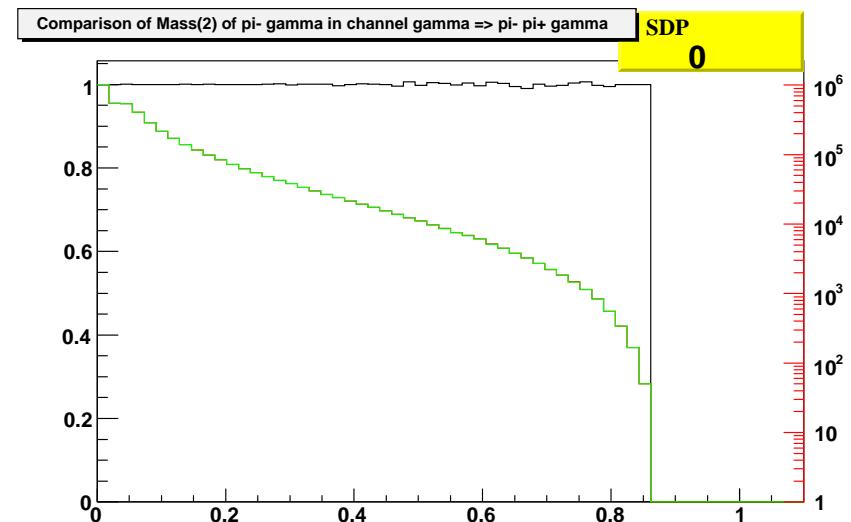
Numerical Results

Comparison of A'_I ($A'_{I'}$) with **Scalar QED kernel** at $\sqrt{S} = 2\text{GeV}$

BR: $4.2278 \pm 0.0021\%$, $4.2279 \pm 0.0021\%$. **Very good agreement**



$M^2_{\pi^+\pi^-}$ distribution

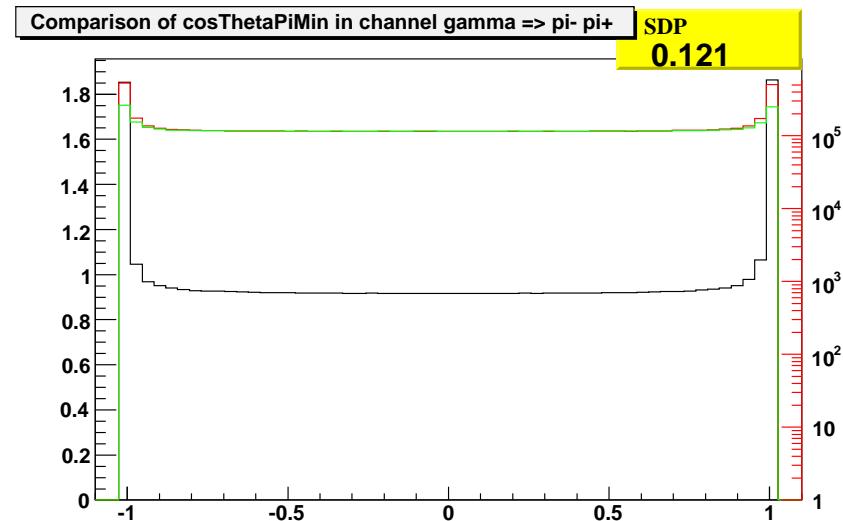
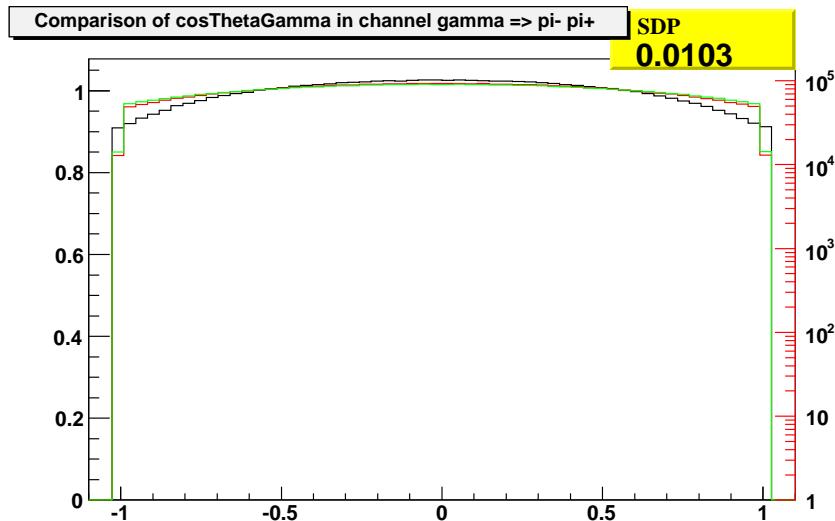


$M^2_{\pi^-\gamma}$ distribution

Note: The distribution variable in all plots is normalized to the virtuality of decaying photon

Numerical Results

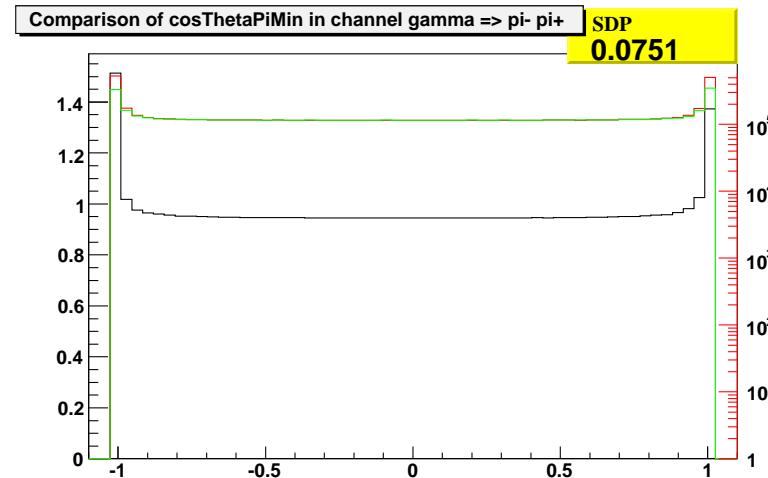
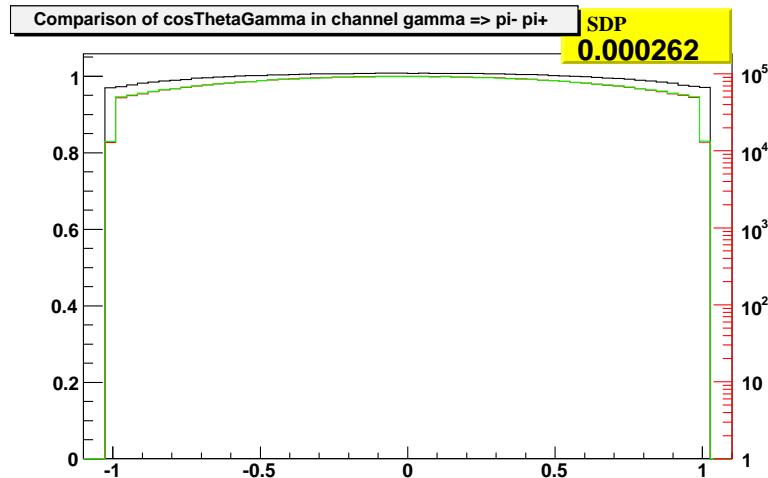
Comparison of A'_I (green) with Scalar QED kernel (red) at $\sqrt{S} = 2\text{GeV}$



A little difference for θ_γ and θ_{π^-} distribution, angles are respect to the beam direction

Numerical Results

Comparison of A'_I , (green) with Scalar QED kernel (red) at $\sqrt{S} = 2\text{GeV}$

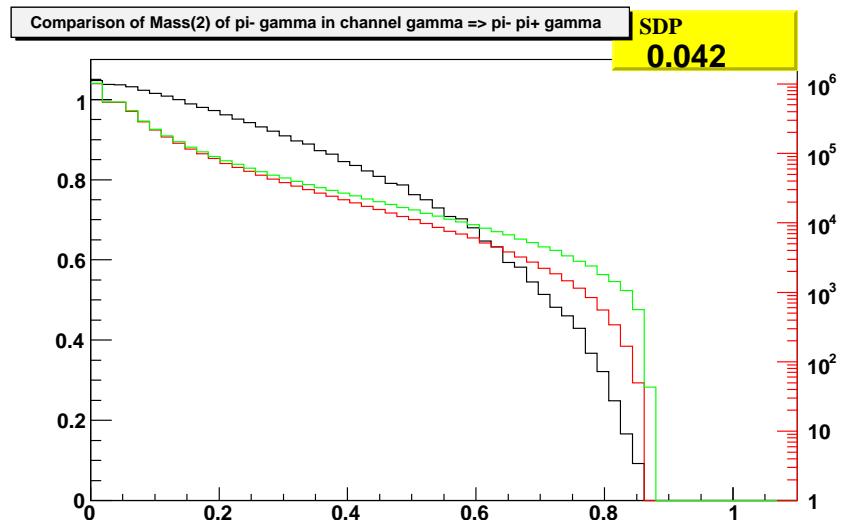
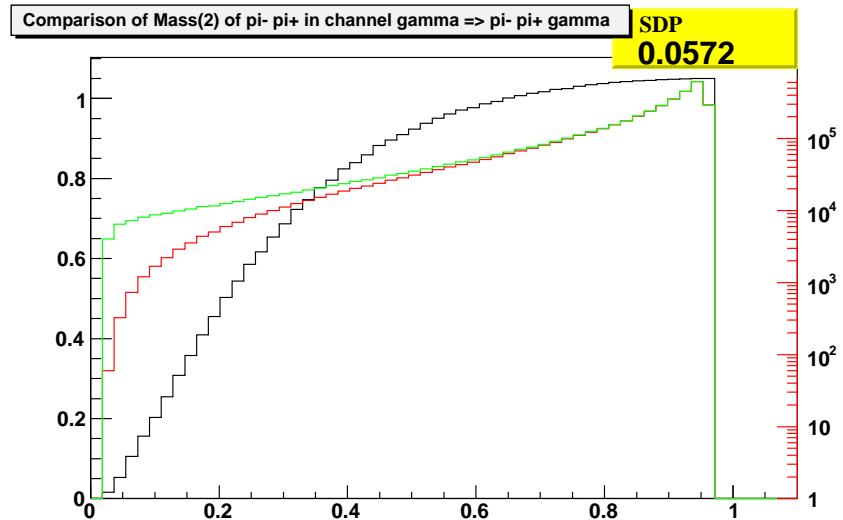


Differences for θ_γ and θ_{π^-} distribution are smaller than that of A'_I

Numerical results (Comparison of $\sum_{\lambda, \epsilon} |M|^2$ with Scalar QED kernel in PHOTOS)

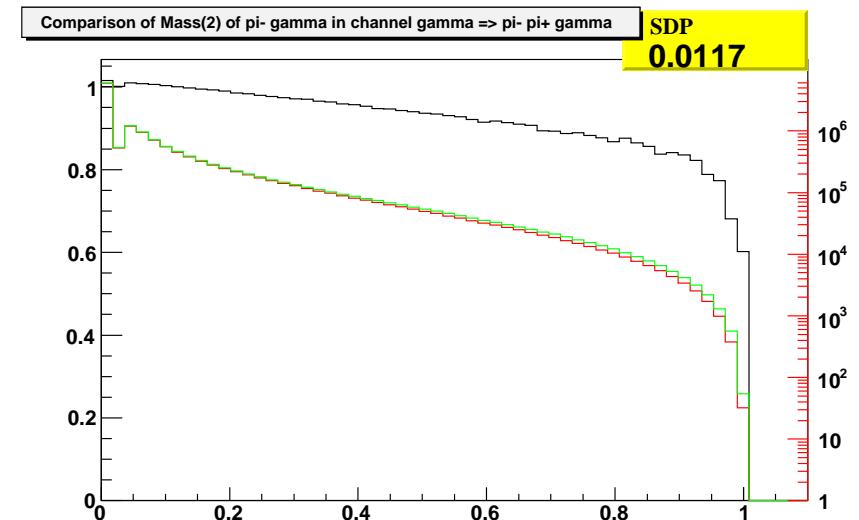
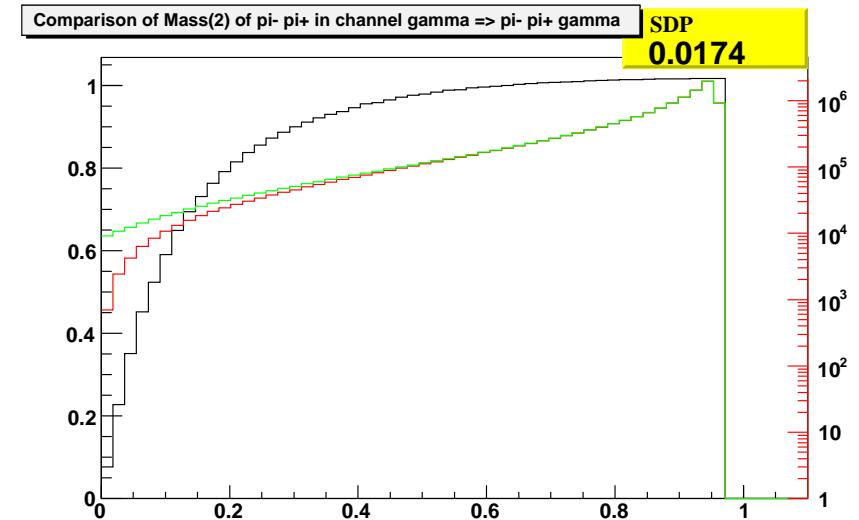
$\sqrt{S} = 2\text{GeV}$

BR: $4.4378 \pm 0.0021\%$, $4.2279 \pm 0.0021\%$



$\sqrt{S} = 200\text{GeV}$

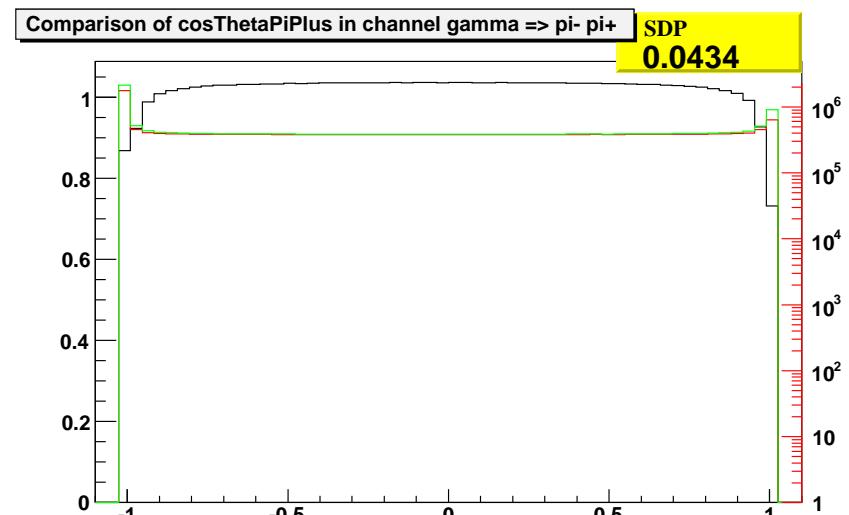
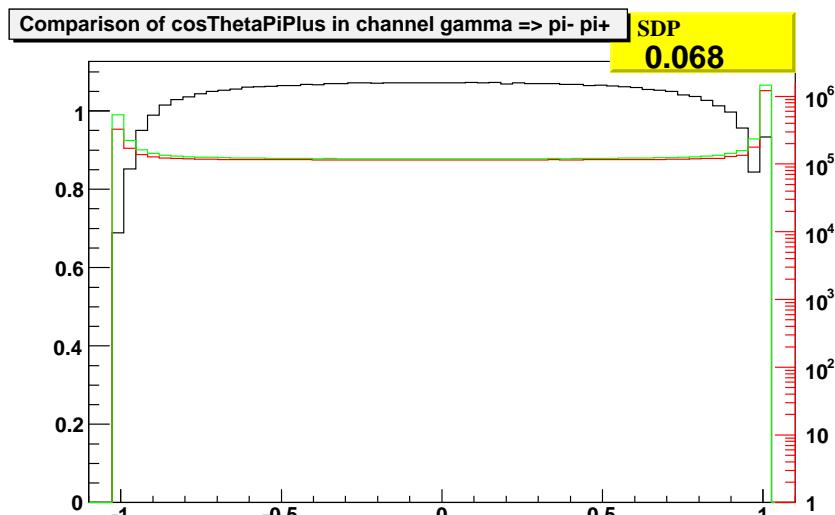
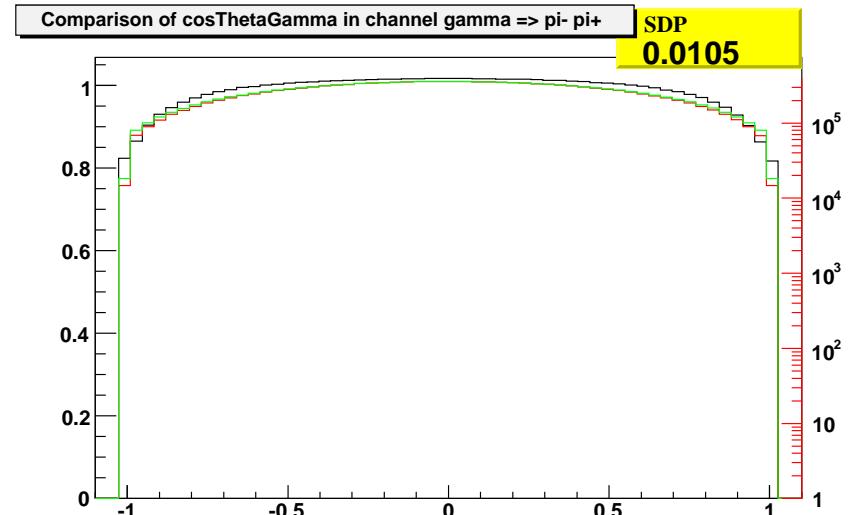
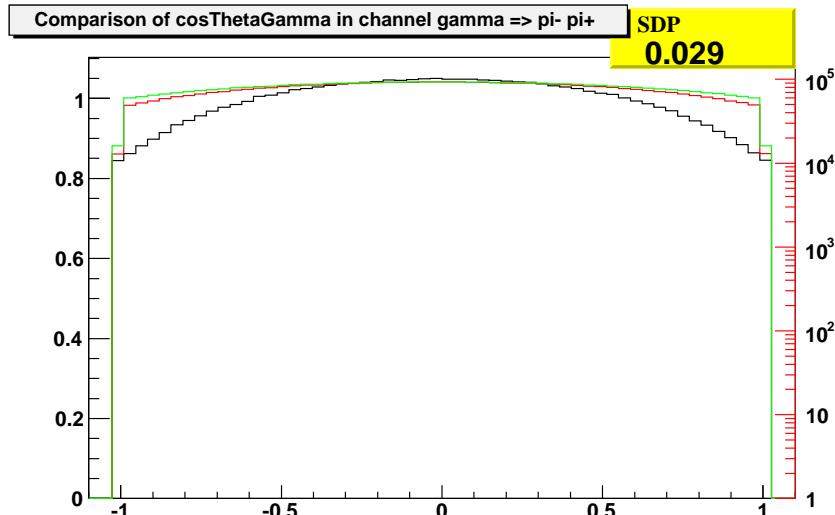
BR: $14.1474 \pm 0.0038\%$, $13.9208 \pm 0.0037\%$



Numerical results

Comparison of $\sum_{\lambda, \epsilon} |M|^2$ (green) with Scalar QED kernel in PHOTOS (red)

$$\sqrt{S} = 2\text{GeV}$$



Summary and outlook

- The cross section of $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-(\gamma)$ can be separated into an **eikonal** part and a **remaining** part using principle of gauge invariance. The **eikonal** part is identical to **Scalar QED kernel** in PHOTOS used for B meson decays
- With PHOTOS our process can be simulated using **exact matrix element** now. Results were compared with the one where spin of γ^* is ignored (**Scalar QED kernel** in PHOTOS) . The approximated, easy to use version is correct up to **0.2%** level. The difference remain constant with increasing γ^* virtuality
- Multi-photos emission can be simulated
- Analogies with QCD amplitudes are visible
- $\tau^\pm \rightarrow l^\pm \nu_\tau \nu_l$