Status of Vacuum Polarisation Corrections and available Codes

UNIVERSITY OF LIVERPOOL

THOMAS TEUBNER

and Daisuke Nomura (KEK)

LIVERPOOL THEORETICAL PHYSICS

I. Introduction. Definitions. Approximations

II. Hadronic Contributions via the Dispersion Integral

III. Comparison of different Compilations

IV. Next steps

I. Introduction. Definitions. Approximations

- Why Vacuum Polarisation / running α corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

— Corrections for data used as input for $g - 2$: 'undressed' σ^0_{had}

$$a_\mu^{\text{had,LO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^\infty \mathrm{d}s\, \sigma^0_{\text{had}}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s} \cdot (0.63 \ldots 1)$$

— Determination of α_s and quark masses from total hadronic cross section R_{had}
at low energies and of resonance parameters.

— Part of higher order corrections in Bhabha scattering important for precise Luminosity
determination.

— $\alpha(M_Z^2)$ a fundamental parameter at the Z scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$),
needed to test the SM via precision fits/constrain new physics.

$\rightarrow$ Ingredient in MC generators for many processes.

- Photon Vacuum Polarisation (VP) a quantum effect which leads to the running of the renormalised (effective) QED coupling α_{QED}.

- Dyson summation of Real part of one-particle irreducible blobs Π into the effective, real coupling α_{QED}:

$$\Pi \;=\; \underset{q}{\overset{\gamma^*}{\text{\small$\sim\!\!\sim\!\!\sim$}}}\!\!\bigotimes\!\!\text{\small$\sim\!\!\sim\!\!\sim$}$$

Full photon propagator $\sim\; 1 \;+\; \Pi \;+\; \Pi\cdot\Pi \;+\; \Pi\cdot\Pi\cdot\Pi \;+\; \ldots$

$$\rightsquigarrow \qquad \alpha(q^2) \;=\; \frac{\alpha}{1 - \mathrm{Re}\Pi(q^2)}$$

- Effect from both leptonic and hadronic loops;
  — leptonic VP calculable in Perturbation Theory,
  — hadronic VP receives contributions from non-perturbative sector
  $\rightsquigarrow$ calculation via dispersion integral using experimental $\sigma_{\mathrm{had}}(e^+e^- \to hadrons)$:
  $$\alpha(q^2) \;=\; \alpha \,/\, \big(1 - \Delta\alpha_{\mathrm{lep}}(q^2) - \Delta\alpha_{\mathrm{had}}(q^2)\big)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha_{\mathrm{had}}^{(5)}(q^2) = -\frac{q^2}{4\pi^2\alpha}\,\mathrm{P}\int_{m_\pi^2}^{\infty} \frac{\sigma_{\mathrm{had}}^0(s)\,\mathrm{d}s}{s - q^2}\,, \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma_{\mathrm{had}}^0(s)}{|1 - \Pi|^2}$$

$$[\to \sigma^0 \text{ requires 'undressing', e.g. via } \cdot(\alpha/\alpha(s))^2 \rightsquigarrow \text{ iteration needed}]$$

- Observable cross sections σ_{had} contain the |full photon propagator|^2, i.e. |infinite sum|^2, including the Imaginary part, $\Pi = e^2(P + iA)$.

- However, (formally) the Imaginary part is suppressed by e^2 w.r.t. the Real part:

$$|1 + e^2(P+iA) + e^4(P+iA)^2 + \ldots|^2 = 1 + e^2\,2P + e^4\,(3P^2 - A^2) + e^6\,4P(P^2 - A^2) + \ldots$$

To account for $\mathrm{Im}\Pi$ we can use the summed form: $\quad \dfrac{1}{|1 - e^2(P+iA)|^2} \equiv \dfrac{1}{|1 - \Pi|^2}$

- Note:

   — At narrow resonance energies, if $|\Pi| \sim 1$, the summation breaks down
      $\rightsquigarrow$ need other formulation, e.g. Breit-Wigner resonance propagator.

   — Summation of bubbles covers only one class of graphs (factorisable blobs); does *not* take other higher order terms from extra photons.

- Leptonic:

  — Leading and next-to-leading order known analytically;
  lepton masses the only *tiny* uncertainty.

  — NNLO available as expansion in the lepton mass, i.e. in m_ℓ^2/q^2 Steinhauser

  ⤳ no limitations from this sector.

- Hadronic:

  — 'All-order' using experimental data and dispersion integral for low energies

      ↪ stat. + sys. uncertainties from input data

  — Non-resonant 'continuum' contributions can be evaluated by perturbative QCD;
  especially above well above (charm and) bottom thresholds.

  — Strongly suppressed top quark contribution added using pQCD.

▶ Uncertainties in running $\alpha_{\mathrm{QED}}(q^2)$ / VP dominated by hadronic contributions
at low (to medium) q^2 (see discussion below).

II. Hadronic Contributions via the Dispersion Integral

- For compilation done and used by the Novosibirsk group see e.g. the excellent talks by Gennadiy Fedotovich & Fedor Ignatov at Beijing meeting Oct. 2008.

- For Fred Jegerlehner's results see e.g. his Nucl. Phys. Proc. Suppl. 181-182 (2008) 135 and references therein.

- HMNT use their data compilation for $g-2$ also for their own $\Delta\alpha(q^2)$ and $R(q^2)$ routines, for details and Refs see

    Hagiwara+Martin+Nomura+T: PRD 69(2004)093003; PLB 649(2007)173.

  — Data compilation uses most of the available data, with the leading hadronic channels 2π, 3π, KK, 4π, but altogether sum of ~ 24 exclusive channels and inclusive data for $\sqrt{s}$ above $1.43 - 2$ GeV to get total σ_{had}^0 with high precision.

  — Some subleading channels via isospin symmetry. Chiral PT for relevant thresholds.

  — Data driven, i.e. use of *state-of-the-art* perturbative QCD only above ~ 11.09 GeV.

 — Note: by using pQCD in a wider range one could improve the error at the expense of a more TH-driven approach.

– Data combination by non-linear χ^2_{min} fit which takes into account correlations through systematic errors; fit of one renormalization factor for each set (within/governed by systematics).

– Radiative corrections ('VP undressing') (re-)done as required in each set; where no reliable information is available an additional error due to radiative corrections has been assigned.
  [HMNT make no attempt at having a 'dressed' VP compilation.]

- Once the data are corrected for VP (and FSR) and suitably combined and continued in the perturbative regime, the numerical dispersion integral is straightforward (but has to take into account the Principal Value description).

- Narrow Resonances J/ψ, ψ' and the Υ family are added separately, see discussion below.

- The error estimate comes through combined statistical, systematic and parametric (α_s, quark masses, renormalisation scale in case of pQCD, resonance parameters for NR) uncertainties:

- $\delta\left(\Delta\alpha_{\text{had}}^{(5)}(s)\right)$ of HMNT compilation

Error of VP in the timelike regime at low and higher energies:
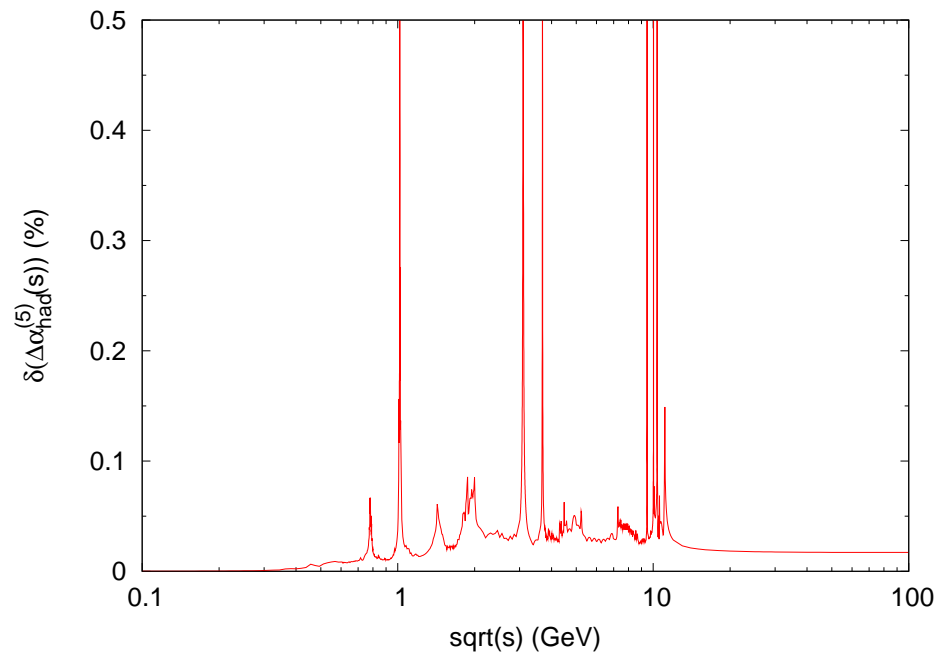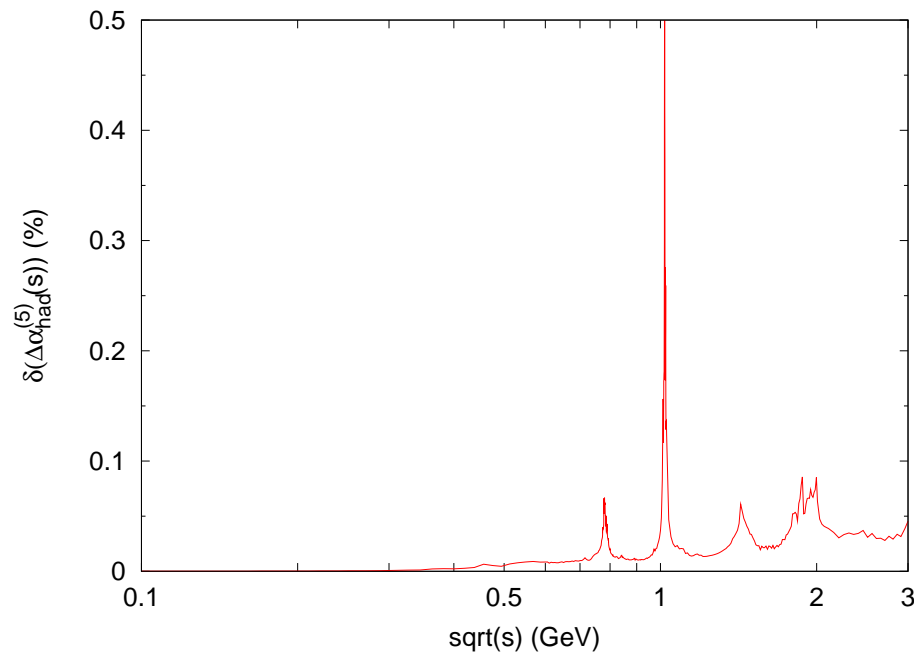
$\rightarrow$ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.
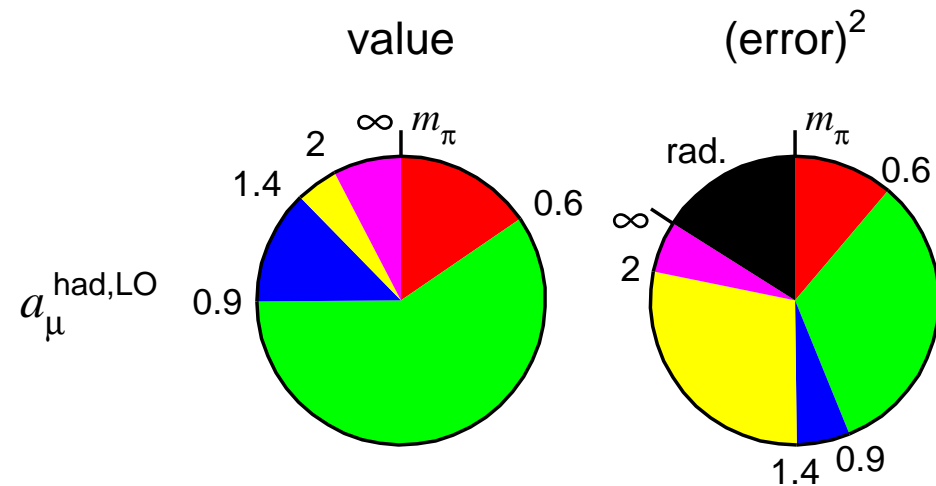
$g - 2$ and $\alpha(M_Z^2)$: Which energy regions contribute most to the error?

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors[2]:
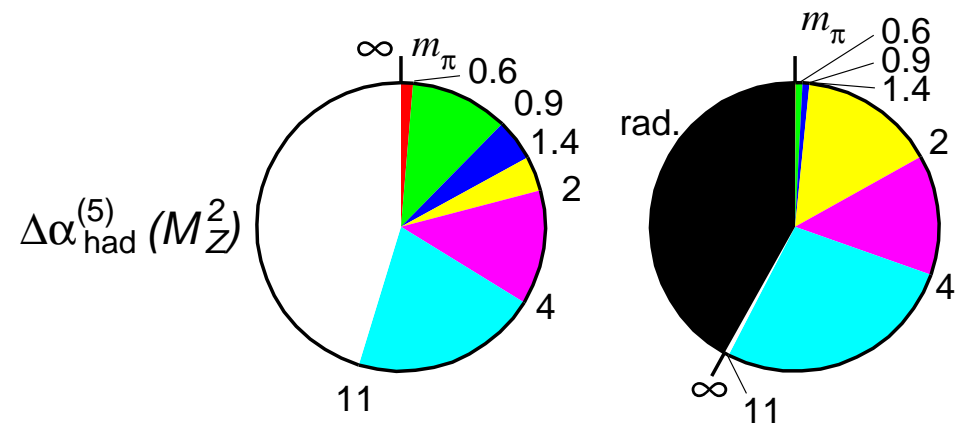
Critical regions:

$\rightarrow a_\mu$:

2π improved a lot, but still ρ central regime, region below 2 GeV !

$\rightarrow \alpha(M_Z)$:

again below 2 GeV, + intermed./large energies, w. *better* control of radcors!

Narrow resonances

- For the ω and ϕ resonances the data is suitable for direct integration, avoiding parametri-sation ambiguities/uncertainties.
  The same is true for the higher charm excitations, $\psi(3770, 4040, 4160, 4415)$.

- For J/ψ, ψ' and the Υ family $(1 - 6S)$ one can easily calculate their contributions to $(g - 2$ and$)$ $\Delta\alpha$ through the Narrow width approximation or via a Breit-Wigner parametrisation.

- However close to resonance the summation in an effective coupling breaks down, signalled by a very large correction.

- Also need to take care of properly undressing the electronic widths Γ_{ee}:

  - Using the dressed width would be inconsistent and introduce sizeable effects (a few percent), undressing via the smooth spacelike running α comes closer numerically but is not fully correct.

  - HMNT have derived the formula

$$\Gamma_{ee}^{V,0} = \frac{\left[\alpha/\alpha_{\mathrm{no\,V}}(M_V^2)\right]^2}{1 + 3\alpha/(4\pi)} \Gamma_{ee}^V$$

III. Comparison of different Compilations

- Timelike $\alpha(s)$ from Fred Jegerlehner's (2003 routine as available from his web-page)

$$\alpha(s) = \alpha / \left(1 - \Delta\alpha_{\text{lep}}(s) - \Delta\alpha_{\text{had}}^{(5)}(s) - \Delta\alpha^{\text{top}}(s)\right)$$
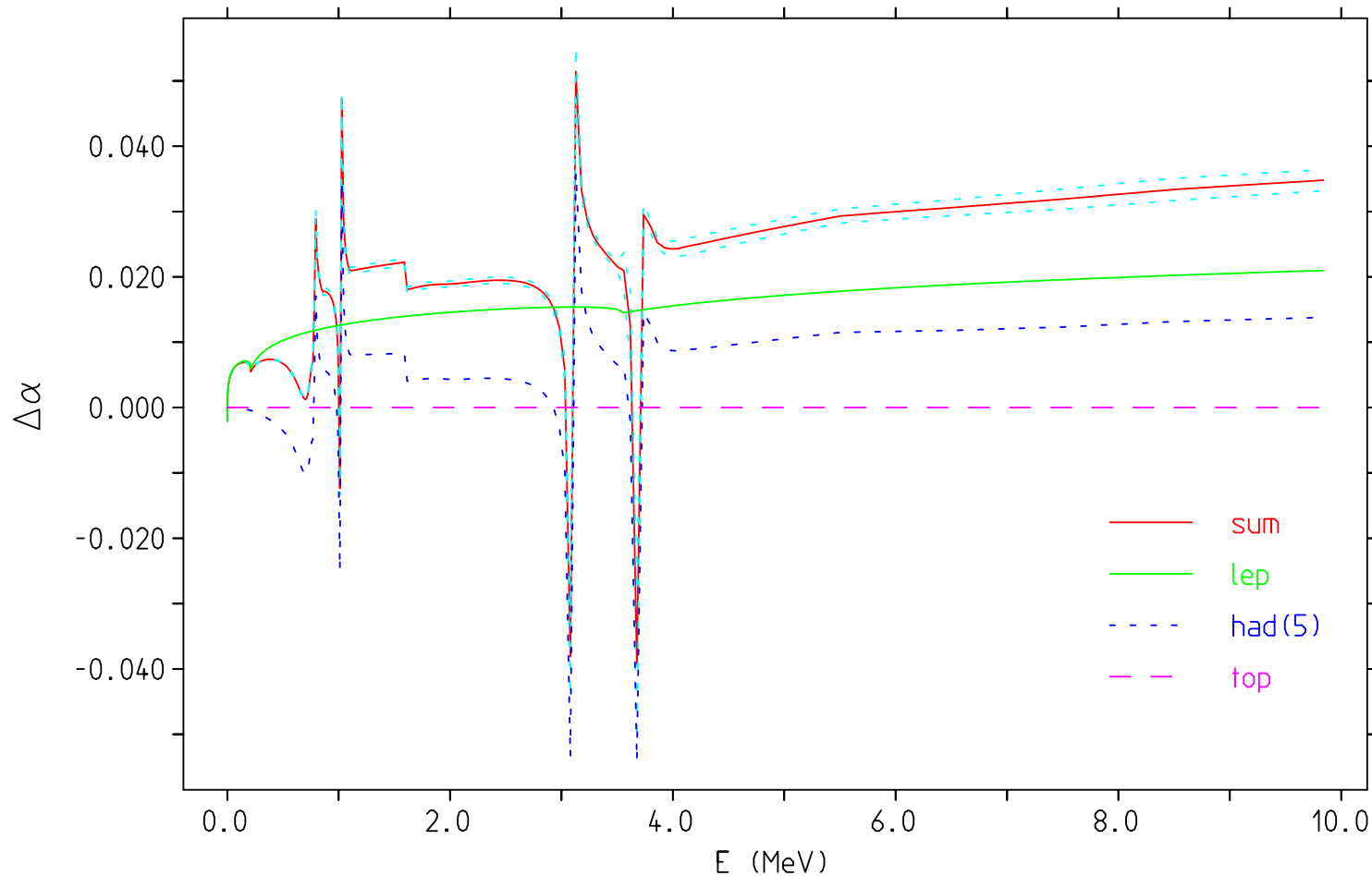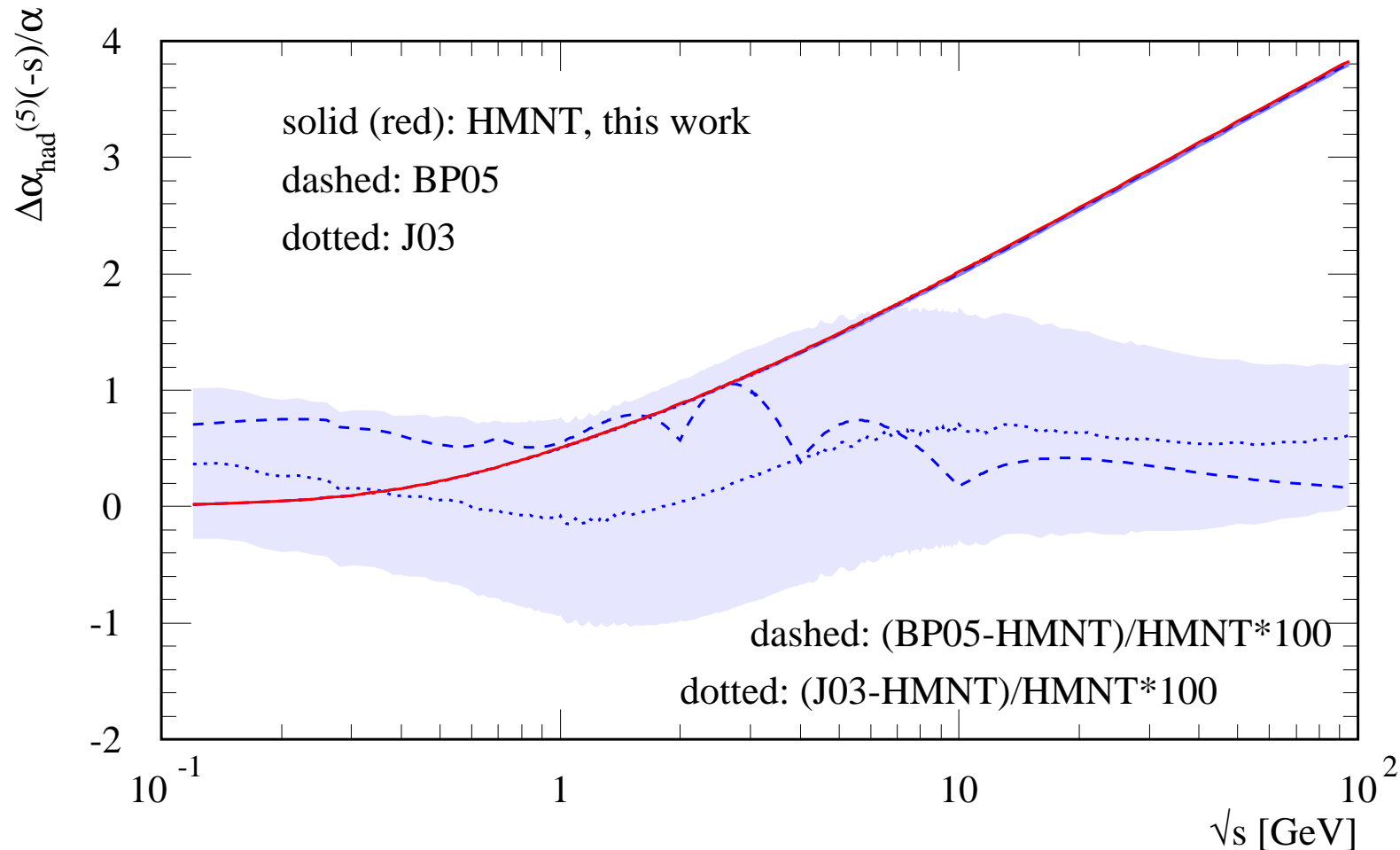

Figure from Fred Jegerlehner

- HMNT's evaluation of $\alpha_{\mathrm{QED}}(q^2)$ compared to other parametrisations: HMNT avail. soon

Spacelike $\Delta\alpha_{\mathrm{had}}^{(5)}(-s)/\alpha$ (smooth $\alpha(q^2 < 0)$)

solid (red): HMNT, this work

dashed: BP05

dotted: J03

dashed: (BP05-HMNT)/HMNT*100

dotted: (J03-HMNT)/HMNT*100

— Differences between parametrisations clearly visible but within error band (of HMNT)

— Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.

— What is in the MCs used by the experiments for Bhabha?

Timelike $\alpha(q^2 > 0)$ follows resonance structure:

solid (red): HMNT, this work
dotted (blue): J03
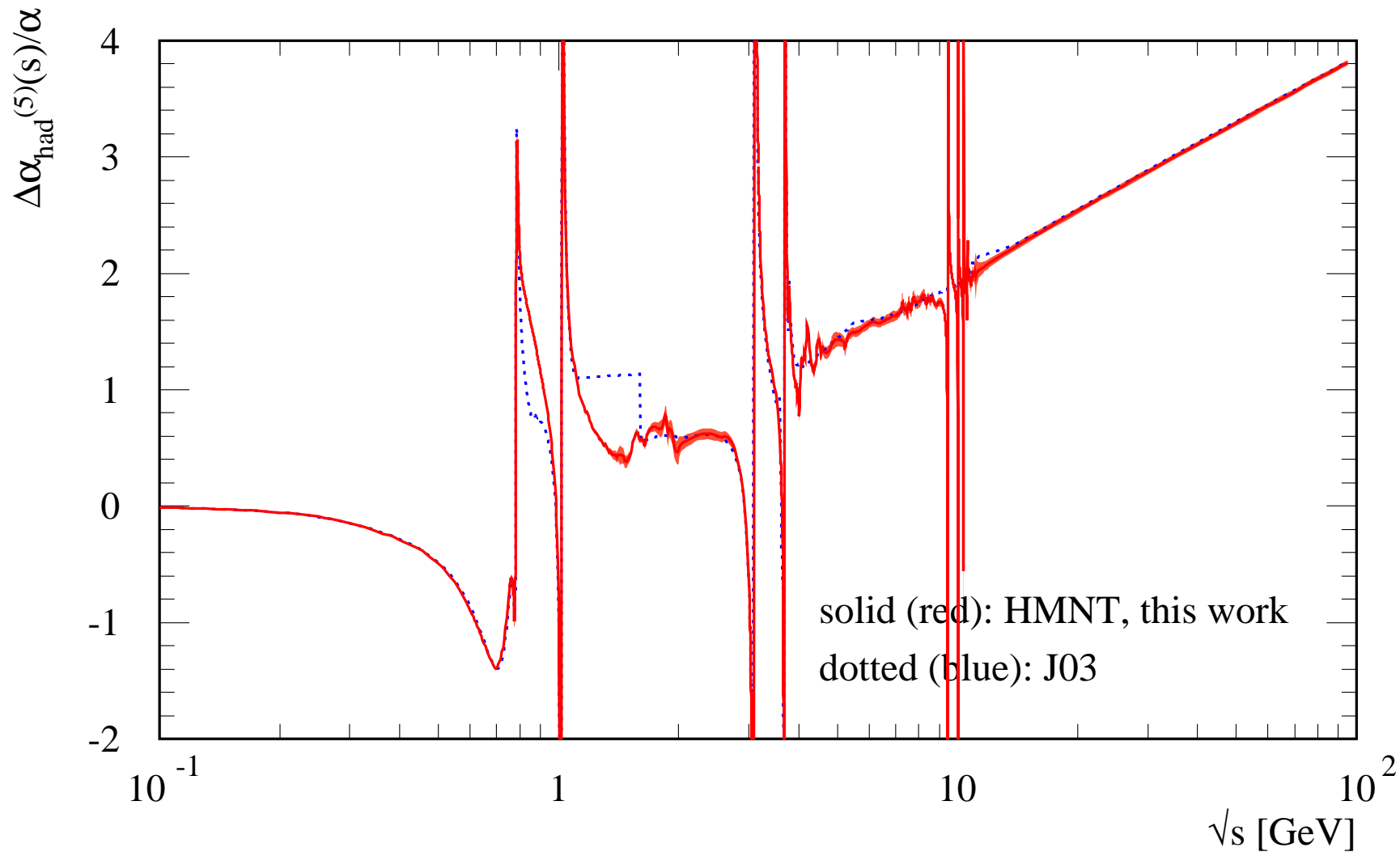
— Step below just a feature of unfortunate grid?

— Difference below 1 GeV not expected from data. [HMNT have done comparison by using RPP data compilation and confirmed their result.] Similar findings by S. Müller.

- **HMNT compared to Novosibirsk**

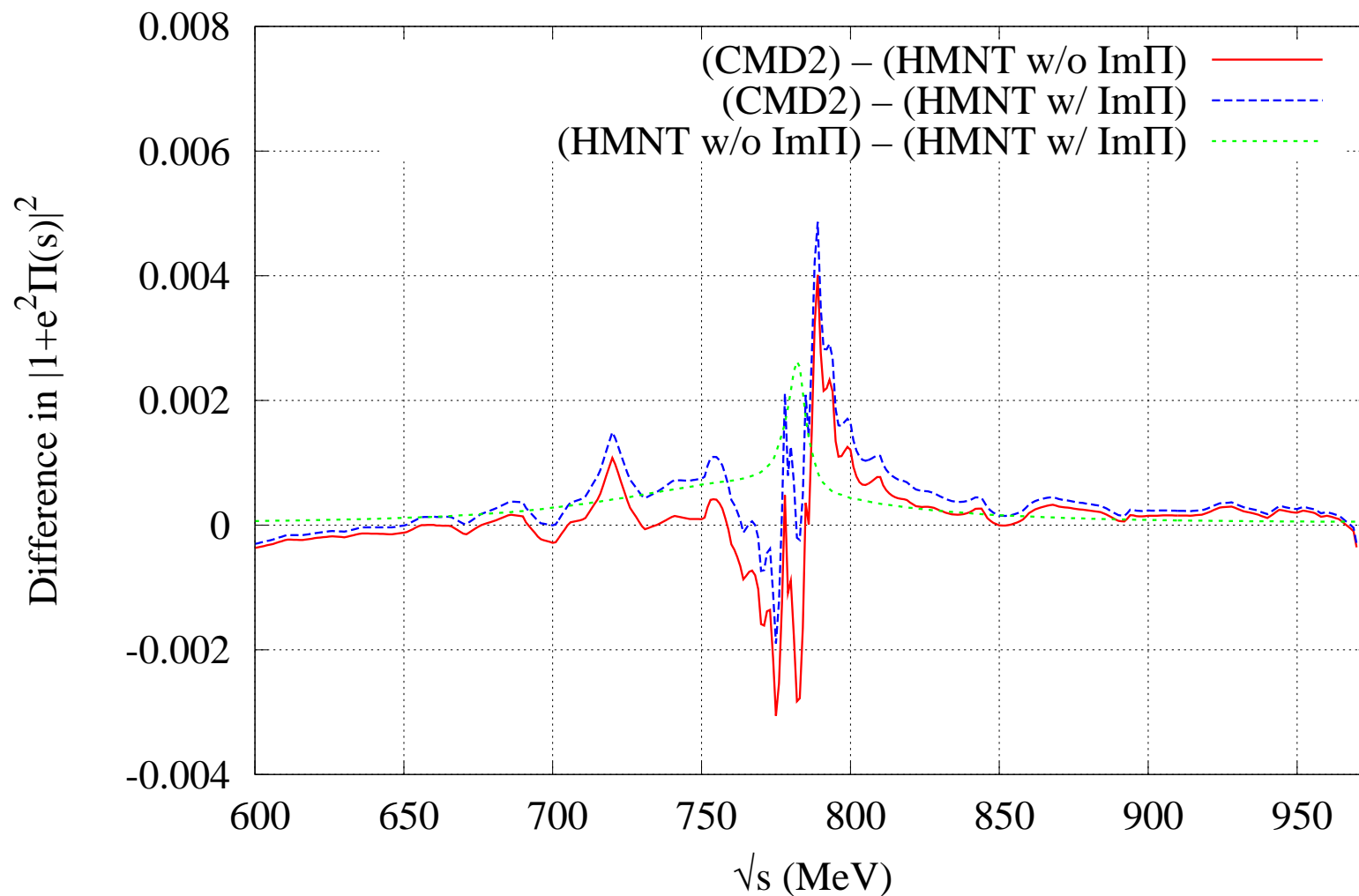Timelike $|1 - \Pi(s)|^2 \sim (\alpha(s)/\alpha)^2$ in ρ central energy region: A relevant correction!
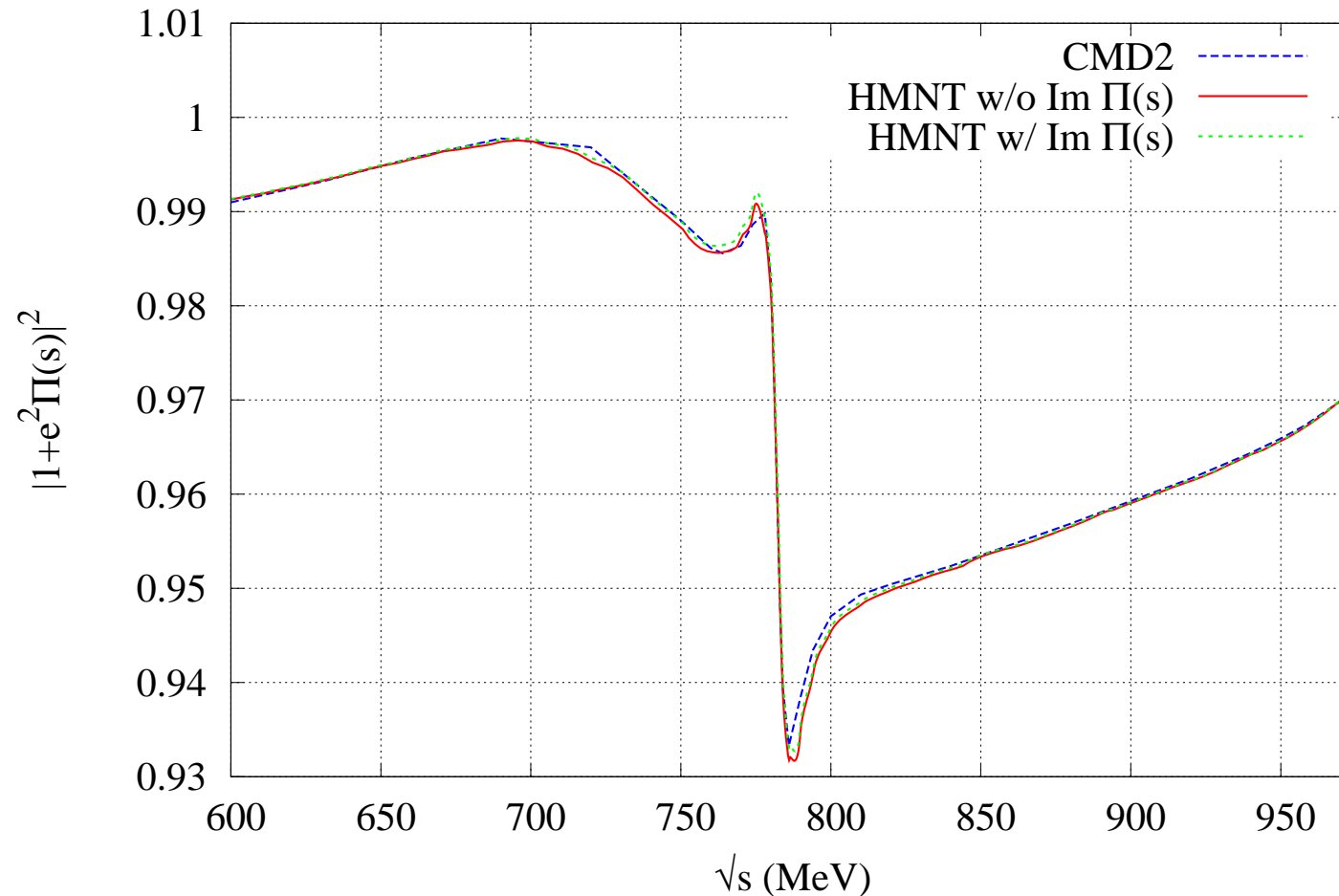
(Different sign and prefactor, $-e^2$, used for Π by HMNT.)

$\rightarrow$ Small but visible differences, as expected from independent compilations.

- **HMNT compared to Novosibirsk**   Timelike, differences in $|1 - \Pi(s)|^2$

$\longrightarrow$ Differences of about one per-mille in the 'undressing' factor, up to -3/+5 per-mille in the $\rho - \omega$ interference regime, but likely to cancel at least partly in applications.

$\longrightarrow$ As expected small negative contribution from $\mathrm{Im}\Pi$.

What about $\Delta\alpha(M_Z^2)$?

- With the same data compilation of σ_{had}^0 as for $g-2$ we find:

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = 0.02768 \pm 0.00022$$

  i.e. $\alpha(M_Z^2)^{-1} = 128.937 \pm 0.030$ (HMNT '06)

Other compilations:

Group	$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '08	0.027594 ± 0.000219	data driven/pQCD
$(M_0 = 2.5~\mathrm{GeV})$	0.027515 ± 0.000149	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

Adler function: $D(-s) = \dfrac{3\pi}{\alpha} s \dfrac{\mathrm{d}}{\mathrm{d}s}\Delta\alpha(s) = -(12\pi^2)s\dfrac{\mathrm{d}\Pi(s)}{\mathrm{d}s}$

allows use of pQCD and minimizes dependence on data.

IV. The next steps for our Proceedings

- What we already have for 'VP':

  — Three+ independent compilations: Jegerlehner, Novosibirsk, HMNT (+BP)

  — 'Semi-public' codes

  — No individual dedicated write-ups yet

- What we want/need/should aim at?!

  — Updated code from Fred Jegerlehner

  — Updated code, also in timelike regime from BP?

  — Papers from Novosibirsk and HMNT (updated code?!)

  — Download-sites with some help for all codes

- Which form should the codes have? NR treatment: two versions? Feedback from users?!

- Already a good starting point for the WG report, but serious work needs to start now.

- Would be nice to have more data/clearer picture from awaited BaBar and KLOE 2π analyses, but will this crash our time schedule? (As of today there is no VP draft yet :-(
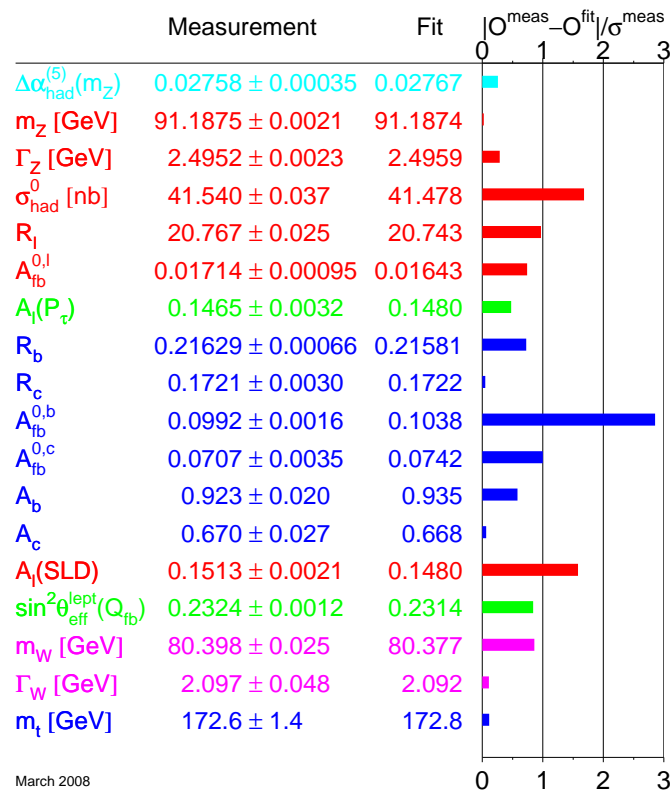
Extras:

EW Precision Fits

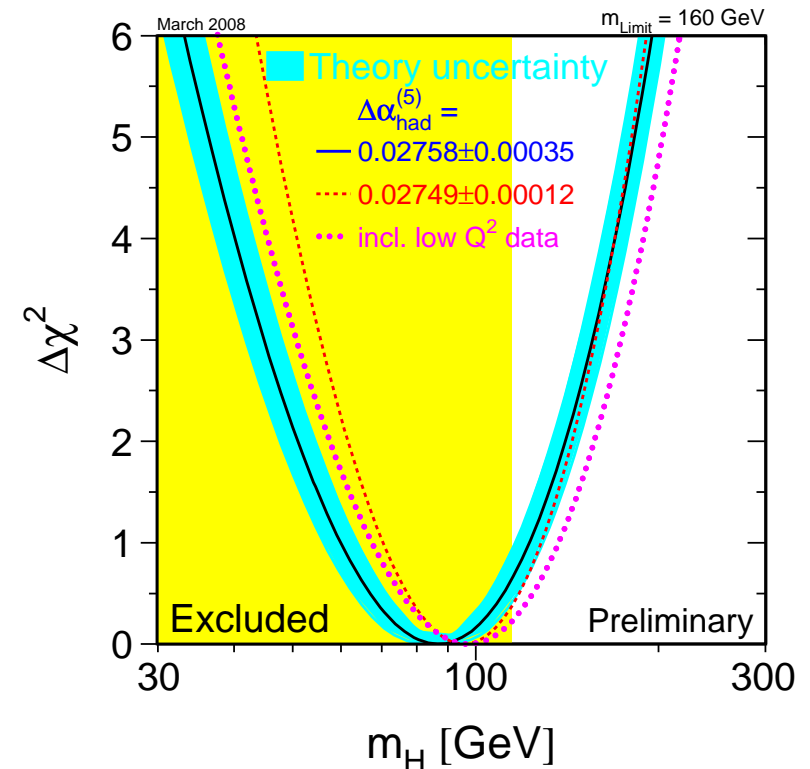- With the same compil. of σ_{had} as for $g-2$ we find:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.02768 \pm 0.00022$$

i.e. $\alpha(M_Z^2)^{-1} = 128.937 \pm 0.030$ (HMNT '06)

LEP EWWG 08:

March 2008

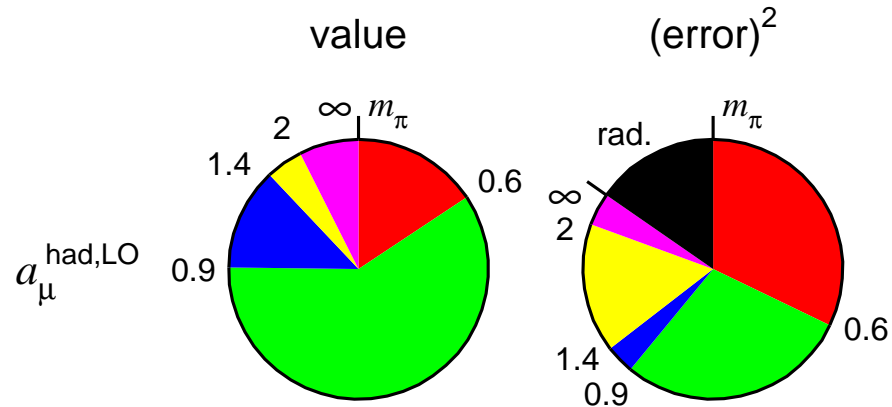## Fit of the SM Higgs mass: EWWG 08

$\longrightarrow$ preferred m_H moves down w. higher $\Delta\alpha$
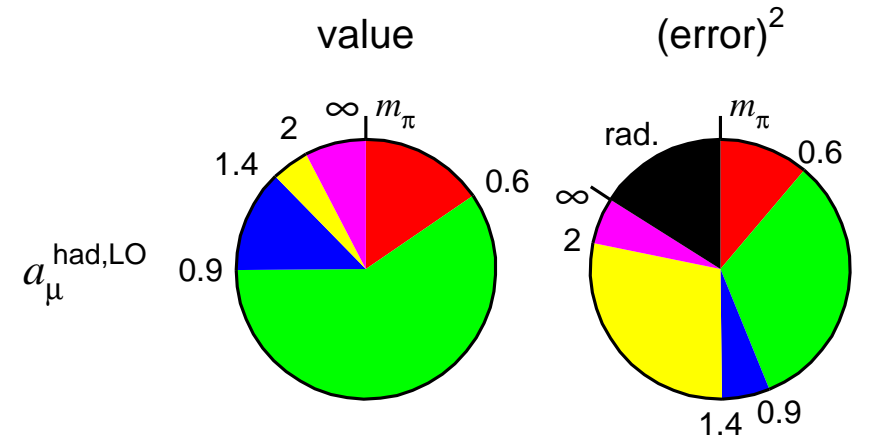
$\longrightarrow$ lower error lowers excl. limit

Difficult to 'cure' $g-2$ and m_H by changing σ_{had}

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors[2]:

HMNT03:

value   (error)^2

$a_\mu^{\text{had,LO}}$

$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

HMNT06:

value   (error)^2

$a_\mu^{\text{had,LO}}$

$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$

By far the biggest change was in $\pi\pi$: $502.78 \pm 5.02 \longrightarrow 498.46 \pm 2.87$

The prospects for further improvements through better data are good:

- Further Radiative Return analyses from KLOE eagerly awaited...

  $\longrightarrow$ check 2π down to threshold and hopefully combine to squeeze error.

- BaBar already very successful with RadRet for higher multiplicity channels

  $\hookrightarrow$ critical region $1.4 \ldots 2$ GeV should improve further.

  $\longrightarrow$ final $\pi\pi\gamma$ analysis on the way..

- More opportunities for BELLE?

- With upcoming VEPP-2000 in Novosibirsk, KLOE2 here in Frascati (and hopefully DAFNE-2) improvement of up to including the critical region below 2 GeV!

- At higher energies, relevant for $\Delta\alpha(M_Z^2)$, possibly more analyses from CLEO at Cornell and BES-II at BEPC in Beijing; soon BEPCII / BES-III !