Metodi di Scattering Multicanale

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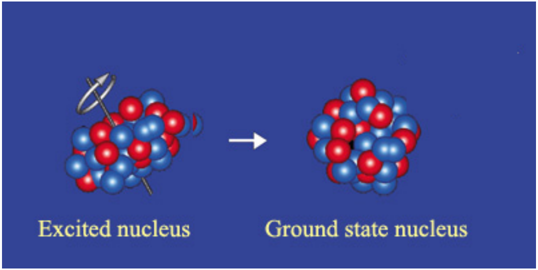
Treating scattering on light-medium nuclei

COUPLED-CHANNEL dynamics: including the collective low-energy excitations of the core.



C13 (n-C12), N13 (p-C12), C15 (n-C14), F15 (p-O14), He7, B7, Be7, Li7, Be9, B9, C17 (n-C16), Na-17 (p-Ne16) C-19 (n-C18), ${}^{9}{}_{\Lambda}Be$, ${}^{13}{}_{\Lambda}C$,
and also ... Ne23 (n-Ne22), Mn23, Na23 (p-Ne22), Al23, O17 (n-O16), F17 (p-O16), O19 ... O16 (alpha-C12), Be10 (alpha-He6)

The multichannel scattering problem

The coupled-channel Schrödinger equation:

$$(H_{0c} - E_c)\,\psi_c(r) = -\sum_{c'=1}^{C} \int_0^\infty U_{cc'}(r, r')\,\psi_{c'}(r')\,\mathrm{d}r' , \qquad c = 1, 2, \dots, C .$$

the *Free* Hamiltonian:

$$H_{0c} = -\frac{1}{\mu}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{l(l+1)}{r^2}\right) + V_c(r) ,$$

the *Interacting* Hamiltonian:

$$U_{cc'}(r, r') = V_{cc'}(r)\,\delta(r - r') + K_{cc'}(r, r') .$$

Model of nuclear interaction

Current description: nucleon-nucleus scattering (light-medium nuclei with 0^+ g.s.) including first core excitations of collective nature (quadrupole, octupole, etc).

$$V_{cc'}(r) = \sum_{n=C,LS,LL,SI} V_n < (\ell s)jI; J^\pi | \mathcal{O}_n f_n(r, R, \theta_{\mathbf{r},\mathbf{R}}) | (\ell' s)j'I'; J^\pi >$$

For all operators, the functional forms are expanded to second order in the core-deformation parameter ($R = R_0(1 + \beta_2 P_2(\theta))$)

$$f_n(r, R, \theta) = f_n^{(0)}(r) - \beta_2 R_0 P_2(\theta) \frac{d}{dr} f_n^{(0)}(r)$$
$$+ \frac{\beta_2^2 R_0^2}{2\sqrt{\pi}} \left(P_0 - \frac{2\sqrt{5}}{7} P_2(\theta) + \frac{2}{7} P_4(\theta) \right) \frac{d^2}{dr^2} f_n^{(0)}(r)$$

we want to determine S-matrices to evaluate:
Total elastic scattering cross section

$$\sigma_{EL} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell+1) \left| S_\ell^+(k) - 1 \right|^2 + \ell \left| S_\ell^-(k) - 1 \right|^2 \right\}$$

Total reaction cross section

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell+1) \left(1 - |S_\ell^+(k)|^2 \right) + \ell \left(1 - |S_\ell^-(k)|^2 \right) \right\}$$

(Energy dependent) Sturmians

Sturmians (*aka* Weinberg states): a *different* way to QM.
Consider a two-body like Hamiltonian:

$$(E - H_o)\Psi_E = V\Psi_E\,, \tag{2}$$

where E is the spectral variable, and Ψ_E is the eigenstate.
Sturmians are the eigensolutions of:

$$(E - H_o)\Phi_i(E) = \frac{V}{\eta_i(E)}\Phi_i(E)\,, \tag{3}$$

where E is a parameter. The eigenvalue η_i is the potential scale.
SPECTRUM: all the potential rescalings that give solution to that equation, for given energy E, and with well-defined boundary conditions.

(Energy dependent) Sturmians

Then, the single-channel S-matrix can be written as

$$S(E) = \frac{\Pi_i(1 - \eta_i(E^{(-)}))}{\Pi_i(1 - \eta_i(E^{(+)}))} \tag{4}$$

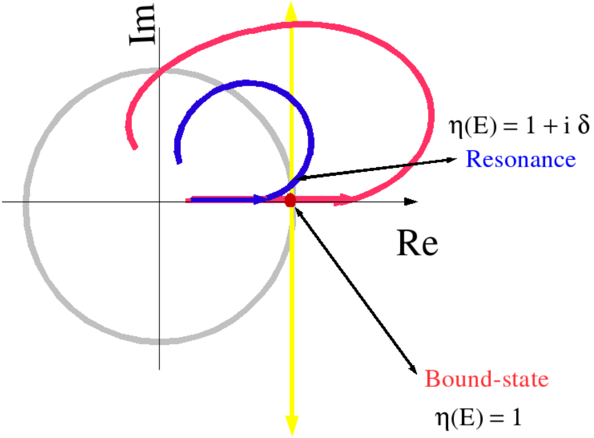
Alternatively, in CC dynamics, introducing the form factor in momentum space

$$\hat{\chi}_{ci}(E^{(+)}; k) =< k, c|V|\Phi_i(E) > , \tag{5}$$

the CC S-matrix can be rewritten also as

$$S_{cc'}(E) = \delta_{cc'} - i\pi\sqrt{k_c k_c'} \sum_i \hat{\chi}_{ci}(E^{(+)}; k_c) \frac{1}{1 - \eta_i(E^{(+)})} \hat{\chi}_{c'i}(E^{(+)}; k_{c'}) \tag{6}$$

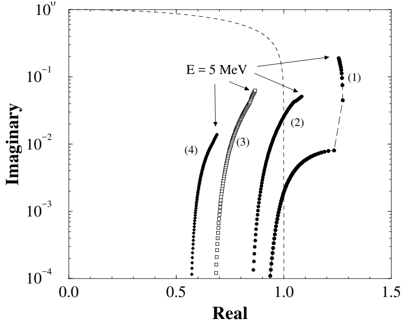
How resonances and bound states are found in Sturmian theory.

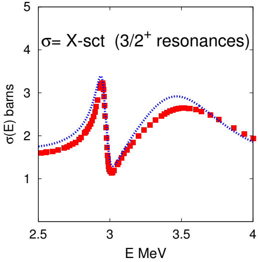


Resonances

The case of neutron-^{12}C in the $3/2^+$ channel

A realistic case: low-energy resonances in $3/2^+$ n-^{12}C system.
Sturmian patterns (left) and $3/2^+$ resonant X-sect (right).





The elastic cross section for n + ^{22}Ne at low energy

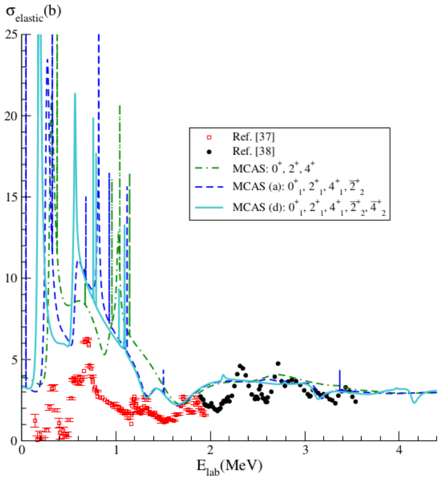


Figure: Experimental data of S. Sikkema et al (1958) and of S. R. Salisbury et al (1966). See publication P.R.Fraser, L.Canton, et al., Phys Rev. C 2014

CC dynamics and the Pauli principle

First application $n - C12/p - C12$ aborted: Why?

Bound states

^{13}C four observed $\rightarrow$ 12 computed

^{13}N one observed $\rightarrow$ 8 computed

The deep forbidden states contaminate the physical solution due to Coupled-Channel dynamics. Problems in CC formalisms (but not only)...

The OPP "potential"

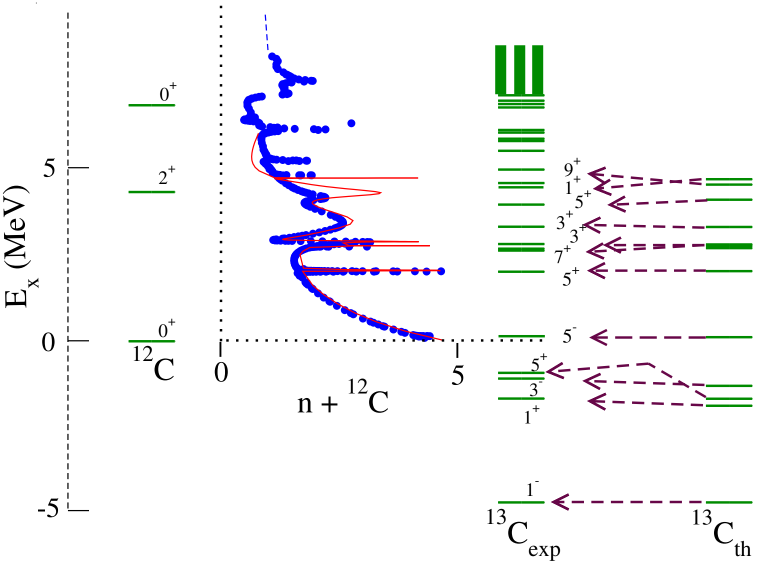
The OPP approach (Kukulin, Pomerantsev et al.) eliminates the deep forbidden states!
The full nuclear potential $\mathcal{V}_{cc}(r)$ is not the local potential $n - C12$:
The "complete" potential is (in partial-wave decomposition)

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r')$$

$$+\delta_{cc'}\lambda_c A_c(r)A_c(r')(\delta_{c=s\frac{1}{2}^+}) + \delta_{cc'}\lambda_c A_c(r)A_c(r')(\delta_{c=p\frac{3}{2}^-})$$

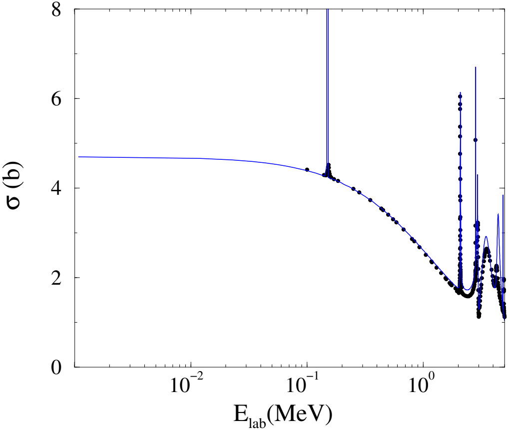
$A_c(r)$ are the Pauli-forbidden deep (CC-uncoupled) bound states.

A state in the OPP approach is:
forbidden in the limit $\lambda \to +\infty$
allowed when $\lambda \to 0$



E_x (MeV)
5
0
-5
0+
2+
0+
12C
0
5
n + 12C
9+
1+
5+
3+
3+
7+
5+
5-
5+
3-
1+
1-
13C_exp
13C_th

n-^{12} C: Low energy details



MCAS calculation

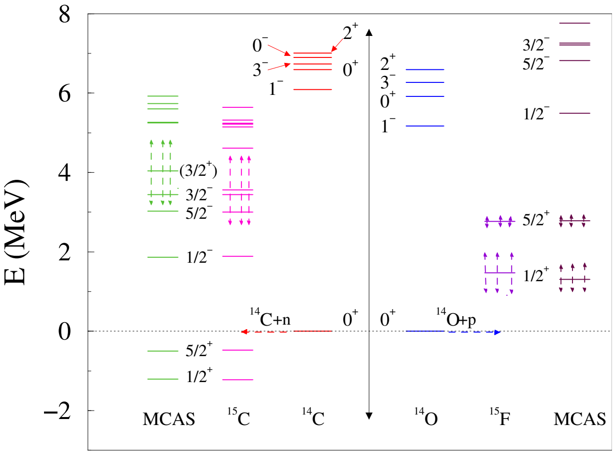
$\frac{5}{2}^{-}$ resonance centroid very sensitive to Pauli blocking

Analysis of ^{15}F (vs. ^{15}C mirror partner)

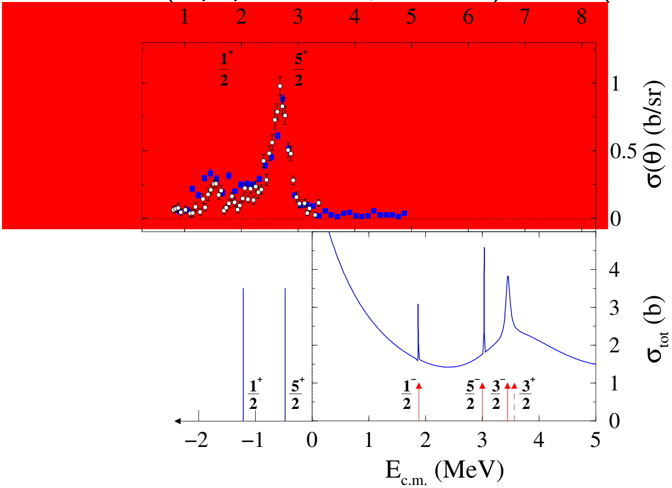
- It started in 2006, we were triggered by recent data by:
  V.Z. Goldberg *et al.* PRC **69** ('04)
  F.Q. Guo *et al.* PRC **72** ('05).
- **OUR STUDY** L.Canton, J.Svenne, K.Amos,*et al.*:
  PRL **96** ('06) used ^{15}C in fit-analysis
  to reproduce the resonant GS and first excited state in ^{15}F

$n - {}^{15}C$ PARAMETERS

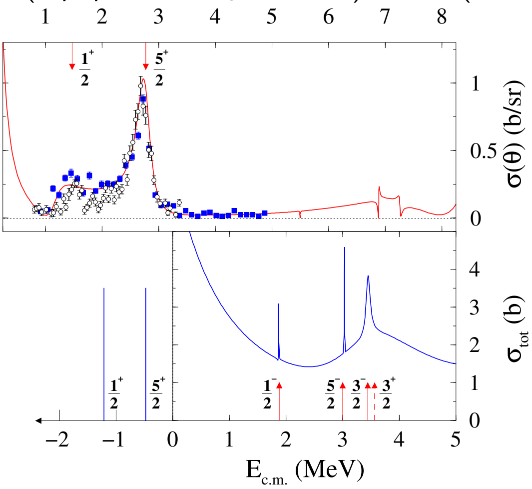
$V_0^{(\pm)} = -45.0$ MeV	$V_{ll}^{(\pm)} = 0.42$ MeV
$V_{ls}^{(\pm)} = 7.0$ MeV	$V_{ss}^{(\pm)} = --$
$R_0[a_0] = 3.1[0.65]$ fm	$\beta_2 = -0.50$



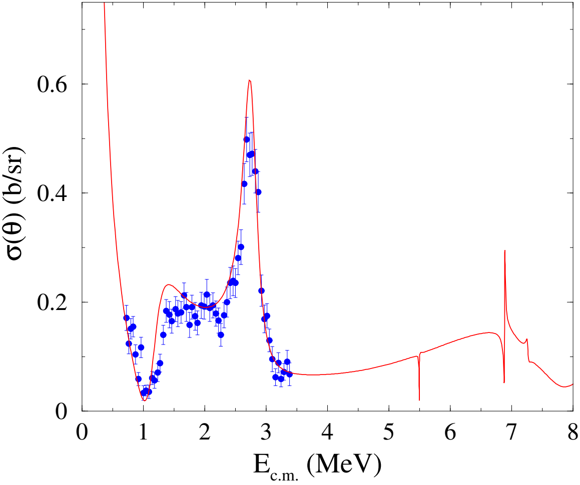
^{15}F (top, $p - ^{14}O$ $\theta_{cm} = 180^o$) & ^{15}C (bottom)



^{15}F (top, $p-^{14}O$ $\theta_{cm}=180^o$) & ^{15}C (bottom)



$^{15}F \ (p -^{14} O \ \ \theta_{cm} = 147^o)$



Voices from theory ...

^{15}F resonant states

J^π	Theory $E, (\frac{1}{2}\Gamma)$	Experiment $E, (\frac{1}{2}\Gamma)$
$\frac{1}{2}^+$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^+$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^-$	5.49 (0.005)	
$\frac{5}{2}^-$	6.88 (0.01)	
$\frac{3}{2}^-$	7.25 (0.04)	
$\frac{1}{2}^+$	7.21 (1.2)	
$\frac{5}{2}^+$	7.75 (0.4)	
$\frac{3}{2}^+$	7.99 (3.6)	

Table: See publication PRL **96** 072502 (2006)

^{15}F resonant states

J^π	Theory $E, (\frac{1}{2}\Gamma)$	Experiment $E, (\frac{1}{2}\Gamma)$
$\frac{1}{2}^+$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^+$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^-$	5.49 (0.005)	4.9 (<0.2)
$\frac{5}{2}^-$	6.88 (0.01)	6.4 (<0.2)
$\frac{3}{2}^-$	7.25 (0.04)	
$\frac{1}{2}^+$	7.21 (1.2)	
$\frac{5}{2}^+$	7.75 (0.4)	7.8 (0.4) ?
$\frac{3}{2}^+$	7.99 (3.6)	?

Table: See publication @GSI Darmstadt Mukha et al. PRC **79** 061301 (2009)

Ten years later... Physics Letters B 758 (2016) @Ganil

An above-barrier narrow resonance in ^{15}F

F. de Grancey [a], A. Mercenne [a], F. de Oliveira Santos [a,*], T. Davinson [b], O. Sorlin [a], J.C. Angélique [c], M. Assié [a,d], E. Berthoumieux [e], R. Borcea [h], A. Buta [h], I. Celikovic [g], V. Chudoba [i], J.M. Daugas [f], G. Dumitru [h], M. Fadil [a], S. Grévy [a,j], J. Kiener [k], A. Lefebvre-Schuhl [k], N. Michel [a], J. Mrazek [i], F. Negoita [h], J. Okołowicz [l], D. Pantelica [h], M.G. Pellegriti [a], L. Perrot [a,d], M. Płoszajczak [a], G. Randisi [a], I. Ray [a], O. Roig [f], F. Rotaru [h], M.G. Saint Laurent [a], N. Smirnova [j], M. Stanoiu [h], I. Stefan [a,d], C. Stodel [a], K. Subotic [g], V. Tatischeff [k], J.C. Thomas [a], P. Ujić [g], R. Wolski [l]

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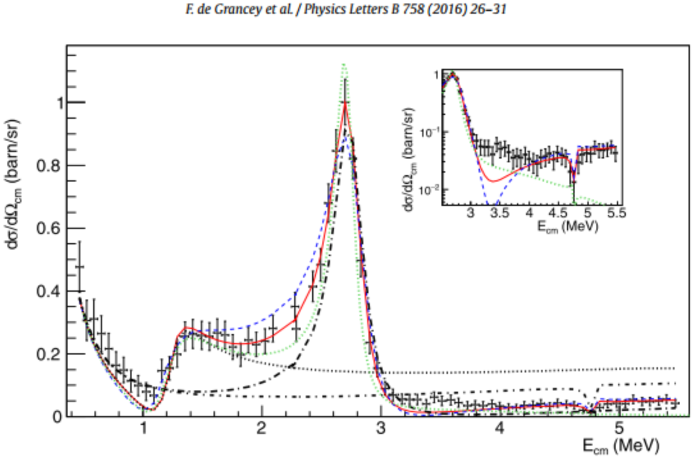
FOUND in proton-^{14}O scattering in inverse kinematics the first narrow state



With determined spin-parity $\frac{1}{2}^{(-)}$

Table 1
Resonance energy, width and spin measured and theoretical predictions for the second excited state of ^{15}F.

	Ref.	Second excited state		
		E_R (MeV)	Γ (keV)	J^π
Measured	[16]	4.800(100)	150(100)	–
	[31]	4.900(200)	200(200)	–
	Present	4.757(16)	36(19)	$\frac{1}{2}^-$
Predicted	[27]	5.49	5	$\frac{1}{2}^-$

Connection to ^{13}N-(2p) threshold!

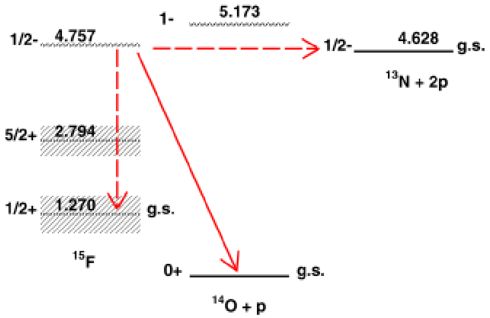
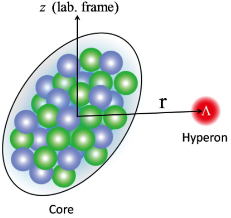


Fig. 2. (Color online.) Level scheme of ^{15}F. The possible decay channels from the $J^{\pi} = 1/2^{-}_{1}$ resonance are: the one proton emission (red arrow), gamma transition and two proton emission (red dashed arrow). The hatched areas correspond to the width of the resonances.

Extrapolating IKEDA's rule, close to that threshold, the system (and therefore that $\frac{1}{2}^{(-)}$ state) becomes a cluster ^{13}N-(2p), which explains why it has little overlap with ^{14}O-(p), and becomes a narrow resonance.

Other examples: hypernuclei



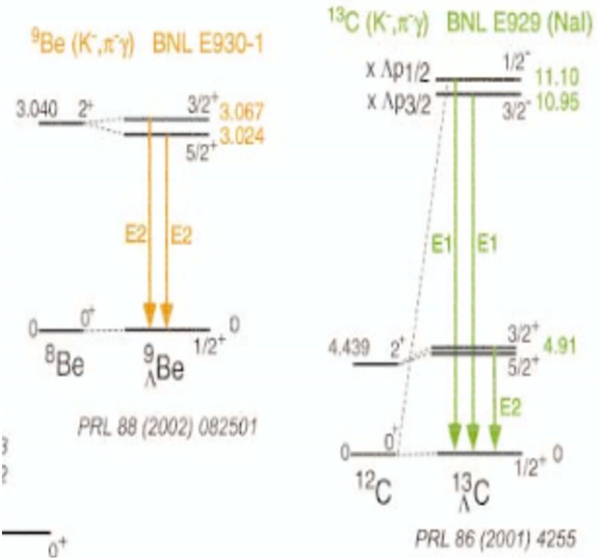


Table: Strengths of the Λ-^{12}C interaction with $R = 2.6$ fm., $a = 0.6$ fm., and β_2 =-0.52

	Case 1		Case 2	
	$\pi = -1$	$\pi = +1$	$\pi = -1$	$\pi = +1$
V_0 (MeV)	-28.9	-30.4	-28.9	-30.4
$V_{\ell s}$ (MeV)	0.35	0.35	0.35	0.35
V_{sl} (MeV)	0.0	0.0	-0.1	-0.1

and no constraints from the Pauli principle!

Table: Spectra of ${}^{13}_{\Lambda}C$

J^π	Exp.	Case 1	Case 2
$\frac{1}{2}^-$	− − −	+4.65 (0.21 MeV)	+4.66 (0.23 MeV)
$\frac{3}{2}^-$	− − −	+4.64 (0.22 MeV)	+4.63 (0.21 MeV)
$\frac{5}{2}^-$	− − −	+4.28 (1.0 keV)	+4.31 (1.0 keV)
$\frac{7}{2}^-$	− − −	+4.17 (1.0 keV)	+4.14 (1.0 keV)
$\frac{3}{2}^-$	− − −	+3.10 (0.1 keV)	+3.15 (0.1 keV)
$\frac{5}{2}^-$	− − −	+3.05 (>0.1 keV)	+3.02 (>0.1 keV)
$\frac{1}{2}^-$	−0.708	−0.74	−0.74
$\frac{3}{2}^-$	−0.86	−0.89	−0.89
$\frac{1}{2}^+$	− − −	−4.12	−4.12
$\frac{3}{2}^+$	−6.81	−7.177	−7.08
$\frac{5}{2}^+$	−6.81	−7.178	−7.24
$\frac{1}{2}^+$	−11.69	−11.68	−11.68

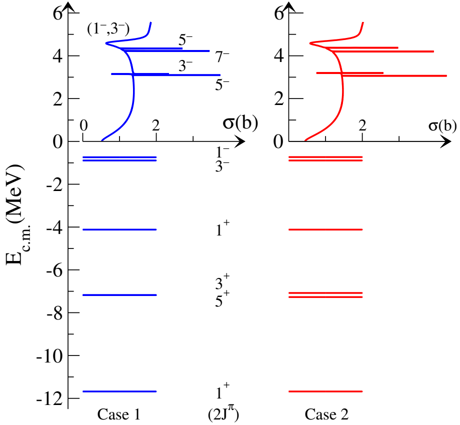


Figure: The spectra and total elastic cross sections for the Λ-^{12}C system.

Low-energy hypernuclear spectra of $^{13}C_\Lambda$

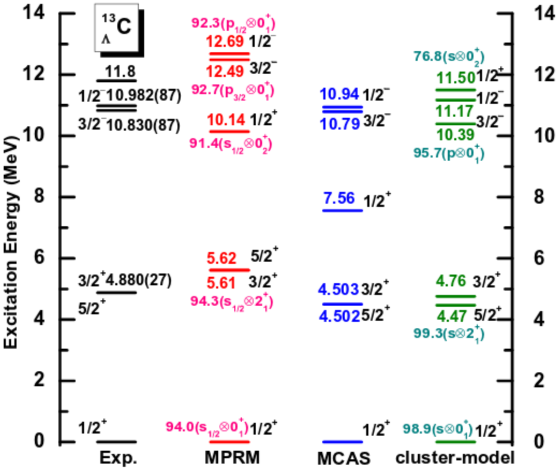


Figure: PRC 91/93 (2015/2016) Mei, Hagino, Yao, Motoba

α-He6 scattering/cross-sections

$$V_{cc'}(r) = \langle \ell I \mid W(r) \mid \ell' I' \rangle = \left[V_0 \delta_{c'c} f(r) + V_{\ell\ell} f(r) [\ell \cdot \ell] + V_{II} f(r) [\mathbf{I} \cdot \mathbf{I}] + V_{\ell I} g(r) [\ell \cdot \mathbf{I}] \right]_{cc'}$$

Table: The states of ^{6}He used in the coupled-channel evaluations All energies are in units of MeV.

state	Centroid	Width
$0^+_{\rm g.s.}$	0.000	0.00
2^+_1	1.797	0.113
2^+_2	5.60	10.0

Treatment of Pauli principle using the OPP technique. Method discussed within the Cluster Approach (Analytical RGM) in Yu.A. Lashko, G.F. Filippov, L. Canton Ukr. J. Phys. 2015.

α-He6 scattering/cross-sections

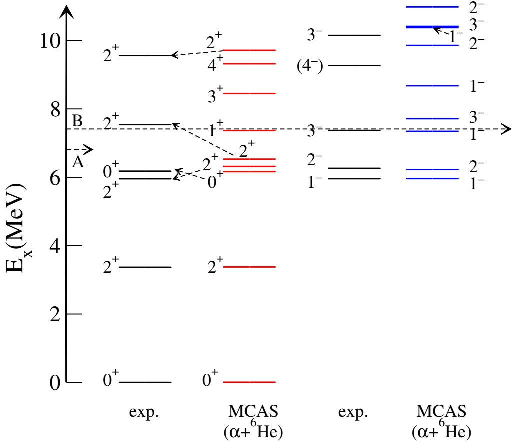


Figure: The spectrum of low-excitation states in ^{10}Be. To aid distinction the positive and negative parity states are shown on the left and right separately. The lines labelled 'A' and 'B' indicate the n+^{9}Be and the α+^{6}He thresholds respectively.

α-He6 scattering/cross-sections

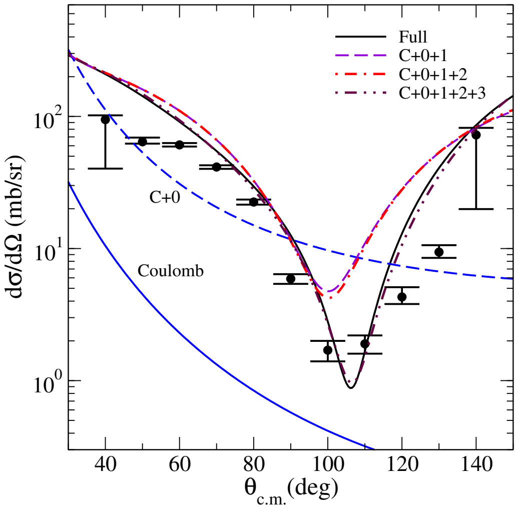


Figure: The differential cross section measured at 3.8 MeV (c.m.) as partial waves are added to the evaluations.

α-He6 scattering/cross-sections

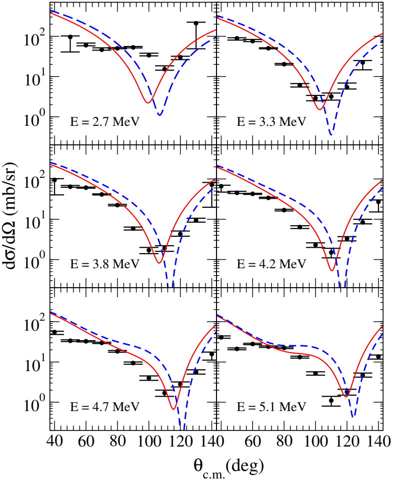


Figure: Exp data D. Suzuki et al., Phys. Rev. C 87, 054301 (2013). The solid curves are the results when a monopole interaction is used, the dashed ones when 2s-orbit enhancement in OPP is applied

C o n c l u s i o n i

E' stato presentato uno schema di calcolo fenomenologico a canali accoppiati adatto per l'analisi dei processi nucleari nella regione di bassa energia. E' applicabile allo studio dello scattering, risonanze e stati legati, con nuclei stabili, instabili, e oltre le driplines. E' applicabile anche allo studio dei processi di cattura radiativa (anche di interesse astrofisico).

MCAS COLLABORATION

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