Unified Description of Neutrino-Nucleus Interactions within Nuclear Many-Body Theory

Omar Benhar

INFN and Department of Physics, “Sapienza” University
I-00185 Roma, Italy

Based on work done in collaboration with
A. Ankowski, A. Cipollone, N. Farina, C. Losa, A. Loreti, A. Lovato, D. Meloni, and E. Vagnoni

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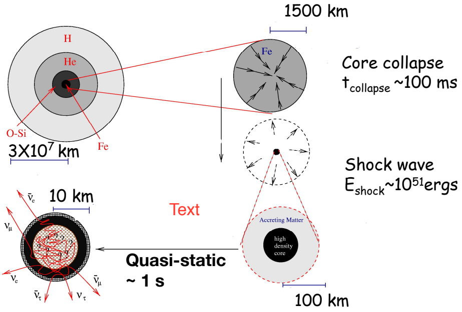
OUTLINE

- ## Motivation
  - Low-energy regime: supernova neutrinos
  - High-energy regime: oscillation signal of accelerator-based experiment

- ## The electroweak nuclear response
  - Low-energy regime: mean field dynamics, short- and long-range correlations
  - High-energy regime: the impulse approximation. Single particle motion in interacting many-body systems

- ## Summary & Outlook

Supernova Neutrinos



- ⋆ Neutrino diffuse out of dense newly born neutron stars, carrying away $\sim 10^{53}$ erg

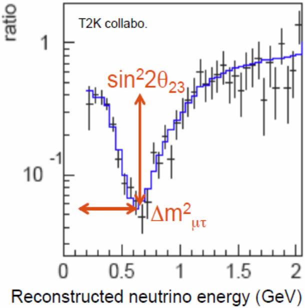
- ⋆ The time structure of the neutrino signal depends on heat transport in the neutron star core, the density of which is $n_B \sim 10^{13} - 10^{15}$ g cm^{-3}
- ⋆ The spectrum is determined by scattering in the hot outer region called neutrino-sphere, with typical temperature and density $T \sim 5 - 10$ MeV and $n_B \sim 10^{11} - 10^{13}$ g cm^{-3}
- ⋆ Energy scale $E_\nu \lesssim 10$ MeV

Neutrino Oscillations

- Neutrinos produced in weak interaction processes, e.g. β-decay, are not in mass-eigenstates $\Rightarrow$ a neutrino created with a specific flavor (ν_e, ν_μ or ν_τ) can be detected at a later time with different flavor
- Probability of appearance of a new flavor after travelling a distance L (consider two flavors, for simplicity)

$$P_{\alpha\to\beta} = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

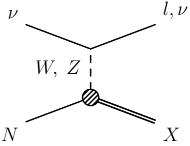
$$\Delta m^2 = m^2_{\nu_1} - m^2_{\nu_2}$$



- The energy scale of long-baseline experiments ($L \sim$ few 100s km) is $E_\nu \sim$ few 100s MeV to few GeV

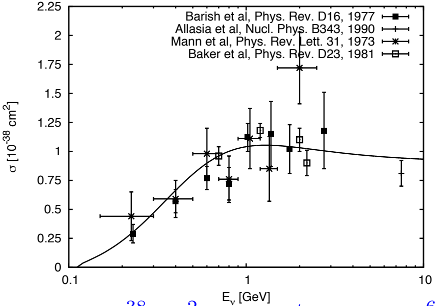
Neutrino Interactions are Weak

- Neutrino-nucleon scattering



- Total cross section of the process

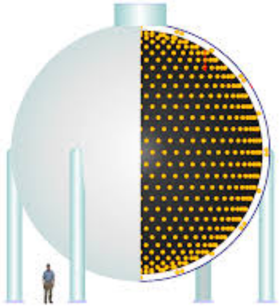
$$\nu_\mu + n \to \mu^- + p$$



$$\sigma_{\nu N} \sim 10^{-38}\,\mathrm{cm}^2 \quad , \quad \sigma_{\nu N}/\sigma_{eN} \sim 10^{-6}$$

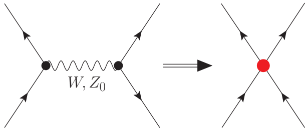
Detecting Neutrinos Requires Big Detectors

- The SuperKamiokande detector, in Japan, is filled with 12.5 million gallon of ultra-clean water
- The MiniBooNE detector,at FNAL, is filled with 800 tons of mineral oil
- The detected signal results from neutrino interactions with Oxygen and Carbon nuclei
- A quantitative understanding of the their response to neutrino interactions is required for the interpretation of the measured cross sections



Neutrino-Nucleon Interactions

- Neutrino interactions are mediated by the gauge bosons $W^{\pm}$ and Z_0, whose masses are in the range $\approx 80 - 90$ GeV
- In all processes to be discussed in this talk the momentum transfer q is such that $q^2 << M^2_{W,Z} \sim 80 - 90$ GeV $\Rightarrow$ Fermi theory of weak interaction works just fine



$$\mathcal{L}_F = \frac{G}{\sqrt{2}} J_{N\mu} J_\ell{}^\mu$$

$$J_\ell{}^\mu = \begin{cases} \bar{u}_{\ell^-} \gamma^\mu (1 - \gamma_5) u_\nu & (CC) \\ \bar{u}_{\nu'} \gamma^\mu (1 - \gamma_5) u_\nu & (NC) \end{cases}$$

- The nucleon current can be cast in the non relativistic limit

$$J_{N\mu} = \begin{cases} \bar{u}_p \gamma_\mu (1 - g_A \gamma_5) u_n & \rightarrow & \chi^\dagger_{s_p} (g^0_\mu + g_A g^\mu_i \sigma_i) \chi_{s_n} & (CC) \\ \bar{u}_{n'} \gamma_\mu (1 - c_A \gamma_5) u_n & \rightarrow & \chi^\dagger_{s'_n} (g^0_\mu + c_A g^\mu_i \sigma_i) \chi_{s_n} & (NC) \end{cases}$$

Neutrino-Nucleon Cross Section

- x-section of the charged-current process $\nu_\ell + n \to \ell^- + X$

$$d\sigma \propto L_{\lambda\mu} W^{\lambda\mu}$$

  - $L_{\lambda\mu}$ is determined by the lepton kinematical variables (more on this later)
  - under general assumptions $W^{\lambda\mu}$ can be written in terms of five structure functions $W_i(q^2, (p \cdot q))$ (p is the nucleon four momentum)

$$W^{\lambda\mu} = -g^{\lambda\mu}\, W_1 + p^\lambda\, p^\mu\, \frac{W_2}{m_N^2} + \varepsilon^{\lambda\mu\alpha\beta}\, q_\alpha\, p_\beta\, + \frac{W_3}{m_N^2} + q^\lambda\, q^\mu\, \frac{W_4}{m_N^2}$$
$$+ (p^\lambda\, q^\mu + p^\mu\, q^\lambda)\, \frac{W_5}{m_N^2}$$

- In principle, the structure functions W_i can be extracted from the measured cross sections

NEUTRINO-NUCLEUS INTERACTION

- Replace the nucleon tensor witht the nuclear response tensor

$$W_{\lambda\mu} = \sum_{n \neq 0} \langle 0 | J_\lambda^\dagger | n \rangle \langle n | J_\mu | 0 \rangle \delta^{(4)}(p + k - p_n - k')$$

- Consider, for example, a neutral current process

$$\nu + A \to \nu' + X$$

- In non relativistic regime

$$W(\mathbf{q}, \omega) \propto \frac{G_F}{4\pi^2} L_{\lambda\mu} W^{\lambda\mu} = \frac{G_F}{4\pi^2} \left[(1 + \cos\theta) \mathcal{S}^\rho + \frac{{c_A}^2}{3} (3 - \cos\theta) \mathcal{S}^\sigma \right]$$

where $\cos\theta = (\mathbf{k} \cdot \mathbf{k}')/(|\mathbf{k}||\mathbf{k}'|)$, while $\mathcal{S}^\rho$ and $\mathcal{S}^\rho$ are the nuclear responses in the density and spin-density channels, respectively.

- density response

$$\mathcal{S}^\rho = \frac{1}{N}\sum_n |\langle 0|J_0|n\rangle\langle n|J_0|0\rangle\delta^{(4)}(P_0+q-P_n)$$

- spin-density response ($\alpha,\ \beta = 1,\ldots 3$)

$$\mathcal{S}^\rho = \sum_\alpha \mathcal{S}^\rho_{\alpha\alpha}$$

$$\mathcal{S}^\rho_{\alpha\beta} = \frac{1}{N}\sum_n |\langle 0|J_\alpha|n\rangle\langle n|J_\beta|0\rangle\delta^{(4)}(P_0+q-P_n)$$

- Neutral weak current

$$J_0 = \sum_i j_i^0 = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \quad , \quad J_\alpha = \sum_i j_i^\mu = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i}\sigma_\alpha$$

- Outstanding issues
  - Model nuclear dynamics
  - Determine the nuclear initial and final states

MODELING NUCLEAR DYNAMICS

- *ab initio* (bottom-up) approach

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk}$$

  - v_{ij} provides a very accurate descritpion of the two-nucleon system, and reduces to Yukawa's one-pion-exchange potential at large distances
  - inclusion of v_{ijk} needed to explain the ground-state energies of the three-nucleon systems
  - v_{ij} is spin and isospin dependent, and strongly repulsive at short distance
  - note: nuclear interactions can not be treated in perturbation theory in the basis of eigenstates of the non interacting system. Either the interaction or the basis states need to be "renormalized" to incorporate non perturbative effetcs
- Mean field (independent particle) approximation

$$\left\{ \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk} \right\} \to \sum_i U_i$$

Effective Interaction

- The effective interaction in nuclear matter at density ρ is defined through the relation [$k_F = (3\pi^2\rho/2)^{1/3}$]

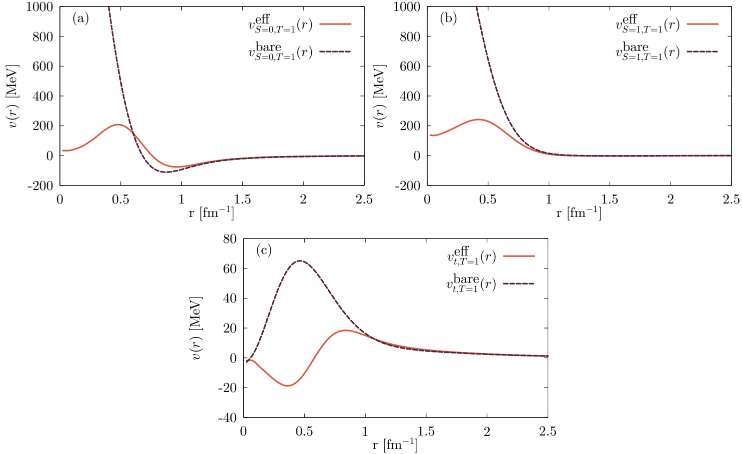
$$\langle 0|H|0\rangle = \frac{3}{5}\frac{k_F^2}{2m} + \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle$$

- unlike the bare NN potential, $V_{\rm eff}$ is well behaved, and can be used to perform perturbative calculations in the basis of eigenstates of the non interacting system
- the response can be also computed using the Fermi gas states and the corresponding effective operators, defined through

$$\langle n|J^\mu|0\rangle = \langle n_{FG}|J^\mu_{\rm eff}|0_{FG}\rangle$$

Effective Interaction

- Comparison between bare and CBF effective interaction at nuclear matter equilibrium density



INTERACTION EFFECTS

- Mean field effects
  - Change of nucleon energy spectrum

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}' | V_{\text{eff}} | \mathbf{k}\mathbf{k}' \rangle_a$$

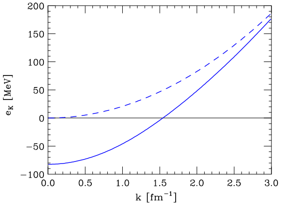
  - Effective mass

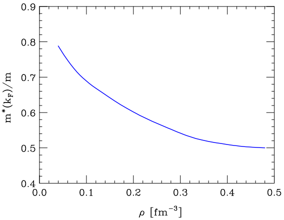
$$\frac{1}{m_k^\star} = \frac{1}{|\mathbf{k}|} \frac{de_k}{d|\mathbf{k}|}$$

- Correlation effects
  - Effective operators couple the ground state to two-particle–two-hole (2p2h) final states, thus removing strength from the 1p1h sector

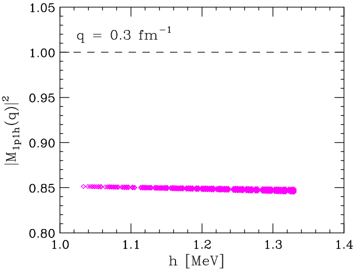
$$M_{2p2h} = \langle 2p2h | J^\mu_{\text{eff}} | 0 \rangle \neq 0 \to M_{1p1h} = \langle 1p1h | J^\mu_{\text{eff}} | 0 \rangle < \langle 1p1h | J^\mu | 0 \rangle$$

⋆ Nucleon energy spectrum and Effective mass in isospin-symmetric matter at equilibrium density



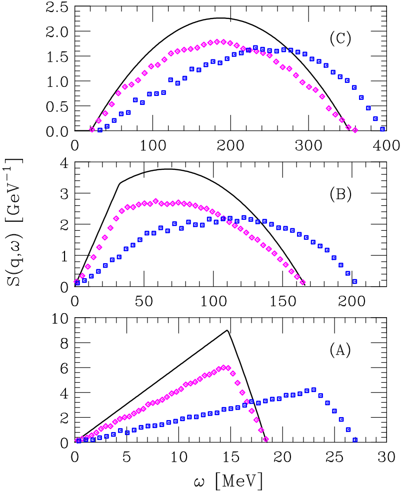


⋆ Quenching of Fermi transition strength in isospin-symmetric matter at equilibrium density



q-Evolution of Interaction Effects

⋆ Density response of isospin-symmetric matter at equilibrium density



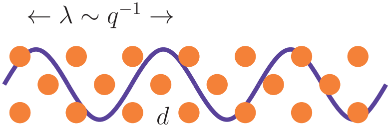
$|\mathbf{q}| = 3.0\ \mathrm{fm}^{-1}$

$|\mathbf{q}| = 1.8\ \mathrm{fm}^{-1}$

$|\mathbf{q}| = 0.3\ \mathrm{fm}^{-1}$

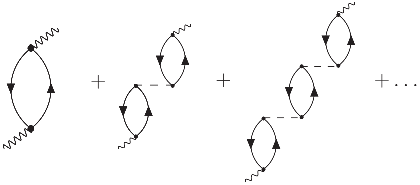
Long-Range Correlations

- At low momentum transfer the space resolusion of the neutrino becomes much larger than the average NN separation distance (~ 1.5 fm), and the interaction involves many nucleons



- Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

$$|n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$



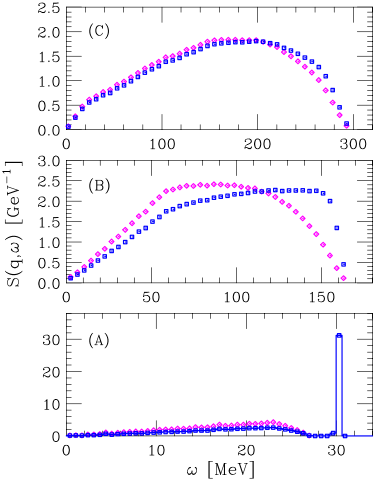
Effects of long-range correlations

- $|\mathbf{q}|$-evolution of the density-response of isospsin-symmetric nuclear matter. Calculation carried out within CBF using a realistic nuclear hamiltonian.

$|\mathbf{q}| \approx$ 480 MeV

$|\mathbf{q}| \approx$ 300 MeV

$|\mathbf{q}| \approx$ 60 MeV



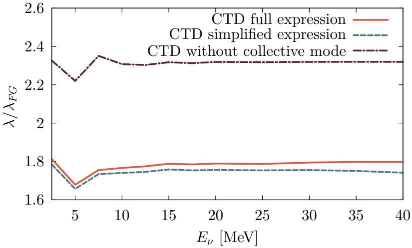
OB and N. Farina, PLB **680** 305, (2009)

Neutrino Mean Free Path in Neutron Matter

- The mean free path of non degenerate neutrinos at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4}\rho \int \frac{d^3q}{(2\pi)^3} \left[(1+\cos\theta)S(\mathbf{q},\omega) + \mathbf{C_A^2}(\mathbf{3} - \cos\theta)\mathcal{S}(\mathbf{q},\omega)\right]$$

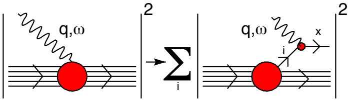
where S and $\mathcal{S}$ are the density (Fermi) and spin (Gamow Teller) response, respectively



- Both short and long range correlations important

High-Energy Regime: the Impulse Approximation

- The IA amounts to replacing



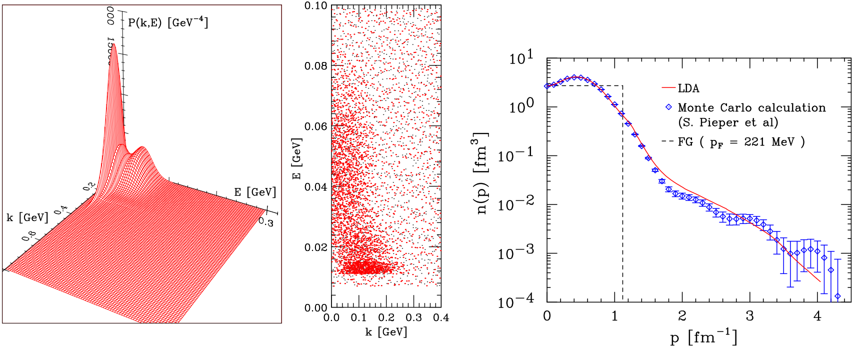
$$J_\mu = \sum_i j_\mu(i) \quad , \quad |X\rangle \to |x, \mathbf{p}_x\rangle \otimes |\mathcal{R}, \mathbf{p}_\mathcal{R}\rangle \ .$$

- Nuclear dynamics and electromagnetic interactions are decoupled. Relativistic effect can be properly accounted for

$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E)$$

  - The electron-nucleon cross section $d\sigma_N$ can be written in terms of stucture functions extracted from electron-proton and electron-deuteron scattering data
  - The *hole* spectral function $P_h(\mathbf{k}, E)$, momentum *and* energy distribution of the knocked out nucleon, can be obtained from *ab initio* many-body calculations

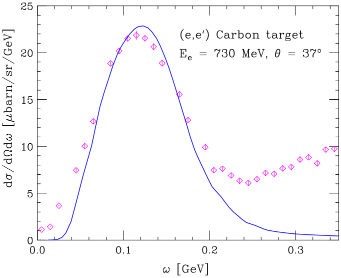
Oxygen Spectral Function



- shell model states account for $\sim$ 80% of the strenght
- the remaining $\sim$ 20%, arising from NN correlations, is located at high momentum *and* large removal energy

Quasi Elastic Electron Scattering

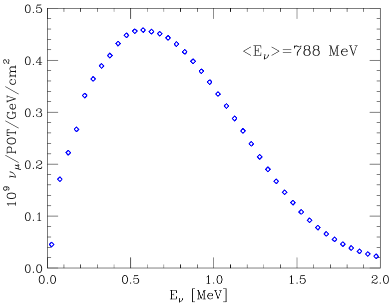
- Elementary interaction vertex described in terms of the vector form factors, $F_1^{(p,n)}$ and $F_2^{(p,n)}$, precisely measured over a broad range of Q^2



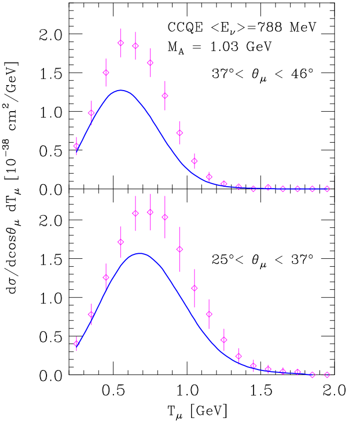
- Position and width of the peak are determined by $P_h(\mathbf{k}, E)$
- The tail extending to the region of high energy loss is due to nucleon-nucleon correlations in the initial state, leading to the occurrence of two particle-two hole (2p2h) final states

Analysis of MiniBooNE CCQE data

- MiniBooNE flux

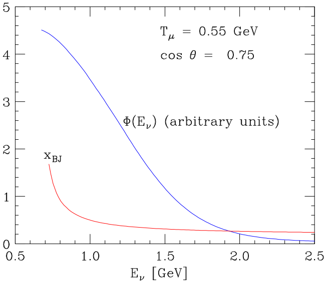


- MiniBooNe CCQE data



Contribution of Different Reaction Mechanisms

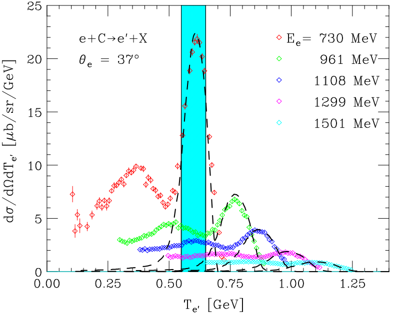
- In neutrino interactions the lepton kinematics is *not* determined. The flux-averaged cross sections at fixed T_μ and $\cos\theta_\mu$ picks up contributions at different beam energies, corresponding to a variety of kinematical regimes in which different reaction mechanisms dominate



- $x = 1 \rightarrow E_\nu$ 0.788 GeV , $x = 0.5 \rightarrow E_\nu$ 0.975 GeV
- $\Phi(0.975)/\Phi(0.788) = 0.83$

"Flux Averaged" Electron-Nucleus x-Section

- The electron scattering x-section off Carbon at $\theta_e = 37°$ has been measured for a number of beam energies



- Theoretical calculations of the flux-averaged cross sections must include all relevant reaction mechanism—one and two -nucleon emission, excitation of nucleon resonances and deep inelastic scattering—within a consistent approach

Summary & Outlook

- ⋆ The study of neutrino interactions with nuclear matter and nuclei is a strongly cross-disciplinary field, with applications in a variety of areas, ranging from the astrophysics of compact stars to the search of neutrino oscillations

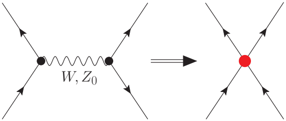
- ⋆ Recent studies carried out within nuclear many-body theory using realistic hamiltonians have revealed a variety of dynamical effects, whose effects on the nuclear response are large

- ⋆ The emerging picture suggests that a unified description of the nuclear response in a broad kinematical range cortresponding to neutrino energies ranging from few MeV to few GeV

Backup slides

Neutrino-Nucleon Interactions

- In the regime of momentum transfer (q) discussed in this talk Fermi theory of weak interaction works just fine



- x-section of the charged-current process $\nu_\ell + n \to \ell^- + X$

$$d\sigma \propto L_{\lambda\mu} W^{\lambda\mu}$$

  - $L_{\lambda\mu}$ is determined by the lepton kinematical variables (more on this later)
  - under general assumptions $W^{\lambda\mu}$ can be written in terms of five structure functions $W_i(q^2, (p \cdot q))$ (p is the nucleon four momentum)

$$W^{\lambda\mu} = -g^{\lambda\mu}\, W_1 + p^\lambda\, p^\mu\, \frac{W_2}{m_N^2} + \varepsilon^{\lambda\mu\alpha\beta}\, q_\alpha\, p_\beta\, \frac{W_3}{m_N^2} + q^\lambda\, q^\mu\, \frac{W_4}{m_N^2} + (p^\lambda\, q^\mu + p^\mu\, q^\lambda)\, \frac{W_5}{m_N^2}$$

- In principle, the structure functions W_i can be extracted from the measured cross sections
- In the charged-current elastic sector $\nu_\ell + n \to \ell^- + p$ they can be expressed in terms of vector ($F_1(q^2)$ and $F_2(q^2)$), axial-vector ($F_A(q^2)$) and pseudoscalar ($F_P(q^2)$) *form factors*

$$W_1 = 2\left[-\frac{q^2}{2}\,(F_1+F_2)^2 + \left(2\,m_N^2 - \frac{q^2}{2}\right)\,{F_A}^2\right]$$

$$W_2 = 4\left[{F_1}^2 - \left(\frac{q^2}{4\,m_N^2}\right)\,{F_2}^2 + {F_A}^2\right] = 2W_5$$

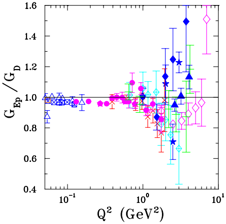
$$W_3 = -4\,(F_1+F_2)\,F_A$$

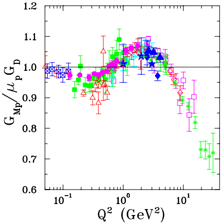
$$W_4 = -2\left[F_1\,F_2 + \left(2\,m_N^2 + \frac{q^2}{2}\right)\,\frac{{F_2}^2}{4\,m_N^2} + \frac{q^2}{2}\,{F_P}^2 - 2\,m_N\,F_P\,F_A\right]$$

- according to the CVC hypothesis, F_1 and F_2 can be related to the electromagnetic form factors, measured by electron-nucleon scattering, while PCAC allows one to express F_P in terms of the axial form factor (more on this later)

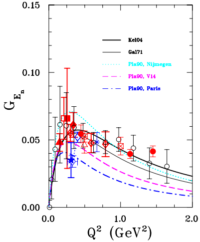
Vector Form Factors

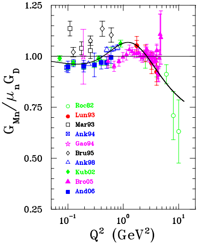
- ⋆ Proton data





- ⋆ Neutron (deuteron) data

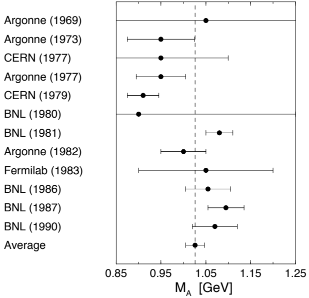




Axial Form Factor

⋆ Dipole parametrization

$$F_A(Q^2) = \frac{g_A}{\left[1 + (Q^2/M_A^2)\right]^2}$$



- ▷ g_A from neutron β-decay
- ▷ axial mass M_A from (quasi) elastic ν- and $\bar{\nu}$-deuteron experiment

TAMM-DANCOFF (RING) APPROXIMATION

- ⋆ Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

$$|ph\rangle \to |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the hamiltonian matrix

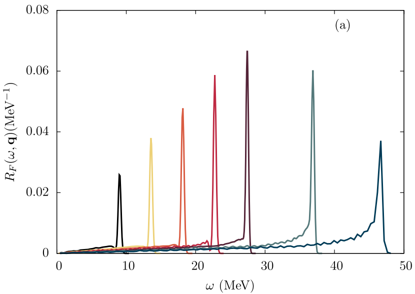
$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i | V_{\text{eff}} | h_j p_j)$$

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}' | V_{\text{eff}} | \mathbf{k}\mathbf{k}' \rangle_a$$

- ⋆ The appearance of an eigenvalue, ω_n, lying outside the particle-hole continuum signals the excitation of a collective mode

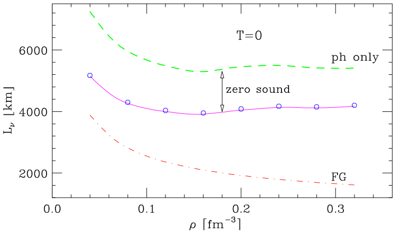
EXCITATION OF COLLECTIVE MODES

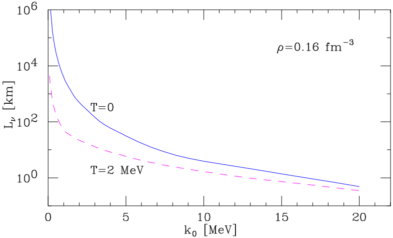
- Density (a) and spin-density (b) responses of isospin-symmetric nuclear matter at equilibrium density



- $|\mathbf{q}|$ = 0.1, 0.15, 0.20, 0.25, 0.30, 0.40 and 0.50 fm^{-1}

⋆ Mean free path of a non degenerate neutrino in neutron matter. Left: density-dependence at $k_0 = 1$ MeV and $T = 0$; Right: energy dependence at $\rho = 0.16$ fm^{-3} and $T = 0, 2$ MeV





$\star$ Density and temperature dependence of the mean free path of a non degenerate neutrino at $k_0 = 1$ MeV and $\rho = 0.16$ fm^{-3}