Unified Description of Neutrino-Nucleus Interactions within Nuclear Many-Body Theory

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Outline

* Motivation
  - Low-energy regime: supernova neutrinos
  - High-energy regime: oscillation signal of accelerator-based experiment

* The electroweak nuclear response
  - Low-energy regime: mean field dynamics, short- and long-range correlations
  - High-energy regime: the impulse approximation. Single particle motion in interacting many-body systems

* Summary & Outlook
**Supernova Neutrinos**

- Neutrino diffuse out of dense newly born neutron stars, carrying away $\sim 10^{53}$ erg

- The time structure of the neutrino signal depends on heat transport in the neutron star core, the density of which is $n_B \sim 10^{13} - 10^{15}$ g cm$^{-3}$

- The spectrum is determined by scattering in the hot outer region called neutrino-sphere, with typical temperature and density $T \sim 5 - 10$ MeV and $n_B \sim 10^{11} - 10^{13}$ g cm$^{-3}$

- Energy scale $E_\nu \lesssim 10$ MeV
Neutrino Oscillations

- Neutrinos produced in weak interaction processes, e.g. $\beta$-decay, are not in mass-eigenstates $\Rightarrow$ a neutrino created with a specific flavor ($\nu_e$, $\nu_\mu$ or $\nu_\tau$) can be detected at a later time with different flavor.
- Probability of appearance of a new flavor after travelling a distance $L$ (consider two flavors, for simplicity)

\[
P_{\alpha \rightarrow \beta} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right)
\]

\[
\Delta m^2 = m_{\nu_1}^2 - m_{\nu_2}^2
\]

- The energy scale of long-baseline experiments ($L \sim$ few $100s$ km) is $E_\nu \sim$ few $100s$ MeV to few GeV
Neutrino Interactions are Weak

▶ Neutrino-nucleon scattering

\[ \nu + W, Z + N \rightarrow l, \nu + X \]

Total cross section of the process

\[ \nu_\mu + n \rightarrow \mu^- + p \]

\[ \sigma_{\nu N} \sim 10^{-38} \text{ cm}^2, \quad \sigma_{\nu N}/\sigma_{eN} \sim 10^{-6} \]
Detecting Neutrinos Requires Big Detectors

- The SuperKamiokande detector, in Japan, is filled with 12.5 million gallon of ultra-clean water
- The MiniBooNE detector, at FNAL, is filled with 800 tons of mineral oil
- The detected signal results from neutrino interactions with Oxygen and Carbon nuclei
- A quantitative understanding of the their response to neutrino interactions is required for the interpretation of the measured cross sections
**Neutrino-Nucleon Interactions**

- Neutrino interactions are mediated by the gauge bosons $W^\pm$ and $Z_0$, whose masses are in the range $\approx 80 - 90$ GeV
- In all processes to be discussed in this talk the momentum transfer $q$ is such that $q^2 << M_{W,Z}^2 \sim 80 - 90$ GeV $\Rightarrow$ Fermi theory of weak interaction works just fine

\[ L_F = \frac{G}{\sqrt{2}} J_{N\mu} J_{\ell\mu} \]

\[ J_{\ell\mu} = \begin{cases} 
\bar{u}_\ell \gamma^\mu (1 - \gamma_5) u_\nu & (CC) \\
\bar{u}_\nu' \gamma^\mu (1 - \gamma_5) u_\nu & (NC) 
\end{cases} \]

- The nucleon current can be cast in the non relativistic limit

\[ J_{N\mu} = \begin{cases} 
\bar{u}_p \gamma_\mu (1 - g_A \gamma_5) u_n & \rightarrow \chi_{s_p}^\dagger (g_0 + g_A g_\mu^i \sigma_i) \chi_{s_n} & (CC) \\
\bar{u}_n \gamma_\mu (1 - c_A \gamma_5) u_n & \rightarrow \chi_{s_n}^\dagger (g_0 + c_A g_\mu^i \sigma_i) \chi_{s_n} & (NC) 
\end{cases} \]
**Neutrino-Nucleon Cross Section**

- x-section of the charged-current process $\nu_{\ell} + n \rightarrow \ell^- + X$

$$d\sigma \propto L_{\lambda\mu} W^{\lambda\mu}$$

- $L_{\lambda\mu}$ is determined by the lepton kinematical variables (more on this later)
- under general assumptions $W^{\lambda\mu}$ can be written in terms of five structure functions $W_i(q^2, (p \cdot q))$ ($p$ is the nucleon four momentum)

$$W^{\lambda\mu} = -g^{\lambda\mu} W_1 + p^\lambda p^\mu \frac{W_2}{m_N^2} + \varepsilon^{\lambda\mu\alpha\beta} q_\alpha p_\beta + \frac{W_3}{m_N^2} + q^\lambda q^\mu \frac{W_4}{m_N^2} + (p^\lambda q^\mu + p^\mu q^\lambda) \frac{W_5}{m_N^2}$$

- In principle, the structure functions $W_i$ can be extracted from the measured cross sections
Neutrino-Nucleus Interaction

- Replace the nucleon tensor with the nuclear response tensor

\[ W_{\lambda\mu} = \sum_{n \neq 0} \langle 0 | J^\dagger_\lambda | n \rangle \langle n | J_\mu | 0 \rangle \delta^{(4)} (p + k - p_n - k') \]

- Consider, for example, a neutral current process

\[ \nu + A \rightarrow \nu' + X \]

- In non relativistic regime

\[ W(q, \omega) \propto \frac{G_F}{4\pi^2} L_{\lambda\mu} W^{\lambda\mu} = \frac{G_F}{4\pi^2} \left[ (1 + \cos \theta) S^\rho + \frac{c_A^2}{3} (3 - \cos \theta) S^\sigma \right] \]

where \( \cos \theta = (\mathbf{k} \cdot \mathbf{k}')/(||\mathbf{k}|| ||\mathbf{k}'||) \), while \( S^\rho \) and \( S^\rho \) are the nuclear responses in the density and spin-density channels, respectively.
* density response

\[ S^\rho = \frac{1}{N} \sum_n |\langle 0 | J_0 | n \rangle \langle n | J_0 | 0 \rangle \delta^{(4)}(P_0 + q - P_n) \]

* spin-density response \((\alpha, \beta = 1, \ldots 3)\)

\[ S^\rho = \sum_\alpha S^\rho_\alpha \alpha \]
\[ S^\rho_{\alpha \beta} = \frac{1}{N} \sum_n |\langle 0 | J_\alpha | n \rangle \langle n | J_\beta | 0 \rangle \delta^{(4)}(P_0 + q - P_n) \]

* Neutral weak current

\[ J_0 = \sum_i j_i^0 = \sum_i e^{i q \cdot x_i} , \quad J_\alpha = \sum_i j_i^\mu = \sum_i e^{i q \cdot x_i} \sigma_\alpha \]

* Outstanding issues
  ▶ Model nuclear dynamics
  ▶ Determine the nuclear initial and final states
MODELING NUCLEAR DYNAMICS

* * * ab initio (bottom-up) approach

\[ H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk} \]

▷ * \( v_{ij} \) provides a very accurate description of the two-nucleon system, and reduces to Yukawa’s one-pion-exchange potential at large distances

▷ inclusion of \( v_{ijk} \) needed to explain the ground-state energies of the three-nucleon systems

▷ * \( v_{ij} \) is spin and isospin dependent, and strongly repulsive at short distance

▷ note: nuclear interactions can not be treated in perturbation theory in the basis of eigenstates of the non interacting system. Either the interaction or the basis states need to be “renormalized” to incorporate non perturbative effects

* Mean field (independent particle) approximation

\[ \left\{ \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk} \right\} \rightarrow \sum_i U_i \]
**Effective Interaction**

★ The effective interaction in nuclear matter at density $\rho$ is defined through the relation $[k_F = (3\pi^2 \rho/2)^{1/3}]$

$$\langle 0 | H | 0 \rangle = \frac{3}{5} \frac{k_F^2}{2m} + \langle 0_{FG} | V_{\text{eff}} | 0_{FG} \rangle$$

★ unlike the bare NN potential, $V_{\text{eff}}$ is well behaved, and can be used to perform perturbative calculations in the basis of eigenstates of the non interacting system

★ the response can be also computed using the Fermi gas states and the corresponding effective operators, defined through

$$\langle n | J^\mu | 0 \rangle = \langle n_{FG} | J_{\text{eff}}^\mu | 0_{FG} \rangle$$
Effective Interaction

* Comparison between bare and CBF effective interaction at nuclear matter equilibrium density

![Graphs showing comparison between bare and effective interaction at different conditions](image)
Interaction Effects

★ Mean field effects

▷ Change of nucleon energy spectrum

\[ e_k = \frac{k^2}{2m} + \sum_{k'} \langle kk'|V_{\text{eff}}|kk'\rangle_a \]

▷ Effective mass

\[ \frac{1}{m_k^*} = \frac{1}{|k|} \frac{d e_k}{d|k|} \]

★ Correlation effects

▷ Effective operators couple the ground state to two-particle–two-hole (2p2h) final states, thus removing strength from the 1p1h sector

\[ M_{2p2h} = \langle 2p2h|J_{\text{eff}}^\mu|0\rangle \neq 0 \rightarrow M_{1p1h} = \langle 1p1h|J_{\text{eff}}^\mu|0\rangle < \langle 1p1h|J^\mu|0\rangle \]
Nucleon energy spectrum and Effective mass in isospin-symmetric matter at equilibrium density

Quenching of Fermi transition strength in isospin-symmetric matter at equilibrium density
q-EVOLUTION OF INTERACTION EFFECTS

Density response of isospin-symmetric matter at equilibrium density

$|q| = 3.0$ fm$^{-1}$

$|q| = 1.8$ fm$^{-1}$

$|q| = 0.3$ fm$^{-1}$
LONG-RANGE CORRELATIONS

* At low momentum transfer the space resolution of the neutrino becomes much larger than the average NN separation distance ($\sim 1.5 \text{ fm}$), and the interaction involves many nucleons

\[ \lambda \sim q^{-1} \]

* Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

\[ |n\rangle = \sum_{i=1}^{N} C_i |p_i h_i\rangle \]
Effects of Long-Range Correlations

- $|q|$-evolution of the density-response of isospin-symmetric nuclear matter. Calculation carried out within CBF using a realistic nuclear Hamiltonian.

$|q| \approx 480$ MeV

$|q| \approx 300$ MeV

$|q| \approx 60$ MeV

OB and N. Farina, PLB 680, 305, (2009)
Neutrino Mean Free Path in Neutron Matter

★ The mean free path of non degenerate neutrinos at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4} \rho \int \frac{d^3q}{(2\pi)^3} \left[ (1 + \cos \theta)S(q, \omega) + C_A^2(3 - \cos \theta)S(q, \omega) \right]$$

where $S$ and $S$ are the density (Fermi) and spin (Gamow Teller) response, respectively

★ Both short and long range correlations important
HIGH-ENERGY REGIME: THE IMPULSE APPROXIMATION

- The IA amounts to replacing

\[ J_\mu = \sum_i j_\mu(i) \text{, \hspace{1cm} } |X\rangle \rightarrow |x, p_x\rangle \otimes |R, p_R\rangle . \]

- Nuclear dynamics and electromagnetic interactions are decoupled. Relativistic effect can be properly accounted for

\[ d\sigma_A = \int d^3k dE \ d\sigma_N P_h(k, E) \]

- The electron-nucleon cross section \( d\sigma_N \) can be written in terms of structure functions extracted from electron-proton and electron-deuteron scattering data.

- The hole spectral function \( P_h(k, E) \), momentum and energy distribution of the knocked out nucleon, can be obtained from \textit{ab initio} many-body calculations.
Oxygen Spectral Function

- shell model states account for $\sim 80\%$ of the strength
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum and large removal energy
**Quasi Elastic Electron Scattering**

- Elementary interaction vertex described in terms of the vector form factors, $F_{1}^{(p,n)}$ and $F_{2}^{(p,n)}$, precisely measured over a broad range of $Q^2$

- Position and width of the peak are determined by $P_h(k, E)$

- The tail extending to the region of high energy loss is due to nucleon-nucleon correlations in the initial state, leading to the occurrence of two particle-two hole (2p2h) final states
ANALYSIS OF MINIBOONE CCQE DATA

Miniboone flux

$<E_{\nu}> = 788$ MeV

$10^9 \nu_\mu/\text{POI}/\text{GeV/cm}^2$

$E_{\nu}$ [MeV]

CCQE $<E_{\nu}> = 788$ MeV
$M_A = 1.03$ GeV
$37^\circ < \theta_\mu < 46^\circ$

$25^\circ < \theta_\mu < 37^\circ$

$df/d\cos\theta_\mu \cdot dT_\mu [10^{-38} \text{ cm}^2/\text{GeV}]$

$T_\mu$ [GeV]
CONTRIBUTION OF DIFFERENT REACTION MECHANISMS

- In neutrino interactions the lepton kinematics is not determined. The flux-averaged cross sections at fixed $T_{\mu}$ and $\cos \theta_{\mu}$ picks up contributions at different beam energies, corresponding to a variety of kinematical regimes in which different reaction mechanisms dominate.

- $x = 1 \rightarrow E_\nu \ 0.788 \text{ GeV}$, $x = 0.5 \rightarrow E_\nu \ 0.975 \text{ GeV}$
- $\Phi(0.975)/\Phi(0.788) = 0.83$
**"Flux Averaged" Electron-Nucleus x-Section**

- The electron scattering x-section off Carbon at $\theta_e = 37^\circ$ has been measured for a number of beam energies.

- Theoretical calculations of the flux-averaged cross sections must include all relevant reaction mechanism—one and two-nucleon emission, excitation of nucleon resonances and deep inelastic scattering—within a consistent approach.
The study of neutrino interactions with nuclear matter and nuclei is a strongly cross-disciplinary field, with applications in a variety of areas, ranging from the astrophysics of compact stars to the search of neutrino oscillations.

Recent studies carried out within nuclear many-body theory using realistic Hamiltonians have revealed a variety of dynamical effects, whose effects on the nuclear response are large.

The emerging picture suggests that a unified description of the nuclear response in a broad kinematical range corresponding to neutrino energies ranging from few MeV to few GeV.
Backup slides
Neutrino-Nucleon Interactions

* In the regime of momentum transfer \( (q) \) discussed in this talk Fermi theory of weak interaction works just fine

\[
W, Z_0
\]

* \( x \)-section of the charged-current process \( \nu_\ell + n \rightarrow \ell^- + X \)

\[
d\sigma \propto L_{\lambda\mu} W^{\lambda\mu}
\]

\( L_{\lambda\mu} \) is determined by the lepton kinematical variables (more on this later)

under general assumptions \( W^{\lambda\mu} \) can be written in terms of five structure functions \( W_i(q^2, (p \cdot q)) \) (\( p \) is the nucleon four momentum)

\[
W^{\lambda\mu} = -g^{\lambda\mu} W_1 + p^\lambda p^\mu \frac{W_2}{m_N^2} + \varepsilon^{\lambda\mu\alpha\beta} q_\alpha p_\beta + \frac{W_3}{m_N^2} + q^\lambda q^\mu \frac{W_4}{m_N^2} \\
+ (p^\lambda q^\mu + p^\mu q^\lambda) \frac{W_5}{m_N^2}
\]
In principle, the structure functions $W_i$ can be extracted from the measured cross sections.

In the charged-current elastic sector $\nu_\ell + n \rightarrow \ell^- + p$ they can be expressed in terms of vector ($F_1(q^2)$ and $F_2(q^2)$), axial-vector ($F_A(q^2)$) and pseudoscalar ($F_P(q^2)$) form factors.

\[
W_1 = 2 \left[ -\frac{q^2}{2} (F_1 + F_2)^2 + \left( 2 m_N^2 - \frac{q^2}{2} \right) F_A^2 \right]
\]

\[
W_2 = 4 \left[ F_1^2 - \left( \frac{q^2}{4 m_N^2} \right) F_2^2 + F_A^2 \right] = 2W_5
\]

\[
W_3 = -4 \left( F_1 + F_2 \right) F_A
\]

\[
W_4 = -2 \left[ F_1 F_2 + \left( 2 m_N^2 + \frac{q^2}{2} \right) \frac{F_2^2}{4 m_N^2} + \frac{q^2}{2} F_P^2 - 2 m_N F_P F_A \right]
\]

According to the CVC hypothesis, $F_1$ and $F_2$ can be related to the electromagnetic form factors, measured by electron-nucleon scattering, while PCAC allows one to express $F_P$ in terms of the axial form factor (more on this later).
Vector Form Factors

- Proton data

- Neutron (deuteron) data
Axial Form Factor

- Dipole parametrization

\[ F_A(Q^2) = \frac{g_A}{[1 + (Q^2/M_A^2)]^2} \]

- \( g_A \) from neutron \( \beta \)-decay
- Axial mass \( M_A \) from (quasi) elastic \( \nu \)- and \( \bar{\nu} \)-deuteron experiment
**Tamm-Dancoff (ring) Approximation**

- Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

  \[ |ph\rangle \rightarrow |n\rangle = \sum_{i=1}^{N} C_i |p_i h_i\rangle \]

- The energy of the state \(|n\rangle\) and the coefficients \(C_i\) are obtained diagonalizing the hamiltonian matrix

  \[ H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i |V_{eff}| h_j p_j) \]

  \[ e_k = \frac{k^2}{2m} + \sum_{k'} \langle kk'|V_{eff}|kk'\rangle_a \]

- The appearance of an eigenvalue, \(\omega_n\), lying outside the particle-hole continuum signals the excitation of a collective mode.
Excitation of Collective Modes

- Density (a) and spin-density (b) responses of isospin-symmetric nuclear matter at equilibrium density

\[ |q| = 0.1, 0.15, 0.20, 0.25, 0.30, 0.40 \text{ and } 0.50 \text{ fm}^{-1} \]
Mean free path of a non-degenerate neutrino in neutron matter. Left: density-dependence at $k_0 = 1\, \text{MeV}$ and $T = 0$; Right: energy dependence at $\rho = 0.16\, \text{fm}^{-3}$ and $T = 0, 2\, \text{MeV}$.
Density and temperature dependence of the mean free path of a non degenerate neutrino at $k_0 = 1$ MeV and $\rho = 0.16$ fm$^{-3}$