

Modelli Teorici per Studi di Struttura Nucleare II

Luigi Coraggio

Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

November 15, 2016

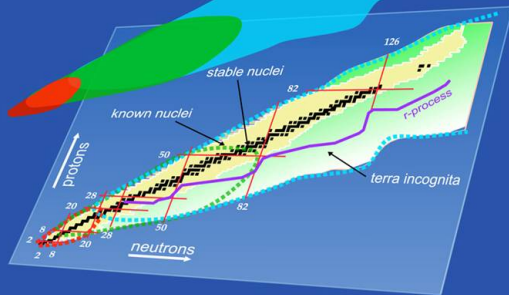


Microscopic nuclear structure calculations

- Common background: the starting point is a **realistic nuclear potential**.
- Main tasks:
 - take into account, as much as possible, **all degrees of freedom** of the interacting nucleons;
 - keep under control the needed approximations: **evaluation of theoretical errors**;
 - **parameter-free** approach to the nuclear structure;
 - enhance the **predictiveness**.
- Notable models: No-core shell model, coupled-cluster approach, realistic shell model.

Nuclear Landscape

- Ab initio
- Configuration Interaction
- Density Functional Theory

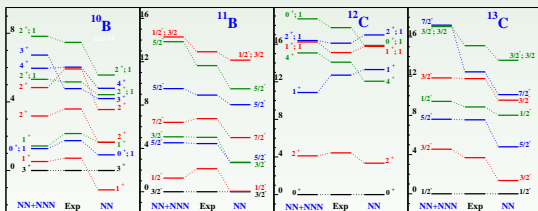


No-Core Shell Model

The starting point: A –nucleons hamiltonian, that includes also the center-of-mass potential

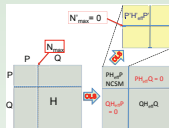
$$H_A^\omega = \sum_{i=1}^A \left[\frac{p_i^2}{2m} + \frac{1}{2} \omega^2 r_i^2 \right] + \sum_{i < j=1}^A \left[V(\mathbf{r}_i - \mathbf{r}_j) - \frac{m\omega^2}{2A} (\mathbf{r}_i - \mathbf{r}_j)^2 \right]$$

- **Problem:** too big to be diagonalized, even for light nuclei
- **Solution:** The Hilbert space is then truncated to those states with harmonic oscillator quanta N less or equal than a fixed N_{\max}
- \Rightarrow An effective hamiltonian H_{eff} that acts in this truncated model space is derived by way of the Lee-Suzuki unitary transformation.

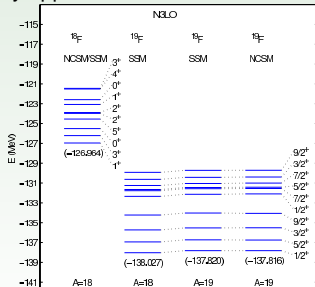


No-Core Shell Model

- **Limitations:** NCSM, within present computer technology, provides reliable nuclear structure only for **light nuclear systems**.
- **Evolution:** with a double-step procedure, an **effective shell-model hamiltonian** for a shell-model calculation (a few valence nucleons in a reduced model space) may be obtained.



- ⇒ This extension may applied to **heavier nuclei**.



Coupled-Cluster Approach

Within the **coupled-cluster theory** (F. Coester, 1958) the correlated function of the many-body system is written as

$$|\Psi\rangle = \exp(-T)|\Psi_0\rangle ,$$

where the correlation operator $T = T_1 + T_2 + \dots + T_A$ is expressed as a sum of correlation operators

$$T_1 = \sum_{i < \epsilon_f, a > \epsilon_f} t_i^a a_a^\dagger a_i \quad T_2 = \sum_{ij < \epsilon_f, ab > \epsilon_f} t_{ij}^{ab} a_a^\dagger a_b^\dagger a_i a_j \quad \dots$$

Coupled-Cluster Approach

The amplitudes t of the correlation operators are calculated using the variational principle. This procedure leads to the following set of equations:

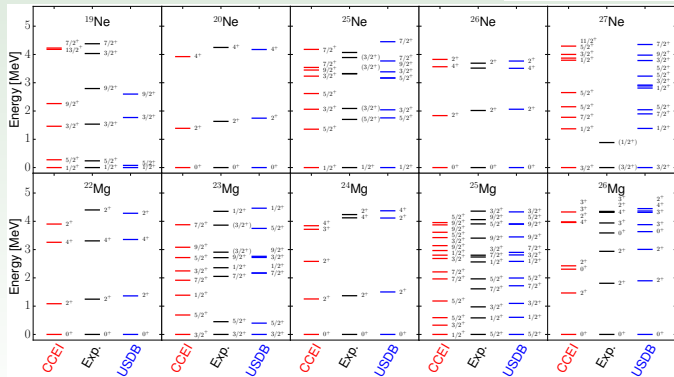
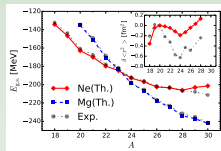
$$\begin{aligned}\langle \Psi_0 | \exp(T) H \exp(-T) | \Psi_i^a \rangle &= 0 \\ \langle \Psi_0 | \exp(T) H \exp(-T) | \Psi_{ij}^{ab} \rangle &= 0 \\ &\dots\end{aligned}$$

These equations may be calculated using the Baker-Hausdorff relation:

$$\begin{aligned}\exp(T) H \exp(-T) &= H + [H, T_1] + [H, T_2] + \frac{1}{2} [[H, T_1], T_1] + \\ &+ \frac{1}{2} [[H, T_2], T_2] + [[H, T_1], T_2]\end{aligned}$$

Coupled-Cluster Approach

This method may be more easily extended to heavier systems



The realistic shell model

- The derivation of the **shell-model effective hamiltonian** using the **many-body perturbation theory** may provide a useful tool to study nuclei in the full range of nuclear masses
- The model space may be “shaped” according to the computational needs of the diagonalization of the **shell-model hamiltonian**
- In such a case, the effects of the **neglected degrees of freedom** are taken into account by the effective hamiltonian H_{eff} theoretically

Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

Workflow for a realistic shell-model calculation

- 1 Choose a realistic NN potential (NNN)
- 2 Set up the model space which is better tailored to study the nuclear system under investigation
- 3 Derive the effective shell-model hamiltonian by way of the many-body theory
- 4 Calculate the physical observables (energies, e.m. transition probabilities, ...)

The shell-model effective hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

with

$$H = H_0 + H_1 = \sum_{i=1}^A (T_i + U_i) + \sum_{i<j} (V_{ij}^{NN} - U_{ij})$$

Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \dots a_n^\dagger]_i |c\rangle \Rightarrow P = \sum_{i=1}^d |\Phi_i\rangle\langle\Phi_i|$$

Model-space eigenvalue problem

$$H_{\text{eff}} P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$

The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \mathcal{H} = X^{-1} H X \Rightarrow \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$$Q\mathcal{H}P = 0$$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^\omega$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$

$$H_1^{\text{eff}}(\omega) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P -$$

$$-PH_1Q \frac{1}{\epsilon - QHQ} \omega H_1^{\text{eff}}(\omega)$$

The shell-model effective hamiltonian

Folded-diagram expansion

\hat{Q} -box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

\Rightarrow Recursive equation for $H_{\text{eff}} \Rightarrow$ iterative techniques
(Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \dots,$$

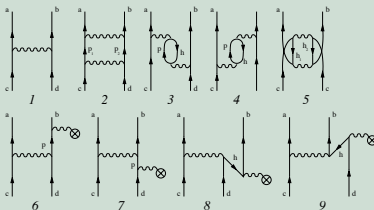
The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QH_0Q} QH_1P$$

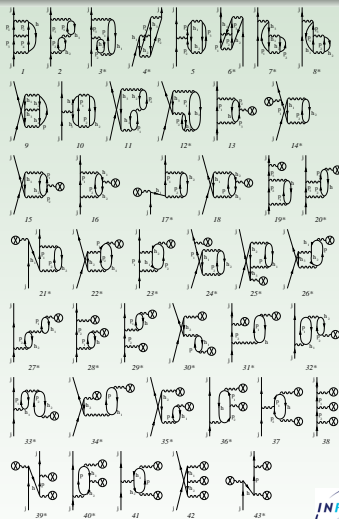
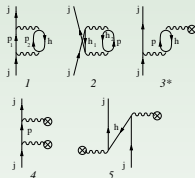
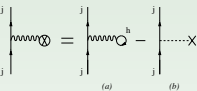
The \hat{Q} -box can be calculated perturbatively

$$\frac{1}{\epsilon - QH_0Q} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

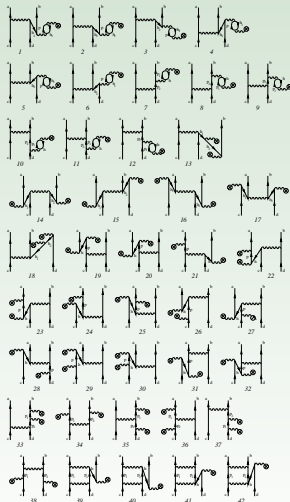
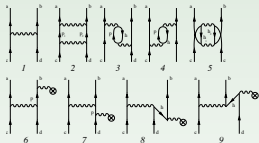
The diagrammatic expansion of the \hat{Q} -box



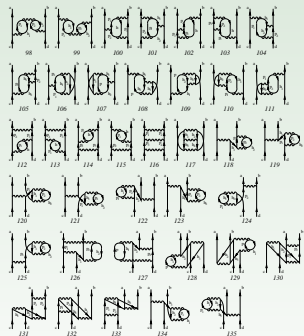
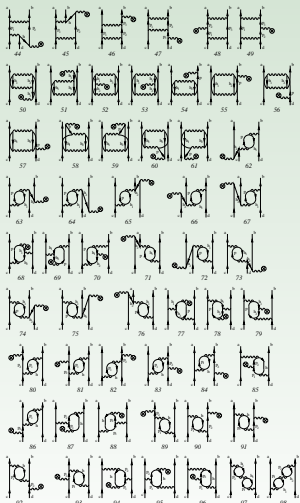
\hat{Q} -box perturbative expansion: 1-body diagrams



\hat{Q} -box perturbative expansion: 2-body diagrams



\hat{Q} -box perturbative expansion: 2-body diagrams



Our recipe for realistic shell model

- Input V_{NN} : $V_{\text{low-k}}$ derived from the high-precision NN CD-Bonn potential with a cutoff: $\Lambda = 2.6 \text{ fm}^{-1}$.
- H_{eff} obtained calculating the Q -box up to the 3rd order in $V_{\text{low-k}}$.
- Effective electromagnetic operators are consistently derived by way of the the MBPT

Large-scale shell model

- **Large-scale shell-model calculations** are, at present, a consolidated tool to investigate nuclear properties.
- The new physics coming from **RIBs** facilities provides a challenging ground, since they are approaching the nuclear driplines.
- **The computational complexity** of dealing with large model spaces and many interacting valence nucleons is the main problematic to be tackled.

Large-scale shell model

Large-scale shell model:
shell model calculations performed within a model space made up by a number of orbitals larger than usual.

An extended model space enables to study exotic (for shell model) properties: **collective motion, deformation, clustering, etc.**

PHYSICAL REVIEW C

VOLUME 50, NUMBER 1

JULY 1994

Full pf shell model study of $A=48$ nuclei

E. Caurier and A. P. Zuker

Groupe de Physique Théorique, Centre de Recherches Nucléaires, Institut National de Physique Nucléaire et de Physique des Particules, Centre National de la Recherche Scientifique, Université Louis Pasteur Boite Postale 20, F-67037 Strasbourg-Cedex, France

A. Poves and G. Martínez-Pinedo

*Departamento de Física Teórica C.XI, Universidad Autónoma de Madrid, E-28049 Madrid, Spain
(Received 16 December 1993)*

VOLUME 77, NUMBER 16

PHYSICAL REVIEW LETTERS

14 OCTOBER 1996

Nuclear Shell Model by the Quantum Monte Carlo Diagonalization Method

Michio Honma,¹ Takahiro Mizusaki,² and Takaharu Otsuka^{2,3}

¹*Center for Mathematical Sciences, University of Aizu, Tsuruga, Itakura-machi, Aizu-Wakamatsu, Fukushima 965, Japan*

²*Department of Physics, University of Tokyo, Hongo, Tokyo 113, Japan*

³*RIKEN, Hirosawa, Wako-shi, Saitama 351-01, Japan*

(Received 29 April 1996)

Collective behavior

PRL 110, 242701 (2013)

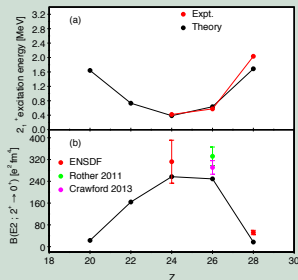
PHYSICAL REVIEW LETTERS

week ending
14 JUNE 2013

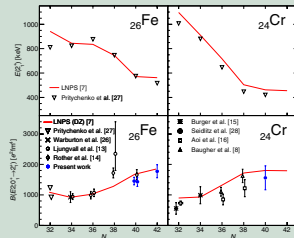
Quadrupole Collectivity in Neutron-Rich Fe and Cr Isotopes

H. L. Crawford,¹ R. M. Clark,¹ P. Fallon,¹ A. O. Macchiavelli,¹ T. Baugher,^{2,3} D. Bazin,² C. W. Beausang,⁴ J. S. Berryman,² D. L. Bleuel,² C. M. Campbell,¹ M. Cromaz,¹ G. de Angelis,⁵ A. Gade,^{2,3} R. O. Hughes,⁴ I. Y. Lee,⁴ S. M. Lenzi,⁷ F. Nowacki,⁸ S. Paschalis,¹ M. Petri,¹ A. Poves,⁹ A. Ratkiewicz,^{2,3} T. J. Ross,⁴ E. Sahin,⁶ D. Weisshaar,² K. Wimmer,^{2,10} and R. Winkler²

Onset of collectivity
at $N = 40$



L. C., A. Covello, A. Gargano, and N. Itaco, *Phys. Rev. C* **89**, 024319 (2014)



Collective behavior

PRL 100, 172501 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Coulomb Excitation of ^{100}Sn and the Strength of the ^{100}Sn Shell Closure

G. Guarata,¹ D. D. Diddis,² M. Görke,³ J. Colekili,⁴ P. Bontasios,^{5,6} P. Gelbach,⁷ S. Platt,⁸ H. Grava,⁹ F. Nowack,¹⁰ K. Suda,¹¹ A. Algora,¹² F. Anst,¹³ T. Aoi,¹⁴ A. Aze,¹⁵ M. A. Bentley,¹⁶ A. Blazhev,¹⁷ D. Blum,¹⁸ S. Brantzae,¹⁹ N. Brasa,²⁰ T. Causa,²¹ Z. Dsanjani,²² C. Duaneau-Pouk,²³ A. Estrada,²⁴ F. Fontana,²⁵ J. Geel,²⁶ N. Geoi,²⁷ J. Gorbunov,²⁸ T. Habermann,²⁹ R. Heitsch,³⁰ K. Kaneko,³¹ J. Jahn,³² A. Janghuan,³³ I. Kojanovic,³⁴ R. Kozuch,³⁵ H. Kume,³⁶ J. Kuznetsov,³⁷ M. Kuz,³⁸ N. Laitinen,³⁹ E. Mankus,⁴⁰ F. Nappi,⁴¹ B. B. Nara Singh,⁴² N. Nibberg,⁴³ C. Nocioni,⁴⁴ A. Oertelt,⁴⁵ M. Pfitzner,⁴⁶ N. Piantala,⁴⁷ Z. Podolyak,⁴⁸ A. Prochaska,⁴⁹ D. Raju,⁵⁰ P. Reiter,⁵¹ D. Rudolph,⁵² H. Schaffner,⁵³ F. Schmitt,⁵⁴ L. Sereno,⁵⁵ D. Schler,⁵⁶ T. Szwed,⁵⁷ J. Taprogge,^{58,59} Zs. Vajta,⁶⁰ R. Wadsworth,⁶¹ N. Warr,⁶² H. Wock,⁶³ A. Wunde,⁶⁴ O. Winkler,⁶⁵ I. S. Wondol,⁶⁶ and H. J. Wollersheim⁶⁷

PHYSICAL REVIEW C 88, 014301 (2013)

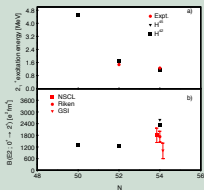
Quadrupole collectivity in neutron-deficient Sn nuclei: ^{100}Sn and the role of proton excitations

V. M. Radts,¹ A. Gade,² D. Wiescher,³ B. A. Brown,⁴ T. Baugher,⁵ D. Bazin,⁶ J. S. Berryman,⁷ A. Ekström,⁸ M. Hjorth-Jensen,^{9,10} S. R. Stroberg,¹¹ W. B. Walters,¹² K. Wimmer,¹³ and R. Winkler¹⁴

PHYSICAL REVIEW C 90, 041301 (2014)

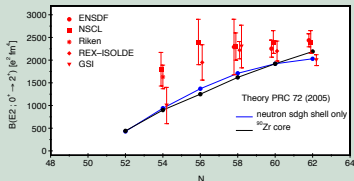
Intermediate-energy Coulomb excitation of ^{100}Sn : Moderate E2 strength decrease approaching ^{100}Sn

F. Dammann,^{1,2} S. Takahashi,³ N. Aoi,⁴ M. Matsushita,⁵ A. Oertelt,⁶ D. Stoppacher,⁷ H. Wang,⁸ L. Andrae,⁹ H. Rab,¹⁰ P. Bhatnagar,¹¹ S. Böttcher,¹² M. Carraway,¹³ A. Corci,¹⁴ T. Franzmeier,¹⁵ T. Jahn,¹⁶ A. Janghuan,¹⁷ V. Lapoux,¹⁸ J. Liu,¹⁹ K. Matsuda,²⁰ T. Minatoyoshi,²¹ D. Nishimura,²² S. Ota,²³ E. C. Pollacco,²⁴ H. Sakurai,²⁵ C. Sannarise,²⁶ Y. Shiga,²⁷ D. Schler,²⁸ and R. Tamachi²⁹



L. C., A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo *Phys. Rev. C* **91**, 041301 (2015)

Quadrupole collectivity in light tin isotopes



Novel collective features

RAPID COMMUNICATIONS

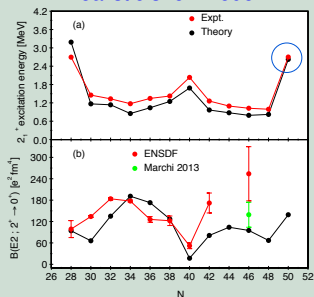
PHYSICAL REVIEW C **89**, 031301(R) (2014)

Novel shape evolution in exotic Ni isotopes and configuration-dependent shell structure

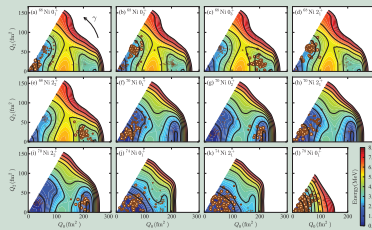
Yusuke Tsunoda,¹ Takaharu Otsuka,^{1,2,3} Noritaka Shimizu,² Michio Honma,⁴ and Yutaka Utsuno⁵

Shape evolution
in Ni isotopes

Realistic shell-model



Monte Carlo shell model



Islands of inversion

PHYSICAL REVIEW C **90**, 014302 (2014)

Merging of the islands of inversion at $N = 20$ and $N = 28$

E. Caurier,¹ F. Nowacki,¹ and A. Poves^{2,3}

¹IPHC, IN2P3-CNRS and Université Louis Pasteur, F-67037 Strasbourg, France

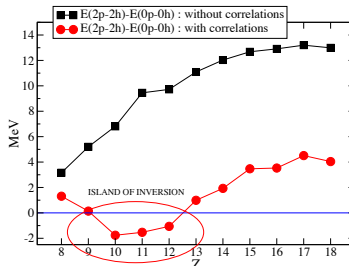
²Departamento de Física Teórica and IFT-UAM/CSIC, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

³Isolde (CERN) 1211 Genève 23, Switzerland

Merging of the
 $N = 20$ and $N = 28$
islands of inversion
in Mg isotopes

Model space:
full **sd** shell

Shell model basis has a
dimension up to 10^{10}



Shell evolution

PHYSICAL REVIEW C **87**, 034309 (2013)



Evolution of single-particle states beyond ^{132}Sn

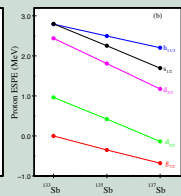
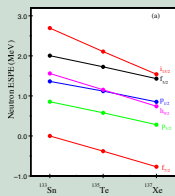
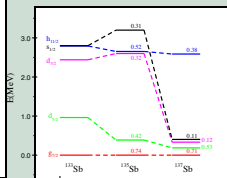
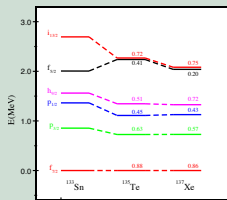
L. Coraggio,¹ A. Covello,^{1,2} A. Gargano,¹ and N. Itaco^{1,2}

¹Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy

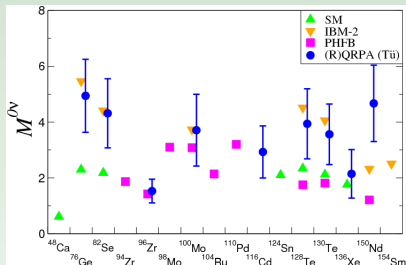
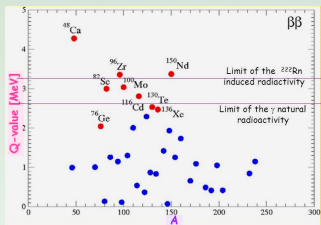
²Dipartimento di Fisica, Università di Napoli Federico II, Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy

(Received 15 January 2013; published 7 March 2013)

Evolution of single-particle structure beyond doubly-closed ^{132}Sn



The detection of the $0\nu\beta\beta$ -decay



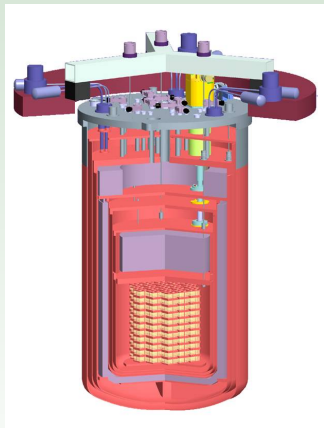
NLDBD Report April 24, 2014

Nuclear matrix elements

In recent years we have witnessed a renaissance of nuclear structure theory which has been driven by progress in solving the nuclear many-body problem by ab-initio methods like effective field theory (EFT) and nuclear lattice EFT, Green's Function Monte Carlo (GFMC), no-core shell model, and coupled-cluster approaches and by deriving NN and 3N interactions systematically on the basis of the symmetries of QCD. Some of these approaches hold the promise to describe nuclear properties including a consistent estimate of the theoretical uncertainties. However, despite the exciting advances, these methods cannot yet be applied to the complex heavy nuclei of experimental interest in $\beta\beta$ decay. Hence $\beta\beta$ decay matrix element evaluations must be taken from models like the configuration-mixed shell model, the QRPA, and others that

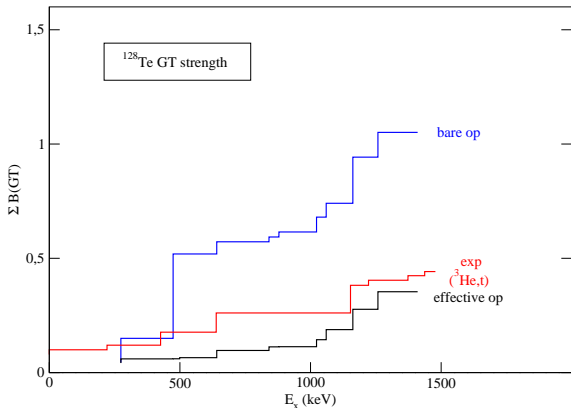
The spread of nuclear structure calculations evidences **inconsistencies** among results obtained with different **models**



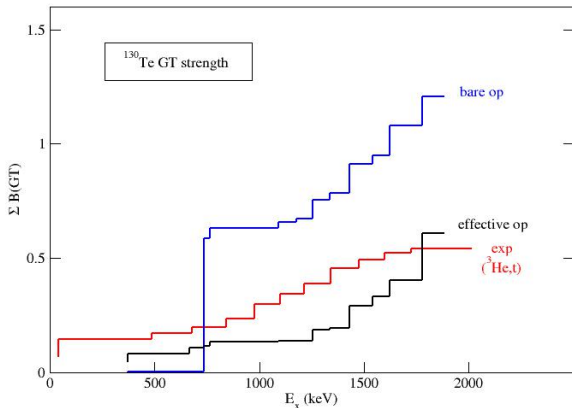


- TeO₂ crystals used as low heat capacity bolometers, arranged into towers and cooled in a large cryostat to approximately 10 m°K with a dilution refrigerator.
- The detectors are isolated from backgrounds by ultrapure low-radioactivity shielding.
- Temperature spikes from electrons emitted in Te $0\beta\beta$ are collected for spectrum analysis.

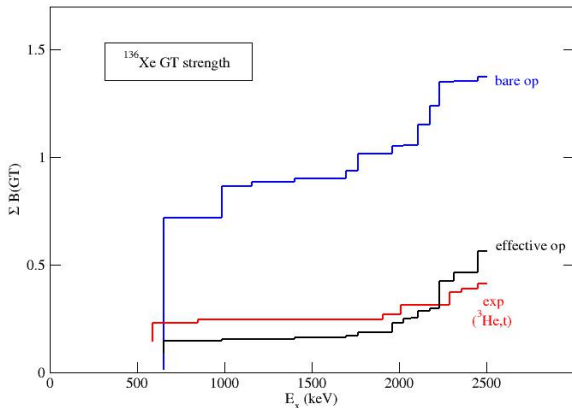
Realistic shell model: ^{128}Te Gamow-Teller strengths



Realistic shell model: ^{130}Te Gamow-Teller strengths



Realistic shell model: ^{136}Xe Gamow-Teller strengths

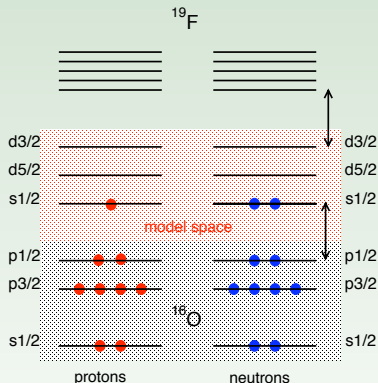


Conclusions and outlook

- The flow of experimental data coming from RIB facilities is stimulating advances in nuclear structure theory.
- Nuclear structure calculations starting from realistic nuclear potentials are now successful instruments to understand a large variety of nuclear phenomena.
- The parameter-free nature of realistic nuclear-structure calculations should provide a more reliable predictive power.
- Realistic shell model at present is the approach that provides microscopic calculations over a wider mass range, and may describe a large set of different observables.
- Application to problems in fundamental physics: $0\nu\beta\beta$ -decay, WIMP-nucleus interaction, electric-dipole moments and T -violation, etc.

Backup slides

An example: ^{19}F



- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

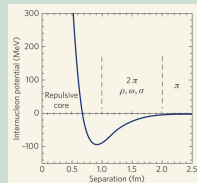
Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

How to handle the short-range repulsion ?

- Brueckner G matrix
- EFT inspired approaches

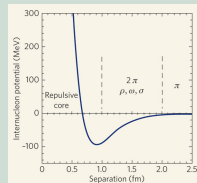
Strong short-range repulsion



Realistic nucleon-nucleon potential: V_{NN}

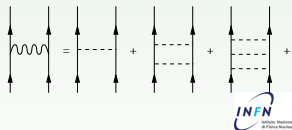
Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

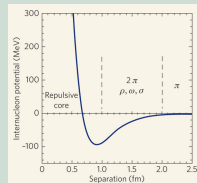
- Brueckner G matrix
- EFT inspired approaches
 - $V_{low k}$, SRG



Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



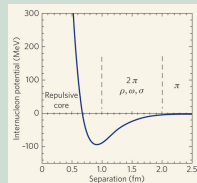
How to handle the short-range repulsion ?

- Brueckner G matrix
- EFT inspired approaches
 - V_{low-k} , SRG
 - chiral potentials

Realistic nucleon-nucleon potential: V_{NN}

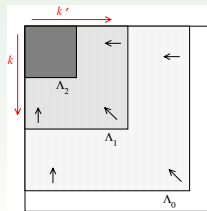
Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

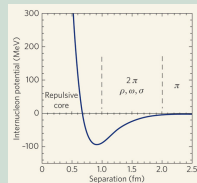
- Brueckner G matrix
- EFT inspired approaches
 - $V_{\text{low-}k}$, SRG
 - chiral potentials



Realistic nucleon-nucleon potential: V_{NN}

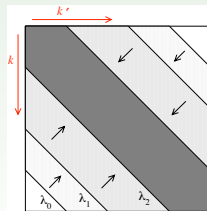
Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

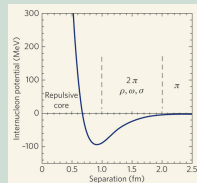
- Brueckner G matrix
- EFT inspired approaches
 - V_{low-k} , SRG
 - chiral potentials



Realistic nucleon-nucleon potential: V_{NN}

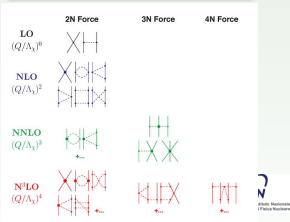
Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner G matrix
- EFT inspired approaches
 - $V_{\text{low-}k}$, SRG
 - chiral potentials



Test case: p -shell nuclei

- $V_{NN} \Rightarrow$ chiral N^3LO potential by Entem & Machleidt (smooth cutoff $\simeq 2.5 \text{ fm}^{-1}$)
- H_{eff} for two valence nucleons outside ${}^4\text{He}$
- Single-particle energies and residual two-body interaction are derived from the theory. **No empirical input**

First, some convergence checks !

*L.C., A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Ann. Phys. **327** , 2125-2151 (2012)*

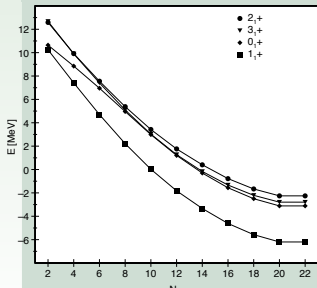
Convergence checks

The intermediate-state space Q

Q -space is truncated: intermediate states whose unperturbed excitation energy is greater than a fixed value E_{max} are disregarded

$$|\epsilon_0 - QH_0Q| \leq E_{max} = N_{max} \hbar\omega$$

${}^6\text{Li}$ yrast states

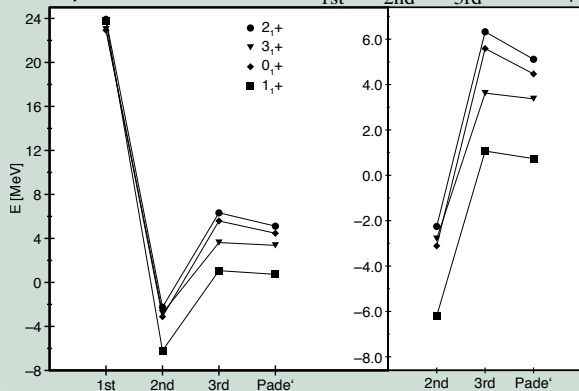


results stable for $N_{max} \geq 20$

Convergence checks

Order-by-order convergence

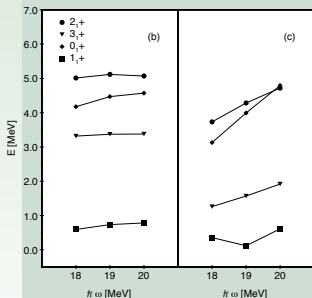
Compare results from H_{1st}^{eff} , H_{2nd}^{eff} , H_{3rd}^{eff} and $H_{Padè}^{eff}$



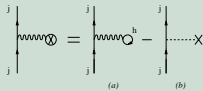
Convergence checks

Dependence on $\hbar\omega$

Auxiliary potential $U \Rightarrow$ harmonic oscillator potential



HF-insertions



- zero in a self-consistent basis
- neglected in most applications
- disregard of HF-insertions introduces relevant dependence on $\hbar\omega$

Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

ab initio no-core shell model (NCSM)

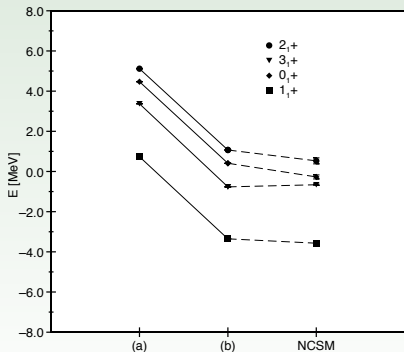
P. Navrátil, E. Caurier, Phys. Rev. C **69**, 014311 (2004)

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)

Benchmark calculation

To compare our results with NCSM we need to start from a translationally invariant Hamiltonian

$$H_{int} = \left(1 - \frac{1}{A}\right) \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i<j=1}^A \left(V_{ij}^{NN} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) =$$
$$= \left[\sum_{i=1}^A \left(\frac{p_i^2}{2m} + U_i \right) \right] + \left[\sum_{i<j=1}^A \left(V_{ij}^{NN} - U_i - \frac{p_i^2}{2mA} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{mA} \right) \right]$$

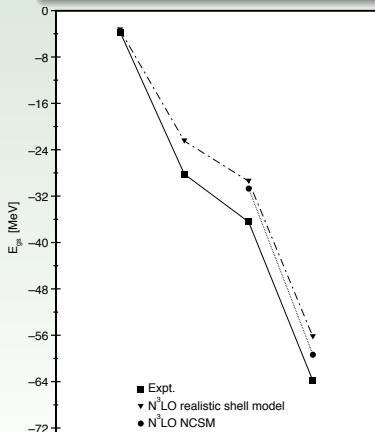


(a) not translationally invariant Hamiltonian
(b) purely intrinsic hamiltonian

Benchmark calculation

Remark

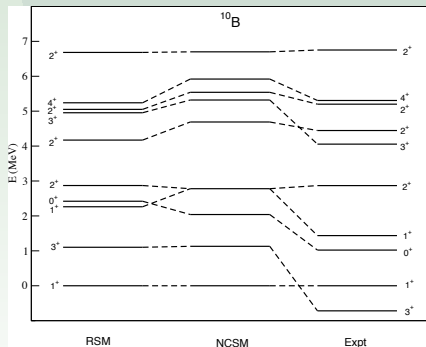
H^{eff} derived for 2 valence nucleon systems \Rightarrow 3-, 4-, .. n -body components are neglected



- ground-state energies for $N = Z$ nuclei
- discrepancy grows with the number of valence nucleons

Benchmark calculation

^{10}B relative spectrum



- discrepancy ≤ 1 MeV
- minor role of many-body correlations