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## Outline

Clusters in the Trojan Horse Method for Nuclear Astrophysics

Triple alpha resonances in 12C and Efimov mechanism in nuclei

Role of Clusters in fusion reactions between light nuclei at low energy

## Clustering features

The concept of clustering has a long history. Even at the inception of nuclear science, even before the neutron was discovered, it was known that conglomerates of nucleons (nuclear clustering) were extremely important in determining the structure of light nuclei




The cluster structure would expect to be manifest close to the cluster decay threshold. In order to be fully formed, the proximity of cluster states to the decay threshold is crucial. This is the message of what is known as the Ikeda diagram (Ikeda et al., 1968)

Quasi-free processes: a way to probe clustering in light nuclei

Quasi-free scattering and reactions to investigate the cluster structure in light nuclei.
$A+a \rightarrow c+C+s \quad a: x \oplus s$ clusters

Quasifiree mechanlsm

$$
\begin{aligned}
& \checkmark \text { only } x-A \text { interaction } \\
& \checkmark s=\text { spectator }\left(p_{s} \sim 0\right)
\end{aligned}
$$



The Impulse Approximation factorizes the three-body cross-section as:
KF kinematical factor
$\frac{d^{3} \sigma}{d \Omega_{c} d \Omega_{c} d E_{c}}=P K F \cdot\left|\Phi\left(p_{s}\right)\right|^{2} \frac{d \sigma^{N}}{d \Omega}$
$|\phi|^{2}$ momentum distribution of $s$ inside a
$\mathrm{d}{ }^{\mathrm{N}} / \mathrm{d} \Omega$ Nuclear cross section for the $A+X \rightarrow C+c$ reaction

P clustering probability

Once we know the $\frac{d^{\prime} \sigma}{} \sigma$ and $d \sigma^{*}$
$d \Omega_{c} d \Omega_{c} d E_{c} \quad d \Omega$
from independent experiments
$\left|\Phi\left(\mathrm{p}_{\mathrm{s}}\right)\right|^{2}$ with the expected behaviour
if clustering exists, we can estimate $P$.
In the $80 ' \mathrm{~s}$, this technique was applied to ${ }^{6} \mathrm{Li}=a \oplus d(B=1.47 \mathrm{MeV})$.

The ${ }^{6} \mathrm{Li}\left({ }^{6} \mathrm{Li}, a \mathrm{a}\right){ }^{4} \mathrm{He}$ at 5.9 MeV (M. Lattuada, C. Spitaleri et al., Il Nuovo Cimento (1984))
${ }^{6}$ Li clustering description confirmed by the agreement between experimental and theoretical curves


Nuclear clusters as virtual projectile/targets for nuclear astrophysics: the THM

The THM is an indirect method to measure cross-sections at ultra-low energies overcoming the main problems of direct measurement:

Coulomb suppression,


Electron screening


Trojan Horse Method
Basic principle: astrophysically relevant two-body o from quasi-free contribution of an appropriate three-body reaction

$$
A+a \rightarrow c+C+s \quad \rightarrow \rightarrow \rightarrow A+x \rightarrow c+C
$$

$a: x \oplus s$ clusters


$$
E_{q . f:}=E_{\text {ex }}=m_{x} /\left(m_{x}+m_{A}\right) E_{A}-B_{x-s} \pm \text { intercluster motion }
$$

plays a key role in compensating for the beam energy

$$
E_{q . f:} \approx 0 \quad!!!
$$

|  | Binary reaction | Indirect reaction | $E_{\text {lab }}$ | Q | Accelerator |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | ${ }^{7} \mathrm{Li}(\mathrm{p}, \alpha)^{4} \mathrm{He}$ | ${ }^{2} H(7 L i, \alpha \alpha) n$ | 19-22 | 15.122 | TANDEM 13 MV LNS-INFN, Catania | Spitaleri et al. PRC,1999, Lattuada et al. ApJ, 2001 |
| 2 | ${ }^{7} \mathrm{Li}(\mathrm{p}, \alpha)^{4} \mathrm{He}$ | ${ }^{7} \mathrm{Li}\left({ }^{3} \mathrm{He}, \alpha \alpha\right) \mathrm{d}$ | 33 | 11.853 | CYCLOTRON, Rez, Praha | Tumino et al. EPJ, 2006 |
| 3 | ${ }^{6} \mathrm{Li}(\mathrm{p}, \alpha)^{3} \mathrm{He}$ | ${ }^{2} \mathrm{H}\left({ }^{6} \mathrm{Li}, \alpha^{3} \mathrm{He}\right) \mathrm{n}$ | 14,25 | 1.795 | TANDEM 13 MV LNS-INFN, Catania | Tumino et al. PRC, 2003 |
| 4 | ${ }^{9} \mathrm{Be}(\mathrm{p}, \alpha)^{6} \mathrm{Li}$ | ${ }^{2} \mathrm{H}\left({ }^{9} \mathrm{Be}, \alpha^{6} \mathrm{Li}\right) \mathrm{n}$ | 22 | -0.099 | TANDEM CIAE, Beijing TANDEM 13 MV LNS-INFN, Catania | Wen et al. PRC, 2008, Wen et al. JPG 2011 |
| 5 | ${ }^{11} \mathrm{~B}(\mathrm{p}, \alpha)^{8} \mathrm{Be}$ | ${ }^{2} \mathrm{H}\left({ }^{11} \mathrm{~B}, \alpha^{8} \mathrm{Be}\right) \mathrm{n}$ | 27 | 6.36 | TANDEM 13 MV LNS-INFN, Catania | Spitaleri et al. PRC, 2004, Lamia et al. JPG, 2011 |
| 6 | ${ }^{15} \mathrm{~N}(p, \alpha){ }^{12} \mathrm{C}$ | ${ }^{2} \mathrm{H}\left({ }^{15} \mathrm{~N}, \alpha^{12} \mathrm{C}\right) \mathrm{n}$ | 60 | 2.74 | CYCLOTRON, TAMU, College Station TANDEM 13 MV LNS-INFN, Catania | La Cognata et al. PRC, 2008 |
| 7 | ${ }^{18} \mathrm{O}(\mathrm{p}, \alpha){ }^{15} \mathrm{~N}$ | ${ }^{2} \mathrm{H}\left({ }^{18} \mathrm{O}, \alpha{ }^{15} \mathrm{~N}\right) \mathrm{n}$ | 54 | 1.76 | (CYCLOTRON, <br> TAMU, College Station TANDEM 13 MV LNS-INFN, Catania | La Cognata et al. PRL 2008, |
| 8 | ${ }^{19} \mathrm{~F}(\mathrm{p}, \alpha)^{16} \mathrm{O}$ | ${ }^{2} \mathrm{H}\left({ }^{19} \mathrm{~F}, \alpha^{16} \mathrm{O}\right) \mathrm{n}$ | 50 | 8.11 | TANDEM 13 MV LNS-INFN, Catania | La Cognata et al. ApJ Lett., 2011) |
| 9 | ${ }^{17} \mathrm{O}(\mathrm{p}, \alpha){ }^{14} \mathrm{~N}$ | ${ }^{2} \mathrm{H}\left({ }^{17} \mathrm{O}, \alpha^{14} \mathrm{~N}\right) \mathrm{n}$ | 45 | -1.032 | TANDEM 13 MV LNS-INFN, Catania TANDEM 11 MV Notre Dame | Sergi et al. PRC (R), 2010 |


|  | Binary reaction | Indirect reaction | $\mathrm{E}_{\text {lab }}$ | Q | Accelerator | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | ${ }^{18} \mathrm{~F}(\mathrm{p}, \alpha)^{15} \mathrm{O}$ | ${ }^{2} H\left({ }^{18} \mathrm{~F}, \alpha^{15} \mathrm{O}\right) \mathrm{n}$ | 48 |  | CYCLOTRON CNS-RIKEN, Tokyo | Cherubini et al. PRC 2015 R.G. Pizzone et al. EPJA 20 |
| 11 | ${ }^{10} B(p, \alpha)^{7} B e$ | ${ }^{2} H\left({ }^{10} B, \alpha^{7} B e\right) n$ | 27 |  | TANDEM 13 MV LNS-INFN, Catania | Spitaleri et al. PRC 2014 |
| 12 | ${ }^{6} \mathrm{Li}(\mathrm{d}, \alpha)^{4} \mathrm{He}$ | ${ }^{6} \mathrm{Li}\left({ }^{6} \mathrm{Li}, \alpha \alpha\right)^{4} \mathrm{He}$ | $\begin{array}{\|l\|} \hline 5 \\ 4.8 \end{array}$ | 22.372 | TANDEM Demoscritos, Atene TANDEM, IRB, Zagreb | Cherubini et al. ApJ, 1996 Spitaleri et al .PRC, 2001 |
| 13 | ${ }^{6} \mathrm{Li}(\mathrm{d}, \alpha)^{4} \mathrm{He}$ | ${ }^{6} \mathrm{Li}\left({ }^{6} \mathrm{Li}, \alpha \alpha\right)^{4} \mathrm{He}$ |  |  | CYCLOTRON Rez, Praha | Pizzone et al. PRC, 2011 |
| 14 | ${ }^{3} \mathrm{He}(\mathrm{d}, \alpha)^{1} \mathrm{H}$ | ${ }^{6} \mathrm{Li}\left({ }^{3} \mathrm{He}, \mathrm{p}^{4} \mathrm{He}\right)^{4} \mathrm{He}$ | 5,6 | 16.878 | DYNAMITRON, Bochum | La Cognata et al. 2005 |
| 15 | ${ }^{2} \mathrm{H}(\mathrm{d}, \mathrm{p})^{3} \mathrm{H}$ | ${ }^{2} \mathrm{H}\left(6 \mathrm{Li}, \mathrm{p}^{3} \mathrm{He}\right){ }^{4} \mathrm{He}$ | 14 | 2.59 | DYNAMITRON, Bochum | Rinollo et al. EPJ 2005 |
| 16 | ${ }^{2} \mathrm{H}(\mathrm{d}, \mathrm{p})^{3} \mathrm{H}$ | ${ }^{2} \mathrm{H}\left({ }^{3} \mathrm{He}, \mathrm{p}^{3} \mathrm{H}\right)^{1} \mathrm{H}$ | 18 | -1.46 | CYCLOTRON, Rez, Praha | Tumino et al. PLB 2011 Tumino et al. APJ 2014 |
| 17 | ${ }^{2} \mathrm{H}(\mathrm{d}, \mathrm{n})^{3} \mathrm{He}$ | ${ }^{2} \mathrm{H}\left({ }^{3} \mathrm{He}, \mathrm{n}^{3} \mathrm{He}\right){ }^{1} \mathrm{H}$ | 18 | -2.224 | CYCLOTRON Rez, Praha | Tumino et al. PLB 2011 Tumino et al. APJ 2014 |
| 18 | ${ }^{9} \mathrm{Be}(\mathrm{p}, \mathrm{d})^{8} \mathrm{Be}$ | ${ }^{9} \mathrm{Be}\left(\mathrm{d}, \mathrm{d}^{8} \mathrm{Be}\right) \mathrm{n}$ |  |  | TANDEM 13 MV CIAE, Beijing | Preliminary results |
| 19 | ${ }^{6} \mathrm{Li}(\mathrm{n}, \mathrm{a})^{3} \mathrm{H}$ | ${ }^{2} \mathrm{H}\left({ }^{6} \mathrm{Li}, \pm \alpha\right)^{1} \mathrm{H}$ | 14 | 2.224 | TANDEM 13 MV LNS-INFN, Catania | Tumino et al.,EPJ A 2005 Gulino et al., JPG 2010 |


|  | Binary reaction | Indirect reaction | $\mathrm{E}_{\text {lab }}$ | Q | Accelerator | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | ${ }^{17} O(n, a){ }^{14} C$ | ${ }^{17} O\left(n, a^{14} C\right)^{1} H$ | 43.5 | $\begin{aligned} & \hline-0.40 \\ & 7 \end{aligned}$ | TANDEM 11 MV Notre Dame TANDEM 13 MV LNS-INFN, Catania | Gulino et al. PRC(R) 2013 |
| 21 | ${ }^{13} \mathrm{C}(\mathrm{a}, n)^{16} \mathrm{O}$ | ${ }^{13} \mathrm{C}\left({ }^{6} \mathrm{Li}, \mathrm{a} \mathrm{n}\right){ }^{16} \mathrm{O}$ |  |  | TANDEM FSU, Tallaassee, Florida, USA | La Cognata et al. PRL 2013 |
| 22 | $\begin{aligned} & { }^{12} C\left({ }^{12} C, a\right)^{20} \mathrm{Ne} \\ & { }^{12} C\left({ }^{12} C, p\right)^{23} \mathrm{Na} \end{aligned}$ | $\begin{aligned} & { }^{12} \mathrm{C}\left({ }^{16} \mathrm{O}, \mathrm{a}^{20} \mathrm{Ne}\right)^{4} \mathrm{He} \\ & { }^{12} \mathrm{C}\left({ }^{4} \mathrm{~N}, \mathrm{a}^{20} \mathrm{Ne}\right)^{2} \mathrm{H} \\ & { }^{12} \mathrm{C}\left({ }^{14} \mathrm{~N}, \mathrm{p}^{23} \mathrm{Na}\right)^{2} \mathrm{H} \end{aligned}$ |  |  | TANDEM 13 MV Bucharest,Catania | Preliminary results |
| 23 | ${ }^{12} C(a, a)^{12} C$ | ${ }^{12} \mathrm{C}\left({ }^{6} \mathrm{Li}, \mathrm{a}{ }^{12} \mathrm{C}\right)^{2} \mathrm{H}$ | 20 | 0 | TANDEM 13 MV LNS-INFN, Catania | Spitaleri et al. EPJ 2000 |
| 24 | ${ }^{1} \mathrm{H}(\mathrm{p}, \mathrm{p})^{1} \mathrm{H}$ | ${ }^{2} \mathrm{H}(\mathrm{p}, \mathrm{pp}) \mathrm{n}$ | 5,6 | 2,224 | CYCLOTRON <br> ATOMKI, Debrecen <br> TANDEM <br> IRB, Zagreb <br> TANDEM 13 MV <br> LNS-INFN, Catania <br> TANDEM 5 MV <br> Nanoli Universitv | Tumino et al. PRL 2007 <br> Tumino et al. PRC 2008 |
| 25 | ${ }^{7} \mathrm{n}\left({ }^{7 \mathrm{Be} e, a)^{4} \mathrm{He}}\right.$ | ${ }^{2} \mathrm{H}\left({ }^{7} \mathrm{Be}, a \mathrm{a}\right){ }^{1} \mathrm{H}$ |  |  | TANDEM LNLINFN | Under analysis |

## Trojan Horse Method For Resonant Reactions

The $A+a(x+s) \rightarrow F^{\star}(c+C)+s$ process is a transfer to the continuum where particle $x$ is the transferred particle

## Standard R-Matrix approach cannot be applied to extract the resonance parameters $\rightarrow$ Modified R-Matrix is introduced instead

In the case of a resonant THM reaction the cross section takes the form

$$
\frac{d^{2} \sigma}{d E_{C c} d \Omega_{s}} \propto \frac{\Gamma_{(C c)_{i}}(E)\left|M_{i}(E)\right|^{2}}{\left(E-E_{R_{i}}\right)^{2}+\Gamma_{i}^{2}(E) / 4}
$$

$M_{i}(E)$ is the amplitude of the transfer reaction (upper vertex) that can be easily calculated $\rightarrow$ The resonance parameters can be extracted and in particular the strenght

IMPORTANT: reduced widths are the same for the extraction of the $S(E)$ factors $\rightarrow$ From the fitting of the experimental THM cross section they can be obtained and used to deduce the OES S(E) factor.

$$
{ }^{13} \mathrm{C}+\alpha \rightarrow \mathrm{n}+{ }^{16} \mathrm{O} \text { : recent experiment at FSU }
$$

Neutron source for the main component of the s-process, responsible for the production of most nuclei in the mass range 90<A<204

Active in He-burning shell in AGB from 140 to $230 \mathrm{keV} \rightarrow$ importance of the higher energy tail of the -3 keV resonance
its new partial width and ANC

$$
\begin{aligned}
& \Gamma_{n}^{1 / 2^{*}}=83_{-12}^{+9} \mathrm{keV} \\
& \left(\tilde{C}_{\alpha^{-1 / \mathrm{C}} \mathrm{C}}^{17\left(2^{+}\right)}\right)^{2}=6.7_{-0.6}^{+09} \mathrm{fm}^{-1}
\end{aligned}
$$

Reaction rate increases by a factor 3 in the relevant region:
$\rightarrow 30 \%$ variation in the abundance of

$$
{ }^{86} \mathrm{Kr},{ }^{87} \mathrm{Rb},{ }^{96} \mathrm{Zr} \text {, and }{ }^{142} \mathrm{Ce}
$$

due to the increased neutron density!


M. La Cognata et al., PRL 2013
M. La Cognata et al., ApJ 2014

The ${ }^{18} \mathrm{~F}(p, a)^{16} \mathrm{O}$ Reaction @ CRIB - CNS - RIKEN and @ Cyclotron Laboratory, TAMU using a ${ }^{18} \mathrm{~F}$ RIB


The ${ }^{18} \mathrm{~F}(p, a)^{15} \mathrm{O}$ astrophysical S factor from the present experiment.

Solid circles: THM experimental data with the assumption of $\mathrm{J}^{\pi}=3 / 2^{+}$for the resonance at $\mathrm{E}=6460 \mathrm{keV}$ (upper limit)

Open circles: ${ }^{18} \mathrm{~F}(\mathrm{p}, \mathrm{a})^{15} \mathrm{O}$ astrophysical S factor corresponding to the assumption of $J^{\pi}=5 / 2^{-}$ (lower limit)
S. Cherubini et al. PRC 92 (2015) 015805
R.G. Pizzone et al. EpJ A (2016)

Blue solid and red dashed lines: calculations reported and discussed in Beer et al. PRC 83 (2011) 042801 smeared to the present experimental resolution ( $\sigma=53 \mathrm{keV}$ ). The difference is given by the alternative interference pattern adopted in the calculations.

Each pair of curves represents the upper and lower limit for each calculation
Around 700 keV : normalization region
Elsewhere: fair agreement with the dashed lines if $J^{\pi}=3 / 2^{+}$is assumed

Triple alpha resonances in the ${ }^{6} \mathrm{Li}+6 \mathrm{Li} \rightarrow 3 \alpha$ at low energy
Coincidence measurement of the ${ }^{6} \mathrm{Li}+{ }^{6} \mathrm{Li} \rightarrow 3 \alpha$ reaction at 3.1 MeV of beam energy
Results: existence of $\alpha$-trimers at an excitation energy of ${ }^{12} \mathrm{C}$ of about 29.6 MeV ( ${ }^{6} \mathrm{Li}+{ }^{6} \mathrm{Li}$ threshold: 28.17 MeV; center-of-mass beam energy at half target: 1.4 MeV

Two-dimensional plots $\theta_{1}$ vs. $E_{2}$ for a fixed value of $\theta_{2}=60^{\circ}$

Intersection between lines:
events with correlated a's fed by two and even three ${ }^{8} \mathrm{Be}$ at the same time can exist.

From this figure, though very illustrative, it is not straightforward to gather a global information from all experimental data.

A. Tumino et al. PLB (2015) 59

Hyperspherical formalism for the low-energy three-body problem:
to better investigate the correlation between the three a particles from the available data in a whole,

$$
\begin{aligned}
& \vec{p}_{i j}=\vec{p}_{i}-\vec{p}_{j}, \\
& \vec{p}_{k, i j}=\vec{p}_{k}-\frac{1}{2}\left(\vec{p}_{i}+\vec{p}_{j}\right) . \\
& \alpha_{k}=\arctan \left(\frac{\sqrt{3} p_{i j}}{2 p_{k, i j}}\right)
\end{aligned}
$$

Jacobi momenta where $(i, j, k)$ is a permutation of $(1,2,3)$ and $p_{i j}$ and $\mathrm{p}_{k, i j}$ are the magnitudes of the momentum vectors. The range of $a_{k}$ is from 0 to $\pi / 2$
k is the undetected third $\alpha$ particle. Thus $\alpha_{k}=\alpha_{3}$

Events within those peaks were projected onto the $\operatorname{Eij}(i, j=1,2,3)$ relative energy axis:
ground state and 16.62 MeV state of ${ }^{8} \mathrm{Be}$ (fed twice with respect to the 90 keV )
three ${ }^{8}$ Be states at 20.1, 11.35 and 3.03 MeV , taking advantage of their huge widths
A. Tumino et al. PLB (2015) 59


Efimov idea: A system of three particles with resonant two-body interactions may form bound states, the so called Efimov trimers, even when any two of the particles are unable to bind.

No observation exists yet in nuclei

Efimov has predicted the possibility of existence of trimers in a system of three $\alpha$-particles, with a maximum radius of attraction of the order of the Bohr radius, $a_{c}=1 / e^{2}$, due to the repulsive longrange Coulomb potential.

His prescription refers mainly to ${ }^{12} \mathrm{C}$ levels in the vicinity of the threshold of breakup into three $\alpha$-particles or $\alpha+{ }^{8} \mathrm{Be}$, and here we are much higher with the excitation energy.


Other interpratation
${ }^{6} \mathrm{Li}+\alpha+d$ Efimov trimer in the entrance channel (zero energy). Thus, systems with correlated a particles can be seen as escape channels from Efimov.

Clustering in light nuclei: do they play a role in the sub-Coulomb fusion processes?

From the $\alpha_{3}$ spectrum: the bulk of recombination into $3 \alpha$ particles pertains to the $90^{\circ}$ region. First reason: contribution of the 22.2 MeV level of ${ }^{8} \mathrm{Be}$.

Side reason: $\alpha_{3}$ around $90^{\circ}$ corresponds to $3 \alpha$ particles in a line and can be linked to two interacting ${ }^{6} \mathrm{Li}$ in a stretched $a-d-d$-a configuration that corresponds to the minimum value of the Coulomb barrier.

Next step: To investigate the role of clustering in the sub-Coulomb fusion processes of light nuclei with pronounced cluster structure, we have started with the ${ }^{6} \mathrm{Li}+{ }^{6} \mathrm{Li} \rightarrow 3 \alpha$ reaction $(Q=20.9 \mathrm{MeV})$. The aim is to investigate the decrease of its cross section while decreasing the interaction energy from above to below the Coulomb barrier. If clustering plays a role, the system should experience a lower decrease of the cross section than expected.

Conjecture for nuclear astrophysics: there might be the possibility that other direct processes never considered at sub-Coulomb energies, triggered by the cluster structures of the interacting nuclei that manage to reduce the overall Coulomb barrier, may contribute to unsolved problems...

## Conclusions

Many years ago the idea that atomic nucleus might show a cluster substructure has stronly supported the development of the physics of clusters, revealing a wide variety of clustering ranging from nuclear molecules to possible chains of a-particles.

This physics is still very attractive and lends itself to many applications:

- Nuclear astrophysics and the THM: virtual projectile/targets to overcome the main issues of nuclear reactions at astrophysical energies
- Strong correlations and a conglomerates possibly connected with the Efimov mechanism in nuclei $\rightarrow$ still a lot to understand
- Role of clustering in the sub-Coulomb fusion processes of light nuclei with pronounced cluster structure

To be continued ...

