Relativistic all-sky analysis with Gaia

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Gaia – main characteristics and status

Science with one billion objects in 3 dimensions

- From structure and evolution of the MW to general relativity
- Astrometry, photometry, spectroscopy (RVS)
  - Astrometry and photometry G < 20.7 mag
  - Stars brighter than G=3 captured with Sky Mapper imager
  - Spectra still $G_{RVS} = 16.2$

Satellite (including payload) by industry, management and operations by ESA, data processing by scientists (DPAC)

Now in 5-year routine operations (since 25/7/2014)
First DR planned for September, science alerts started
Data validation started
Data Release Scenario
(http://www.cosmos.esa.int/web/gaia/release)

First release end of summer 2016 - Subject to successful validation:

- Positions ($\alpha$, $\delta$) and G magnitudes for all stars with acceptable formal
  standard errors on positions

- Photometric data of RR Lyrae and Cepheids from high-cadence measurements

The Tycho-Gaia astrometric solution

How to get 2.5 million parallaxes with less than one year of Gaia data

Daniel Michalik, Lennart Lindegren, and David Hobbs

Second release summer 2017 - Potentially:

- Five-parameter astrometric solutions of objects with single-star behaviour

- Integrated BP/RP photometry, for sources where basic astrophysical
  parameter estimation has been verified

- Mean radial velocities will be released for “well behaved objects” objects

Third release summer 2018 (TBC)….
The location of an object in astrometry is considered reliable if its relative error is less 10%.
The Gaia’s look into the Milky Way

end-of-mission astrometric accuracies better than 5-10µas for the brighter stars and 130-600µas for faint targets

http://www.cosmos.esa.int/web/gaia/science
Our laboratory: the Solar System

\[ g = \eta + h \]

h perturbations at \( \mu\text{-arcsec} \) due to the solar system bodies
### Detectable relativistic deflections at L2

<table>
<thead>
<tr>
<th></th>
<th>$\delta \chi_{PN}$</th>
<th>$\delta \chi_{J2}$</th>
<th>$\delta \chi_{L}$</th>
<th>$\chi_{\text{max}}$</th>
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<tbody>
<tr>
<td>Sun</td>
<td>1''75</td>
<td>$\sim 1,\mu\text{as}$</td>
<td>0.7 $\mu\text{as}$</td>
<td>(180°)</td>
</tr>
<tr>
<td>Mercury</td>
<td>83 $\mu\text{as}$</td>
<td>–</td>
<td>–</td>
<td>(7')</td>
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<tr>
<td>Venus</td>
<td>493</td>
<td>–</td>
<td>–</td>
<td>(4.0°)</td>
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<tr>
<td>Earth</td>
<td>574</td>
<td>0.6</td>
<td>–</td>
<td>(101°)</td>
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<tr>
<td>Moon</td>
<td>26</td>
<td>–</td>
<td>–</td>
<td>(2.3°)</td>
</tr>
<tr>
<td>Mars</td>
<td>116</td>
<td>0.2</td>
<td>–</td>
<td>(17°)</td>
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<td>Jupiter</td>
<td>16290</td>
<td>240</td>
<td>0.2</td>
<td>(87°/3')</td>
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<tr>
<td>Saturn</td>
<td>5772</td>
<td>94</td>
<td>–</td>
<td>(16°/51'')</td>
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<tr>
<td>Uranus</td>
<td>2030</td>
<td>7</td>
<td>–</td>
<td>(67'/4'')</td>
</tr>
<tr>
<td>Neptune</td>
<td>2487</td>
<td>8</td>
<td>–</td>
<td>(50'/3'')</td>
</tr>
<tr>
<td>Pluto</td>
<td>7</td>
<td>–</td>
<td>–</td>
<td>(0''3)</td>
</tr>
</tbody>
</table>
micro-arcsecond accuracy + dynamical gravitational fields = relativistic models of Light propagation

RELATIVISTIC ASTROMETRY
Gaia,
WG REMAT: RELativistic Models And Tests

Inside the Consortium constituted for the Gaia data reduction (Gaia CU3, Core Processing, DPAC), two models have been developed:

1. **GREM** (Gaia RElativistic Model), baselined for the Astrometric Global Iterative Solution for Gaia (AGIS)

2. **RAMOD** (Relativistic Astrometric MODel) implemented in the Global Sphere Reconstruction (GSR) of the Astrometric Verification Unit (AVU) at the Italian data center (DPCT)
Why to look for a comparison?

At first glance **no evidence that** GREM (IAU based) = RAMOD even if they are based on a “GR framework” -> relativistic astrometry opens a largely uncharted territory

both models will work on the same Gaia data: we need a validation process!
RAMODs & Gaia: from the “measurement” to the star

RAMOD is a framework of general relativistic astrometric models with increasing intrinsic accuracy, adapted to many different observer’s settings, interfacing numerical and analytical relativity.

**RAMOD applies the measurement protocol (MP) in GR**


6. **RAMODINO1-2-3**: satellite-observer model for Gaia (Bini et al., 2003, Class.Quantum Grav., 20, 2251/4695); ray tracing error budget (de Felice, F.; Preti, G. 2006CQGra..23.5467D and 2008CQGra..25p5015D)
7. **RAMOD vs PM/PN approach**: Crosta 2011 Class. Quantum Grav. 28 235013;
8. **RAMOD analytical solutions for Gaia-like case**: 2015Crosta, Vecchaito, de Felice , Lattanzi Classum Quantum Gravity

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RAMOD Measurement Protocol (MP)

1. Specify the phenomenon under investigation.
2. Identify the covariant equations which describe it.
3. Identify the observer who makes the measurements.
4. **Chose a frame adapted to that observer allowing the space-time splitting into the observer’s space and time.**
5. **Understand the locality properties of the measurement under consideration** (local or non-local with respect to the background curvature).
6. **Identify the frame components of those quantities which are the observational targets.**
7. **Find a physical interpretation of the above components following a suitable criterium.**
8. Verify the degree of the residual ambiguity in the interpretation of the measurements and decide the strategy to evaluate it (i.e. comparing what already is known).
Light crossing the metric of the Solar System

\[ g_{\alpha \beta} = \eta_{\alpha \beta} + h_{\alpha \beta} + O(h^2) \]

\[ |h_{\alpha \beta}| \ll 1 \]

\[ |v^2/c^2| \sim \text{GM/rc}^2 \sim \text{mas accuracy} \]

which requires determination of:
- \( g_{oo} \) even terms in \( \varepsilon \), lowest order \( \varepsilon^2 \)-mas
- \( g_{oj} \) odd terms in \( \varepsilon \), lowest order \( \varepsilon^3 \)-\( \mu \)-as
- \( g_{ij} \) even terms in \( \varepsilon \), lowest order \( \varepsilon^2 \)-mas

Time variation of the order of

IAU metric!

(MP step 1) Specify the phenomenon under investigation.
(MP step 2) Identify the covariant equations which describe it.
Retarded distances contributions

\[ h_{00} = \sum_a \frac{2 M(a)}{r(a)} + O(\epsilon^4) \]
\[ h_{0i} = -\sum_a \frac{4 M(a)}{r(a)} \tilde{\beta}_{i(a)} + O(\epsilon^5) \]
\[ h_{ij} = \sum_a \frac{2 M(a)}{r(a)} \delta_{ij} + O(\epsilon^4), \]

\[ t' - t = r(\epsilon)/c \]

\[ \tilde{\beta}^i = \left(1 - \frac{h_{00}}{2}\right) \tilde{\beta}^i(\tilde{\sigma}) + O(h^2) \]
✓ Family of **fiducial observers (FIDO)** to set the reference frames:

- **a time-like congruences of curves** \( u \)

\( u \) is defined everywhere

the fiducial observer **locally and only locally** is at rest w.r.t. the spatial coordinates

-> **static observer at rest locally w.r.t.**

the barycenter of the Barycentric Celestial Reference System (B) ->

\[ u \sim \frac{1}{\sqrt{-g_{00}}} \partial_0 \]

(MP step 3) Identify the observer who makes the measurements.

(MP step 4) Chose a frame adapted to that observer allowing the space-time splitting into the observer’s space and time.

\[ g_{0i} \sim \mu\text{-arcsecond} \]
The vorticity $\omega(u)$ cannot be neglected at the order of microarcsecond, the possibility to ignore it locally, and only locally, can be applied only to a small neighborhood with respect to the scale of the vorticity itself.

\[ \omega_{ij} = \partial_{[j} h_{i]} + O \left( \frac{1}{c^4} \right) \]

Our aim is to have a hypersurface of simultaneity w.r.t. the barycenter $B$ everywhere, i.e. the possibility to map the null geodesic on the slice (the rest-space of the observer) that extends from the observer up to the star that emits light.
Local line-of-sight

Tangent to null geodesic

Projector operator in the rest-space of \( \mathbf{u} \)

\[
P(u')_{\alpha \beta} = g_{\alpha \beta} + u'_{\alpha} u'_{\beta}
\]

\[ k^\alpha k_\alpha = 0, \quad \frac{dk^\alpha}{d\lambda} + \Gamma^\alpha_{\rho \sigma} k^\rho k^\sigma = 0 \]

A general solution

(MP step 6) Identify the frame components of those quantities which are the observational targets.

\[ \ell^i(\sigma) = \ell^i(\ell(\sigma_0), h_{\alpha \beta}(\sigma)) \]

\( \ell^k_{\text{obs}} \)

- boundary condition to solve uniquely the differential equations
- link to the parameters of the star in the astrometric measurements (condition equation)

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One more equation to integrate: spatial time component!

With appropriate assumptions adapted to the case of the Solar System and the accuracy of a Gaia-like observer there exists analytical solutions:

R.A.MOD. models (Crosta et al., Classum Quantum Gravity, 32 (2015) 1655008 and references thererein)
**RAMOD3 MASTER EQUATION**

n-monopoles
1. RAMOD3a (R3a) and, the spatial derivatives of the metric are considered while $h_{0i}$ are neglected

2. RAMOD3b (R3b), the spatial and time derivatives of the metric are considered while $h_{0i}$ are neglected

n-monopoles+ quadrupole
3. RAMOD3aQ (R3aQ)
4. RAMOD3bQ (R3bQ)

**RAMOD4 MASTER EQUATION**

n-monopoles
1. RAMOD4a (R4a), the spatial derivatives of the metric are considered including $h_{0i}$

2. RAMOD4b (R4b), the spatial and time derivatives of the metric are considered including $h_{0i}$

n-monopoles+ quadrupole

1. RAMOD4aQ (R4aQ)

The implementation of RAMOD models and the need of testing them through a self-consistency check at different levels of accuracy, will benefit form this explicit classification.

(MP step 7)

Find a physical interpretation of the above components following a suitable criterium.

\[ r(t', t) \approx r(t) - r \cdot v \]

At first order in $v$

Crosta 2011 Class. Quantum Grav.
Crosta et al., 2015 Class. Quantum. Grav.
The RAMOD local-line-of-sight is not exactly equal to the light direction used in the semi-classical approximation.
The astrometric observable in RAMOD/AVU

Projector operator onto the rest space of the satellite

$\{E_{\alpha}\}$

"attitude tetrad" -> ESSENTIAL to define the boundary condition

(Bini, Crosta, and de Felice, Class. Quant. Grav. 20, 4695, 2003)

\[ \cos \psi (E_{\alpha}, \ell_{obs}) \equiv e_{\alpha} = \frac{P(u')_{\alpha \beta} \ell_{\alpha}}{(P(u')_{\alpha \beta} k_{\alpha} k_{\beta})^{1/2}} \]

Observation equation

$-\sin \phi d\phi = \left\{ \frac{\partial F}{\partial \alpha_*} \delta \alpha_* + \frac{\partial F}{\partial \delta_*} \delta \delta_* + \frac{\partial F}{\partial \omega_*} \delta \omega_* + \cdots \right\}$

All derivatives are calculated at appropriate "catalog" values

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The concept of the Global Sphere Reconstruction

\[
\cos \phi \equiv F \left( \begin{array}{c} \alpha_*, \delta_*, \bar{\omega}_*, \mu_\alpha*, \mu_\delta*, \sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(3)}, \sigma_2^{(3)}, \sigma_3^{(3)}, c_1, c_2, \ldots, \gamma, \ldots \end{array} \right)
\]

Astrometric parameters  \hspace{1cm} Attitude parameters  \hspace{1cm} Instrument \hspace{1cm} Global

1 obs. \implies 1 condition eq. (linearized) system of solution with dimensions \( \sim 10^{10} \times 10^8 \)

\[
\begin{align*}
\text{Know} & \quad \sin \psi_i^{(1)} \Delta \psi_i^{(1)} = \frac{\partial f}{\partial \alpha_i} \Delta \alpha_i + \frac{\partial f}{\partial \delta_i} \Delta \delta_i + \frac{\partial f}{\partial \pi_i} \Delta \pi_i + L + \frac{\partial f}{\partial \gamma} \Delta \gamma \\
\text{Unknown} & \quad \sin \psi_i^{(2)} \Delta \psi_i^{(2)} = \frac{\partial f}{\partial \alpha_i} \Delta \alpha_i + \frac{\partial f}{\partial \delta_i} \Delta \delta_i + \frac{\partial f}{\partial \pi_i} \Delta \pi_i + L + \frac{\partial f}{\partial \gamma} \Delta \gamma \\
\text{Unknown} & \quad \sin \psi_i^{(n)} \Delta \psi_i^{(n)} = \frac{\partial f}{\partial \alpha_i} \Delta \alpha_i + \frac{\partial f}{\partial \delta_i} \Delta \delta_i + \frac{\partial f}{\partial \pi_i} \Delta \pi_i + L + \frac{\partial f}{\partial \gamma} \Delta \gamma
\end{align*}
\]

\( \rightarrow \) A real Galilean experiment in space: a massive repetition of the Eddington et al. astrometric test of GR with 21st century technology, thank to the interfacing of analytical&numerical relativity methods
Relativistic scheme to transform pair $(OBT_k, UTC_k)$ into pair $(OBT_k, TG_k)$ in order to calibrate the Gaia clock (here $TT$ is the Terrestrial Time and is related to UTC by a simple relation).
Italian Data Processing Center

All Gaia operations activities (daily and cyclic) done in Italy are implemented at the DPCT, the Italian provided HW and SW operations system designed, built and run by ALTEC (To) and INAF-OATo for ASI.

**DPCT at full capacity.**
Accumulated other than 50 TB of data

**Size at completion ~ 1.2 PB**

The DPCT host the systems AVU:

- CCD-level precision and accuracy (Astrometric Instrument Monitoring - AIM)
- Accuracy at the Optical System level (Basic Angle Monitoring - BAM/AVU)
- Precision & accuracy on the celestial sphere (Global Sphere Reconstruction - GSR)

**Essential components of Gaia’s astrometric error budget**

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DPCT was established through a specific ASI contract via a partnership between INAF-OATo and ALTEC S.p.A.
- M. Castronuovo (RC, MLA-SC repr.)
- B. Negri (EOS Head)

This is the only Data Processing Center, within the network of 6 DPCs dedicated to Gaia, which specializes in the treatment of the satellite astrometric data.
Global astrometry: the evaluation of deviations from GR depends on the particular scalar-tensor theory adopted -> quantum theory of gravity, verification of inflationary models, violation of the principle of equivalence, constancy of the physical constants, low-energy limits of string theories, \( f (R) \) gravity with no need of dark matter and dark energy, accelerated cosmological expansion, Galaxy cluster dynamics, Galaxy rotation curves and DM halos

Differential Astrometry:
extrapolation of the evaluation of the quadrupole contribution to second order deflection effects, gravitomagnetic and post-Newtonian effects of higher order
Milky Way..relativisticly tuned (kinematically)

The thick disk and the local halo of Milky Way as chemo-dynamical lab for testing galactic models and (Λ)CDM predictions.

Comparison of simulations Lambda-CDM on the scale of the Milky Way with the data of Gaia (3-4 kpc)

©P. Harding e H. Morrison

POSITION / VELOCITY

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Finding (and counting!) streams in the Galactic inner halo (within 3-5 kpc from the Sun) - Simulations -

True ‘simulated’ data set

Simulations from Sanderson et al. (2014)
Finding streams in the Galactic inner (within 3 kpc from the Sun) halo - Simulations -

Accuracy of the current ground-based catalogs

Expected Gaia observations

True ‘simulated’ data set

Simulations from Sanderson et al. (2014) and error model from Re Fiorentin et al. (2015)
The Galactic Warp (via O-B stars)

Sun

$\mu_b \ (\omega_p = 0 \text{ km s}^{-1} \text{ kpc}^{-1})$

R = 10 kpc
R = 7 kpc

(Courtesy of R. Drimmel - OATo)

HIP DATA (mv < 7.5)
Simulation: Warp
Simulation: No warp

(Courtesy of R. Drimmel - OATo)
Conclusions

- DR1 processing and validation is indicating that the Gaia mission is fulfilling most of the science promised.
  - DR1 is **only** the first Gaia data release and full confirmation has to wait for the next DR (i.e., for a **full-Gaia-only solution**).

Reaching 10-20 µas accuracy on individual parallax and annual proper motions for bright stars (V<16) is the key

- **possibly to perform the largest GR experiment ever attempted from space:**
  - the realization of the celestial sphere is not only a scientific validation of the absolute parallax and proper motions in Gaia, but also, *given the number of celestial objects (a real Galilean method applied on the sky!) and directions involved (the whole celestial sphere!), the largest experiment in General Relativity ever made with astrometric methods (since 1919)*

- to fully probe the MW (outer) halo (mass content and distribution) and compare the prediction of Lambda-CDM models
But all the goals of Gaia will not be achieved without the correct characterization and exploitation of the "relativistic" astrometric data.

The Gaia-like observer is positioned inside the Solar System, a weak gravitational regime which turns out to be "strong" when one has to perform high accurate measurements.

Any discrepancy between the relativistic models, if it can not be attributed to errors of different nature, will mean either a limit in the modeling/interpretation - that a correct application of GR should fix - and therefore a validation of GR, or, maybe, a clue that we need to refine our approach to GR.
Conclusions

- In tracing back light rays we need to keep consistency, at any level of approximations, with GR.
- This implies a new rendition of the astronomical observables and it may open, at the sub-muas level, a new detection window of many subtle relativistic effects naturally folded in the light while it propagates through the geometry of space-time up to the “local” observer.

Beyond the micro-arcsecond? Gaia represents ONLY the 0-step… increasing the level of the measurement precision requires to refine consistently the metric of the solar system, the solution for the null geodesic and so on..

- Once a relativistic model for the data reduction has been implemented, any subsequent scientific exploitation should be consistent with the precepts of the theory underlying such a model.

One century after General Relativity we must rethink the Mach’s principle: how much the local universe can affect on our knowledge of the global universe?

The method introduced by RAMOD extends beyond the scope of Gaia, after Gaia Astrometry becomes part of the fundamental physics and, in particular, in that of gravitation.
Thanks for your attention!

http://www.cosmos.esa.int/web/gaia/home