Beyond Einstein: gravitational waves from extended gravities

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Open Fundamental Issues

- What is the true theory of gravity?
- Is spacetime with torsion or torsionless?
- Further Gravitational Waves Modes?
- Do Geodesic and Causal structures coincide or not?
- Quantum Gravity?
- Do DM and DE really exist or are geometric effects?

Tools for setting the “right” gravity

3 key tools for Gravitational Physics
that should be combined:

- Equivalence Principle (selects the Theory)
- Weak Field Limit (astronomical dynamics)
- Gravitational Waves (strong field)

The role of the Weak Field Limit

As general solution:

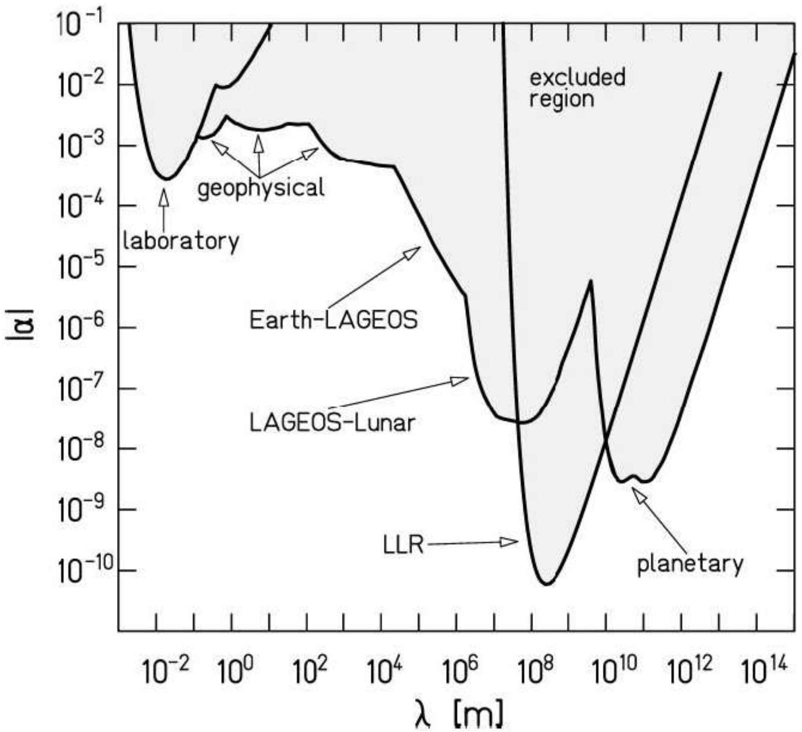
$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha e^{-r/\lambda}]$$

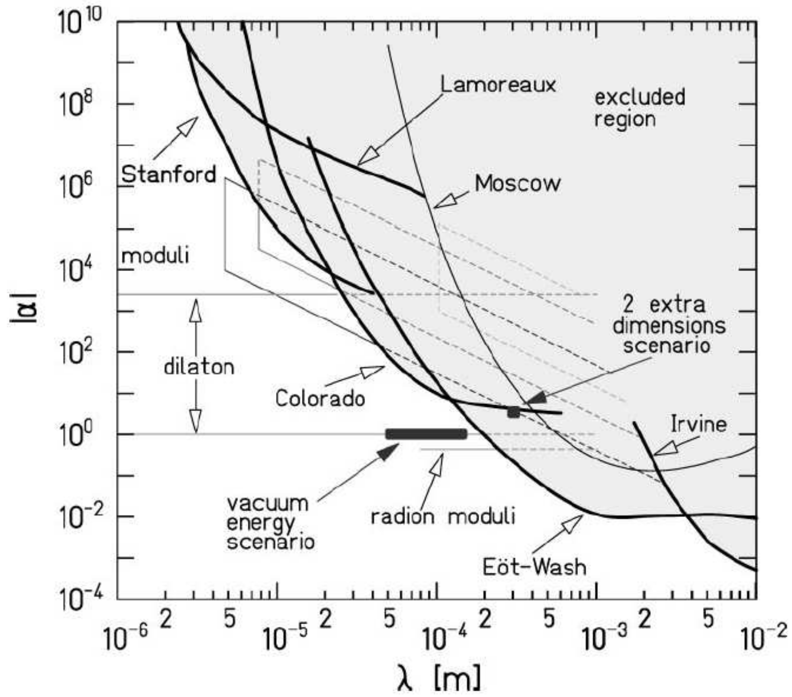
"Fifth force" emerges in any theory of gravity beyond GR

α is a dimensionless strength parameter

λ is a length scale or range

Experimental bounds





Gravitational radiation after GW150914

- GWs really exist!!
- GWs in GR and Alternative Gravity
- Upper bound on the graviton mass?
- Further polarizations?
- Selecting affine or metric theories by GWs?
- Testing EP by GWs?
- Testing extreme gravitational fields
- Cosmology by GWs

Gravitational Astronomy

- Survey of relativistic compact objects
- Systematic search of BHs by GWs
- Graviton as realistic candidate for DM?
- Structure and dynamics of NSs
- Stochastic Background of GWs
- Multimessengers (EM,GWs,ν)
- Testing extreme gravitational fields

Main role of Gravitational Radiation

- Extended Gravity: extending GR in view of DM and DE
- New polarization states of GWs
- Massive, massles and ghost modes
- Response of GW detectors
- The key role of sthocastic background

Field equations in Extended Gravity

Let us consider the action with curvature invariants (the most general action!)

$$S = \int d^4x\sqrt{-g}f(R,P,Q)$$

$$P \equiv R_{ab}R^{ab}$$
$$Q \equiv R_{abcd}R^{abcd}$$

We need only two invariants thanks to the Gauss-Bonnet topological term *G= Q-4P+R*^{2}.Varying with respect to the metric, one gets the field equations

$$\begin{aligned} FG_{\mu\nu} = & \frac{1}{2}g_{\mu\nu}\left(f - R\ F\right) - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F \\ & -2\left(f_P R^a_\mu R_{a\nu} + f_Q\ R_{abc\mu}R^{abc}{}_\nu\right) \\ & -g_{\mu\nu}\nabla_a\nabla_b(f_P R^{ab}) - \Box(f_P R_{\mu\nu}) \\ & +2\nabla_a\nabla_b\left(f_P\ R^a{}_{(\mu}\delta^b{}_{\nu)} + 2f_Q\ R^a{}_{(\mu\nu)}{}^b\right) \end{aligned}$$

Field equations in Extended Gravity

The trace

$$\Box\left(F+\frac{f_P}{3}R\right)=\frac{1}{3}\left(2f-RF-2\nabla_a\nabla_b((f_P+2f_Q)R^{ab})-2(f_PP+f_QQ)\right)$$

If we define

$$\Phi\equiv F+\frac{2}{3}(f_P+f_Q)R$$

and

$$\frac{dV}{d\Phi}\equiv \mathrm{RHS}$$

We get a Klein-Gordon equation for the scalar field

$$\Box\Phi=\frac{dV}{d\Phi}$$

To find the GW modes, we need to linearize gravity about a Minkowski background:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$
$$\Phi=\Phi_0+\delta\Phi$$

SC & MDL Phys.Rept. 509 (2011) 167

Field equations in Extended Gravity

We get

$$\delta\Phi = \delta F + \frac{2}{3}(\delta f_P + \delta f_Q)R_0 + \frac{2}{3}(f_{P0} + f_{Q0})\delta R$$

$$R_0 \equiv R(\eta_{\mu\nu}) = 0$$

$$f_{P0} = \frac{\partial f}{\partial P}|_{\eta_{\mu\nu}}$$

denotes the first order perturbation in the Ricci scalar that, after perturbing the Riemann and Ricci tensors, are given by

$$\delta R_{\mu\nu\rho\sigma} = \frac{1}{2}(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma})$$

$$\delta R_{\mu\nu} = \frac{1}{2}(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu})$$

$$\delta R = \partial_\mu\partial_\nu h^{\mu\nu} - \Box h$$

$$h = \eta^{\mu\nu}h_{\mu\nu}$$

The first term is

$$\delta F = \frac{\partial F}{\partial R}|_0\,\delta R + \frac{\partial F}{\partial P}|_0\,\delta P + \frac{\partial F}{\partial Q}|_0\,\delta Q$$

$$\delta F \simeq F_{,R0}\,\delta R$$

are second order

Gravitational Waves

And...

$$\delta\Phi = \left(F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0})\right)\delta R$$

The Klein-Gordon equation for the scalar perturbation

$$\Box\delta\Phi = \frac{1}{3}\frac{F_0}{F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0})}\delta\Phi - \frac{2}{3}\delta R^{ab}\partial_a\partial_b(f_{P0} + 2f_{Q0}) - \frac{1}{3}\delta R\Box(f_{P0} + 2f_{Q0}) = m_s^2\delta\Phi$$

=0 since $f_{P0},\ f_{Q0}$ are constants.

The effective scalar mass of gravitational modes can be defined as

$$m_s^2 \equiv \frac{1}{3}\frac{F_0}{F_{,R0}+\frac{2}{3}(f_{P0}+f_{Q0})}$$

Gravitational Waves

Perturbing the field equations, we get:

$$F_0(\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta R) = -(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)(\delta\Phi - \frac{2}{3}(f_{P0} + f_{Q0})\delta R)$$
$$-\eta_{\mu\nu}\partial_a\partial_b(f_{P0}\delta R^{ab}) - \Box(f_{P0}\delta R_{\mu\nu}) + 2\partial_a\partial_b(f_{P0}\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)} + 2f_{Q0}\ \delta R^a{}_{(\mu\nu)}{}^b)$$

It is convenient to work in Fourier space so that

$$\partial_\gamma h_{\mu\nu} \rightarrow ik_\gamma h_{\mu\nu}$$ and $$\Box h_{\mu\nu} \rightarrow -k^2 h_{\mu\nu}$$

Then the above equations become

$$F_0(\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta R) = (\eta_{\mu\nu}k^2 - k_\mu k_\nu)(\delta\Phi - \frac{2}{3}(f_{P0} + f_{Q0})\delta R)$$
$$+\eta_{\mu\nu}k_a k_b(f_{P0}\delta R^{ab}) + k^2(f_{P0}\delta R_{\mu\nu}) - 2k_a k_b(f_{P0}\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)}) - 4k_a k_b(f_{Q0}\ \delta R^a{}_{(\mu\nu)}{}^b)$$

Gravitational Waves

We can rewrite the metric perturbation as

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\,\eta_{\mu\nu} + \eta_{\mu\nu} h_f$$

and use the gauge freedom:

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \quad \text{and} \quad \bar{h} = 0$$

The first of these conditions implies that

$$k_\mu \bar{h}^{\mu\nu} = 0$$

the second gives

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \eta_{\mu\nu} h_f$$
$$h = 4 h_f$$

and we have

$$\delta R_{\mu\nu} = \frac{1}{2}\left(2 k_\mu k_\nu h_f + k^2 \eta_{\mu\nu} h_f + k^2 \bar{h}_{\mu\nu}\right)$$
$$\delta R = 3 k^2 h_f$$

$$k_\alpha k_\beta\ \delta R^{\alpha}{}_{(\mu\nu)}{}^{\beta} = -\frac{1}{2}\left((k^4 \eta_{\mu\nu} - k^2 k_\mu k_\nu) h_f + k^4 \bar{h}_{\mu\nu}\right)$$
$$k_a k_b\ \delta R^{a}{}_{(\mu}\delta^{b}{}_{\nu)} = \frac{3}{2} k^2 k_\mu k_\nu h_f$$

Gravitational Waves

…after some algebra, we get

$$\frac{1}{2}\left(k^2 - k^4\frac{f_{P0}+4f_{Q0}}{F_0}\right)\bar{h}_{\mu\nu} = (\eta_{\mu\nu}k^2 - k_\mu k_\nu)\frac{\delta\Phi}{F_0} + (\eta_{\mu\nu}k^2 - k_\mu k_\nu)h_f$$

$$h_f \equiv -\frac{\delta\Phi}{F_0}$$

the equation for the perturbations is

$$\left(k^2 + \frac{k^4}{m^2_{spin2}}\right)\bar{h}_{\mu\nu} = 0$$

$$m^2_{spin2} \equiv -\frac{F_0}{f_{P0}+4f_{Q0}}$$

we have a modified dispersion relation which corresponds to a massless spin-2 field ($k^2=0$) and massive 2-spin ghost mode

$$k^2 = \frac{F_0}{\frac{1}{2}f_{P0}+2f_{Q0}} \equiv -m^2_{spin2}$$

Gravitational Waves

Note that the propagator for $\bar{h}_{\mu\nu}$ can be rewritten as

$$G(k) \propto \frac{1}{k^2} - \frac{1}{k^2 + m^2_{spin2}}$$

the second term has the opposite sign, which indicates the presence of a ghost

The Gauss-Bonnet term can be adopted as a constraint

$$\mathcal{L}_{GB} = Q - 4P + R^2$$

$$f_{Q0} = 1 \qquad f_{P0} = -4$$

We have

Finally we obtain

$$k^2 \bar{h}_{\mu\nu} = 0$$

in this case we have no ghosts as expected

Gravitational Waves

$$\left(k^2 + \frac{k^4}{m^2_{spin2}}\right)\bar{h}_{\mu\nu} = 0$$

$$\Box h_f = m_s^2 h_f$$

GR

Solutions are plane waves

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p}) \cdot exp(ik^\alpha x_\alpha) + cc$$

$$h_f = a(\overrightarrow{p}) \cdot exp(iq^\alpha x_\alpha) + cc$$

Massive mode

Dispersion relations

$$k^\alpha \equiv (\omega_{m_{spin2}}, \overrightarrow{p}) \qquad \omega_{m_{spin2}} = \sqrt{m^2_{spin2} + p^2}$$

$$q^\alpha \equiv (\omega_{m_s}, \overrightarrow{p}) \qquad \omega_{m_s} = \sqrt{m_s^2 + p^2}.$$

Note:

The velocity of any "ordinary" mode $\bar{h}_{\mu\nu}$ is the light speed c

The dispersion law for the h_f mode is that of a massive field which is a wave-packet

The h_f wave-packet group-speed is centred in p with an upper limit for *m* that can be confronted with GW150914 that is *$m < 10^{-22}$ eV (Ligo & Virgo PRL 2016)*

$$\vec{v}_G = \frac{\vec{p}}{\omega},$$

$$v_G = \frac{\sqrt{\omega^2 - m^2}}{\omega}.$$

$$m = \sqrt{1 - v_G^2}\,\omega.$$

Polarization states of GWs

The above two conditions depend on the value of k^2

For $k^2=0$ modes → massless spin-2 field with two independent polarizations (GR)

For $k^2 \neq 0$ modes → massive spin-2 ghost modes
+
scalar modes

OUTSTANDING RESULT

$M=(2s+1)$ with $s=0,2$ and $M=6$
6 modes instead of 2!

Polarization states of GWs

Riemann's Theorem:

A n-dimensional metric has $f=n(n-1)/2$ degrees of freedom $n=4$, $f=6$!

In GR we have only 2 polarizations.
The full budget is 6!! We MUST search for the further polarizations.
This issue comes from first principles.

First case: massless spin-2 modes

In the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different from zero can be chosen. The condition $h = 0$ gives $A_{11} = -A_{22}$.

In this frame we may take the bases of polarizations defined as

$$e_{\mu\nu}^{(+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(\times)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(s)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

…the characteristic amplitude

$$h_{\mu\nu}(t,z) = A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)} + h_s(t - v_G z)e_{\mu\nu}^{s}$$

two standard polarizations of GW arise from GR

the massive field arising from the generic high-order theory

Second case: massive spin-2 modes

We get 6 polarizations defined as the number of independent components of metric!!

$$e_{\mu\nu}^{(+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(\times)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(B)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(C)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(D)} = \frac{\sqrt{2}}{3}\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad e_{\mu\nu}^{(s)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

…. the amplitude in terms of the 6 polarization states is

$$h_{\mu\nu}(t,z) = A^{+}(t - v_{G_{s2}}z)e_{\mu\nu}^{(+)} + A^{\times}(t - v_{G_{s2}}z)e_{\mu\nu}^{(\times)} + B^{B}(t - v_{G_{s2}}z)e_{\mu\nu}^{(B)} + C^{C}(t - v_{G_{s2}}z)e_{\mu\nu}^{(C)} + D^{D}(t - v_{G_{s2}}z)e_{\mu\nu}^{(D)} + h_s(t - v_G z)e_{\mu\nu}^{s}.$$

the group velocity of the massive spin-2 field is given by

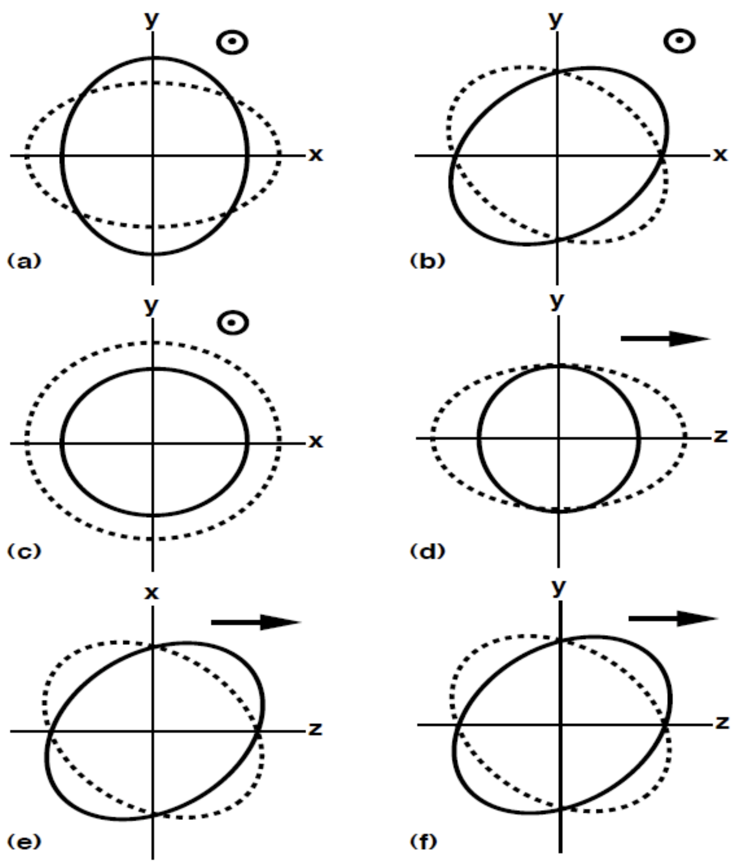
$$v_{G_{s2}} = \frac{\sqrt{\omega^2 - m_{s2}^2}}{\omega}$$

Polarization modes of GWs

Displacement induced by each mode on a sphere of test particles.

The wave propagates out of the plane in (a), (b), (c), and it propagates in the plane in (d), (e) and (f).

In (a) and (b) we have respectively the plus mode and cross mode, in (c) the scalar mode, in (d), (e) and (f) the D, B and C mode.



C. Bogdanos, S.Capozziello, M. De Laurentis, S. Nesseris, *Astrp.Phys*. 34 (2010) 236
S. Capozziello and A. Stabile, *Astr. Sp. Sc*. 358 (2015) 27

Ghost modes

The presence of ghost modes may seem a pathology
from a quantum-mechanical viewpoint

The ghost mode can be viewed as either a particle state of

positive energy and negative probability density

or

positive probability density state with a negative energy

Like gaps and electrons in semiconductivity!!

Ghost modes

- The presence of such quasi-particle induce violation of unitarity (however GR is not a unitary theory!).

- The negative energy states lead to an unbounded theory where there is no minimal energy and the system thus becomes unstable.

- The vacuum can decay into pairs of ordinary and ghost gravitons leading to a catastrophic instability

A way out

- Imposing a very weak coupling of ghosts with the rest of particles in the theory, the decay rate of the vacuum could become comparable to the inverse of the Hubble scale coming out of the horizon.

- The today observed vacuum state would become sufficiently stable (this is a straightforward explanation of cosmological Λ).

- This is a viable option if the ghost states couple differently than the standard massless gravitons to matter (exotic forms of matter could be constituted by massive gravitons). DM as gravitons!!!

Another way out

- Assuming that this picture does not hold up at arbitrarily high energies and that at some cutoff scale M_{cutoff} the theory gets modified appropriately as to ensure a ghost-free behavior and a stable ground state

- This can happen for example if we assume that Lorentz invariance is violated at M_{cutoff}, thereby restricting any potentially harmful decay rates (R. Emparan and J. Garriga, JHEP 0603 (2006) 028)

Possible answers at LHC (CERN) where gravitational massive states can be expected at $M_{cutoff} \approx 5 \div 10$ TeV
S. Capozziello, G. Basini, M. De Laurentis, EPJC 71(2011) 1679

Ghost modes

- The presence of massive ghost gravitons would induce on an interferometer the same effects as an ordinary massive graviton transmitting the perturbation, but with the opposite sign in the displacement.

- Tidal stretching from a polarized wave on the polarization plane will be turned into shrinking and viceversa.
- This signal will, at the end, be a superposition of the displacements coming from the ordinary massless spin-2 gravitons and the massive ghosts
- Since these induce two competing effects, this will lead to a less pronounced signal than the one we would expect if the ghost mode was absent, setting in this way less severe constraints on the theory

- The presence of the new modes will also affect the total energy density carried by the gravitational waves and this may also appear as a candidate signal in stochastic backgrounds, as we will see in the following

Detector response

Now one can study the detector response to each GW polarization without specifying, a priori, the theoretical model

…the angular pattern function of a detector to GWs is

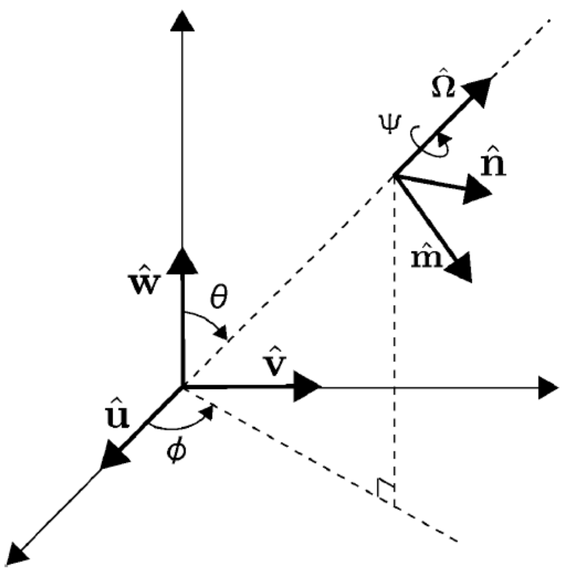
$$F_A(\hat{\Omega}) = \mathbf{D} : \mathbf{e}_A(\hat{\Omega}) \, ,$$
$$\mathbf{D} = \frac{1}{2}\left[\hat{\mathbf{u}} \otimes \hat{\mathbf{u}} - \hat{\mathbf{v}} \otimes \hat{\mathbf{v}}\right]$$

holds only when the arm length of the detector is smaller and smaller than the GW wavelength that we are taking into account

$A = +, \times, B, C, D, s$

detector tensor = response of a laser-interferometric detector

maps the metric perturbation in a signal on the detector



This is relevant for dealing with ground-based laser interferometers but this condition could not be valid when dealing with space interferometers like LISA

Detector response

the coordinate system for the GW, rotated by angles (θ,φ), is

$$\begin{cases} \hat{\mathbf{u}}' = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\ \hat{\mathbf{v}}' = (-\sin\phi, \cos\phi, 0) \\ \hat{\mathbf{w}}' = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \end{cases}$$

The rotation with respect to the angle ψ , around the GW propagating axis, gives the most general choice for the coordinate system

$$\begin{cases} \hat{\mathbf{m}} = \hat{\mathbf{u}}'\cos\psi + \hat{\mathbf{v}}'\sin\psi \\ \hat{\mathbf{n}} = -\hat{\mathbf{v}}'\sin\psi + \hat{\mathbf{u}}'\cos\psi \\ \hat{\mathbf{\Omega}} = \hat{\mathbf{w}}' \end{cases}$$

the polarization tensors are

$$\begin{aligned} \mathbf{e}_+ &= \frac{1}{\sqrt{2}}(\hat{\mathbf{m}}\otimes\hat{\mathbf{m}} - \hat{\mathbf{n}}\otimes\hat{\mathbf{n}}) \ , \\ \mathbf{e}_\times &= \frac{1}{\sqrt{2}}(\hat{\mathbf{m}}\otimes\hat{\mathbf{n}} + \hat{\mathbf{n}}\otimes\hat{\mathbf{m}}) \ , \\ \mathbf{e}_B &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{m}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{m}}\right) \ , \\ \mathbf{e}_C &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{n}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{n}}\right) \ . \\ \mathbf{e}_D &= \frac{\sqrt{3}}{2}\left(\frac{\hat{\mathbf{m}}}{2}\otimes\frac{\hat{\mathbf{m}}}{2} + \frac{\hat{\mathbf{n}}}{2}\otimes\frac{\hat{\mathbf{n}}}{2} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \\ \mathbf{e}_s &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \ , \end{aligned}$$

Detector response

the angular patterns for each polarization

$$F_+(\theta,\phi,\psi) = \frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi\ ,$$

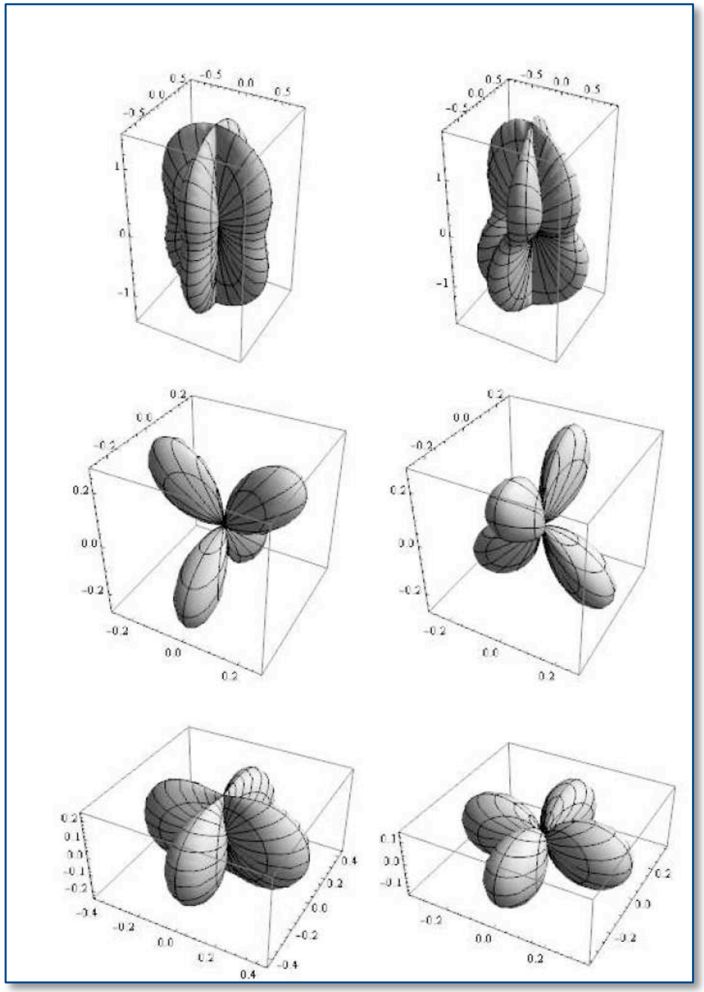
$$F_\times(\theta,\phi,\psi) = -\frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi\sin 2\psi - \cos\theta\sin 2\phi\cos 2\psi\ ,$$

$$F_B(\theta,\phi,\psi) = \sin\theta\,(\cos\theta\cos 2\phi\cos\psi - \sin 2\phi\sin\psi)\ ,$$

$$F_C(\theta,\phi,\psi) = \sin\theta\,(\cos\theta\cos 2\phi\sin\psi + \sin 2\phi\cos\psi)\ ,$$

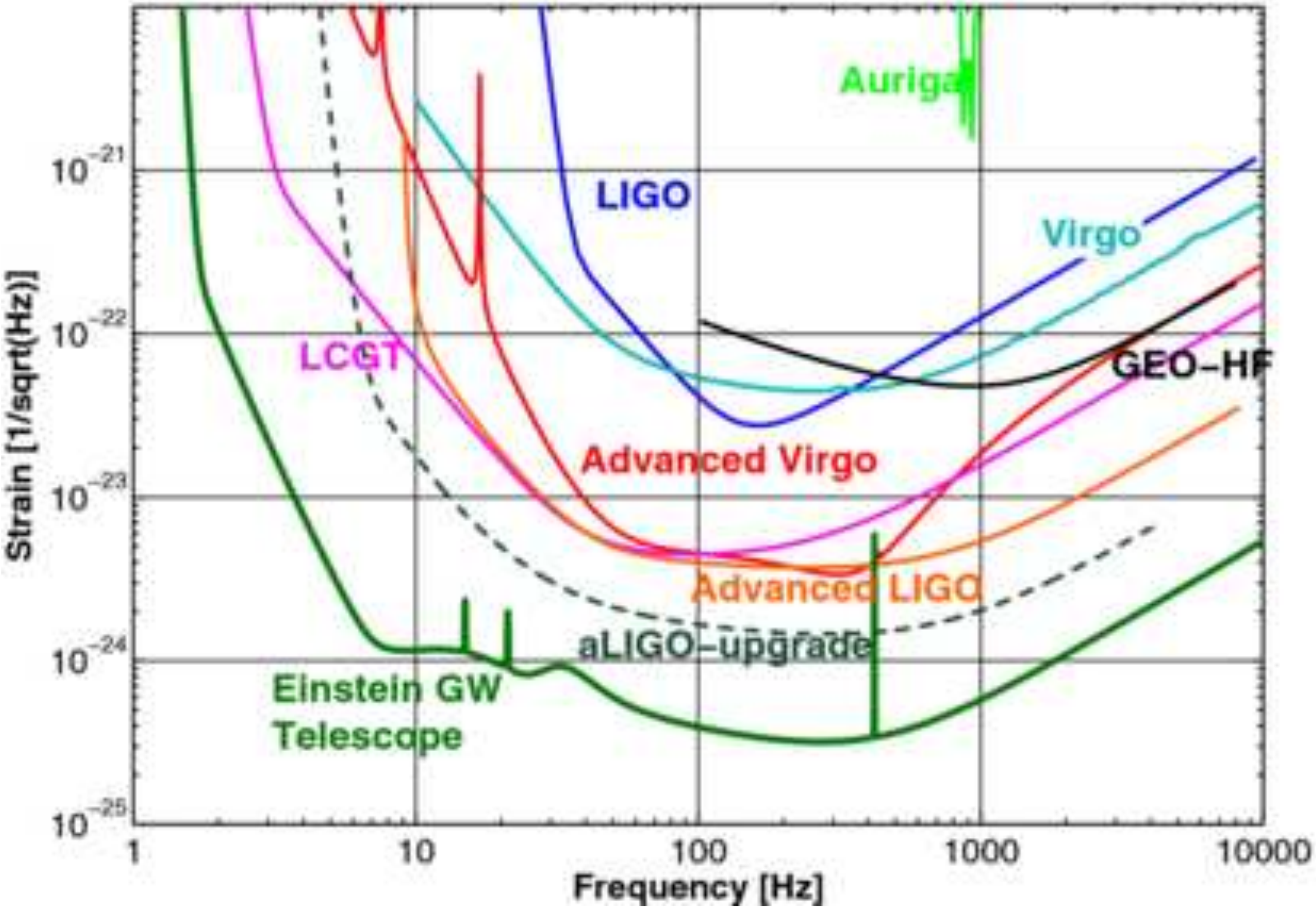
$$F_D(\theta,\phi) = \frac{\sqrt{3}}{32}\cos 2\phi\left(6\sin^2\theta + (\cos 2\theta + 3)\cos 2\psi\right)$$

$$F_s(\theta,\phi) = \frac{1}{\sqrt{2}}\sin^2\theta\cos 2\phi\ .$$



Detector response

Combining detectors is the right strategy to reveal further modes



The stochastic background of GWs

The contributions to the gravitational radiation coming from Extended Gravity could be efficiently selected by investigating gravitational sources in extremely strong field regimes.

The further polarizations coming from the higher order contributions could be, in principle, investigated by the response of combined GW detectors
K. Hayama and A. Nishizawa, PRD 87 (2013) 062003

Another approach is to investigate these further contributions from the cosmological background

The stochastic background of GWs

The stochastic background of GWs

GW background can be roughly divided into two main classes of phenomena:

- the background generated by the incoherent superposition of gravitational radiation emitted by large populations of astrophysical sources;

- the primordial GW background generated by processes in the early cosmological eras (see also BICEP2 and PLANCK results)

it can be described and characterized by a dimensionless spectrum

$$\Omega_{sgw}(f) = \frac{1}{\rho_c}\frac{d\rho_{sgw}}{d\ln f},$$

→ energy density of part of the gravitational radiation contained in the frequency range f to f + df.

$$\rho_c \equiv \frac{3H_0^2}{8\pi G}$$

→ H_0: today observed Hubble expansion rate

→ ρ_c: today critical energy density of the Universe

The stochastic background of GWs

The GW stochastic background energy density of all modes can be written as

$$\Omega_{\rm gw}^{A} \equiv \Omega_{\rm gw}^{+} + \Omega_{\rm gw}^{\times} + \Omega_{\rm gw}^{B} + \Omega_{\rm gw}^{C} + \Omega_{\rm gw}^{D} + \Omega_{\rm gw}^{s}$$

GR

$$\Omega_{\rm gw}^{GR} = \Omega_{\rm gw}^{+} + \Omega_{\rm gw}^{\times}$$

$$\Omega_{\rm gw}^{+} = \Omega_{\rm gw}^{\times}$$

Higher-order

$$\Omega_{\rm gw}^{HOG} = \Omega_{\rm gw}^{B} + \Omega_{\rm gw}^{C} + \Omega_{\rm gw}^{D}$$

$$\Omega_{\rm gw}^{B} = \Omega_{\rm gw}^{C} = \Omega_{\rm gw}^{D}$$

Scalar term

The stochastic background of GWs

The equation for the characteristic amplitude adapted to one of the components of the GWs can be used

$$h_A(f) \simeq 8.93 \times 10^{-19} \left(\frac{1Hz}{f}\right) \sqrt{h_{100}^2 \Omega_{gw}(f)}$$

and then we obtain for the GR modes

$$h_{GR}(100Hz) < 1.3 \times 10^{-23}$$

while for the higher-order modes

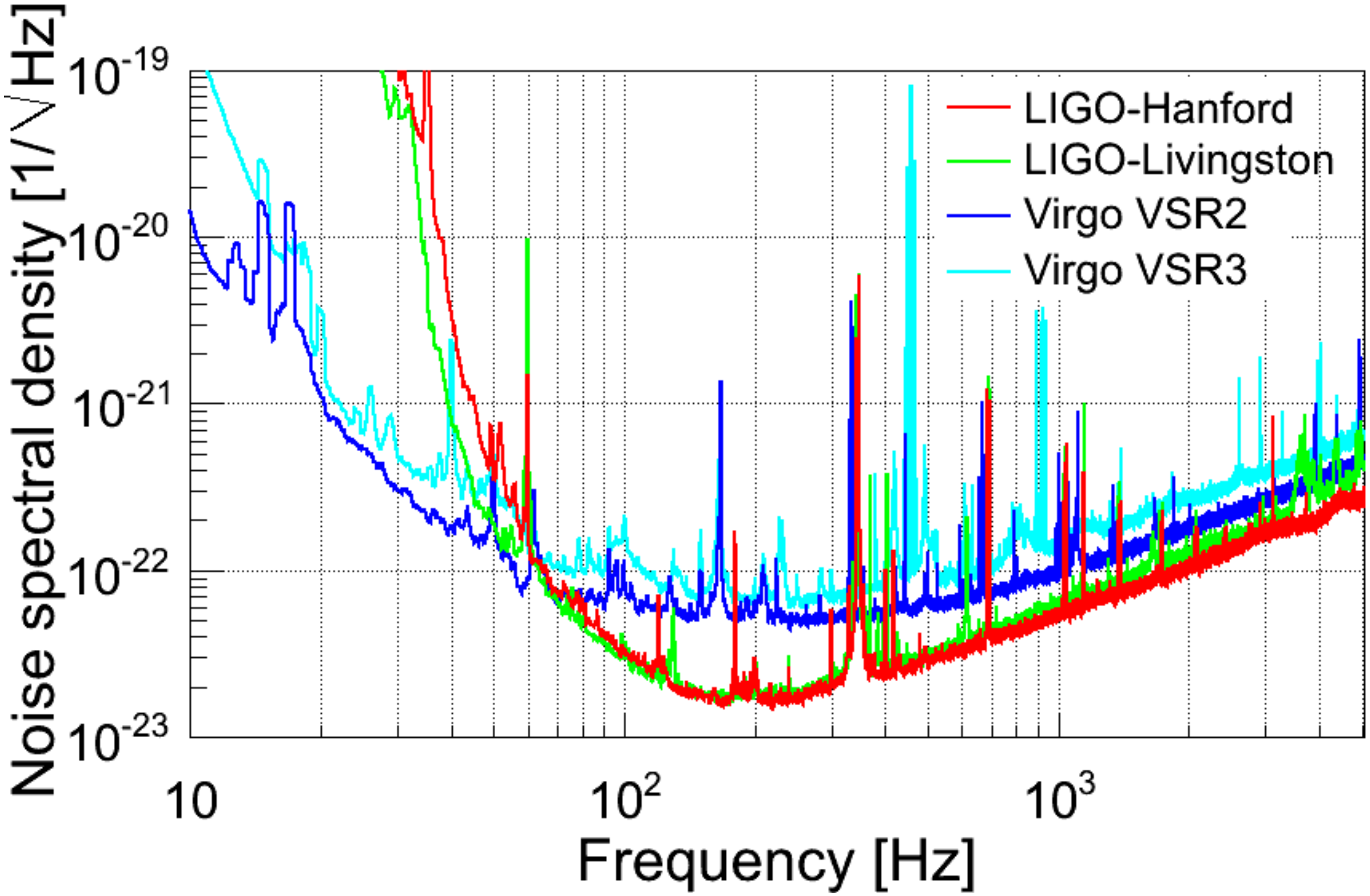
$$h_{HOG}(100Hz) < 7.3 \times 10^{-25}$$

and for scalar modes

$$h_s(100Hz) < 2 \times 1.410^{-26}$$

The stochastic background of GWs

Then, since we expect a sensitivity of the order of 10^{-22} for the above interferometers at ≈ 100Hz, we need to gain at least three orders of magnitude.



The stochastic background of GWs

Let us analyze the situation also at smaller frequencies.

The sensitivity of the VIRGO interferometer is of the order of 10^{-21} at ≈ 10Hz and in that case it is for the GR modes

$$h_{GR}(100Hz) < 1.3 \times 10^{-22}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-24}$$

and for scalar modes

$$h_s(100Hz) < 1.4 \times 10^{-25}$$

Still, these effects are below the sensitivity threshold to be observed today but new generation interferometers could be suitable (e.g. Advanced VIRGO-LIGO)

The stochastic background of GWs

The sensitivity of the LISA interferometer should be of the order of 10^{-22} at ≈ 10^{-3}Hz and in that case it is

$$h_{GR}(100Hz) < 1.3 \times 10^{-18}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-20}$$

and for scalar modes

$$h_s(100Hz) < 1.4 \times 10^{-21}$$

This means that a stochastic background of relic GWs could be, in principle, detected by the LISA interferometer, including the additional modes.

Conclusions and outlooks

- The above analysis covers the most extended gravity models with a generic class of Lagrangian density with higher-order terms as f(R,P,Q), where $P \equiv R_{ab}R^{ab}$ and $Q \equiv R_{abcd}R^{abcd}$

- Linearized field equations about Minkowski background gives, besides a massless spin-2 field (the GR graviton), also spin-0 and spin-2 massive modes. One gets 5+1=6 polarizatons

- The detectability of additional polarization modes could be possible, in principle, combining more than 2 interferometers. Possible signal in the GW stochastic background. New polarization modes, in general, can be used to constrain theories beyond GR.

Conclusions and outlooks

….a point has to be discussed in detail !!!

if the source is coherent → The interferometer is directionally sensitive and we also know the orientation of the source

The massive mode coming from the simplest extension, *f(R)* gravity, would induce longitudinal displacements along the direction of propagation which should be detectable and only the amplitude due to the scalar mode would be the true, detectable, "new" signal

we could have a second scalar mode inducing a similar effect, coming from the massive ghost, although with a minus sign.

Conclusions and outlooks

….another point !!!

if the source is not coherent (case of the stochastic background) and no directional detection of the gravitational radiation

The background has to be isotropic, the signal should be the same regardless of the orientation of the interferometer, no matter how the plane is rotated, it would always record the characteristic amplitude h_c.

There is no way to disentangle the modes in the background, being hc related to the total energy density of the gravitational radiation, which depends on the number of modes available

Every mode contributes in the same way, at least in the limit where the mass of massive and ghost modes are very small

It should be the number of available modes that makes the difference, not their origin.

Conclusions and outlooks

- The massive modes are certainly of interest for direct detection by the LIGO-VIRGO experiment

- Massive GW modes could be produced in more significant quantities in cosmological or early astrophysical processes.

- They could constitute the new frontier of physics also at LHC.

- GWs as natural DM candidates?