The Power Spectrum of Cosmic Ray Arrival Directions

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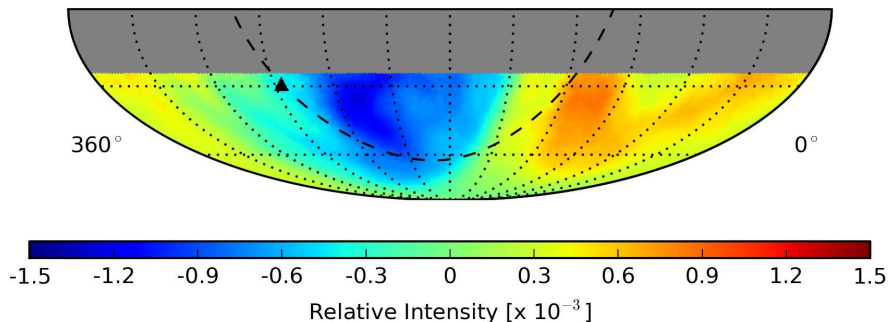
Vulcano, May 24, 2016

Map of CR Arrival Directions

→ Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies [Super-Kamiokande'07; Milagro'08; ARGO-YBJ'09,'13;EAS-TOP'09]
[Tibet AS-γ'05,'06,'15;IceCube'10,'11,'16; HAWC'13,'14]

Relative Intensity [x 10^{-3}]

$E_{CR} \simeq 10$ TeV, $N_{CR} \sim 3.2 \times 10^{11}$ [IceCube (IC59-IC86-IV)'16]
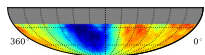
Map of CR Arrival Directions

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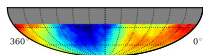[Tibet AS-γ'05,'06,'15;IceCube'10,'11,'16; HAWC'13,'14]

$360°$ $0°$

-16   -12   -8   -4   0   4   8   12   16

relative intensity [x 10^{-4}]
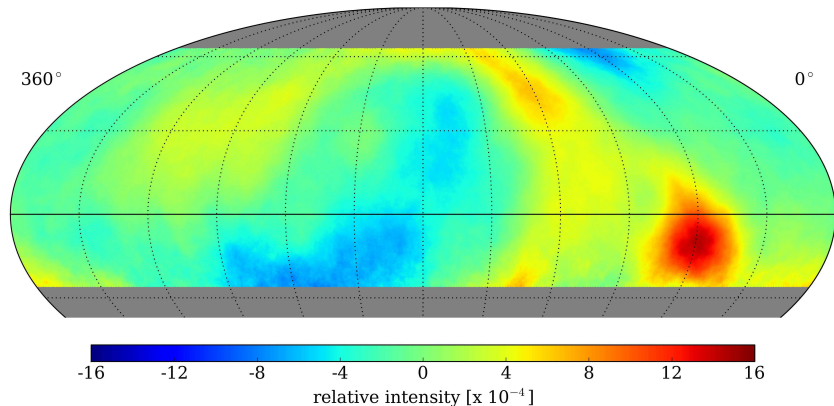
$E_{CR} \simeq 1$ TeV, $N_{CR} \sim 4.9 \times 10^{10}$ [HAWC'14]

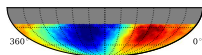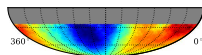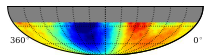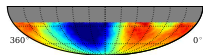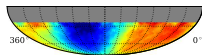# Time-Dependence of Anisotropy

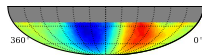

(a) Period 1

(b) Period 2

(c) Period 3

(d) Period 4

(e) Period 5

(f) Period 6

(g) Period 7

(h) Period 8

IceCube Preliminary

(i) Period 9

(l) Period 12

Relative Intensity [$\times 10^{-3}$]
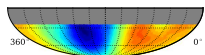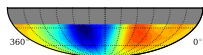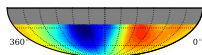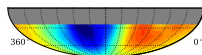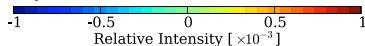
[AMANDA (7 yrs) & IceCube (5 yrs)'13]]

Expected CR Anisotropies

- turbulent motion of charged particles in Galactic environment described by **diffusion**

- diffusion tensor $\mathbf{K}$ (in general anisotropic):

$$K_{ij} = \frac{\hat{B}_i \hat{B}_j}{3\nu_\parallel} + \frac{\delta_{ij} - \hat{B}_i \hat{B}_j}{3\nu_\perp} + \frac{\epsilon_{ijk}\hat{B}_k}{3\nu_A}$$

- expected **dipole anisotropy** is small and depends on:

  - (local) source distribution [Erlykin & Wolfendale'06] [Blasi & Amato'12; Sveshnikova *et al.*'13; Pohl & Eichler'13]

  - (local) ordered magnetic field $\mathbf{B}$ [*e.g.* Schwadron *et al.*'14; Mertsch & Funk'14]

  - rigidity dependence of diffusion, $\mathbf{K} \propto \rho^{0.3-0.6}$

  - relative velocity of the medium [Compton & Getting'35]

  ✗ observational limitations

∇n_{CR}
density gradient

$-\mathbf{K}\nabla n_{\mathrm{CR}}$
dipole anisotropy

relative intensity

excess   deficit
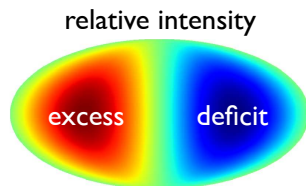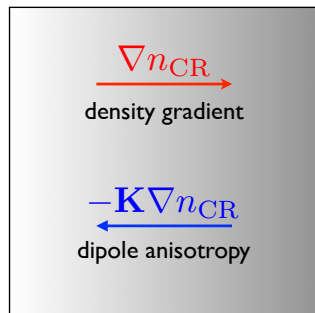
Observational Limitations
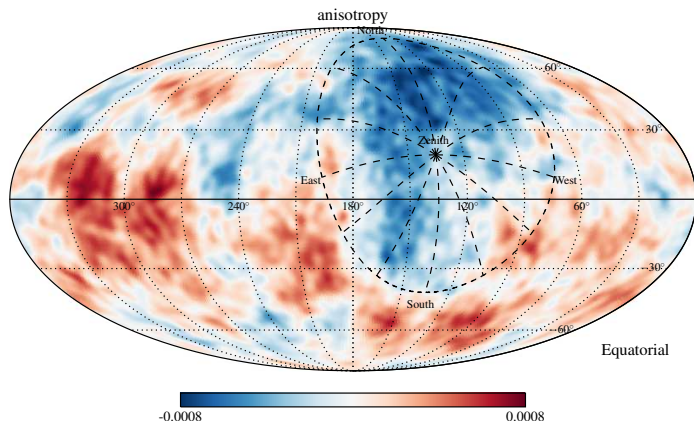
[MA *et al.*'16]

- ground-based detectors are calibrated by CR data ➜ **reduces anisotropies**
- true CR dipole defined by amplitude D, and orientation (RA,DEC) = (α, δ)
- ✘ observable only **projected dipole** with amplitude $D \cos \delta$ and orientation $(\alpha, 0)$
- ✘ further problems by limited field of view (cross-talk with small-scale structure)

Observed Dipole Amplitude and Phase

[MA'16]

Local Magnetic Field and Dipole Anisotropy

- strong ordered magnetic fields in the local environment

→ diffusion tensor reduces to projector:

$$K_{ij} \rightarrow \frac{\hat{B}_i \hat{B}_j}{3\nu_{\parallel}}$$

- TeV–PeV dipole data consistent with magnetic field direction inferred by IBEX data   [McComas *et al.*'09]

- 1–100 TeV phase indicates a local gradient within longitudes:

$$120^{\circ} \lesssim l \lesssim 300^{\circ}$$

→ phase flip induces by Vela supernova remnant?   [MA'16]

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[MA'16]

Suggested Origin of Small-Scale Anisotropy

- CR acceleration in nearby SNRs   [Salvati & Sacco'08]

- local magnetic field structure with energy-dependent magnetic mirror leakage
  [Drury & Aharonian'08]

- preferred CR transport directions   [Malkov, Diamond, Drury & Sagdeev'10]

- magnetic reconnections in the heliotail   [Lazarian & Desiati'10]

- non-isotropic particle transport in the heliosheath   [Desiati & Lazarian'11]

- heliospheric electric field structure   [Drury'13]

- magnetized outflow from old SNRs   [Biermann, Becker, Seo & Mandelartz'12]

- strangelet production in molecular clouds or neutron stars
  [Kotera, Perez-Garcia & Silk '13]

- interplay between regular and turbulent magnetic fields
  [Battaner, Castellano & Masip'14; Mertsch & Funk'14; Schwadron *et al.*'14; MA'16]

�→ small-scale anisotropies from local magnetic field mapping of a global dipole
  [Giacinti & Sigl'12; MA'14; MA & Mertsch'15]

Analogy to Gravitational Lensing

CMB temperature fluctuations

Cosmic Ray Gradient

Large Scale Structure

Local Magnetic Turbulence

small scale temperature fluctuations

small scale anisotropies
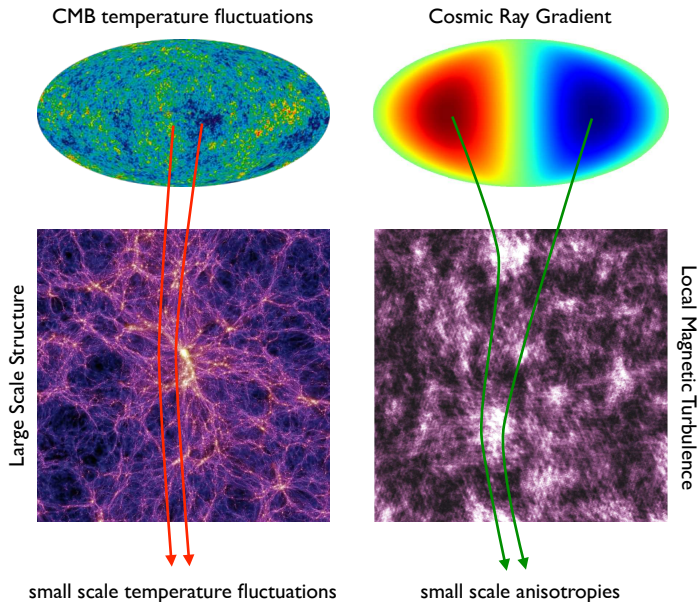
Gedankenexperiment

- **Idea:** local realization of magnetic turbulence introduces small-scale structure

  [Giacinti & Sigl'11]

- Particle transport in (static) magnetic fields is governed by Liouville's equation of the CR's phase-space distribution f:

$$\frac{\mathrm{d}}{\mathrm{d}t} f(t, \mathbf{x}, \mathbf{p}) = 0$$

- "trivial" solution:

$$f(0, \mathbf{0}, \mathbf{p}) = f(-T, \mathbf{x}(-T), \mathbf{p}(-T))$$

- *Gedankenexperiment*:
  Assume that at look-back time $-T$ initial condition is **homogenous, but not isotropic**:

$$f(0, \mathbf{0}, \mathbf{p}) = \widetilde{f}(\mathbf{p}(-T))$$

Angular Power Spectrum

- Every smooth function $g(\theta, \phi)$ on a sphere can be decomposed in terms of spherical harmonics Y_m^ℓ:

$$g(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta, \phi) \qquad \leftrightarrow \qquad a_{\ell m} = \int d\Omega (Y_\ell^m)^*(\theta, \phi) g(\theta, \phi)$$

- **angular power spectrum**:

$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

- related to the **two-point auto-correlation function**: $(\mathbf{n}_{1/2} : \text{unit vectors}, \mathbf{n}_1 \cdot \mathbf{n}_2 = \cos\eta)$

$$\xi(\eta) = \frac{1}{8\pi^2} \int d\mathbf{n}_1 \int d\mathbf{n}_2 \delta(\mathbf{n}_1\mathbf{n}_2 - \cos\eta) g(\mathbf{n}_1) g(\mathbf{n}_2) = \frac{1}{4\pi} \sum_\ell (2\ell+1) C_\ell P_\ell(\cos\eta)$$

→ Note that individual C_ℓ's are **independent** of coordinate system (assuming full sky coverage).

Gedankenexperiment (continued)

- Initial configuration has power spectrum $\widetilde{C}_\ell$.

- For small correlation angles η flow remains correlated even beyond scattering sphere.

- Correlation function for $\eta = 0$:

$$\xi(0) = \frac{1}{4\pi} \int d\hat{\mathbf{p}}_1 \widetilde{f}^2(\mathbf{p}_1(-T))$$
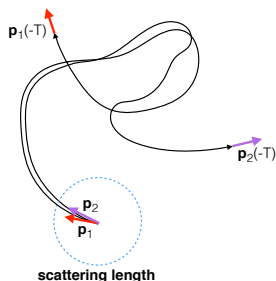
- On **average**, the rotation in an *isotropic* random rotation in the turbulent magnetic field leaves an isotropic distribution on a sphere **invariant**:

$$\langle \xi(0) \rangle = \frac{1}{4\pi} \int d\hat{\mathbf{p}}_1 \widetilde{f}^2(\mathbf{p}_1)$$

→ The weighted sum of $\langle C_\ell \rangle$'s remains constant:

$$\frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell+1)\widetilde{C}_\ell = \frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell+1) \langle C_\ell(T) \rangle$$

Evolution Model

- Diffusion theory motivates that each $\langle C_\ell \rangle$ decays exponentially with an effective **relaxation rate** [Yosida'49]

$$\nu_\ell \propto \mathbf{L}^2 \propto \ell(\ell+1)$$

- A **linear** $\langle C_\ell \rangle$ evolution equation with generation rates $\nu_{\ell \to \ell'}$ requires:

$$\partial_t \langle C_\ell \rangle = -\nu_\ell \langle C_\ell \rangle + \sum_{\ell' \geq 0} \nu_{\ell' \to \ell} \frac{2\ell'+1}{2\ell+1} \langle C_{\ell'} \rangle \quad \text{with} \quad \nu_\ell = \sum_{\ell' \geq 0} \nu_{\ell \to \ell'}$$

- For $\nu_\ell \simeq \nu_{\ell \to \ell+1}$ and $\widetilde{C}_\ell = 0$ for $l \geq 2$ this has the analytic solution:

$$\langle C_\ell \rangle(T) \simeq \frac{3\widetilde{C}_1}{2\ell+1} \prod_{m=1}^{\ell-1} \nu_m \sum_n \prod_{p=1(\neq n)}^{\ell} \frac{e^{-T\nu_n}}{\nu_p - \nu_n}$$

- For $\nu_\ell \simeq \ell(\ell+1)\nu$ we arrive at a finite asymptotic ratio:

$$\lim_{T \to \infty} \frac{\langle C_\ell \rangle(T)}{\langle C_1 \rangle(T)} \simeq \frac{18}{(2\ell+1)(\ell+2)(\ell+1)}$$

Comparison with CR Data

[MA'14; updated with HAWC data]
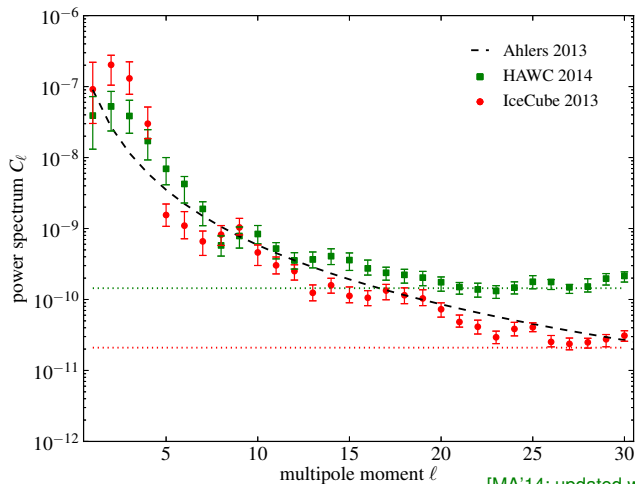
$$\lim_{T \to \infty} \frac{\langle C_\ell \rangle (T)}{\langle C_1 \rangle (T)} \simeq \frac{18}{(2\ell + 1)(\ell + 2)(\ell + 1)}$$

Simulation via Backtracking

- Consider a local (quasi-)stationary solution of the **diffusion approximation**:

$$4\pi \langle f \rangle \simeq n + \underbrace{\mathbf{r}\nabla n - 3\,\widehat{\mathbf{p}}\,\mathbf{K}\nabla n}_{\text{1st linear correction}}$$

- backtracking over long time-scales T and evaluating in mean field

$$4\pi f_i \simeq 4\pi \underbrace{\delta f(-T, \mathbf{r}_i(-T), \mathbf{p}_i(-T))}_{\text{deviation from } \langle f \rangle} + n + [\mathbf{r}_i(-T) - 3\hat{\mathbf{p}}_i(-T)\mathbf{K}]\nabla n$$

- Ensemble-averaged C_ℓ's ($\ell \geq 1$):   [MA & Mertsch'15]

$$\frac{\langle C_\ell \rangle}{4\pi} \simeq \int \frac{\mathrm{d}\hat{\mathbf{p}}_1}{4\pi} \int \frac{\mathrm{d}\hat{\mathbf{p}}_2}{4\pi} P_\ell(\hat{\mathbf{p}}_1\hat{\mathbf{p}}_2) \lim_{T \to \infty} \underbrace{\langle \mathbf{r}_{1i}(-T)\mathbf{r}_{2j}(-T) \rangle}_{\textit{relative} \text{ diffusion}} \frac{\partial_i n \partial_j n}{n^2}$$

- Sum of $\langle C_\ell \rangle$ **related to diffusion tensor**:

$$\frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell + 1)\langle C_\ell \rangle \simeq 2TK_{ij}^s \frac{\partial_i n \partial_j n}{n^2}$$

Simulation via Backtracking

- CR arrival direction determined by backtracking of CRs towards a **homogeneous initial dipole anisotropy** (ballistic ➜ laminar ➜ turbulent)
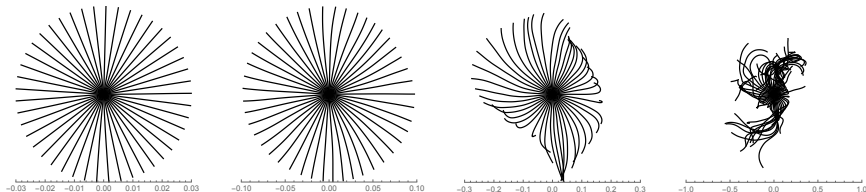
- Kolmogorov turbulence with energy density comparable to $\mathbf{B}_0$

- two cases: dipole vector aligned with or perpendicular to $\mathbf{B}_0$

- asymptotically **limited by simulation noise**:

$$\mathcal{N} \simeq \frac{4\pi}{N_{\text{pix}}} 2TK_{ij}^{s} \frac{\partial_i n \partial_j n}{n^2}$$

Comparison to CR Data

$\sigma^2 = 1$, $r_L/L_c = 0.1$, $\lambda_{\min}/L_c = 0.01$, $\lambda_{\max}/L_c = 100$, $\Omega T = 100$

Legend:
- model ($p = 2/3$)
- model ($p = 1/2$)
- model ($p = 1/3$)
- simulation ($\mathbf{B}_0 \parallel \nabla n$)
- simulation ($\mathbf{B}_0 \perp \nabla n$)
- IceCube (rescaled)
- HAWC (rescaled)

y-axis: relative power spectrum $\widehat{C}_\ell/\widehat{C}_1$

x-axis: multipole moment ℓ

HAWC noise level

IceCube noise level

Ahlers & Mertsch (2015)
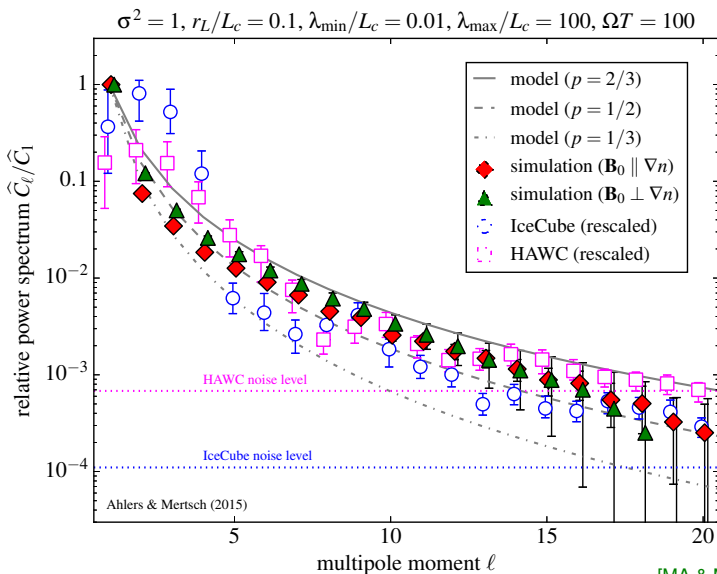
[MA & Mertsch'15]

Local Description: Relative Scattering

- evolution of C_ℓ's: [MA & Mertsch'15]

$$\partial_t \langle C_\ell \rangle = -\frac{1}{2\pi} \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (\mathbf{p}_1 \nabla f_1 + i\boldsymbol{\omega} \mathbf{L} f_1) f_2 \rangle$$

- large-scale dipole anisotropy gives an effective "source term":

$$-\frac{1}{2\pi} \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (\mathbf{p}_1 \nabla f_1) f_2 \rangle \to Q_1 \delta_{\ell 1}$$

- BGK-like Ansatz for scattering term ($\langle i\boldsymbol{\omega} \mathbf{L} f \rangle \to -\frac{\nu}{2} \mathbf{L}^2 \langle f \rangle$) [Bhatnagaer, Gross & Krook'54]

$$-\frac{1}{2\pi} \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle (i\boldsymbol{\omega} \mathbf{L} f_1) f_2 \rangle \to \frac{1}{2\pi} \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \tilde{\nu}(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \mathbf{L}^2 \langle f_1 f_2 \rangle$$

- Note that $\tilde{\nu}(1) = 0$ for vanishing regular magnetic field.

$$\tilde{\nu}(x) \simeq \nu_0 (1-x)^p$$

Summary

- Large- and small-scale anisotropy can be understood in the context of standard diffusion theory.

- Local magnetic turbulence is a source of small-scale anisotropies.

- **Analogous** to induced high-ℓ multipoles in CMB temperature power spectrum from **gravitational lensing** in small scale structure.

- Ensemble-averaged C_ℓ's ($\ell \geq 1$) related to relative diffusion:

$$\frac{\langle C_\ell \rangle}{4\pi} \simeq \int \frac{d\hat{\mathbf{p}}_1}{4\pi} \int \frac{d\hat{\mathbf{p}}_2}{4\pi} P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \lim_{T \to \infty} \langle \mathbf{r}_{1i}(-T) \mathbf{r}_{2j}(-T) \rangle \frac{\partial_i n \partial_j n}{n^2}$$

- Microscopic description of $\langle C_\ell \rangle$ evolution via modified BGK-like Ansatz.

- **Open issues:** damping effects for CR rigidity distributions, partial sky coverage of experiments, angular resolution, . . .

Thank you for your attention!

Appendix

Simulated Turbulence

- 3D-isotropic turbulence: [Giacalone & Jokipii'99]

$$\delta \mathbf{B}(\mathbf{x}) = \sum_{n=1}^{N} A(k_n)(\mathbf{a}_n \cos \alpha_n + \mathbf{b}_n \sin \alpha_n) \cos(\mathbf{k}_n \mathbf{x} + \beta_n)$$

- α_n and β_n are random phases in $[0, 2\pi)$, unit vectors $\mathbf{a}_n \propto \mathbf{k}_n \times \mathbf{e}_z$ and $\mathbf{b}_n \propto \mathbf{k}_n \times \mathbf{a}_n$
- with amplitude

$$A^2(k_n) = \frac{2\sigma^2 B_0^2 G(k_n)}{\sum_{n=1}^{N} G(k_n)} \qquad \text{with} \qquad G(k_n) = 4\pi k_n^2 \frac{k_n \Delta \ln k}{1 + (k_n L_c)^\gamma}$$

- Kolmogorov-type turbulence: $\gamma = 11/3$
- $N = 160$ wavevectors $\mathbf{k}_n$ with $|\mathbf{k}_n| = k_{\min} e^{(n-1)\Delta \ln k}$ and $\Delta \ln k = \ln(k_{\max}/k_{\min})/N$
- $\lambda_{\min} = 0.01 L_c$ and $\lambda_{\max} = 100 L_c$ [Fraschetti & Giacalone'12]
- rigidity: $r_L = 0.1 L_c$
- turbulence level: $\sigma^2 = \mathbf{B}_0^2/\langle \delta \mathbf{B}^2 \rangle = 1$

Powerspectrum of CR Arrival Directions

➜ Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies

[Tibet AS-γ'05,'06; Super-Kamiokande'07; Milagro'08; ARGO-YBJ'09,'13;EAS-TOP'09]

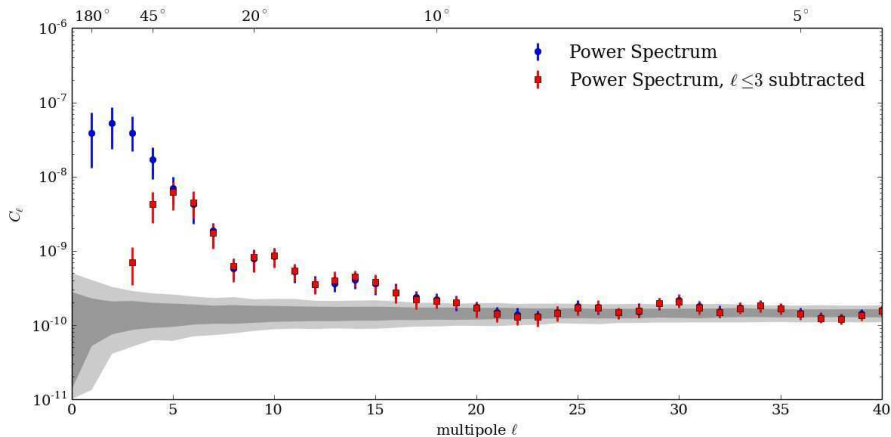[IceCube'10,'11; HAWC'13,'14]
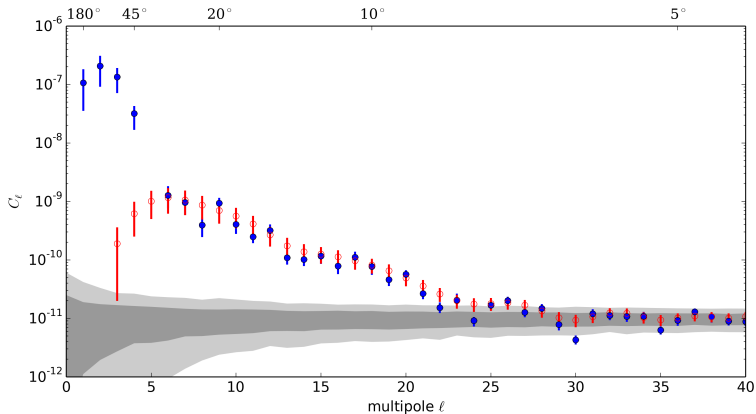
[IceCube [arXiv:1603.01227]]

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[IceCube'10,'11; HAWC'13,'14]

[HAWC [arXiv:1408.4805]; note: low$-\ell$ power under-estimated]