HOLOGRAPHIC THEORY OF GRAVITY AND COSMOLOGY

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References:

Ng, Intl. J. Mod. Phys. D 22, 1342022 (2013).

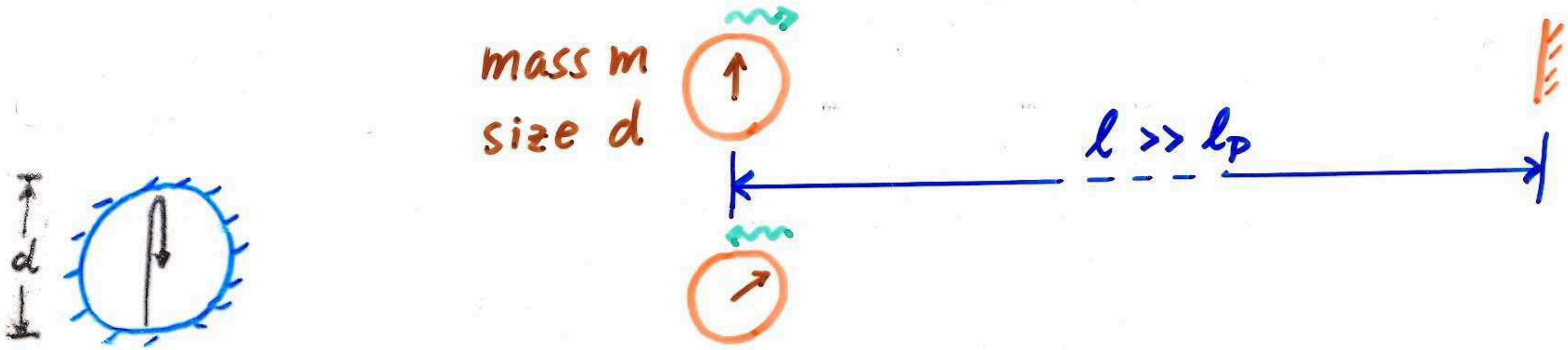
Ho, Minic and Ng, Phys. Lett. B693, 567 (2010); Phys. Rev. D85, 104033 (2012).

Edmonds, Farrah, Ho, Minic, Ng, and Takeuchi, ApJ 793, 41 (2014);
arXiv:1601.00662.

& references contained therein

PREAMBLE

In Vulcano 2004, in a talk titled "Space-time fluctuations," I discussed some aspects of "space-time foam" – a foamy structure of spacetime arising from quantum fluctuations. To examine how large the fluctuations are, I considered a gedankan experiment in which a light signal is sent from a clock to a mirror (at a distance l away) and back to the clock in a timing experiment to measure l:



From the jiggling of the clock's position alone, the Heisenberg uncertainty principle yields $\delta l^2 \gtrsim \frac{\hbar l}{mc}$, where m is the mass of the clock. On the other hand, the clock must be large enough not to collapse into a black hole; this requires $\delta l \gtrsim \frac{Gm}{c^2}$. We conclude that the fluctuations of a distance l scales as $\delta l \gtrsim l^{1/3} l_P^{2/3}$ (where $l_P \sim 10^{-33}$cm is the Planck length).

I further showed that this scaling of δl is exactly what the holographic principle demands, according to which the maximum amount of information stored in a region of space scales as the area of its two-dimensional surface, like a hologram. (Heuristically, this comes about because a cube with side l contains $\sim l^2/l_P^2$ number of small cubes with side δl.)

It will be useful to rederive this scaling of δl by another method which can be generalized to the case of an expanding universe...

Note: In a recent paper "New Constraints on Quantum Gravity from X-ray and Gamma-Ray Observations" by Perlman, Rappaport, Christiansen, Ng, DeVore, and D. Pooley (ApJ. 805, 10 (2015)), it was claimed that detections of quasars at GeV energies with *Fermi*,and at TeV energies with ground-based Cherenkov telescopes seem to have ruled out the holographic spacetime foam model (with δl scaling as $l^{1/3}l_P^{2/3}$). But this conclusion is conceivably premature when correct averaging is carried out.

CONTENTS

From spacetime foam to cosmological constant Λ

From Λ to modified dark matter (MDM)

Observational tests of MDM

Summary and future work

Quanta of the dark sector obey infinite statistics? [short discussion if time permits]

Units (most of the time): $c = 1, \hbar = 1, k_B = 1$

From spacetime foam to cosmological constant Λ

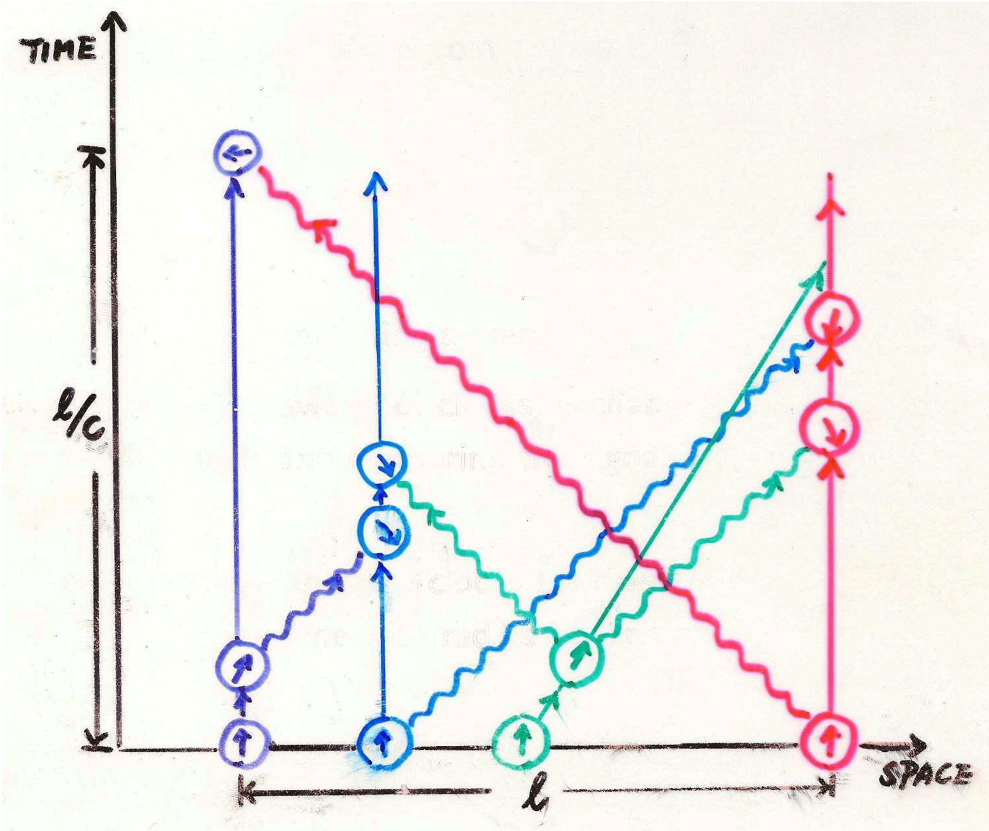
- Space-time Foam via the Mapping the geometry of spacetime
- Effective cosmological constant Λ

Space-time Foam via Mapping the geometry of spacetime

[Lloyd & Ng]

Use a global positioning system: Fill space with a swarm of clocks, exchanging signals with the other clocks and measuring the signals' time of arrival

How accurately can these clocks (of mass M) map out a volume of space-time with radius l over time l/c it takes light to cross the volume



Ticks & clicks of clocks in spacetime volume l^4/c

The process of mapping the geometry of spacetime is a kind of computational operation

• Margolus-Levitin theorem (rate of operations $\leq E/\hbar$; essentially energy-time Heisenberg uncertainty)

$$\Rightarrow \# \text{ operations} < (E/\hbar) \times \text{ time} = \frac{Mc^2}{\hbar}\frac{l}{c}$$

• To prevent black-hole formation $\Rightarrow M < \frac{lc^2}{G}$

$$\Downarrow$$

$$\# \text{ ops or events (i.e., } \# \text{ spacetime ``cells'')} < l^2 \frac{c^3}{\hbar G} = \frac{l^2}{l_P^2}$$

For max. spatial resolution, each clock ticks only ONCE

$\Rightarrow$ Each "cell" occupies spatial vol. $\frac{l^3}{l^2/l_P^2} = l l_P^2$

$\Rightarrow$ Average spatial separation of "cells" $\simeq l^{1/3} l_P^{2/3}$

- Consistent with the holographic principle
- Interpretation: $\delta l \gtrsim l^{1/3} l_P^{2/3}$, holographic spacetime foam model

Maximum spatial resolution

- requires max. energy density $\rho \sim \frac{3}{8\pi}(l l_P)^{-2}$
- yields # of bits $\sim l^2/l_P^2$

STF $\Rightarrow$ HOLOGRAPHIC FOAM COSMOLOGY

Let us generalize the above discussion for a static spacetime region with low spatial curvature to the case of an expanding universe by the substitution of l by $1/H$ (with H being the Hubble parameter).

2 main features: (H, R_H = Hubble parameter, radius)

- critical cosmic energy $\rho = \frac{3}{8\pi}\left(\frac{H}{l_P}\right)^2 \sim (R_H l_P)^{-2}$
- Universe contains $I \sim (R_H/l_P)^2$ bits of info

Average energy carried by each bit/"particle" is $\rho R_H^3/I \sim R_H^{-1}$

- Dark energy acts like a dynamical cosmological constant $\Lambda \sim 3H^2$

From Λ to modified dark matter (MDM)

- Review of Verlinde's entropic gravity (for $\Lambda = 0$)
- Constructing MDM via entropic gravity (for $\Lambda \neq 0$)

Constructing MDM via (Verlinde's) entropic gravity*

Verlinde's "recipe":

Verlinde derives

(I) Newton's 2nd law $\vec{F} = m\vec{a}$, by using

(1) First law of thermodynamics $\Rightarrow$ entropic force $F_{entropic} = T\frac{\Delta S}{\Delta x}$,

and invoking Bekenstein's original arguments concerning the entropy S of black holes: $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$.

(2) The formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer.

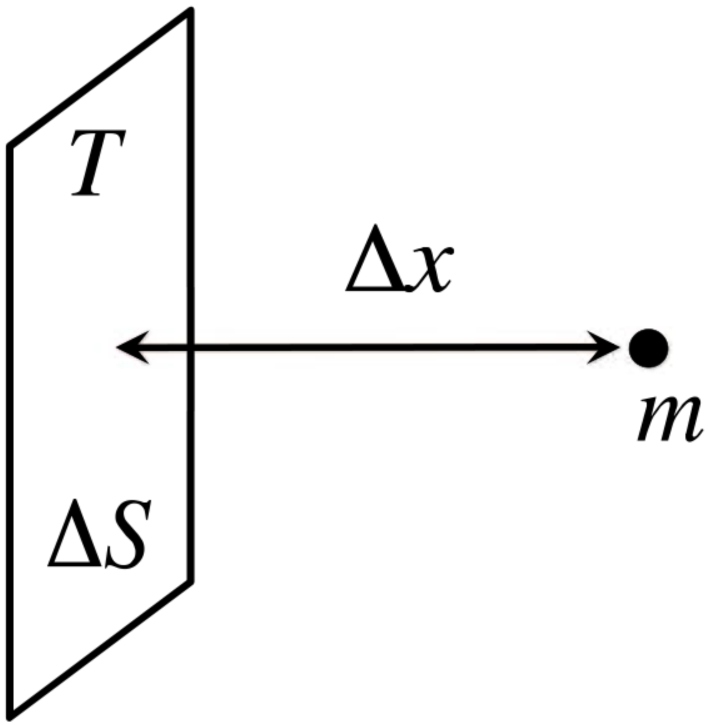
(II) Newton's law of gravity $a = GM/r^2$ by considering an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T, and using

(1) Equipartition of energy $E = \frac{1}{2}Nk_B T$ with $N = Ac^3/(G\hbar)$ being the total number of degrees of freedom (bits) on the screen;

(2) The Unruh temperature formula and the fact that $E = Mc^2$.

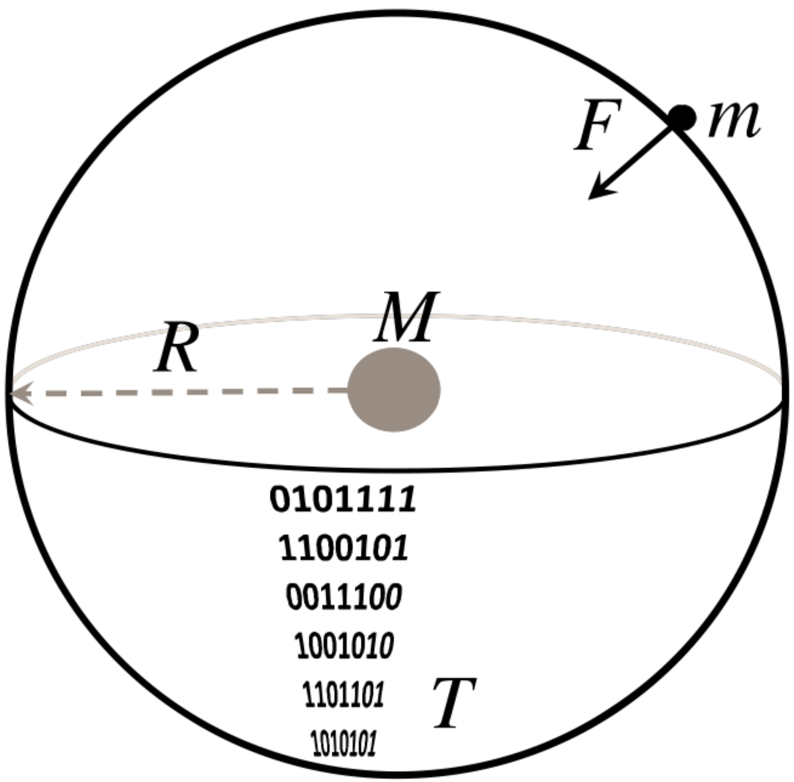
*which happens to provide a convenient framework for its construction

A particle with mass approaches a part of the holographic screen



A particle with mass approaches a part of the holographic screen.

A particle with mass m near a spherical holographic screen



A particle with mass m near a spherical holographic screen.

Constructing MDM

Need a generalization of Verlinde's proposal to de Sitter (dS) space with $+$ve cosmological constant Λ in an accelerating universe.

Note: Unruh-Hawking temperature, as measured by an inertial observer, is $T_{dS} = \frac{1}{2\pi k_B} a_0$ where $a_0 = \sqrt{\frac{\Lambda}{3}} \sim H$ (numerically).

Net temperature as measured by the non-inertial observer (due to some matter sources that cause the acceleration a) is
$\tilde{T} \equiv T_{dS+a} - T_{dS} = \frac{1}{2\pi k_B}[\sqrt{a^2 + a_0^2} - a_0]$.

(I) Verlinde's approach $\Rightarrow$ the entropic force in de Sitter space is

$$F_{entropic} = \tilde{T}\,\nabla_x S = m[\sqrt{a^2 + a_0^2} - a_0].$$

For $a \gg a_0$, we have $F_{entropic} \approx ma$.
For $a \ll a_0$: $F_{entropic} \approx m\frac{a^2}{2\,a_0}$, so the terminal velocity v of the test mass m should be determined from $ma^2/(2a_0) = mv^2/r$.

For the small acceleration $a \ll a_0$ regime: The observed flat galactic rotation curves (v is independent of r) and the observed Tully-Fisher relation ($v^4 \propto M$) now require (recall $a_N = GM/r^2$) that $a \approx \left(2\,a_N\,a_0^3\,/\pi\right)^{\frac{1}{4}}$.

But that means

$$F_{entropic} \approx m\frac{a^2}{2\,a_0} = F_{Milgrom} \approx m\sqrt{a_N a_c}\,.$$

We have recovered MoND — provided we identify $a_0 \approx 2\pi a_c$, with the (observed) critical galactic acceleration $a_c \sim \sqrt{\Lambda/3} \sim H \sim 10^{-8} cm/s^2$. Thus from our perspective, MoND is a phenomenological consequence of quantum gravity.

(II) For an imaginary holographic screen of radius r, Verlinde's argument $\Rightarrow$

$$2\pi k_B \tilde{T} = 2\pi k_B \left(\frac{2\tilde{E}}{N k_B} \right) = 4\pi \left(\frac{\tilde{M}}{A/G} \right) = \frac{G\,\tilde{M}}{r^2},$$

where $\tilde{M}$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$.

$\tilde{M} = M + M'$ where M' is some unknown mass, i.e., dark matter; consistency $\Rightarrow$

$$M' = \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M.$$

$$\Rightarrow F_{entropic} = m[\sqrt{a^2 + a_0^2} - a_0] = m\,a_N \left[1 + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 \right]$$

For $a \gg a_0$, $F_{entropic} \approx ma \approx ma_N$, and hence $a = a_N$. ($M' \approx 0$)

For $a \ll a_0$, $F_{entropic} \approx m\frac{a^2}{2\,a_0} \approx ma_N(1/\pi)(a_0/a)^2$, yielding $a = \left(2\,a_N\,a_0^3/\pi\right)^{\frac{1}{4}}$, as required. ($M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$)

DARK MATTER MASS DENSITY PROFILE ($\rho\prime(r)$)

Consider an ordinary (visible) matter source of radius r_0 with total mass $M(r_0)$:

$$M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r \Rightarrow \rho\prime(r) = \frac{M^{1/2}(r_0)(\sqrt{\Lambda}/G)^{1/2}}{r^2}.$$

This result can be compared with the distribution associated with an isothermal Newtonian sphere in hydrostatic equilibrium (used by some dark matter proponents):

$$\rho(r) = \frac{\sigma}{r^2 + r_0^2}.$$

Asymptotically the two expressions agree with σ identified as $M^{1/2}(r_0)(\sqrt{\Lambda}/G)^{1/2}$.
A phenomenological check.

Observational tests of MDM

- In galaxies
- In clusters
- Strong lensing

Fitting rotation curves with MDM mass profiles

Modified Dark Matter:

$$F_{entropic} = m[\sqrt{a^2 + a_0^2} - a_0] = m\, a_N \left[1 + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right]$$

To determine rotation curves:

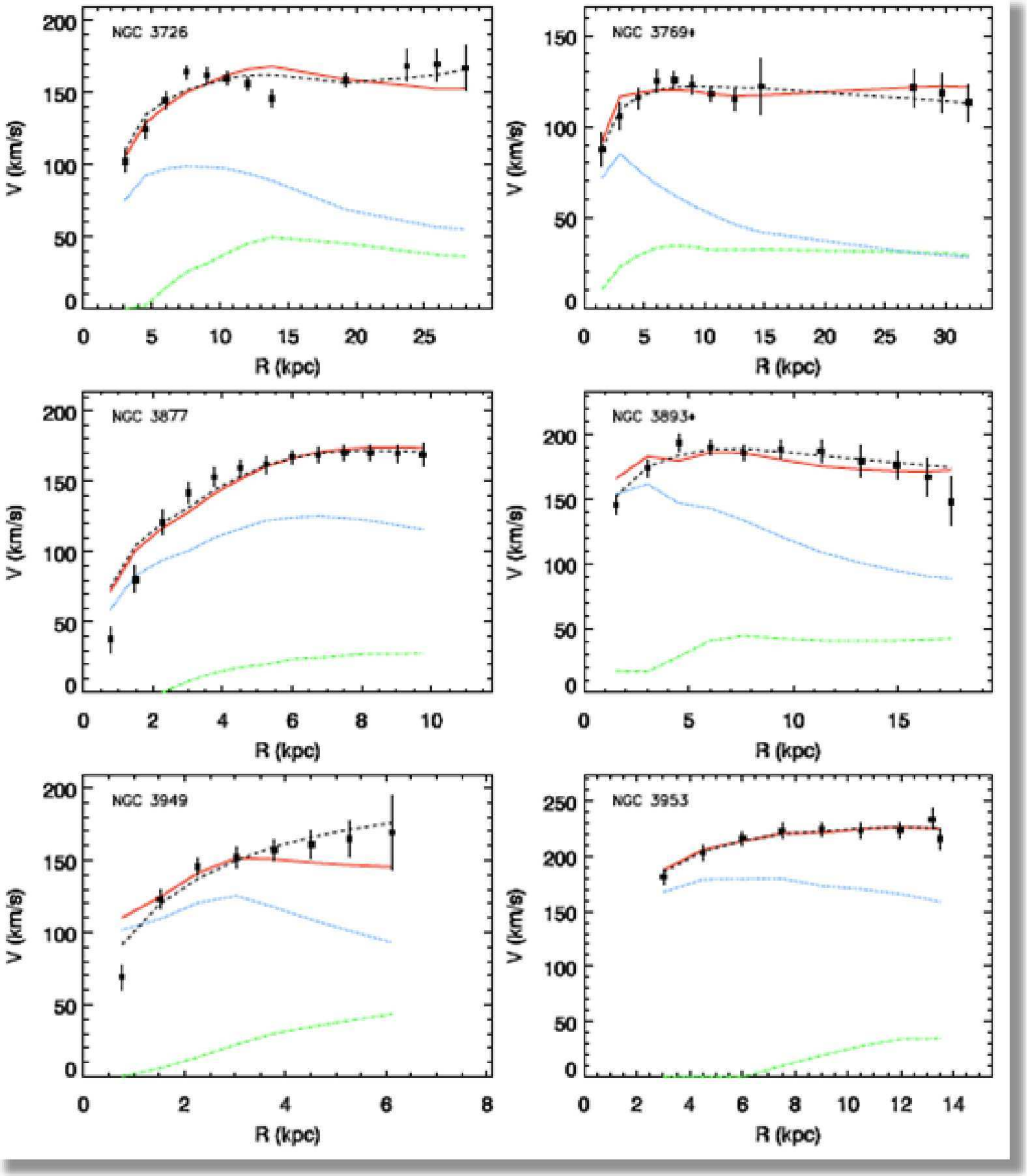
$$F_{entropic} = mv^2/r$$

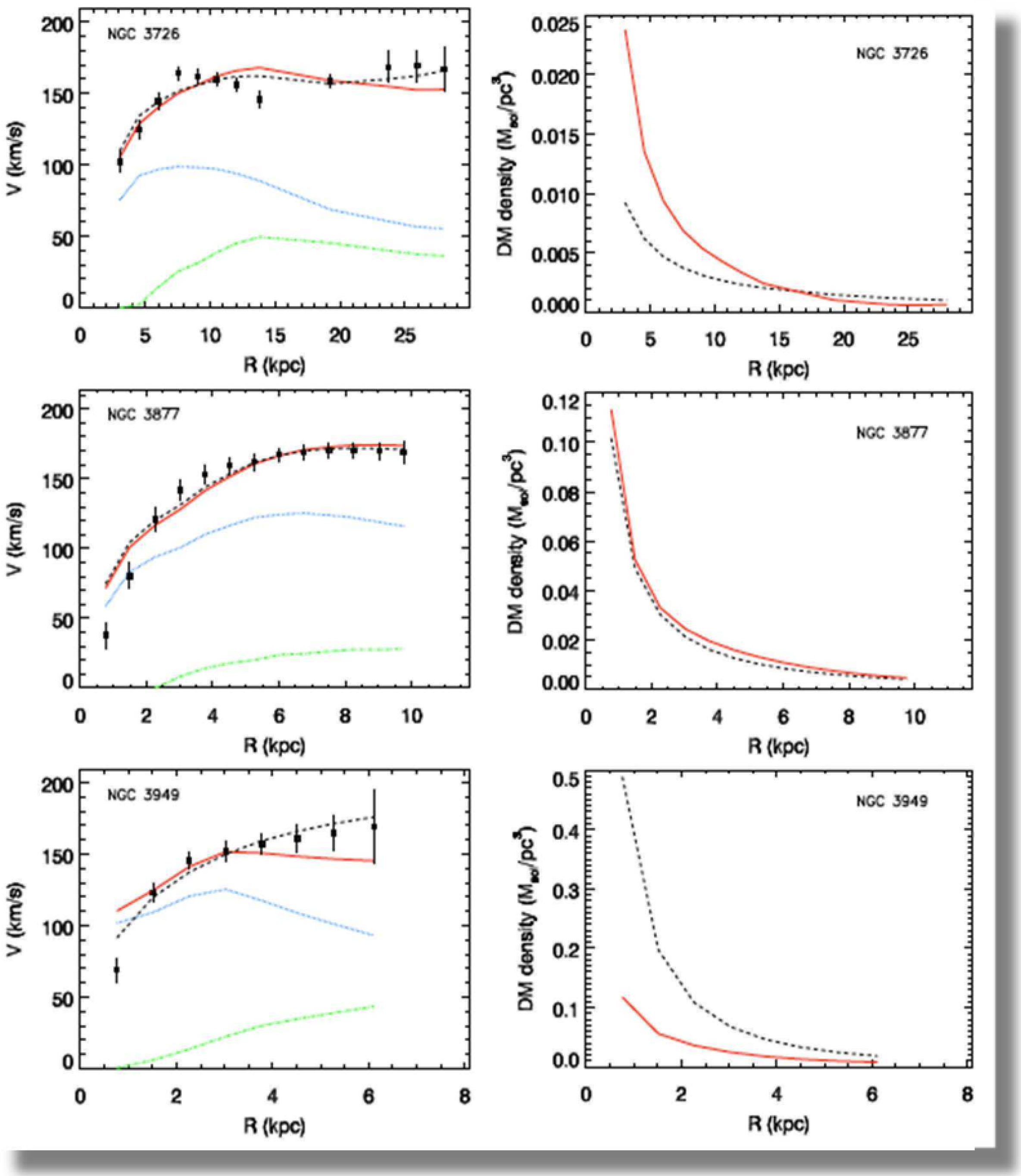
We fit rotation curves for 30 local spiral (HSB as well as LSB) galaxies.

Next 4 slides: Samples of rotation curves and dark matter density profiles.

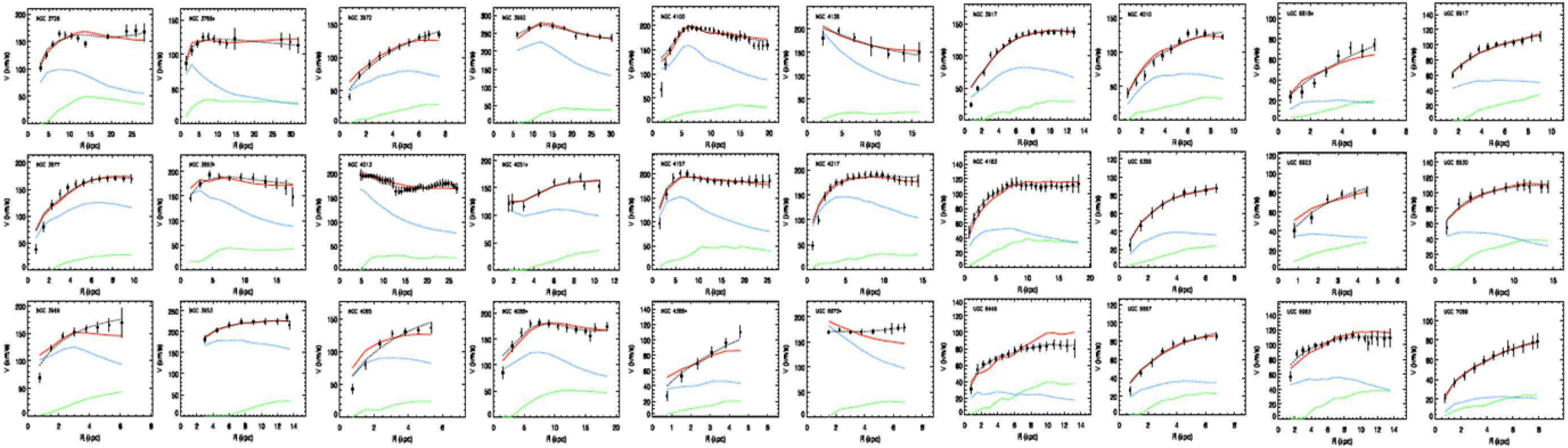
Data - black squares; Stars - blue line; Gas - green line. [Sanders & Verheijen]
MDM - red line; CDM - black dashed line (using NFW profile).

Fitting parameters: MDM (1): mass-to-light ratio M/L; CDM (3): c, v_{200}, M/L.
MDM uses the minimum number of parameters: hence the name Minimal DM.


NGC 3726
NGC 3769
NGC 3877
NGC 3893
NGC 3949
NGC 3953
V (km/s)
R (kpc)



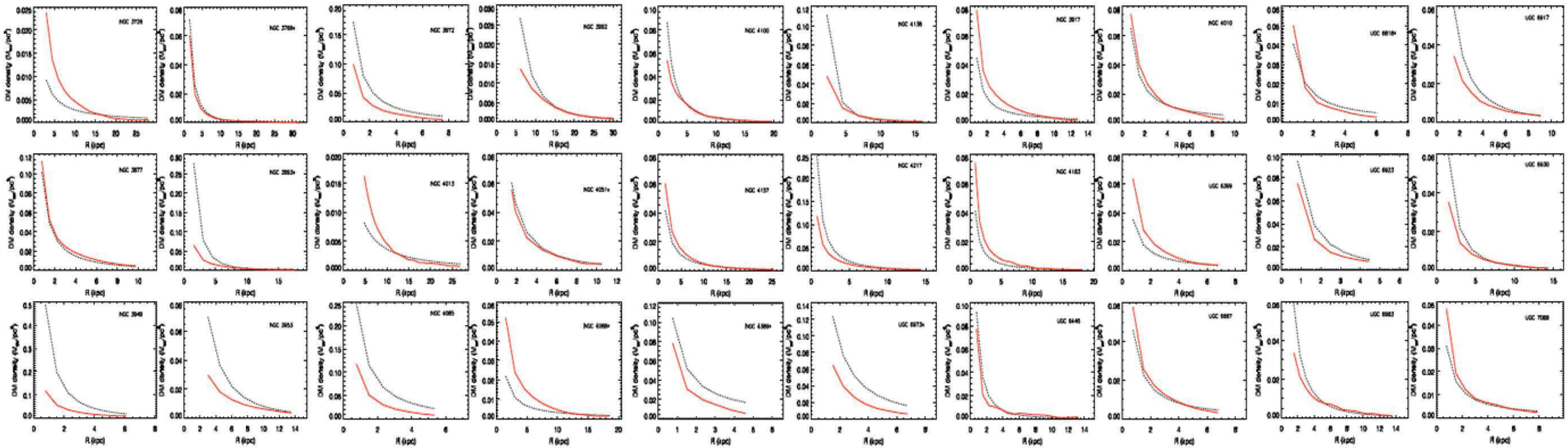
NGC 3726
NGC 3877
NGC 3949
V (km/s)
R (kpc)
DM density ($M_{\odot}/pc^3$)



We fit rotation curves for 30 local spiral galaxies, providing the first astrophysical test of MDM

IT PASSED!

Dark matter density profiles for 30 local spiral galaxies (HSB/LSB)



While the Modified Dark Matter profile given by $M' = \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 M$ reproduces the correct force laws (to the leading order) in both regimes of $a \gg a_0$ and $a \ll a_0$, we expect a more generic profile of the form

$$M' = \left[\lambda \left(\frac{a_0}{a} \right) + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right] M \,,$$

with $\lambda > 0$ (and of order 1) which ensures that $M' > 0$ when $a \gg a_0$.

(One can easily check that this more generic expression for M' will also lead to the correct predictions for force laws in both regimes of $a \gg a_0$ and $a \ll a_0$; for the former regime, $a \approx a_N + (1 + \lambda)\, a_0$.)

As a function of r, the dark matter profile now reads (for $a \gg a_0$):

$$M' \approx \left[\lambda \left(\frac{a_0}{a_N} \right) + \left(\frac{1}{\pi} - \lambda\,(1 + \lambda) \right) \left(\frac{a_0}{a_N} \right)^2 \right] M \,.$$

In principle, this mass profile can be checked ...

MDM IN CLUSTERS

The total gravitating mass in Newtonian, MOND, and MDM dynamics:

Newtonian dynamical mass: In Newtonian dynamics, the mass enclosed within radius r_{out} may be estimated: $M_N = -\frac{r_{out}}{G}\left(\frac{kT}{\mu m_p}\right)\left(\frac{d\ln\rho}{d\ln r}\right)$, where $\rho = \rho_0\left[1+\frac{r^2}{r_c^2}\right]^{-1.5\beta}$, with observed values $\beta \approx 0.65; r_c \approx 0.25$ Mpc; μ the mean molecular weight (0.62); m_p the mass of the proton.

$M_N \approx 4.4\, M_{obs} \Rightarrow$ the old missing mass problem.

MOND dynamical mass: $M_{MOND} = \frac{M_N}{\sqrt{1+(a_c/a)^2}}$

(Here we use $\mu(x) = x\left(1+x^2\right)^{1/2}$ for the interpolating formula.)

MDM dynamical mass: $M_{MDM} = \frac{M_N}{1+\lambda(a_0/a)+(a_0/a)^2/\pi}$ (The observed (effective) acceleration is given by $a_{obs} = \sqrt{a^2+a_0^2} - a_0$. With the aid of the more general expression for the MDM profile, we have $a_{obs} = \frac{GM_{MDM}}{r^2}\{1+\lambda\left(\frac{a_0}{a}\right)+\frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\}$. We also recall that $a_{obs} = GM_N/r^2$ for Newtonian dyanmics.)

Galactic Clusters: the sample

White, Jones & Forman (1997, MNRAS 292) tabulated observed temperatures and mass estimates of the hot gas for 207 clusters from X-ray data collected by the *Einstein satellite*.

Mass of stars is estimated using the rough correlation found by David et al. (1990, ApJ, 356). $M_{gas}/M_{stars} \approx 0.5 T_{keV} h_{50}^{-1.5}$.

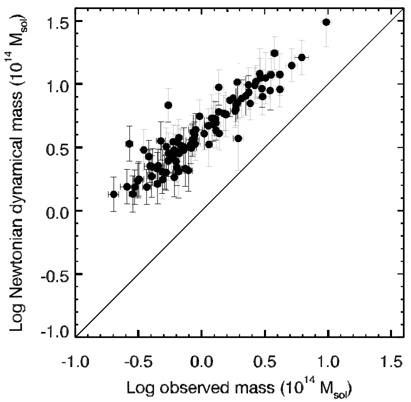
David's correlation and beta-models are imprecise for clusters with small outer radius. We therefore consider only clusters with outer radius ≥ 0.75 Mpc.

We are left with 93 clusters.

We have adapted Sanders' approach (for MOND) to the case of MDM (to compare MOND with MDM, formerly known as MONDian dark matter). (Note: Preliminary results. Work in progress.)

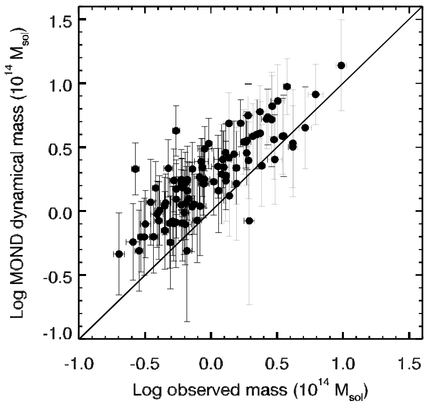
Galactic Clusters: data fits

Newtonian dynamics



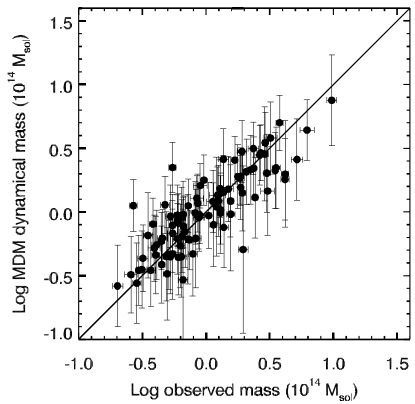
$$\frac{M_{Newton}}{M_{obs}} \approx 4.4$$

MoNDian dynamics



$$\frac{M_{MoND}}{M_{obs}} \approx 2.1$$

MoNDian dark matter



$$\frac{M_{MDM}}{M_{obs}} \approx 1.0$$

DE, Farrah, Ho, Minic, Ng & Takeuchi, 2015 [in preparation];
Sanders 1999, ApJ 512

A comparison of mass profiles

Proposed form of MDM mass:

$$M' = \left[\lambda \left(\frac{a_0}{a} \right) + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right] M \, ,$$

In 2014 ApJ paper, we chose $\lambda = 0$, which gave us good fits to galactic rotation curves.

Fits to the cluster data suggests $\lambda \approx 1/2$

So we re-fit the galactic rotation curves with $\lambda = 1/2$. The fits are nearly identical. The fitting parameter M/L is reduced by about 15%, and the values are still physically reasonable.

A comment on strong lensing

Strong lensing: the formation of multiple images of background sources by the central regions of some clusters.

Critical surface density required for strong lensing is $\Sigma_c = \frac{1}{4\pi}\frac{cH_0}{G}F(z_l, z_s)$, with $F \approx 10$, typical observations.

Deep MOND limit: $\Sigma_{MOND} \approx a_c/G$

Numerical values: $a_c \approx cH_0/6$

So, as noted by Sanders, MOND cannot produce strong lensing on its own: $\Sigma_c \approx 5\Sigma_{MOND}$

But MDM mass distribution appears to be sufficient for strong lensing: $a_0 = cH_0 = 2\pi a_c \approx 6a_c$

SUMMARY

- A spatial region of size l can contain no more than $\sim l^3/(ll_P^2) = (l/l_P)^2$ cells that are allowed by the holographic principle.
- Applied to cosmology, the cosmic energy density has the critical value $\rho \sim (H/l_P)^2$ and the universe of Hubble size R_H contains $(R_H/l_P)^2$ bits/particles of information.
- Long-wave constituents of dark energy act as a dynamical cosmological constant with the observed small but non-zero value $\Lambda \sim 3H^2$.
- By generalizing entropic gravity to de Sitter space, and accounting for Milgrom's scaling, we are led to a new form of dark matter.
- Modified dark matter (MDM) behaves like MOND at galactic scales but like CDM at cluster and cosmic scales.
- We fit rotation curves for 30 local spiral galaxies, it PASSES!
- We also test MDM at cluster scales, and again it fares well. (work in progress)
- Preliminary work on strong gravitational lensing is promising.
- No time to discuss: Speculation: "particles" constituting DM obey ∞ statistics.

FUTURE WORK:

1. Gravitational lensing; Can it distinguish MDM from CDM?

2. Study interactions of MDM (quanta obeying infinite statistics) with ordinary (baryonic) matter $\Rightarrow$ particle phenomenology. The Bullet Cluster; How strongly coupled is MDM to baryonic matter? How does MDM self-interact?

3. Tests at cosmological scales (acoustic oscillations measured in the CMB...); Simulations of structure formation?

4. Stars made of quanta obeying infinite statistics?

5. Can quantum gravity be actually the origin of particle statistics and the underlying statistics is infinite statistics in that ordinary particles obeying Bose or Fermi statistics are actually some sort of collective degrees of freedom? (What are the effects on the idea of grand unification?)

STAY TUNED FOR FUTURE DEVELOPMENTS

Back-up slides:

Wigner-Salecker thought experiment

holographic foam model

"derivation" of holographic principle

heuristic counting: partitioning l^3 into $(\delta l)^3$ cubes

a disclaimer

(short discussion of) cosmology with MDM

infinite statistics for quanta of dark energy and modified dark matter

properties of infinite statistics

NFW density profile

more detailed discussion of cosmology

more detailed discussion of gravitational Born-Infeld theory and infinite statistics for quanta of MDM

QUANTUM FLUCTUATIONS OF SPACETIME

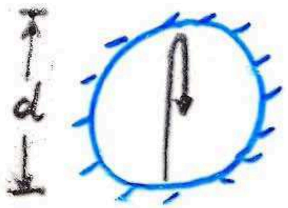
Distance measurements: [Ng & van Dam; Karolyhazy]

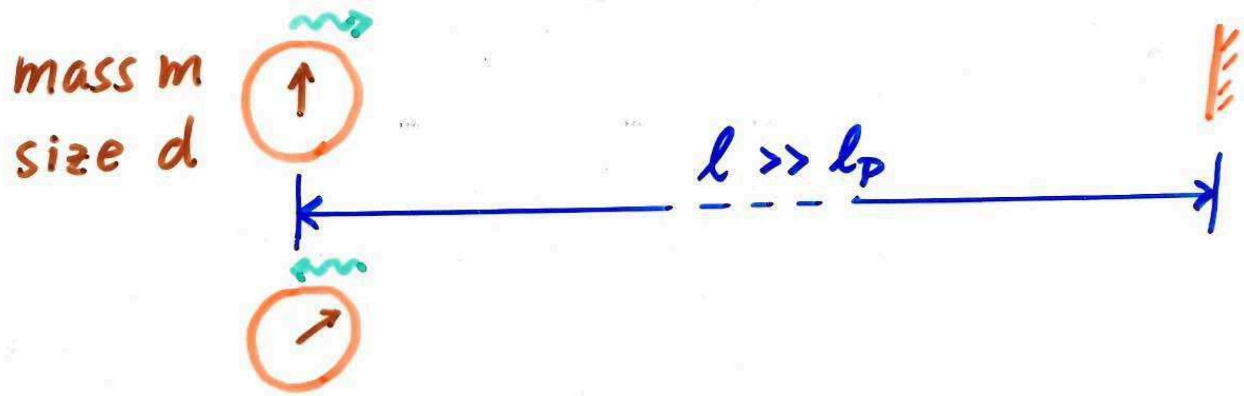
$\delta l =$ accuracy with which distance l can be measured

$\sim$ uncertainty/ fluctuation in l

Method 1: gedankan experiment

[a crude argument]





Quantum mechanics: [following Salecker & Wigner]

$$\delta l \left(\frac{2l}{c} \right) = \delta l + \frac{2l}{c} \frac{1}{m} \frac{\hbar}{2\delta l} \Rightarrow \delta l^2 \gtrsim \frac{\hbar l}{mc}$$

Quantum mechanics:

$$\delta l \left(\frac{2l}{c}\right) = \delta l + \frac{2l}{c}\frac{1}{m}\frac{\hbar}{2\delta l} \Rightarrow \delta l^2 \gtrsim \frac{\hbar l}{mc}$$

General relativity:

$$\delta l \gtrsim d; \qquad d \gtrsim \frac{Gm}{c^2} \Rightarrow \delta l \gtrsim \frac{Gm}{c^2}$$

QM + GR:

$$\delta l \gtrsim l^{1/3} l_P^{2/3} >> l_P$$

Interpretation: Spacetime undergoes quantum fluctuations

Holographic principle (h.p.)

The Universe which we perceive to have 3 spatial dimensions can be encoded on a 2-dimensional surface, like a hologram.

Max. amount of information in region is bounded by area

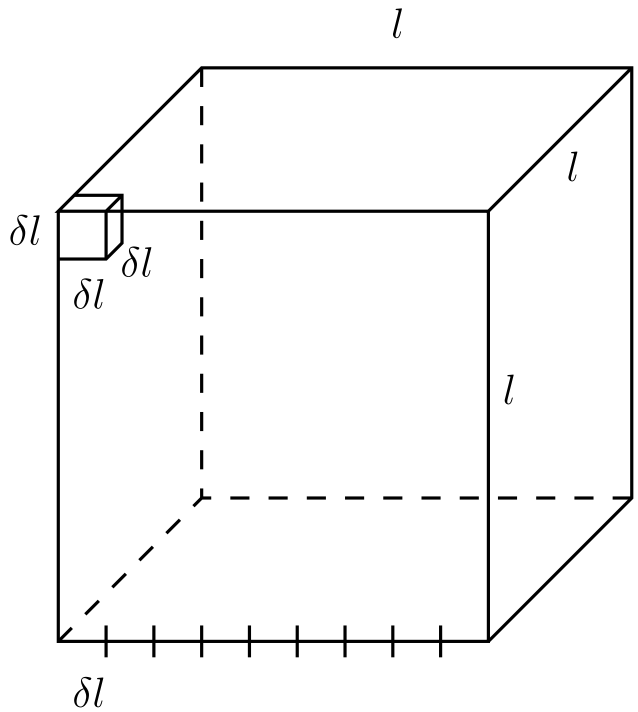
I.e., $\#$ of degrees of freedom $\lesssim$ area in l_P^2 ['tHooft; Susskind]

Origin of h.p.: Black holes are hot: Entropy $\sim \left(\frac{r_s}{l_P}\right)^2 \propto$ area [Bekenstein; Hawking]

A heuristic "derivation" of Holographic Principle

In essence, the holographic principle ('tHooft, Susskind ...) stipulates that although the world around us appears to have three spatial dimensions, its contents can actually be encoded on a two-dimensional surface, like a hologram. In other words, the maximum entropy of a region of space is given (aside from multiplicative factors of order 1) by its surface area in Planck units. This result can be derived by appealing to black hole physics and the second law of theromodynamics as follows. Consider a system with entropy S_0 inside a spherical region Γ bounded by surface area A. Its mass must be less than that of a black hole with horizon area A (otherwise it would have collapsed into a black hole). Now imagine a spherically symmetric shell of matter collapsing onto the original system with just the right amount of energy so that together with the original mass, it forms a black hole which just fills the region Γ. The black hole so formed has entropy $S \sim A/l_P^2$. But according to the second law of thermodynamics, $S_0 \leq S$. It follows immediately that $S_0 \lesssim A/l_P^2$, and hence the maximum entropy of a region of space is bounded by its surface area, as asserted by the holographic principle.

Consider partitioning l^3 into cubes [average size $= (\delta l)^3$] as small as physical laws allow, so intuitively one degree of freedom is associated with each small cube.



No. of degrees of freedom inside $l^3 = \#$ small cubes
$= \left(\frac{l}{\delta l}\right)^3 \lesssim \frac{l^2}{l_P^2}$ by h.p. $\Longrightarrow \delta l \gtrsim l^{1/3} l_P^{2/3}$

Spacetime foam model corresponding to $\delta l \gtrsim l^{1/3} l_P^{2/3}$ is now known as the holographic model of STF. [Ng & van Dam]

A Cautionary Disclaimer (speaking on my own behalf only)

In a recent paper "New Constraints on Quantum Gravity from X-ray and Gamma-Ray Observations" by Perlman, Rappaport, Christiansen, Ng, DeVore, and D. Pooley (ApJ. 805, 10 (2015)), it was claimed:

We reassess previous proposals to use astronomical observations of distant quasars and AGN to test models of spacetime foam. We show explicitly how wavefront distortions on small scales cause the image intensity to decay to the point where distant objects become undetectable when the path-length fluctuations become comparable to the wavelength of the radiation. (C)onstraints can be set utilizing detections of quasars at GeV energies with *Fermi*, and at TeV energies with ground-based Cherenkov telescopes: (they seem to rule out the holographic spacetime foam model.)

There are, however, a number of caveats to this idea for constraining models of spacetime foam. E.g.,

In this work the authors have considered the instantaneous fluctuations in the distance between the location of the emission and a given point on the telescope aperture. Perhaps one should average over both the huge number of Planck timescales during the time it takes light to propagate through the telescope system, and over the equally large number of Planck squares across the detector aperture. It is then possible that the fluctuations the authors have been calculating are vanishingly small, but at the moment there is no formalism for carrying out such averages.

- Cosmology: Friedmann's Equations

In a fully relativistic situation, we should use the active gravitational (Tolman-Komar) mass, i.e., replace a non-relativistic source of gravity by a fully relativistic source

⇒ Friedmann's Equations:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3},$$

and

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

I.e., One can in principle have Einstein's gravity together with a(n additional) Modified Dark Matter source.

If we naively use MoND at the cluster scale, we would be missing the pressure and cosmological constant terms which could be significant. This may explain why MoND doesn't work well at the cluster scale, despite the CDM-MoND duality realized at the galactic scale.

Quanta of the dark sector obey infinite statistics?

- Infinite statistics
- Quanta of MDM

A (LOGICAL) SPECULATION

Assume DE is composed of long wavelength "particles"

How different are these "particles"?

Consider $N \sim (R_H/l_P)^2$ such "particles" obeying Boltzmann statistics in volume $V \sim R_H^3$ at $T \sim R_H^{-1}$

The partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N \Rightarrow$ Entropy of the system is $S = N[ln(V/N\lambda^3) + 5/2]$ with $\lambda \sim T^{-1}$

But $V \sim \lambda^3$, so S becomes negative unless $N \sim 1$ which is equally nonsensical

Solution: The N inside the log in S, i.e, the Gibbs factor $(N!)^{-1}$ in Z_N, must be absent $\Rightarrow$ the N "particles" are distinguishable!

Then $S = N[ln(V/\lambda^3) + 3/2]$, $+$ ve $S \sim N$

The only known consistent statistics in greater than 2 space dimensions without the Gibbs factor is the quantum Boltzmann statistics, aka infinite statistics

A logical speculation: The “particles” constituting dark energy obey infinite statistics, rather than the familiar Fermi or Bose statistics. This is the overrideing difference between DE and conventional matter.

In the framework of M-theory, V. Jejjala, M. Kavic and D. Minic [hep-th:0705.4581] have made a similar suggestion

INFINITE STATISTICS

[Doplicher, Haag, & Roberts; Govorkov; Greenberg; ...]

• q-deformation of the Heisenberg algebra ($-1 \leq q \leq 1$)

$$a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{kl}$$

($q = \pm 1$ corresponds to bosons/fermions)

• Take $q = 0 \Rightarrow a_k a_l^\dagger = \delta_{kl}$

• Any 2 states obtained by acting on $|0>$ with creation operators in different order are orthogonal to each other

$$< 0|a_{i1} ... a_{iN} a_{jN}^\dagger ... a_{j1}^\dagger |0 >= \delta_{i1,j1} ... \delta_{iN,jN}$$

implying that particles obeying inf. stat. are virtually distinguishable

• The partition function is

$$Z = \Sigma e^{-\beta H}, \text{ NO Gibbs factor}$$

In infinite statistics, all representations of the particle permutation group can occur.

Theories of particles obeying ∞ statistics are non-local

[Fredenhagen; Greenberg]

Number operator

$$n_i = a_i^\dagger a_i + \sum_k a_k^\dagger a_i^\dagger a_i a_k + \sum_l \sum_k a_l^\dagger a_k^\dagger a_i^\dagger a_i a_k a_l + ...,$$

and Hamiltonian, etc., are both nonlocal and nonpolynomial in the field operators

• TCP theorem and cluster decomposition still hold

QFTs with ∞ statistics remain unitary

Nonlocality in ∞ statistics can be a virtue

• may be related to nonlocality in holographic principle

- **Modified Dark Matter via Gravitational Born-Infeld Theory**

A particularly useful reformulation of MDM is via an effective gravitational dielectric medium, motivated by the analogy between Coulomb's law in a dielectric medium and Milgrom's law for MoND. [E.g., write Milgrom's μ as $1 + \chi$ with χ being interpretted as "gravitational susceptibility".]

$\Rightarrow$ MoNDian force law is recovered if the quanta of MDM obey the so-called infinite statistics (as described by the Cuntz algebra (a curious average of the bosonic and fermionic algebras) $a_i \, a_j^\dagger = \delta_{ij}$.)

Note: Theories of particles obeying ∞ statistics are non-local [Fredenhagen; Greenberg]

(The fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields.)

Can expect unusual dynamics and interactions with ordinary matter (?)

Perhaps this explains the difficulty in detecting dark matter.

For the CDM fits, we use the NFW density profile:

$$\rho'(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}.$$

Here $r_s = \frac{r_{200}}{c}$ which designates the 'edge' of the halo, within which objects are assumed to be virialized, usually taken to be the boundary at which the halo density exceeds 200 times that of the background. The parameter c is a dimensionless number that indicates how centrally concentrated the halo is.

The velocity curves are then determined by

$$v(r) = v_{200} \sqrt{\frac{\ln(1 + cx) - cx/(1 + cx)}{x\left[\ln(1 + c) - c/(1 + c)\right]}},$$

where v_{200} is the Newtonian velocity at r_{200}. This equation is fit to the data with c, v_{200}, and M/L as free parameters.

Cosmology: Friedmann's Equations.

The FRW metric: $ds^2 = -dt^2 + R(t)(dr^2 + r^2\, d\Omega^2)$, where $R(t)$ is the scale factor.

Assume that the matter sources in the universe form a perfect fluid, with four velocity $u_\mu (= (1, \vec{0})$ in rest frame of fluid).

Consider Verlinde's imaginary holographic screen of comoving radius r (with physical radius $\tilde{r} = rR(t)$).

In a fully relativistic situation, we replace $\tilde{M}$ by the active gravitational (Tolman-Komar) mass $\mathcal{M} = \frac{1}{4\pi G} \int dV\, R_{\mu\nu} u^\mu u^\nu$, and by Einstein's field equation, $\mathcal{M} = 2 \int dV\, \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \frac{\Lambda}{8\pi G} g_{\mu\nu} \right) u^\mu u^\nu = \left(\frac{4}{3}\pi r^3 R^3 \right) \left[(\rho + 3p) - \frac{\Lambda}{4\pi G} \right]$

$\Rightarrow$ Friedmann's Equations:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3},$$

and

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

• Thus one can in principle have Einstein's gravity together with a(n additional) MoNDian dark matter source.

The departure from MoND happens when (we replace $\tilde{M}$ with $\mathcal{M}$, i.e. when) a non-relativistic source is replaced by a fully relativistic source. In that case $\sqrt{a^2 + a_0^2} - a_0 = \frac{G\,\mathcal{M}}{\tilde{r}^2}$, where $\tilde{r} = rR(t)$ is the physical radius, i.e.,

$$\sqrt{a^2 + a_0^2} - a_0 = \frac{G\,(\,M(t) + M'(t)\,)}{\tilde{r}^2} + 4\pi G\,p\,\tilde{r} - \frac{\Lambda}{3}\,\tilde{r}.$$

If we naively use MoND at the cluster scale, we would be missing $4\pi G\,p\,\tilde{r} - \frac{\Lambda}{3}\,\tilde{r}$ which could be significant. This may explain why MoND doesn't work well at the cluster scale, despite the CDM-MoND duality realized at the galactic scale.

Modified Dark Matter via Gravitational Born-Infeld Theory

A particularly useful reformulation of MoND is via an effective gravitational dielectric medium, motivated by the analogy between Coulomb's law in a dielectric medium and Milgrom's law for MoND. It has been known to (Blanchet, Milgrom and) others that the MoNDian force law can be formulated as being governed by a nonlinear generalization of Poisson's equation which describes the nonlinear electrostatics embodied in the Born-Infeld theory.

Consider the Born-Infeld (BI) theory defined with the following Lagrangian density (b being a dimensionful parameter; the second form is for the case of $\vec{B} = 0$):

$$L_{BI} = b^2 \left(1 - \sqrt{1 - \frac{E^2 - B^2}{b^2} - \frac{(\vec{E} \cdot \vec{B})^2}{b^4}} \right) \longrightarrow b^2(1 - \sqrt{1 - E^2/b^2}),$$

(a nonlinear electrodynamics motivated by the analogy with relativistic mechanics given by $L_{particle} = mc^2(1 - \sqrt{1 - v^2/c^2})$ with c $\Longleftrightarrow b$).

Let us set $\vec{B} = 0$, concentrate on the Hamiltonian density H, supply an extra overall factor of $\frac{1}{4\pi}$ and use the notation $\vec{D} = \epsilon \vec{E}$.

Then the corresponding gravitational Hamiltonian density reads:

$$H_g = \frac{b^2}{4\,\pi} \left(\sqrt{1 + \frac{D_g^2}{b^2}} - 1 \right) .$$

Let $A_0 \equiv b^2$ and $\vec{A} \equiv b\,\vec{D}_g$, then the Hamiltonian density becomes

$$H_g = \frac{1}{4\,\pi} \left(\sqrt{A^2 + A_0^2} - A_0 \right) .$$

Assume there exists an energy equipartition, then the effective gravitational Hamiltonian density is equal to

$$H_g = \frac{1}{2}\, k_B\, T_{\rm eff} \; ,$$

where $T_{\rm eff}$ is an effective temperature associated which the energy.

(Note that this energy density is energy per unit volume. But we can regard it as energy per degree of freedom by recalling that volume, which usually scales as entropy S, scales as the number of degrees of freedom N in a holographic setting. Parenthetically $S \sim N$ is one of the features of infinite statistics.)

The Unruh temperature formula $T_{\text{eff}} = \frac{\hbar}{2\,\pi\,k_B}\,a_{\text{eff}}$ implies that the effective acceleration is given by

$$a_{eff} = \sqrt{A^2 + A_0^2} - A_0\,.$$

The equivalence principle suggests that we should identify (at least locally) the local accelerations $\vec{a}$ and $\vec{a}_0$ with the local gravitational fields $\vec{A}$ and $\vec{A}_0$ respectively, viz., $\vec{a} \equiv \vec{A}, \quad \vec{a}_0 \equiv \vec{A}_0\,.$ Then a_{eff} should be identified as:

$$a_{\text{eff}} \equiv \sqrt{a^2 + a_0^2} - a_0\,,$$

which, in turn, implies that the Born-Infeld inspired force law takes the form (for a given test mass m)

$$F_{\text{BI}} = m\left(\sqrt{a^2 + a_0^2} - a_0\right)\,,$$

which is precisely the MoNDian force law derived previously!

(Note: For a typical acceleration of order 10 ms^{-2}, the corresponding effective temperature is of order $k_B\, T_{\mathrm{eff}} \sim 10^{-24}$ eV, much smaller than the mass of any viable *cold* dark matter candidate.)

To be a viable cold dark matter candidate, the quanta of our MoNDian dark matter are expected to be much heavier than $k_B\, T_{\mathrm{eff}}$.

Recall that the equipartition theorem in general states that the average of the Hamiltonian is given by

$$\langle H \rangle = - \frac{\partial \log Z(\beta)}{\partial \beta}\,,$$

where $\beta^{-1} = k_B T$ and Z denotes the partition function. To obtain $\langle H \rangle = \frac{1}{2}\, k_B\, T$ per degree of freedom, we require Z to be of the Boltzmann form

$$Z = \exp(\, -\beta\, H\,)\,.$$

(The conventional quantum-mechanical Bose-Einstein or Fermi-Dirac statistics would not lead to $\langle H \rangle = \frac{1}{2}\, k_B\, T$ per degree of freedom at low temperataure.)

Thus the validity of $H_g = \frac{1}{2}\, k_B\, T_{\rm eff}$ for very low temperature $T_{\rm eff}$ somehow requires a unique quantum statistics with a Boltzmann partition function.
This is precisely what is called the infinite statistics as described by the Cuntz algebra (a curious average of the bosonic and fermionic algebras)

$$a_i\, a_j^\dagger = \delta_{ij}\ .$$

Thus, by invoking infinite statistics, the assumption of energy equipartition $H_g = \frac{1}{2}\, k_B\, T_{\rm eff}$, even for very low temperature $T_{\rm eff}$, is justified.

Recap: (i) the relation between our force law that leads to MoNDian phenomenology and an effective gravitational Born-Infeld theory; (ii) the need for infinite statistics of some microscopic quanta which underly the thermodynamic description of gravity implying such a MoNDian force law.

(A side remark: From the Matrix theory point of view, we expect infinite statistics and an effective theory of the gravitational Born-Infeld type to be closely related.)