NON-MINIMALLY COUPLED GRAVITY AND PLANETARY MOTION

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TESTING NMC GRAVITY WITH MERCURY

- Observation of Mercury orbit (MESSENGER) can be used to constrain new theories of gravitational physics.
- Nonminimally coupled (NMC) gravity is a modification of General Relativity that has been applied to astrophysical and cosmological problems as a possible alternative to the standard scenario of dark matter and dark energy.
- The nonrelativistic limit of NMC gravity consists of the Newtonian potential plus a Yukawa perturbation.
- Measurement of perihelion precession of Mercury orbit, resulting from MESSENGER data, can be converted into constraints on NMC gravity parameters.

NONMINIMALLY COUPLED GRAVITY

- The action functional of NMC gravity is (Bertolami et al. 2007):

$$\int \left[f^1(R)/2 + \left(1 + f^2(R)\right)\mathcal{L}_m \right] \sqrt{-g}\, d^4x,$$

- R is the spacetime curvature, g is the metric determinant, $\mathcal{L}_m = -\rho c^2$ is the Lagrangian density of matter, ρ is mass density.

- General Relativity (GR) is recovered by taking

$$f^1(R) = 2\kappa R, \quad f^2(R) = 0, \quad \kappa = \frac{c^4}{16\pi G}, \; G = \text{Newton's constant}$$

- $f^2(R) = 0$ corresponds to $f(R)$ gravity theory.
- $f^2(R)$ yields the NMC between geometry and matter.

METRIC AND ENERGY-MOMENTUM TENSOR

- The metric tensor is of the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \; |h_{\mu\nu}| << 1$

  $\eta_{\mu\nu}$ is the Minkowski tensor; *1/c* expansion of $g_{\mu\nu}$ (as in PPN):

$$g_{00} = -1 + h_{00}^{(2)} + h_{00}^{(4)} + O(1/c^6), \quad g_{0i} = h_{0i}^{(3)} + O(1/c^5),$$

$$g_{ij} = \delta_{ij} + h_{ij}^{(2)} + O(1/c^4).$$

- Components of energy momentum tensor $T_{\mu\nu}$ (as in PPN):

$$T_{00} = \rho c^2 \left(1 + \frac{v^2}{c^2} + \frac{\Pi}{c^2} - h_{00}^{(2)} \right) + O(1/c^2), \quad T_{0i} = -\rho c v_i + O(1/c),$$

$$T_{ij} = \rho v_i v_j + p\delta_{ij} + O(1/c^2),$$

- where matter is considered as a perfect fluid with density ρ, velocity ***v***, pressure *p*, and specific energy density Π.

ASSUMPTIONS ON FUNCTIONS OF CURVATURE

- We assume the functions $f^1(R)$ and $f^2(R)$ to be analytic at *R=0*. Hence f^1 admits the Taylor expansion:

$$f^1(R) = 2\kappa \sum_{i=1}^{\infty} a_i R^i , \qquad a_1 = 1, \qquad \kappa = \frac{c^4}{16\pi G}$$

- f^2 admits the Taylor expansion:

$$f^2(R) = \sum_{j=1}^{\infty} q_j R^j$$

- If $a_i = 0 \; \forall i > 1$ and $q_j = 0 \; \forall j,$ the action of GR is recovered.
- The coefficients a_2, a_3, q_1, q_2 (parameters of the NMC model) will be used to compute the metric at the required order.

FIELD EQUATIONS OF NMC GRAVITY

- The first variation of the action functional with respect to the metric yields the field equations:

$$\left(f_R^1 + 2 f_R^2 \mathfrak{L}_m\right) R_{\mu\nu} - \frac{1}{2} f^1 g_{\mu\nu} = \nabla_{\mu\nu}\left(f_R^1 + 2 f_R^2 \mathfrak{L}_m\right) + \left(1 + f^2\right) T_{\mu\nu}$$

$$\nabla_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} g^{\sigma\eta} \nabla_\sigma \nabla_\eta$$

- $f_R^i = d f^i / dR$, $\mathfrak{L}_m = -\rho c^2$ is the Lagrangian density of matter, ρ is mass density, $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ is the energy-momentum tensor, ∇_μ is the covariant derivative.

- The field equations are solved by a perturbative method.

NONRELATIVISTIC LIMIT

- At order *O(1/c²)* we obtain the equations of Yukawa type:

$$\nabla^2 R^{(2)} - \frac{1}{6a_2} R^{(2)} = -\frac{4\pi G}{3c^2 a_2}\left(\rho - 6q_1 \nabla^2 \rho\right),$$

$$\nabla^2\left(h_{00}^{(2)} - 2a_2 R^{(2)} + 16\frac{\pi G}{c^2} q_1 \rho\right) = -\frac{8\pi G}{c^2}\rho,$$

- $R^{(2)}$ is curvature at order *O(1/c²)*, a_2, q_1 are NMC parameters.

- the solution for the *0-0* component of the metric at order *O(1/c²)* is the Newtonian potential plus a Yukawa potential:

$$h_{00}^{(2)} = 2\frac{U}{c^2} + \left(1 - \frac{q_1}{a_2}\right)\frac{2}{3c^2} Y, \qquad Y = G\int \rho(t, \vec{x}') \frac{e^{-|x-\vec{x}'|/\lambda}}{|x - \vec{x}'|} d^3x'$$

YUKAWA POTENTIAL

- $h_{00}^{(2)}$ is the Newtonian potential plus a Yukawa potential:

$$h_{00}^{(2)} = 2\frac{U}{c^2} + (1-\theta)\frac{2}{3c^2}Y, \qquad Y = G\int \rho(t,\vec{x}')\frac{e^{-|x-\vec{x}'|/\lambda}}{|x-\vec{x}'|}d^3x'$$

- The range λ of the Yukawa potential is

$$\lambda = \sqrt{6a_2}$$

- The strength α of the Yukawa potential is

$$\alpha = \frac{1}{3}(1-\theta), \qquad \theta = \frac{q_1}{a_2}, \qquad q_1 = a_2 \Rightarrow \alpha = 0.$$

Long range (astronomical) effects are possible if $q_1 \cong a_2$.

PPNY APPROXIMATION

- The *i-j* components of field equations at order *O(1/c²)* are:

$$\nabla^2\left(\frac{1}{2}h_{ij}^{(2)} - 2a_2\delta_{ij}R^{(2)} + \frac{16\pi G}{c^2}q_1\rho\delta_{ij}\right) + \frac{1}{2}\delta_{ij}R^{(2)} + 2a_2R_{,ij}^{(2)} = \frac{c^2}{\kappa}q_1\rho_{,ij}$$

- $R^{(2)}$ is curvature at order *O(1/c²)*, a_2, q_1 are NMC parameters.

- Diagonal solution after gauge transformation:

$$h_{ij}^{(2)} = \left[2\frac{U}{c^2} - (1-\theta)\frac{2}{3c^2}Y\right]\delta_{ij}, \qquad Y = G\int \rho(t,\vec{x}')\frac{e^{-|x-\vec{x}'|/\lambda}}{|x-\vec{x}'|}d^3x'$$

$$\theta = \frac{q_1}{a_2}$$

0-0 COMPONENT AT ORDER *O(1/c^4)*

- The largest term of $h_{00}^{(4)}$ is:

$$h_{00}^{(4)} = h_{00}^{(4)-GR} + \frac{8\pi G^2}{c^4}\theta\left(-2q_1 + \frac{a_3 q_1}{a_2^2} - \frac{4}{3}\frac{q_2}{a_2}\right)X\left(\rho^2\right) + \ldots$$

$$X\left(\rho^2\right) = \int \rho^2\left(t,\vec{x}'\right)\frac{e^{-|\vec{x}-\vec{x}'|/\lambda}}{\left|x-\vec{x}'\right|}d^3x', \qquad \theta = \frac{q_1}{a_2}$$

- a_2, a_3, q_1, q_2 are NMC parameters; $h_{00}^{(4)-GR}$ is the GR term;

dots … denote the sum of further potentials; some are of type

$$\psi_{i,k}\left(t,\vec{x}\right) = G^2 \int \frac{\rho(t,\vec{y})\rho(t,\vec{z})\left(\vec{x}-\vec{y}\right)\cdot\left(\vec{y}-\vec{z}\right)}{\left|\vec{x}-\vec{y}\right|^i\left|\vec{y}-\vec{z}\right|^k} e^{-(|\vec{x}-\vec{y}|+|\vec{y}-\vec{z}|)/\lambda} d^3y d^3z$$

$$i,k = \in \{2,3\}$$

METRIC AROUND A STATIC, SPHERICAL BODY

- Uniform density is assumed (M_S,R_S: mass, radius of the body):

$$g_{00} = -1 + \frac{2}{c^2}\left[U(r) + \alpha Y(r)\right] - \frac{2}{r}\left(\frac{GM_S}{c^2}\right)^2 F(r), \qquad g_{0i=0},$$

$$g_{ij} = \left\{1 + \frac{2}{c^2}\left[U(r) - \alpha Y(r)\right]\right\}\delta_{ij}$$

- *U(r)* is the Newtonian potential, *Y(r)* is the Yukawa potential; *F(r)* is a further potential depending on exponential functions.

The largest term in the Yukawa strength α for $\lambda >> R_S$ is

$$\alpha = \frac{1}{3}(1-\theta) + \frac{GM_S}{c^2 R_S}\theta\left[\theta\left(\frac{\mu}{2} - 1\right) - \frac{2}{3}\nu\right]\left(\frac{\lambda}{R_S}\right)^2 + \ldots$$

$$\theta = \frac{q_1}{a_2}, \quad \mu = \frac{a_3}{a_2^2}, \quad \nu = \frac{q_2}{a_2^2}, \quad \lambda = \sqrt{6a_2}$$

DEVIATION FROM GEODESICS

- The energy-momentum tensor is not covariantly conserved:

$$\nabla_\mu T^{\mu\nu} = \frac{f_R^2}{1+f_2}\left(g^{\mu\nu}\mathcal{L}_m - T^{\mu\nu}\right)\nabla_\mu R \neq 0 \quad \text{if} \quad f_2 \neq 0,$$

- Consequently, the trajectories deviate from geodesics:

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = -\frac{f_R^2}{1+f_2} g^{\alpha\beta}\partial_\beta R.$$

Moreover, geodesics are different with respect to GR.

- Computation of the orbit of a planet around the Sun when

$$\lambda >> L,$$

L = semilatus rectum of the unperturbed orbit.

PERIHELION PRECESSION

- Largest term in formula for perihelion precession of a planet:

$$\delta\phi_P = \frac{6\pi G M_S}{Lc^2} + (1-\theta)^2 \frac{\pi}{3}\left(\frac{L}{\lambda}\right)^2 e^{-L/\lambda}$$

$$+(1-\theta)\frac{\pi G M_S}{3Lc^2}\theta\left[3\theta\left(\frac{\mu}{2}-1\right)-2\nu\right]\left(1-\frac{L}{\lambda}\right)\left(\frac{L}{R_S}\right)^3+\dots$$

$\theta = \frac{q_1}{a_2}, \mu = \frac{a_3}{a_2^2}, \nu = \frac{q_2}{a_2^2}, \lambda = \sqrt{6a_2},$ M_S, R_S, L=mass, radius, semilatus rectum

- The first row contains the GR precession + Yukawa precession;

the second row contains the NMC relativistic correction;

dots … correspond to contribution from further potentials.

Constraints on a_2, a_3, q_1, q_2 from observation of Mercury orbit.

CONSTRAINTS FROM MERCURY OBSERVATION

- Prediction of perihelion precession assuming a PPN metric:

$$\delta\phi_P^{PPN} = \left[\frac{2(1+\gamma)-\beta}{3} + 3\times 10^3 J_2\right]\frac{6\pi G M_S}{Lc^2}$$

γ, β are PPN parameters, J_2 is the quadrupole moment of the Sun

- Cassini bound on γ and bound on β from fits to planetary data including data from MESSENGER (Fienga et al., 2011) yield

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5}, \qquad \beta - 1 = (-4.1 \pm 7.8)\times 10^{-5}$$

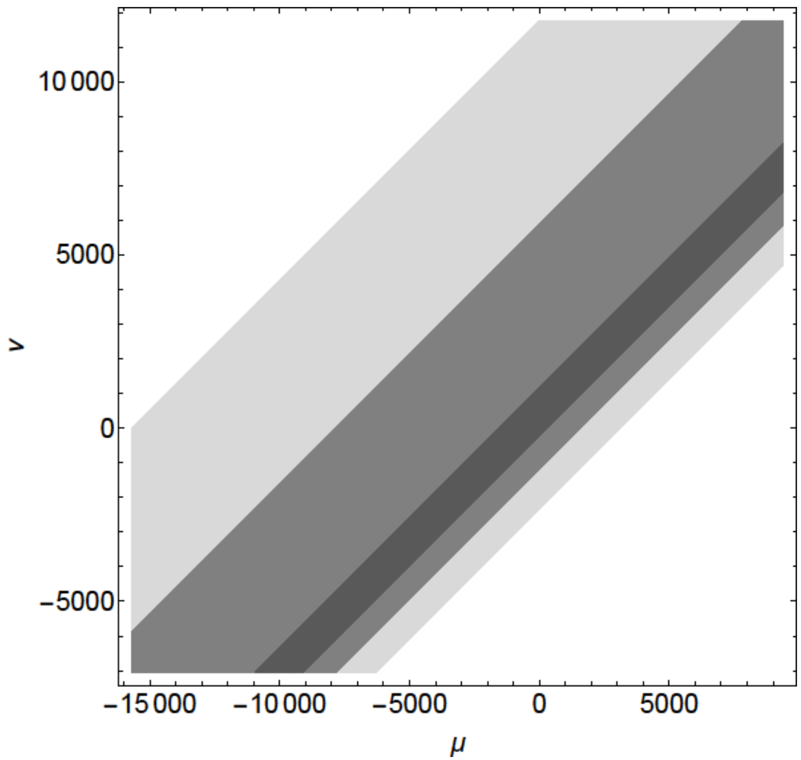
- We assume that the additional perihelion precession due to NMC deviations from GR is given by

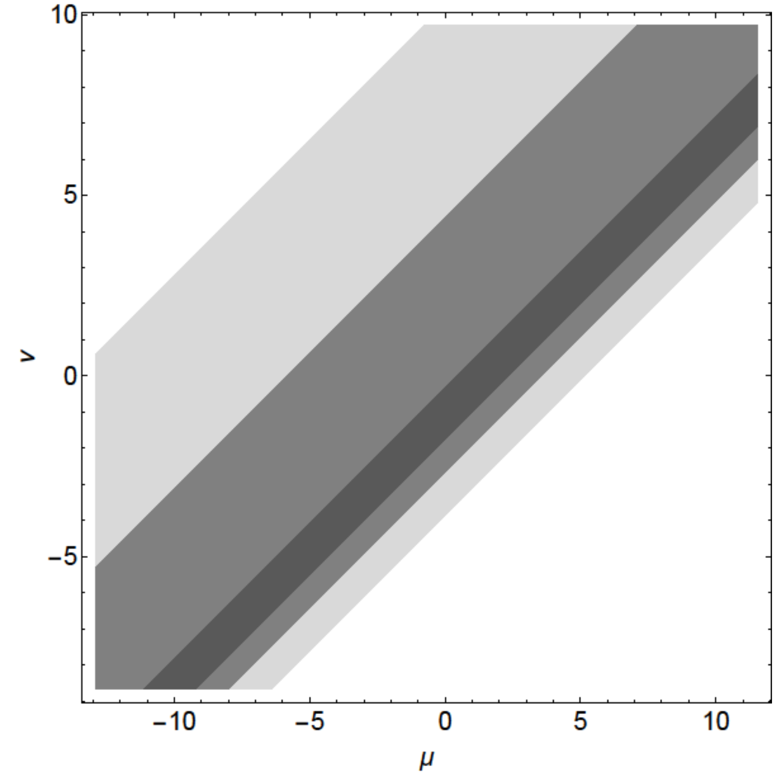
$$-5.8753\times 10^{-4} < \delta\phi_P - 42.98'' < 2.96631\times 10^{-3}$$

Formula for $\delta\phi_P$ then yields exclusion plots for NMC parameters:

$$\theta = \frac{q_1}{a_2}, \quad \mu = \frac{a_3}{a_2^2}, \quad \nu = \frac{q_2}{a_2^2}, \quad \frac{L}{\lambda}$$

EXCLUSION PLOTS IN THE PLANE (μ, ν)

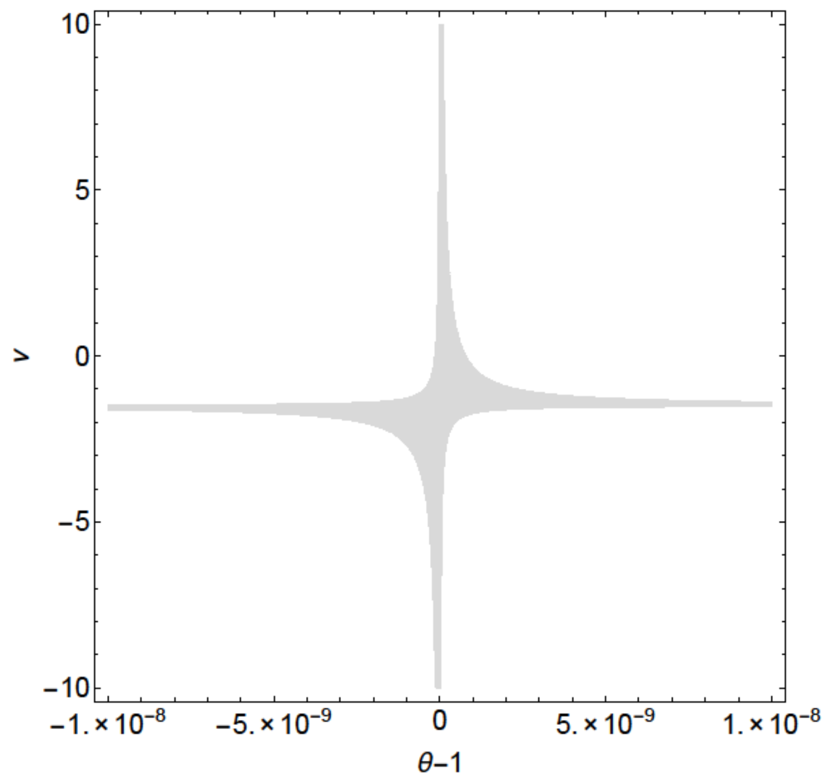


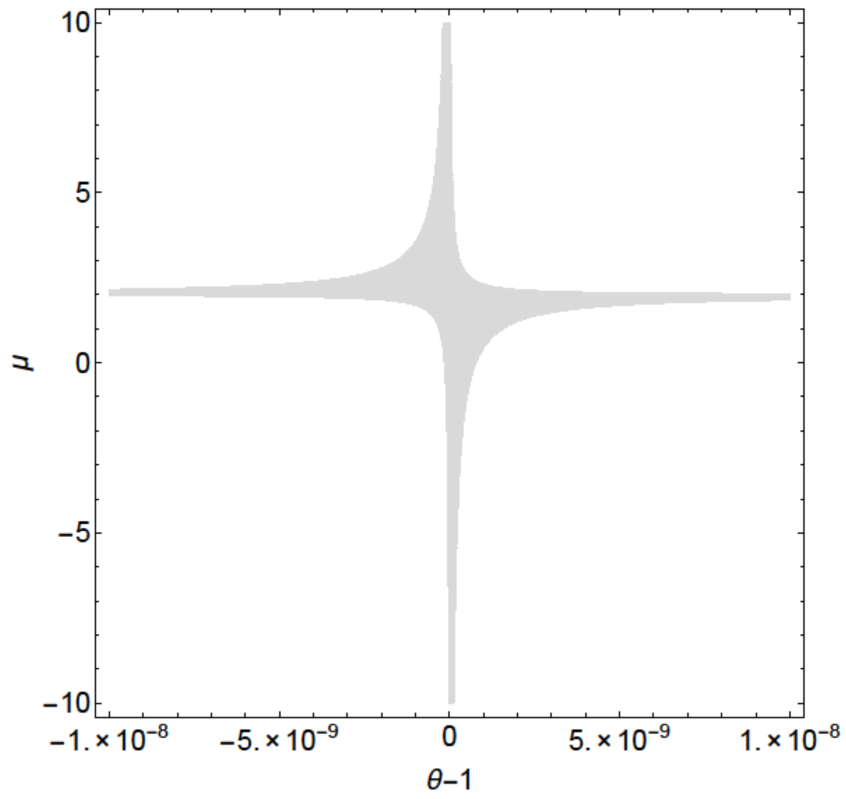


Left: λ=50L, θ-1={10^{-13},$2\text{x}10^{-13}$,10^{-12}} (light, medium, dark grey)

Right: λ=50L, θ-1={10^{-10},$2\text{x}10^{-10}$,10^{-9}} (light, medium, dark grey)

EXCLUSION PLOTS IN PLANES (θ, ν) AND (θ, μ)

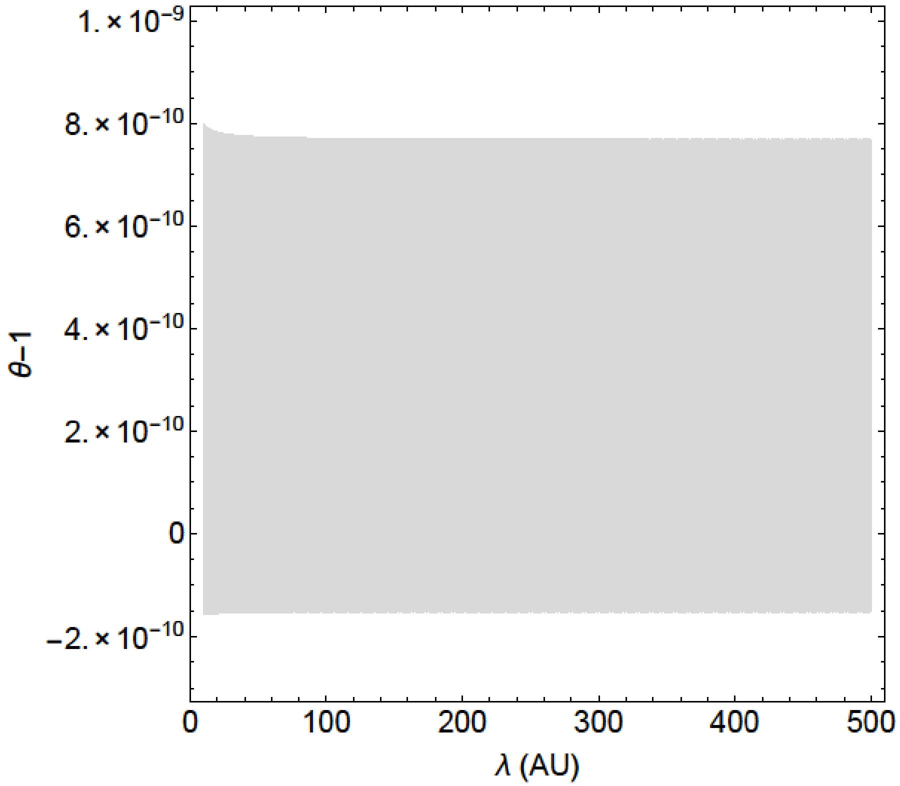


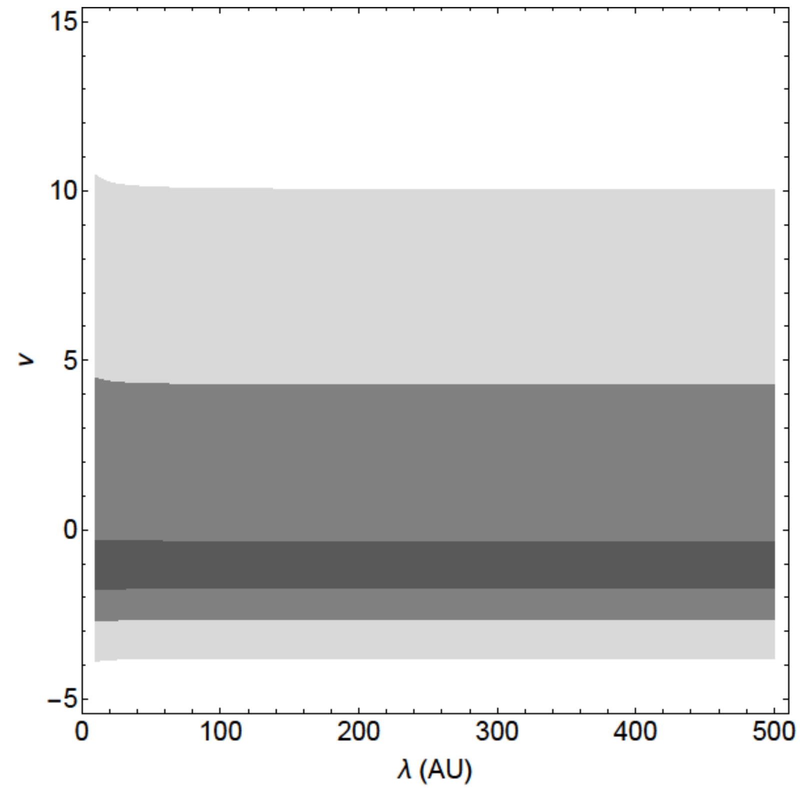


Left: λ=50L, μ=0

Right: λ=50L, ν=0

EXCLUSION PLOTS IN PLANES (λ,θ) AND (λ,ν)





Left: $\mu=\nu=0$

Right: $\mu=0,\ \theta-1=\{10^{-10},2\text{x}10^{-10},10^{-9}\}$ (light, medium, dark grey)