D-Branes moduli in Open String Field Theory

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based on

1601.04046, *(w/ **Matej Kudrna**)*
1506.03723, JHEP 1508 (2015)149 *(w/ **Martin Schnabl**)*
1402.3546, JHEP 1405 (2014) 004

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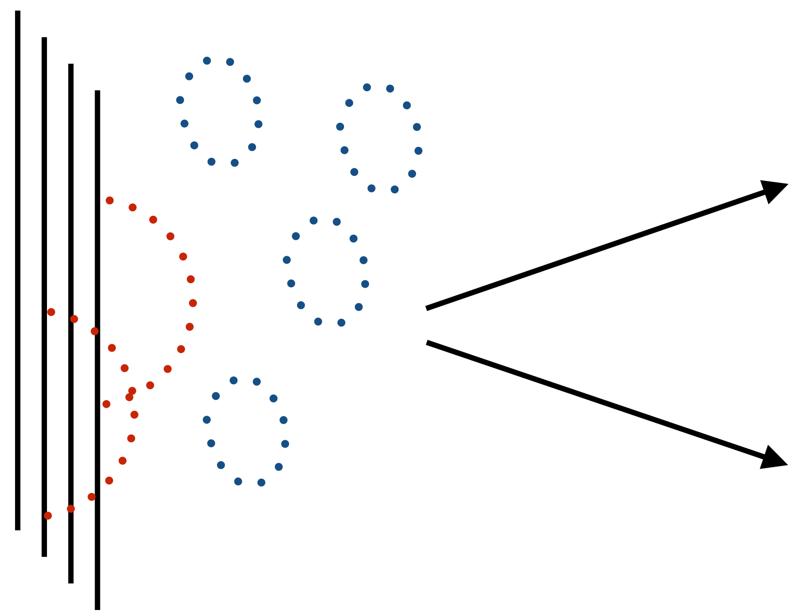
- String Theory gives important tools to better understand QFT and Gravity

- Most fundamental degrees of freedom: ***D-branes***



Open strings: Gauge Theory

$\alpha' \to 0$ ⇕

Closed strings: Gravity

- ***Open string field theory:*** exact theory for **all** open string modes

- ***Full non abelian DBI=OSFT w/ massive states integrated out***.

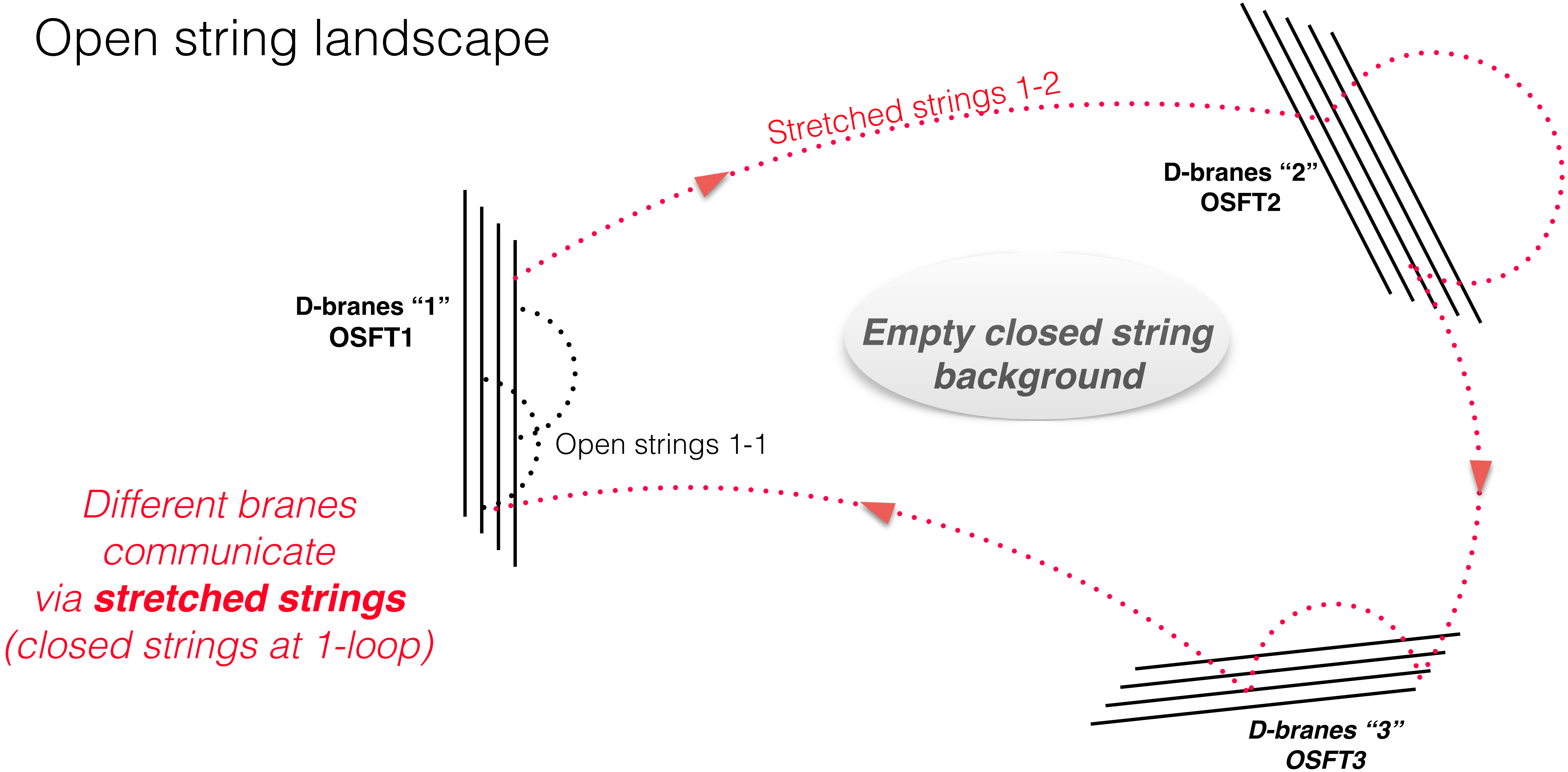
$$\lim_{\alpha' \to 0} \text{OSFT} = \text{ (Super) Yang-Mills}$$

- Recent progress: **full covariant SFT actions for the superstring**

Sen; Okawa-Kunitomo; Erler-Okawa-Takezaki; Konopka-Sachs 2015-16

D-Branes and OSFT

- Open string landscape



- Different D-branes, different OSFT's
- OSFT1=Full second quantized theory of 11 strings

OPEN STRING FIELD THEORY

- Fix a bulk CFT (closed string background)
- Fix a reference $BCFT_0$ (open string background, D-brane's system)
- The string field is a state in $BCFT_0$

$$|\psi\rangle = \sum_i t_i\, \psi^i(0)|0\rangle_{SL(2,R)}$$

- There is a non-degenerate inner product (BPZ)

$$\langle \psi, \phi \rangle = \langle \psi(-1)\phi(1)\rangle^{Disk}_{\mathrm{BCFT}_0}$$

- The bpz-inner product allows to write a target-space action

$$S[\psi] = -\frac{1}{2}\,\langle \psi, Q\psi\rangle_{\mathrm{BCFT}_0} - \frac{1}{3}\,\langle \psi, \psi * \psi\rangle_{\mathrm{BCFT}_0} = S_{eff}[t_i]$$

- Witten product *: associative product between states (OPE+conf. map
- ***Equation of motion***

$$Q\Psi + \Psi * \Psi = 0$$

- Just like ordinary gauge theories have classical solutions so does OSFT. The solitons of OSFT on a given D-brane system are just the other possible D-branes (*strongest formulation of Sen's Conjectures*)

$$S[\Psi] = -\frac{1}{g_o^2}\int_{\rm W}\left(\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3\right) \qquad Q\Psi + \Psi^2 = 0 \qquad Q = Q_{\rm BRST}$$

- Most basic solution: ***Tachyon Vacuum*** (no D-branes) ***Sen-Zwiebach '99 , Schnabl '05***

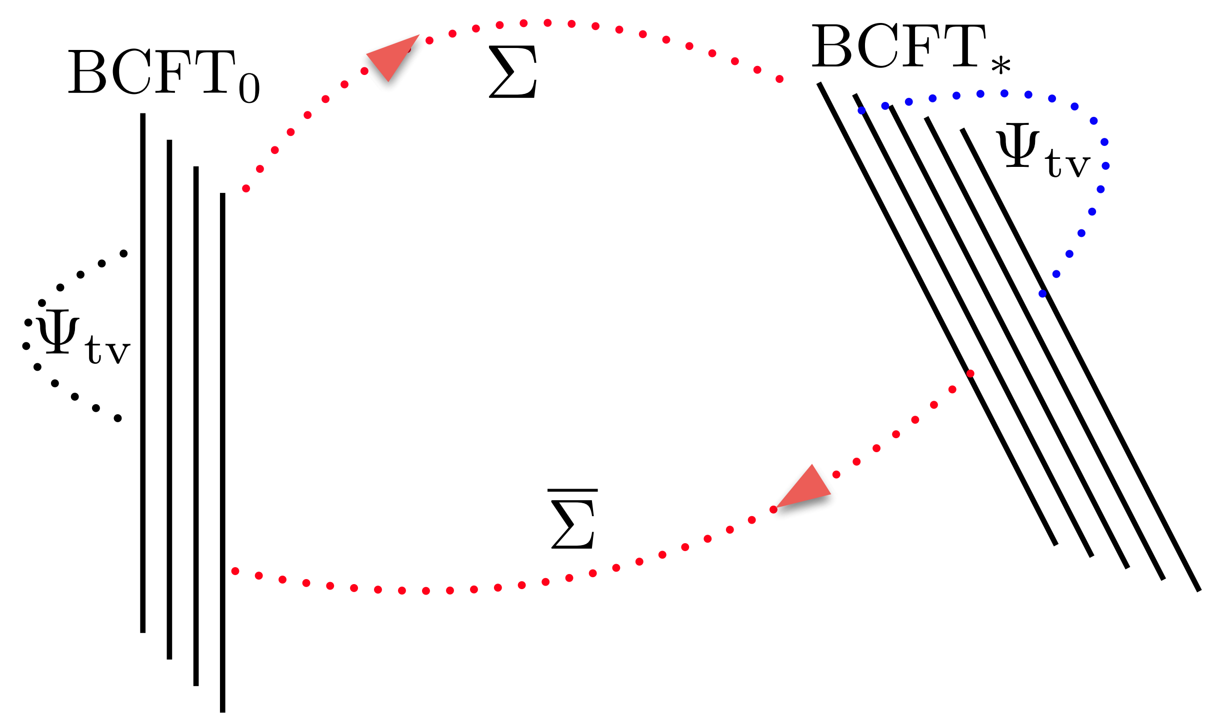
$$\Psi_{\rm tv} = F(K)c\frac{KB}{1-F^2(K)}cF(K) = t\, c_1|0\rangle + \ldots$$

$$Bc + cB = 1 \qquad [B,K] = 0 \qquad QB = K \qquad B^2 = c^2 = 0 \qquad Qc = cKc$$

- Solutions representing ***any D-brane configuration*** ***Erler, CM , 2014***

$$\Psi_{\rm EM} = \Psi_{\rm tv} - \Sigma\Psi_{\rm tv}\overline{\Sigma}$$

$$\overline{\Sigma}\Sigma = 1 \qquad Q\Sigma + [\Psi_{\rm tv},\Sigma] = 0 \qquad Q\overline{\Sigma} + [\Psi_{\rm tv},\overline{\Sigma}] = 0$$

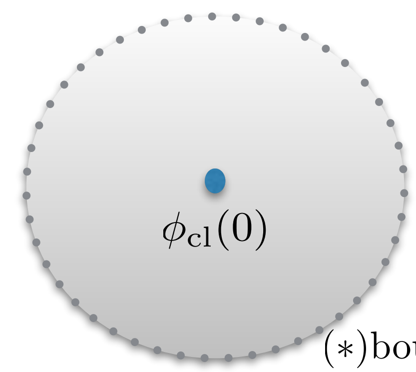


- ***Background independence through classical solutions***

D-branes: two microscopic descriptions

- Open strings: *D-branes are backgrounds.* ***OSFT SOLUTIONS*** Ψ_*
- Closed strings: *D-branes are sources.* ***BOUNDARY STATES*** $|B_*\rangle$

$$|B_*\rangle : \quad \langle B_*|\phi_{\rm cl}\rangle \equiv \langle \phi_{\rm cl}(0)\rangle^{\rm BCFT_*}_{\rm disk} =$$



- It has been possible to ***construct*** $|B_*\rangle$ ***from*** Ψ_* ***(Kudrna, CM, Schnabl, 2012)***

See also Kiermaier, Okawa, Zwiebach

$$Q\Psi_* + \Psi_*^2 = 0$$

$$|B_*\rangle = \sum_\alpha n_*^\alpha \, |V_\alpha\rangle\rangle$$

$$\Longrightarrow \quad n_*^\alpha = \langle V^\alpha | B_*\rangle = \langle V^\alpha(0)\rangle^{\rm BCFT_*}_{\rm disk} = W_{V^\alpha}[\Psi_* - \Psi_{\rm tv}]$$

"Ellwood" Gauge invariant operator in OSFT

- ***Closed string description = gauge invariant observables of OSFT***

D-branes moduli space

- D-branes have associated moduli (relative positions, Wilson lines etc..)
- On the world sheet: continuous family of conformal boundary conditions all related by ***exactly marginal boundary deformations***

$$S^{\lambda}_{\rm ws} = S^{\rm bulk}_0 + \lambda \int_{\partial {\rm ws}} ds\, j(s) \qquad\qquad j(s_1)j(s_2) = \frac{1}{(s_1-s_2)^2} + (reg.)$$

- Closed strings: continuous family of boundary states

$$|B_\lambda\rangle = \exp\left[-\lambda \oint ds\, j(s)\right] |B_0\rangle \qquad \lambda = \lambda_{\rm BCFT}$$

- Open strings: continuous family of OSFT classical solutions *(gauge)*

$$\Psi_{\tilde\lambda} = \tilde\lambda\, cj(0)|0\rangle + O(\tilde\lambda^2) \qquad \tilde\lambda = \lambda_{\rm SFT}$$

- We typically have $\lambda_{\rm SFT} \neq \lambda_{\rm BCFT}$!

BCFT vs SFT moduli

A problem with a long history!

- **2000**, Sen, Zwiebach ***SFT moduli space mysteriously truncate***
- **2004**, Sen ***SFT vs BCFT moduli via the construction of OSFT EM tensor***
- **2012**, Kudrna, Masuda, Okawa, Schnabl, Yoshida (KMOSY)
  SFT vs BCFT moduli via OSFT gauge invariant observables (cfr KMS boundary state)
- **2015**, CM, Schnabl ***SFT vs BCFT moduli analytically related for the first time***
- **2016**, Kudrna, CM ***Better method to search for marginal deformations in LT***
- **20xx …**

So let's tell the story…

1- Marginal effective potential in SFT

Sen-Zwiebach 2000

- OSFT on a D-brane with an exactly marginal boundary operator $j(s)$, SZ searched for a ***numerical*** solution in ***level truncation*** of the form

$$|\Psi^{(L)}_{\tilde{\lambda},r_i}\rangle = \tilde{\lambda}\,|cj\rangle + \sum_{i}^{i_{\max}} r_i |s_i\rangle \qquad b_0\Psi = 0$$

Feynman-Siegel gauge

- Plug in the action up to a given level $L \sim i_{\max}$

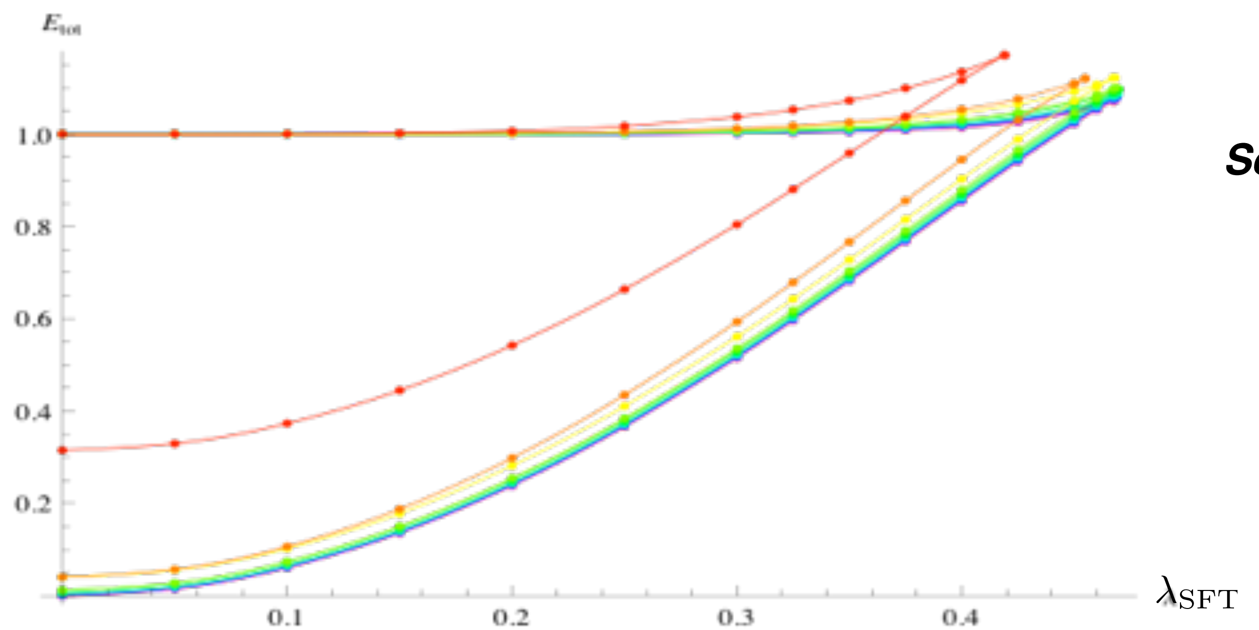
$$S_{\text{OSFT}}[\Psi^{(L)}_{\tilde{\lambda},r_i}] = S^{(L)}(\tilde{\lambda}, r_i)$$

- Integrate out the massive fields

$$\frac{\partial S}{\partial r_i}^{(L)} = 0 \quad \rightarrow \quad r_i = r_i^{(L)}(\tilde{\lambda})$$

- Get a level truncated effective potential for $\tilde{\lambda}$

$$S^{(L)}_{\text{eff}}(\tilde{\lambda}) = S^{(L)}\left(\tilde{\lambda}, r_i^{(L)}(\tilde{\lambda})\right)$$



Sen Zwiebach(2000): up to L=4
KMOSY (2012): up to L=12

- A flat branch (moduli space) clearly forms, **but it truncates!**
- Critical SFT parameter: where is it in the CFT moduli space?
- **Sen, 2004**: Noether construction of the energy momentum tensor in OSFT, to compare with the CFT boundary state. The critical SFT parameter seems to correspond to a ***finite*** point in the CFT moduli space. ***If so, SFT doesn't reach all D-branes configurations!***

2- Gauge invariant definition of $\lambda_{\rm BCFT}$

KMOSY - KMS 2012

- The BCFT boundary state can be explicitly constructed from a solution Ψ_*

$$|B_*\rangle = |B^{gh}\rangle \otimes \sum_{\alpha=\text{spinless}} n_*^\alpha \, \|V^\alpha\rangle\!\rangle$$

Generic form of a boundary state (Ishibashi)

$$n_*^\alpha = (2\pi i)\,\langle I|\mathcal{V}^\alpha(i,-i)|\tilde\Psi_* - \tilde\Psi_{\rm tv}\rangle$$

OSFT gauge invariant coefficients (KMS, from Ellwood)

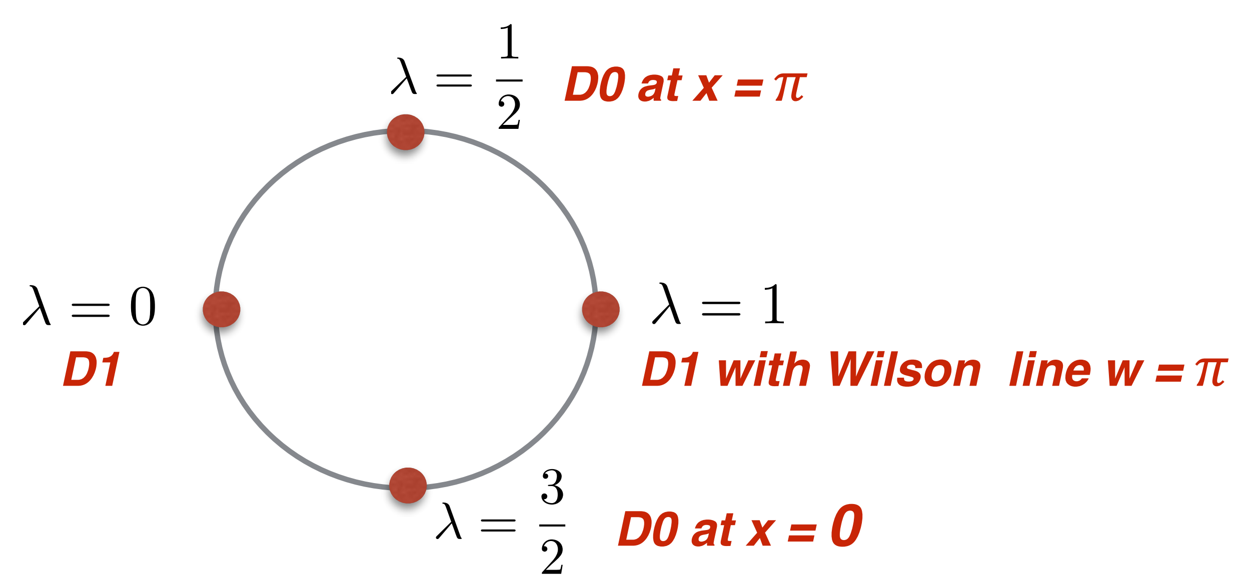
- Example of D1 wrapped on a circle at ***self-dual*** radius, with boundary deformation

$$\lambda \int_{\partial \text{ws}} ds\, j(s) = \lambda \int_{\partial \text{ws}} ds\, \cos X(s)$$

Callan, Klebanov, Ludwig, Maldacena '94

- Periodic moduli space interpolating from Neumann (D1) to Dirichlet (D0)

$$\lambda \sim \lambda + 2$$



- Compute the boundary state coefficients from OSFT and compare it with the known BCFT coefficients.

$$(2\pi i)\,\langle I|\mathcal{V}^{\alpha}(i,-i)|\tilde{\Psi}_*(\lambda_{\rm SFT})-\tilde{\Psi}_{\rm tv}\rangle = n_*^{\alpha}(\lambda_{\rm SFT})$$

$$V^{\alpha}(z,\bar{z})=\frac{1}{2i}\partial X\bar{\partial}X(z,\bar{z}) \longrightarrow n_*(\lambda_{\rm BCFT})=\cos 2\pi\lambda_{\rm BCFT}$$

- Use the above relation to express the CFT modulus as a function of the SFT parameter.

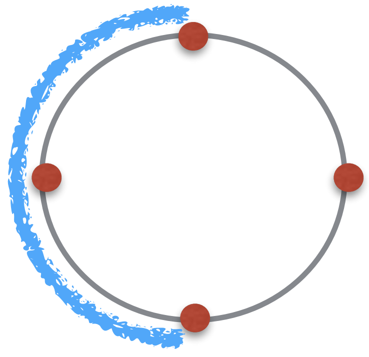
$$f^{(\rm SFT)}(\lambda_{\rm SFT})=f^{(\rm BCFT)}(\lambda_{\rm BCFT})$$

- The critical SFT parameter corresponds indeed to a finite CFT point!

$$\lambda_{\rm SFT}=\lambda_{\rm SFT}^{\rm crit} \quad \rightarrow \quad \lambda_{\rm BCFT}\sim\frac{1}{2}$$

Kudrna-Masuda-Okawa-Schnabl-Yoshida

Only HALF of the CFT moduli space is covered!



3- Relation from analytic solution *CM-SCHNABL 2015*

- ***Exact analytic*** solution for self-local marginal deformations

$$\Psi_\lambda = \frac{1}{1+K}\Phi_\lambda\frac{1}{1+K+J_\lambda} - Q\left(\frac{1}{1+K}\Phi_\lambda\frac{B}{1+K+J_\lambda}\right)$$

(CM, 2014) ***NOT GAUGE-FIXED***

$$Q\Phi_\lambda + \Phi_\lambda^2 = 0$$

Formal identity-based solution (observables not directly computable)
Takahashi-Tanimoto (2001)

$$B^2 = 0$$
$$QB = K$$
$$[B, \Phi_\lambda] = J_\lambda$$

$$\Phi_\lambda = \int_{-i\infty}^{i\infty}\frac{dz}{2\pi i}\left(\lambda f(z)cj(z) + \frac{1}{2}\lambda^2 f^2(z)c(z)\right)$$

$$\int_{-i\infty}^{i\infty}\frac{dz}{2\pi i}f(z) = 1$$

f(z) is a gauge freedom

- The boundary state can be exactly computed from Ψ_λ

$$|B_{\Psi_\lambda}\rangle = \exp\left[-\lambda\oint\frac{dz}{2\pi i}j(z)\right]|B_0\rangle$$

- The solution is already parametrized by the BCFT modulus!
- The BCFT moduli space is therefore fully covered.

- Interesting to look at the coefficient of the low-levels fields

$$\Psi_\lambda = t(\lambda)\, c_1|0\rangle + \lambda_{\rm SFT}(\lambda)\, j_{-1}c_1|0\rangle + \ldots$$

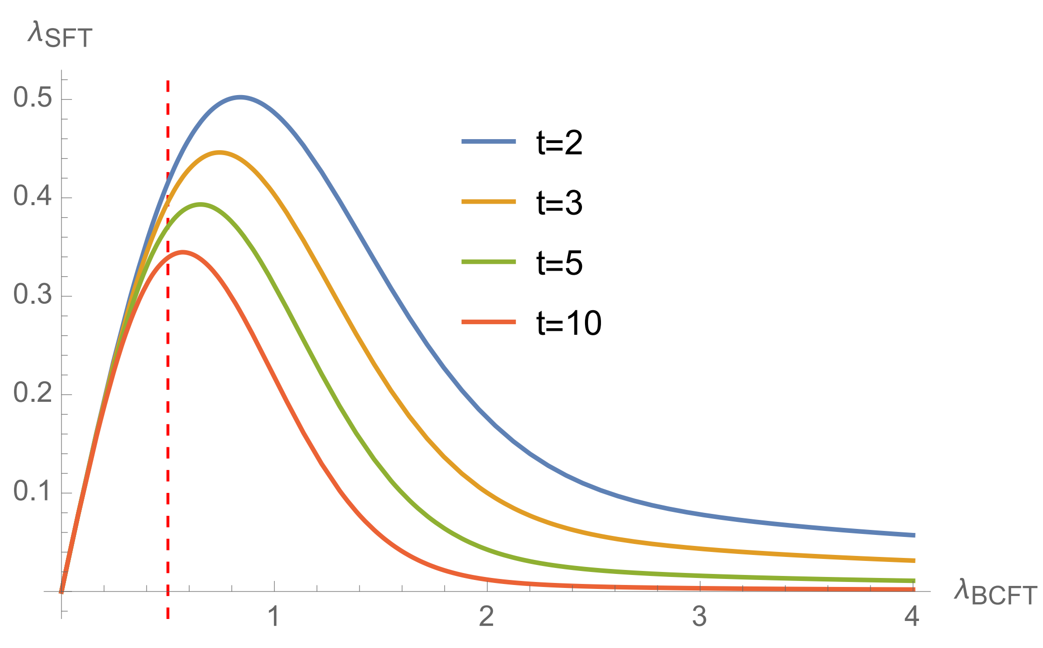
- The tachyon and marginal coefficients can be exactly computed

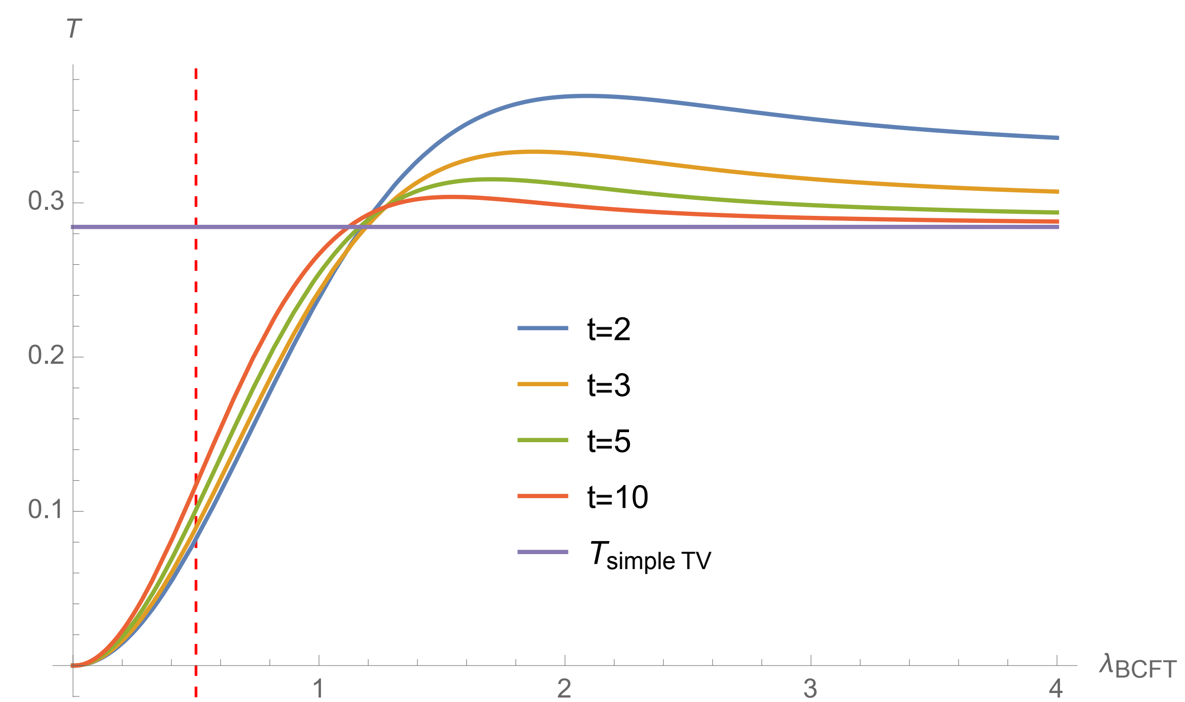
$$\lambda_{\rm SFT}(\lambda) = \lambda \int_0^\infty dx\, e^{-x} \int_0^x dy\, \left(1+\lambda^2 \mathcal{F}_f(x,y)\right) e^{-\lambda^2 \mathcal{G}_f(x,y)}$$

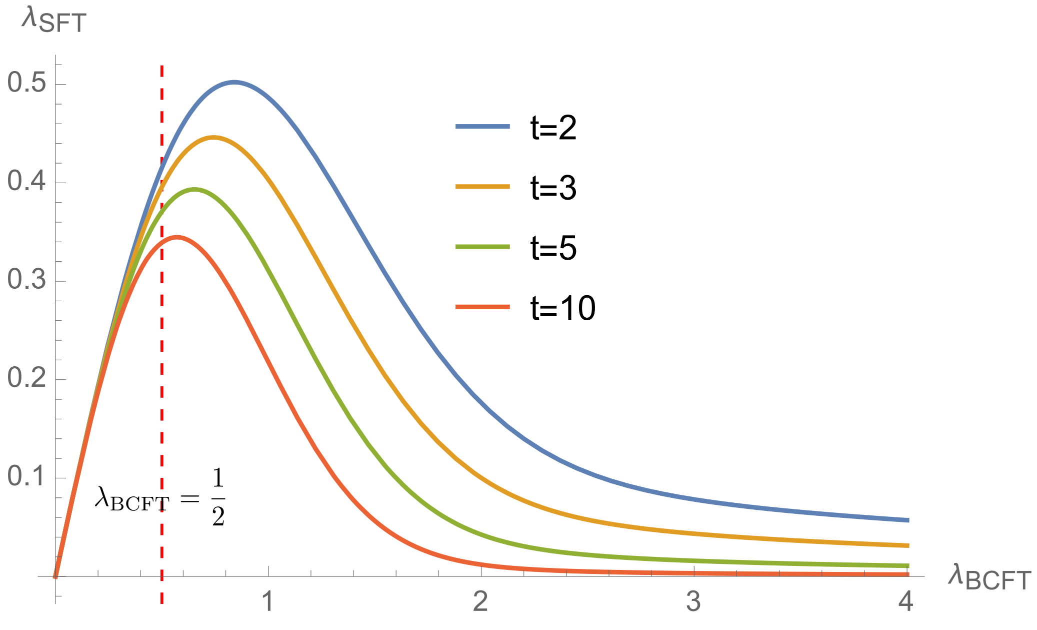
CM, Schnabl (2015)

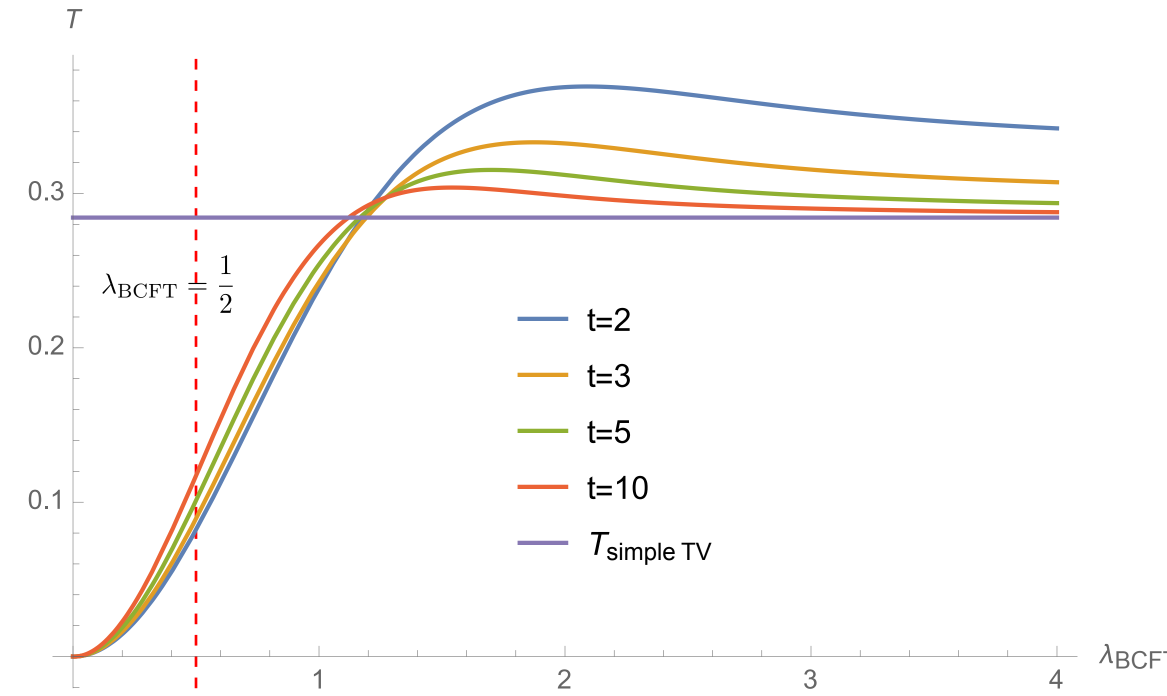
$$t(\lambda) = \frac{1}{2}\lambda^2 \int_0^\infty dx\, x e^{-x} \int_0^x dy\, \mathcal{P}_f(x,y)\, e^{-\lambda^2 \mathcal{G}_f(x,y)}$$

$$f(z) = 2\sqrt{\pi} t\, e^{(tz)^2}$$







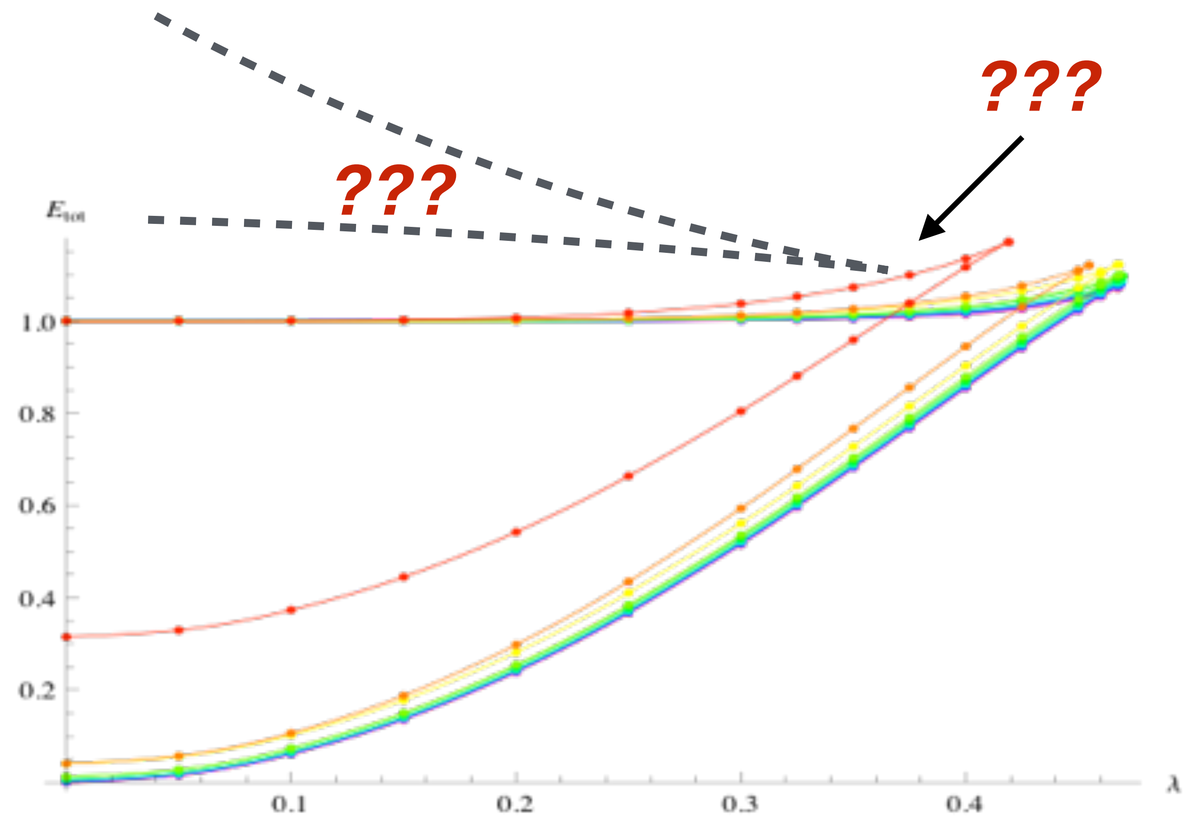


- If we would parametrize this solution with λ_{SFT} it would necessarily truncate at the maximum, as it happens in Siegel gauge!
- As for numerical solution the maximum is close to $\lambda_{\mathrm{BCFT}} = \frac{1}{2}$!
- The marginal parameter goes to ***zero*** at high modulus!
- On the other hand the ***tachyon*** coefficient tends to the tachyon vacuum!
- Sen-Zwiebach puzzle looks much clearer now… but the analytic solution ***is not in Siegel gauge***: NEED FOR EXPERIMENT!

4- “Experiment” in Siegel Gauge

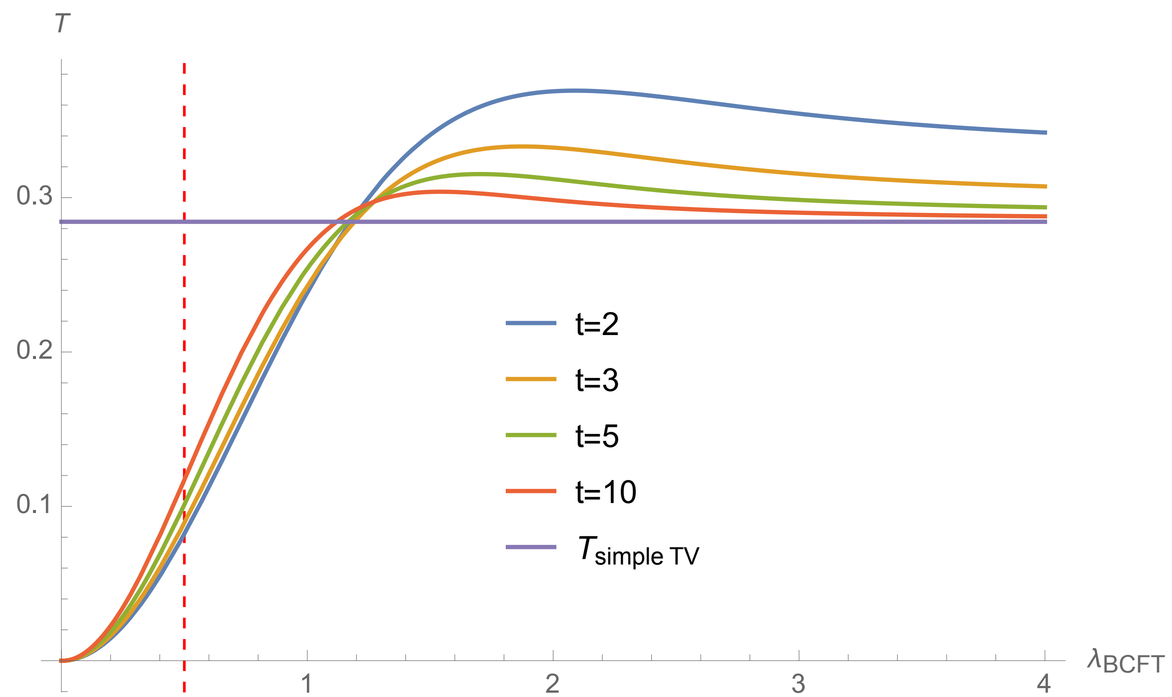
KUDRNA-CM 2016

- Is there a new large-moduli branch in Feynman-Siegel gauge $b_0\Psi = 0$??



- Very difficult to directly search for it level truncation (***where to start?***)

- Lesson from the analytic solution: ***the tachyon coefficient is one-to-one with the BCFT modulus in a quite vast region***

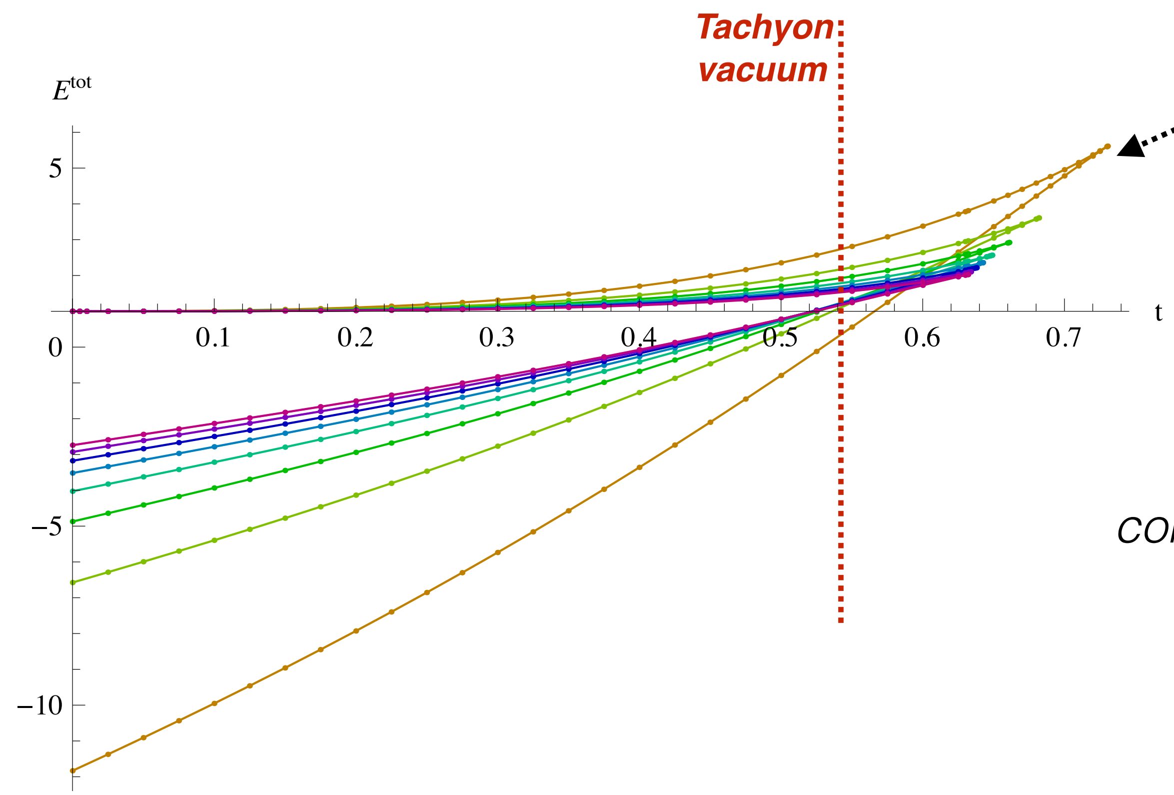


- Therefore it makes sense to parametrize the solution with the VEV of the tachyon and expect to cover a much larger region of moduli space

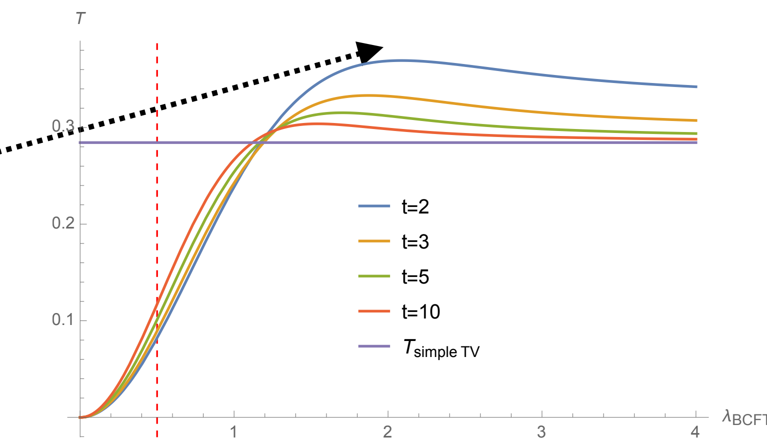
$$|\Psi^{(L)}_{t,r_i}\rangle = t\, c_1|0\rangle + \sum_i^{i_{\max}} v_i|s_i\rangle$$

- Level by level solve the ***v***'s (which *include* the marginal field) in terms of the tachyon.

- We found the following effective tachyon potential



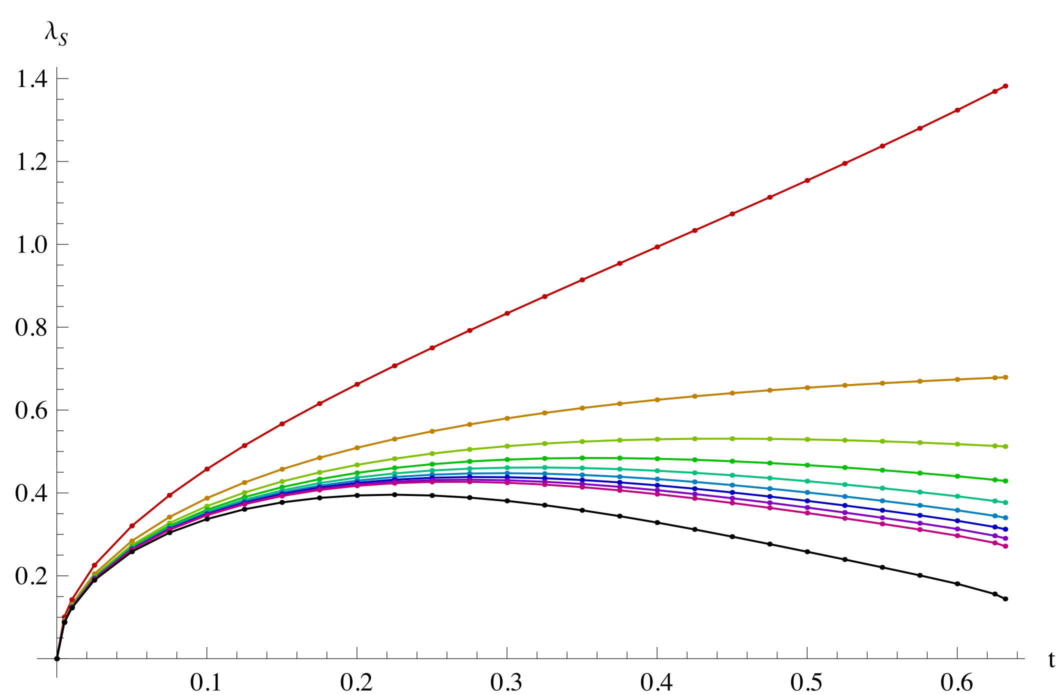
Tachyon bump



Analytic solution

*computation to L=18, **34842** fields*
about a week
with C++ on parallel clusters
(M. Kudrna)

- The marginal field is determined in terms of the tachyon and is given by



A maximum shows up
at level 5!

$$\lambda_{\text{SFT}}^{\text{max}} \sim \lambda_{\text{SFT}}^{\text{crit}} \sim 0.4 \pm 0.05$$

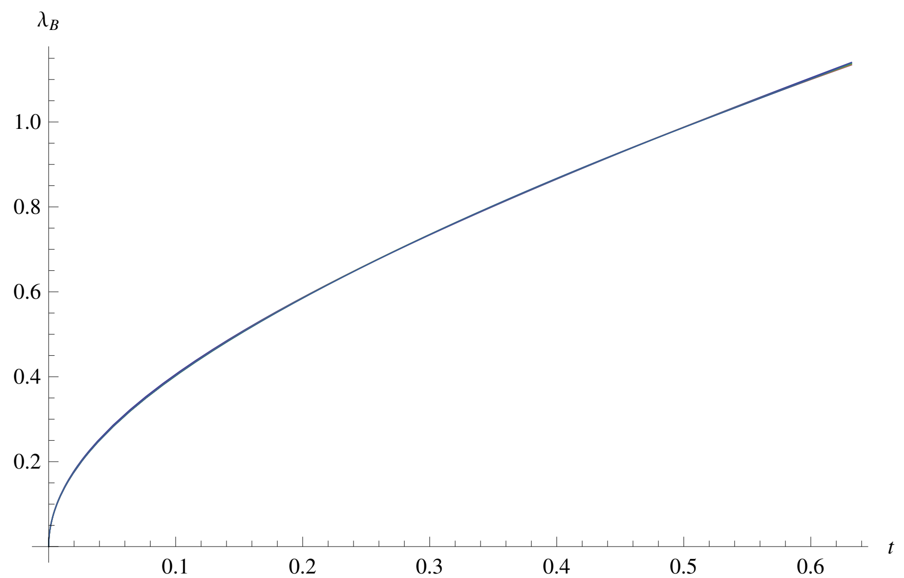
- Let’s extract the BCFT modulus. Boundary state coefficient of the lowest weight momentum mode (best converging in LT)

$$E_1(t) = \langle I| w\, c\bar{c}\, \cos X(i,-i) |\Psi(t) - \Psi_{\rm tv}\rangle = -\sin \pi\lambda_{\rm BCFT}$$

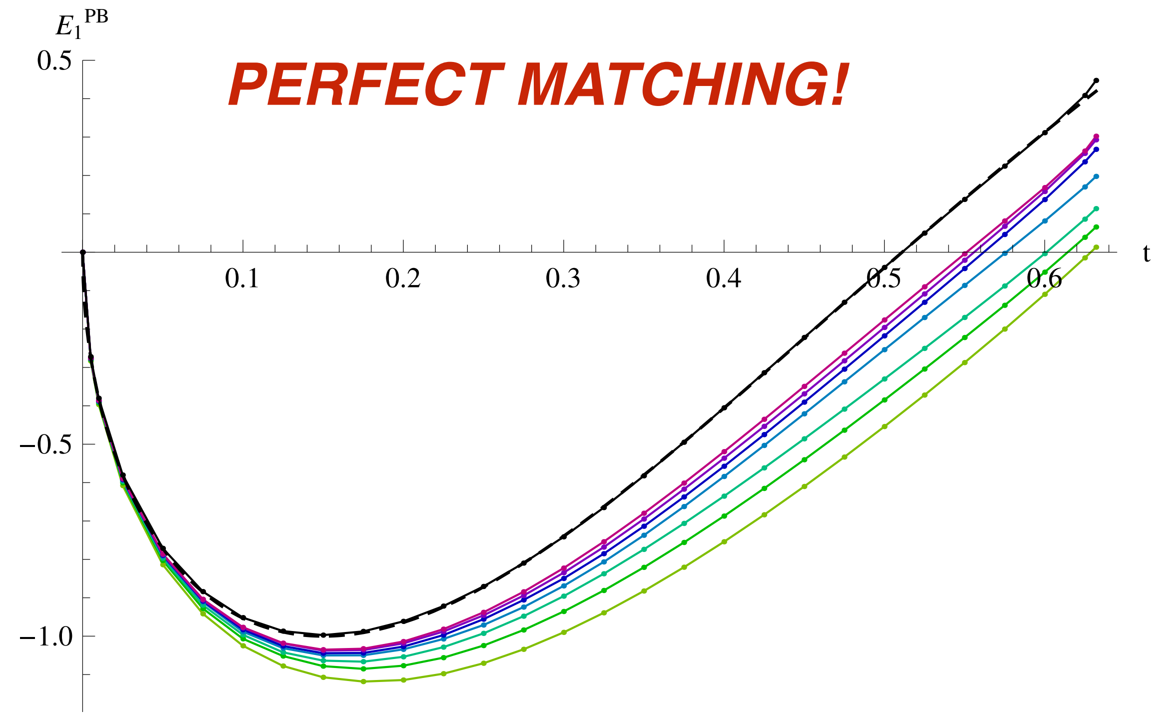
Ansatz from perturbative construction of the solution

$$\lambda_{\rm BCFT}(t) = \sqrt{t}\,(a_0 + a_1 t + ...)$$

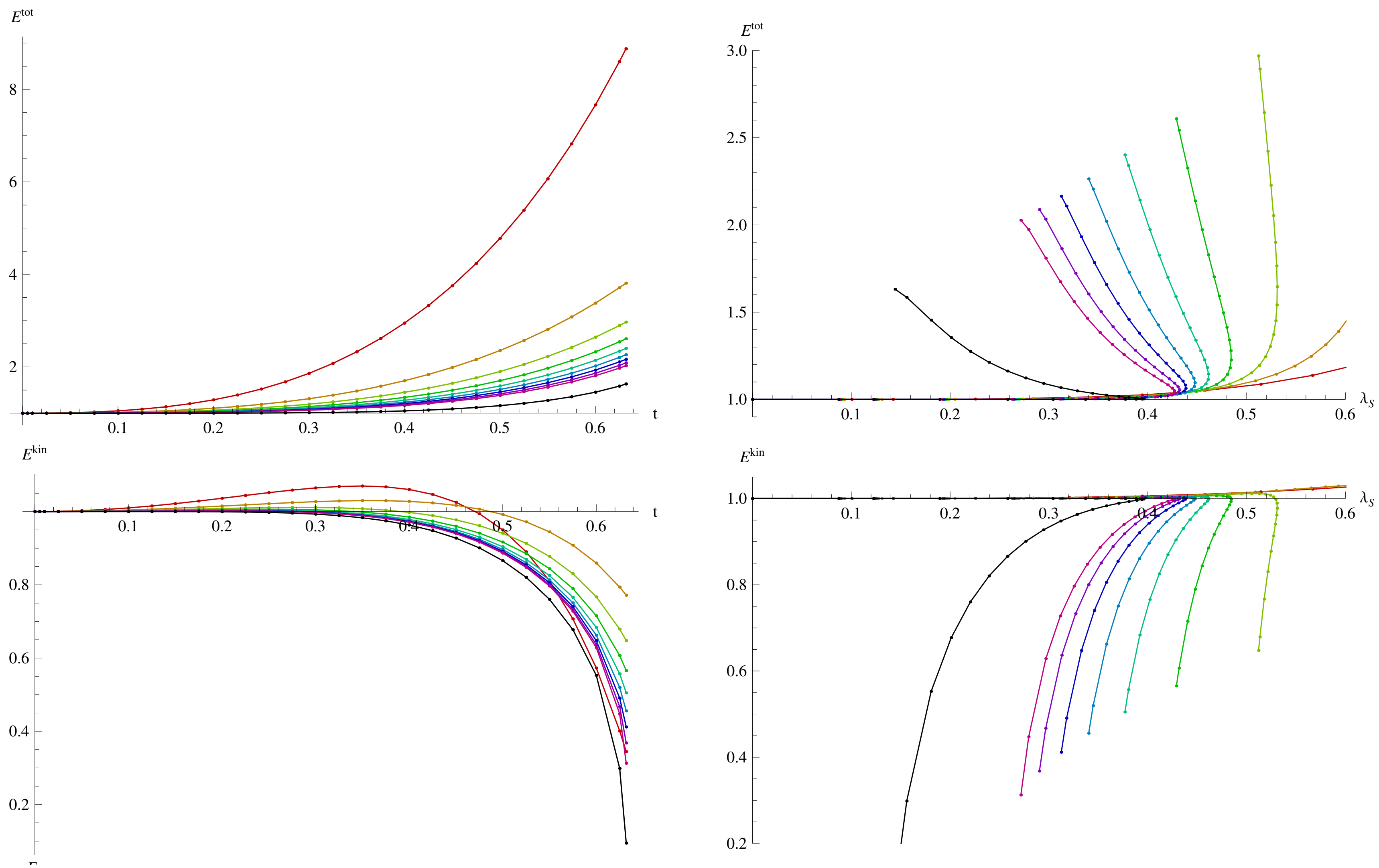
Determine the ***a***’s by best fit



The curve is insensitive of how many ***a***’s one chooses



- However at finite level the full equation of motion is not well satisfied after the marginal field starts decreasing…



- ***ISSUE***: Should we just go to (very much) higher level to flatten the potential **OR** Feynman-Siegel gauge is too restrictive for a full solution, in the large moduli region???

- ***Unfortunately this question cannot be answered in level truncation at least not today.***

Conclusions

- OSFT as a theory for D-branes. D-branes are open string's solitons (***$1/g^2$***)
- "Far" D-branes moduli are reachable but not through the Goldstone modes, rather by the ***whole string field*** . High level fields are essential. (This is a *field* rather than *string* phenomenon: *cfr* ϕ^3 *toy model by Zwiebach, 2000 and Kudrna-CM, 2016*)
- Curiously the tachyon mode describes the moduli space much better than the Goldstone mode. ***Simple physical reason?***
- ***Ultimate goal***: can we ***discover*** new D-branes by solving the OSFT equations? (D-branes moduli could have been discovered from OSFT, both numerically and analytically)
- Level Truncation: is Feynmann-Siegel gauge too restrictive for "far away" backgrounds??
- Can we classify OSFT solutions from string-field-theoretic first-principles (cfr ADHM)??
- A small step forward: ***topological defects*** in OSFT (*Kojita, CM, Masuda, Schnabl*)
- Don't forget the ***really ultimate*** goal: ***Open Super String Field Theory path integral could give a non perturbative definition of String Theory (at least when D-branes are around)***

Grazie!