Physics beyond the Standard Model with trapped atoms in the LHC era

M. Anholm[1], D. Ashery[2], O. Aviv[2], S. Behling [3], J.A. Behr[4], I. Cohen[2] , B. Fenker[3], A. Gorelov[4], G. Gwinner[5], K.P. Jackson[4], T. Kong[1], D. Melconian[3], M.R. Pearson[4], R.A. Pitcairn[1], P. Shidling[3], M. Mehlman[3]

1. Department of Physics, University of British Columbia, Vancouver, British Columbia, Canada
2. School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel
3. Department of Physics, Texas A & M University, College Station, Texas, USA
4. TRIUMF, Vancouver, British Columbia, Canada
5. Department of Physics, University of Manitoba, Winnipeg, Manitoba, Canada

$SM\ \ \beta{-}decay$	$Beyond\ \ SM\ \ \beta{-}decay$
$V-A\ \ interactions$	$S,\ \ T\ \ interactions$
$Left-handed$	$Right-handed$
$Time\ \ Reversal\ \ Conserving$	$Time\ \ Reversal\ \ Violation$

Will Discuss:

S interactions
Right - Handed ν
Time Reversal Violation

β-decay rate (Jackson, Treiman, Wyld 1957):

$$dW =$$

$$dW_o(1 + \frac{\vec{p_\beta} \cdot \vec{p_\nu}}{E_\beta E_\nu} a_{\beta\nu} + \frac{\Gamma m_e}{E_\beta} b + \frac{\vec{J}}{J} \cdot [\frac{\vec{p_\beta}}{E_\beta} A_\beta + \frac{\vec{p_\nu}}{E_\nu} B_\nu + \frac{\vec{p_\beta} \times \vec{p_\nu}}{E_\beta E_\nu} D]$$

$$+c[\frac{\vec{p_\beta} \cdot \vec{p_\nu}}{3E_\beta E_\nu} - \frac{(\vec{p_\beta} \cdot \vec{j})(\vec{p_\nu} \cdot \vec{j})}{E_\beta E_\nu}][\frac{J(J+1) - 3 < (\vec{J} \cdot \vec{j})^2 >}{J(2J-1)}])$$

Allows for: V - A, Scalar, Tensor Interactions

Left, Right-handed currents

Time-reversal violation

$a_{\beta\nu},\ b,\ c,\ A_\beta,\ B_\nu,\ D$: values predicted by the Standard Model

Recent review: S. Severijins and M. Beck, Rev. Mod. Phys. 78 991 (2006)

Measurements feasible using Atom traps and Radioactive Beams.

$$\begin{aligned}
\xi &= |M_F|^2(|C_S|^2+|C_V|^2+|C'_S|^2+|C'_V|^2)+|M_{GT}|^2(|C_T|^2+|C_A|^2+|C'_T|^2+|C'_A|^2) \\
a_{\beta\nu}\xi &= |M_F|^2(-|C_S|^2+|C_V|^2-|C'_S|^2+|C'_V|^2)+\frac{|M_{GT}|^2}{3}(|C_T|^2-|C_A|^2+|C'_T|^2-|C'_A|^2) \\
b\xi &= \pm 2Re[|M_F|^2(C_S C_V^*+C'_S C'^*_V)+|M_{GT}|^2(C_T C_A^*+C'_T C'^*_A)] \\
c\xi &= |M_{GT}|^2\Lambda_{J'J}(|C_T|^2-|C_A|^2+|C'_T|^2-|C'_A|^2) \\
A_\beta\xi &= 2\mathrm{Re}[\pm|M_{GT}|^2\lambda_{J'J}(C_T C'^*_T-C_A C'^*_A)+\delta_{J'J}|M_{GT}||M_F|\sqrt{J/(J+1)}(C_S C'^*_T \\
&\quad +C'_S C^*_T-C^*_V C'^*_A-C'_V C^*_A)] \\
B_\nu\xi &= 2\mathrm{Re}\{|M_{GT}|^2\lambda_{J'J}[\frac{m_e}{E_e}(C_T C'^*_A+C'_T C^*_A)\pm(C_T C'^*_T+C_A C'^*_A)] \\
&\quad -\delta_{J'J}|M_{GT}||M_F|\sqrt{J/(J+1)}\times[(C_S C'^*_T+C'_S C^*_T+C_V C'^*_A+C'_V C^*_A) \\
&\quad \pm\frac{m}{E_e}(C_S C'^*_A+C'_S C^*_A+C_V C'^*_T+C'_V C^*_T)]\} \\
D\xi &= 2\mathrm{Im}\{\delta_{JJ'}|M_F||M_{GT}|\sqrt{\frac{J}{J+1}}(C_S C^*_T+C'_S C'^*_T-C_V C^*_A-C'_V C'^*_A)\}
\end{aligned}$$

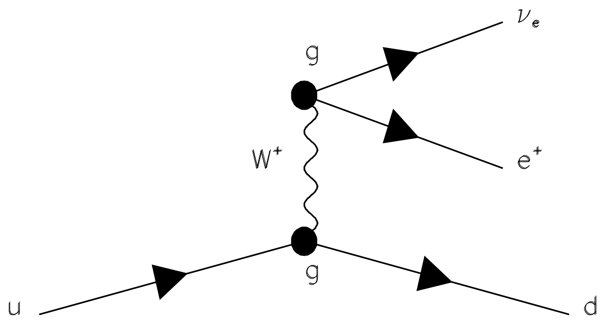
$$\lambda_{J'J}=\begin{cases} 1, & J\to J'=J-1 \\ \frac{1}{J+1}, & J\to J'=J \\ -\frac{J}{J+1}, & J\to J'=J+1 \end{cases}$$

$$\Lambda_{J'J}=\begin{cases} 1, & J\to J'=J-1 \\ -\frac{2J-1}{J+1}, & J\to J'=J \\ \frac{J(2J-1)}{(2J+3)(J+1)}, & J\to J'=J+1 \end{cases}$$

C_i: Interaction Amplitudes (complex)

$$
\begin{aligned}
C_V &= g_V(a_{LL} + a_{LR} + a_{RR} + a_{RL}) & C_V' &= g_V(a_{LL} + a_{LR} - a_{RR} - a_{RL}) \\
C_A &= g_A(a_{LL} - a_{LR} + a_{RR} - a_{RL}) & C_A' &= g_A(a_{LL} - a_{LR} - a_{RR} + a_{RL}) \\
C_S &= g_S(A_{LL} + A_{LR} + A_{RR} + A_{RL}) & C_S' &= g_S(A_{LL} + A_{LR} - A_{RR} - A_{RL}) \\
C_T &= 2g_T(\alpha_{LL} + \alpha_{RR}) & C_T' &= 2g_T(\alpha_{LL} - \alpha_{RR})
\end{aligned}
$$

g_i **: Hadronic Form Factors** a_{ij} **: Chirality coupling constants** i **:** ν j **: quark**



Standard Model: V - A, left handed

g_V **= 1,** g_A **= -1.27 (n decay)**

$a_{LL} = V_{ud}\frac{g^2}{8M_W^2} \cong 8 \cdot 10^{-6} GeV^{-2}$

$a_{ij}, A_{i,j}, \alpha_{i,j} = 0 \quad i,j \neq L,L$

$a_{\beta\nu} = \frac{y^2 - \frac{1}{3}}{y^2+1}, \quad y = \frac{C_V M_F}{C_A M_{GT}}$

$b = 0$

$c = \frac{-\Lambda_{JJ'}}{1+y^2}$

$A_\beta = \frac{\mp\lambda_{JJ'} - 2\delta_{JJ'} y \sqrt{J/(J+1)}}{y^2+1}$

$B_\nu = \frac{\pm\lambda_{JJ'} - 2\delta_{JJ'} y \sqrt{J/(J+1)}}{y^2+1}$

$D, R = 0$

Limits on Scalar Boson Interaction

$$dW =$$

$$dW_o(1 + \frac{\vec{p_\beta} \cdot \vec{p_\nu}}{E_\beta E_\nu} a_{\beta\nu} + \frac{m_e}{E_\beta} b + \frac{\vec{J}}{J} \cdot [\frac{\vec{p_\beta}}{E_\beta} A_\beta + \frac{\vec{p_\nu}}{E_\nu} B_\nu + \frac{\vec{p_\beta} \times \vec{p_\nu}}{E_\beta E_\nu} D]$$

$$+c[\frac{\vec{p_\beta} \cdot \vec{p_\nu}}{3E_\beta E_\nu} - \frac{(\vec{p_\beta} \cdot \vec{j})(\vec{p_\nu} \cdot \vec{j})}{E_\beta E_\nu}][\frac{J(J+1) - 3 < (\vec{J} \cdot \vec{j})^2 >}{J(2J-1)}])$$

For pure Fermi $0^+ \to 0^+$ decay $\beta - \nu$ angular correlation:

$$P(\theta) = 1 + b\frac{\mathrm{m}_\beta}{\mathrm{E}_\beta} + a_{\beta\nu}\frac{\mathrm{v}_\beta}{\mathrm{c}} cos(\theta)$$

$$a_{\beta\nu} = 1 - 4\frac{g_S^2}{g_V^2}\left(|a_L^S|^2 + |a_R^S|^2\right) \qquad b = \pm\frac{g_S}{g_V}\frac{\mathcal{R}e(a_{LL}a_R^S)}{|a_{LL}|^2}$$

$$a_L^S = A_{LL} + A_{LR} \qquad a_R^S = A_{RR} + A_{RL}$$

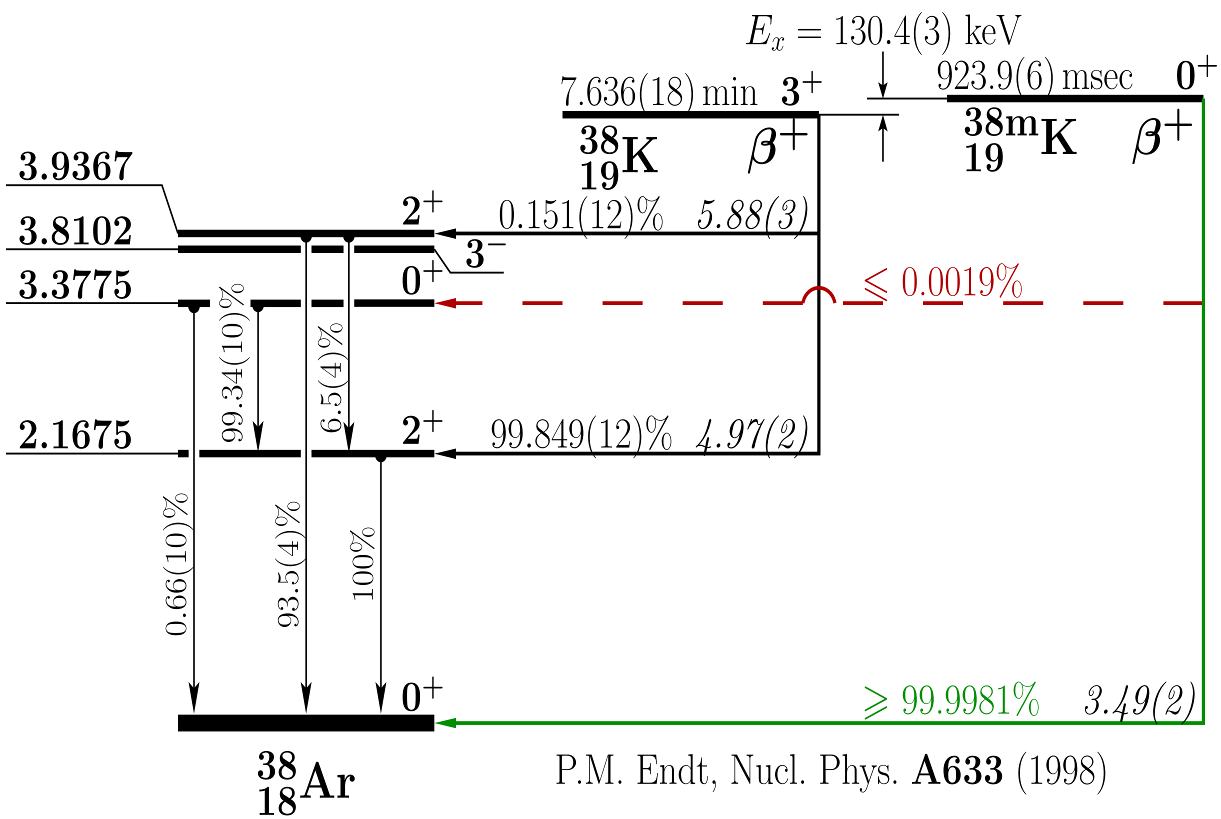
SM: b = 0, $a_{\beta\nu}$ = 1.0.

$C_S + C_S' \leq$ **0.001 in MSSM, Profumo et al., PRD 75 075017**

$g_s = 1.02 \pm 0.11$ **Gonzalez-Alonso and Camalich PRL 112 042501 (2014)**

Measurement of $\beta - \nu$ Angular Correlation in $^{38m}K \xrightarrow{\beta^+} {}^{38}Ar$

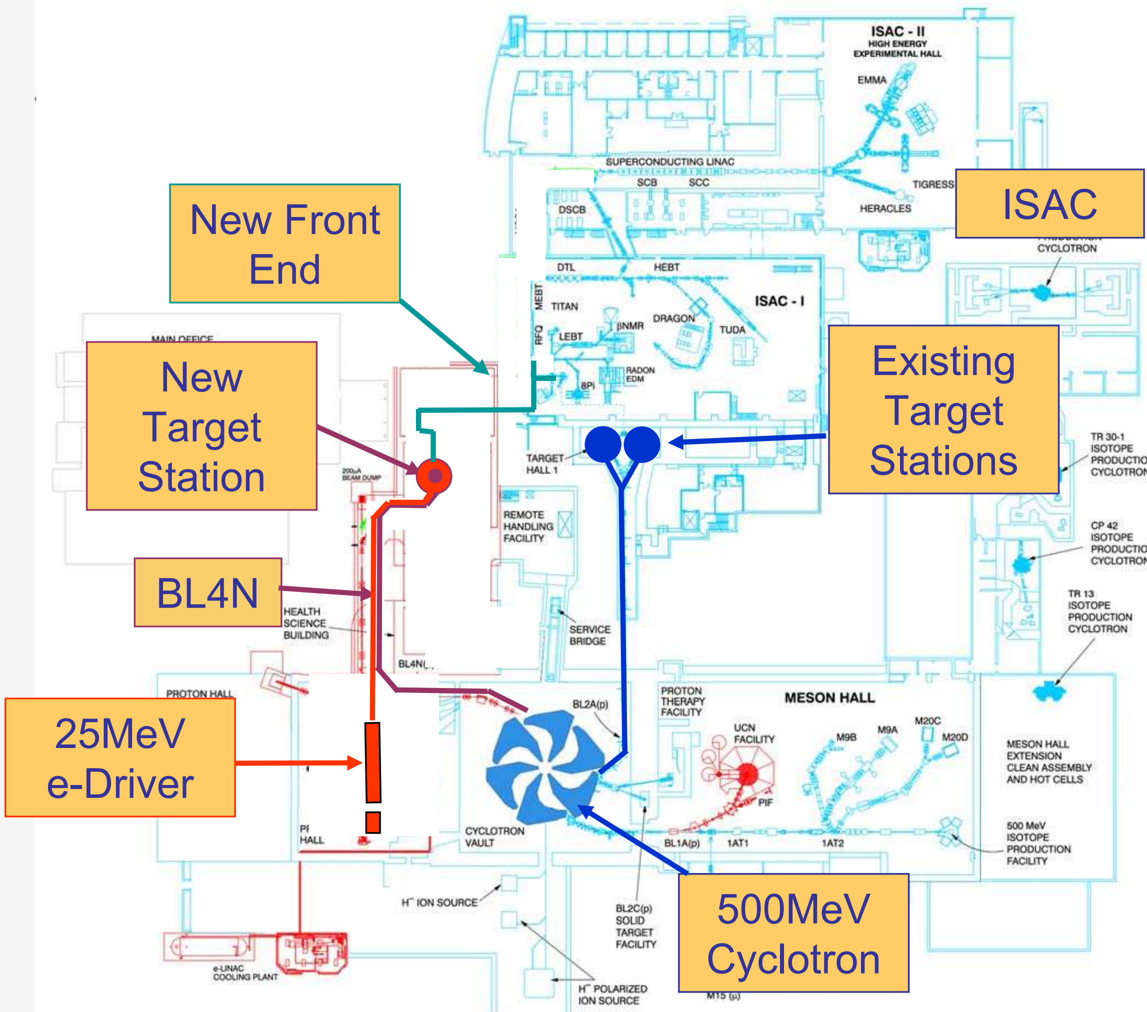
$$Q(^{38m}_{19}\mathrm{K}) = 5.02234(12)\ \mathrm{MeV}$$



P.M. Endt, Nucl. Phys. **A633** (1998)

TRIUMF

Future (2010-2015)



22

Proposal:

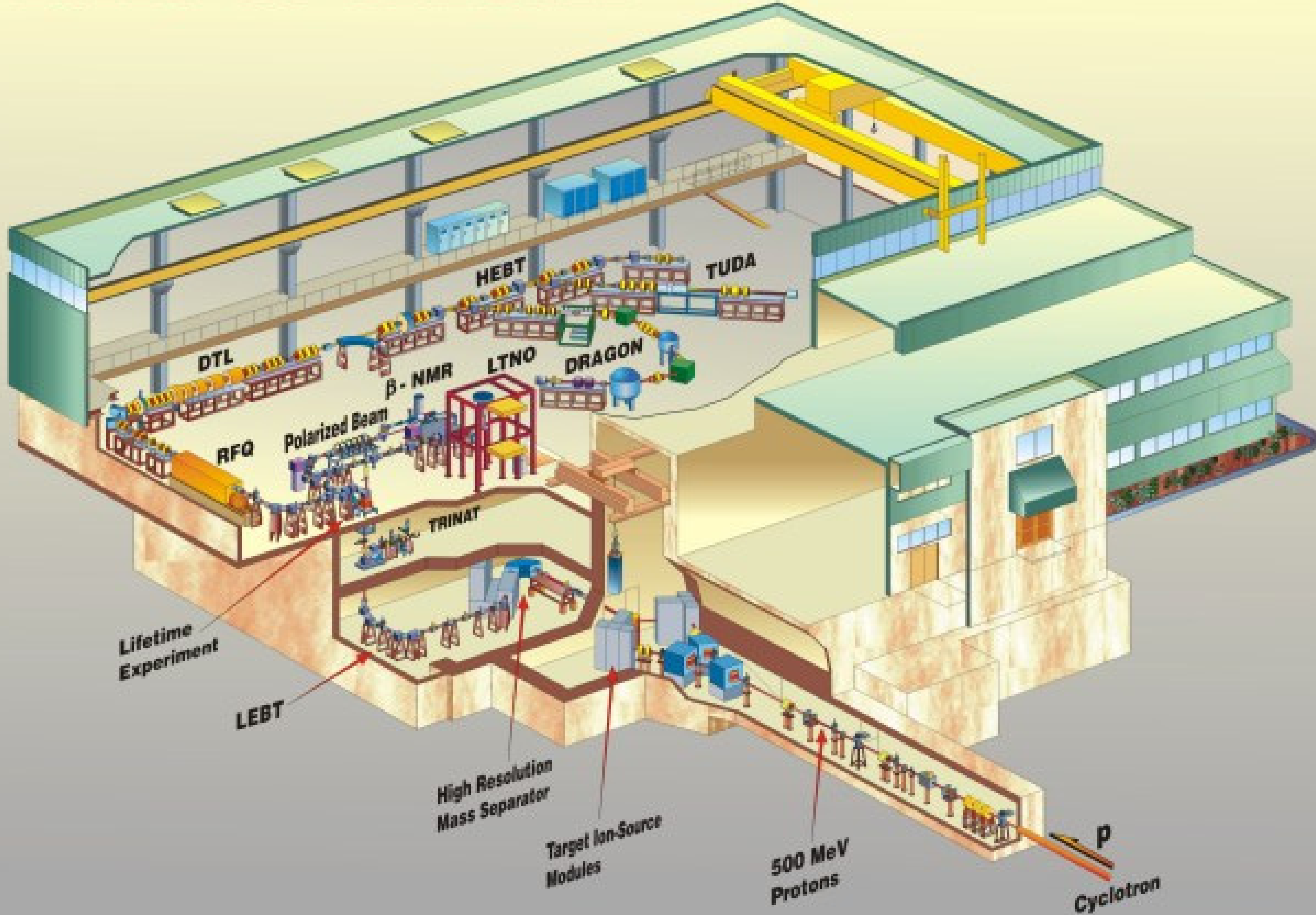
By 2013:

- **Add a 25MeV electron driver to supply electrons to one new target**
- **Add a new ISAC front-end to deliver a second RIB beam to ISAC**

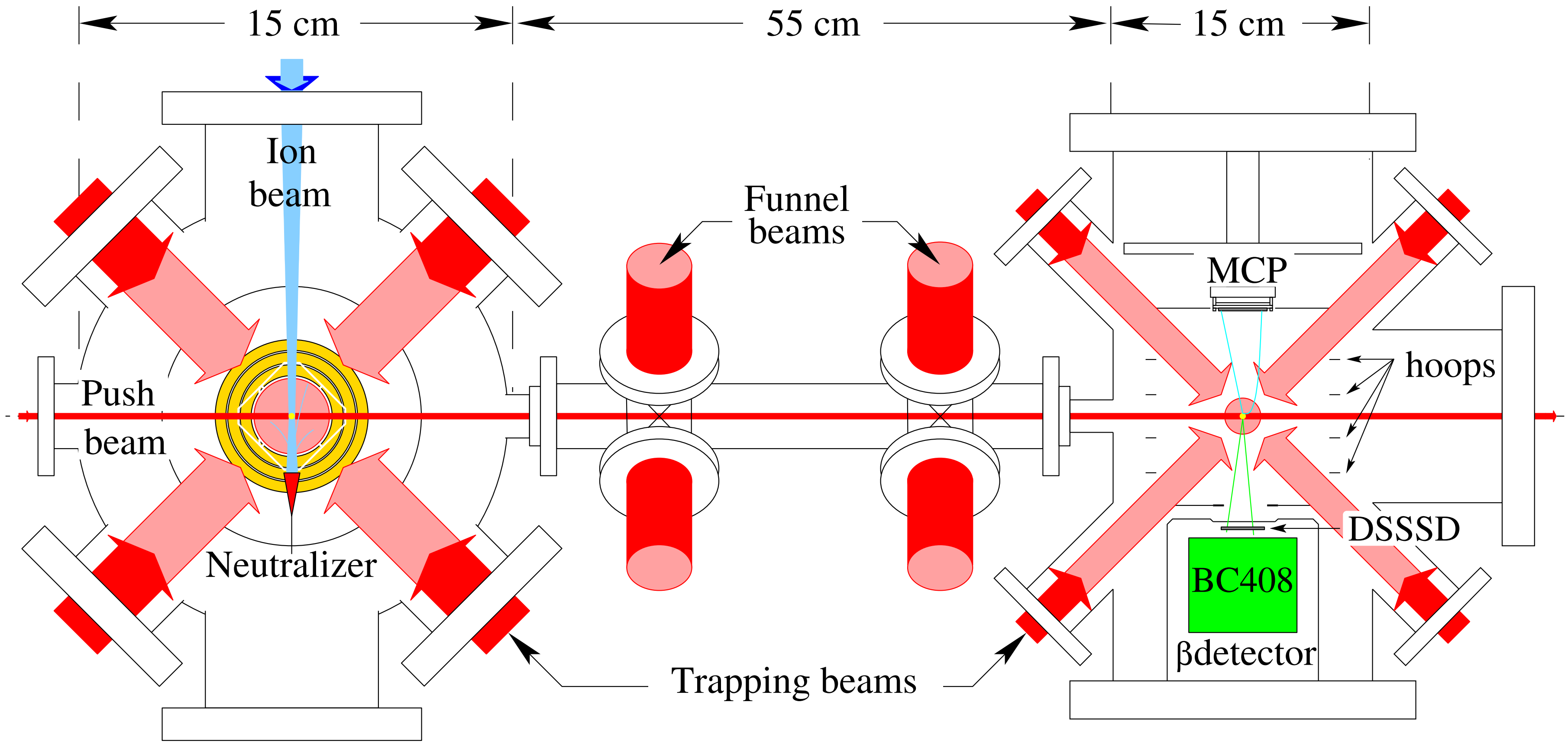
By 2015:

- **Add a new beam line from the cyclotron to deliver 500MeV protons to the new target**

ISAC at TRIUMF



TRINAT DOUBLE MOT TRAPPING SYSTEM

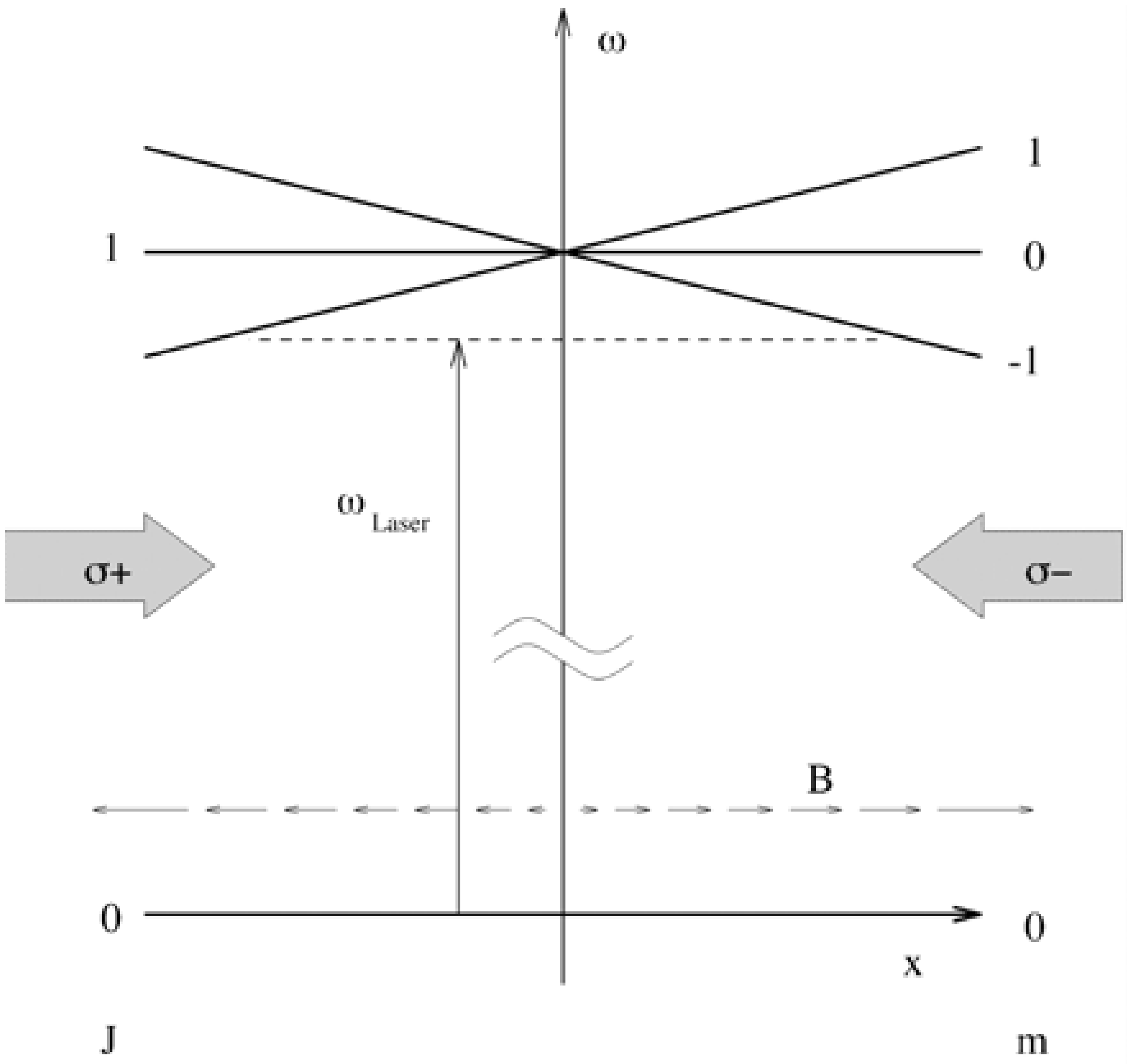


Collection chamber

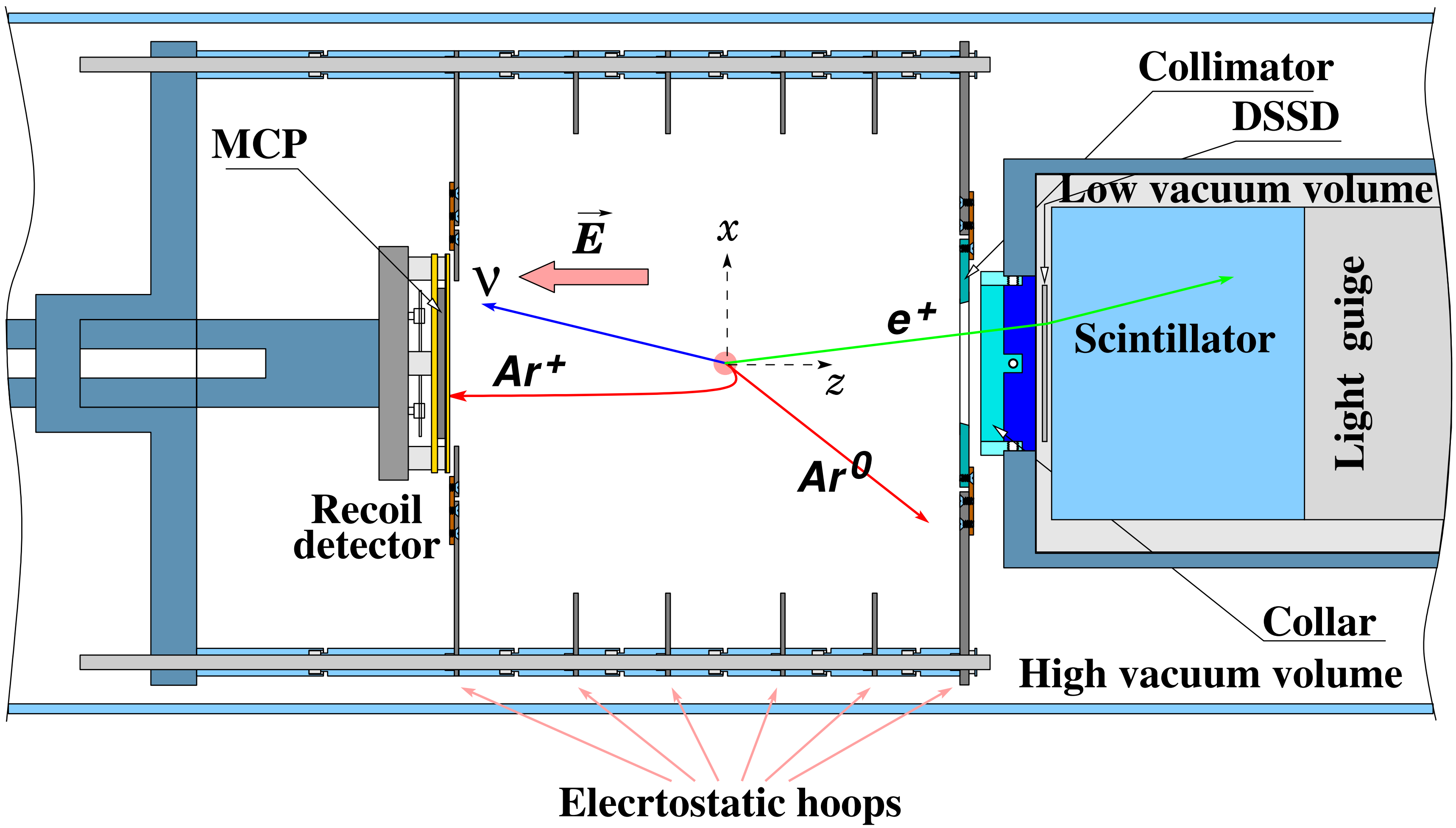
- 95% $^{38gs}K^{+}$($t_{1/2}$= 7.64 min) + 5% $^{38m}K^{+}$($t_{1/2}$= 0.924 s)
- neutralization of $^{38}K^{+}$
- vapor cell trap
- 10^{-8} Torr
- 0.1% of ^{38m}K trapped
- 75% of trapped ^{38m}K moved

Detection chamber

- 100% ^{38m}K, $t_{1/2}$= 0.924 s
- retrap from atomic beam
- $3{\cdot}10^{-10}$ Torr, $t_{1/2}^{trap}$= 30 s
- 0.75 mm FWHM trap size
- 2000 atoms in trap
- photoionization of ^{38m}K

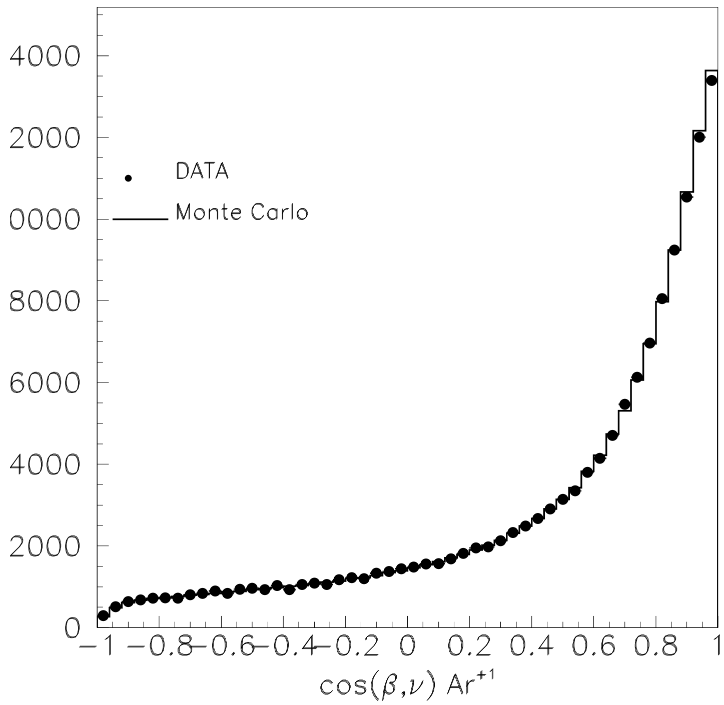

ω
1
1
0
-1
ω Laser
σ+
σ−
B
0
0
x
J
m

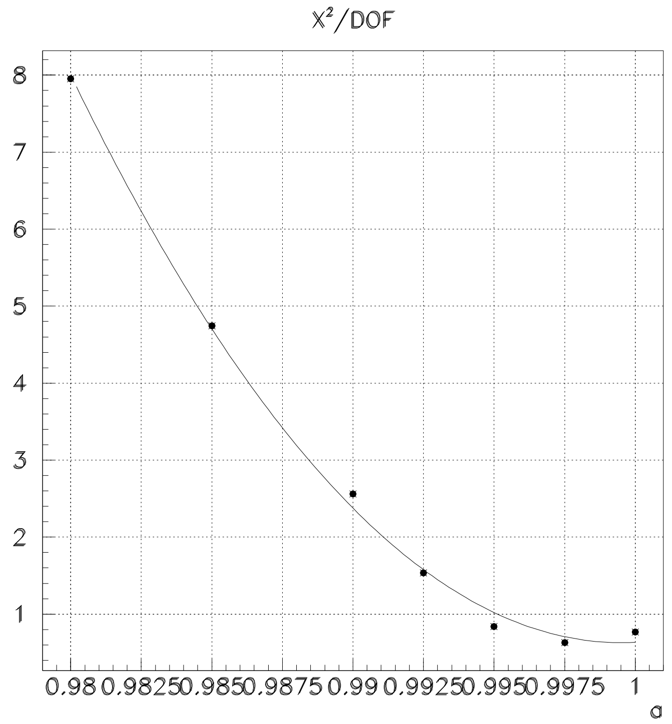
TRINAT DETECTION SYSTEM FOR ^{38m}K DECAY



- High recoil collection and detection efficiencies due to $\boldsymbol{E}$-field
- Coincident detection of e^+ and recoils back-to-back
- Position information both from e^+ and recoil detectors
- Possibility to measure p_e and p_{recoil} and using them to determine p_ν.
- Chamber geometry suppresses recoiling ion detection from decays on walls and electrostatic hoops

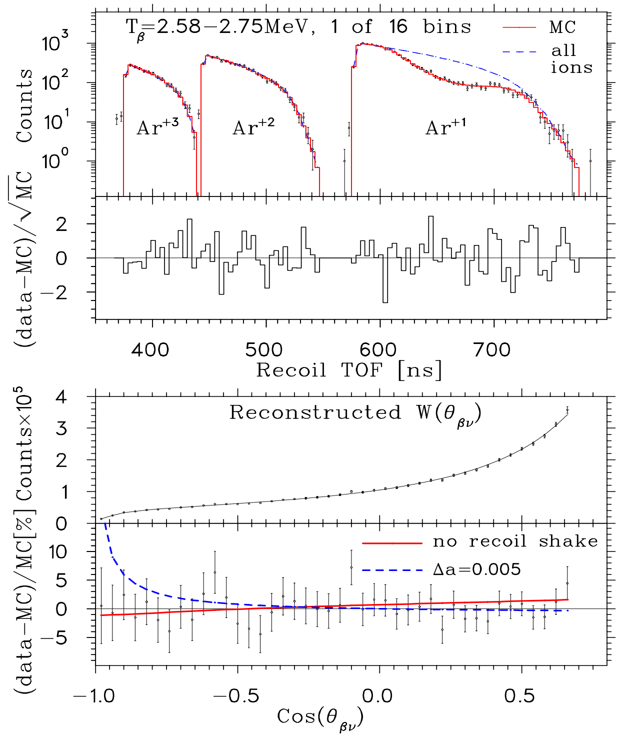
Resulting $\beta - \nu$ Angular Correlation





Results: A. Gorelov *et al.*, PRL 94, 142501 (2005)

$$P(\theta) = 1 + b\frac{m_\beta}{E_\beta} + a_{\beta\nu}\frac{v_\beta}{c}cos(\theta)$$



For $|b| < 0.04$, $\langle E_\beta \rangle$=3.3 MeV

Define:

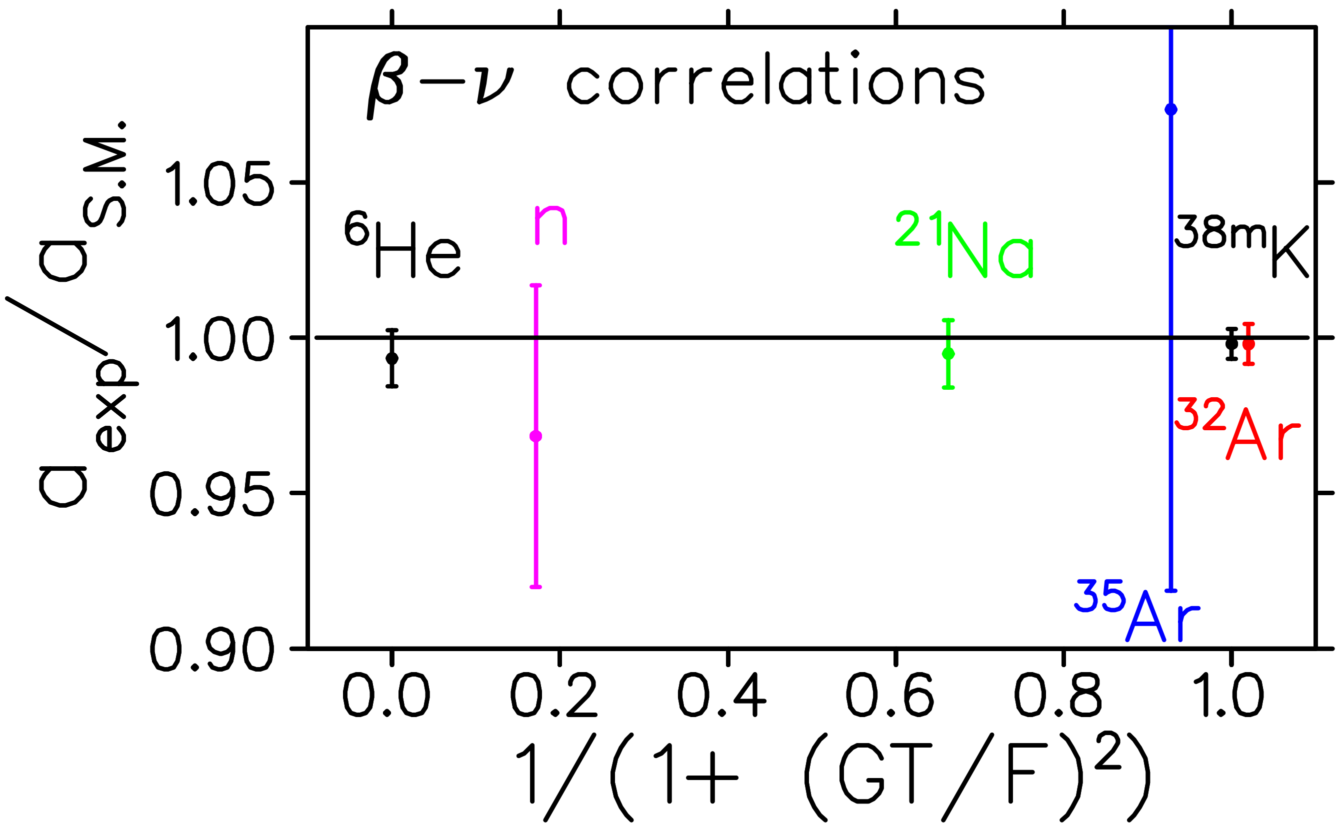
$$\tilde{a} = \frac{a_{\beta\nu}}{1+b\frac{m_\beta}{\langle E_\beta \rangle}}$$

$$\tilde{a} = 0.9981 \pm 0.0030^{+0.0032}_{-0.0037}$$

In agreement with the Standard Model.

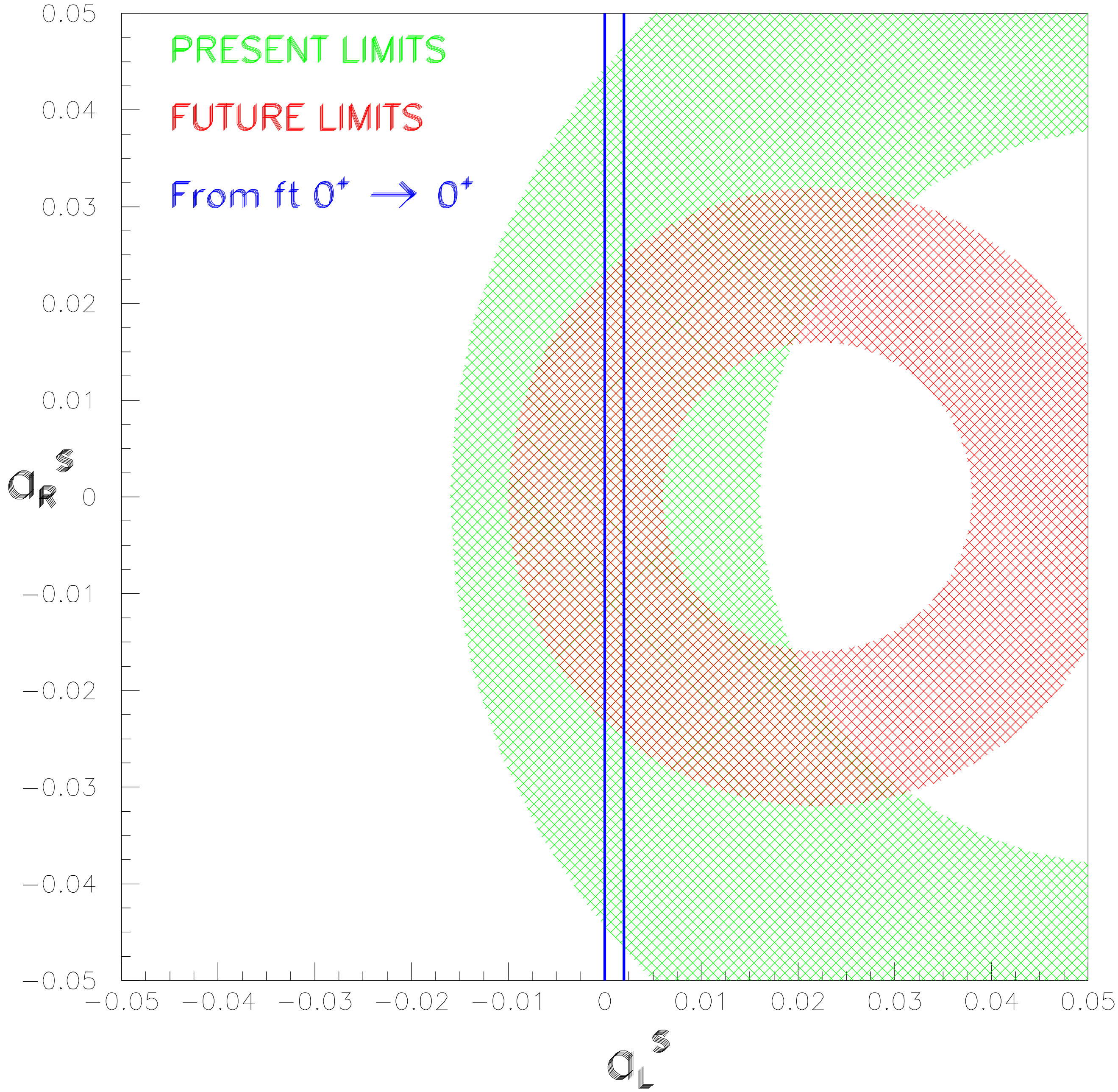
$M_S > 250 GeV$ with $g_S \sim 1$

Summary of results for a

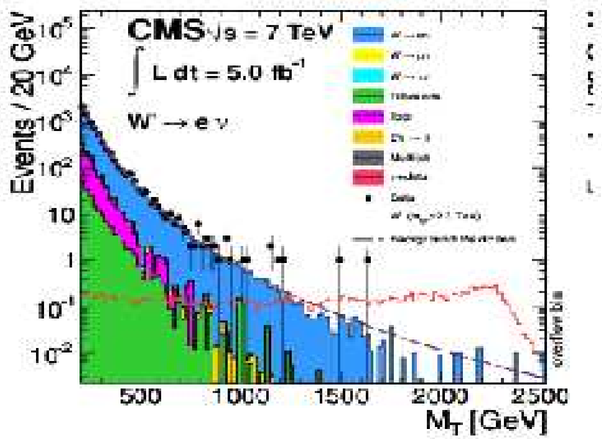


^{32}Ar: E. G. Adelberger et al., Phys. Rev. Lett. 83, 1299(1999)
^{38m}K: A. Gorelov *et al.*, PRL 94, 142501 (2005)
^{21}Na: P.A. Vetter et al., Phys. Rev. C77, 035502 (2008)

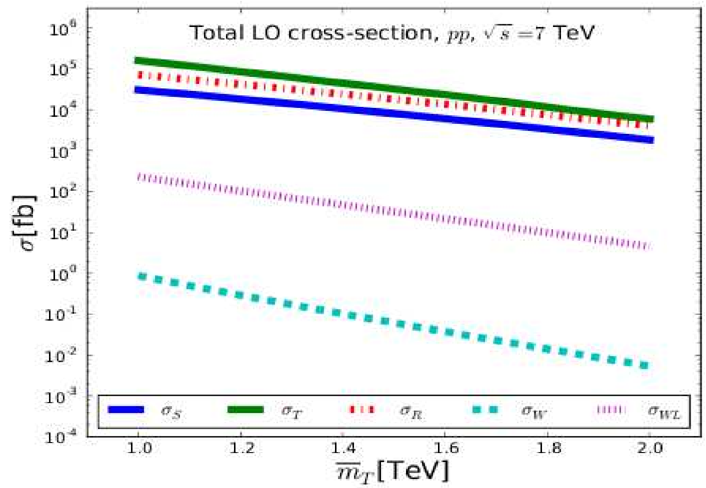
Limits on Scalar Interaction



Limits from the LHC: $pp \to e + m_T + X$



CMS Col. JHEP 08 (2012) 023



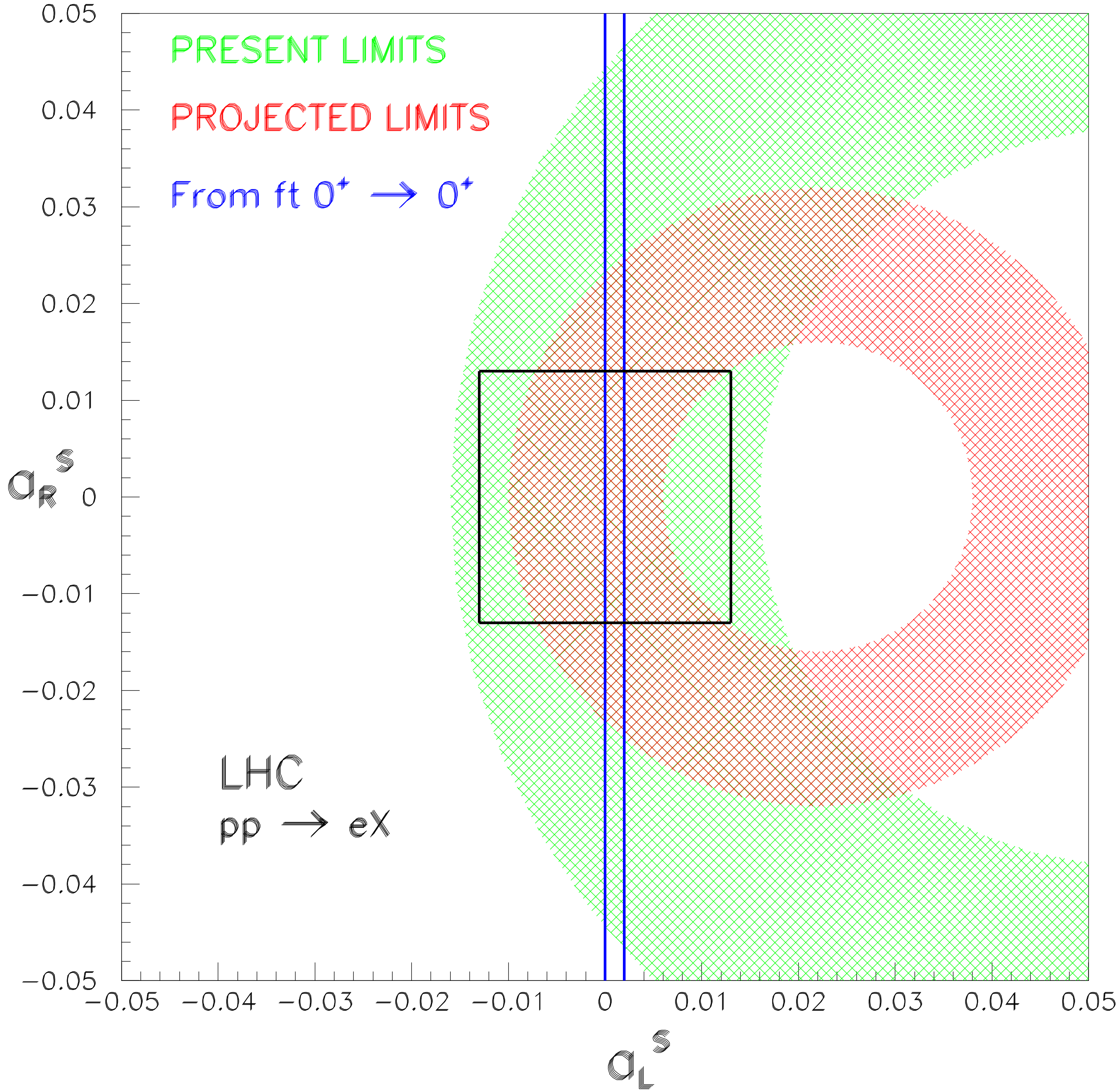
V. Cirigliano *et. al.* JHEP 02 (2013) 046

Data accounted for by known procecces (W,Z etc. decays)

$\sigma(m_T > \overline{m_T}) = f(\sigma_i, a_{i,j})$, i,j = V, A, S, T, L, R

Upper limits from data at $m_T = 1.2\ TeV$ on $a_{i,j}$. For Scalar: $|a_L^S|, |a_R^S| < 1.3 \cdot 10^{-2}$

Problem of scale in comparing with β decay.


PRESENT LIMITS
PROJECTED LIMITS
From ft 0+ → 0+
LHC
pp → eX
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.05 −0.04 −0.03 −0.02 −0.01 0 0.01 0.02 0.03 0.04 0.05
a_R^S
a_L^S

Observables with Polarized Nuclei

$$dW =$$

$$dW_o(1 + \frac{\vec{p_\beta} \cdot \vec{p_\nu}}{E_\beta E_\nu} a_{\beta\nu} + \frac{\Gamma m_e}{E_\beta} b + \frac{\vec{J}}{J} \cdot [\frac{\vec{p_\beta}}{E_\beta} A_\beta + \frac{\vec{p_\nu}}{E_\nu} B_\nu + \frac{\vec{p_\beta} \times \vec{p_\nu}}{E_\beta E_\nu} D]$$

$$+c[\frac{\vec{p_\beta} \cdot \vec{p_\nu}}{3E_\beta E_\nu} - \frac{(\vec{p_\beta} \cdot \vec{j})(\vec{p_\nu} \cdot \vec{j})}{E_\beta E_\nu}][\frac{J(J+1) - 3 < (\vec{J} \cdot \vec{j})^2 >}{J(2J-1)}])$$

Asymmetry $= \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$

$\vec{J} \parallel \vec{P_\beta} \Longrightarrow$ **measure** A_β **(**β **singles or coin. with recoil)**

$\vec{J} \perp \vec{P_\beta}, \quad \vec{P_\nu} = \vec{P_R} - \vec{P_\beta} \quad \Longrightarrow dW \propto \frac{\vec{J}}{J} \cdot \left[B_\nu \vec{P_R} + D \frac{(\vec{P_\beta} \times \vec{P_R})}{E_\beta} \right]$

Measure B_ν **from Recoil Asymmetry in** $\vec{P_R} \parallel \vec{J}$ **plane**

Measure D **from Recoil Asymmetry in** $\vec{P_R} \perp \vec{J}$ **plane**

Right-handed Currents

$$|W_L >= cos\zeta|W_1 > -sin\zeta|W_2 >$$

$$|W_R >= sin\zeta|W_1 > +cos\zeta|W_2 >$$

Define: $x = (M_L/M_R)^2 - \zeta$ **and** $y = (M_L/M_R)^2 + \zeta$

$\lambda \equiv g_A M_{GT}/g_V M_F$

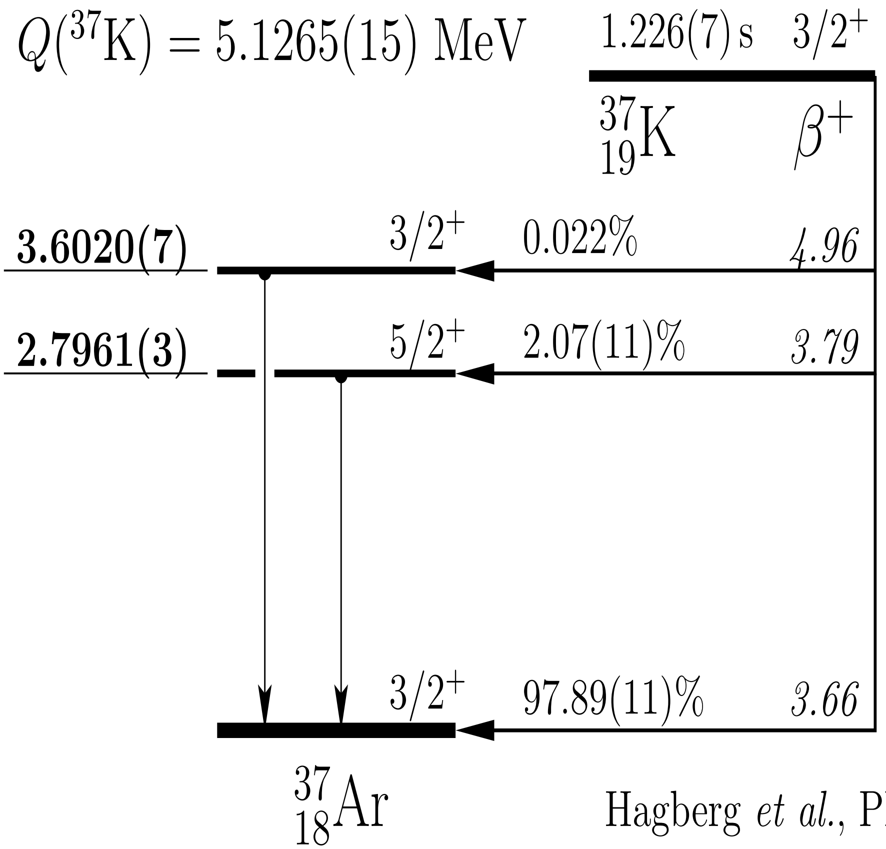
$$A_\beta = \frac{-2\lambda}{1+\lambda^2}\left[\frac{\lambda(1-y^2)}{5(1+y^2)} - (1-xy)\sqrt{\frac{3(1+x^2)}{5(1+y^2)}}\,\right]$$

$$B_\nu = \frac{-2\lambda}{1+\lambda^2}\left[\frac{\lambda(1-y^2)}{5(1+y^2)} + (1-xy)\sqrt{\frac{3(1+x^2)}{5(1+y^2)}}\,\right]$$

$$R_{slow} \equiv \frac{dW(\vec{J}\cdot\vec{p}_\beta = -1)}{dW(\vec{J}\cdot\vec{p}_\beta = +1)} = \frac{1-a-2c/3-(A+B)}{1-a-2c/3+(A+B)} = y^2$$

Measurement of $\beta - \nu$ Angular Correlation in Polarized $^{37}K \xrightarrow{\beta^+} {}^{37}Ar$

$Q(^{37}\mathrm{K}) = 5.1265(15)$ MeV



Hagberg *et al.*, PRC **56** (1997)

$M_{GT}/M_F = -0.5874(71)$

Severijns *et al.*, PRC **78** (2008)

Coefficients of $\beta - \nu$ Angular Correlation in Polarized $^{37}K \xrightarrow{\beta^+} {}^{37}Ar$

Calculated with the Standard Model assuming

$\lambda \equiv g_A M_{GT}/g_V M_F = -0.5754 \pm 0.0018$

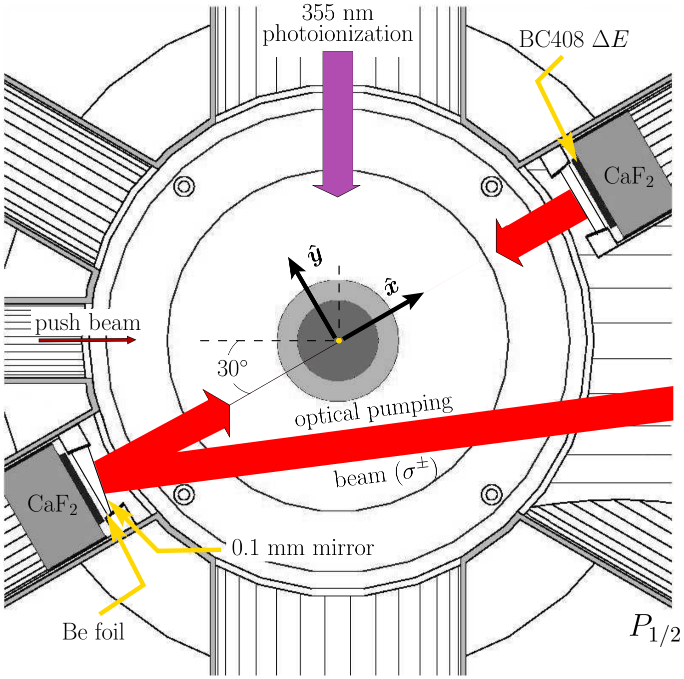
Maximal Parity Violation

observable	$a_{\beta\nu}$	A_β	B_ν	c
value	0.6683	–0.5702	–0.7692	0.1990
error[1]	0.0013	0.0005	0.0013	0.0008

[1] **Due to error in** λ

$$b = D = R_{slow} = 0$$

Optical Pumping



$\hat{x}$ = polarization axis

= phoswich detector axis

$\hat{z}$ = MCP – β–telescope axis

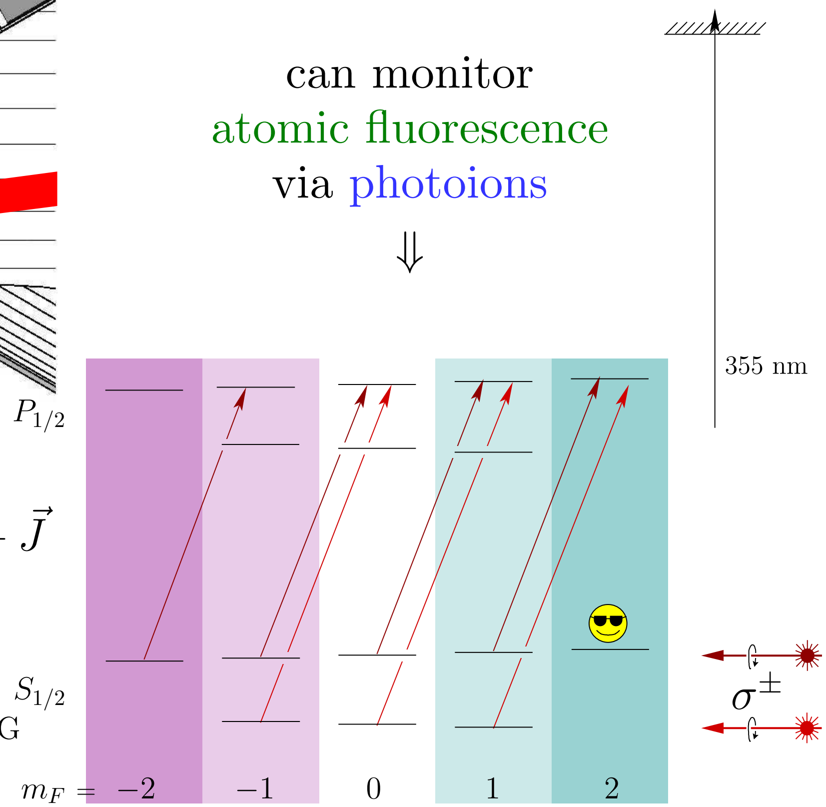
can monitor
atomic fluorescence
via photoions
⇓

$$\vec{F} = \vec{I} + \vec{J}$$

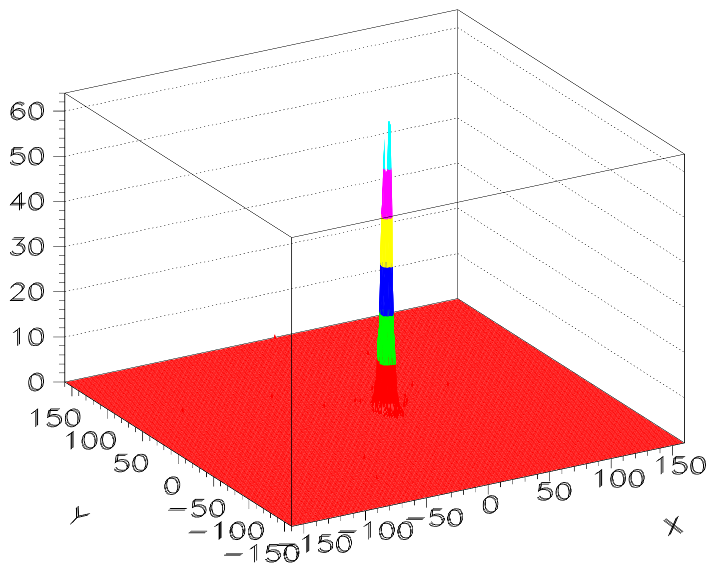
$I = \frac{3}{2}$

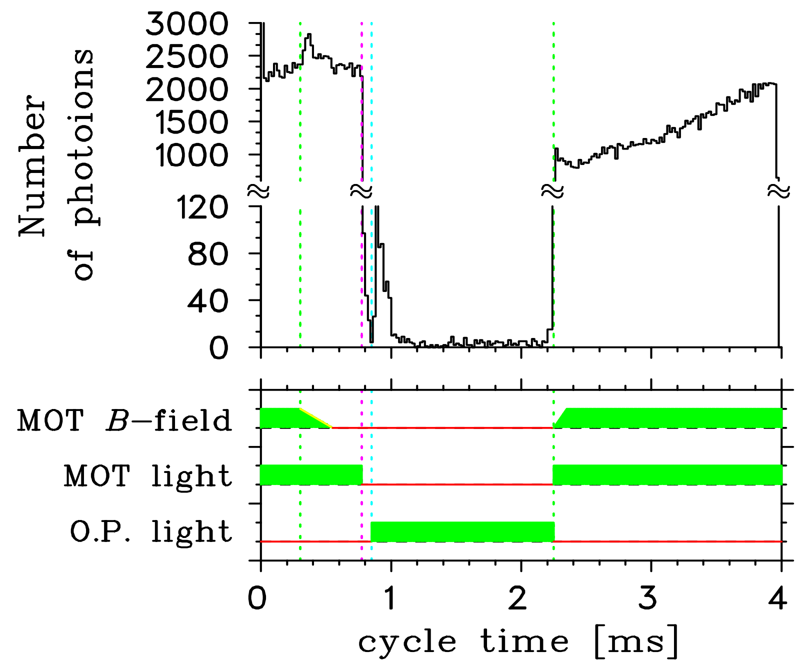
$J = \frac{1}{2}$

$\vec{B}_{OP} = 2.5$ G



Determination of the Polarization

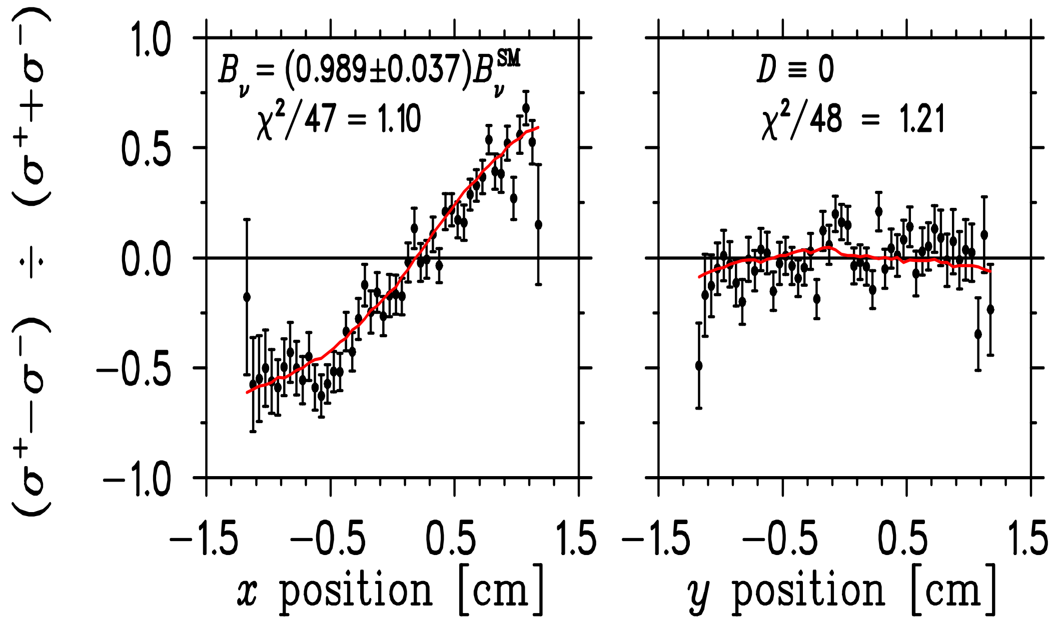




Photoions detected in MCP

Trap Cycle

Measure B_ν from Recoil Asymmetry in $\hat{x} - \hat{z}$ plane
Measure D from Recoil Asymmetry in $\hat{y} - \hat{z}$ plane

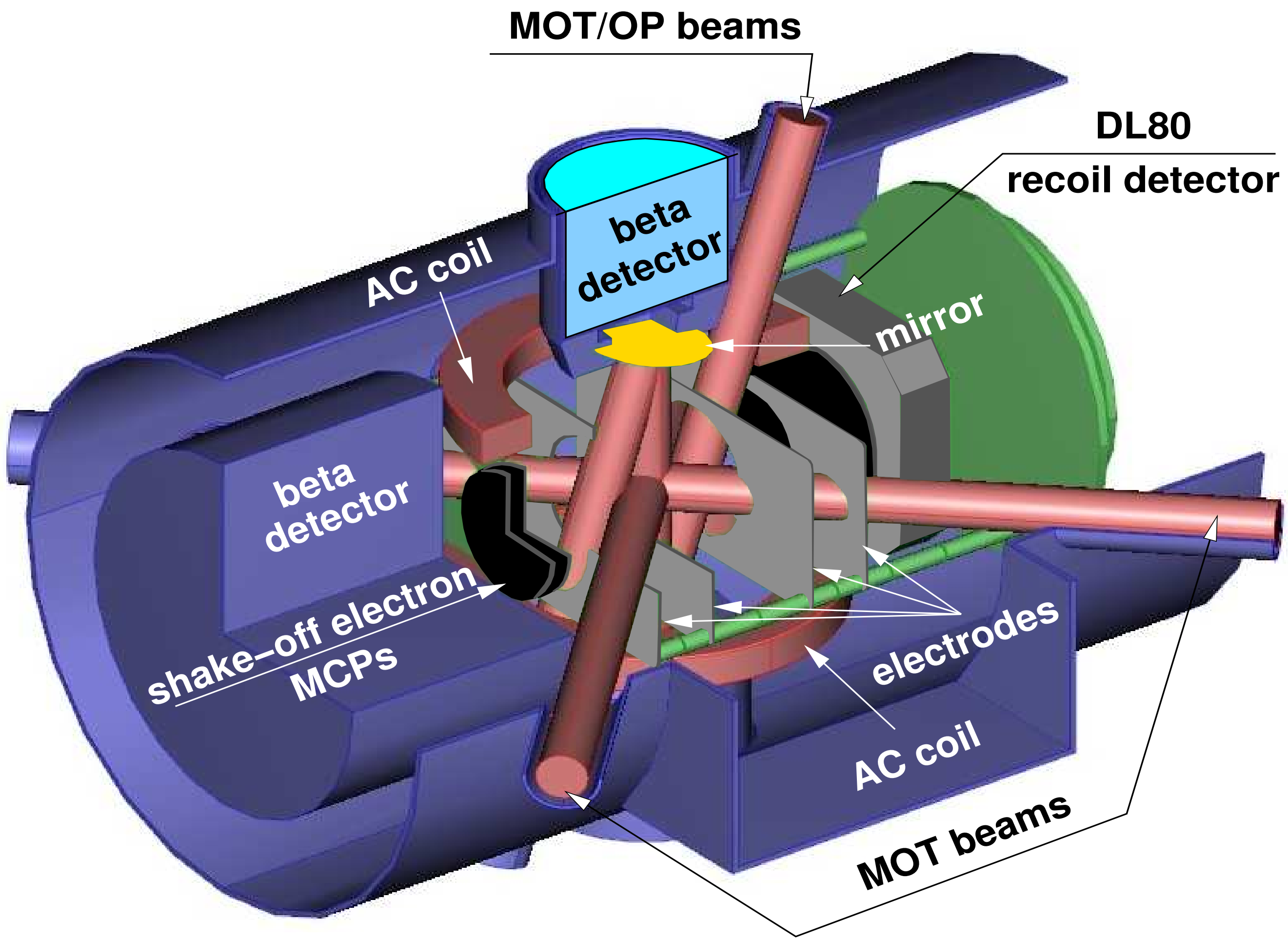


$$B_\nu \quad = \quad 0.755 \pm 0.020(stat) \pm 0.013(syst)$$

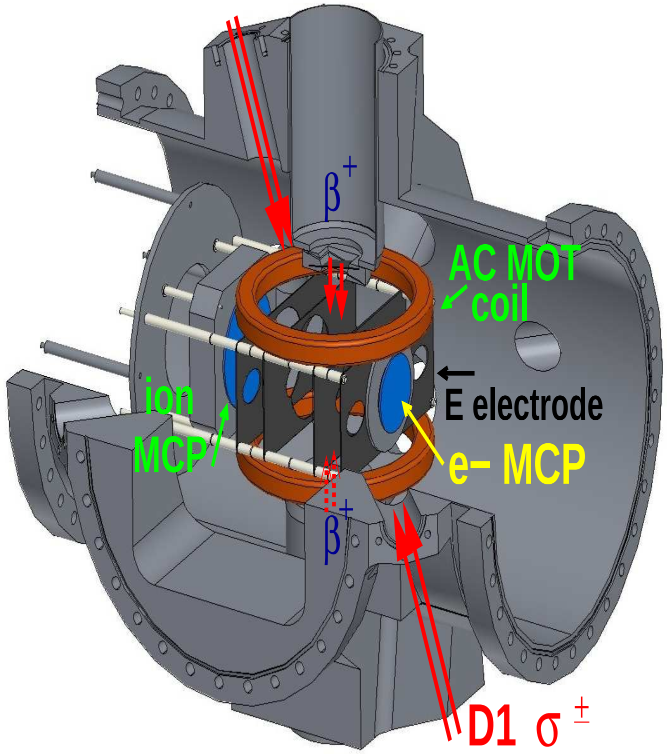
D. Melconian *et al.* PL B 649, 370 (2007)

D: **test only**

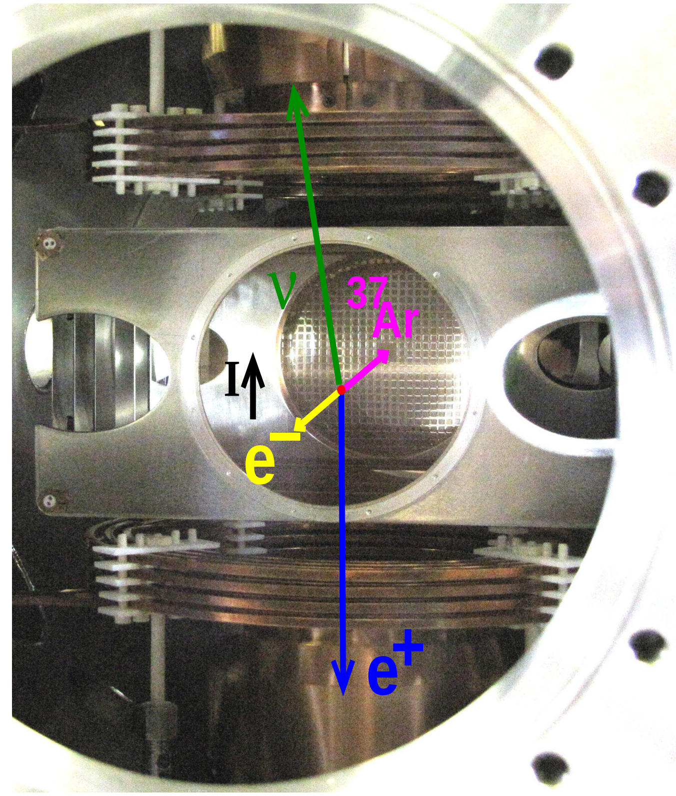
NEW DETECTION CHAMBER FOR ^{37}K



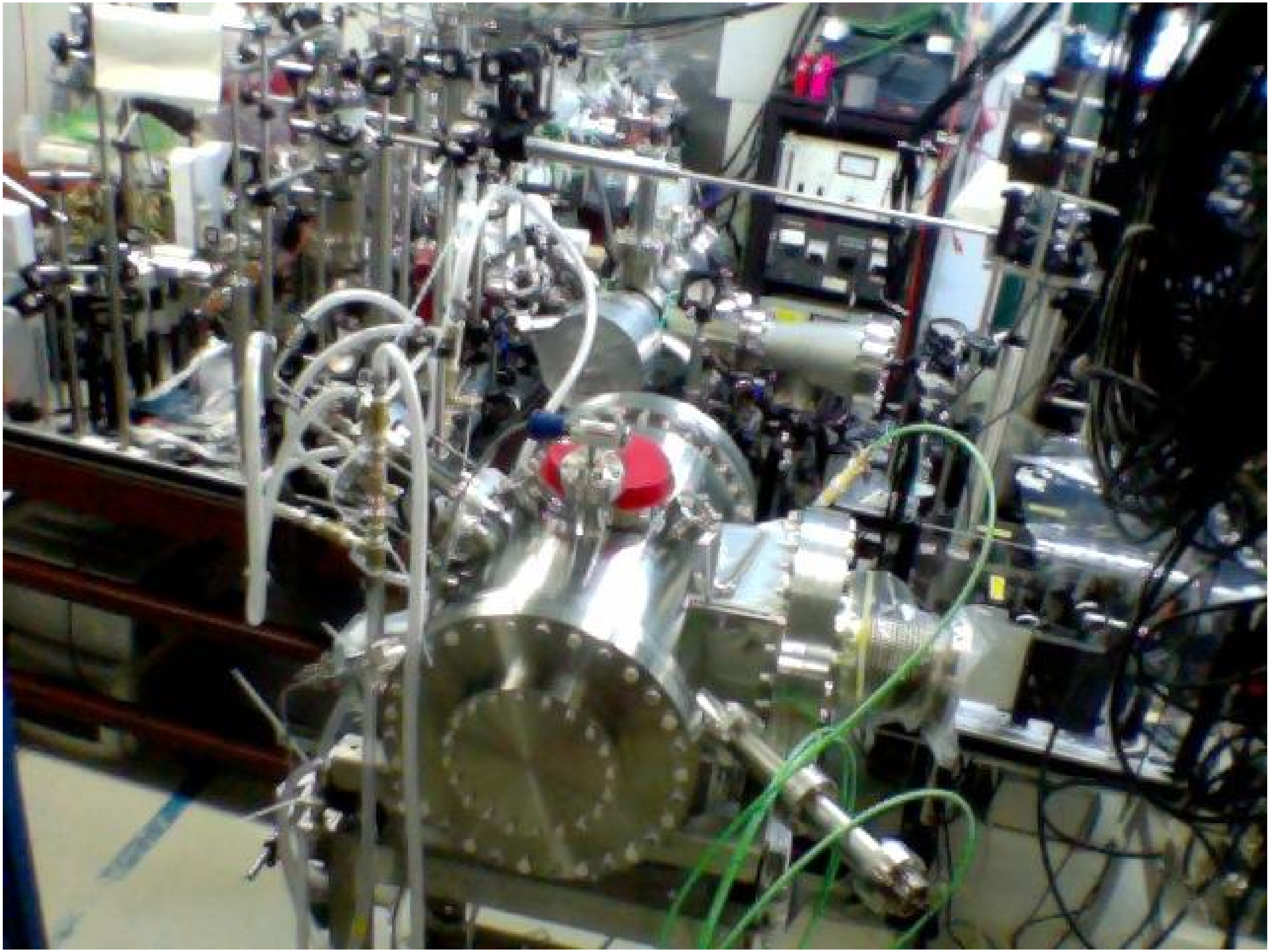
- Position sensitivity on all beta and recoil detectors
- Larger beta and recoil detectors will improve statistics
- AC MOT will speed up switching from MOT cycle to OP cycle
- Improvement of a weak magnetic field during OP will improve polarization
- Coincidences with shake off electron MCP will reduce background for competitive measurements of beta asymmetry



β+
AC MOT
coil
ion
MCP
E electrode
e− MCP
β+
D1 σ ±



ν
37Ar
I
e−
e+



Commissioning experiment (Dec. 12)

$$dW =$$

$$dW_o(1 + \frac{\vec{p_\beta}\cdot\vec{p_\nu}}{E_\beta E_\nu}a_{\beta\nu} + \frac{\Gamma m_e}{E_\beta}b + \frac{\vec{J}}{J}\cdot[\frac{\vec{p_\beta}}{E_\beta}A_\beta + \frac{\vec{p_\nu}}{E_\nu}B_\nu + \frac{\vec{p_\beta}\times\vec{p_\nu}}{E_\beta E_\nu}I$$

$$+c[\frac{\vec{p_\beta}\cdot\vec{p_\nu}}{3E_\beta E_\nu} - \frac{(\vec{p_\beta}\cdot\vec{j})(\vec{p_\nu}\cdot\vec{j})}{E_\beta E_\nu}][\frac{J(J+1)-3<(\vec{J}\cdot\vec{j})^2>}{J(2J-1)}])$$

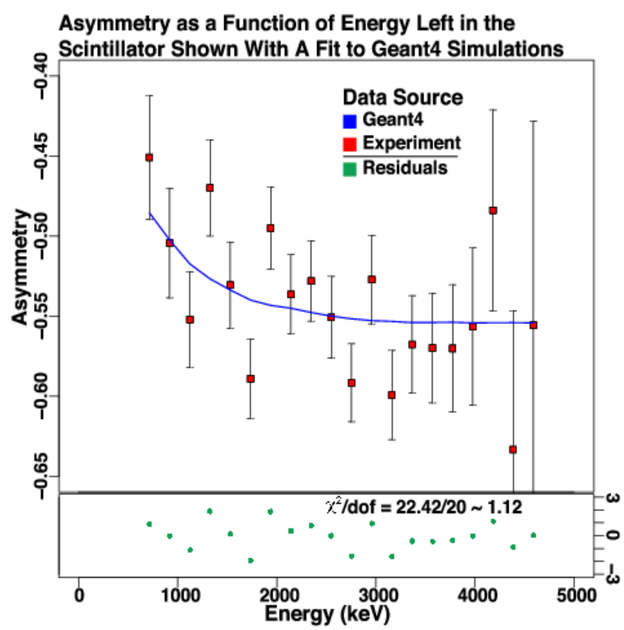
Asymmetry $= \frac{\sigma(\uparrow)-\sigma(\downarrow)}{\sigma(\uparrow)+\sigma(\downarrow)}$

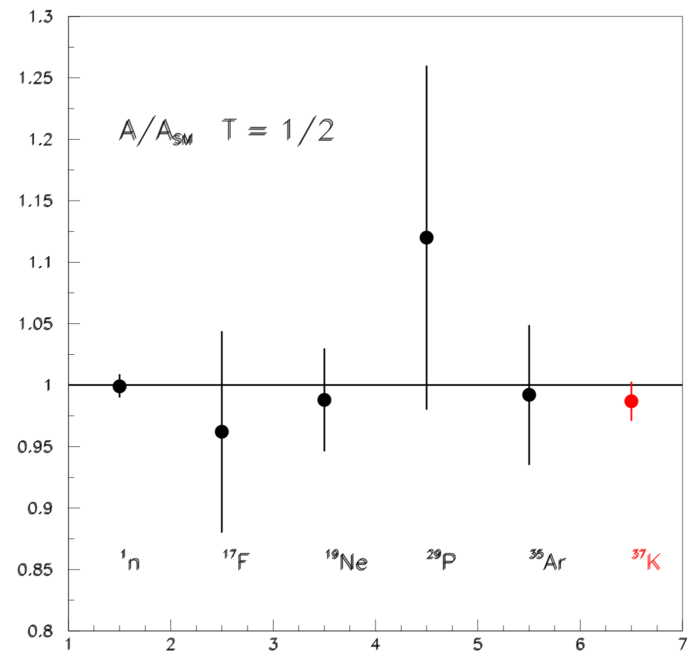
$A_\beta = -0.563 \pm 0.009$, **PRELIMINARY**

Nuclear Polarization (Dec. 12) = 0.99 ± 0.01

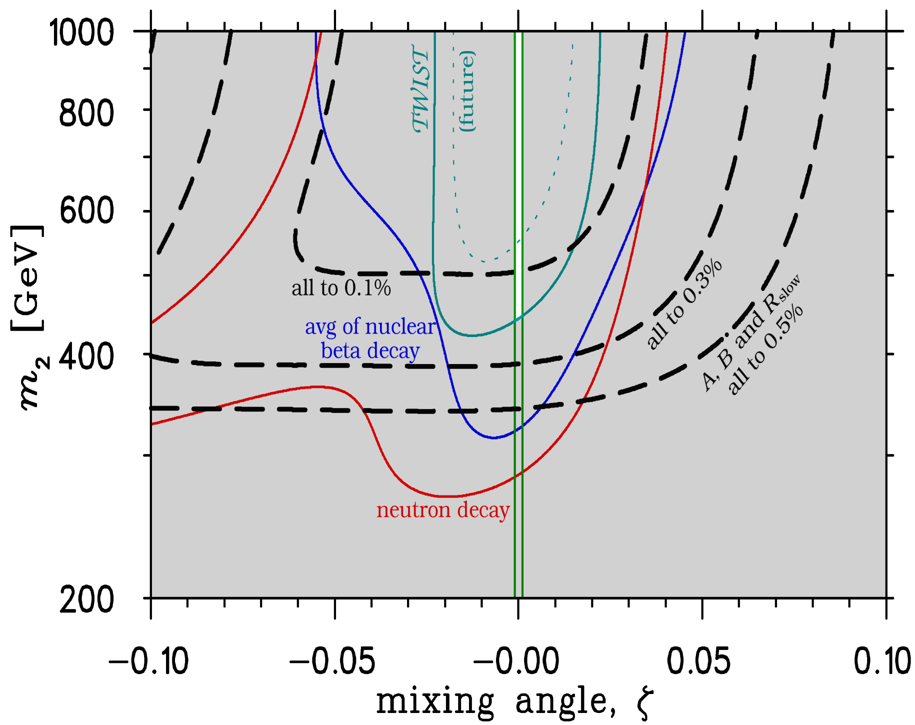
R.S. Behling, PhD thesis Texas A&M (2015)

Nuclear Polarization (Jun. 14) = 0.9913 ± 0.0008


Asymmetry as a Function of Energy Left in the Scintillator Shown With A Fit to Geant4 Simulations
Data Source
Geant4
Experiment
Residuals
χ^2/dof = 22.42/20 ~ 1.12
Asymmetry
Energy (keV)


A/A_{SM} T = 1/2
^{1}n
^{17}F
^{19}Ne
^{29}P
^{35}Ar
^{37}K

Projected Limits on Right-Handed Currents



β-decay rate (Jackson, Treiman, Wyld 1957):

$$dW =$$

$$dW_o\xi(1 + \frac{\vec{p_\beta}\cdot\vec{p_\nu}}{E_\beta E_\nu}a_{\beta\nu} + \frac{\Gamma m_e}{E_\beta}b + \frac{\vec{J}}{J}\cdot[\frac{\vec{p_\beta}}{E_\beta}A_\beta + \frac{\vec{p_\nu}}{E_\nu}B_\nu + \frac{\vec{p_\beta}\times\vec{p_\nu}}{E_\beta E_\nu}D]$$

$$+c[\frac{\vec{p_\beta}\cdot\vec{p_\nu}}{3E_\beta E_\nu} - \frac{(\vec{p_\beta}\cdot\vec{j})(\vec{p_\nu}\cdot\vec{j})}{E_\beta E_\nu}][\frac{J(J+1)-3<(\vec{J}\cdot\vec{j})^2>}{J(2J-1)}])$$

With β polarization measured:

$$dW = dW_o\xi\,(\,1 + \frac{\Gamma m_e}{E_e}b + \frac{\vec{p_e}}{E_e}(\frac{\vec{J}}{J}A + \vec{\sigma_e}G) +$$

$$\vec{\sigma_e}\cdot[\,\frac{\vec{J}}{J}N + \frac{\vec{p_e}}{E_e+m_e}(\frac{\vec{J}}{J}\cdot\frac{\vec{p_e}}{E_e})Q + \frac{\vec{J}}{J}\times\frac{\vec{p_e}}{E_e}R])$$

$a_{\beta\nu}$, b, c, A_β, B_ν, D, R: values predicted by the Standard Model

Review: S. Severijins and M. Beck, Rev. Mod. Phys. 78 991 (2006)

Measurements feasible using Atom traps and Radioactive Beams.

Theoretical Expectations

Within SM D, R $\sim\ 10^{-12}$

Beyond SM: Calculations based on EDM limits:

D - using left-right symmetric models: $\sim 10^{-5} - 3 \times 10^{-7}$

Leptoquark exchange models: 3×10^{-5}

R - using Scalar Interactions: $10^{-2} - 10^{-5}$

EM Final State Interaction: $10^{-4} - 10^{-5}$

Experimental Limits from β Decay

D - ^{19}Ne β **decay:** $(4 \pm 8)10^{-4}$

Cold Neutrons: $(-0.94 \pm 1.89 \pm 0.97)10^{-4}$

Cold Neutrons: $(-2.8 \pm 6.4 \pm 3)10^{-4}$

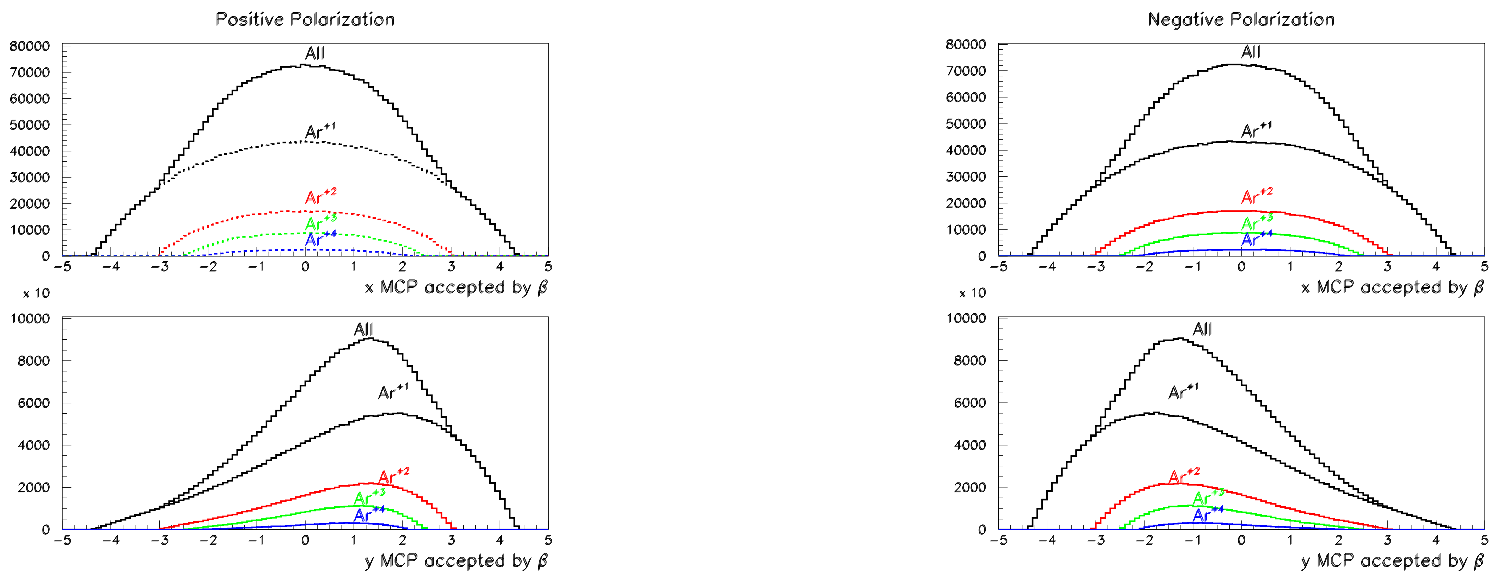
R - σ_e **in** ^{8}**LI decay:** $(1.6 \pm 2.2)10^{-4}$

Cold Neutrons $(4 \pm 12 \pm 5)10^{-3}$

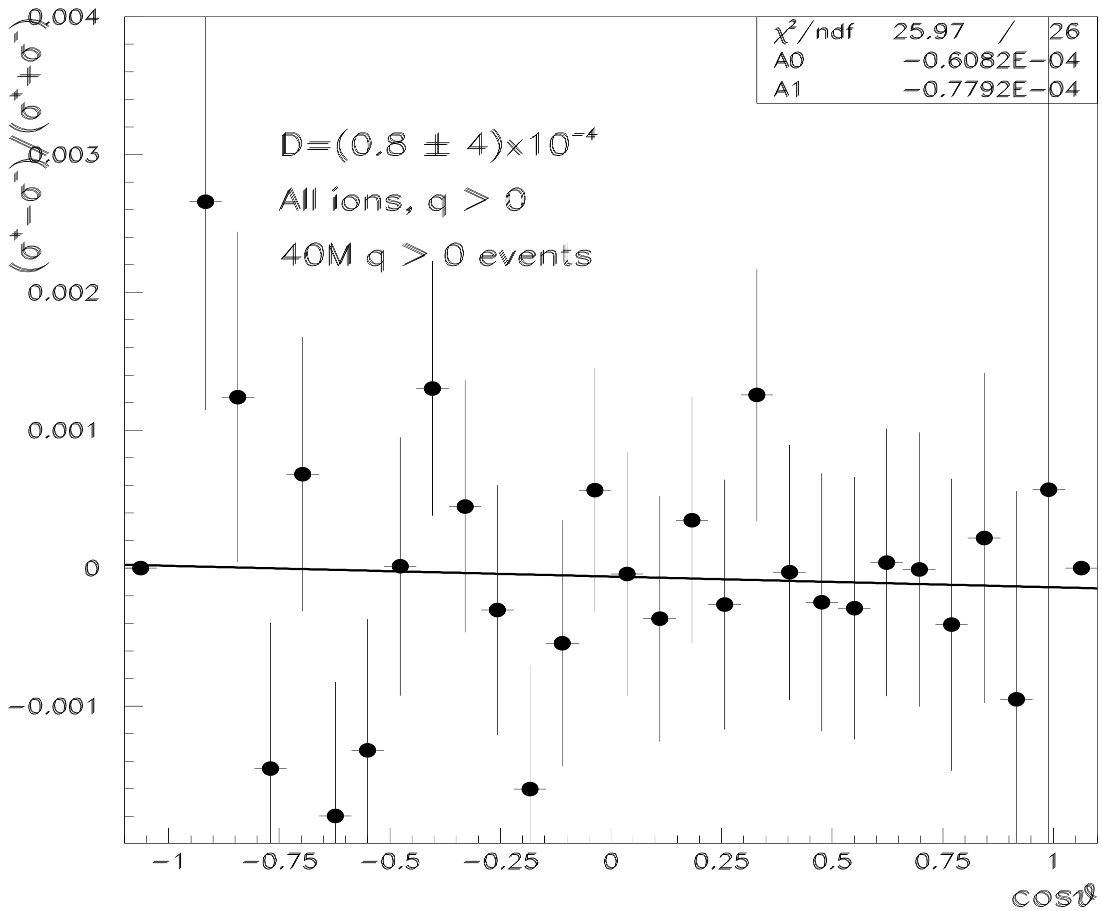
TRV Experimental System

Very Large Detection Acceptance

Ion MCP: 100% acceptance for all ions, E = 1 KV/cm



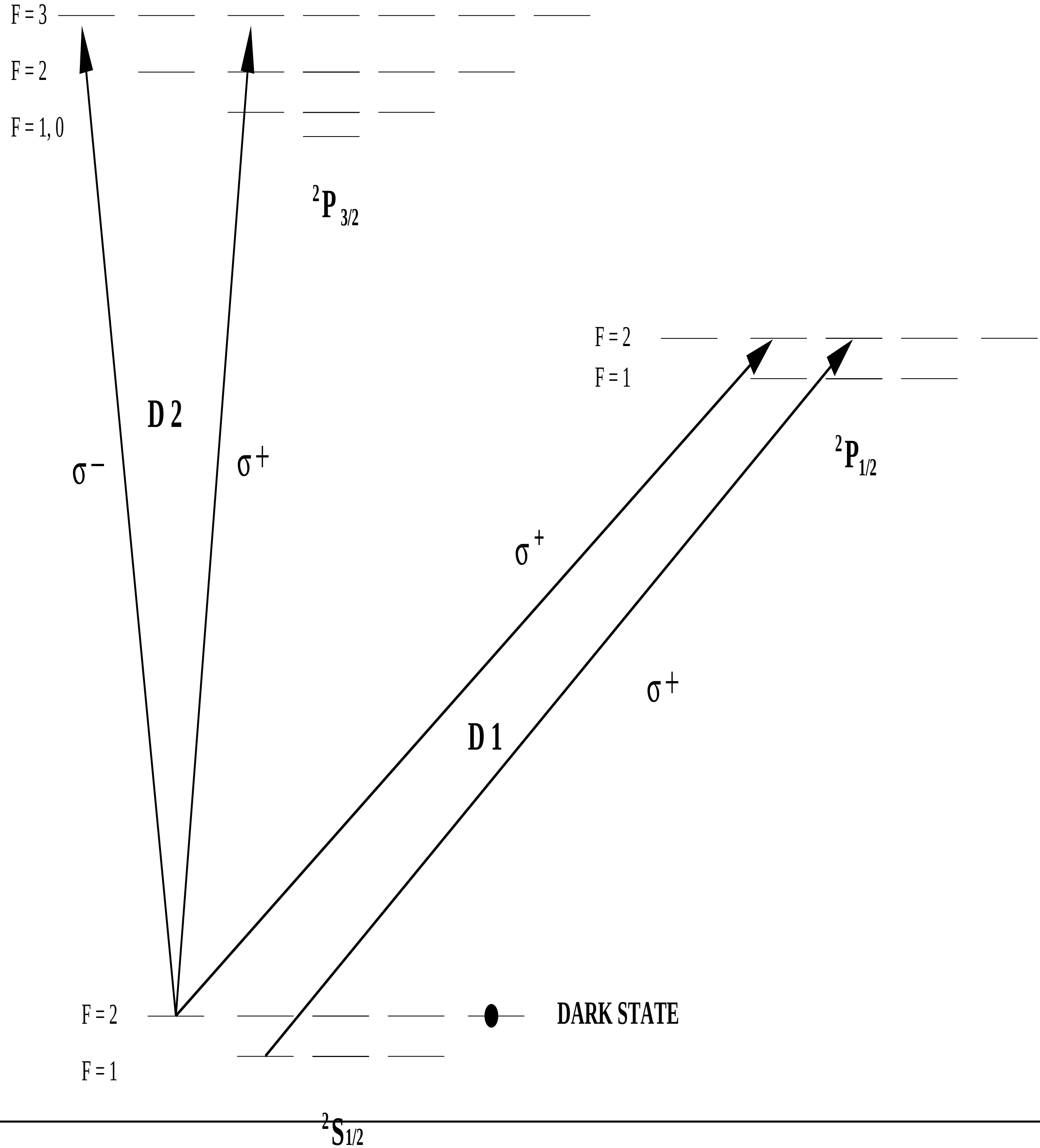
TRV Results for 200M decays $D < 4 \times 10^{-4}$

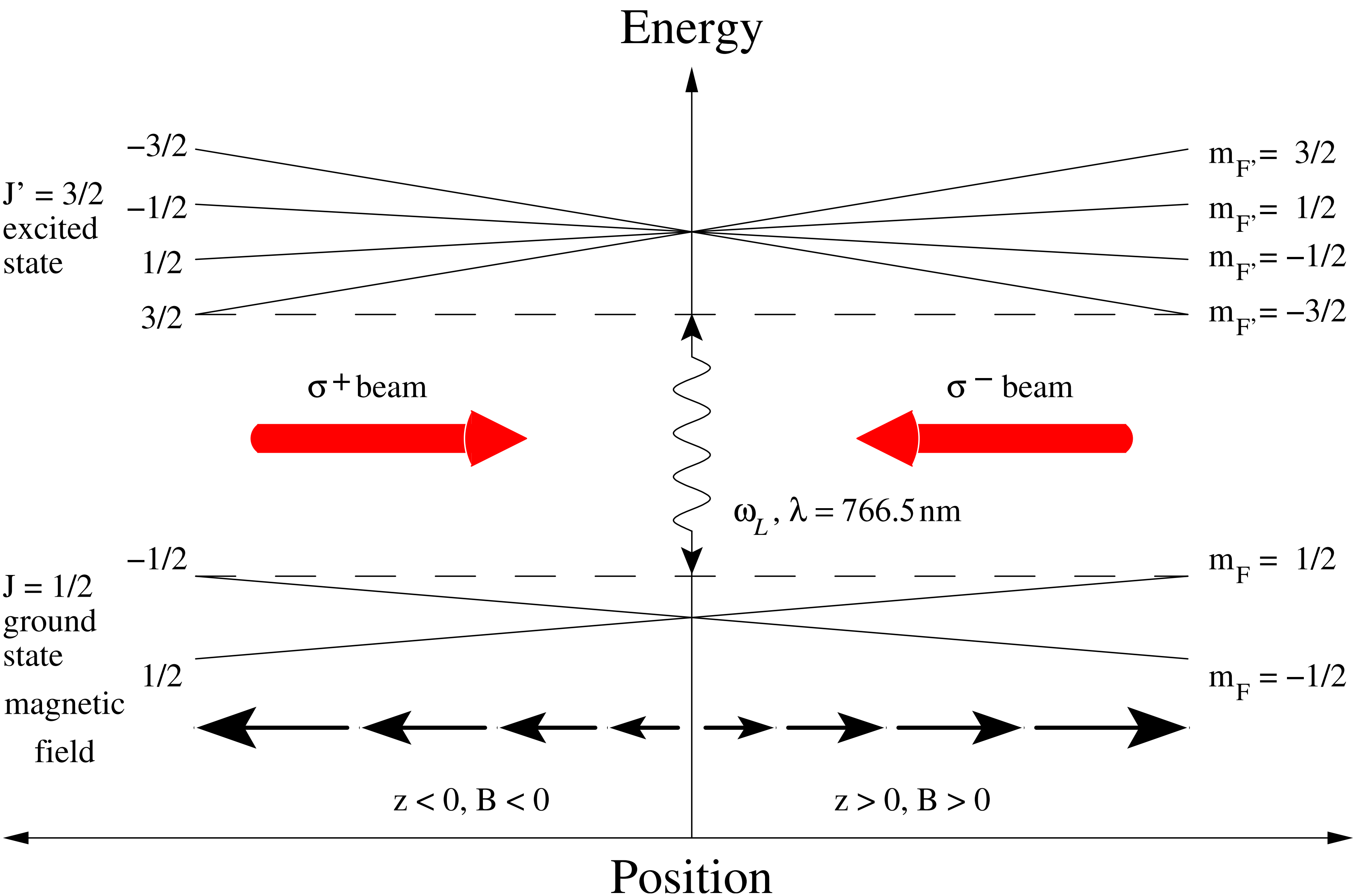


SUMMARY

- Studies of β decay of trapped radioactive nuclei provide constraints on the Standard Model, complementary to measurements with HE accelerators

- Next generation experiments will provide tighter constraints


F = 3
F = 2
F = 1, 0
$^2P_{3/2}$
D 2
σ−
σ+
F = 2
F = 1
$^2P_{1/2}$
σ+
σ+
D 1
F = 2
F = 1
DARK STATE
$^2S_{1/2}$



Energy
J' = 3/2
excited
state
−3/2
−1/2
1/2
3/2
m F' = 3/2
m F' = 1/2
m F' = −1/2
m F' = −3/2
σ + beam
σ − beam
ω L , λ = 766.5 nm
J = 1/2
ground
state
−1/2
1/2
m F = 1/2
m F = −1/2
magnetic
field
z < 0, B < 0
z > 0, B > 0
Position

LHC and β decay limits on Scalar and Tensor interactions

T. Bhattacharya *et. al.*, Phys. Rev. D85, 054512 (2012)