# Global analysis of $b \to s\ell\ell$ anomalies: SM versus New Physics

Joaquim Matias

**Universitat Autònoma de Barcelona**

Lathuile

*In collaboration with:* **B. Capdevila, S. Descotes-Genon, L. Hofer and J. Virto**

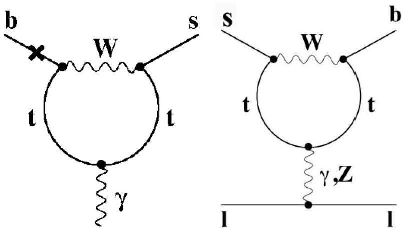
*Based on:* DMV'13 **PRD88 (2013) 074002**, DHMV'14 **JHEP 1412 (2014) 125**, JM'12 **PRD86 (2012) 094024** HM'15 **JHEP 1509(2015)104**, DHMV'15 **1510.04239 (updated with final data)** and CDHMV'16 (to appear)

**All originated in Frank Krueger, J.M., Phys. Rev. D71 (2005) 094009**

# Motivation

Analysis of FCNC in a model-independent approach, effective Hamiltonian:

$$b \to s\gamma(^*) : \mathcal{H}^{SM}_{\Delta F=1} \propto \sum_{i=1}^{10} V^*_{ts} V_{tb} \mathcal{C}_i \mathcal{O}_i + \ldots$$




- $\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}$
- $\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)\ (\bar{\ell}\gamma_\mu \ell)$
- $\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)\ (\bar{\ell}\gamma_\mu \gamma_5 \ell)$, ...

• **SM** Wilson coefficients up to NNLO + e.m. corrections at $\mu_{ref} = 4.8$ GeV [Misiak et al.]:

$$\mathcal{C}_7^{\rm SM} = -0.29,\ \mathcal{C}_9^{\rm SM} = 4.1,\ \mathcal{C}_{10}^{\rm SM} = -4.3$$

• **NP** changes short distance $\mathcal{C}_i - \mathcal{C}_i^{\rm SM} = \mathcal{C}_i^{\rm NP}$ and induces new operators, like

$\mathcal{O}'_{7,9,10} = \mathcal{O}_{7,9,10}\ (P_L \leftrightarrow P_R)$ ... also scalars, pseudoescalar, tensor operators...

The way to obtain information on those Wilson coefficients is via a GLOBAL FIT to the relevant processes.

DHMV'15 1510.04239 (updated with final LHCb data 1512.04442 )

# Updated GLOBAL FIT 2015:

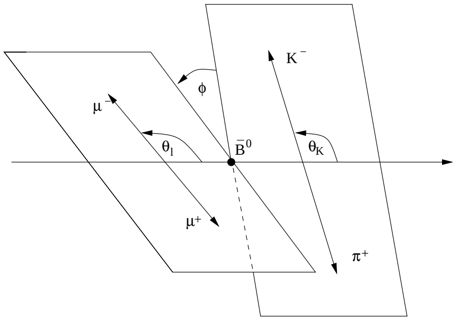
## THE OBSERVABLES

- Inclusive
  - $B \to X_s\gamma$ ($BR$) .................................................... $\mathcal{C}_7^{(\prime)}$
  - $B \to X_s\ell^+\ell^-$ ($dBR/dq^2$) ........................................ $\mathcal{C}_7^{(\prime)}, \mathcal{C}_9^{(\prime)}, \mathcal{C}_{10}^{(\prime)}$
- Exclusive leptonic
  - $B_s \to \ell^+\ell^-$ ($BR$) ................................................ $\mathcal{C}_{10}^{(\prime)}$
- Exclusive radiative/semileptonic
  - $B \to K^*\gamma$ ($BR$, $S$, $A_I$) ........................................ $\mathcal{C}_7^{(\prime)}$
  - $B \to K\ell^+\ell^-$ ($dBR/dq^2$) ........................................ $\mathcal{C}_7^{(\prime)}, \mathcal{C}_9^{(\prime)}, \mathcal{C}_{10}^{(\prime)}$
  - $\mathbf{B \to K^*\ell^+\ell^-}$ ($dBR/dq^2$, **Optimized Angular Obs.**) .. $\mathcal{C}_7^{(\prime)}, \mathcal{C}_9^{(\prime)}, \mathcal{C}_{10}^{(\prime)}$
  - $B_s \to \phi\ell^+\ell^-$ ($dBR/dq^2$, Angular Observables) ............. $\mathcal{C}_7^{(\prime)}, \mathcal{C}_9^{(\prime)}, \mathcal{C}_{10}^{(\prime)}$
  - $\Lambda_b \to \Lambda\ell^+\ell^-$ (None so far)
  - etc.

# Optimized Basis of Angular Observables for $B \to K^*\mu\mu$

The optimized observables $P_i^{(\prime)}$ come from the angular distribution $\bar{\mathbf{B}}_\mathbf{d} \to \bar{\mathbf{K}}^{*0}(\to \mathbf{K}^-\pi^+)\mathbf{l}^+\mathbf{l}^-$ with the $K^{*0}$ on the mass shell. It is described by $\mathbf{s} = \mathbf{q}^2$ and three angles $\theta_\ell$, $\theta_\mathbf{K}$ and $\phi$

$$\frac{d^4\Gamma(\bar{B}_d)}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi}\mathbf{J}(\mathbf{q}^2, \theta_\ell, \theta_K, \phi) = \sum_i J_i(q^2) f_i(\theta_\ell, \theta_K, \phi\}$$




$\theta_\ell$: Angle of emission between $\bar{K}^{*0}$ and $\mu^-$ in di-lepton rest frame.
$\theta_\mathbf{K}$: Angle of emission between $\bar{K}^{*0}$ and $K^-$ in di-meson rest frame.
$\phi$: Angle between the two planes.

$\mathbf{q}^2$: dilepton invariant mass square.

**See talk T. Blake**

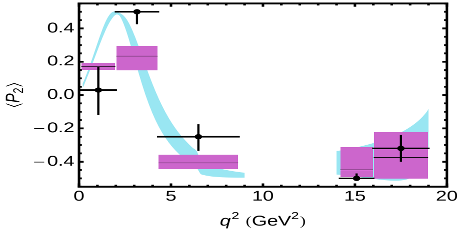
$$\frac{1}{\Gamma'_{full}} \frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Bigg[\frac{3}{4}\mathbf{F_T}\sin^2\theta_K + \mathbf{F_L}\cos^2\theta_K + (\frac{1}{4}\mathbf{F_T}\sin^2\theta_K - \mathbf{F_L}\cos^2\theta_K)\cos 2\theta_l$$

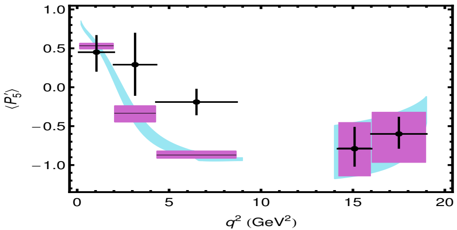
$$+\sqrt{\mathbf{F_T}\mathbf{F_L}}\left(\frac{1}{2}\mathbf{P'_4}\sin 2\theta_K \sin 2\theta_l \cos\phi + \mathbf{P'_5}\sin 2\theta_K \sin\theta_l \cos\phi\right) + 2\mathbf{P_2}\mathbf{F_T}\sin^2\theta_K\cos\theta_l + \frac{1}{2}\mathbf{P_1}\mathbf{F_T}\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$

$$-\sqrt{\mathbf{F_T}\mathbf{F_L}}\left(\mathbf{P'_6}\sin 2\theta_K \sin\theta_l \sin\phi - \frac{1}{2}\mathbf{P'_8}\sin 2\theta_K \sin 2\theta_l \sin\phi\right) - \mathbf{P_3}\mathbf{F_T}\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Bigg](1 - \mathbf{F_S}) + \frac{1}{\Gamma'_{full}}\mathbf{W_S}$$

Why so much excitement in Flavour Physics? What changed in and after 2013?

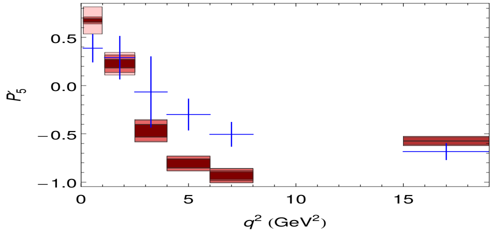
- First measurement by LHCb of the basis of optimized observables with 1 fb$^{-1}$:



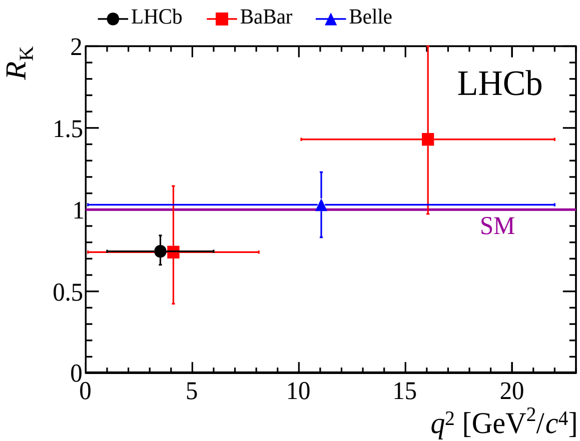

$\Rightarrow$ $P_2$ exhibited a **2.9$\sigma$** deviation in the bin [2,4.3] and $P_5'$ exhibits a **3.7$\sigma$** in the [4.3,8.7] bin.

- In 2015 the so called anomaly in $P_5'$ is confirmed with 3fb$^{-1}$ in **2 bins** with **2.9$\sigma$** each:




$\Rightarrow$ $P_2$ will require a bit of patience to become more interesting (... a bit more of data)

# Brief flash on the anomalies




$$R_K = \frac{\mathrm{Br}\,(B^+ \to K^+\mu^+\mu^-)}{\mathrm{Br}\,(B^+ \to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

- It deviates **2.6**$\sigma$ from SM.
- Data on $BR(B^+ \to K^+\mu^+\mu^-)$ is below SM in **all bins** at large and low-recoil.

Also BR of neutral mode:

| $10^7 \times BR(B^0 \to K^0\mu^+\mu^-)$ | Standard Model | Experiment | Pull |
|---|---|---|---|
| $[0.1, 2]$ | $0.62 \pm 0.19$ | $0.23 \pm 0.11$ | $+1.8$ |
| $[2, 4]$ | $0.65 \pm 0.21$ | $0.37 \pm 0.11$ | $+1.2$ |
| $[4, 6]$ | $0.64 \pm 0.22$ | $0.35 \pm 0.10$ | $+1.2$ |
| $[6, 8]$ | $0.63 \pm 0.23$ | $0.54 \pm 0.12$ | $+0.4$ |
| $[15, 19]$ | $0.91 \pm 0.12$ | $0.67 \pm 0.12$ | $+1.4$ |

# Brief flash on the anomalies

| $10^7 \times BR(B^0 \to K^{*0}\mu^+\mu^-)$ | Standard Model | Experiment | Pull |
|---|---|---|---|
| $[0.1, 2]$ | $1.30 \pm 1.00$ | $1.14 \pm 0.18$ | $+0.2$ |
| $[2, 4.3]$ | $0.85 \pm 0.59$ | $0.69 \pm 0.12$ | $+0.3$ |
| $[4.3, 8.68]$ | $2.62 \pm 4.92$ | $2.15 \pm 0.31$ | $+0.1$ |
| $[16, 19]$ | $1.66 \pm 0.15$ | $1.23 \pm 0.20$ | $+\mathbf{1.7}$ |
| $10^7 \times BR(B^+ \to K^{*+}\mu^+\mu^-)$ | Standard Model | Experiment | Pull |
| $[0.1, 2]$ | $1.35 \pm 1.05$ | $1.12 \pm 0.27$ | $+0.2$ |
| $[2, 4]$ | $0.80 \pm 0.55$ | $1.12 \pm 0.32$ | $-0.5$ |
| $[4, 6]$ | $0.95 \pm 0.70$ | $0.50 \pm 0.20$ | $+0.6$ |
| $[6, 8]$ | $1.17 \pm 0.92$ | $0.66 \pm 0.22$ | $+0.5$ |
| $[15, 19]$ | $2.59 \pm 0.24$ | $1.60 \pm 0.32$ | $+\mathbf{2.5}$ |
| $10^7 \times BR(B_s \to \phi\mu^+\mu^-)$ | Standard Model | Experiment | Pull |
| $[0.1, 2.]$ | $1.81 \pm 0.36$ | $1.11 \pm 0.16$ | $+\mathbf{1.8}$ |
| $[2., 5.]$ | $1.88 \pm 0.32$ | $0.77 \pm 0.14$ | $+\mathbf{3.2}$ |
| $[5., 8.]$ | $2.25 \pm 0.41$ | $0.96 \pm 0.15$ | $+\mathbf{2.9}$ |
| $[15, 18.8]$ | $2.20 \pm 0.17$ | $1.62 \pm 0.20$ | $+\mathbf{2.2}$ |

Also $BR(B \to V\mu\mu)$ exhibit a systematic deficit with respect to SM, particularly $B_s \to \phi\mu\mu$.

# Theory and experimental updates in 2015 fit

- $BR(B \to X_s\gamma)$
  - New theory update: $\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \pm 0.23) \cdot 10^{-4}$ (Misiak et al 2015)
  - $+6.4\%$ shift in central value w.r.t 2006 $\to$ excellent agreement with WA
- $BR(B_s \to \mu^+\mu^-)$
  - New theory update (Bobeth et al 2013), New LHCb+CMS average (2014)
- $BR(B \to X_s\mu^+\mu^-)$
  - New theory update (Huber et al 2015)
- $BR(B \to K\mu^+\mu^-)$ :
  - LHCb 2014 + Lattice form factors at large $q^2$ (Bouchard et al 2013, 2015)
- $B_{(s)} \to (K^*, \phi)\mu^+\mu^-$ : BRs & Angular Observables
  - LHCb 2015 + Lattice form factors at large $q^2$ (Horgan et al 2013)
- $BR(B \to Ke^+e^-)_{[1,6]}$ (or $R_K$) and $B \to K^*e^+e^-$ at very low $q^2$
  - LHCb 2014, 2015

Frequentist approach:

$$\chi^2(C_i) = [O_{\rm exp} - O_{\rm th}(C_i)]_j \, [Cov^{-1}]_{jk} \, [O_{\rm exp} - O_{\rm th}(C_i)]_k$$

- **Cov** $=$ **Cov**$^{\rm exp}$ $+$ **Cov**$^{\rm th}$. We have $Cov^{\rm exp}$ for the first time
- Calculate $Cov^{\rm th}$: correlated multigaussian scan over all nuisance parameters
- $Cov^{\rm th}$ depends on $C_i$: Must check this dependence

For the Fit:

- Minimise $\chi^2 \to \chi^2_{\rm min} = \chi^2(C_i^0)$ (Best Fit Point = $C_i^0$)
- Confidence level regions: $\chi^2(C_i) - \chi^2_{\rm min} < \Delta\chi_{\sigma,n}$

**Definition of Pull$_{SM}$:**

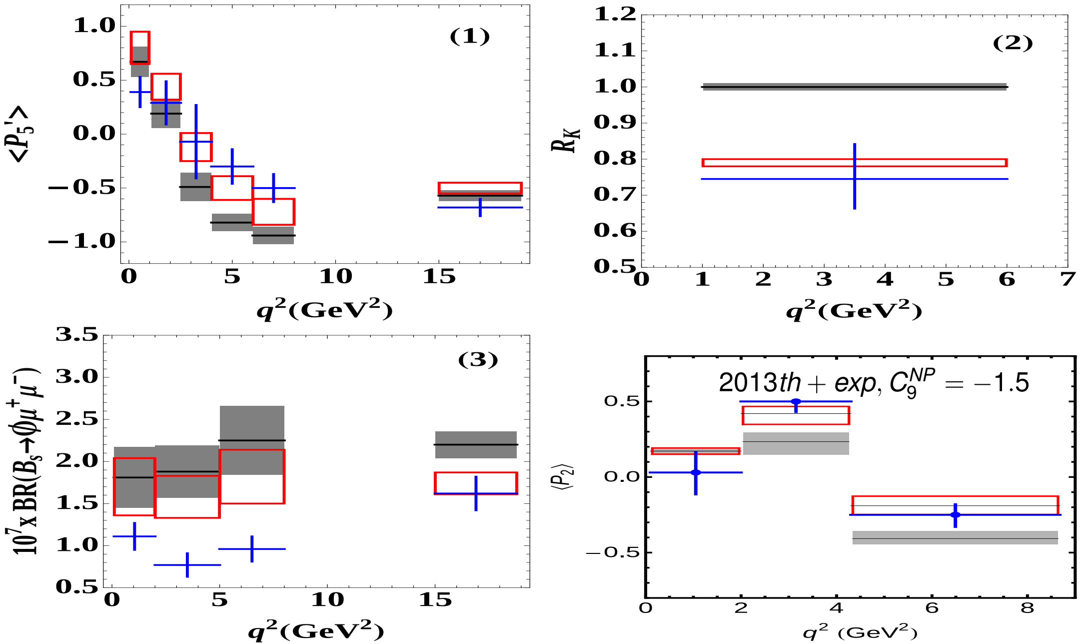
**Pull$_{SM}$** tells you how much in a model defined by a set of free Wilson coefficients $C_i$ the value preferred by data for these Wilson coefficients is in tension with $C_i^{SM}$.

# Result of the fit with 1D Wilson coefficient 2015 ($e^+e^-$ mode not included)

This is the first analysis: - using the basis of **optimized observables** ($B \to K^* \mu\mu$ and $B_s \to \phi\mu\mu$)
- using the **full dataset** of 3fb$^{-1}$:

| Coefficient $\mathcal{C}_i^{NP} = \mathcal{C}_i - \mathcal{C}_i^{SM}$ | Best fit | $1\sigma$ | $3\sigma$ | $\text{Pull}_{\text{SM}}$ |
|---|---|---|---|---|
| $\mathcal{C}_7^{\text{NP}}$ | $-0.02$ | $[-0.04, -0.00]$ | $[-0.07, 0.03]$ | 1.2 |
| $\mathcal{C}_9^{\text{NP}}$ | $-1.09$ | $[-1.29, -0.87]$ | $[-1.67, -0.39]$ | **4.5** $\Leftarrow$ |
| $\mathcal{C}_{10}^{\text{NP}}$ | 0.56 | $[0.32, 0.81]$ | $[-0.12, 1.36]$ | 2.5 |
| $\mathcal{C}_{7'}^{\text{NP}}$ | 0.02 | $[-0.01, 0.04]$ | $[-0.06, 0.09]$ | 0.6 |
| $\mathcal{C}_{9'}^{\text{NP}}$ | 0.46 | $[0.18, 0.74]$ | $[-0.36, 1.31]$ | 1.7 |
| $\mathcal{C}_{10'}^{\text{NP}}$ | $-0.25$ | $[-0.44, -0.06]$ | $[-0.82, 0.31]$ | 1.3 |
| $\mathcal{C}_9^{\text{NP}} = \mathcal{C}_{10}^{\text{NP}}$ | $-0.22$ | $[-0.40, -0.02]$ | $[-0.74, 0.50]$ | 1.1 |
| $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}}$ | $-0.68$ | $[-0.85, -0.50]$ | $[-1.22, -0.18]$ | **4.2** $\Leftarrow$ |
| $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}}$ | $-1.06$ | $[-1.25, -0.86]$ | $[-1.60, -0.40]$ | 4.8 (low recoil) |
| $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}} = -\mathcal{C}_{10'}^{\text{NP}}$ | $-0.69$ | $[-0.89, -0.51]$ | $[-1.37, -0.16]$ | 4.1 |

# Impact on the anomalies of a contribution from NP $C_9^{NP} = -1.1$

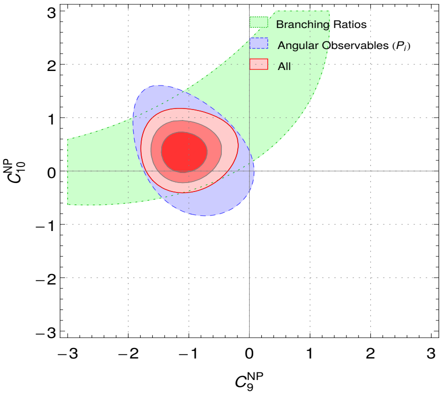



(1),(2) and (3) use 3 fb$^{-1}$ dataset and latest theory prediction for SM (gray) and NP ($C_9^{NP} = -1.1$).

**All anomalies and tensions gets solved or alleviated with $C_9^{NP} \sim \mathcal{O}(-1)$**

# Result of the fit with 2D Wilson coefficient constrained and unconstrained

| Coefficient | Best Fit Point | $\text{Pull}_{\text{SM}}$ | |
|---|---|---|---|
| $(\mathcal{C}_7^{\rm NP}, \mathcal{C}_9^{\rm NP})$ | $(-0.00, -1.07)$ | **4.1** | |
| $(\mathcal{C}_9^{\rm NP}, \mathcal{C}_{10}^{\rm NP})$ | $(-1.08, 0.33)$ | **4.3** | |
| $(\mathcal{C}_9^{\rm NP}, \mathcal{C}_{7'}^{\rm NP})$ | $(-1.09, 0.02)$ | **4.2** | |
| $(\mathcal{C}_9^{\rm NP}, \mathcal{C}_{9'}^{\rm NP})$ | $(-1.12, 0.77)$ | **4.5** | |
| $(\mathcal{C}_9^{\rm NP}, \mathcal{C}_{10'}^{\rm NP})$ | $(-1.17, -0.35)$ | **4.5** | |
| $(\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP}, \mathcal{C}_{10}^{\rm NP} = \mathcal{C}_{10'}^{\rm NP})$ | $(-1.15, 0.34)$ | **4.7** | no-$Z'$ |
| $(\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP}, \mathcal{C}_{10}^{\rm NP} = -\mathcal{C}_{10'}^{\rm NP})$ | $(-1.06, 0.06)$ | **4.4** | $Z'$ |
| $(\mathcal{C}_9^{\rm NP} = \mathcal{C}_{9'}^{\rm NP}, \mathcal{C}_{10}^{\rm NP} = \mathcal{C}_{10'}^{\rm NP})$ | $(-0.64, -0.21)$ | 3.9 | $Z'$ |
| $(\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{10}^{\rm NP}, \mathcal{C}_{9'}^{\rm NP} = \mathcal{C}_{10'}^{\rm NP})$ | $(-0.72, 0.29)$ | 3.8 | no-$Z'$ |




- $C_9^{NP}$ always play a dominant role
- All 2D scenarios above $4\sigma$ are quite indistinguishable. We have done a systematic work to check what are the most relevant Wilson Coefficients to explain all deviations, by allowing progressively different WC to get NP contributions and compare the pulls.

# Result of the fit to the SIX Wilson coefficients free

| Coefficient | $1\sigma$ | $2\sigma$ | $3\sigma$ | |
|---|---|---|---|---|
| $\mathcal{C}_7^{\rm NP}$ | $[-0.02, 0.03]$ | $[-0.04, 0.04]$ | $[-0.05, 0.08]$ | no preference |
| $\mathcal{C}_9^{\rm NP}$ | $[-1.4, -1.0]$ | $[-1.7, -0.7]$ | $[-2.2, -0.4]$ | **negative** |
| $\mathcal{C}_{10}^{\rm NP}$ | $[-0.0, 0.9]$ | $[-0.3, 1.3]$ | $[-0.5, 2.0]$ | positive |
| $\mathcal{C}_{7'}^{\rm NP}$ | $[-0.02, 0.03]$ | $[-0.04, 0.06]$ | $[-0.06, 0.07]$ | no preference |
| $\mathcal{C}_{9'}^{\rm NP}$ | $[0.3, 1.8]$ | $[-0.5, 2.7]$ | $[-1.3, 3.7]$ | positive |
| $\mathcal{C}_{10'}^{\rm NP}$ | $[-0.3, 0.9]$ | $[-0.7, 1.3]$ | $[-1.0, 1.6]$ | $\sim$ positive |

- $C_9$ is consistent with SM only **above 3**$\sigma$
- All other are consistent with zero at $1\sigma$ except for $C_9'$ (at 2 $\sigma$).
- The Pull$_{SM}$ for the 6D fit is $3.6\sigma$. (**See plots in Back-up slides**)

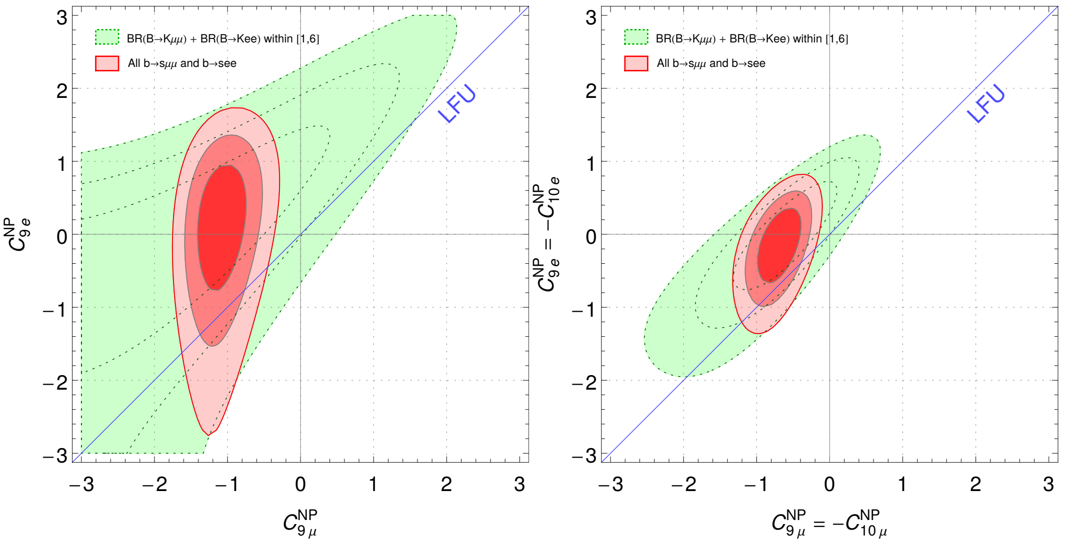
# Impact of $B \to Ke^+e^-$ under hypothesis of maximal Lepton Flavour Universal Violation

| 1D-Coefficient | Best fit | $1\sigma$ | $3\sigma$ | $\text{Pull}_{\text{SM}}$ |
|---|---|---|---|---|
| $\mathcal{C}_9^{\text{NP}}$ | $-1.11$ | $[-1.31, -0.90]$ | $[-1.67, -0.46]$ | **4.5 → 4.9** |
| $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}}$ | $-0.65$ | $[-0.80, -0.50]$ | $[-1.13, -0.21]$ | **4.2 → 4.6** |
| $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}}$ | $-1.07$ | $[-1.25, -0.86]$ | $[-1.60, -0.42]$ | 4.9 |
| $\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}} = -\mathcal{C}_{10'}^{\text{NP}}$ | $-0.66$ | $[-0.84, -0.50]$ | $[-1.25, -0.20]$ | **4.1 → 4.5** |

| 2D-Coefficient | Best Fit Point | $\text{Pull}_{\text{SM}}$ |
|---|---|---|
| $(C_7^{\text{NP}}, C_9^{\text{NP}})$ | $(-0.00, -1.10)$ | **4.1 → 4.6** |
| $(C_9^{\text{NP}}, C_{10}^{\text{NP}})$ | $(-1.06, 0.33)$ | **4.3 → 4.8** |
| $(C_9^{\text{NP}}, C_{7'}^{\text{NP}})$ | $(-1.16, 0.02)$ | **4.2 → 4.7** |
| $(C_9^{\text{NP}}, C_{9'}^{\text{NP}})$ | $(-1.15, 0.64)$ | **4.5 → 4.9** |
| $(C_9^{\text{NP}}, C_{10'}^{\text{NP}})$ | $(-1.23, -0.29)$ | **4.5 → 4.9** |
| $(C_9^{\text{NP}} = -C_{9'}^{\text{NP}}, C_{10}^{\text{NP}} = C_{10'}^{\text{NP}})$ | $(-1.18, 0.38)$ | **4.7 → 5.1** |
| $(C_9^{\text{NP}} = -C_{9'}^{\text{NP}}, C_{10}^{\text{NP}} = -C_{10'}^{\text{NP}})$ | $(-1.11, 0.04)$ | **4.5** |
| $(C_9^{\text{NP}} = C_{9'}^{\text{NP}}, C_{10}^{\text{NP}} = C_{10'}^{\text{NP}})$ | $(-0.64, -0.11)$ | **3.9 → 4.3** |
| $(C_9^{\text{NP}} = -C_{10}^{\text{NP}}, C_{9'}^{\text{NP}} = C_{10'}^{\text{NP}})$ | $(-0.69, 0.27)$ | **3.8 → 4.2** |

- The strong correlations among form factors of $B \to K\mu\mu$ and $B \to Kee$ assuming no NP in $B \to Kee$ enhances the NP evidence in muons.
- Notice that we use all bins in $B \to K\mu\mu$ while $R_K$ is only [1,6]. **All theory correlations included**.
- Only scenarios explaining $R_K$ get an extra enhancement of +0.4-0.5 $\sigma$

# Fits considering Lepton Flavour (non-) Universality




- If NP-LFUV is assumed, NP may enter both $b \to see$ and $b \to s\mu\mu$ decays with different values.

⇒ For each scenario, we see that there is no clear indication of a NP contribution in the electron sector, whereas one has clearly a non-vanishing contribution for the muon sector, with a deviation from the Lepton Flavour Universality line.

# Prediction for LFU tests observables

| | $R_K[1,6]$ | $R_{K^*}[1.1,6]$ | $R_\phi[1.1,6]$ |
|---|---|---|---|
| SM | $1.00 \pm 0.01$ | $1.00 \pm 0.01$ $[1.00 \pm 0.01]$ | $1.00 \pm 0.01$ |
| $\mathcal{C}_9^{\rm NP} = -1.11$ | $0.79 \pm 0.01$ | $0.87 \pm 0.08$ $[0.84 \pm 0.02]$ | $0.84 \pm 0.02$ |
| $\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP} = -1.09$ | $1.00 \pm 0.01$ | $0.79 \pm 0.14$ $[0.74 \pm 0.04]$ | $0.74 \pm 0.03$ |
| $\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{10}^{\rm NP} = -0.69$ | $0.67 \pm 0.01$ | $0.71 \pm 0.03$ $[0.69 \pm 0.01]$ | $0.69 \pm 0.01$ |
| $\mathcal{C}_9^{\rm NP} = -1.15, \mathcal{C}_{9'}^{\rm NP} = 0.77$ | $0.91 \pm 0.01$ | $0.80 \pm 0.12$ $[0.76 \pm 0.03]$ | $0.76 \pm 0.03$ |
| $\mathcal{C}_9^{\rm NP} = -1.16, \mathcal{C}_{10}^{\rm NP} = 0.35$ | $0.71 \pm 0.01$ | $0.78 \pm 0.07$ $[0.75 \pm 0.02]$ | $0.76 \pm 0.01$ |
| $\mathcal{C}_9^{\rm NP} = -1.23, \mathcal{C}_{10'}^{\rm NP} = -0.38$ | $0.87 \pm 0.01$ | $0.79 \pm 0.11$ $[0.75 \pm 0.02]$ | $0.76 \pm 0.02$ |
| $\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP} = -1.14$ $\mathcal{C}_{10}^{\rm NP} = -\mathcal{C}_{10'}^{\rm NP} = 0.04$ | $1.00 \pm 0.01$ | $0.78 \pm 0.13$ $[0.74 \pm 0.04]$ | $0.74 \pm 0.03$ |
| $\mathcal{C}_9^{\rm NP} = -\mathcal{C}_{9'}^{\rm NP} = -1.17$ $\mathcal{C}_{10}^{\rm NP} = \mathcal{C}_{10'}^{\rm NP} = 0.26$ | $0.88 \pm 0.01$ | $0.76 \pm 0.12$ $[0.71 \pm 0.04]$ | $0.71 \pm 0.03$ |

Table: *Predictions for $R_K$, $R_{K^*}$, $R_\phi$ at the best fit point of different scenarios of interest, assuming that NP enters only in the muon sector, and using the inputs of our reference fit , in particular the KMPW form factors for $B \to K$ and $B \to K^*$, and BSZ for $B_s \to \phi$. In $B \to K^*$, we indicate in brackets predictions using the form factors in BSZ.*

**Relative ordering between the three may help to disentangle some scenarios from others.**

# How much the fit results depend on the details?

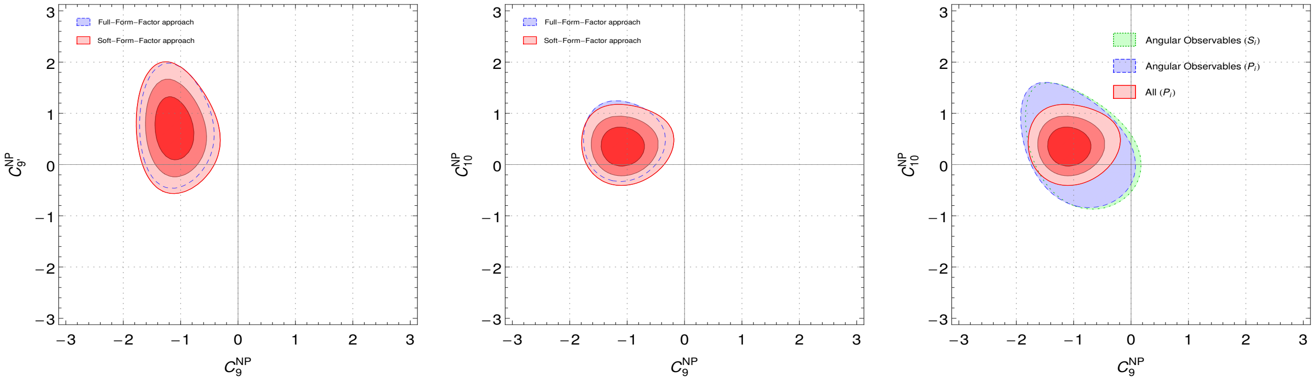



Figure: *We show the 3 $\sigma$ regions allowed using form factors in BSZ'15 in the full form factor approach (long-dashed blue) compared to our reference fit with the soft form factor approach (red, with 1,2,3 $\sigma$ contours).*

- The results of the fit using (IQCDF-KMPW) or (Full-FF-BSZ) and/or different set of observables are perfectly consistent once all correlations are included. But the individual observables...

| anomaly [4,6] bin | $P_5'$ error SIZE [pull] | $S_5$ error SIZE [pull] |
|---|---|---|
| Full-FF-**BSZ** (1503.05534) | 8.6% [2.7$\sigma$] | 12% [2.0$\sigma$] |
| IQCDF-**KMPW** (1510.04239) | 10% [2.9$\sigma$] | 40% [1.2$\sigma$] |

# Theoretical description of $B \to K^*\mu\mu$ in a nutshell:

# systematic treatment of hadronic uncertainties

# and **deconstruction** of incorrect criticisms

Discussion of Criticism from 3 papers:

Lyon-Zwicky, arXiv: **1406.0566** (**LZ'14**)
Jaeger-Camalich, arXiv: **1412.3183** (**JC'14**)
Ciuchini-Silvestrini-Valli et al. arXiv: **1512.07157** (**CFFMPSV'15**)

**Improved-QCDF approach**: QCDF+exploit **symmetry relations** at large-recoil (limit) among FF:

$$\frac{m_B}{m_B+m_{K^*}}\mathbf{V}(\mathbf{q^2}) = \frac{m_B+m_{K^*}}{2E}\mathbf{A_1}(\mathbf{q^2}) = \mathbf{T_1}(\mathbf{q^2}) = \frac{m_B}{2E}\mathbf{T_2}(\mathbf{q^2}) = \xi_\perp(E)$$

$$\frac{m_{K^*}}{E}\mathbf{A_0}(\mathbf{q^2}) = \frac{m_B+m_{K^*}}{2E}\mathbf{A_1}(\mathbf{q^2}) - \frac{m_B-m_{K^*}}{m_B}\mathbf{A_2}(\mathbf{q^2}) = \frac{m_B}{2E}\mathbf{T_2}(\mathbf{q^2}) - \mathbf{T_3}(\mathbf{q^2}) = \xi_\|(E)$$

Our approach is completed with 4 types of corrections. First two are related to FF decomposition:

$$\mathbf{F}(\mathbf{q^2}) = F^\infty(\xi_\perp, \xi_\|) + \Delta F^{\alpha_s}(q^2) + \Delta F^\Lambda(q^2)$$

- I. $\Delta F^{\alpha_s}(q^2)$: Known **Factorizable** $\alpha_s$ breaking corrections at NLO from QCDF.
- II. $\Delta F^\Lambda(q^2)$: **Factorizable power corrections** (using a systematic procedure for each FF)

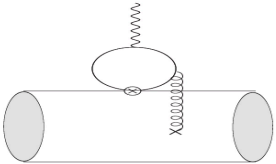
- III. Known **Non-factorizable** $\alpha_s$ corrections: spectator hard-scattering + 4-quark matrix elements & $O_8$.

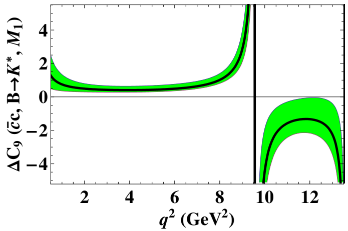
$$\langle \ell^+\ell^- \bar{K}^*_a | H_{\rm eff} | \bar{B} \rangle = \mathcal{C}_a \xi_a + \Phi_B \otimes T_a \otimes \Phi_{K^*} \text{ with } a = \perp, \|$$

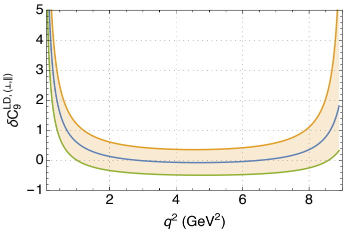
- IV. **Non-factorizable power corrections** including charm-quark loops.

# Non-factorizable power corrections

- **Non-factorizable power corrections (amplitudes)**: subleading new unknown non-perturbative. BEYOND SCET/QCDF at leading power in $1/m_b$. Multiply each amplitude $i = 0, \perp, \|$ with a complex $q^2$-dependent factor. $\mathcal{T}_i^{\rm had} \to (1 + r_i(q^2))\, \mathcal{T}_i^{\rm had}$ with $\mathcal{T}_i^{\rm had} = \mathcal{T}_i|_{C_7^{(\prime)} \to 0}$ entering $\langle K^* \gamma^* | H_{eff} | B \rangle$.
- **Charm-loops:** At large-recoil two type of contributions: $\Delta C_9^{BK^*} = \delta C_{9,\rm pert}^{BK(*)} + \mathbf{s_i} \delta C_{9,\rm non\ pert}^{BK(*),i}$
  - Short distance (hard-gluons): $\delta \mathbf{C}_{\mathbf{9},\rm pert}^{\mathbf{BK}(*)}$
    - LO included in $C_9 \to C_9 + Y(q^2)$
    - higher-order corrections via QCDF/HQET.
  - Long distance (soft-gluons): $\delta \mathbf{C}_{\mathbf{9},\rm non\ pert}^{\mathbf{BK}(*),\mathbf{i}}$
    - Only existing computation KMPW'10 using LCSR.
    - Partial computation yields $\Delta C_9^{BK^*} > 0$ ($s_i = 1$) $\Rightarrow$ enlarges the anomaly.
      **We obtain the LD from KMPW AND allow FOR ANY SIGN** $s_i = 0 \pm 1$





**What are Factorizable power corrections and how they emerge?**

Appear when expressing the full form factor in a soft form factor piece + corrections:

$$F^{full}(q^2) = F^{\rm soft}(\xi_{\perp,\|}(q^2)) + \Delta F^{\alpha_s}(q^2) + \Delta F^{\Lambda} \quad \text{with} \quad \Delta F^{\Lambda} = a_F + b_F \frac{q^2}{m_B^2} + c_F \frac{q^4}{m_B^4}$$

*How one can obtain power corrections?*

(DHMV'14)

$\Delta F^{\Lambda}$ is obtained from a 2nd order fit in $q^2/m_B^2 \Rightarrow$ central values $a_F$, $b_F$, $c_F$.

Errors are taken **uncorrelated** to be $\mathcal{O}(\Lambda/m_b) \times \mathrm{FF} \simeq 0.1\mathrm{FF}$.

Why? to minimize sensitivity/dependence on FF computational details.

| | $\hat{a}_F^{(1)}$ | $\hat{b}_F^{(1)}$ | $\hat{c}_F^{(1)}$ | $r(0\,\mathrm{GeV}^2)$ | $r(4\,\mathrm{GeV}^2)$ | $r(8\,\mathrm{GeV}^2)$ |
|---|---|---|---|---|---|---|
| $A_1$(KMPW) | $-0.01 \pm 0.03$ | $-0.06 \pm 0.02$ | $0.16 \pm 0.02$ | 5% | 6% | 5% |
| $A_1$(BZ) | $-0.01 \pm 0.03$ | $0.04 \pm 0.02$ | $0.08 \pm 0.02$ | 3% | 1% | 3% |

$r = (a_F + b_F q^2/m_B^2 + c_F q^4/m_B^4)/FF(q^2)$ is the percentage of p.c. found to be $\leq 10\%$

$\rightarrow$ Later on JC'14 followed same strategy and considered also uncorrelated errors but central values were set to zero.

# What do they missed in JC'14?

**In JC'14**: *It is implicitly assumed that the* **prediction** *of an observable like $P_5'$ is scheme independent.*

Scheme choice here means the way $\xi_{\perp,\|}$ are fixed to all orders in terms of full FF. Example:

$$\xi_\perp^{(1)}(q^2) \equiv \frac{m_B}{m_B+m_{K^*}} V(q^2) \quad \xi_\|^{(1)}(q^2) \equiv \frac{m_B+m_{K^*}}{2E} A_1(q^2) - \frac{m_B-m_{K^*}}{m_B} A_2(q^2), \ \text{(Beneke et al. 05)}$$

or

$$\xi_\perp^{(2)}(q^2) \equiv T_1(q^2), \qquad \xi_\|^{(2)}(q^2) \equiv \frac{m_{K^*}}{E} A_0(q^2). \ \text{(old Beneke et al. 01)}$$

**ALERT: THIS is ONLY TRUE if correlations are included.**

Illustrative example (using for instance BSZ):

| $\langle P_5'\rangle_{[4,6]}$ | error of f.f.+p.c. scheme-1 in transversity basis **DHMV'14** | error of f.f.+p.c. scheme-2 in helicity basis **JC'14** |
|---|---|---|
| **NO correlations** among errors of p.c. (hyp. 10%) | $\pm 0.05$ | $\pm$**0.12** |
| **WITH correlations** among errors of p.c. | $\pm 0.03$ | $\pm 0.03$ |

| | |
|---|---|
| **FULL FF** scheme indep. | $\pm 0.03$ |

Conclusions:

• If p.c. are taken uncorrelated to reduce the sensitivity to details of FF computation, which is fine, **not any arbitrary scheme choice is appropriate.**

Example: A bad choice like in JC'14 inflated artificially the errors **x 4** above.

# Criticism 2: A huge charm-loop or unknown non-factorizable correction?

Two attempts:

Attempt 1 (Lyon, Zwicky'14 unpublished):

- Using $e^+e^- \to$ hadrons to build a model of $c\bar{c}$ resonances at low-recoil in $B \to K\mu\mu$. **Conceptual problem**: **extrapolate result** at large-recoil and assume it holds the same for $B \to K^*\mu\mu$.

$\Rightarrow$ **Interesting observation:** Phase of helicity amplitudes $e^{i\delta_{J/\Psi K^*}}$ from $\delta_{J/\Psi K^*} \simeq 0$ (KMPW) to $\pi$ $\to$ we introduce $s_i$.

Attempt 2 (Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli'15 -CFFMPSV):

- Introduce a fully arbitrary parametrization for non-factorizable power correction:

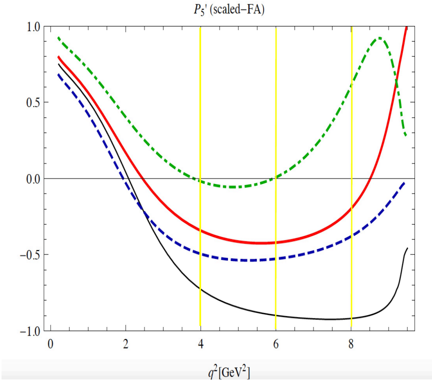
$$H_\lambda \to H_\lambda + h_\lambda \text{ where } h_\lambda = h_\lambda^{(0)} + h_\lambda^{(1)} q^2 + h_\lambda^{(2)} q^4 \qquad \text{and} \quad h_\lambda^{(0)} \to C_7^{NP}, h_\lambda^{(1)} \to C_9^{\rm NP}$$

with $(\lambda = 0, \pm)$ (copied from JC'14).
**Fundamental problems:** complete lack of theory input/output $\Rightarrow$ **no predictivity** with 18 free parameters (any shape). **Specific problems...**

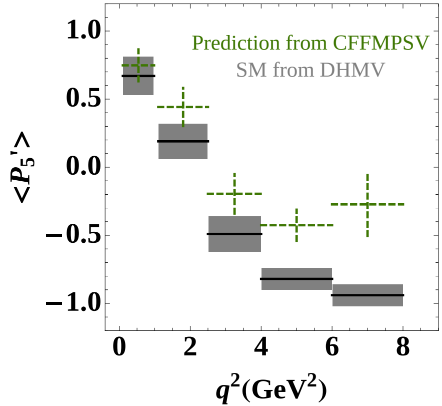
(CAUTION: They only considered $B \to K^*\mu\mu$ at large-recoil)

1. BOTH LZ'14 and CFFMPSV'15 exhibit the **same uptrend behaviour**:
**Predict $\langle P_5' \rangle_{[6,8]}$ to be above $\langle P_5' \rangle_{[4,6]}$ but data favours the opposite (more significance needed)**

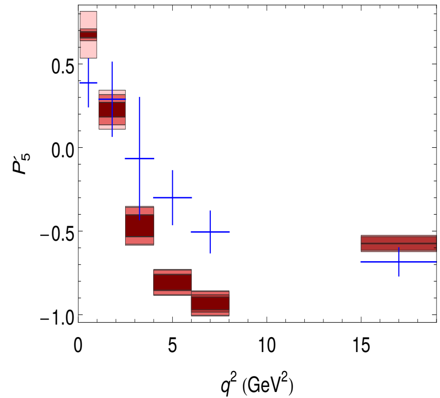
Lyon-Zwicky'14




Ciuchini-Silvestrini-Valli et al.'15




Data (blue) and DHMV (red).




Different hypothesis (colors RBG)

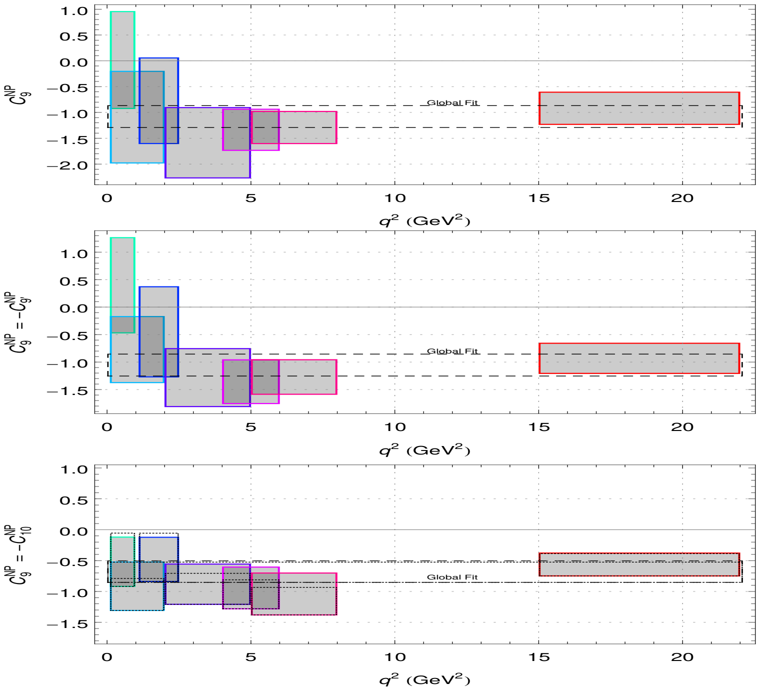
From Table 6 of predictions'

Descending trend of data.

2. **If the answer would be unknown $h_\lambda^{(i)}$ you cannot explain many data, while $C_9^{NP} = -1.1$ can:**

- nor $R_K$ (solved with $C_9^{NP} = -1.1$) neither any future LFVU observable like $R_{K^*}$ due to charm universality.
- any tiny tension in the low-recoil region of $B^0 \to K^{*0}\mu\mu$ ($1.7 \to 0.3\sigma$), $B^+ \to K^{*+}\mu\mu$ ($2.5 \to 1.2\sigma$), $B_s \to \phi\mu\mu$ ($2.3 \to 0.5\sigma$) cannot be explained.
- Also the old bin [2,4.3] of $P_2$ of 2013 is difficult to explain by charm.

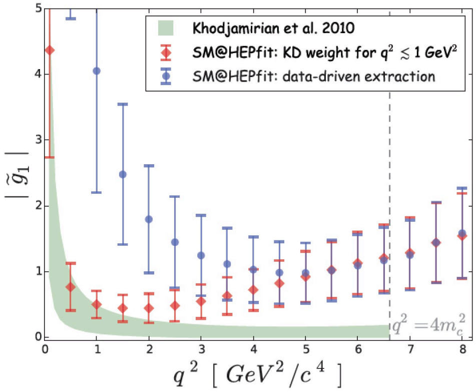
# Cross check: Bin by Bin analysis of $C_9$ in three scenarios




Result of bin-by-bin analysis of $C_9$ in 3 scenarios.

- **Notice the excellent agreement of bins [2,5], [4,6], [5,8].** *Strong argument in favour of including the [5,8] region-bin.*
- **First bin is afflicted by lepton-mass effects.** (see Back-up slides)
- **We do not find indication for a $q^2$-dependence in $C_9$ neither in the plots nor in a 6D fit adding $a^i + b^i s$ to $C_9^{\rm eff}$ for $i = K^*, K, \phi$.**
  $\rightarrow$ disfavours again charm explanation.
- 2nd and 3rd plots test if you allow for NP in other WC the agreement of $C_9$ bin by bin improves as compared to 1st plot.

# Specific problems of CFFMPSV'15




- $\tilde{g} = \Delta C_9^{non\,pert.}/(2C_1)$
- They force the fit (red points) to agree on the very low-q$^2$ with KMPW. This has two problems:
  - At very low-q$^2$ there are other problems **they forgot (lepton mass effects)**.
  - By forcing the fit to agree at very low-q$^2$ can induce an artificial tilt of your fit.
- More interestingly the blue points where KMPW is not imposed is perfectly compatible with

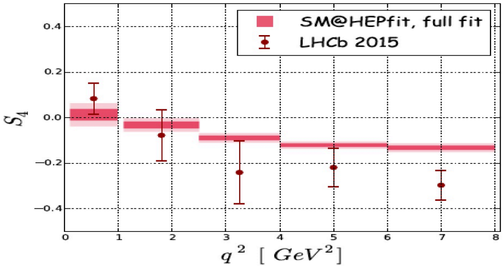
$C_9 - C_9^{SM} \simeq$ constant+KMPW **similar to us!!**.

So what is this constant $C_9^{\rm NP}$ or $h_\lambda^{(1)}$?

Contradictory statements:

**3.** "*No deviation is present once all the theoretical uncertainties are taken into account*".

$\Rightarrow$ Indeed they have a (**2.7**$\sigma$) deviation in $S_4$, a fully SM-like observable for us (us and also BSZ find good agreement with SM in all bins! **See table from DHMV'15**)




| $S_4(B \to K^* \mu^+ \mu^-)$ | Standard Model | Experiment | Pull |
|---|---|---|---|
| [0.1, 0.98] | $-0.08 \pm 0.05$ | $-0.08 \pm 0.07$ | $-0.0$ |
| [1.1, 2.5] | $-0.01 \pm 0.03$ | $0.08 \pm 0.11$ | $-0.8$ |
| [2.5, 4] | $0.11 \pm 0.07$ | $0.23 \pm 0.14$ | $-0.8$ |
| [4, 6] | $0.18 \pm 0.08$ | $0.22 \pm 0.09$ | $-0.3$ |
| [6, 8] | $0.22 \pm 0.07$ | $0.30 \pm 0.07$ | $-0.8$ |
| [15, 19] | $0.30 \pm 0.01$ | $0.28 \pm 0.04$ | $+0.5$ |

**4.** Symmetries transformations of $A_{\perp,\|,0}$ led to a **consistency relation**: **[Serra-Matias'14]**

$$P_2^{rel} = \frac{1}{2}\left[P_4' P_5' + \delta_a + \frac{1}{\beta}\sqrt{(-1 + P_1 + P_4'^2)(-1 - P_1 + \beta^2 P_5'^2) + \delta_b}\right] \qquad P_i \to \langle P_i \rangle \,(\Delta)$$

where $\delta_a$ and $\delta_b$ are function of product of tiny $P_6'$, $P_8'$, $P_3$.

*This* **must hold** *independently of any crazy non-factorizable, factorizable, or New Physics (with no weak phases $P_i^{CP} = 0$ or new scalars) that is included inside the $H_\lambda$ (or $A_{\perp,\|,0}$)*

**Example:** $\Rightarrow$ Using theory predictions (DHMV'15) for **bin [4,6]** one has:

$$\langle P_1\rangle = 0.03 \quad \langle P_4'\rangle = +0.82 \quad \langle P_5'\rangle = -0.82 \quad \langle P_2\rangle = \mathbf{-0.18}$$

consistency relation $\Rightarrow \langle P_2\rangle^{rel} = \mathbf{-0.17}$ ($\Delta = \mathbf{0.01}$ from binning). Perfect agreement.

The previous relation can be rewritten in terms of $A_{\rm FB} = f(F_L, S_i)$:

| | | $CFFMPSV_{predictions}$ | $CFFMPSV_{full\ fit}$ | SM-BSZ ($\delta_i = 0$) | SM-DHMV |
|---|---|---|---|---|---|
| [4,6] | $\langle A_{\rm FB}\rangle^{rel}$ | $\mathbf{-0.14 \pm 0.04}$ | $\mathbf{-0.16 \pm 0.03}$ | $+0.11 \pm 0.05$ | $+0.05 \pm 0.19$ |
| | $\langle A_{\rm FB}\rangle$ | $\mathbf{+0.05 \pm 0.04 \Rightarrow 3.4\sigma}$ | $\mathbf{+0.04 \pm 0.03 \Rightarrow 4.7\sigma}$ | $+0.12 \pm 0.04 \Rightarrow \mathbf{0.2\sigma}$ | $+0.08 \pm 0.11 \Rightarrow \mathbf{0.1\sigma}$ |
| [6,8] | $\langle A_{\rm FB}\rangle^{rel}$ | $-0.27 \pm 0.08$ | $-0.15 \pm 0.05$ | $--$ | $+0.17 \pm 0.18$ |
| | $\langle A_{\rm FB}\rangle$ | $+0.12 \pm 0.08 \Rightarrow 3.4\sigma$ | $+0.13 \pm 0.03 \Rightarrow 4.8\sigma$ | $--$ | $+0.21 \pm 0.21 \Rightarrow \mathbf{0.1\sigma}$ |

This table is computed assuming that central values of all predictions of observables correspond to the same set of theory parameters. No correlations included yet here.

**Summary:** SM-BSZ and SM-DHMV present **excellent consistency**. In CFFMPSV the internal consistency gets reduced in the most interesting bins, and unexpectedly even more in the full-fit.

# A glimpse into the future: looking at $C_{10}$

Having established with high significance a New Physics contribution to $C_9^{NP}$ what about $C_{10}^{NP}$?

$\mathcal{B}_{B_s\to\mu\mu}$ is an excellent observable to measure $C_{10} - C'_{10}$, but this can be nicely complemented:

From large-recoil expression:

$$P_2 = \frac{1}{\mathcal{N}}\left\{C_{10}s\left(2C_7^{\rm eff}\mathbf{m_b m_B} + {\rm Re}\left[C_9^{\rm eff}\right]\mathbf{s}\right) - C'_{10}s\left(2C'_7 m_b m_B + C_9^{\prime\,\rm eff}s\right)\right\}$$

where

$$\begin{aligned}\mathcal{N} = \quad & +4\left(C_7^{\rm eff\,2} + C_7^{\prime\,\rm eff\,2}\right)m_b^2 m_B^2 \\ & +4\left(C_7^{\rm eff}{\rm Re}\left[C_9^{\rm eff}\right] + C_7^{\prime\,\rm eff}C_9^{\prime\,\rm eff}\right)m_b m_B s + \left(|C_9^{\rm eff}|^2 + C_{10}^2 + C_9^{\prime\,\rm eff\,2} + C_{10}^{\prime\,2}\right)s^2\end{aligned}$$

In CDHMV'16 we point that $\mathbf{P_2}$ **in the first bin [0.1,0.98]** exhibits unique properties:

- Large sensitivity to $C_{10}^{NP}$ and extra shielding against $C_9$ in a very safe region.
- Sensitivity to any unknown non-factorizable p.c. hidden in $C_9^{\rm eff}$ is strongly $q^2$-suppressed.

**A $C_{10}^{NP} > 0$ improves agreement between data and SM**

Subtilities related to lepton masses have to be considered!

# Fits to magnetic operators $O_7 - O_{7'}$ at very low $q^2$

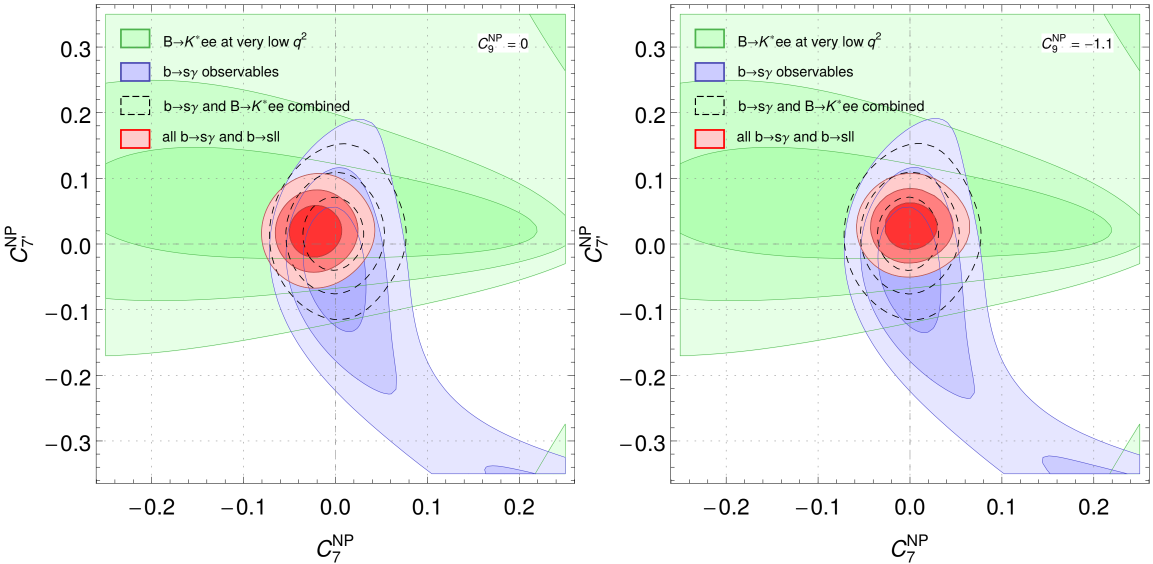



Figure: *Separate fits to $b \to s\gamma$ (blue) and $b \to see$ observables at very low $q^2$ (green). The combined fit to both sets of data is shown with dashed contours (1,2,3 $\sigma$ regions). The result of the global fit to all $b \to s\gamma$, $b \to s\ell\ell$ data is shown by the red contours (1,2,3 $\sigma$ regions). It is assumed that all the other Wilson coefficients have their SM values, except for the plot on the right, where $\mathcal{C}_{9\mu}^{NP} = -1.1$.*

# Conclusions

- The global analysis of $b \to s\ell^+\ell^-$ with 3 $\mathrm{fb}^{-1}$ dataset **shows that the solution** we proposed in 2013 to solve the anomaly with a contribution $\mathbf{C_9^{NP} \simeq -1}$ **is confirmed** and reinforced.

- The **fit result is very robust** and does not show a significant dependence nor on the theory approach used neither on the observables used once correlations are taken into account.
  $\Rightarrow$ **IQCDF and FULL-FF** are nicely complementary methods.

- We have shown that the **treatment of uncertainties** entering the observables in $B \to K^*\mu\mu$ is indeed **under excellent control** and the **alternative explanations** to New Physics are indeed **not in very solid ground**. We have proven (redressing the reassessing...) :
  - **Factorizable p.c.**: While using power corrections with uncorrelated errors is perfectly fine we have shown that an inadequate scheme's choice (JC'14) inflates artificially errors.
  - **Charm-loops**: They all predict bin [6,8] above [4,6] against data. They cannot explain LFVU. Also fundamental consistency problems detected.

- Near future? Maybe $C_{10}^{NP}$ or the prime coefficients can become significant soon.
  We pointed out an observable particularly clean in this respect.

# Thank you

# Back-up slides

**Statement 2**: *In JC'14 $P_5'$ is argued to be "accidentally" scheme independent even with uncorrelated p.c:*

In helicity basis we find:

$$P_5' = P_5'|_\infty \Big[ 1 + \frac{\mathbf{aV}_- - \mathbf{aT}_-}{\xi_\perp} \frac{m_B}{|k|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\parallel} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} - \frac{\mathbf{aV}_+}{\xi_\perp} \frac{\mathbf{2C_{9,\parallel}}}{\mathbf{C_{9,\perp} + C_{9,\parallel}}}$$
$$+ \frac{aV_0 - aT_0}{\xi_\parallel} 2 C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\parallel} - C_{10}^2}{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} + \mathcal{O}\left(\frac{m_{K^*}^2}{m_B^2}, \frac{q^2}{m_B^2}\right) \Big]$$

OK with JC'14 except for the missing term $\mathbf{aV}_+$. Choosing a scheme with $\mathbf{aV}_-$ or $\mathbf{aT}_-$ is equivalent.

**ALERT: Only apparently scheme independent in helicity basis for a subset of schemes!**

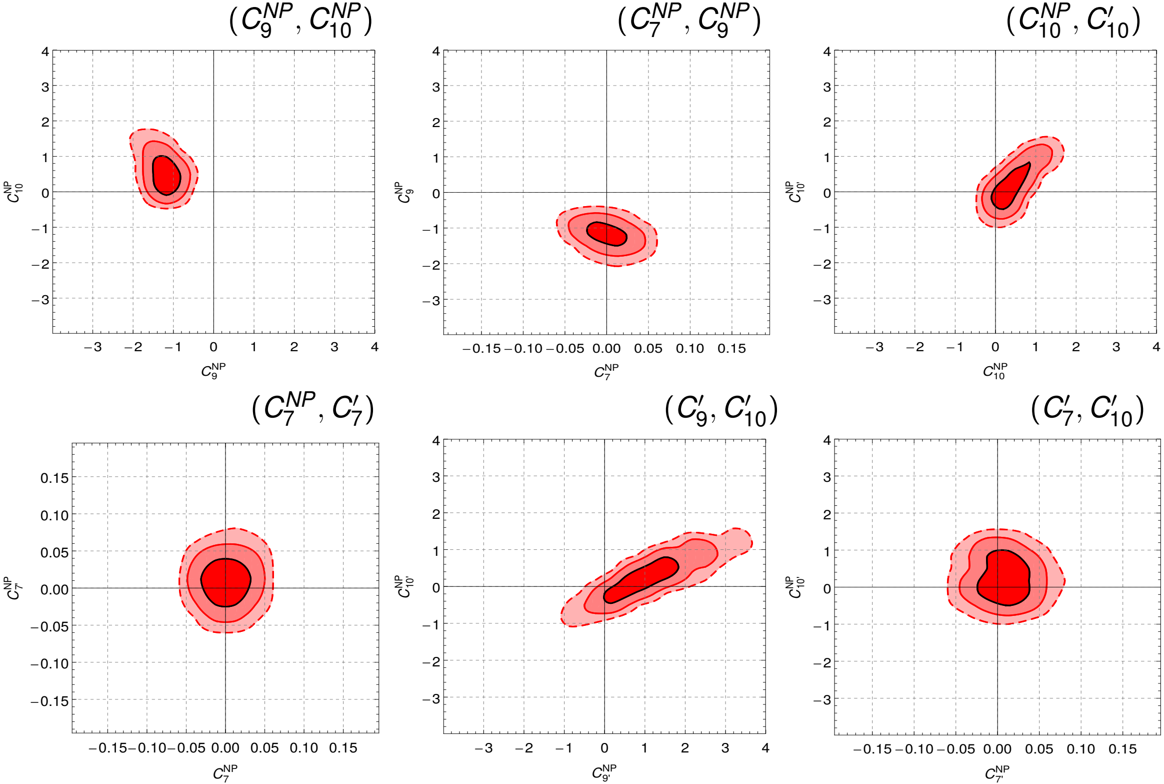
*Counterexample: In transversity basis becomes obvious that the choice of scheme matters*

$$P_5' = P_5'|_\infty \Big[ 1 + \frac{\mathbf{aV}}{\xi_\perp} \frac{C_{9,\parallel}}{C_{9,\perp} + C_{9,\parallel}} + \frac{\mathbf{aV} - 2\mathbf{aT_1}}{\xi_\perp} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\parallel} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} - \frac{aA_1}{\xi_\perp} \frac{C_{9,\perp} C_{9,\parallel} + C_{10}^2}{2(C_{9,\perp}^2 + C_{10}^2)} + \ldots$$

The weights of $\mathbf{aV}$ & $\mathbf{aT_1}$ are MANIFESTLY different: $P_5'^{(q^2=6)} = P_5'|_\infty (1 + [\mathbf{0.82}\,\mathbf{aV} - \mathbf{0.24}\,\mathbf{aT_1}]/\xi_\perp(6) + \ldots$

$$\xi_\perp^{(1)}(q^2) \equiv \frac{m_B}{m_B + m_{K^*}} V(q^2) \quad \Rightarrow \mathbf{aV} = 0 \quad \textit{or} \quad \xi_\perp^{(2)}(q^2) \equiv T_1(q^2) \quad \Rightarrow \mathbf{aT_1} = \mathbf{0}$$

Point also completely missed in CFFMPSV!!



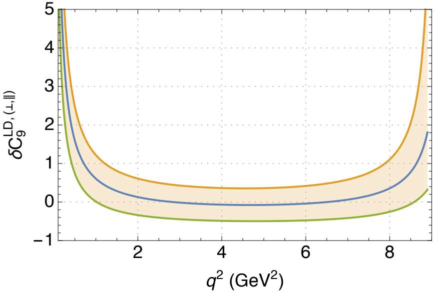
$(C_9^{NP}, C_{10}^{NP})$
$C_9^{NP}$
$C_{10'}^{NP}$
$(C_7^{NP}, C_9^{NP})$
$C_7^{NP}$
$C_9^{NP}$
$(C_{10}^{NP}, C'_{10})$
$C_{10}^{NP}$
$C_{10'}^{NP}$
$(C_7^{NP}, C'_7)$
$C_7^{NP}$
$C_{7'}^{NP}$
$(C'_9, C'_{10})$
$C_{9'}^{NP}$
$C_{10'}^{NP}$
$(C'_7, C'_{10})$
$C_{7'}^{NP}$
$C_{10'}^{NP}$

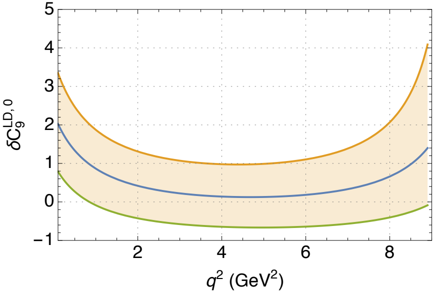
Inspired by Khodjamirian et al (KMPW): $C_9 \to C_9 + s_i\, \delta C_9^{\mathrm{LD}(i)}(q^2)$

Notice that KMPW implies $s_i = 1$, but we vary it independently $s_i = 0 \pm 1$, $i = 0, \perp, \|$ (Zwicky)

$$\delta C_9^{\mathrm{LD},(\perp,\|)}(q^2) = \frac{a^{(\perp,\|)} + b^{(\perp,\|)} q^2 [c^{(\perp,\|)} - q^2]}{b^{(\perp,\|)} q^2 [c^{(\perp,\|)} - q^2]}$$

$$\delta C_9^{\mathrm{LD},0}(q^2) = \frac{a^0 + b^0 [q^2 + s_0][c^0 - q^2]}{b^0 [q^2 + s_0][c^0 - q^2]}$$




Obtaining from fitting the long-distance part to KMPW.

The distribution (massless case) including the **S-wave** and normalized to $\Gamma'_{full}$:

$$\frac{1}{\Gamma'_{full}}\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[\frac{3}{4}\mathbf{F_T}\sin^2\theta_K + \mathbf{F_L}\cos^2\theta_K + (\frac{1}{4}\mathbf{F_T}\sin^2\theta_K - \mathbf{F_L}\cos^2\theta_K)\cos 2\theta_l$$
$$+\sqrt{\mathbf{F_T F_L}}\left(\frac{1}{2}\mathbf{P'_4}\sin 2\theta_K \sin 2\theta_l \cos\phi + \mathbf{P'_5}\sin 2\theta_K \sin\theta_l \cos\phi\right) + 2\mathbf{P_2 F_T}\sin^2\theta_K\cos\theta_l + \frac{1}{2}\mathbf{P_1 F_T}\sin^2\theta_K \sin^2\theta_l \cos 2\phi$$
$$-\sqrt{\mathbf{F_T F_L}}\left(\mathbf{P'_6}\sin 2\theta_K \sin\theta_l \sin\phi - \frac{1}{2}\mathbf{P'_8}\sin 2\theta_K \sin 2\theta_l \sin\phi\right) - \mathbf{P_3 F_T}\sin^2\theta_K\sin^2\theta_l \sin 2\phi\Big](1-\mathbf{F_S}) + \frac{1}{\Gamma'_{full}}\mathbf{W_S}$$

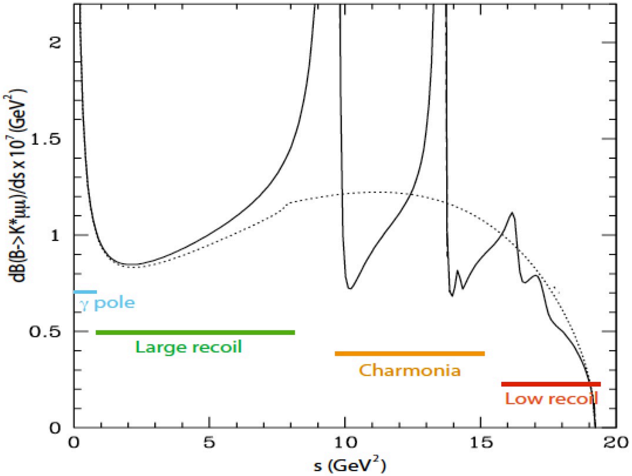
- in **blue** the set of relevant observables $\mathbf{P_{1,2}}$, $\mathbf{P'_{4,5}}$ that are functions of $A^{L,R}_{\perp,\|,0}$.
- the S-wave terms are (see discussion [M'12] & [HM'15]) not all free observables:

$$\frac{\mathbf{W_S}}{\Gamma'_{full}} = \frac{3}{16\pi}\Big[\mathbf{F_S}\sin^2\theta_\ell + \mathbf{A_S}\sin^2\theta_\ell\cos\theta_K + \mathbf{A_S^4}\sin\theta_K\sin 2\theta_\ell\cos\phi$$
$$+\mathbf{A_S^5}\sin\theta_K\sin\theta_\ell\cos\phi + \mathbf{A_S^7}\sin\theta_K\sin\theta_\ell\sin\phi + \mathbf{A_S^8}\sin\theta_K\sin 2\theta_\ell\sin\phi\Big]$$

Symmetries tell you that a complete basis (lepton masses to zero) is, for instance:

$\{\Gamma'_{K^*}, F_L, P_1, P_2, P_3, P'_4, P'_5, P'_6\}$ and only 4 of $\{F_S, A_S, A_S^4, A_S^5, A_S^7, A_S^8\}$ are independent.

# Four regions in $q^2$




Four regions in $q^2$:

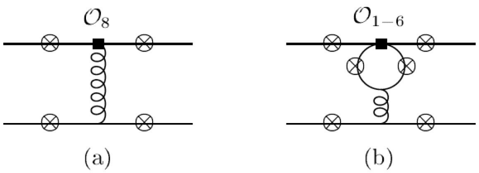
- **very large** $K^*$-recoil ($4m_\ell^2 < q^2 < 1$ GeV$^2$): $\gamma$ almost real.
- **large $K^*$-recoil/low-q$^2$**: $E_{K^*} \gg \Lambda_{QCD}$ or $4m_\ell^2 \leq q^2 < 9$ GeV$^2$: LCSR-FF
- charmonium region ($q^2 = m_{J/\Psi}^2, \ldots$) betwen $9 < q^2 < 14$ GeV$^2$.
- **low $K^*$-recoil/large-q$^2$**: $E_{K^*} \sim \Lambda_{QCD}$ or $14 < q^2 \leq (m_B - m_{K^*})^2$: LQCD-FF

QCDF provides a systematic framework to include $\alpha_s$ (factorizable and non-factorizable) corrections. Amplitude is represented by:

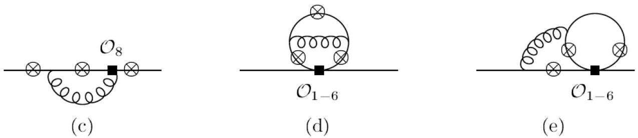
$$\langle \ell^+ \ell^- \bar{K}^*_a | H_{\rm eff} | \bar{B} \rangle = \mathcal{C}_a \xi_a + \Phi_B \otimes T_a \otimes \Phi_{K^*} \text{ with } a = \perp, \parallel$$

- III. **Non-factorizable** $\alpha_s$ corrections:

⇒ First class: spectator quark in the B meson participates in the hard scattering: ($T_a$)




⇒ Second class: Matrix elements of four-quark operators and the chromomagnetic dipole op.: ($\mathcal{C}_a$)




BUT also **we include** a second type of power corrections:

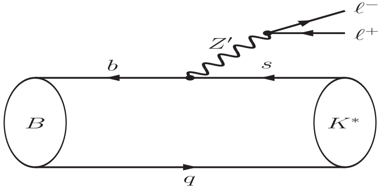
- IV. **Non-factorizable power corrections** including charm-quark loops.

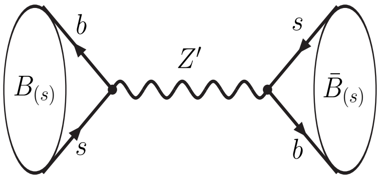
**All four (non-)factorizable $\alpha_s$ and power corrections are included in our predictions.**

In [**DMV'13**] we proposed to explain the anomaly in $B \to K^* \mu\mu$ with a $Z'$ **gauge boson** contributing to

$$\mathcal{O}_9 = e^2/(16\pi^2)\,(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)\,,$$

with specific couplings as a possible explanation of the anomaly in $P_5'$.




Using the notation of Buras'12,'13

$$\mathcal{L}^q = \left(\bar{s}\gamma_\nu P_L b \Delta_L^{sb} + \bar{s}\gamma_\nu P_R b \Delta_R^{sb} + h.c.\right) Z'^\nu \quad \mathcal{L}^{lep} = \left(\bar{\mu}\gamma_\nu P_L \mu \Delta_{\mathbf{L}}{}^{\mu\bar{\mu}} + \bar{\mu}\gamma_\nu P_R \mu \Delta_{\mathbf{R}}{}^{\mu\bar{\mu}} + \ldots\right) Z'^\nu$$

The Wilson coefficients of the semileptonic operators are:

$$\mathcal{C}^{\rm NP}_{\{9,10\}} = -\frac{1}{s_W^2 g_{SM}^2}\frac{1}{M_{Z'}^2}\frac{\Delta_L^{sb}\Delta^{\mu\mu}_{\{\mathbf{V},\mathbf{A}\}}}{\lambda_{ts}}\,, \quad \mathcal{C}^{\rm NP}_{\{9',10'\}} = -\frac{1}{s_W^2 g_{SM}^2}\frac{1}{M_{Z'}^2}\frac{\Delta_R^{sb}\Delta^{\mu\mu}_{\{\mathbf{V},\mathbf{A}\}}}{\lambda_{ts}}\,,$$

with the vector and axial couplings to muons: $\Delta^{\mu\mu}_{\mathbf{V},\mathbf{A}} = \Delta^{\mu\mu}_{\mathbf{R}} \pm \Delta^{\mu\mu}_{\mathbf{L}}$.

$\Delta_L^{sb}$ with same phase as $\lambda_{ts} = V_{tb}V_{ts}^*$ (to avoid $\phi_s$) like in MFV. Main constraint from $\Delta M_{B_s}$ ($\Delta^{sb}_{L,R}$).

A $Z'$ model can belong to the following categories:

| | no-coupling | non-zero couplings | $\text{Pull}_{\text{SM}}$ |
|---|---|---|---|
| $C_9$ | **no**-right-handed quark & no-muon-axial coupling | $\Delta_L^{sb} \neq 0,\ \Delta_V^{\mu\mu} \neq 0$ | $5.0\sigma$ |
| $(C_9, C_{10})$ | **no**-right-handed quark coupling | $\Delta_L^{sb} \neq 0,\ \Delta_V^{\mu\mu} \neq 0,\ \Delta_A^{\mu\mu} \neq 0$ | $4.8\sigma$ |
| $(C_9, C_9')$ | **no**-muon-axial coupling | $\Delta_L^{sb} \neq 0,\ \Delta_R^{sb} \neq 0,,\ \Delta_V^{\mu\mu} \neq 0$ | $4.9\sigma$ |
| $(C_{10}, C_{10}')$ | **no**-muon-vector coupling | $\Delta_L^{sb} \neq 0,\ \Delta_R^{sb} \neq 0,\ \Delta_A^{\mu\mu} \neq 0$ | ... |
| $(C_9', C_{10}')$ | **no**-left-handed quark coupling | $\Delta_R^{sb} \neq 0,\ \Delta_V^{\mu\mu} \neq 0,\ \Delta_A^{\mu\mu} \neq 0$ | ... |

Example: $C_9^{\text{NP}} = -1.1$, $\Delta_V^{\mu\mu}/M_Z' = -0.6\ \text{TeV}^{-1}$ and $\Delta_L^{bs}/M_Z' = 0.003\ \text{TeV}^{-1}$

- If NP enters **all** four semileptonic coefficients, the following relationships hold:

$$\frac{\mathcal{C}_9^{\text{NP}}}{\mathcal{C}_{10}^{\text{NP}}} = \frac{\mathcal{C}_{9'}^{\text{NP}}}{\mathcal{C}_{10'}^{\text{NP}}} = \frac{\Delta_V^{\mu\mu}}{\Delta_A^{\mu\mu}}, \qquad \frac{\mathcal{C}_9^{\text{NP}}}{\mathcal{C}_{9'}^{\text{NP}}} = \frac{\mathcal{C}_{10}^{\text{NP}}}{\mathcal{C}_{10'}^{\text{NP}}} = \frac{\Delta_L^{sb}}{\Delta_R^{sb}}.$$

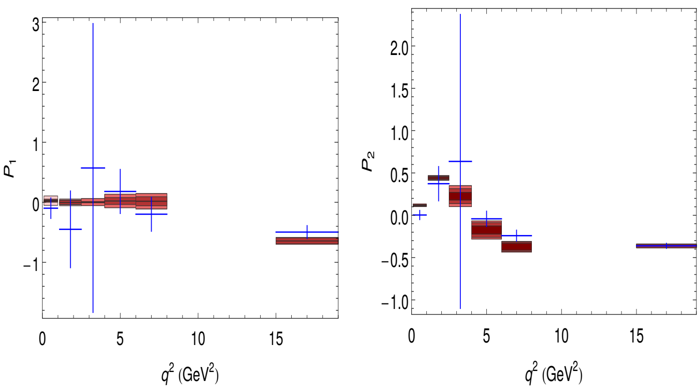
Many ongoing attempts to embed this kind of $Z'$ inside a model [U.Haisch, W.Altmannshofer, A.Buras, D. Straub,..]

# A few properties of the relevant observables $P_{1,2}$

The idea of **exact cancellation of the poorly known soft form factors at LO** at the zero of $A_{FB}$ was incorporated in the construction of the $P_i$ (this is why they are "**clean**" compared to the $S_i$)

$P_1$ and $P_2$ observables function of $A_\perp$ and $A_{||}$ amplitudes

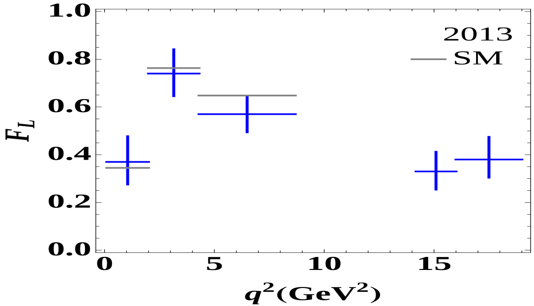
- **$P_1$**: Proportional to $|A_\perp|^2 - |A_{||}|^2$
  - Test the LH structure of SM.
    The existence of RH currents breaks the SM relation $A_\perp \sim -A_{||}$
- **$P_2$**: Proportional to $\mathrm{Re}(A_i A_j)$
  - Zero of $P_2$ at the same position as the zero of $A_{FB}$
  - $P_2$ is the clean version of $A_{FB}$. Their different normalizations offer different sensitivities.

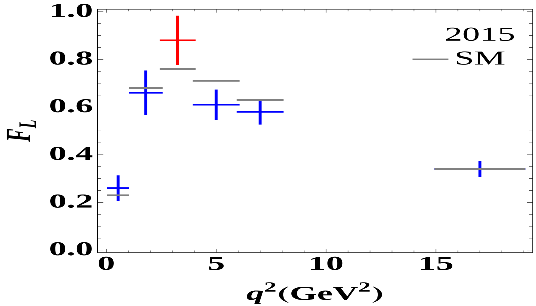



- $P_3$ and $P'_{6,8}$ are proportional to $\mathrm{Im} A_i A_j$ and small if there are no large phases. All are $< 0.1$.
- $P_i^{CP}$ are all negligibly small if there is no New Physics in weak phases.

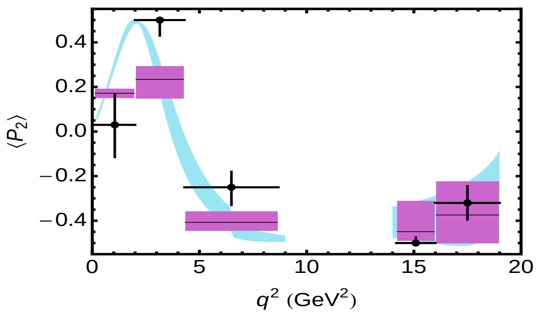
# What happened to $P_2$ in 2015?

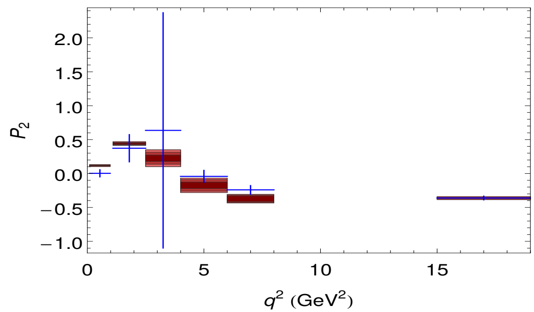
The new binning of $F_L$ in 2015 had a temporary effect on the very interesting bin [2.5,4]




Tiny unfortunate fluctuation up.




$$P_2 \propto \frac{1}{(1 - F_L)}$$

More data (in this bin) is crucial.

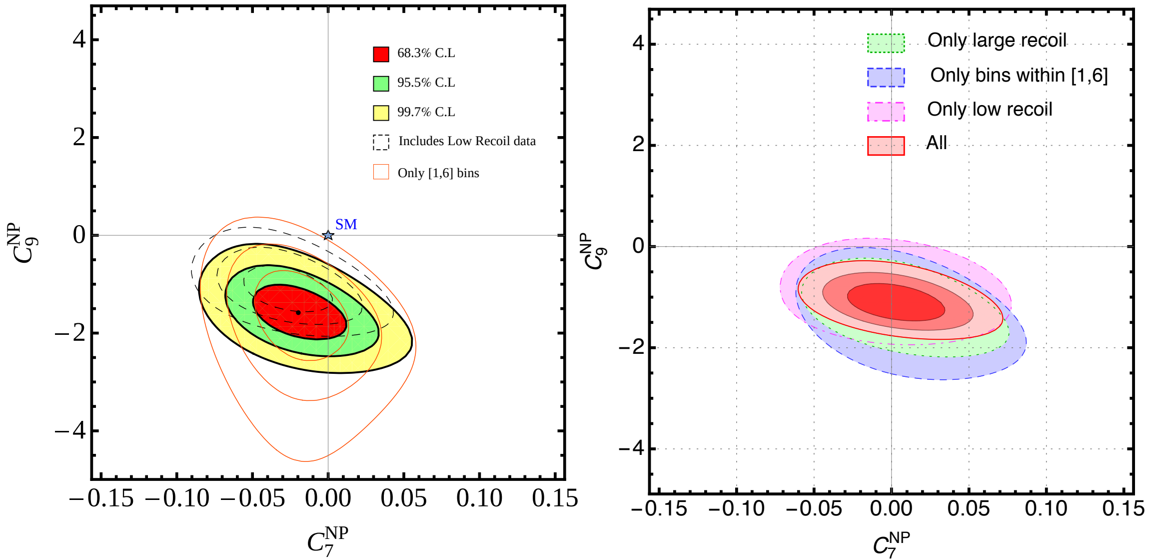



Figure: *For the scenario where NP occurs in the two Wilson coefficients $\mathcal{C}_7$ and $\mathcal{C}_9$, we compare the situation from the analysis in Fig. 1 of Ref. DMV'13(on the left) and the current situation (on the right). On the right, we show the 3 $\sigma$ regions allowed by large-recoil only (dashed green), by bins in the [1-6] range (long-dashed blue), by low recoil (dot-dashed purple) and by considering all data (red, with 1,2,3 $\sigma$ contours).*

# Bin (0.1,0.98) lepton-mass effect

LHCb naturally given the limited statistics takes the massless lepton limit. They measure:

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^3(\Gamma+\bar{\Gamma})}{d\Omega} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L^{LHCb})\sin^2\theta_K + F_L^{LHCb}\cos^2\theta_K + \frac{1}{4}(1-F_L^{LHCb})\sin^2\theta_K\cos 2\theta_l - F_L^{LHCb}\cos^2\theta_K\cos 2\theta_l + ...\Big]$$

which is modified once lepton masses are considered

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^3(\Gamma+\bar{\Gamma})}{d\Omega} = \frac{9}{32\pi}\Big[\frac{3}{4}\hat{F}_T\sin^2\theta_K + \hat{F}_L\cos^2\theta_K + \frac{1}{4}F_T\sin^2\theta_K\cos 2\theta_l - F_L\cos^2\theta_K\cos 2\theta_l + ...\Big]$$

where $\hat{F}_{T,L}$ and $F_{L,T}$ are [**JM'12**]. All our observables are thus written and computed in terms of the longitudinal and transverse polarisation fractions $F_{L,T}$

$$F_L = -\frac{J_{2c}}{d(\Gamma+\bar{\Gamma})/dq^2} \qquad F_T = 4\frac{J_{2s}}{d(\Gamma+\bar{\Gamma})/dq^2} \qquad \Rightarrow \qquad \hat{F}_L = \frac{J_{1c}}{d(\Gamma+\bar{\Gamma})/dq^2}$$

WHEN measured value $\hat{F}_L$ is used instead of $F_L$ SM prediction is shifted towards the data in 1st bin

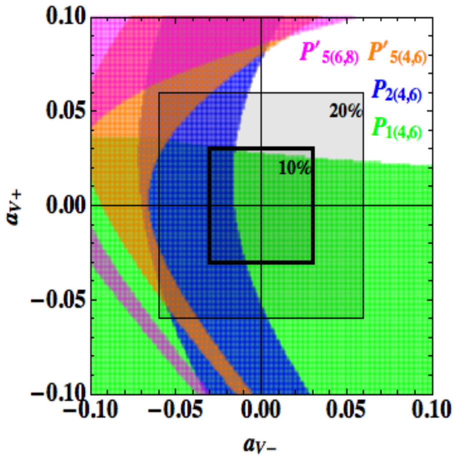
$$\langle F_L\rangle_{[0.1,0.98]} = 0.21 \to 0.26\,, \qquad \langle P_2\rangle_{[0.1,0.98]} = 0.12 \to 0.09\,,$$
$$\langle P_4'\rangle_{[0.1,0.98]} = -0.49 \to -0.38\,, \qquad \langle P_5'\rangle_{[0.1,0.98]} = 0.68 \to 0.53\,.$$

| | | $\lvert\delta\mathcal{C}_7\rvert = 0.1$ | $\lvert\delta\mathcal{C}_9\rvert = 1$ | $\lvert\delta\mathcal{C}_{10}\rvert = 1$ | $\lvert\delta\mathcal{C}_{7'}\rvert = 0.1$ | $\lvert\delta\mathcal{C}_{9'}\rvert = 1$ | $\lvert\delta\mathcal{C}_{10'}\rvert = 1$ |
|---|---|---|---|---|---|---|---|
| $\langle P_1\rangle_{[0.1,.98]}$ | $+\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $-0.53$ | $-0.05$ | $--$ |
| | $-\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $+0.52$ | $+0.05$ | $--$ |
| $\langle P_1\rangle_{[6,8]}$ | $+\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $+0.11$ | $+0.16$ | $-\mathbf{0.37}$ |
| | $-\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $-\mathbf{0.12}$ | $-\mathbf{0.17}$ | $+0.37$ |
| $\langle P_1\rangle_{[15,19]}$ | $+\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $+\mathbf{0.03}$ | $+\mathbf{0.15}$ | $-0.14$ |
| | $-\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $-0.03$ | $-0.11$ | $+\mathbf{0.19}$ |
| $\langle P_2\rangle_{[2.5,4]}$ | $+\lvert\delta C_i\rvert$ | $-0.31$ | $-0.21$ | $+\mathbf{0.05}$ | $--$ | $--$ | $--$ |
| | $-\lvert\delta C_i\rvert$ | $+\mathbf{0.19}$ | $+\mathbf{0.15}$ | $-0.04$ | $-0.03$ | $--$ | $--$ |
| $\langle P_2\rangle_{[6,8]}$ | $+\lvert\delta C_i\rvert$ | $-0.07$ | $-0.09$ | $-0.06$ | $--$ | $--$ | $--$ |
| | $-\lvert\delta C_i\rvert$ | $+\mathbf{0.11}$ | $+\mathbf{0.17}$ | $+\mathbf{0.05}$ | $--$ | $--$ | $--$ |
| $\langle P_2\rangle_{[15,19]}$ | $+\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $--$ | $-0.05$ | $+0.06$ |
| | $-\lvert\delta C_i\rvert$ | $--$ | $+0.04$ | $--$ | $--$ | $+0.05$ | $-0.06$ |
| $\langle P_4'\rangle_{[6,8]}$ | $+\lvert\delta C_i\rvert$ | $+\mathbf{0.04}$ | $--$ | $--$ | $-0.11$ | $-0.10$ | $+\mathbf{0.17}$ |
| | $-\lvert\delta C_i\rvert$ | $-0.05$ | $--$ | $--$ | $+\mathbf{0.09}$ | $+\mathbf{0.10}$ | $-0.20$ |
| $\langle P_4'\rangle_{[15,19]}$ | $+\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $--$ | $-\mathbf{0.06}$ | $+0.05$ |
| | $-\lvert\delta C_i\rvert$ | $--$ | $--$ | $--$ | $--$ | $+0.04$ | $-\mathbf{0.08}$ |
| $\langle P_5'\rangle_{[4,6]}$ | $+\lvert\delta C_i\rvert$ | $-0.11$ | $-0.15$ | $-0.10$ | $-0.11$ | $-0.06$ | $+\mathbf{0.21}$ |
| | $-\lvert\delta C_i\rvert$ | $+\mathbf{0.16}$ | $+\mathbf{0.28}$ | $+\mathbf{0.09}$ | $+\mathbf{0.15}$ | $+\mathbf{0.10}$ | $-0.21$ |
| $\langle P_5'\rangle_{[6,8]}$ | $+\lvert\delta C_i\rvert$ | $-0.04$ | $-0.07$ | $-0.07$ | $-0.08$ | $-0.08$ | $+\mathbf{0.19}$ |
| | $-\lvert\delta C_i\rvert$ | $+\mathbf{0.07}$ | $+\mathbf{0.19}$ | $+\mathbf{0.09}$ | $+\mathbf{0.10}$ | $+\mathbf{0.11}$ | $-0.18$ |

*If one wants to solve the anomalies exhibited in $b \to s\mu\mu$ processes through power corrections, it is important not to focus on one single observable, like $P_5'$, alone but on the full set.*

Illustrative example. Let's do the following exercise: Assume you take the non-optimal scheme-2 as in (JC'14) and helicity basis

$$a_{V_\pm} = \frac{1}{2}\left[\left(1+\frac{m_{K^*}}{m_B}\right)a_1 \mp \left(1-\frac{m_{K^*}}{m_B}\right)a_V\right].$$




- Notice that taking $a_{V-}$ in a range $\pm 0.1$ correspond to an absurd 33% power correction in KMPW.
  - $\to$ because a 10% in KMPW corresponds to 0.03 in $a_{V-}$.
  - $\to$ accepting values like ($a_{V-} = -0.1$, $a_{V+} = 0$) would imply that **BSZ computation of $A_1(q^2)$ is wrong by several sigmas.**
- An explanation of $\langle P_5' \rangle_{[4,6]}$, $\langle P_2 \rangle_{[4,6]}$ and $\langle P_1 \rangle_{[4,6]}$ within SM requires a 20% correction. Adding $\langle P_5' \rangle_{[6,8]}$ no common solution found even beyond 20%.

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R^*} + A_\parallel^L A_\parallel^{R^*}\right),$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R^*})\right] + \beta_\ell^2|A_S|^2,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right], \quad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + (L\to R)\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L\to R)\right], \quad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L^*}) + (L\to R)\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L^*}) - (L\to R) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^R A_S^*)\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L^*}) - (L\to R)\right], \quad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}\left[A_0^L A_S^* + (L\to R)\right],$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L^*}) - (L\to R) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* + A_\perp^R A_S^*)\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L^*}) + (L\to R)\right], \quad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L^*} A_\perp^L) + (L\to R)\right]$$

In red lepton mass terms and $\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$

# A glimpse into the future: Wilson coefficients versus Anomalies

| | | $R_K$ | $\langle P_5' \rangle_{[4,6],[6,8]}$ | $\mathcal{B}_{B_s\to\phi\mu\mu}$ | $\mathcal{B}_{B_s\to\mu\mu}$ | best-fit-point of global fit |
|---|---|---|---|---|---|---|
| $\mathcal{C}_9^{NP}$ | $+$ | | | | | |
| | $-$ | ✓ | ✓ [100%] | ✓ | | X |
| $\mathcal{C}_{10}^{NP}$ | $+$ | ✓ | [36%] | ✓ | ✓ | X |
| | $-$ | | ✓ [32%] | | | |
| $\mathcal{C}_{9'}$ | $+$ | | [21%] | ✓ | | X |
| | $-$ | ✓ | ✓ [36%] | | | |
| $\mathcal{C}_{10'}$ | $+$ | ✓ | ✓ [75%] | | | |
| | $-$ | | [75%] | ✓ | ✓ | X |

Table: *A checkmark (✓) indicates that a shift in the Wilson coefficient with this sign moves the prediction in the right direction to solve the corresponding anomaly. $\mathcal{B}_{B_s\to\mu\mu}$ is not an anomaly but a very mild tension.*

- $\mathcal{C}_9^{NP} < 0$ is consistent with all anomalies. This is the reason why it gives a strong pull.
- $\mathcal{C}_{10}^{NP}$, $C'_{9,10}$ fail in some anomaly. BUT
  - $\Rightarrow$ $\mathcal{C}_{10}^{NP}$ is the most promising coefficient after $\mathcal{C}_9$.
  - $\Rightarrow$ $C_9'$, $C_{10}'$ seems quite inconsistent between the different anomalies and the global fit.

# More technical arguments why scheme-2 **is not** an appropriate scheme

In the old scheme used by (also JC'14): $\xi_\perp^{(2)}(\mathbf{q^2}) \equiv \mathbf{T_1}(\mathbf{q^2}),\ \xi_\parallel^{(2)}(\mathbf{q^2}) \equiv \frac{\mathbf{m_{K^*}}}{\mathbf{E}}\mathbf{A_0}(\mathbf{q^2})$.

$\Rightarrow$ Power corrections associated to $\Delta T_1^\Lambda(q^2)$ and $\Delta A_0^\Lambda(q^2)$ are absorbed in $\xi_{\perp,\parallel}$.

**Problems of $T_1$ choice:**

- Extracting $T_1(0)$ from data on $B \to K^*\gamma$ is plagued of assumptions (as done in JC'12):
  1) assumption of no NP in $C_7^{(\prime)}$ + ignoring possible non-factorizable power corrections.
- Taking $T_1$ from LCSR and use it to define $\xi_\perp$ is also **non-optimal** (as done in JC'14).

$$A_\perp^{L,R} = \mathcal{N}_\perp \left[ \mathcal{C}_{9\pm10}^{+} [\mathbf{V}^{\mathbf{sff}+\alpha_\mathbf{s}}(\mathbf{q^2}) + \Delta V^\Lambda] + \mathcal{C}_7^{+} [\mathbf{T}_\mathbf{1}^{\mathbf{sff}+\alpha_\mathbf{s}}(\mathbf{q^2}) + \Delta T_1^\Lambda] \right] + \mathcal{O}(\alpha_s, \Lambda/m_b, ...)$$

If one is interested in obtaining accurated predictions for observables dominated by $C_9$ (like $P_5'$) better to have a good control of p.c on $V$ than in $T_1$.

$\Rightarrow$ $T_1$ may be a good choice for observables dominated by $C_7$.

**Problem of $A_0$ choice:**

$P_i$ observables do not depend on $A_0(q^2)$ FF. $\Rightarrow$ $A_0$ choice would be a good choice for lepton-mass suppressed observables.

# Theoretical description of $B \to K^* \ell^+ \ell^-$ @ low-$q^2$

2. **Full FF approach**: (Bharucha, Straub, Zwicky):
   Less general, attached to **details** of a particular LCSR computation.

   $\Rightarrow$ $\Delta F^{\alpha_s}$ and $\Delta F^{\Lambda}$ are included.

   $\Rightarrow$ BUT **BE CAREFUL** one should add **also** to be complete:

   - Non-factorizable $\alpha_s$ corrections from QCDF.
   - Non-factorizable power corrections and charm-quark loop effects

   Usually applied to $\mathbf{S_i} = (J_i + \bar{J}_i)/(d\Gamma + \bar{d}\Gamma)$

   $\to$ **observables highly dependent on FF-error estimate and internal assumptions of FF computation. A small error in FF induces a small error in $S_i$**

   Why we prefer to work within lQCDF:

   - NATURAL FRAMEWORK for optimized observables $P_i$
   - CORRELATIONS ARE TRANSPARENT and easy to REPRODUCE
   - It allows us to predict observables from different set of FORM FACTORS (BZ,BSZ,KMPW) and to compare results.

- **Amplitude analysis** (Petridis, Egede, ...). Not a FF treatment but a different approach to data based on **exploiting the symmetries of the distribution**.

2. **Full FF approach**: (Bharucha, Straub, Zwicky):
   Less general, attached to **details** of a particular LCSR computation.
   - $\Rightarrow$ $\Delta F^{\alpha_s}$ and $\Delta F^{\Lambda}$ are included.
   - $\Rightarrow$ BUT **BE CAREFUL** one should add **also** to be complete:
     - Non-factorizable $\alpha_s$ corrections from QCDF.
     - Non-factorizable power corrections and charm-quark loop effects

   Usually applied to $\mathbf{S_i} = (J_i + \bar{J}_i)/(d\Gamma + \bar{d}\Gamma)$
   $\to$ **observables highly dependent on FF-error estimate and internal assumptions of FF computation. A small error in FF induces a small error in $S_i$**

   Why we prefer to work within lQCDF:

   - NATURAL FRAMEWORK for optimized observables $P_i$
   - CORRELATIONS ARE TRANSPARENT and easy to REPRODUCE
   - It allows us to predict observables from different set of FORM FACTORS (BZ,BSZ,KMPW) and to compare results.

- **Amplitude analysis** (Petridis, Egede, ...). Not a FF treatment but a different approach to data based on **exploiting the symmetries of the distribution**.

*B-meson distribution amplitudes.*

| FF-KMPW | $F^i_{BK^{(*)}}(0)$ | $b^i_1$ |
|---|---|---|
| $f^+_{BK}$ | $0.34^{+0.05}_{-0.02}$ | $-2.1^{+0.9}_{-1.6}$ |
| $f^0_{BK}$ | $0.34^{+0.05}_{-0.02}$ | $-4.3^{+0.8}_{-0.9}$ |
| $f^T_{BK}$ | $0.39^{+0.05}_{-0.03}$ | $-2.2^{+1.0}_{-2.00}$ |
| $V^{BK^*}$ | $\mathbf{0.36^{+0.23}_{-0.12}}$ | $-4.8^{+0.8}_{-0.4}$ |
| $A^{BK^*}_1$ | $\mathbf{0.25^{+0.16}_{-0.10}}$ | $0.34^{+0.86}_{-0.80}$ |
| $A^{BK^*}_2$ | $0.23^{+0.19}_{-0.10}$ | $-0.85^{+2.88}_{-1.35}$ |
| $A^{BK^*}_0$ | $0.29^{+0.10}_{-0.07}$ | $-18.2^{+1.3}_{-3.0}$ |
| $T^{BK^*}_1$ | $0.31^{+0.18}_{-0.10}$ | $-4.6^{+0.81}_{-0.41}$ |
| $T^{BK^*}_2$ | $0.31^{+0.18}_{-0.10}$ | $-3.2^{+2.1}_{-2.2}$ |
| $T^{BK^*}_3$ | $0.22^{+0.17}_{-0.10}$ | $-10.3^{+2.5}_{-3.1}$ |

Table: The $B \to K^{(*)}$ form factors from LCSR and their $z$-parameterization.

*Light-meson distribution amplitudes+EOM.*

- Interestingly in BSZ (update from BZ) most relevant FF from BZ moved towards KMPW. For example:

$$V^{BZ}(0) = 0.41 \to 0.37 \quad T_1^{BZ}(0) = 0.33 \to 0.31$$

- The size of uncertainty in *BSZ* = size of error of p.c.

| FF-BSZ | $B \to K^*$ | $B_s \to \phi$ | $B_s \to K^*$ |
|---|---|---|---|
| $A_0(0)$ | $0.391 \pm 0.035$ | $0.433 \pm 0.035$ | $0.336 \pm 0.032$ |
| $A_1(0)$ | $\mathbf{0.289 \pm 0.027}$ | $0.315 \pm 0.027$ | $0.246 \pm 0.023$ |
| $A_{12}(0)$ | $0.281 \pm 0.025$ | $0.274 \pm 0.022$ | $0.246 \pm 0.023$ |
| $V(0)$ | $\mathbf{0.366 \pm 0.035}$ | $0.407 \pm 0.033$ | $0.311 \pm 0.030$ |
| $T_1(0)$ | $0.308 \pm 0.031$ | $0.331 \pm 0.030$ | $0.254 \pm 0.027$ |
| $T_2(0)$ | $0.308 \pm 0.031$ | $0.331 \pm 0.030$ | $0.254 \pm 0.027$ |
| $T_{23}(0)$ | $0.793 \pm 0.064$ | $0.763 \pm 0.061$ | $0.643 \pm 0.058$ |

Table: Values of the form factors at $q^2 = 0$ and their uncertainties.

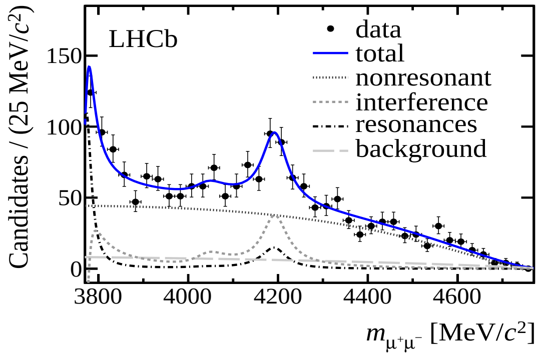
All FF determinations are computed in the transversity basis ($A_{\perp,\|,0}$) and correspond to $V, A_{0,1,2}, T_{1,2,3}$.

But some people prefer (at their own risk) to use an helicity basis:

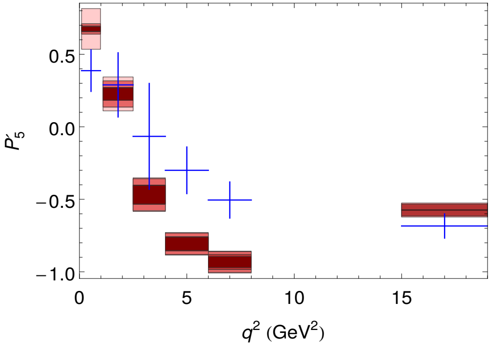
$$
\begin{aligned}
V_\pm(q^2) &= \frac{1}{2}\left[\left(1+\frac{m_V}{m_B}\right)A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B(m_B+m_V)}V(q^2)\right], \\
V_0(q^2) &= \frac{1}{2m_V\lambda^{1/2}(m_B+m_V)}\left[(m_B+m_V)^2(m_B^2-q^2-m_V^2)A_1(q^2)-\lambda A_2(q^2)\right], \\
T_\pm(q^2) &= \frac{m_B^2-m_V^2}{2m_B^2}T_2(q^2) \mp \frac{\lambda^{1/2}}{2m_B^2}T_1(q^2), \\
T_0(q^2) &= \frac{m_B}{2\,m_V\lambda^{1/2}}\left[(m_B^2+3m_V^2-q^2)T_2(q^2)-\frac{\lambda}{(m_B^2-m_V^2)}T_3(q^2)\right], \\
S(q^2) &= A_0(q^2),
\end{aligned}
\tag{31}
$$

- It corresponds to **large** $\sqrt{q^2} \sim \mathcal{O}(m_b)$ above $\Psi'$ mass, i.e., $E_K$ is around GeV or below.
- OPE in $E_K/\sqrt{q^2}$ or $\Lambda_{QCD}/\sqrt{q^2}$ (Buchalla et al). **NLO QCD correct.** to the OPE coeffs (Greub et al)
- **Lattice QCD form factors with correlations** (Horgan et al proceeding update)
- Estimates on BR from GP (5%) and BBF (2%) using Shifman's model.
  $\Rightarrow \pm 10\%$ on angular observables to account for possible Duality Violations.

Existence of $c\bar{c}$ **resonances** in this region (clearly seen $\psi(4160)$ in $B^- \to K^- \mu^+ \mu^-$),
$\Rightarrow$ require to take a long bin.




... but this region is neither the most sensitive to New Physics nor where interesting things happen!

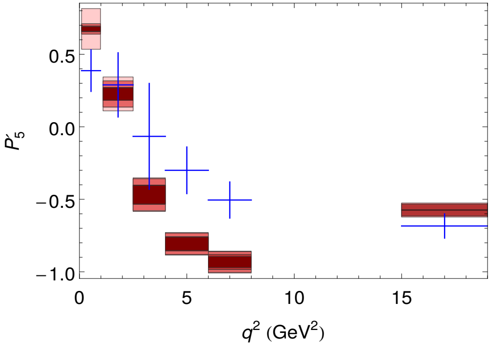



$P_5'$ was proposed for the first time in **DMRV, JHEP 1301(2013)048**

$$P_5' = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2+|A_\parallel|^2)}} = \sqrt{2}\frac{\mathrm{Re}[n_0 n_\perp^\dagger]}{\sqrt{|n_0|^2(|n_\perp|^2+|n_\parallel|^2)}}.$$

with $n_0 = (A_0^L, A_0^{R*})$, $n_\perp = (A_\perp^L, -A_\perp^{R*})$ and $n_\parallel = (A_\parallel^L, A_\parallel^{R*})$

- If no-RHC $|n_\perp| \simeq |n_\parallel|$ ($H_{+1} \simeq 0$) $\Rightarrow P_5' \propto \cos\theta_{\mathbf{0},\perp}(\mathbf{q^2})$




$P_5'$ was proposed for the first time in **DMRV, JHEP 1301(2013)048**

$$P_5' = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2 + |A_\|\|^2)}} = \sqrt{2}\frac{\mathrm{Re}[n_0 n_\perp^\dagger]}{\sqrt{|n_0|^2(|n_\perp|^2 + |n_\|\|^2)}}.$$

with $n_0 = (A_0^L, A_0^{R*})$, $n_\perp = (A_\perp^L, -A_\perp^{R*})$ and $n_\| = (A_\|^L, A_\|^{R*})$

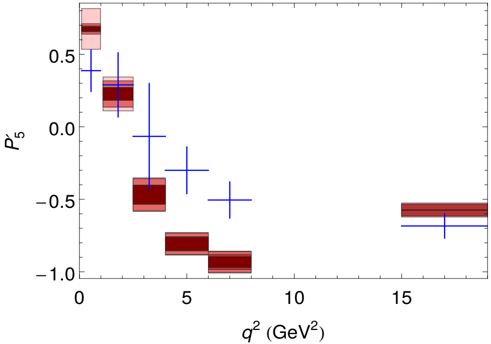
- If no-RHC $|n_\perp| \simeq |n_\|\|$ $(H_{+1} \simeq 0) \Rightarrow P_5' \propto \cos\theta_{\mathbf{0},\perp}(\mathbf{q^2})$

In the large-recoil limit with no RHC

$$A_{\perp,\|}^L \propto (1,-1)\left[\mathcal{C}_9^{\rm eff} - \mathcal{C}_{10} + \frac{2\hat{m}_b}{\hat{s}}\mathcal{C}_7^{\rm eff}\right]\xi_\perp(E_{K^*}) \qquad A_{\perp,\|}^R \propto (1,-1)\left[\mathcal{C}_9^{\rm eff} + \mathcal{C}_{10} + \frac{2\hat{m}_b}{\hat{s}}\mathcal{C}_7^{\rm eff}\right]\xi_\perp(E_{K^*})$$

$$A_0^L \propto -\left[\mathcal{C}_9^{\rm eff} - \mathcal{C}_{10} + 2\hat{m}_b\mathcal{C}_7^{\rm eff}\right]\xi_\|(E_{K^*}) \quad A_0^R \propto -\left[\mathcal{C}_9^{\rm eff} + \mathcal{C}_{10} + 2\hat{m}_b\mathcal{C}_7^{\rm eff}\right]\xi_\|(E_{K^*})$$

- In SM $\mathcal{C}_9^{SM} + \mathcal{C}_{10}^{SM} \simeq 0 \to |A_{\perp,\|}^R| \ll |A_{\perp,\|}^L|$
- **In $P_5'$: If $C_9^{NP} < 0$** then $A_{0,\|}^R \uparrow$, $|A_\perp^R| \uparrow$ and $|A_{0,\|}^L| \downarrow$, $A_\perp^L \downarrow$ and due to $-$, $|P_5'|$ gets **strongly** reduced.




$P'_5$ was proposed for the first time in **DMRV, JHEP 1301(2013)048**

$$P'_5 = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2 + |A_\parallel|^2)}} = \sqrt{2}\frac{\mathrm{Re}[n_0 n_\perp^\dagger]}{\sqrt{|n_0|^2(|n_\perp|^2 + |n_\parallel|^2)}}.$$

with $n_0 = (A_0^L, A_0^{R*})$, $n_\perp = (A_\perp^L, -A_\perp^{R*})$ and $n_\parallel = (A_\parallel^L, A_\parallel^{R*})$

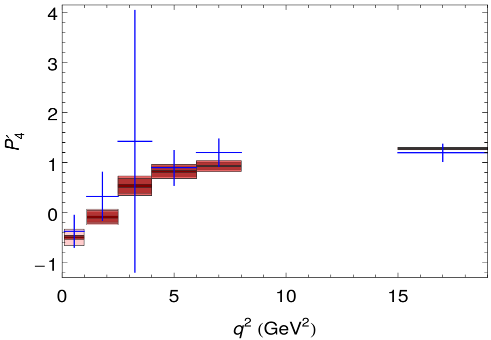
- If no-RHC $|n_\perp| \simeq |n_\parallel|$ $(H_{+1} \simeq 0) \Rightarrow P'_5 \propto \cos\theta_{\mathbf{0},\perp}(\mathbf{q^2})$

In the large-recoil limit with no RHC

$$A^L_{\perp,\parallel} \propto (1,-1)\left[\mathcal{C}_9^{\rm eff} - \mathcal{C}_{10} + \frac{2\hat{m}_b}{\hat{s}}\mathcal{C}_7^{\rm eff}\right]\xi_\perp(E_{K^*}) \qquad A^R_{\perp,\parallel} \propto (1,-1)\left[\mathcal{C}_9^{\rm eff} + \mathcal{C}_{10} + \frac{2\hat{m}_b}{\hat{s}}\mathcal{C}_7^{\rm eff}\right]\xi_\perp(E_{K^*})$$

$$A^L_0 \propto -\left[\mathcal{C}_9^{\rm eff} - \mathcal{C}_{10} + 2\hat{m}_b\mathcal{C}_7^{\rm eff}\right]\xi_\parallel(E_{K^*}) \quad A^R_0 \propto -\left[\mathcal{C}_9^{\rm eff} + \mathcal{C}_{10} + 2\hat{m}_b\mathcal{C}_7^{\rm eff}\right]\xi_\parallel(E_{K^*})$$

- In SM $\mathcal{C}_9^{SM} + \mathcal{C}_{10}^{SM} \simeq 0 \to |A^R_{\perp,\parallel}| \ll |A^L_{\perp,\parallel}|$
- **In $P'_5$: If $C_9^{NP} < 0$ then** $A^R_{0,\parallel} \uparrow$, $|A^R_\perp| \uparrow$ and $|A^L_{0,\parallel}| \downarrow$, $A^L_\perp \downarrow$ and due to $-$, $|P'_5|$ gets **strongly** reduced.




$P_4'$ was proposed for the first time in **DMRV, JHEP 1301(2013)048**

$$P_4' = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\|^{L*} + A_0^R A_\|^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2+|A_\||^2)}} = \sqrt{2}\frac{\mathrm{Re}[n_0 n_\|^\dagger]}{\sqrt{|n_0|^2(|n_\perp|^2+|n_\||^2)}}.$$

with $n_0 = (A_0^L, A_0^{R*})$, $n_\perp = (A_\perp^L, -A_\perp^{R*})$ and $n_\| = (A_\|^L, A_\|^{R*})$

- If no-RHC $|n_\perp| \simeq |n_\|| \; (H_{+1} \simeq 0) \Rightarrow P_4' \propto \cos\theta_{\mathbf{0},\|}(\mathbf{q^2})$

In the large-recoil limit with no RHC

$$A_{\perp,\|}^L \;\propto\; (1,-1)\left[\mathcal{C}_9^{\rm eff} - \mathcal{C}_{10} + \frac{2\hat{m}_b}{\hat{s}}\mathcal{C}_7^{\rm eff}\right]\xi_\perp(E_{K^*}) \qquad A_{\perp,\|}^R \propto (1,-1)\left[\mathcal{C}_9^{\rm eff} + \mathcal{C}_{10} + \frac{2\hat{m}_b}{\hat{s}}\mathcal{C}_7^{\rm eff}\right]\xi_\perp(E_{K^*})$$

$$A_0^L \;\propto\; -\left[\mathcal{C}_9^{\rm eff} - \mathcal{C}_{10} + 2\hat{m}_b\mathcal{C}_7^{\rm eff}\right]\xi_\|(E_{K^*}) \quad A_0^R \propto -\left[\mathcal{C}_9^{\rm eff} + \mathcal{C}_{10} + 2\hat{m}_b\mathcal{C}_7^{\rm eff}\right]\xi_\|(E_{K^*})$$

- In SM $\mathcal{C}_9^{SM} + \mathcal{C}_{10}^{SM} \simeq 0 \to |A_{\perp,\|}^R| \ll |A_{\perp,\|}^L|$
- **In** $P_4'$ :If $C_9^{NP} < 0$ then $A_{0,\|}^R \uparrow$, $|A_\perp^R| \uparrow$ and $|A_{0,\|}^L| \downarrow$, $A_\perp^L \downarrow$ due to $+$ what L loses R gains (little change).

Two last important problems in JC'14:

I) $P_5'$ is claimed to be scheme independent **in their approach** in JC'14.

**This is wrong** consequence of using helicity basis + restricted set of schemes.

Proven numerically in DLMV'14 and analytically in (CDLMV'16) $\Rightarrow$ missing term.

II) Undervalutation of the error of $\xi_\perp$ in JC'14 (affects $F_L$ and $S_i$):

- $\xi_\perp = 0.31 \pm 0.04$ in JC'14: from spread of **only** central values of BZ,KMPW,DSE.
- $\xi_\perp = 0.31^{+0.20}_{-0.10}$ is our input using KMPW but including errors!

- **Positive outcome**: New ingredient added in JC'12: **factorizable power corrections.**
- **Error of JC'12 and JC'14**: **missing the keypoint of scheme dependence that leads them to** artificially inflate errors.
- Our contribution DHMV'14:
  - **Systematic computation of p.c.**
  - **Identification of the relevance of the scheme choice with uncorrelated p.c.**
  - **Correct evaluation of impact in observables**

*In summary, we have shown that to take power corrections uncorrelated and $\mathcal{O}(\Lambda/m_b)$ is perfectly fine (even recommended to be on a conservative side)* **but always** *using an appropriate scheme choice.*

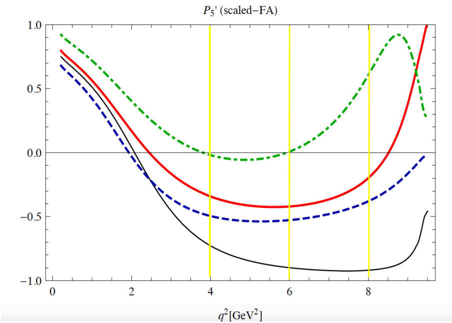
# Criticism 2: A huge non-factorizable (charm contribution) can explain $P_5'$?

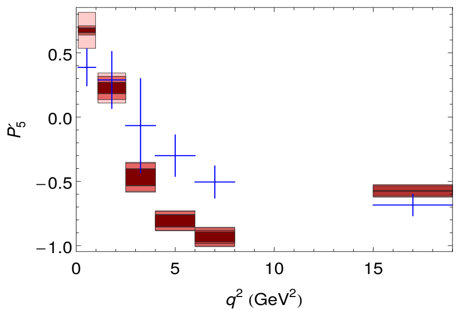
Attempt 1 (Lyon, Zwicky'14 unpublished):

- Using $e^+e^- \to$ hadrons to build a model of $c\bar{c}$ resonances at low-recoil in $B \to K\mu\mu$.
  **Two problems**: extrapolate result at large-recoil and assume it holds the same for $B \to K^*\mu\mu$.

  **Left**: Different predictions from LZ'14 for $P_5'$ corresponding to different hypothesis of extrapolation from high-q$^2$ to low-q$^2$: in all cases LZ'14 predicts bin [6,8] above [4,6].

  • **Positive outcome:** Phase of helicity amplitudes $e^{i\delta_{J/\Psi K^*}}$ from $\delta_{J/\Psi K^*} \simeq 0$ (KMPW) to $\pi$.




**Data tell us:** Smooth behaviour of 3 fb$^{-1}$ data where bin [6,8] is not above [4,6] does not favour claims on large-long distance charm $q^2$ effects in [6,8] bin.

• Our contribution DHMV'14&15: **We include a free parameter $s_i$ for each amplitude from -1 to 1**

Indeed, **our charm error estimate @anomaly is more conservative than BSZwicky estimate.**

**2. If the answer is $h_\lambda^{(1)}$ you are unable to explain many data, if it is $C_9^{NP} = -1.1$ "yes you can":**

- nor $R_K$ (solved with $C_9^{NP} = -1.1$) neither any LFVU observable like $R_{K^*}$ due to charm universality.
- any tiny tension in the low-recoil region of $B^0 \to K^{*0}\mu\mu$ ($1.7 \to \mathbf{0.3}\sigma$), $B^+ \to K^{*+}\mu\mu$ ($2.5 \to \mathbf{1.2}\sigma$), $B_s \to \phi\mu\mu$ ($2.3 \to \mathbf{0.5}\sigma$). Also the old bin [2,4.3] of $P_2$ of 2013 cannot be explained.
- ... (stay tunned)

Contradictory statements:

- "*No deviation is present once all the theoretical uncertainties are taken into account*".

$\Rightarrow$ By forcing the fit they induce a problem ($\mathbf{2.7}\sigma$) in $S_4$ a fully SM-like observable (us and BSZ we both find good agreement with SM in all bins!)