SUMMER INSTITUTE: USING PARTICLE PHYSICS TO UNDERSTAND AND IMAGE THE EARTH

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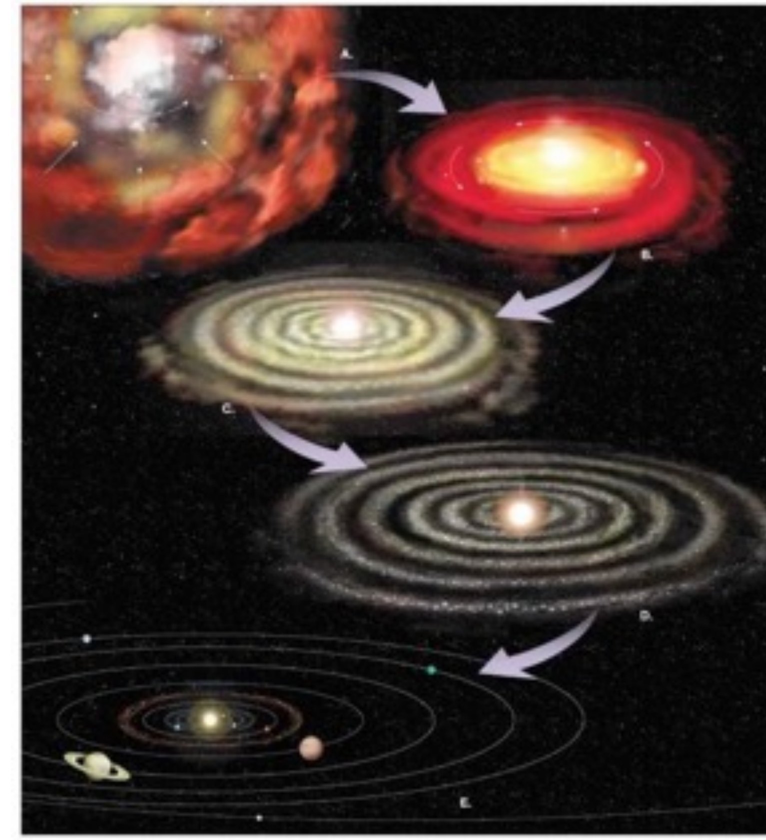
Thermal evolution of the Earth

Ondřej Šrámek
Department of Geophysics
Charles University in Prague
ondrej.sramek@gmail.com
geo.mff.cuni.cz/~sramek

Hot beginning



Planetary formation

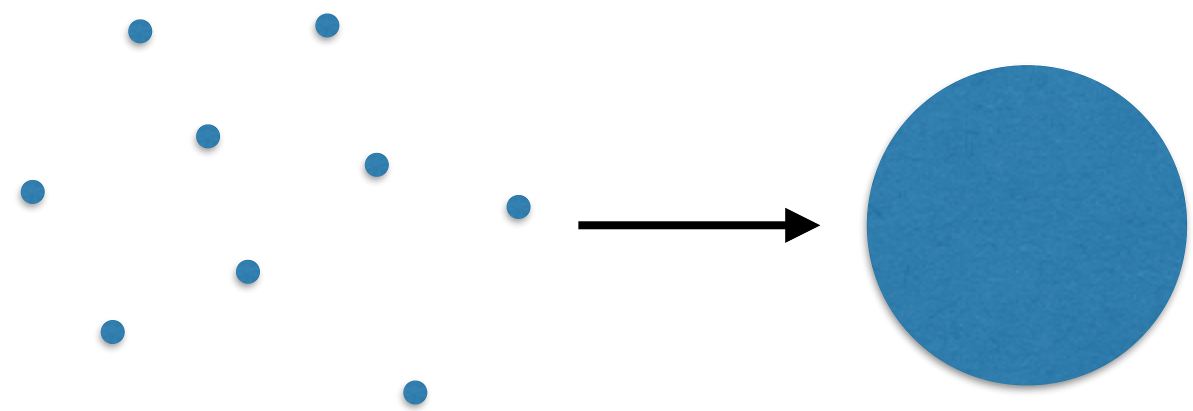
Release of gravitational potential energy

→ "Primordial" heat

+ short-lived radioactivity (^{26}Al, ^{60}Fe)

Gravitational binding energy

To form a uniform density Earth

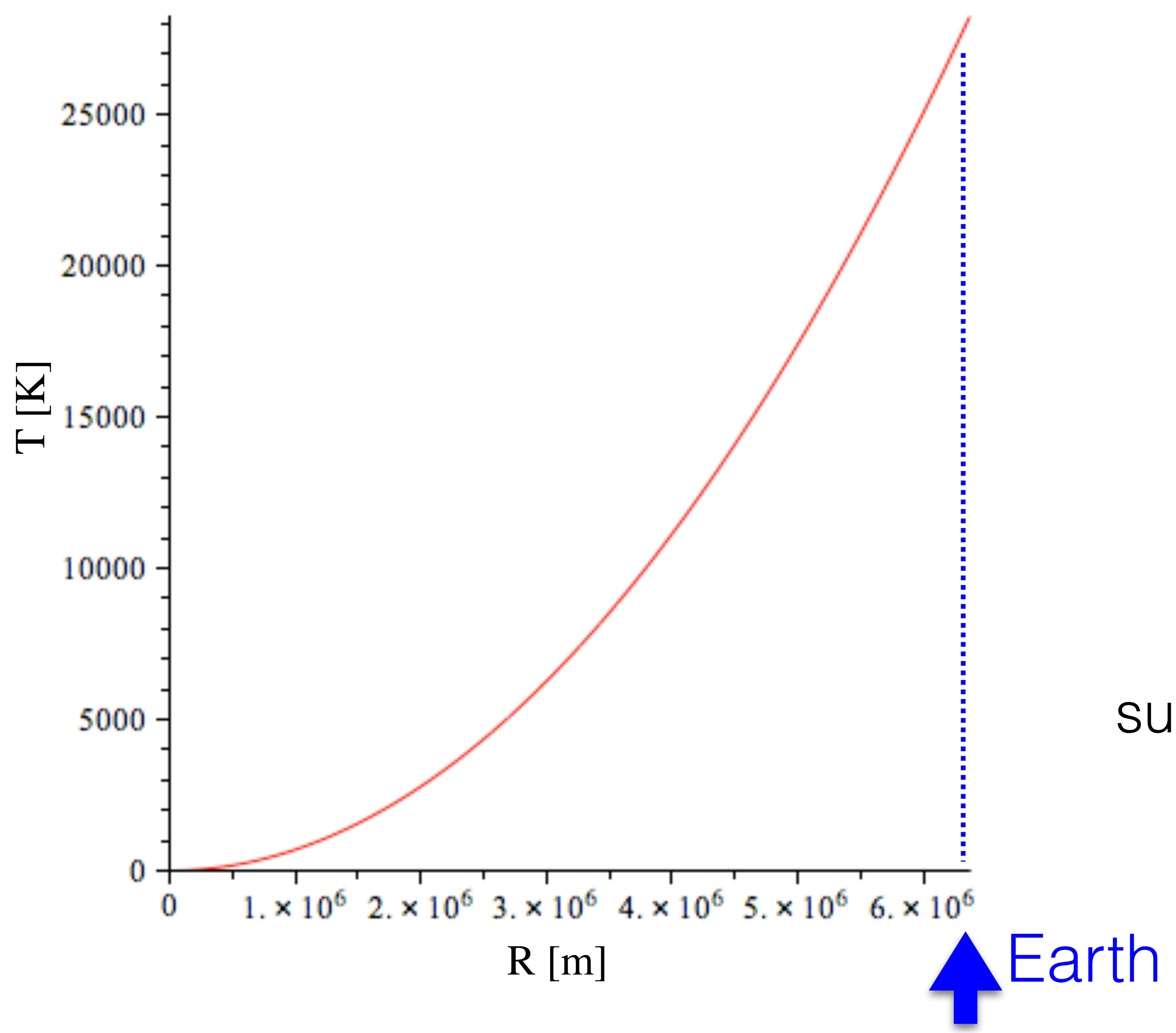


- Two point masses: $E = G\frac{m_1 m_2}{|\vec{r}_1 - \vec{r}_2|}$
- Number of point masses: $E = \frac{1}{2}\sum_i \sum_{j \neq i} G\frac{m_i m_j}{|\vec{r}_i - \vec{r}_j|}$
- 3-D body: $E = \frac{1}{2}\int\int G\frac{\rho(\vec{r}_1)\rho(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}\mathrm{d}\vec{r}_1\mathrm{d}\vec{r}_2$
- Uniform density sphere:
  - $\mathrm{d}E = G\frac{m_{\mathbf{sphere}}(r)\mathrm{d}m_{\mathbf{shell}}(r)}{r} = \frac{G}{r}\ \frac{4}{3}\pi r^3\rho\ 4\pi r^2\rho\mathrm{d}r = \frac{16}{3}\pi^2 G\rho^2 r^4\mathrm{d}r$
  - $E = \int_0^R \mathrm{d}E = \frac{16}{15}\pi^2 G\rho^2 R^5 = \frac{3GM^2}{5R}$

Equivalent temperature increase

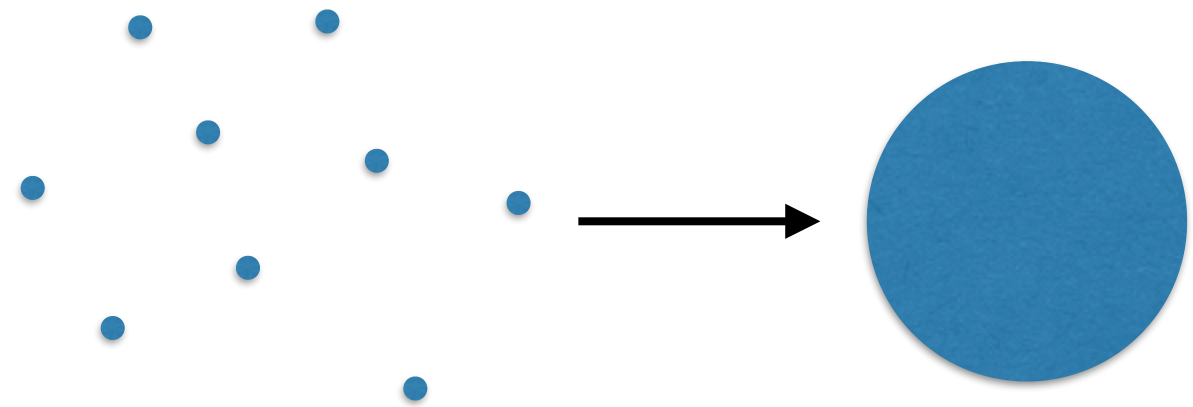
$$E \sim M\, C\, \Delta T$$

Forming a uniform density planet



sufficient energy to vaporize the planet…

Gravitational binding energy



To form a uniform density Earth

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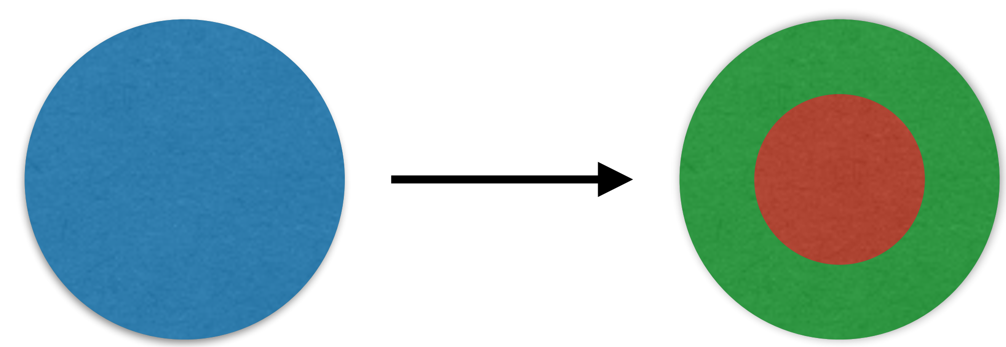


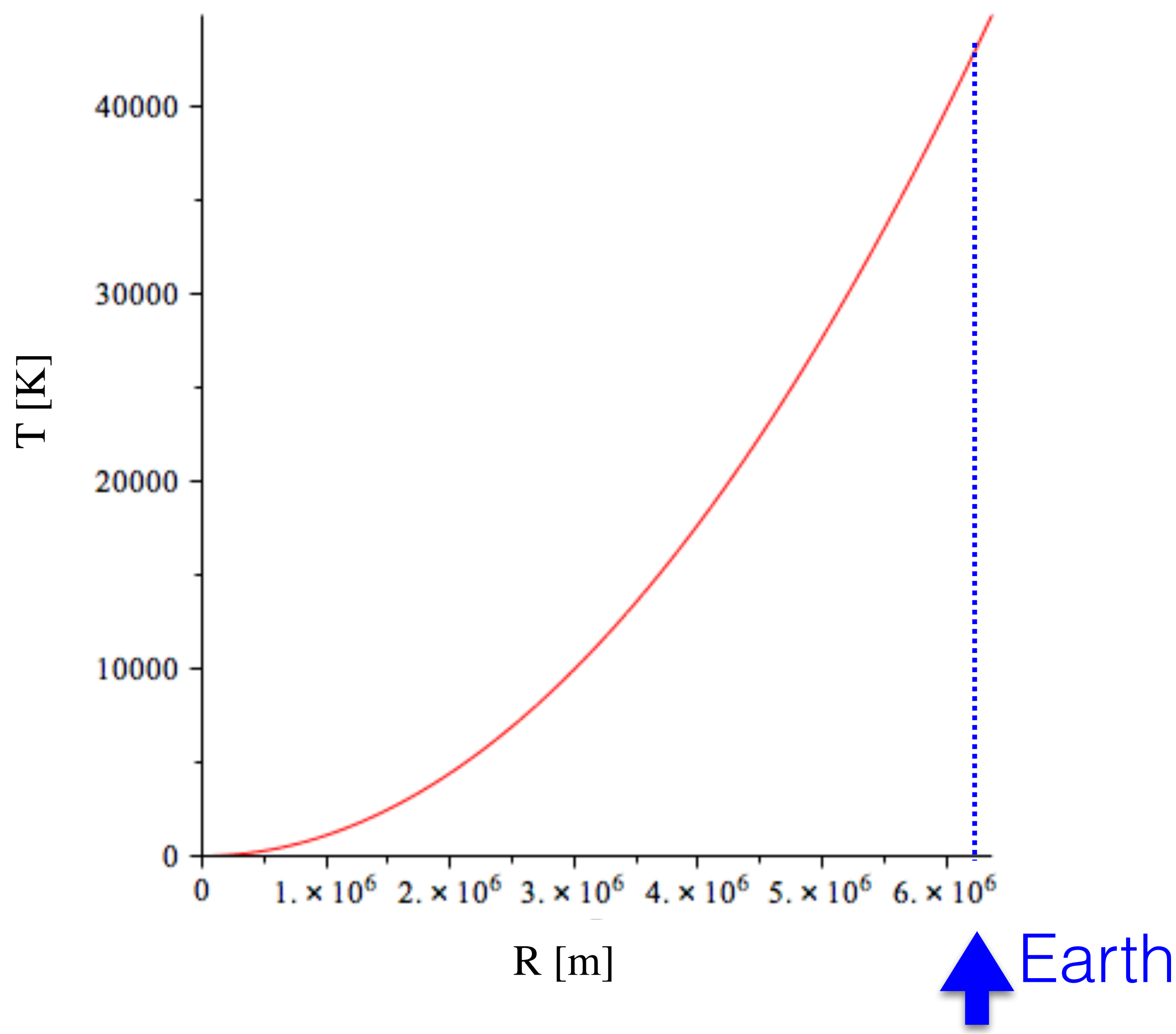
To differentiate a core

- Additional binding energy increase due to differentiation, for $\rho_{\text{core}} = c\rho_{\text{mantle}}$, $V_{\text{core}} = b^3 V_{\text{total}}$,
  $\frac{\Delta E}{E_{\text{unif}}} = \frac{1}{2}(c-1)b^3(1-b)\frac{1+b+2b^2(c-1)}{[1+b^3(c-1)]^2}$

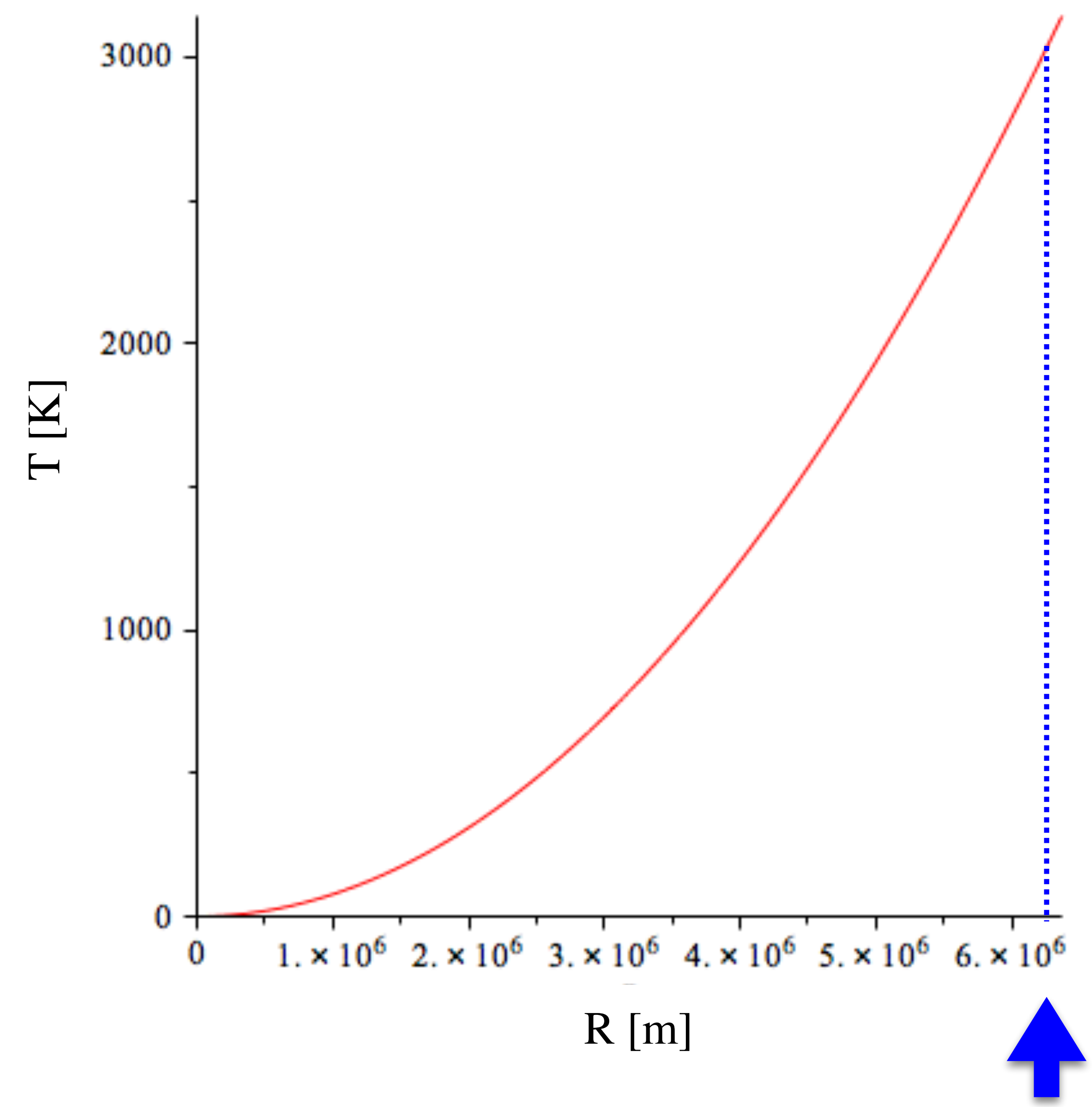
Equivalent temperature increase

$E \sim M\,C\,\Delta T$

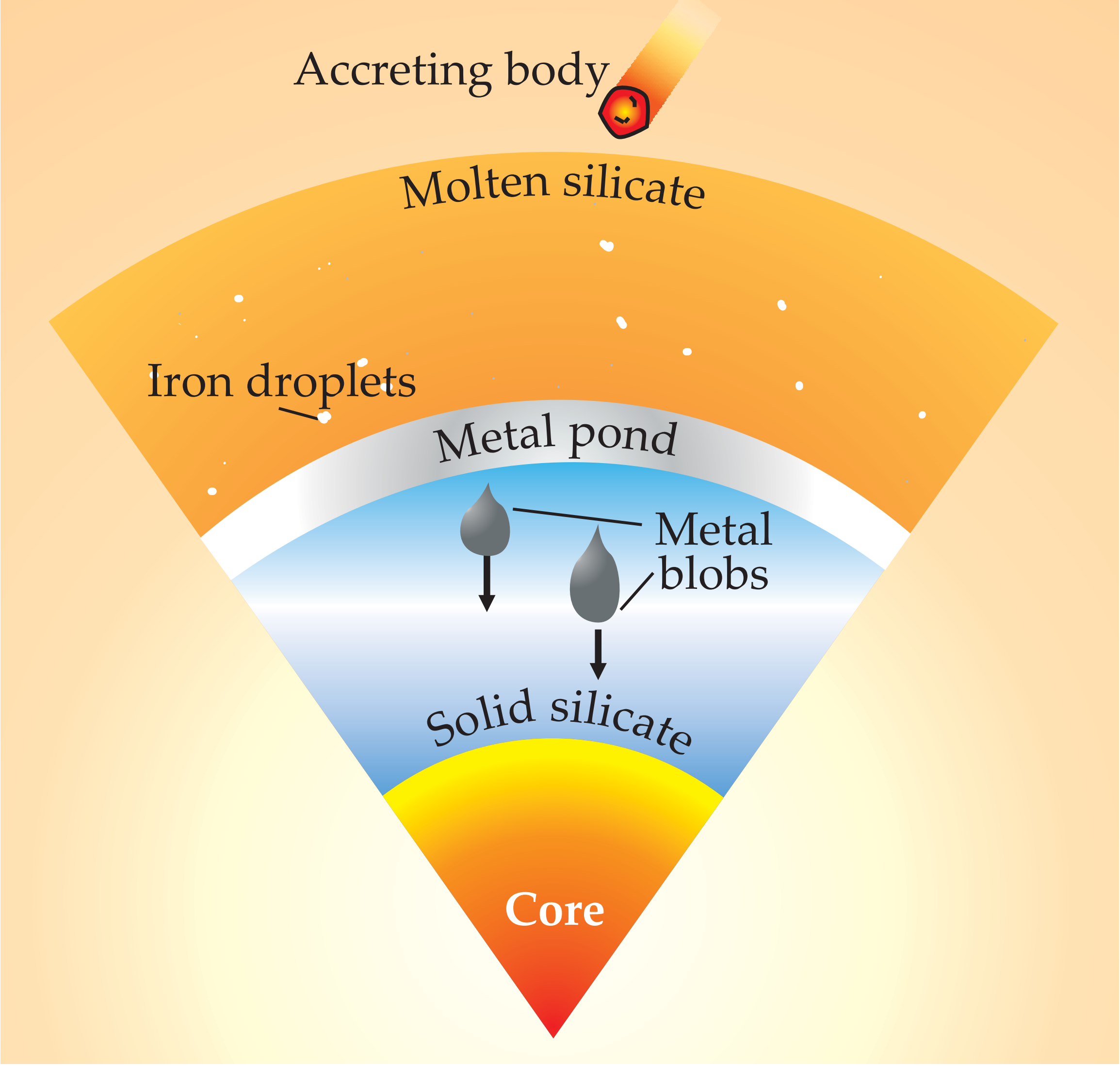
Forming a uniform density planet



Differentiating a core



Deep magma ocean on early Earth?



Wood 2011

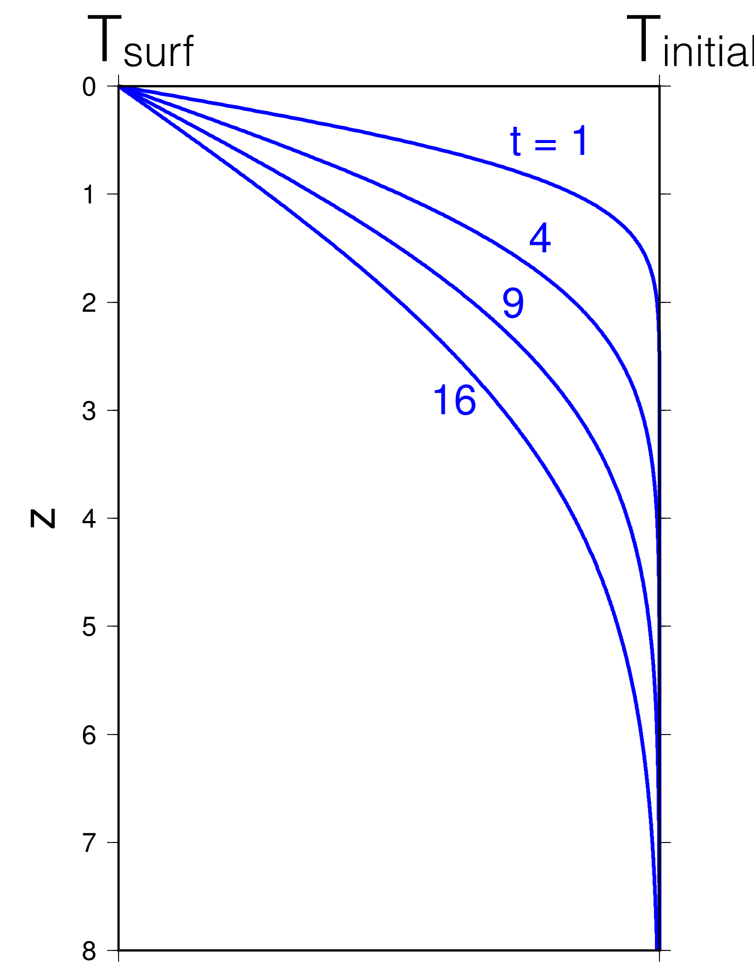
- formation of the core within ~30 Myr after t_0
- sedimentary rocks by ~4.3 Ga

Conductive cooling model

or lord Kelvin's 1863 estimate of age of the Earth

Assumptions:

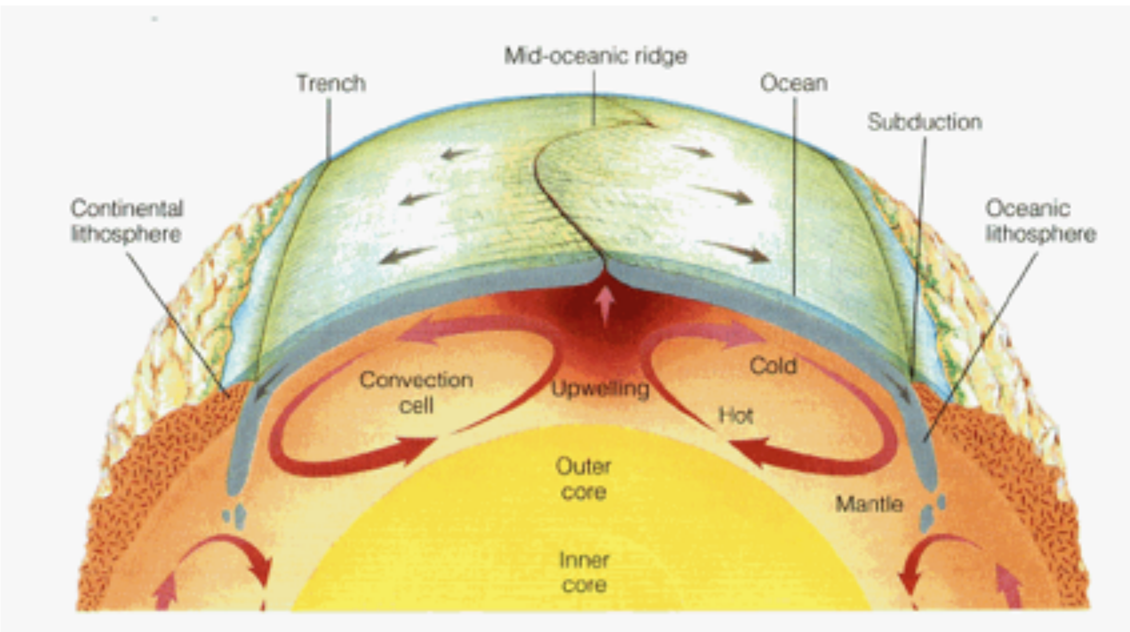
- Earth cools by conduction
- Inputs:
  - Surface temperature gradient 1/50 °F per foot (~36 K/km)
  - Initial temperature 7000 °F (3900 K)
  - Thermal diffusivity 2×10^{-6} m^2/s
- ⇒ age of ~100 Myr

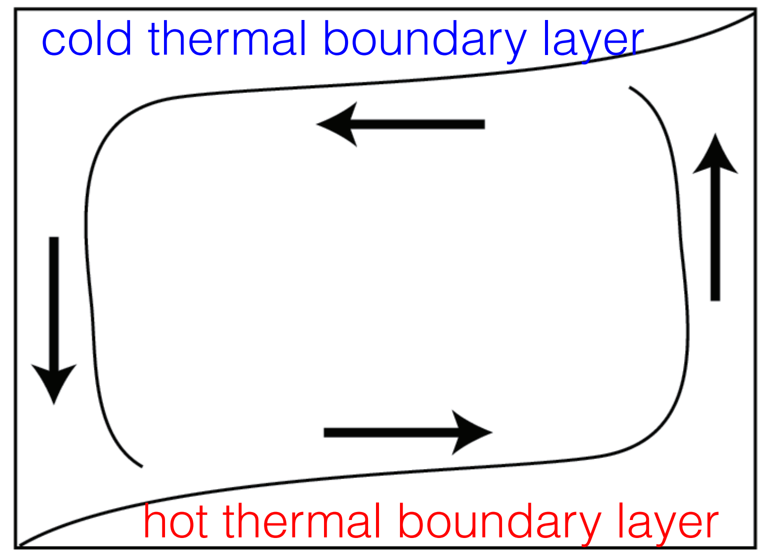


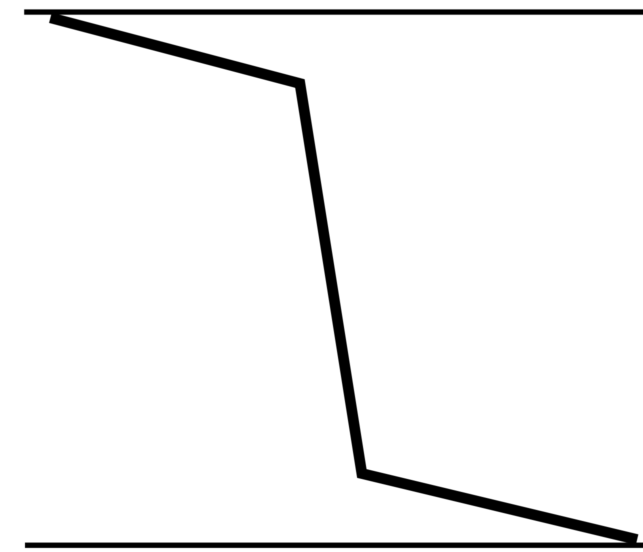
Osmond Fisher, 1881: If Earth's interior were 'plastic' (fluid-like), then a much greater reservoir of heat would be tapped and the resulting age could be much greater.

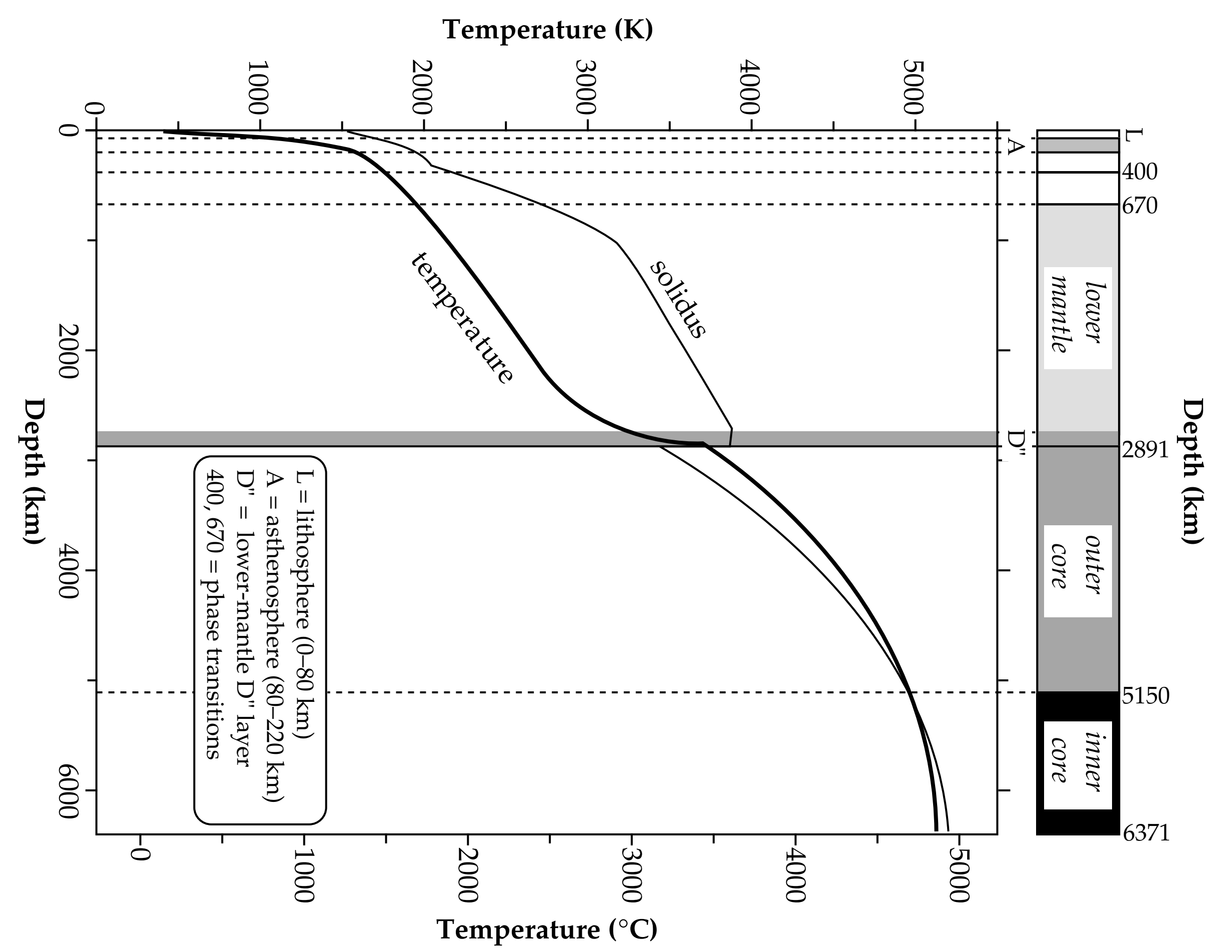
John Perry, 1895: ... much internal fluidity would practically mean infinite conductivity for our purpose.

Steep temperature gradient at the surface: thermal boundary layer of mantle convection

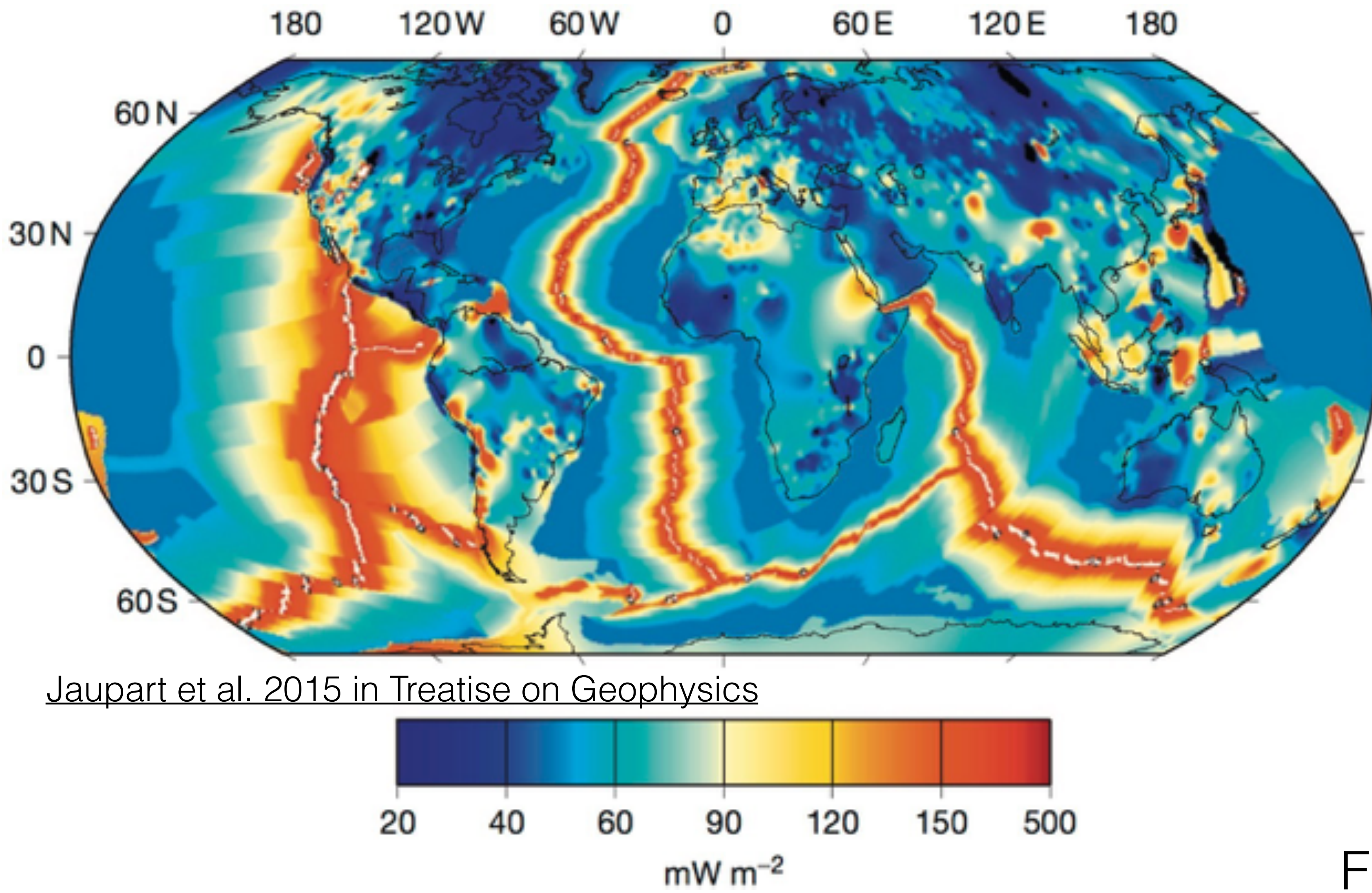








Global surface heat loss



Jaupart et al. 2015 in Treatise on Geophysics

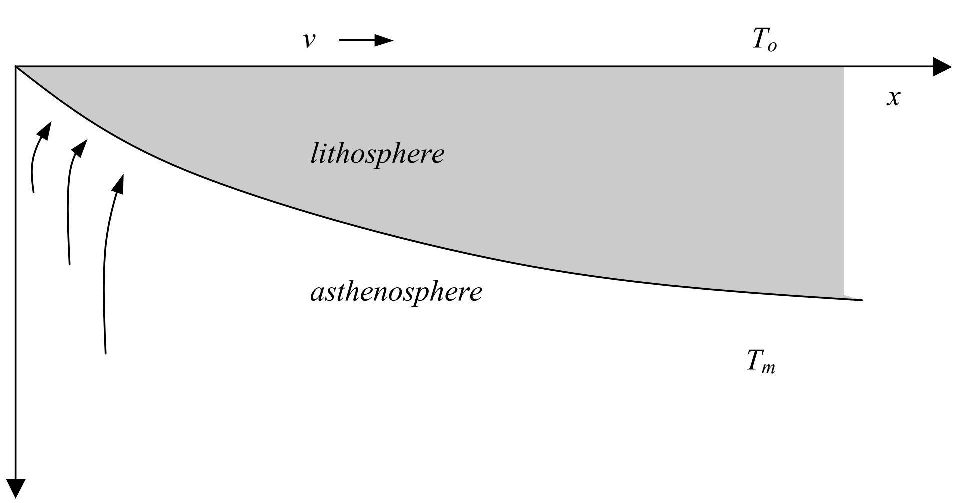
Table 3 Estimates of the continental and oceanic heat flux and global heat loss

	Continental ($mW\ m^{-2}$)	*Oceanic ($mW\ m^{-2}$)*	*Total (TW)*
Williams and von Herzen (1974)	61	93	43
Davies (1980a,b)	55	95	41
Sclater et al. (1980)	57	99	42
Pollack et al. (1993)	65	101	44
Davies and Davies (2010)	71	105	47
This study[a]	65	94	46

[a]The average oceanic heat flux does not include the contribution of hot spots. The total heat loss estimate does include 3 TW from oceanic hot spots.

46 ± 3 TW

From global heat flow measurements combined with lithospheric cooling model in oceans

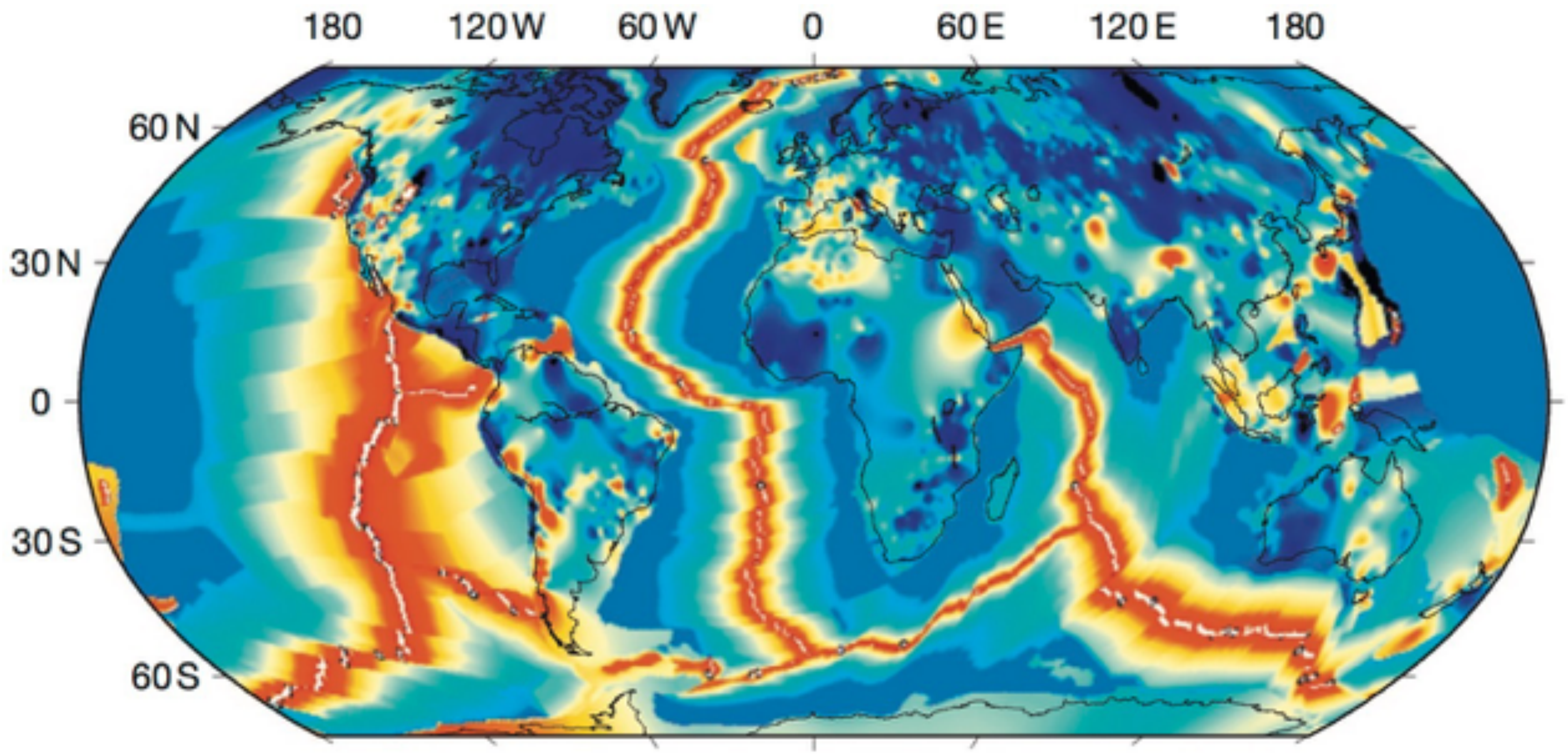


$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

$$\frac{T - T_0}{T_1 - T_0} = \mathrm{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

$$q\big|_{z=0} = -k\frac{\partial T}{\partial z}\bigg|_{z=0} = -\frac{k(T_1 - T_0)}{\sqrt{\pi \kappa t}}$$

Global surface heat loss



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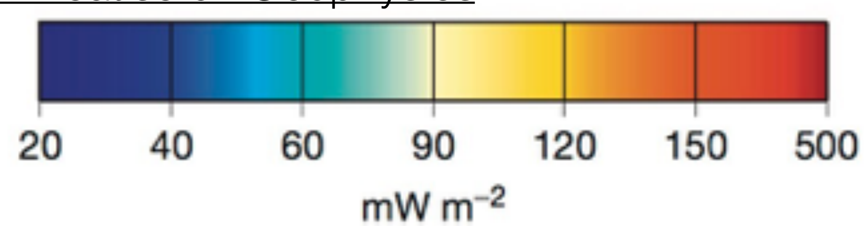

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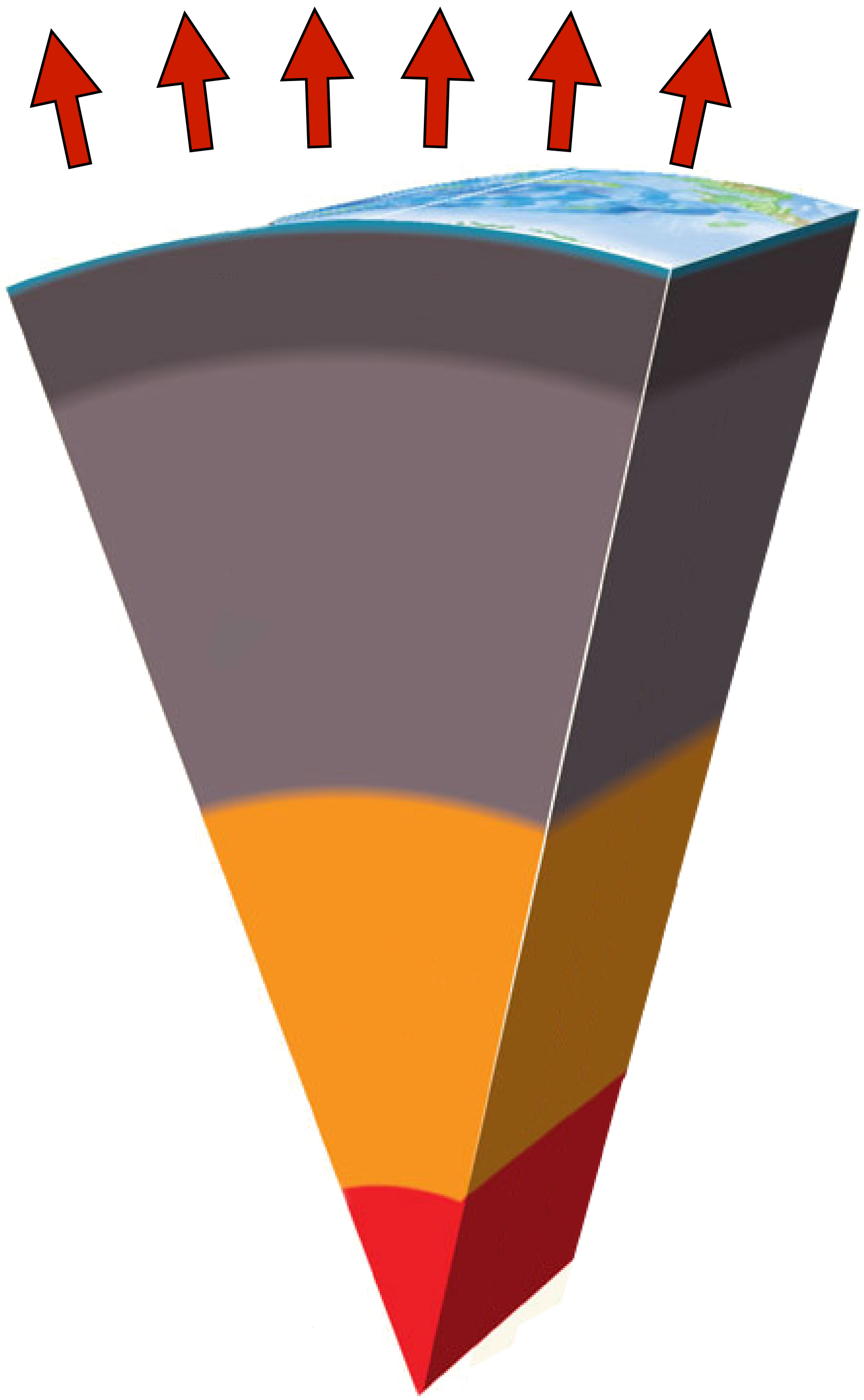
46 ± 3 TW

From global heat flow measurements combined with lithospheric cooling model in oceans

- What cooling rate of the Earth corresponds to 46 TW?
- Heat sources in the Earth?

Secular cooling equation



Basic energy balance:

Surface heat flow (46±3 TW)

=

Cooling of the interior

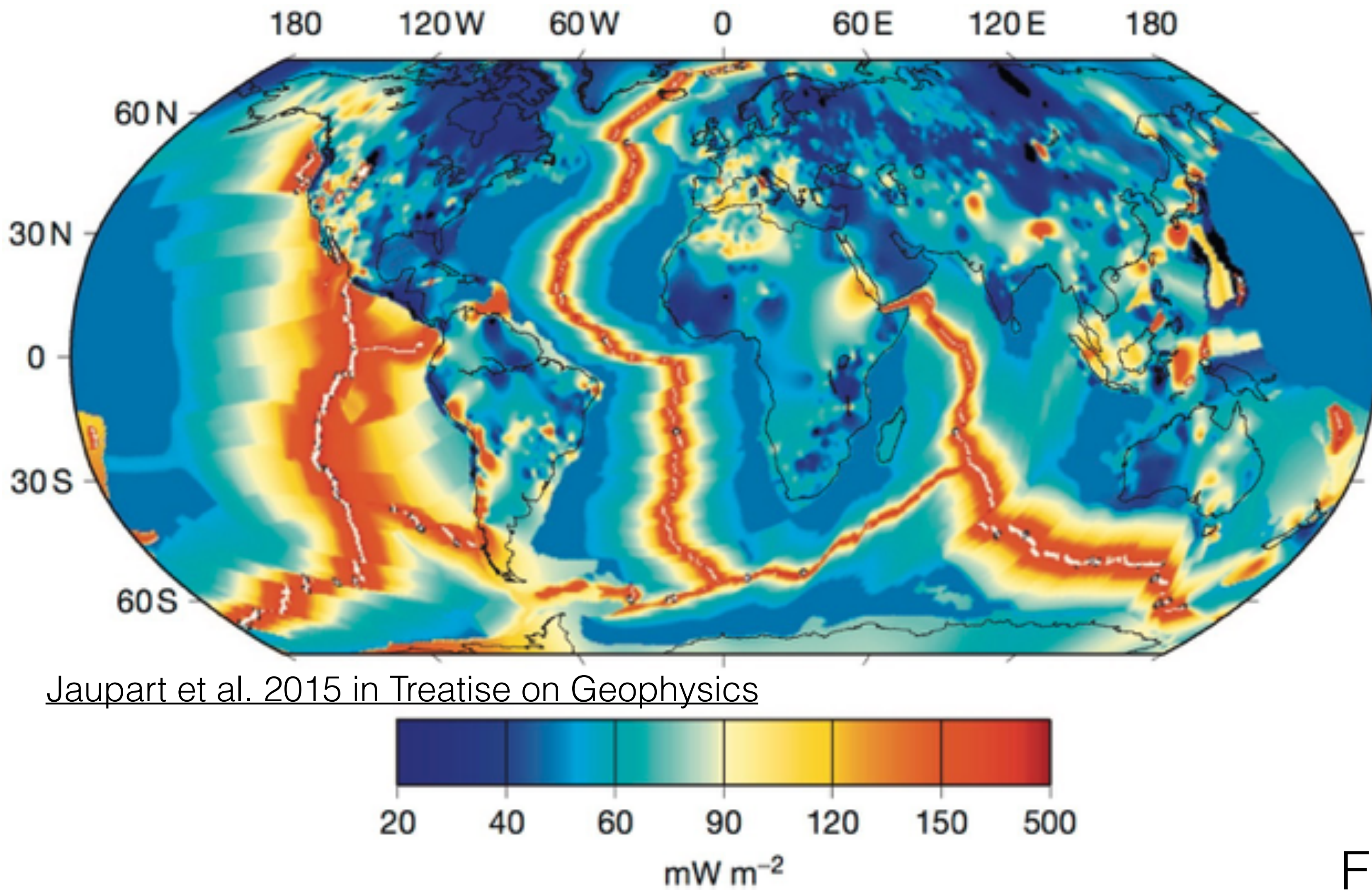
+

Heat sources

$$\int_V \rho C \frac{\partial T}{\partial t} \mathrm{d}V = -\oint_S \mathbf{q} \cdot \hat{\mathbf{n}} \mathrm{d}S + \int_V h \mathrm{d}V$$

$$Q = -MC \frac{\mathrm{d}\langle T \rangle}{\mathrm{d}t} + H$$

Global surface heat loss



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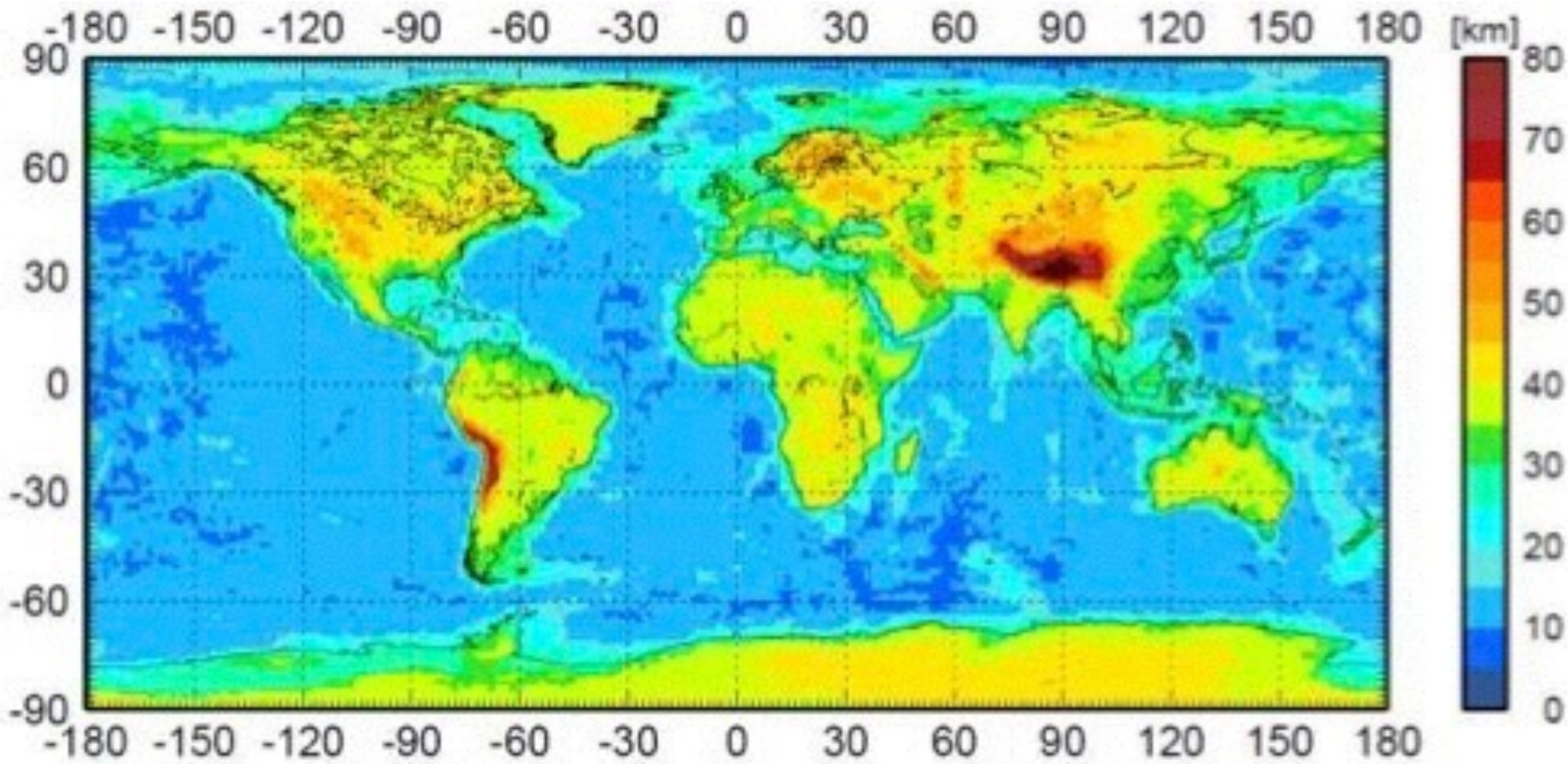
46 ± 3 TW

From global heat flow measurements combined with lithospheric cooling model in oceans

- What cooling rate of the Earth corresponds to 46 TW?
- Heat sources in the Earth?
- Where are the heat sources?

Thickness and density of Earth's crust



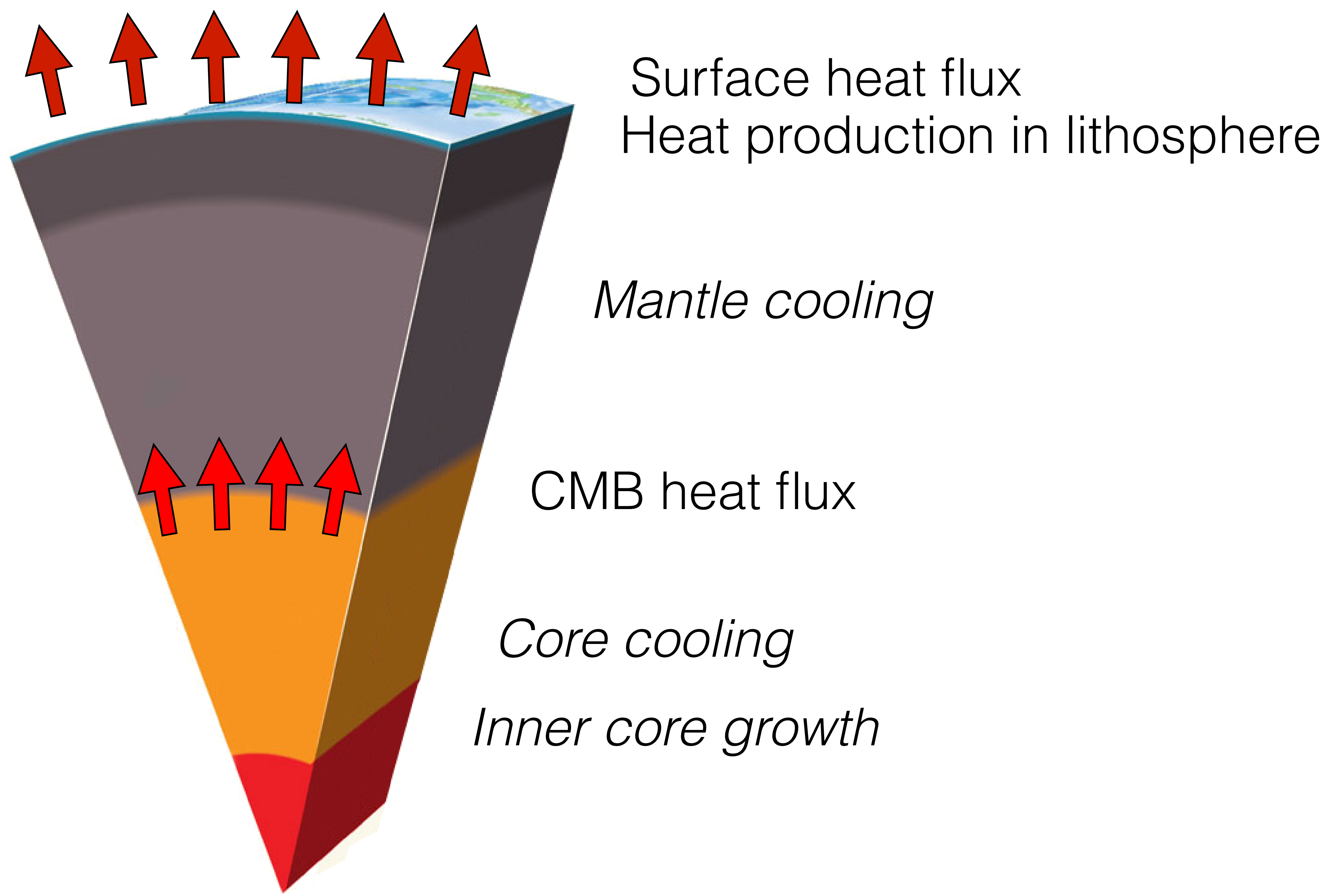
+ model of chemical composition in the crust

⇒ model of heat production in the crust

Continental Crust	**Oceanic Crust**
6.8 (+1.4/−1.1) TW	**0.22 ± 0.03 TW**

Model by Yu Huang et al. 2013

- Rate of heat loss from the Earth … 46 ± 3 TW
- Radiogenic power produced in Silicate Earth … estimates from 9 TW to 36 TW
- Radiogenic power produced in the lithosphere … 8 ± 1 TW
- Heat flow from the convecting mantle … 38 ± 3 TW
- Radiogenic power available in the mantle … from 1 TW to 28 TW Or anywhere from no internal heating to mostly (75%) internal heating.
- Heat flow from the core to the mantle … up to 40 TW



Surface heat flux
Heat production in lithosphere
Mantle cooling
CMB heat flux
Core cooling
Inner core growth

“Parametrized” convection models

Solving the secular cooling equation

$$Q = -MC\frac{\mathrm{d}\langle T\rangle}{\mathrm{d}t} + H$$

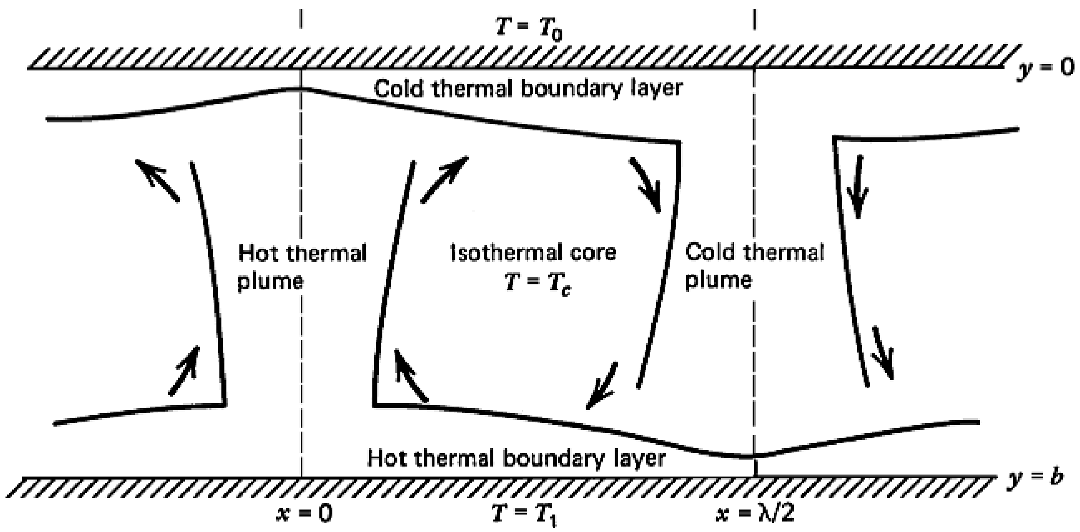
Use scaling to get heat flux from convecting mantle as a function of temperature

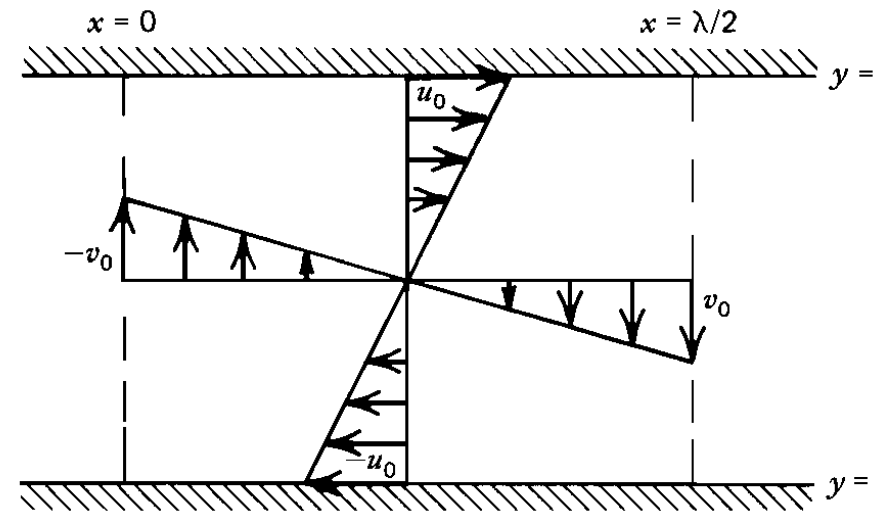
Assume a (present-day) ratio of radiogenic heat production to surface heat flux

Urey ratio

$$Ur = \frac{H}{Q}$$

"Parametrized" convection models





Relationship between heat flux out of the convecting mantle and the vigor of thermal convection

Nusselt number – Rayleigh number scaling

$$Ra = \frac{\rho \alpha \Delta T g d^3}{\kappa \eta}$$

$$\frac{\text{buoyancy}}{\text{(thermal diffusion)} \times \text{(momentum diffusion)}}$$

Ra scales with temperature
Viscosity *η* strong function of *T*

$$Nu = \frac{q}{k \frac{\Delta T}{d}}$$

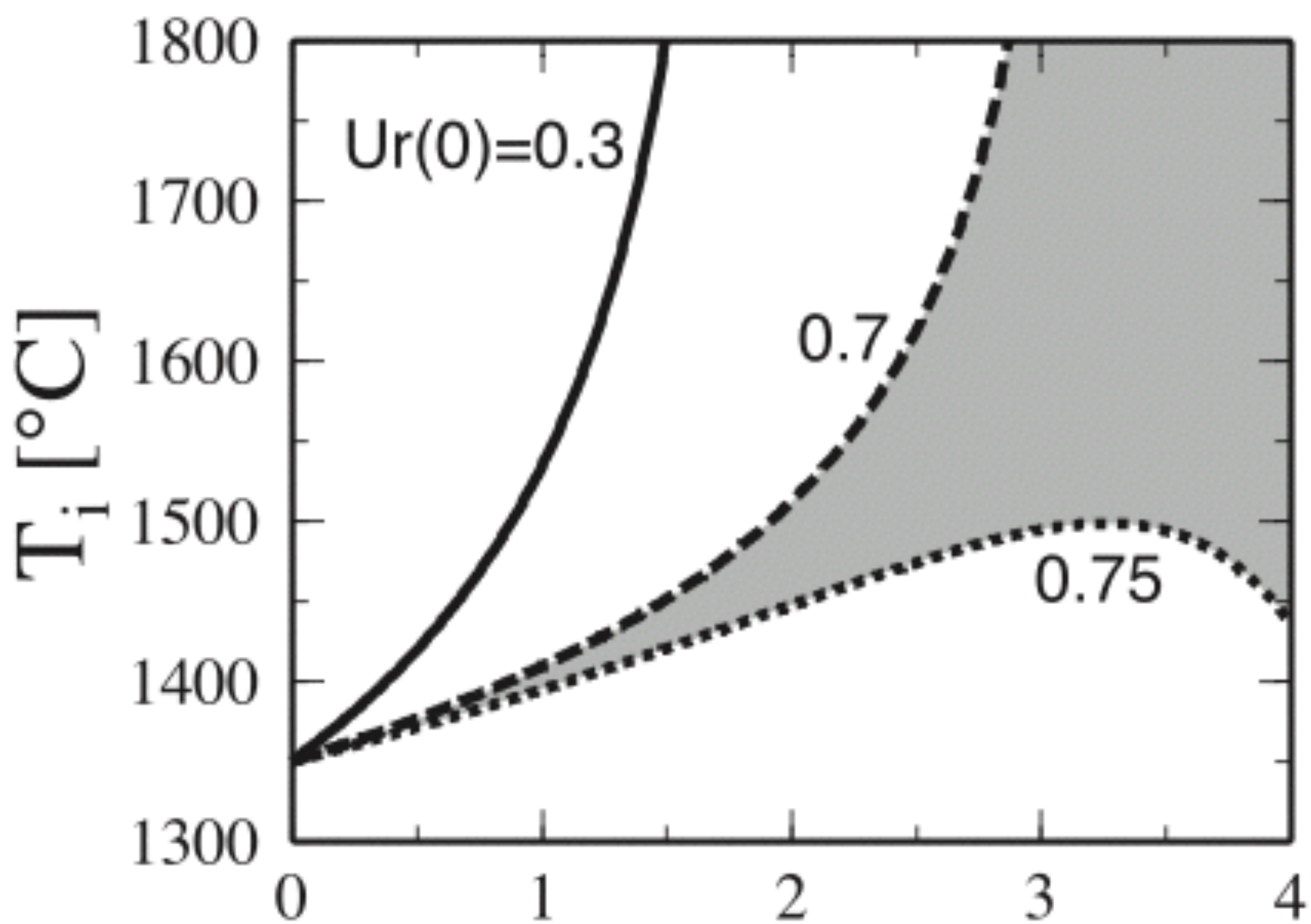
$$\frac{\text{heat flux from convecting layer}}{\text{diffusive heat flux}}$$

$$Nu \propto Ra^{1/3}$$

"Parametrized" convection models

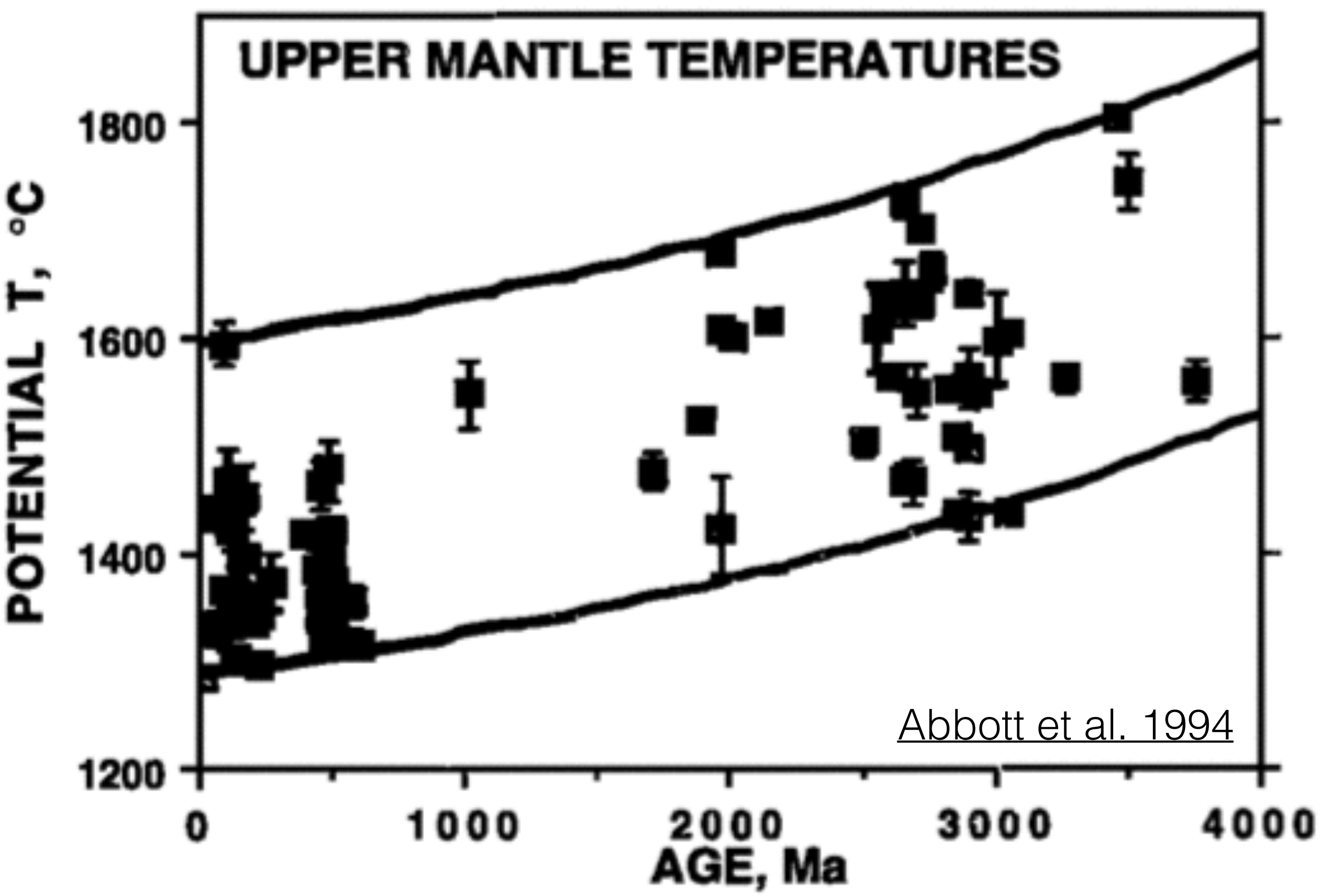
Solving the secular cooling equation

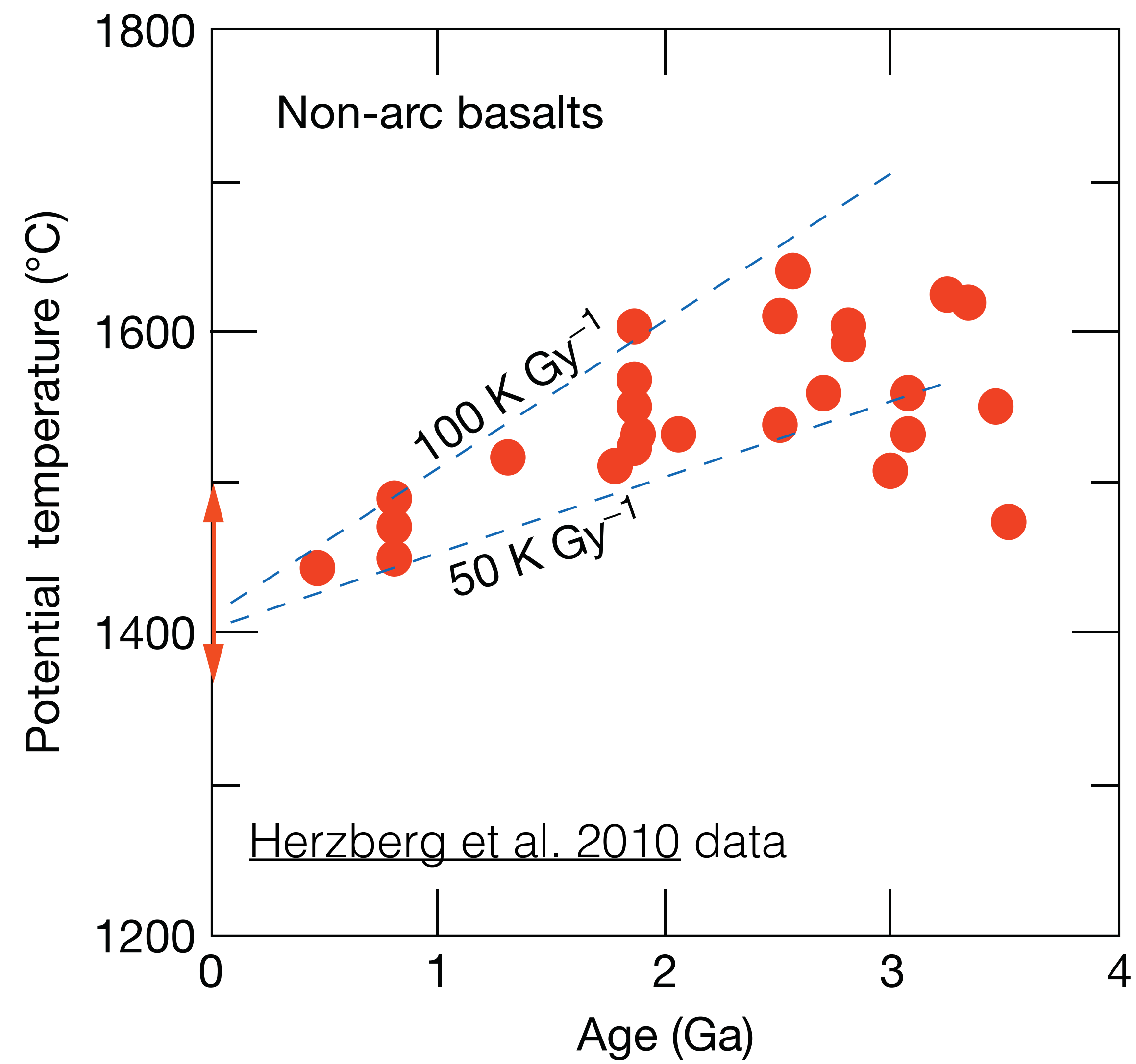
$$Q = -MC\frac{\mathrm{d}\langle T\rangle}{\mathrm{d}t} + H$$



Mantle cooling rate

petrological constraint based on determination
of melting temperature of primitive mantle melts





Cooling rate of 50±25 K/Gyr

Translates into 8±4 TW mantle heat output due to cooling

Cooling rate of 50–100 K/Gyr

Translates into 8–16 TW mantle heat output due to cooling

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  Or anywhere from no internal heating to mostly (75%) internal heating.
- Heat flow from the core to the mantle … up to 40 TW
- Urey ratio 0.75 requires 0.75×38 TW = 28 TW radiogenic power in the mantle, that is 36 TW in Silicate Earth

High mantle Urey ratio?

$$\text{Urey ratio} = \frac{\text{radiogenic power}}{\text{surface heat loss}}$$

20 TW radiogenic power in BSE $\leftrightarrow$ Mantle Ur ~ 0.3

- Deschamps et al. 2010, *Nu–Ra* scaling based on 3-D spherical shell numerical simulations of convection: *"Applied to the Earth's mantle, the mixed heating scaling predicts a Urey ratio between 0.4 and 0.6, depending on the Rayleigh number."* (between 23 and 31 TW radiogenic power in BSE)

- Nakagawa & Tackley 2012: *"The Urey ratio that is calculated purely from convective heat flow is always higher than 0.5* [19 TW in BSE]*. When magmatic heat flow is included, the Urey ratio is slightly lower at the present day"*

- Lenardic et al. 2011: Including continents is important. Results from numerical models relax the tension between classical convection models and lower Urey ratio estimated from geochemical models.