ISAPP
International School on AstroParticle Physics

SUMMER INSTITUTE: USING PARTICLE PHYSICS TO UNDERSTAND AND IMAGE THE EARTH

11-21 July 2016 *Gran Sasso Science Institute*
Europe/Rome timezone

Geodynamic picture of the Earth

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GRAN SASSO SCIENCE INSTITUTE
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JÜLICH
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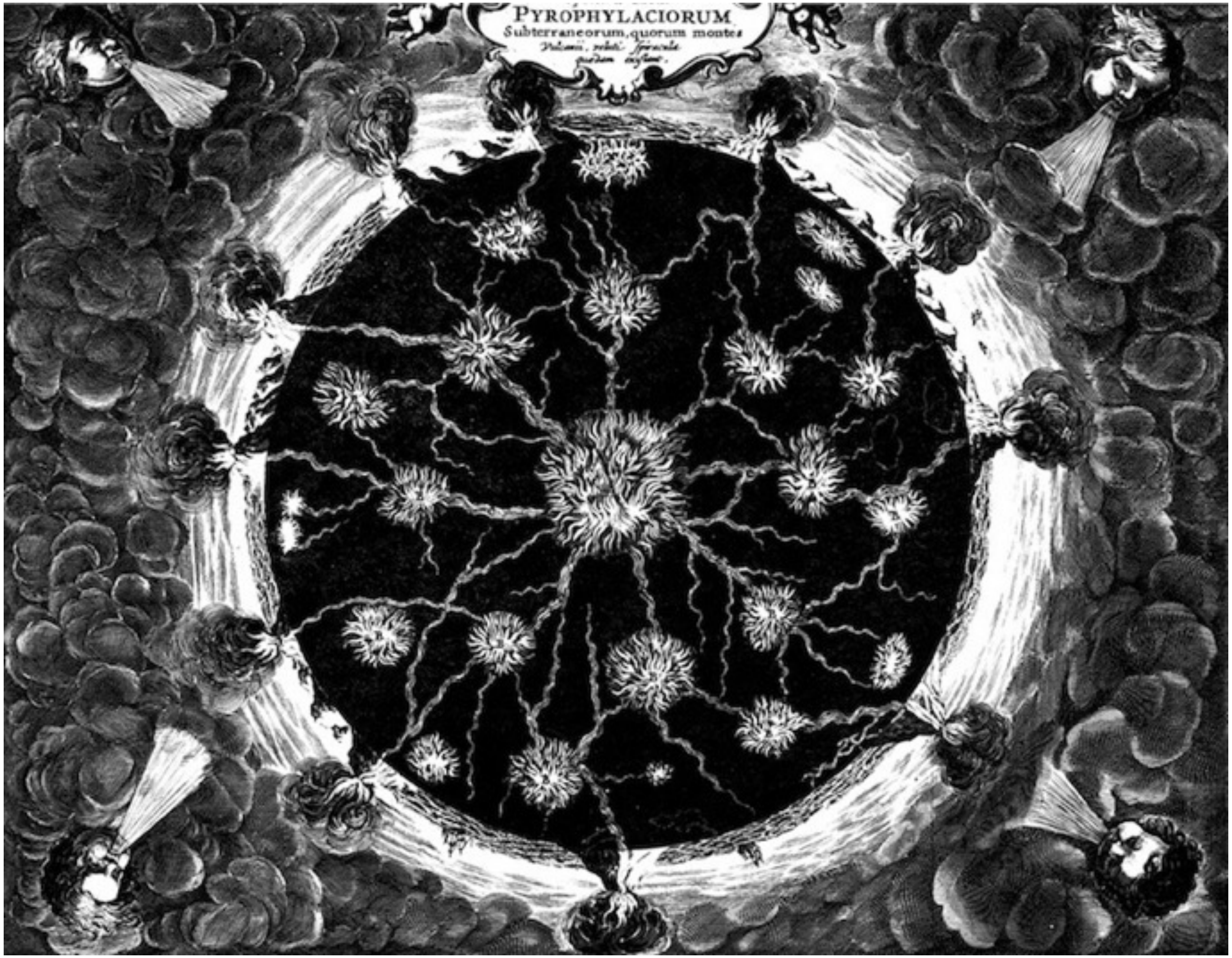


Athanaseus Kircher (~1665):
Systema Ideale Pyrophylaciorum Subterraneorum, Mundus Subterraneus.

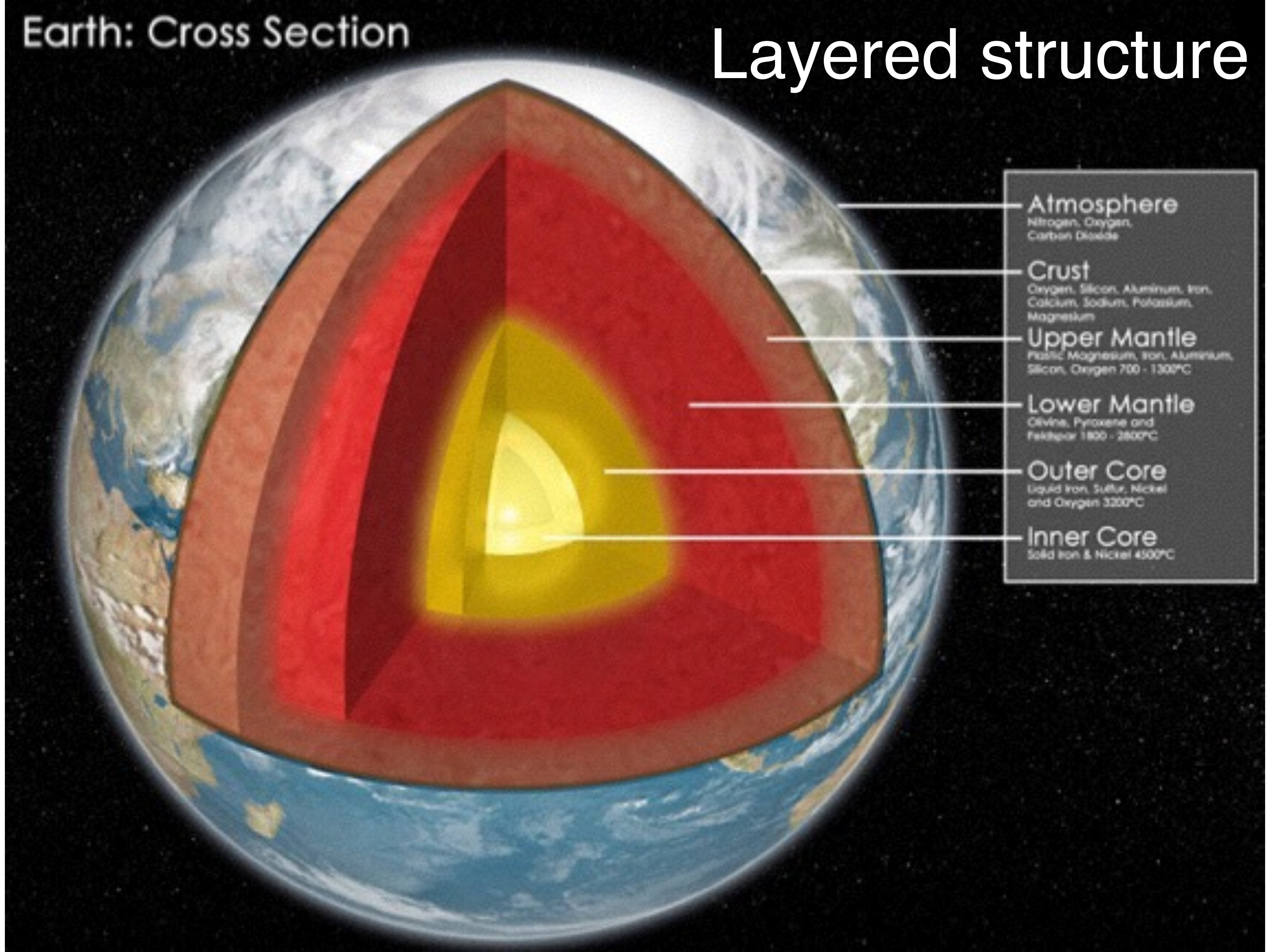
0^{th} order scaling

0^{th} order scaling

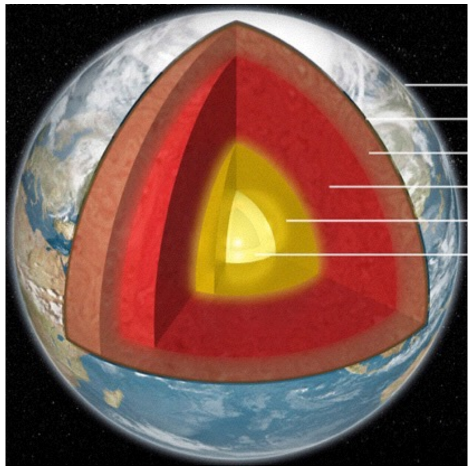
- Mass … M ~ 6×10^{24} kg
- Radius … R ~ 6400 km
- ⇒ Average material density … $\rho = 5600$ kg/m^3
- Acceleration of gravity: $g(r) = G\, m(r) / r^2$
  Assume uniform sphere: $g(r) = 4/3\, \pi\, G\, \rho\, r$
  $G(R) = 10$ m/s^2
- Pressure: $dP = \rho\, g\, dz = -\rho\, g\, dr$
  $P(r) = P_0 - 2/3\, \pi\, G\, \rho^2\, r^2$
  $P(R) = P_0 - 178$ GPa (100 GPa = 1,000,000 × 0.1 MPa)

Atmospheric pressure

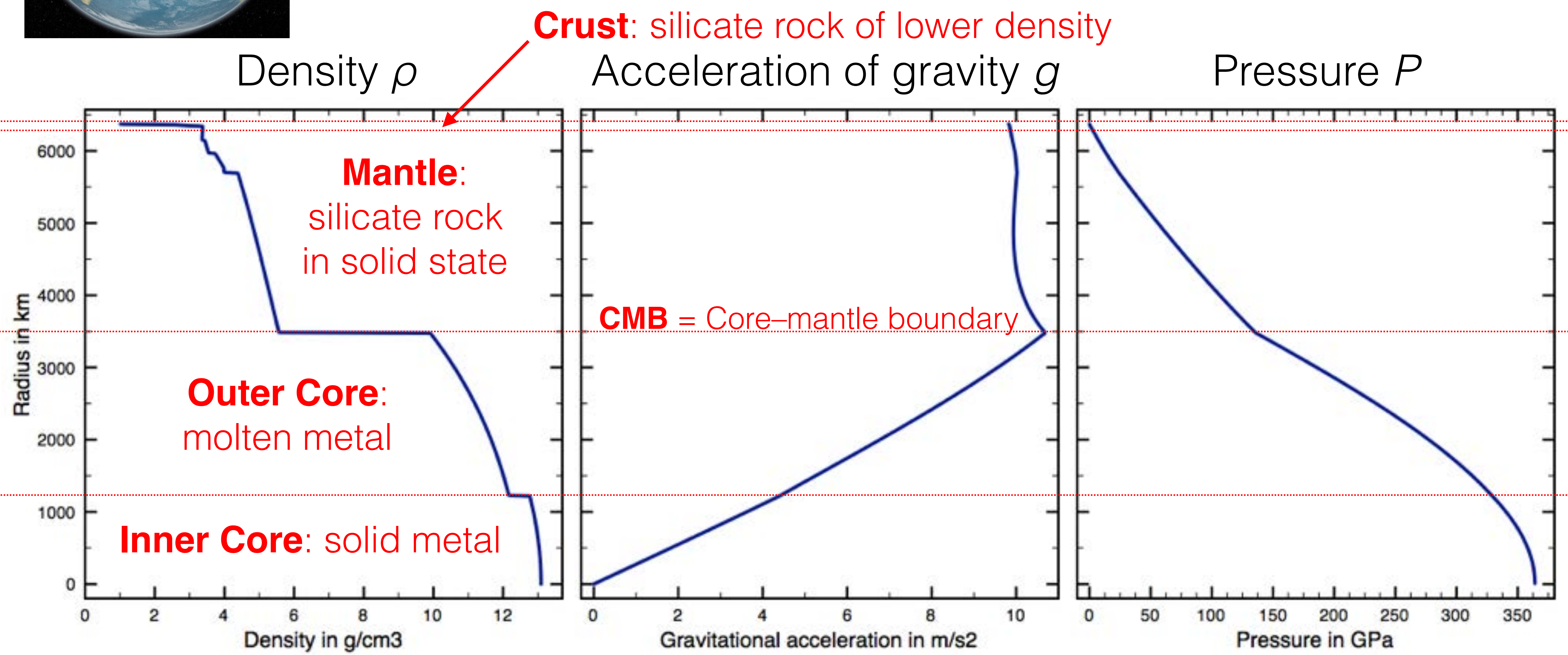
Layered structure



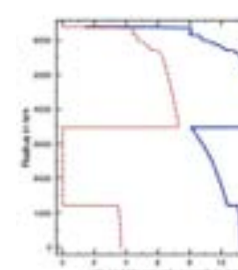
Spherically symmetric Earth model



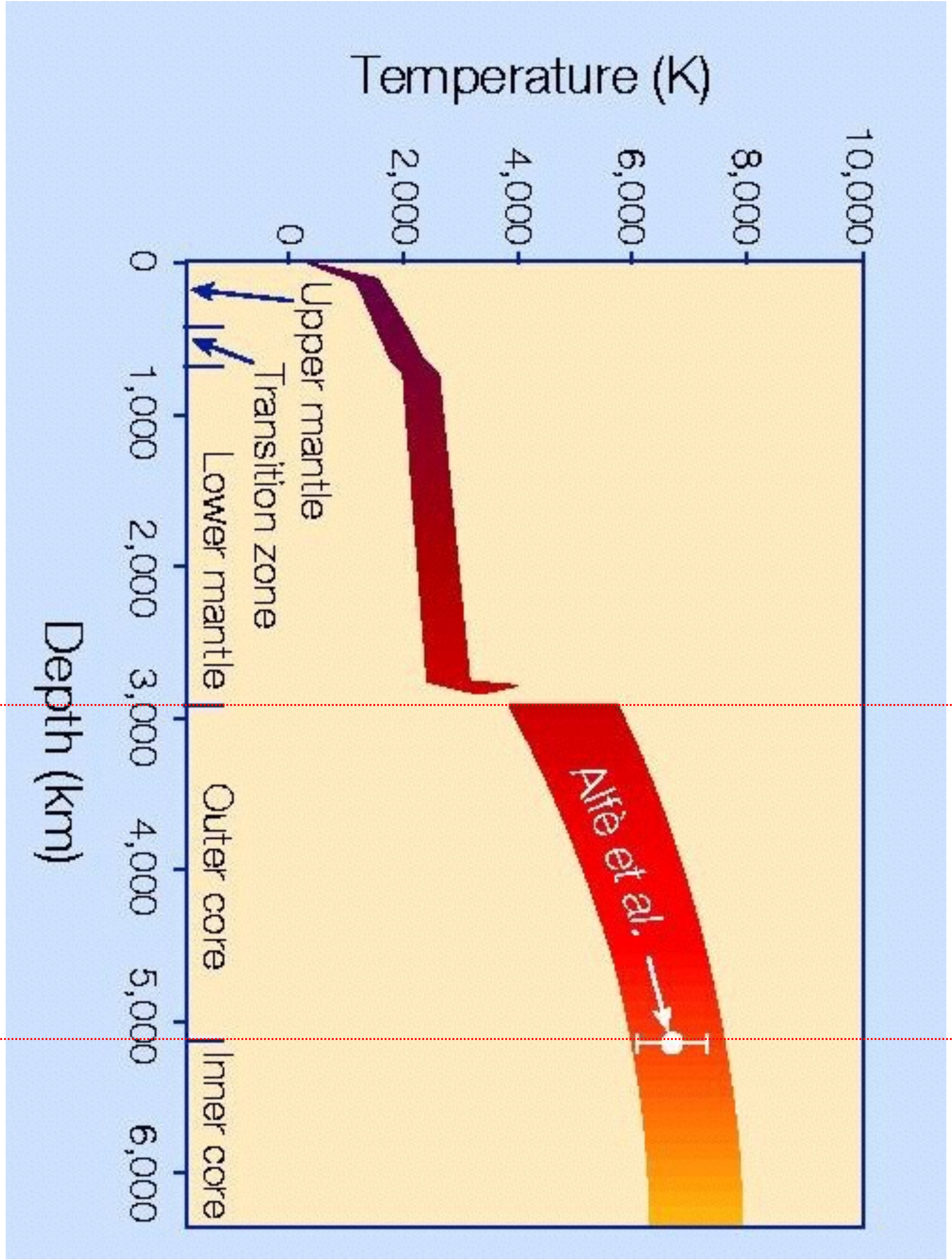
"PREM": Dziewonski & Anderson (1981):
"**P**reliminary **R**eference **E**arth **M**odel"
Phys. Earth Planet. Int. 25, doi:10.1016/0031-9201(81)90046-7



Notice:
- non-uniform density: layering + effects of compressibility
- corresponding depth-dependence of g, P



Temperature in the Earth



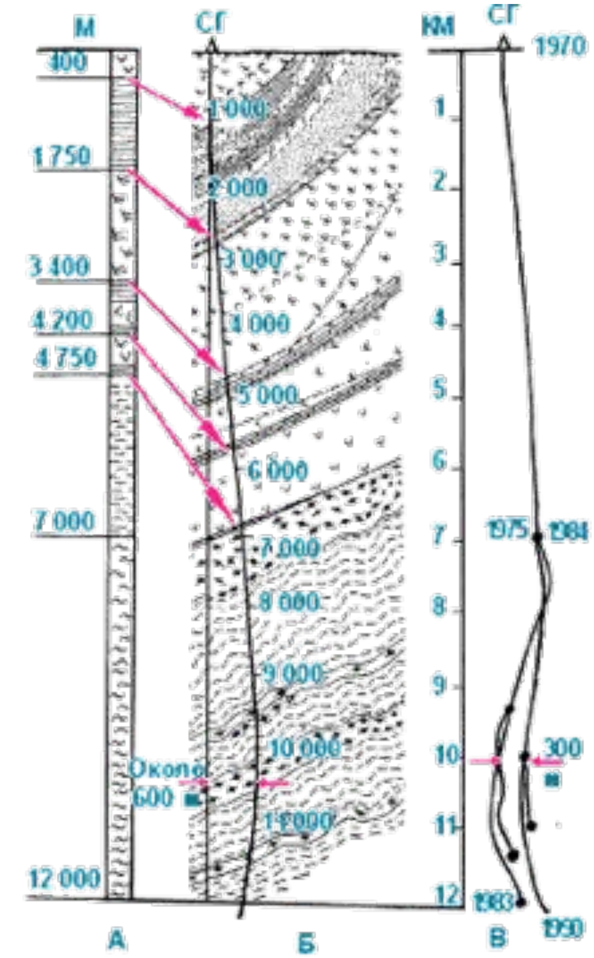
Mantle

Outer core

Inner core

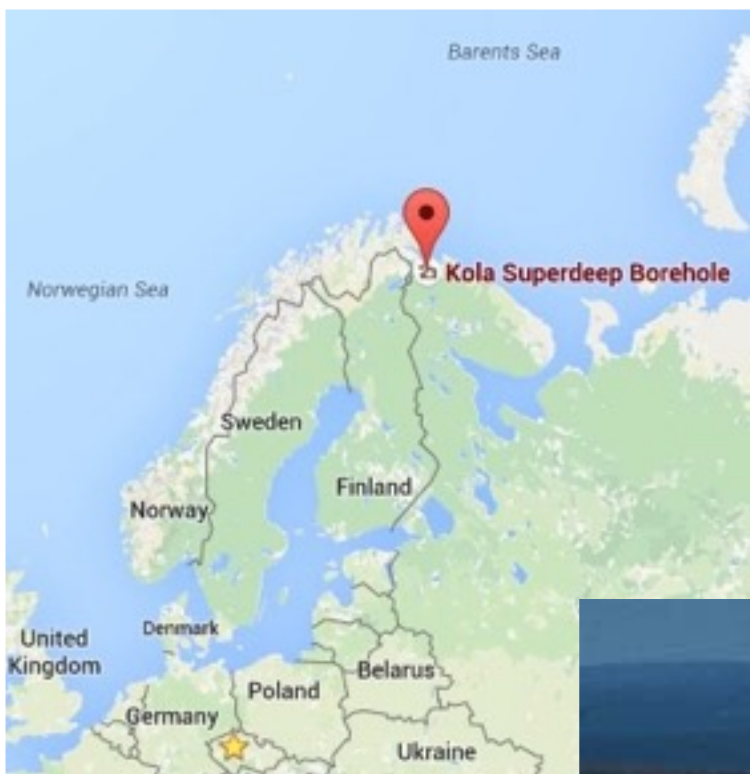
Bukowinski 1999

Scientific drilling



Kola Superdeep Borehole
12.262 km deep

compare with Earth radius
~6371 km

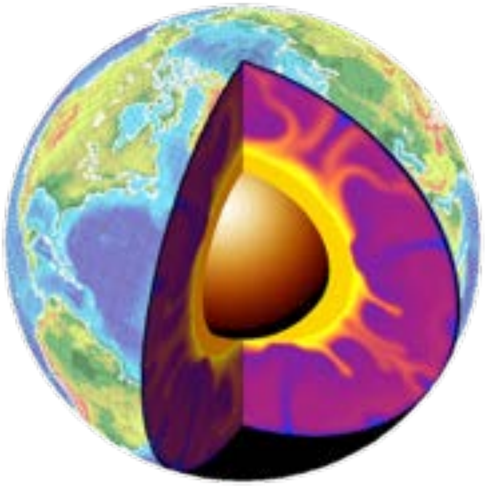


Study of (deep) Earth

Observations and sample collections possible at surface

Measurement and analysis of gravity field

Study of earthquakes and propagation of seismic waves



Numerical modeling of dynamic flow in the interior

Experiments in minerals at high pressure and temperature

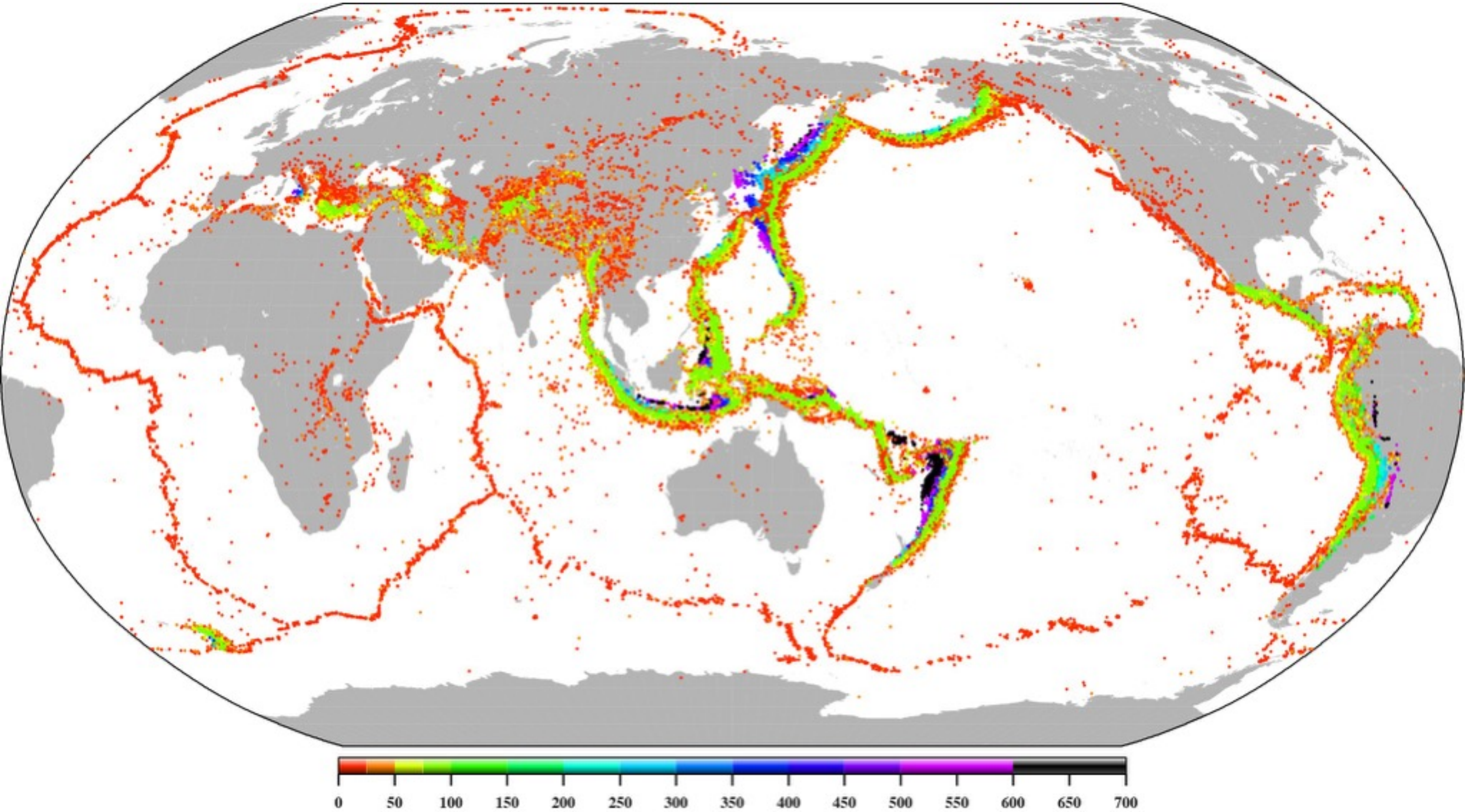
Geochemical analyses of Earth and meteorite samples

First principles (“ab initio”) calculations of material properties

Fluid mechanics experiments in laboratory

Detection of geoneutrinos, “particle geoscience“

Seismicity of the Earth

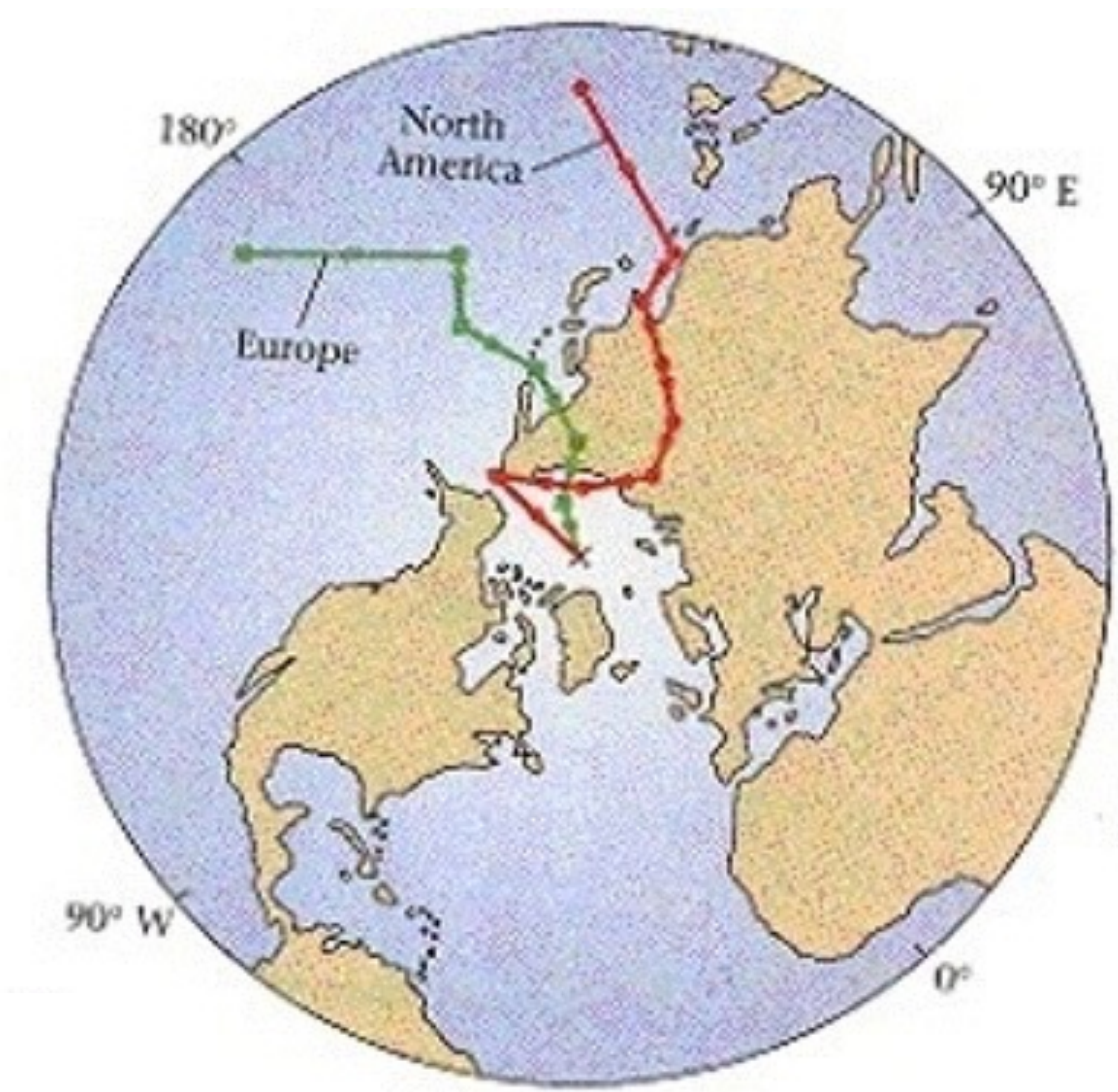


marc.fournier.free.free.fr/enseignement/world_seismicity_map.jpg

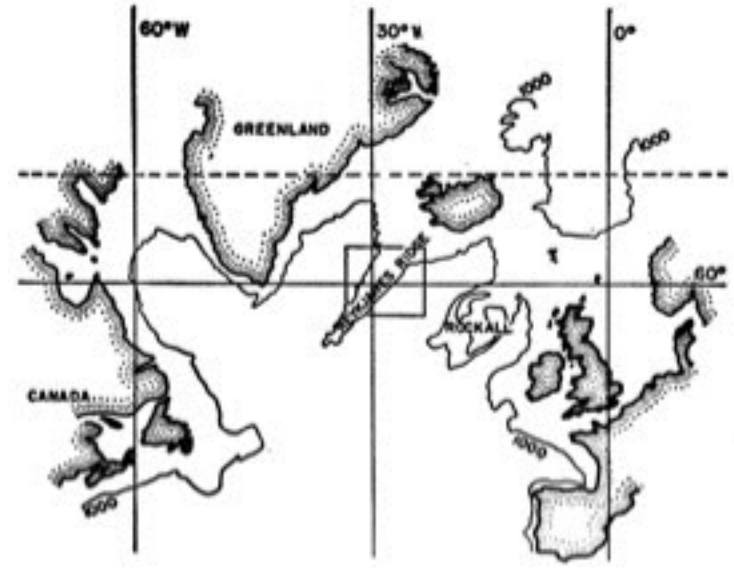
Clues toward plate tectonics hypothesis

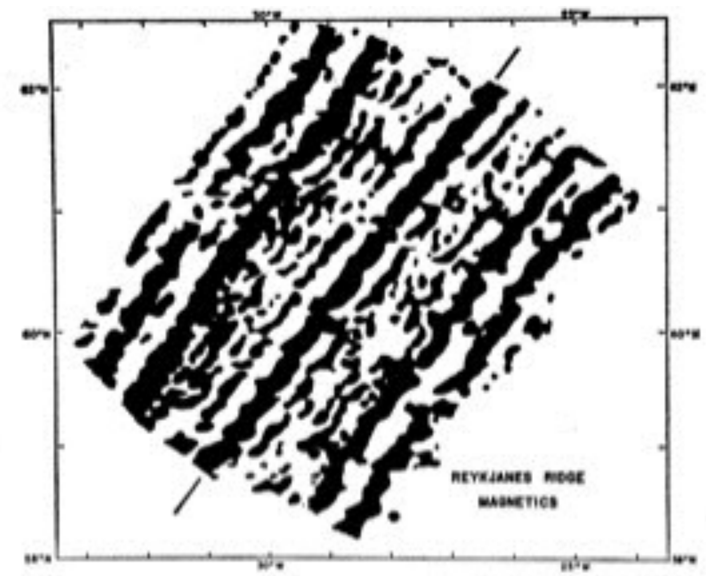
- Apparent wander of geomagnetic pole
- Continental drift
- Mapping of ocean floor ~1950's
- Mid-oceanic ridges (MOR)
- Pattern of magnetic anomalies at MOR
- Depth of ocean floor

Apparent polar wander

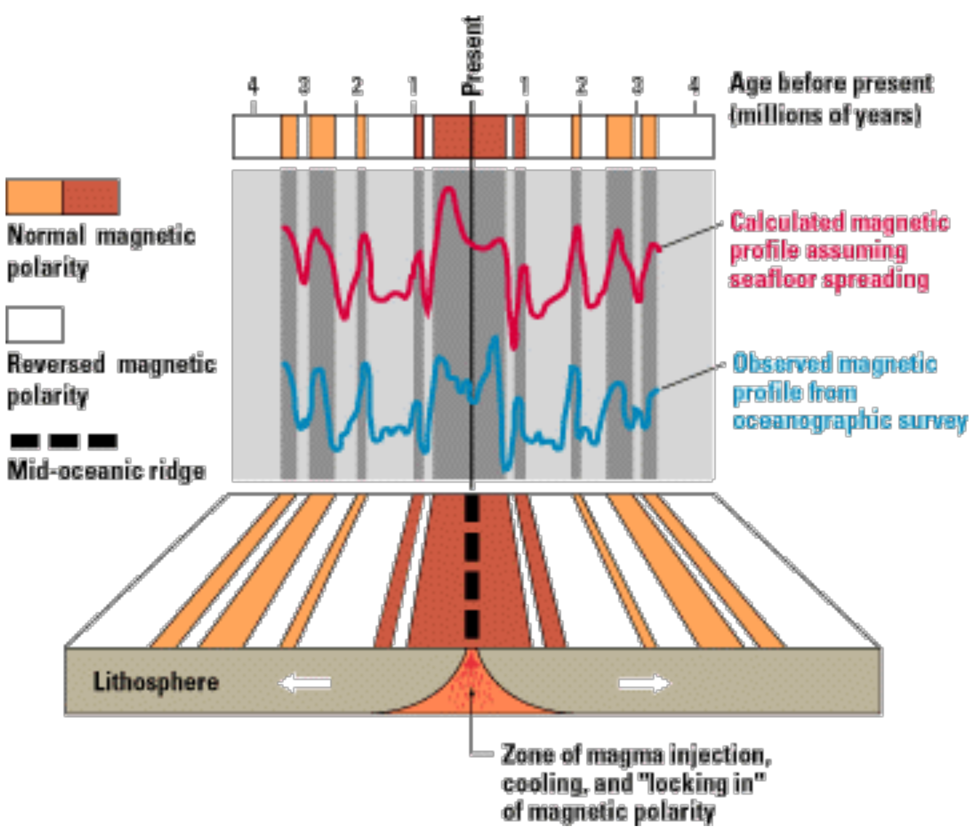


Vine 1966 Science

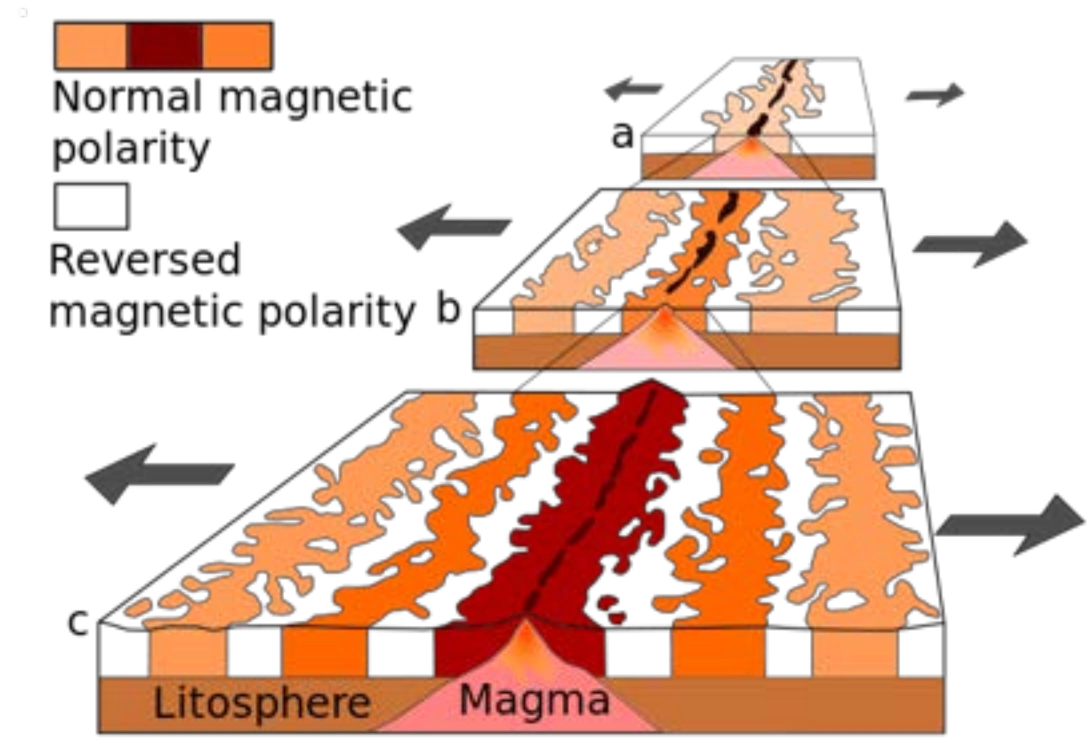




Vine–Matthews–Morley hypothesis

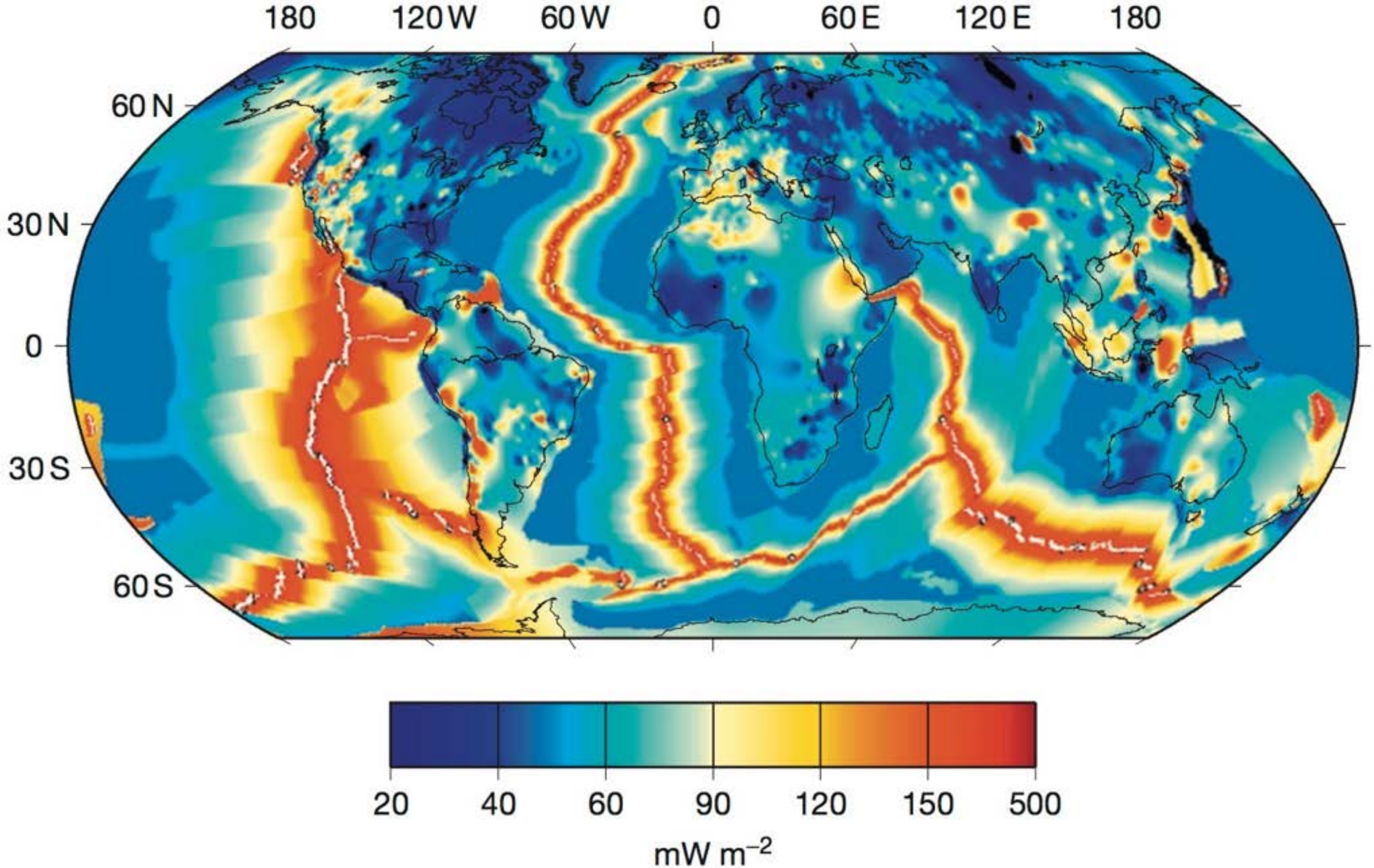


Wikipedia



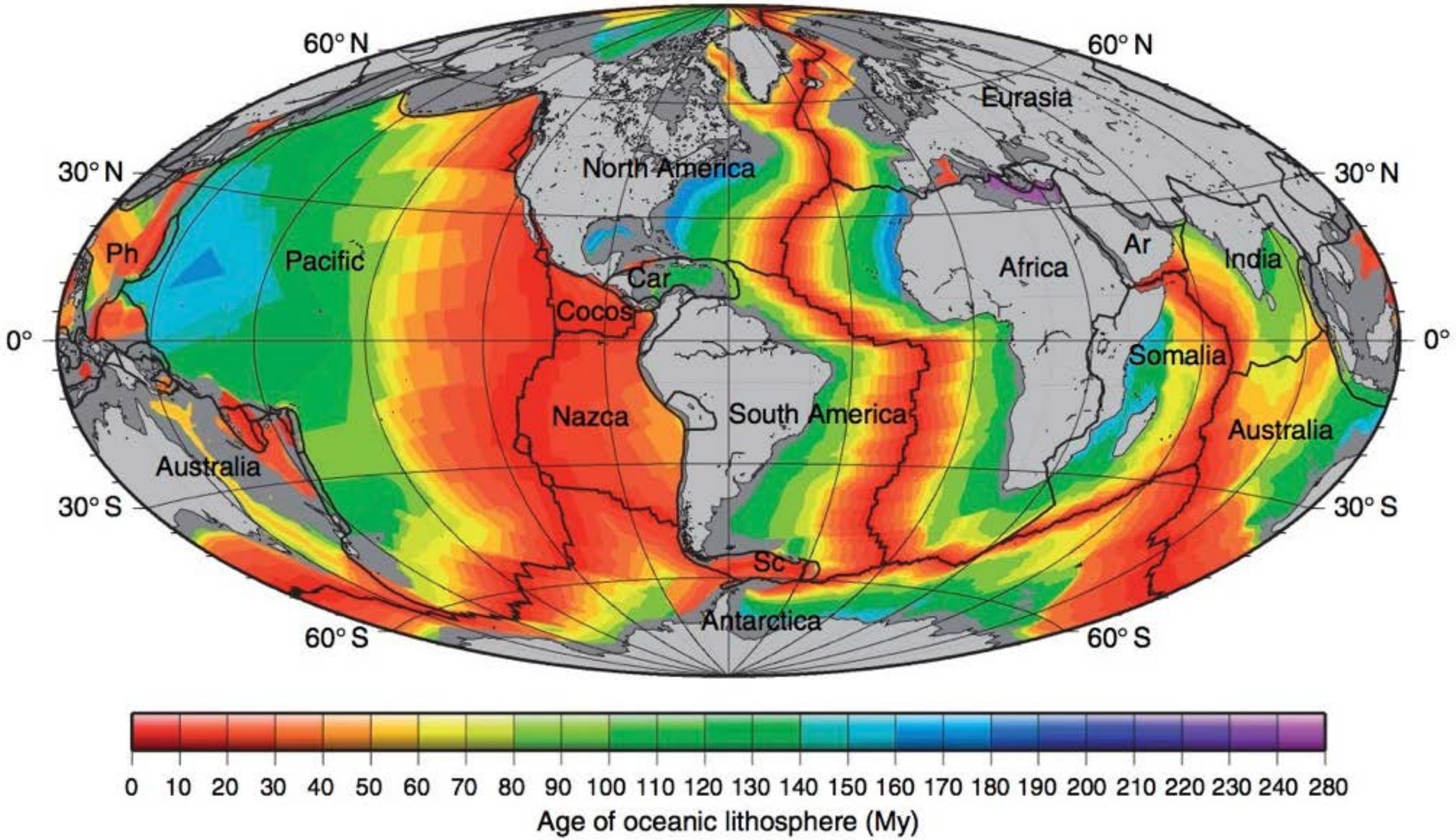
Wikimedia

Surface heat flux of the Earth



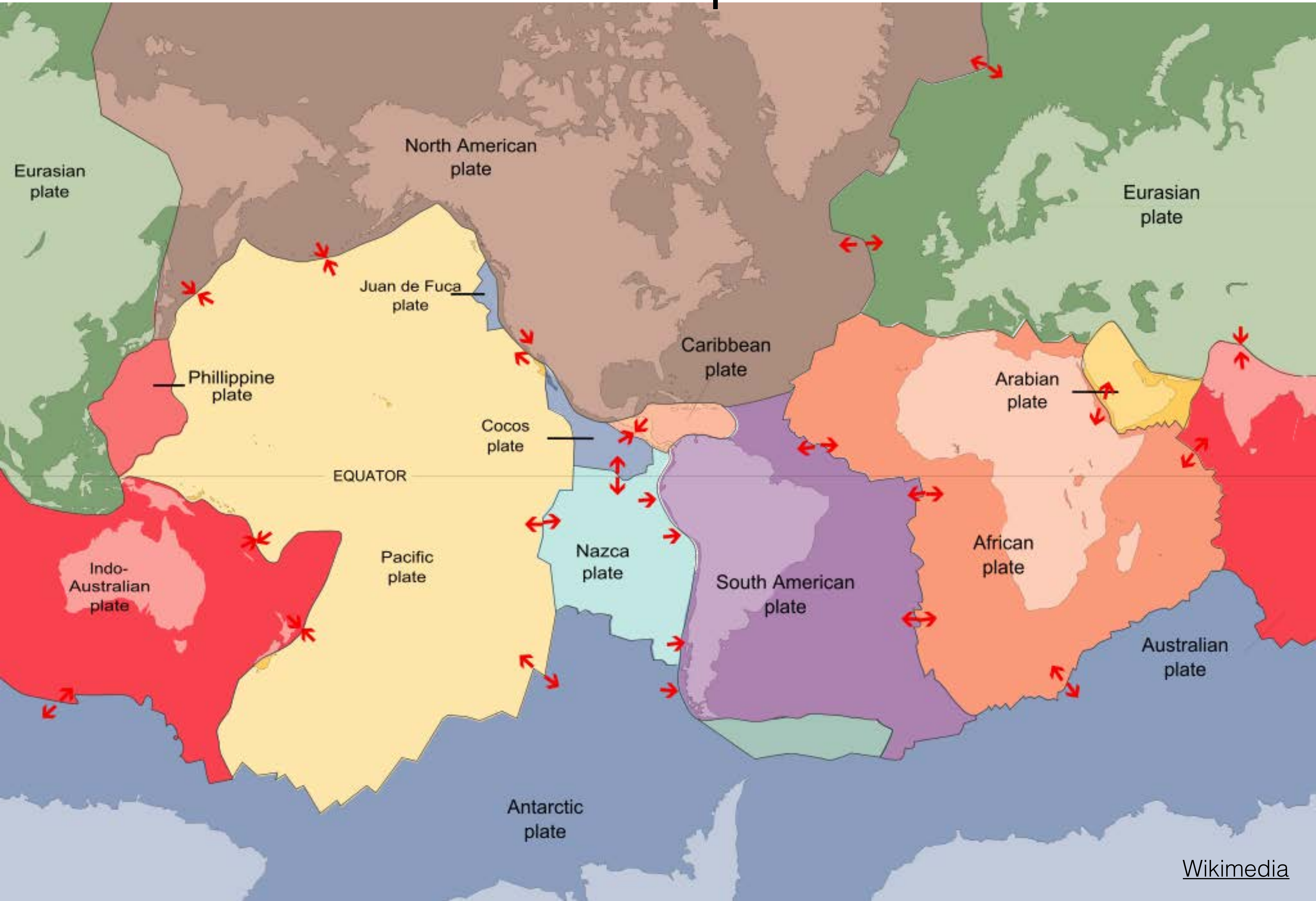
Jaupart et al. 2015 in Treatise on Geophysics

Sea floor age



Wessel & Müller 2015 in Treatise on Geophysics

Tectonic plates



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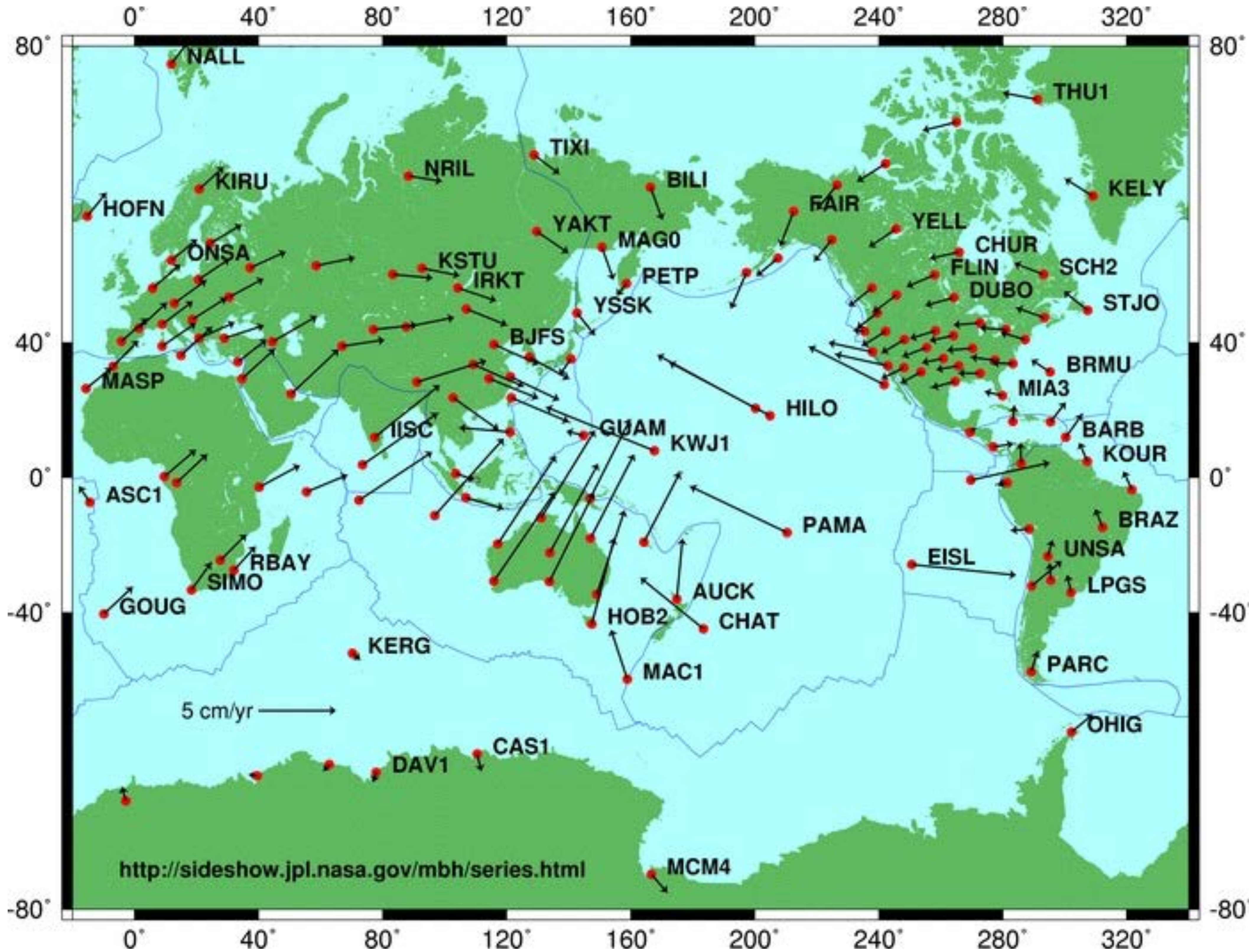
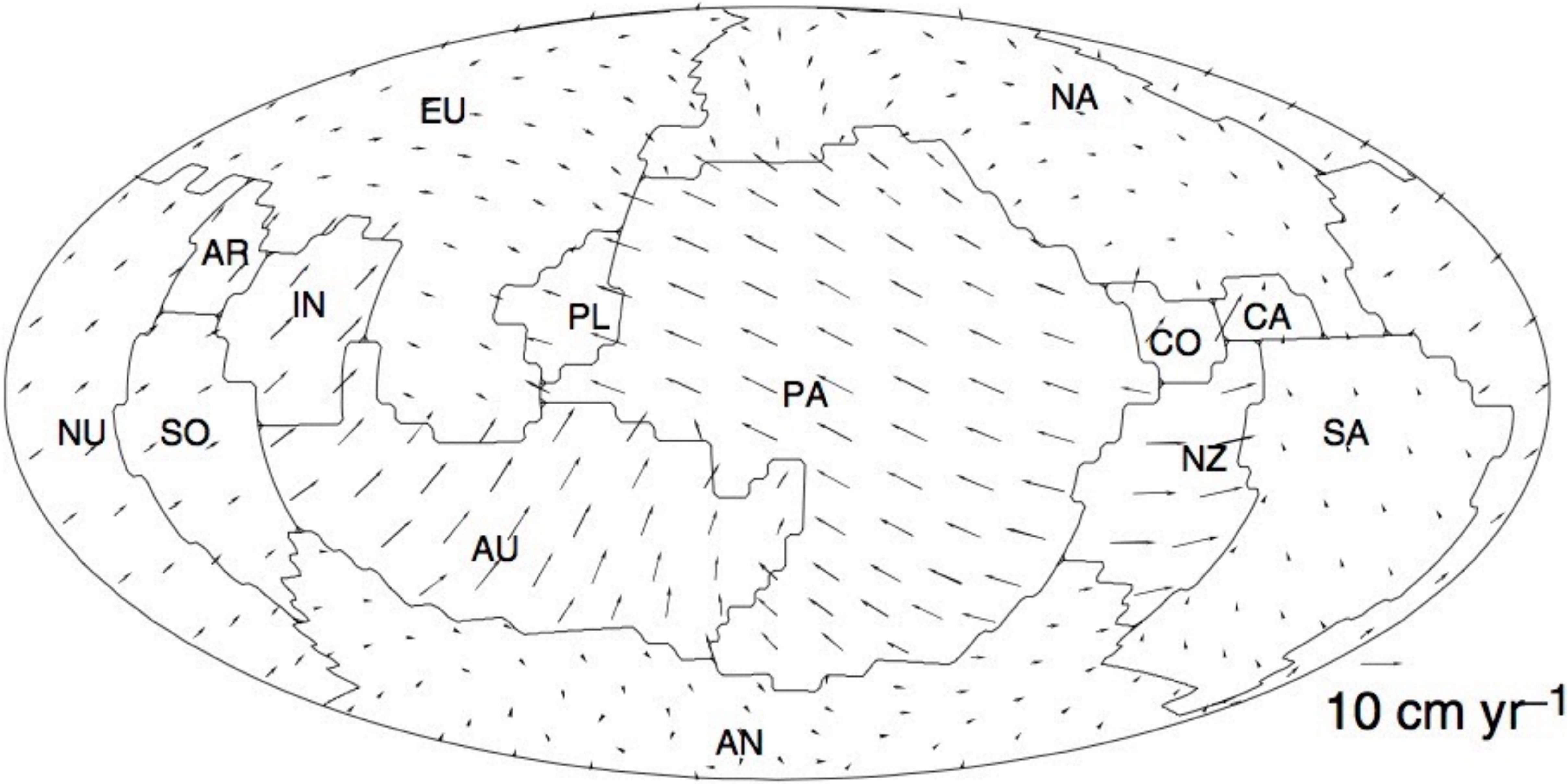
GPS measured plate velocities



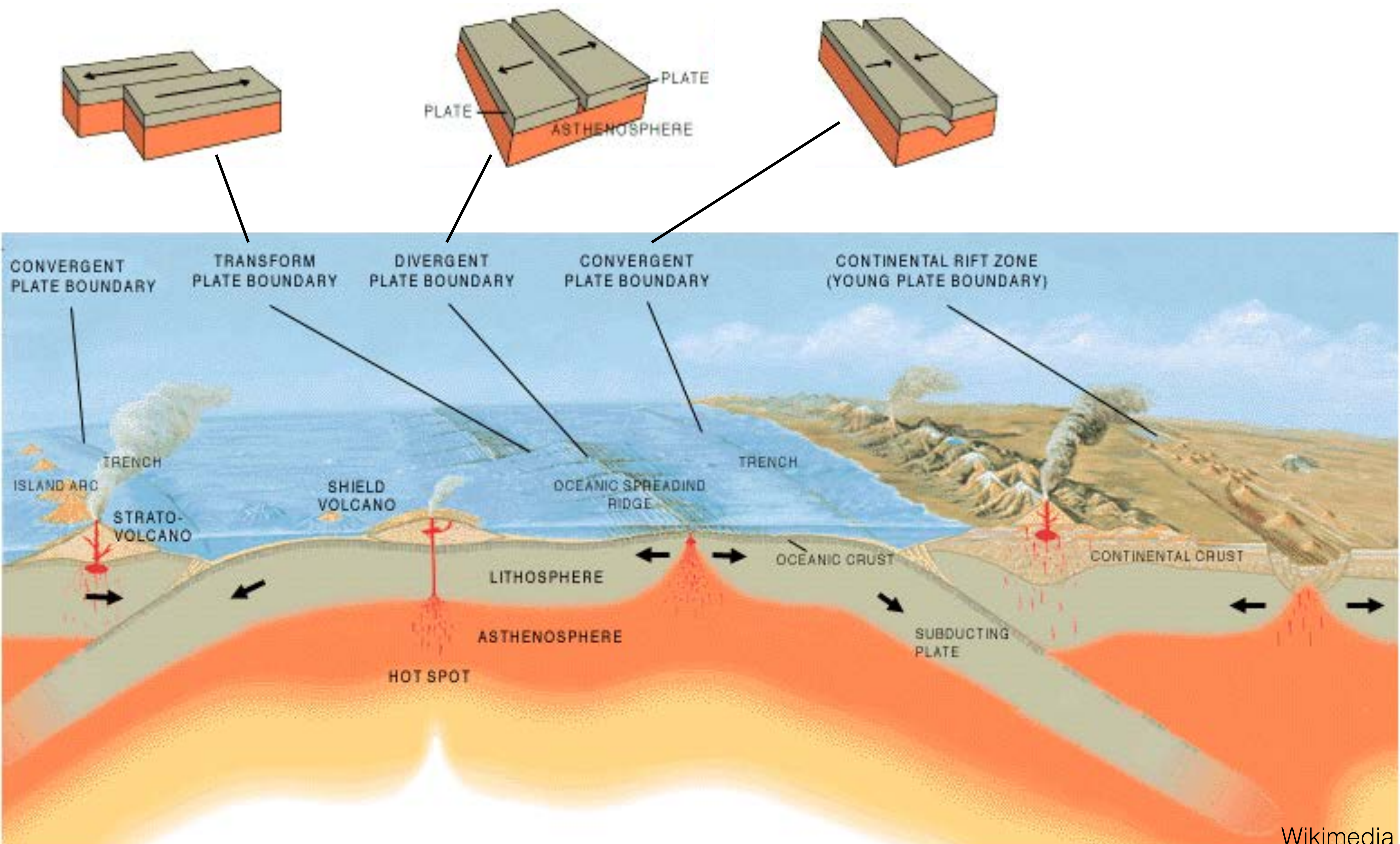
Wikimedia

Model of plate velocities



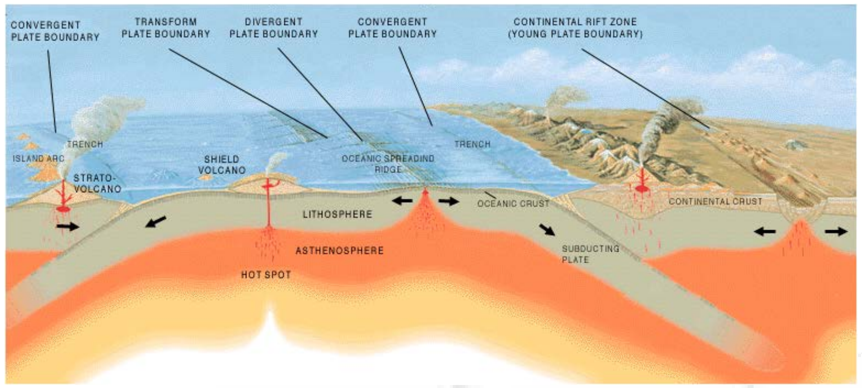
Queré & Forte 2006 GJI

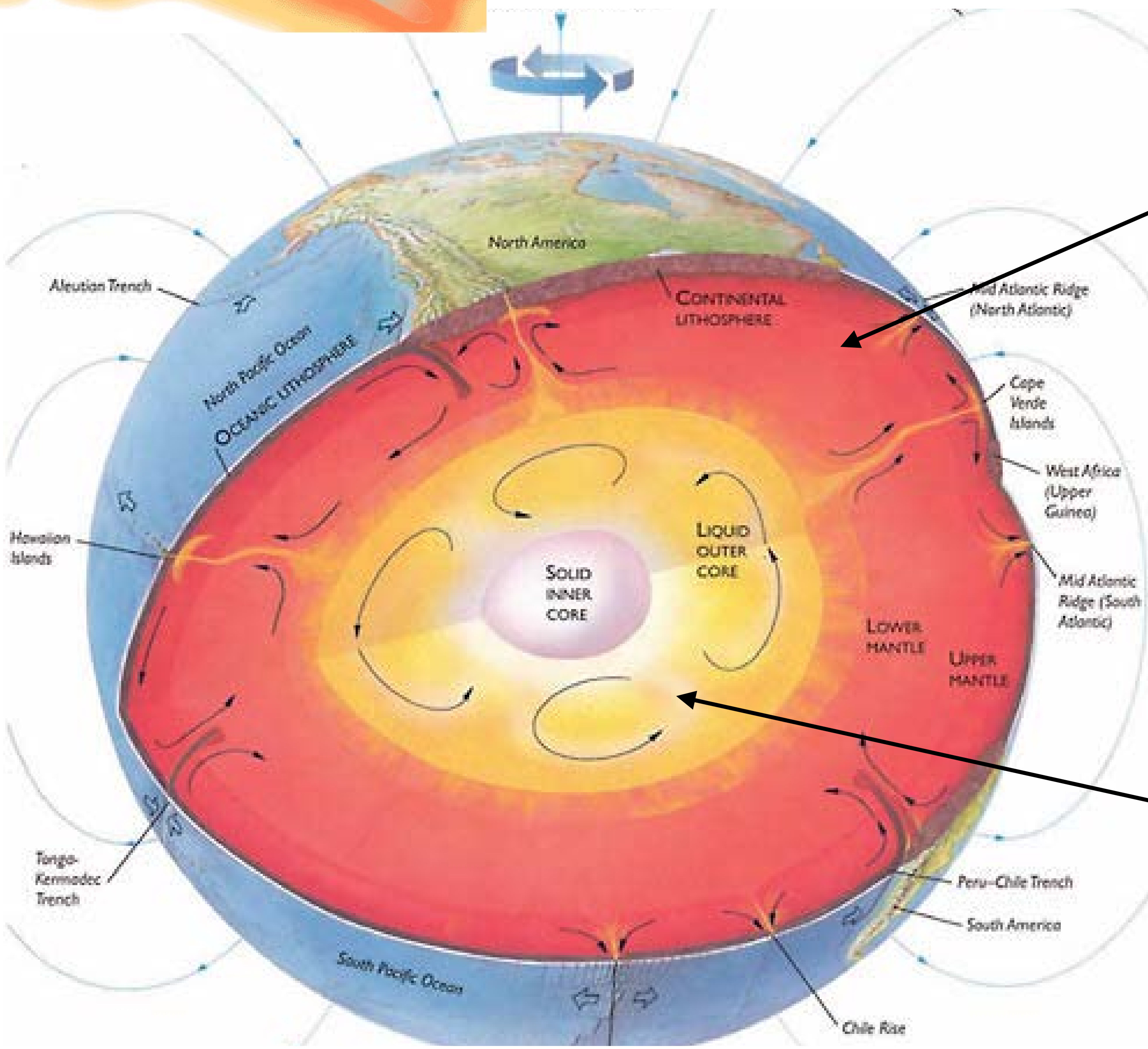
Looking deeper



Wikimedia

Plate tectonics ⇔ Mantle convection





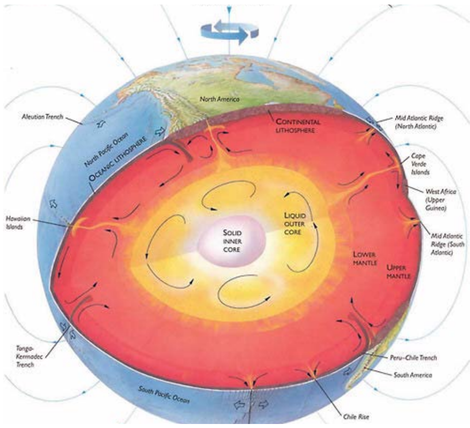
Convection in the mantle

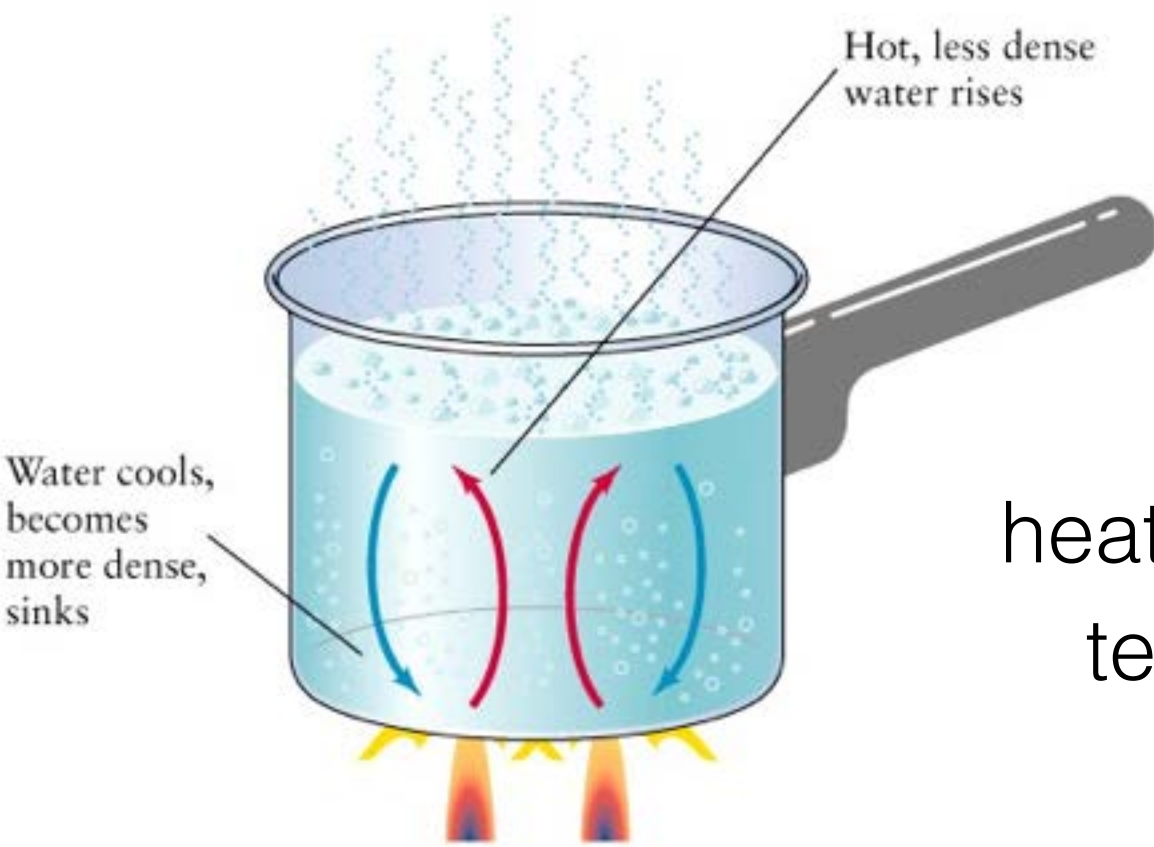
Convection in the outer core

What powers the dynamics

- Earth's cooling
- Energy sources: long-lived radioactivity
  - ^{238}U, ^{232}Th, ^{40}K
  - how much radiogenic heat?
  - how spatially distributed?

Thermal convection





[James Imamura]

viscous fluid

heated from below / cooled from top

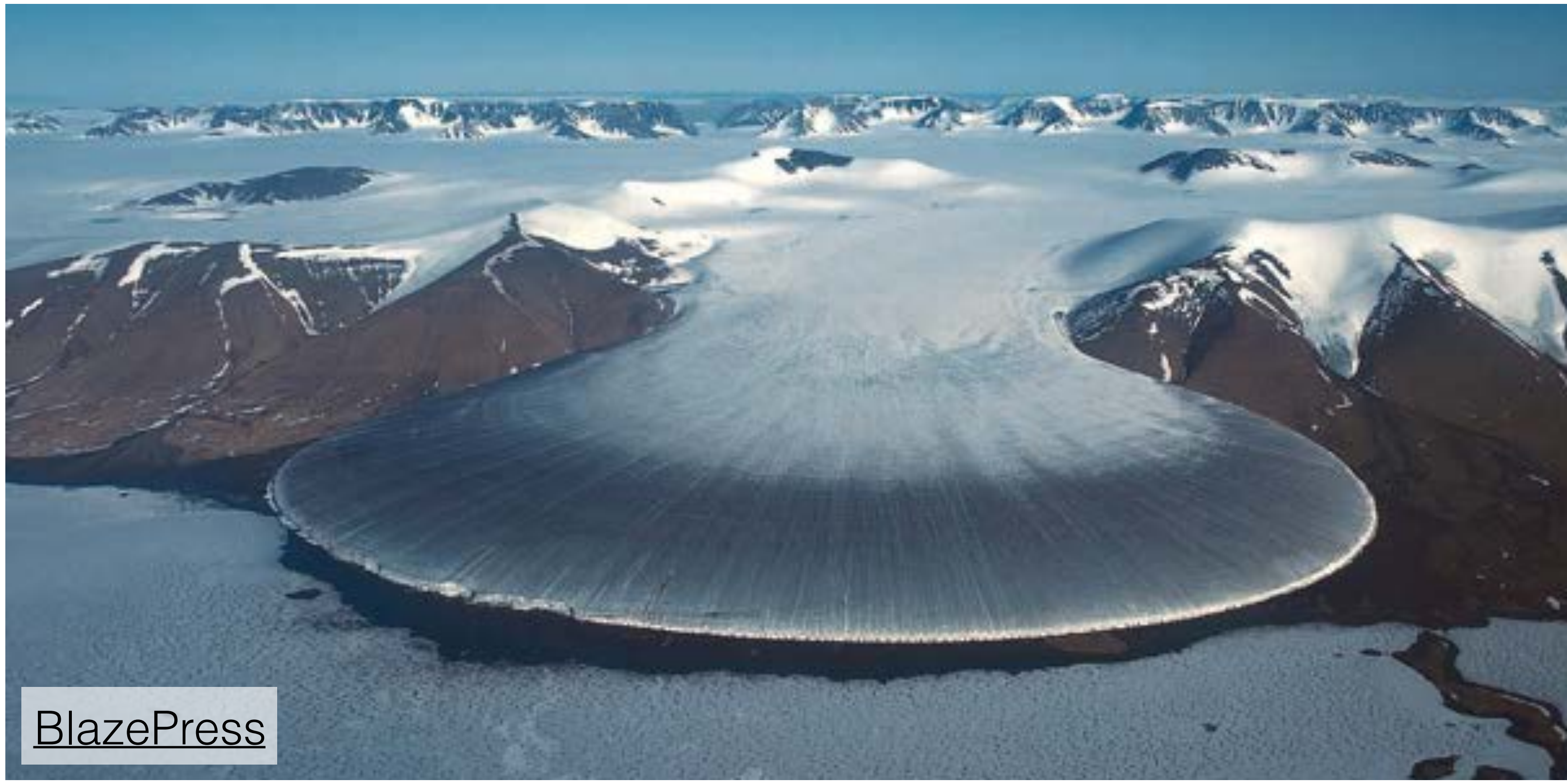
temperature-dependent density

Mantle: solid-state convection

Most of the mantle is solid, i.e., below melting temperature

Ice behaves like solid at short time scale

…flows at much longer time scale

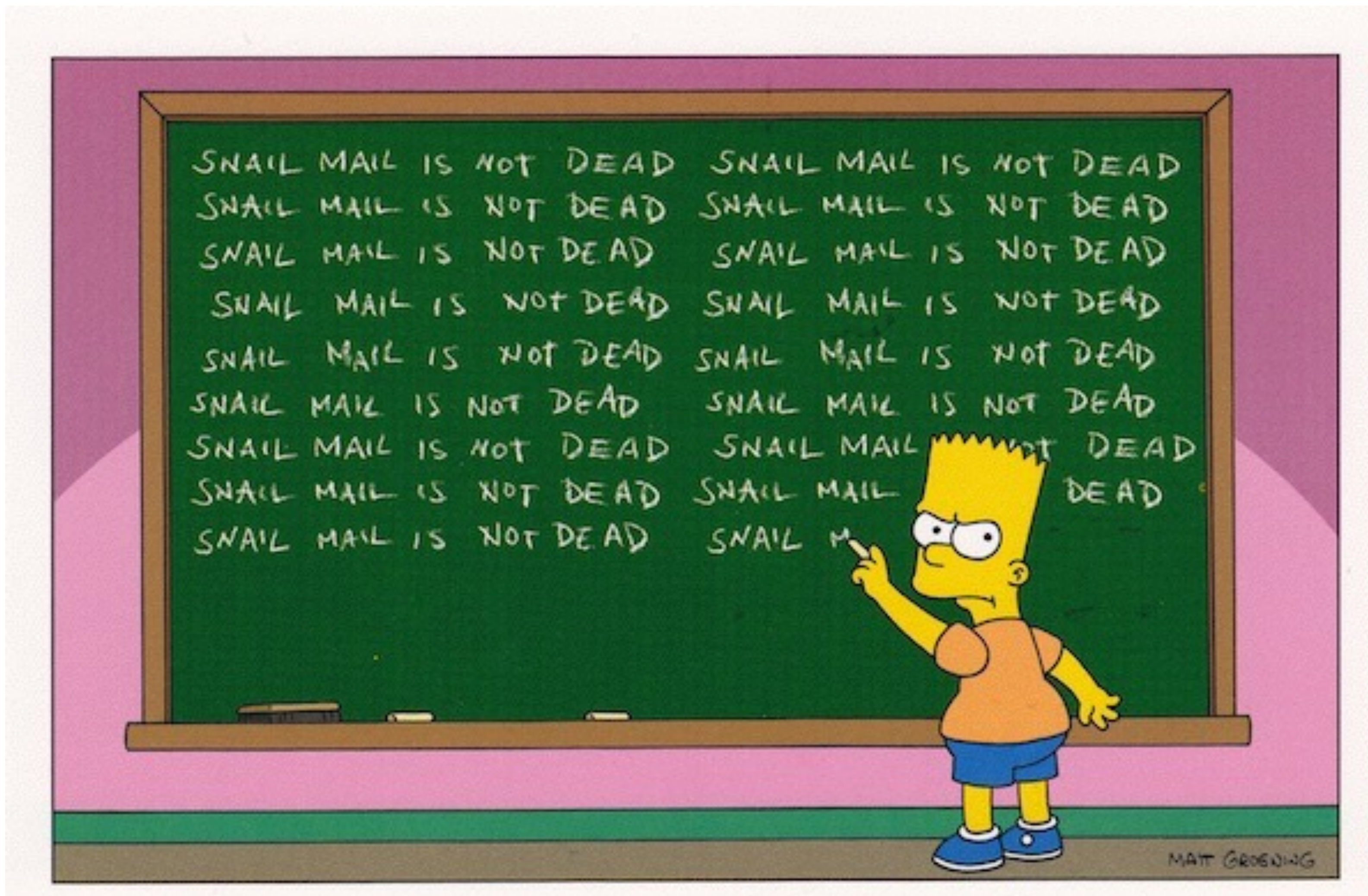


Viscosity in Pa s

water	10^{-3}
honey	10^{1}
ice	10^{10}
Earth's mantle	10^{21} Pa s

→ at time scales ≫ 10^3 years, fluid-like behavior

Equations for viscous fluid



Equations for incompressible viscous fluid

Mass $$\boldsymbol{\nabla} \cdot \mathbf{v} = 0$$

Momentum $$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla}\mathbf{v}\right) = -\boldsymbol{\nabla}P + \eta\nabla^2\mathbf{v} + \rho\mathbf{g}$$

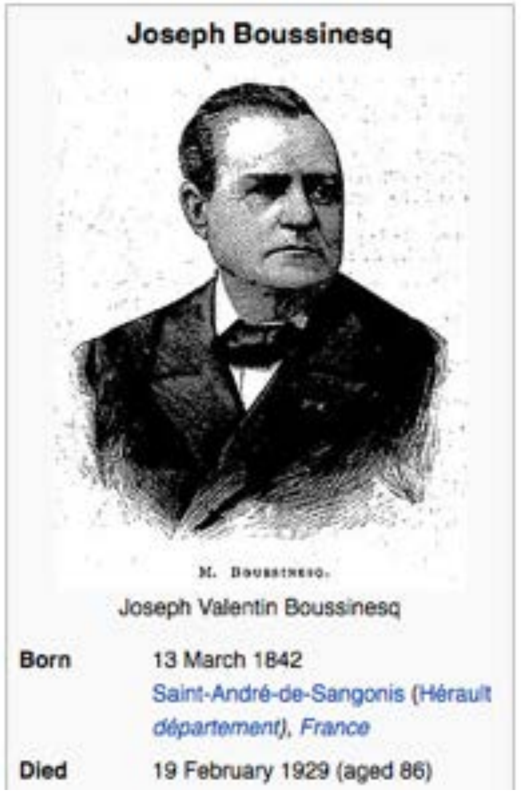
Energy $$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \boldsymbol{\nabla}T + \kappa\nabla^2 T + \frac{Q}{\rho C_P}$$

Boussinesq approximation:

Only consider density variations in the buoyancy term

$$\Delta\rho = -\rho\alpha\Delta T \qquad \text{(EOS)}$$

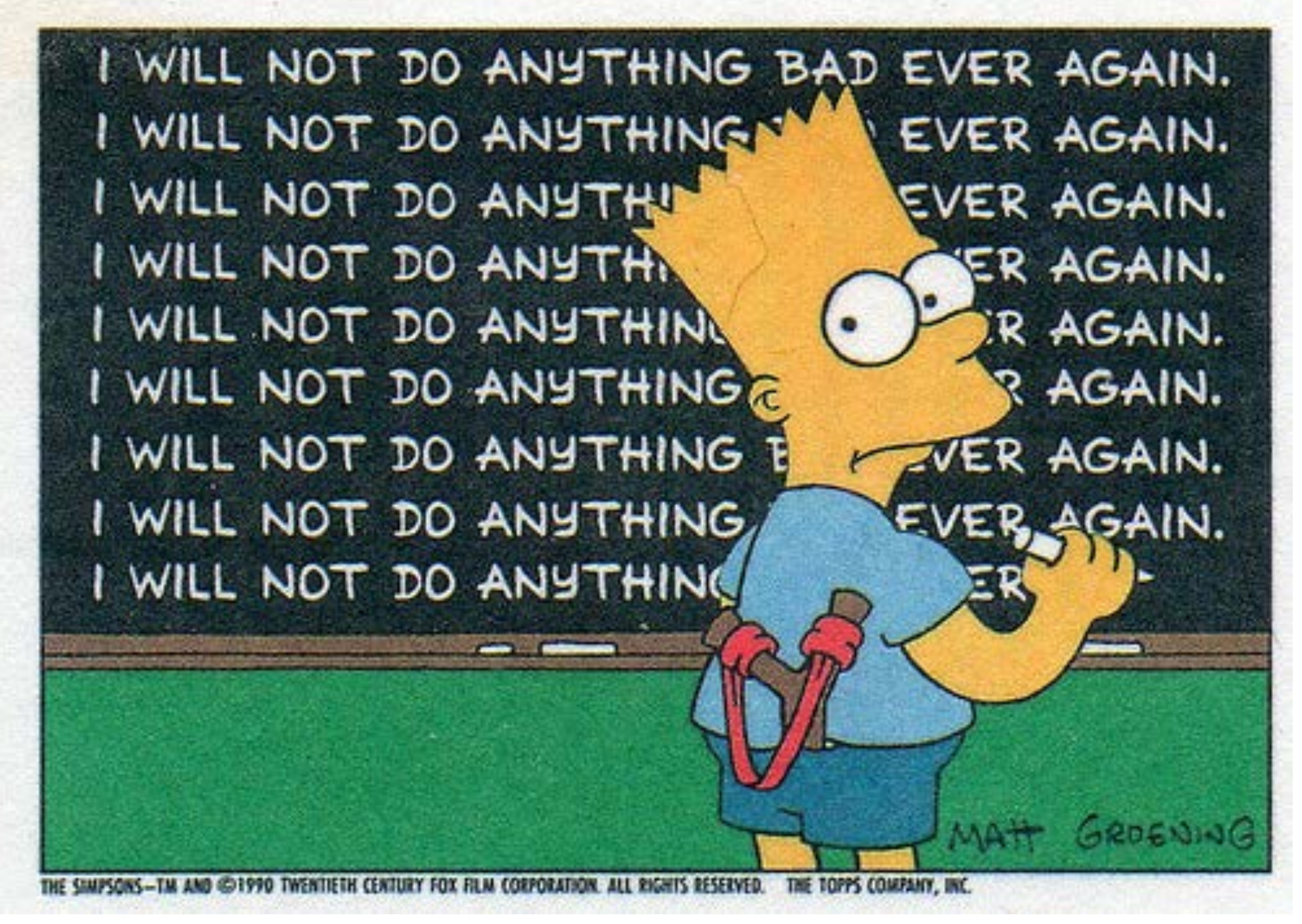
Also uniform viscosity, thermal conductivity



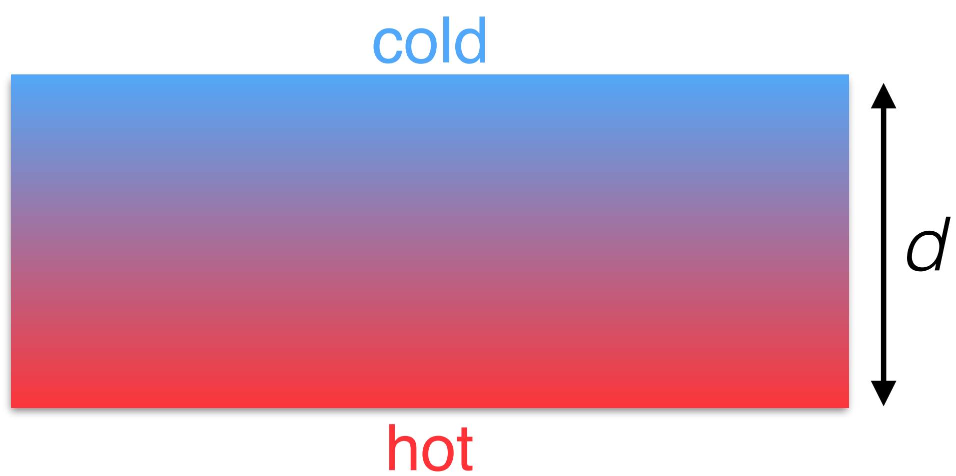
Modes of heat transfer

Rayleigh number

Onset of convection



Modes of heat transfer



$$\frac{\partial T}{\partial t} = \underbrace{-\mathbf{v}\cdot\boldsymbol{\nabla}T}_{\text{advection}} + \underbrace{\kappa\nabla^2 T}_{\text{conduction}} + \frac{Q}{\rho C_P}$$

1. Conduction — time scale $\tau_c = \dfrac{d^2}{\kappa}$ $\qquad \kappa = \dfrac{k}{\rho C_p}$

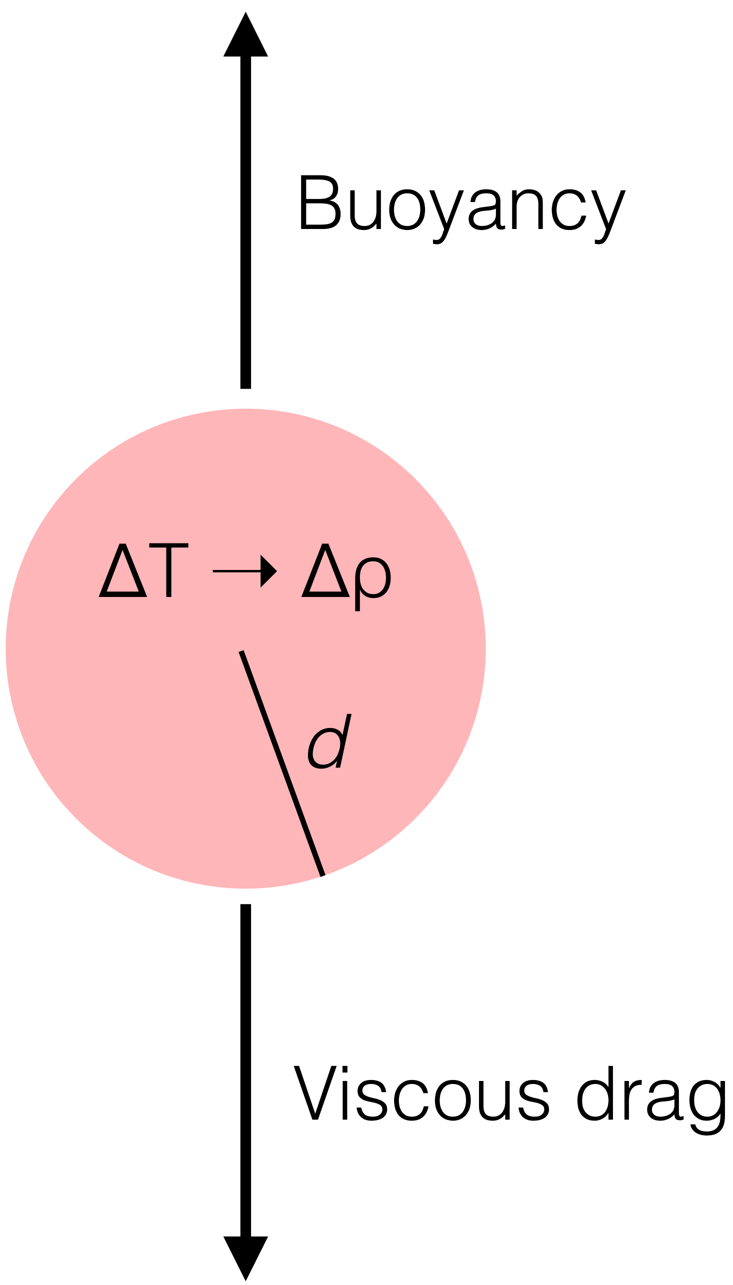
2. Advection — time scale $\tau_a = \dfrac{d}{v}$

$$\downarrow$$

$$\frac{\tau_c}{\tau_a} = \frac{vd}{\kappa}$$ $\approx 10^4$ … mantle

3. ~~Radiation~~

Rayleigh number



Buoyancy F_b balanced by viscous drag F_d

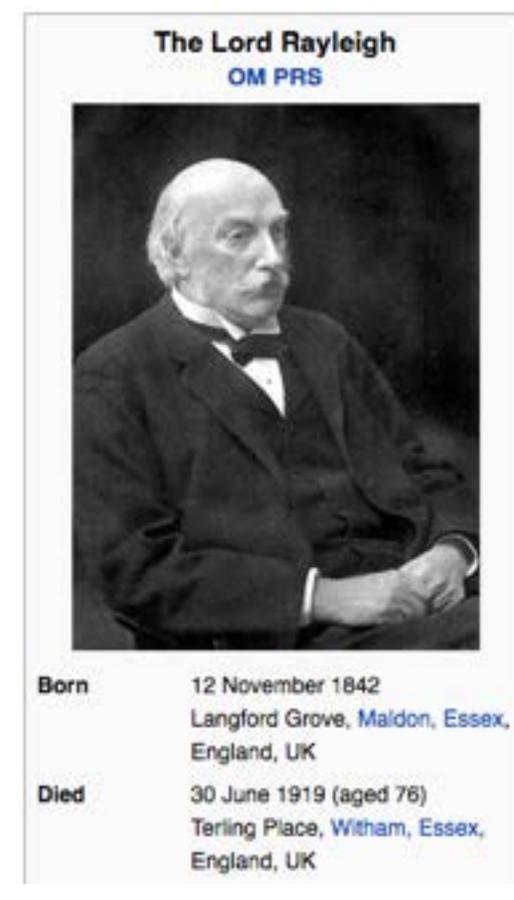
$$F_d = 6\pi\eta v d \qquad \text{Stokes' law}$$

$$F_b = \Delta\rho g V = \frac{4}{3}\pi d^3 \rho\alpha\Delta T g$$

$$\rightarrow \qquad v = \frac{2}{9}\frac{\rho\alpha\Delta T g d^2}{\eta}$$

$$\frac{\tau_c}{\tau_a} = \frac{vd}{\kappa} \approx \frac{\rho\alpha\Delta T g d^3}{\kappa\eta} \equiv Ra$$

- low *Ra* … no convection
- high *Ra* … convection



Onset of convection

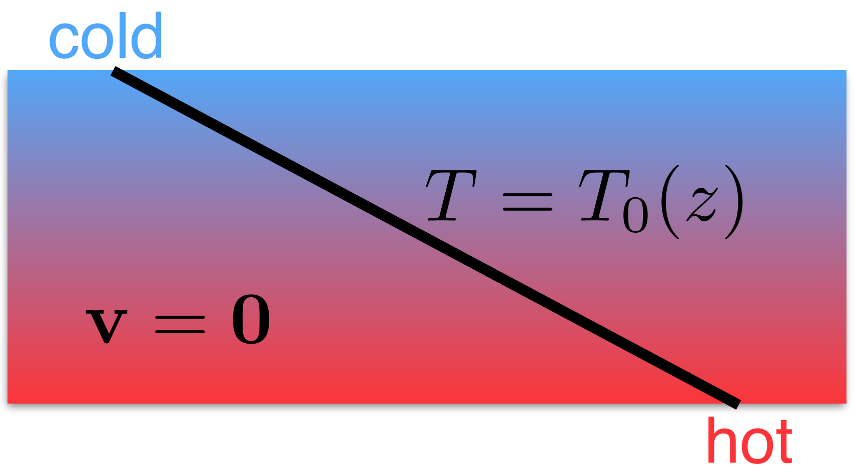
The diffusive solution is a valid solution to the equations of thermal convection.

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0$$

$$-\boldsymbol{\nabla} P + \nabla^2 \mathbf{v} - Ra(T - T_0)\mathbf{e}_z = 0$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} T = \nabla^2 T$$

(non-dimensional form)



Look for another solution in the form: $T = T_0(z) + \theta(t)\sin(\pi z)\sin(\pi x)$

$$\rightarrow \quad \dot{\theta} = \left[\frac{k^2 Ra}{(k^2+\pi^2)^2} - (k^2+\pi^2)\right]\theta$$

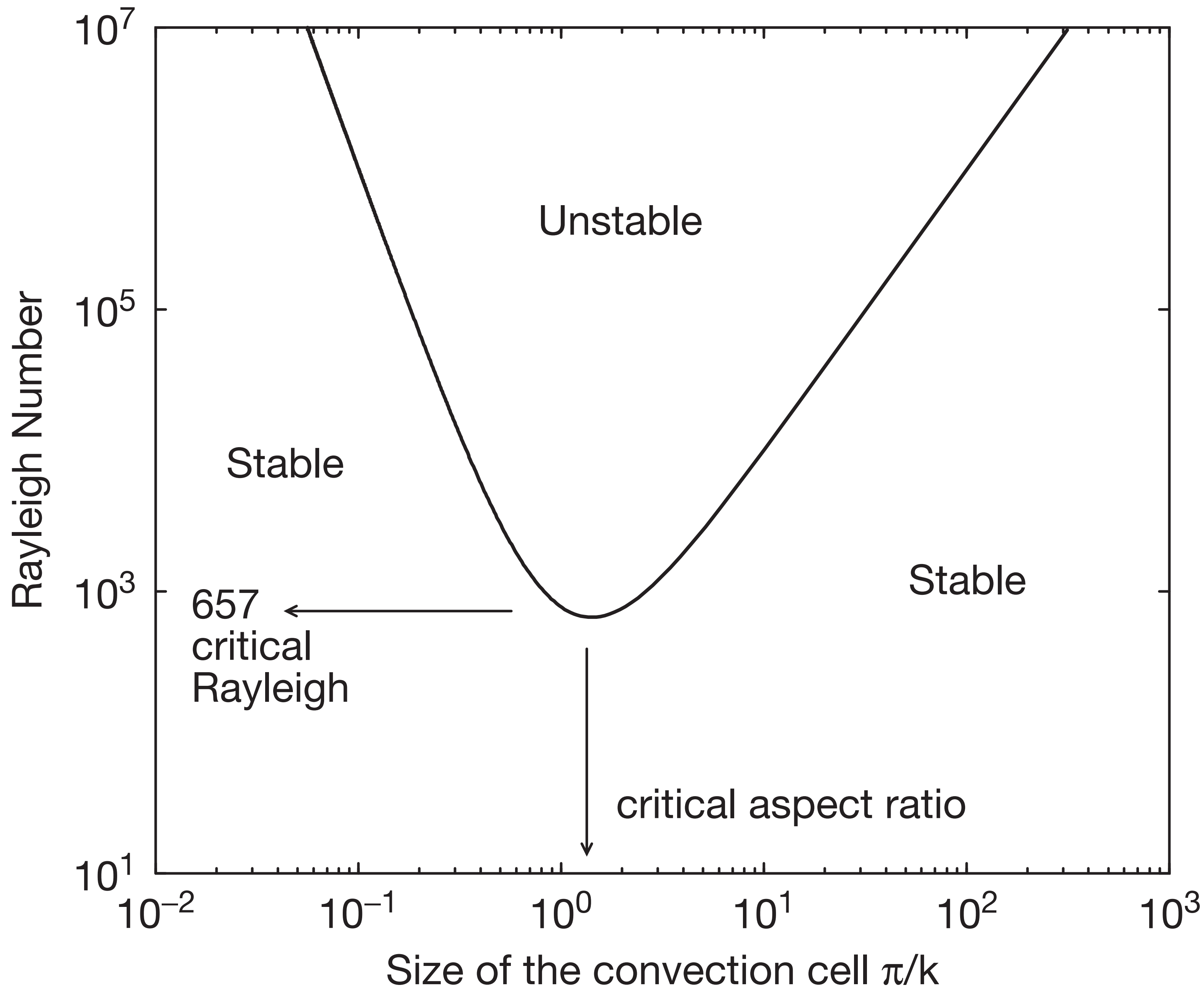
Convection develops if perturbation grows in time (expression in brackets > 0):

$$Ra > Ra_s = \frac{(k^2+\pi^2)^3}{k^2}$$

$$\text{Minimum} \quad Ra_s \quad \text{at} \quad k = \frac{\pi}{\sqrt{2}} \quad \text{or} \quad \lambda = 2\sqrt{2}, \quad Ra_c = \frac{27}{4}\pi^4 \approx 657$$

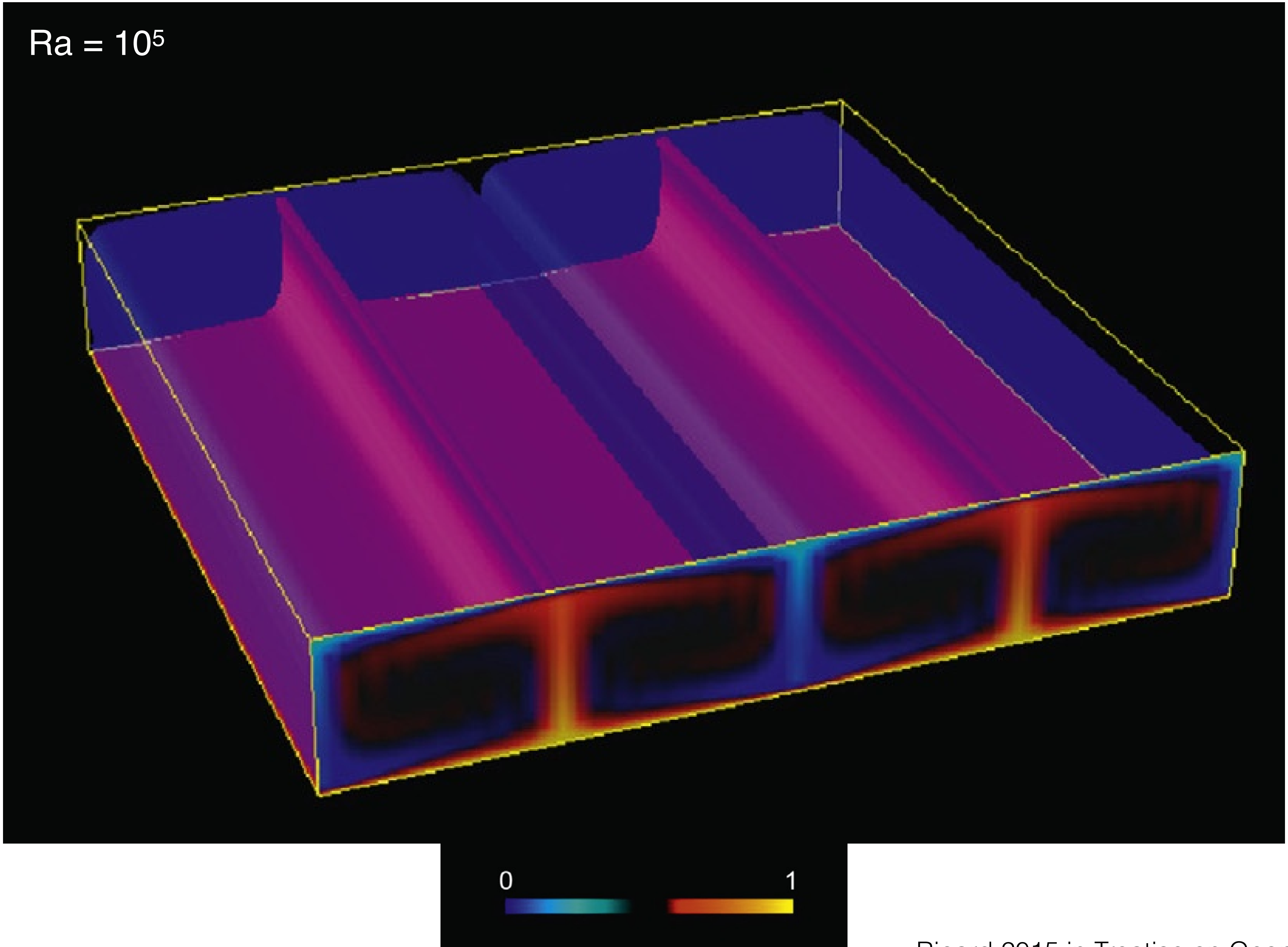
Mantle Ra $\approx 10^8$

Critical Rayleigh number



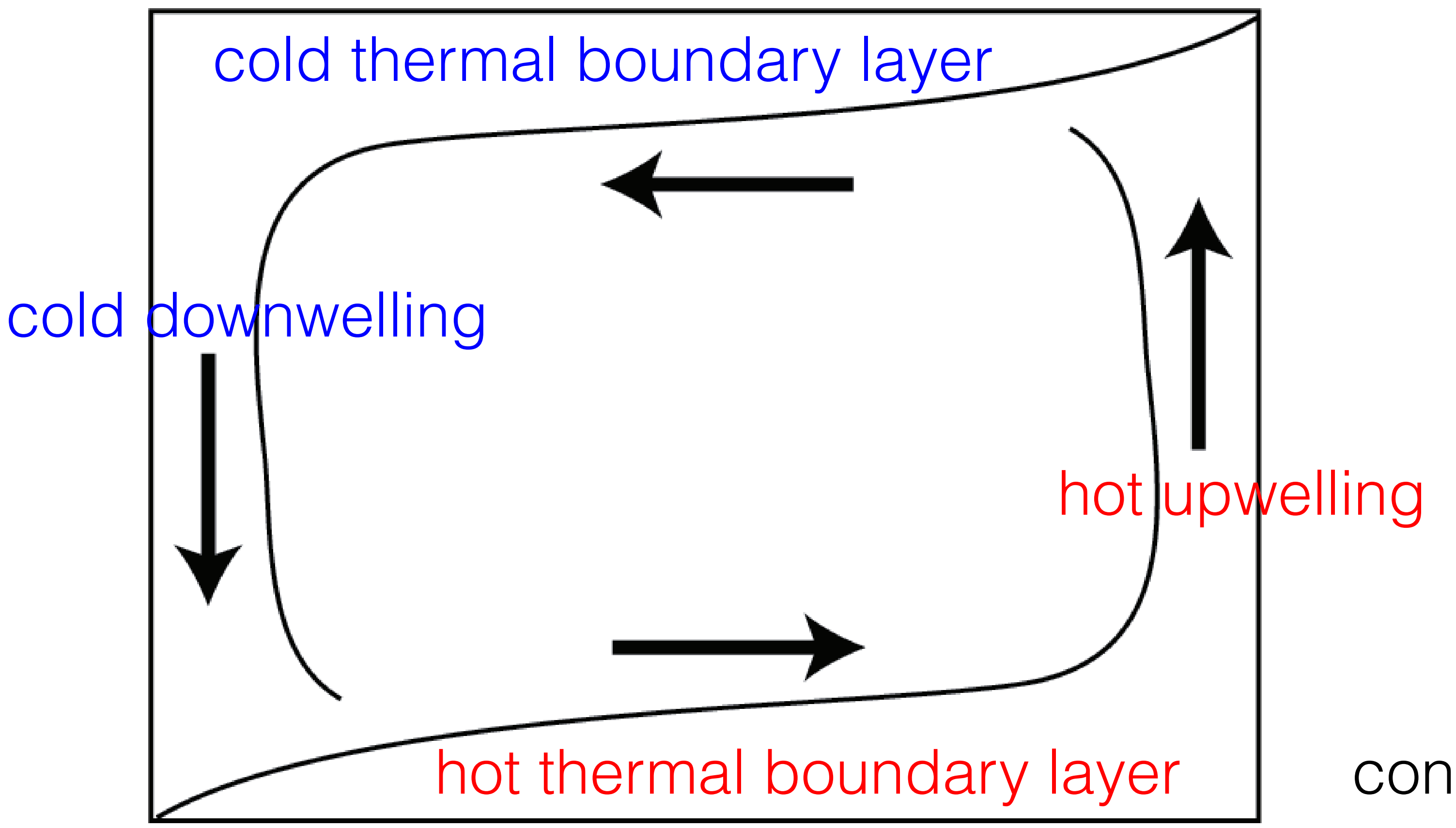
Ricard 2015 in Treatise on Geophysics

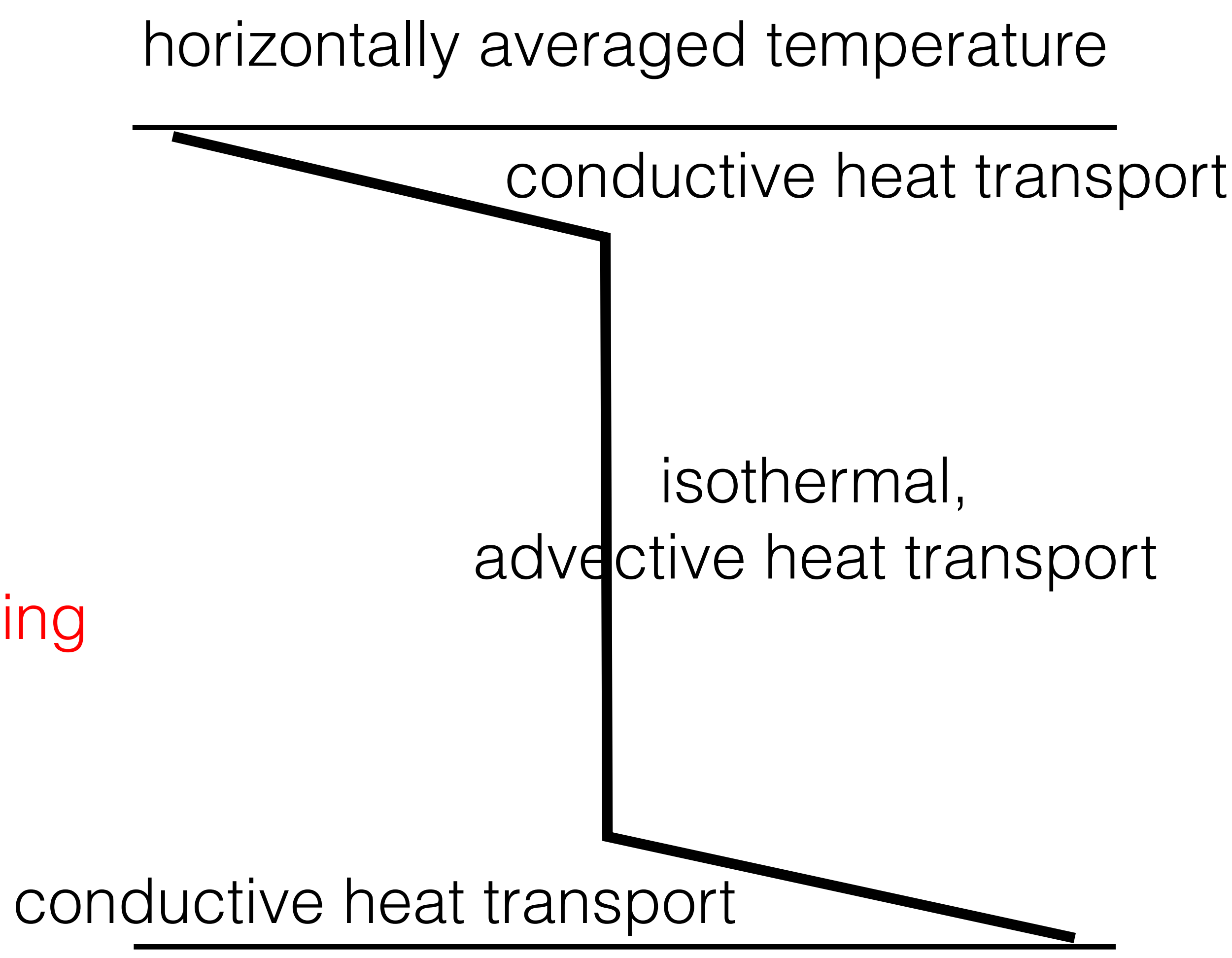
Convection cells



Ricard 2015 in Treatise on Geophysics

Convection cell





Adiabatic temperature gradient

$$\mathrm{d}S(T,P) = \left.\frac{\partial S}{\partial T}\right|_P \mathrm{d}T + \left.\frac{\partial S}{\partial P}\right|_T \mathrm{d}P = 0$$

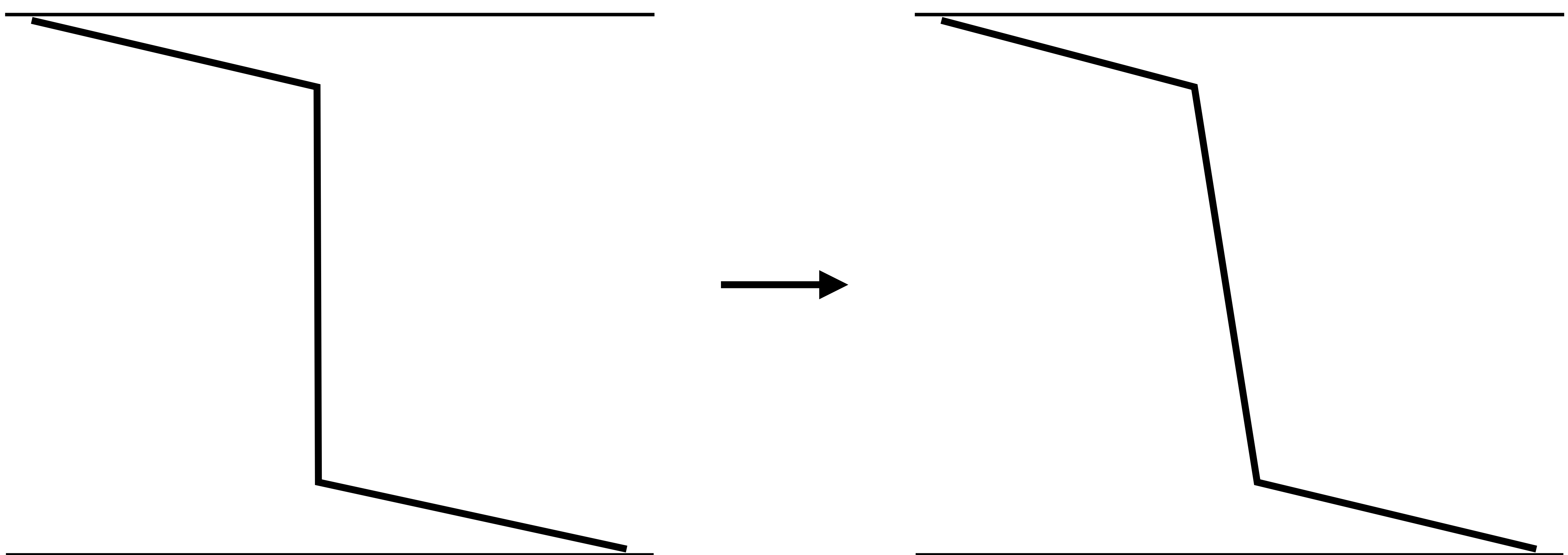
Entropy

assume adiabatic

$$\left.\frac{\partial T}{\partial P}\right|_S = \frac{\alpha T}{\rho C_p}$$

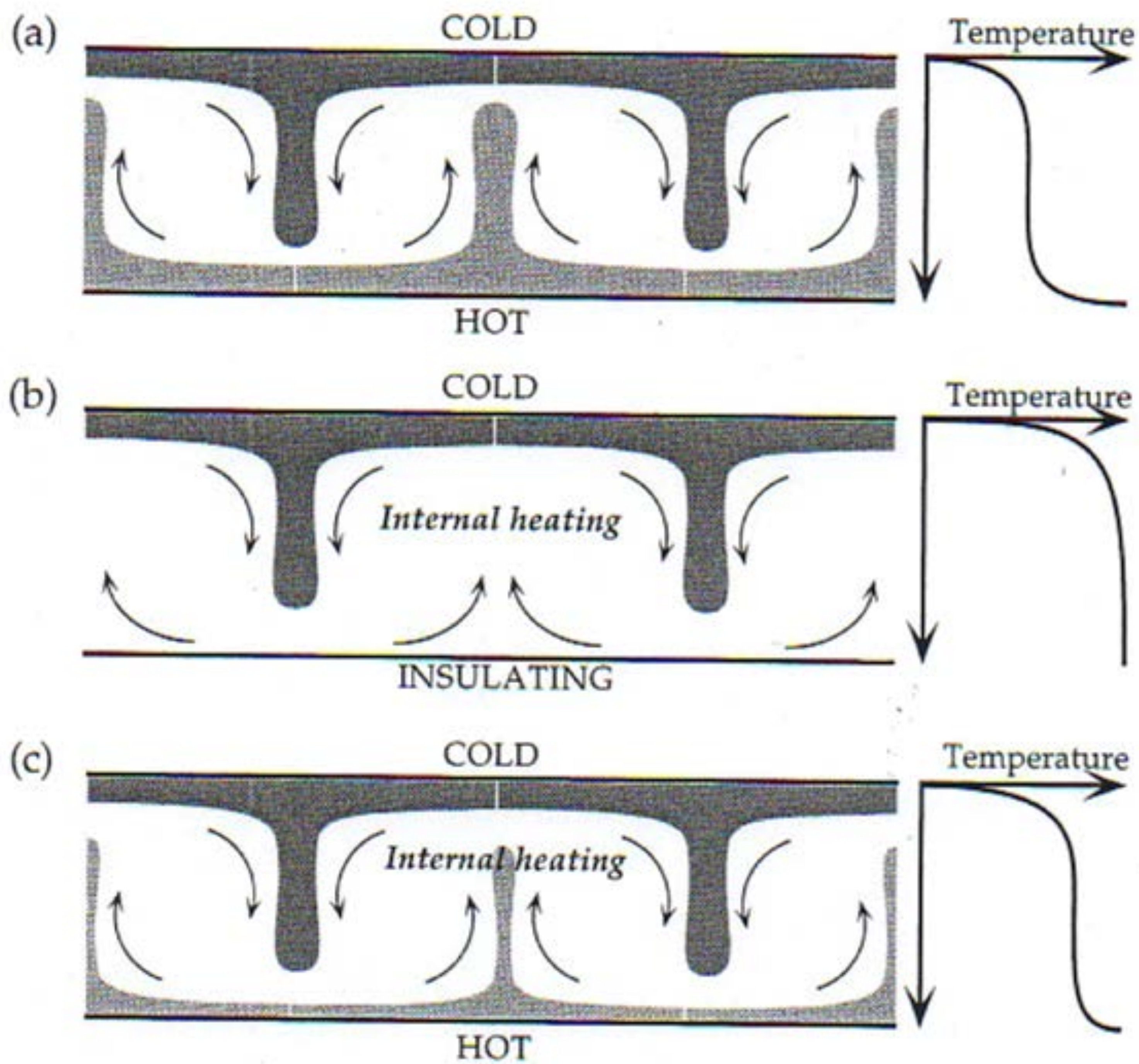
or

$$\left.\frac{\partial T}{\partial z}\right|_S = \frac{\alpha T g}{C_p}$$



Internal vs. basal heating

- thermal boundary layers
- upwellings, downwellings
- convection cells

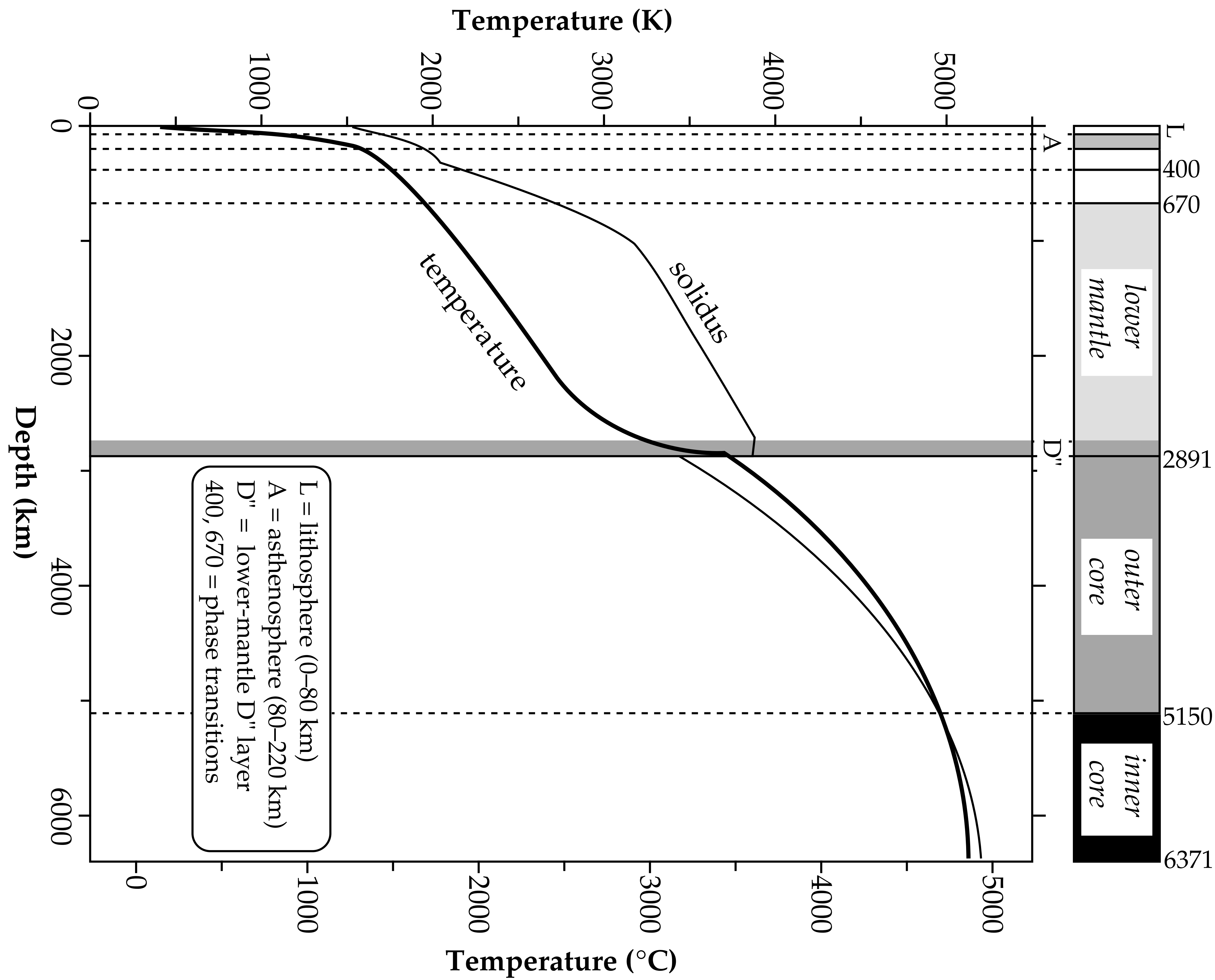


no internal heating

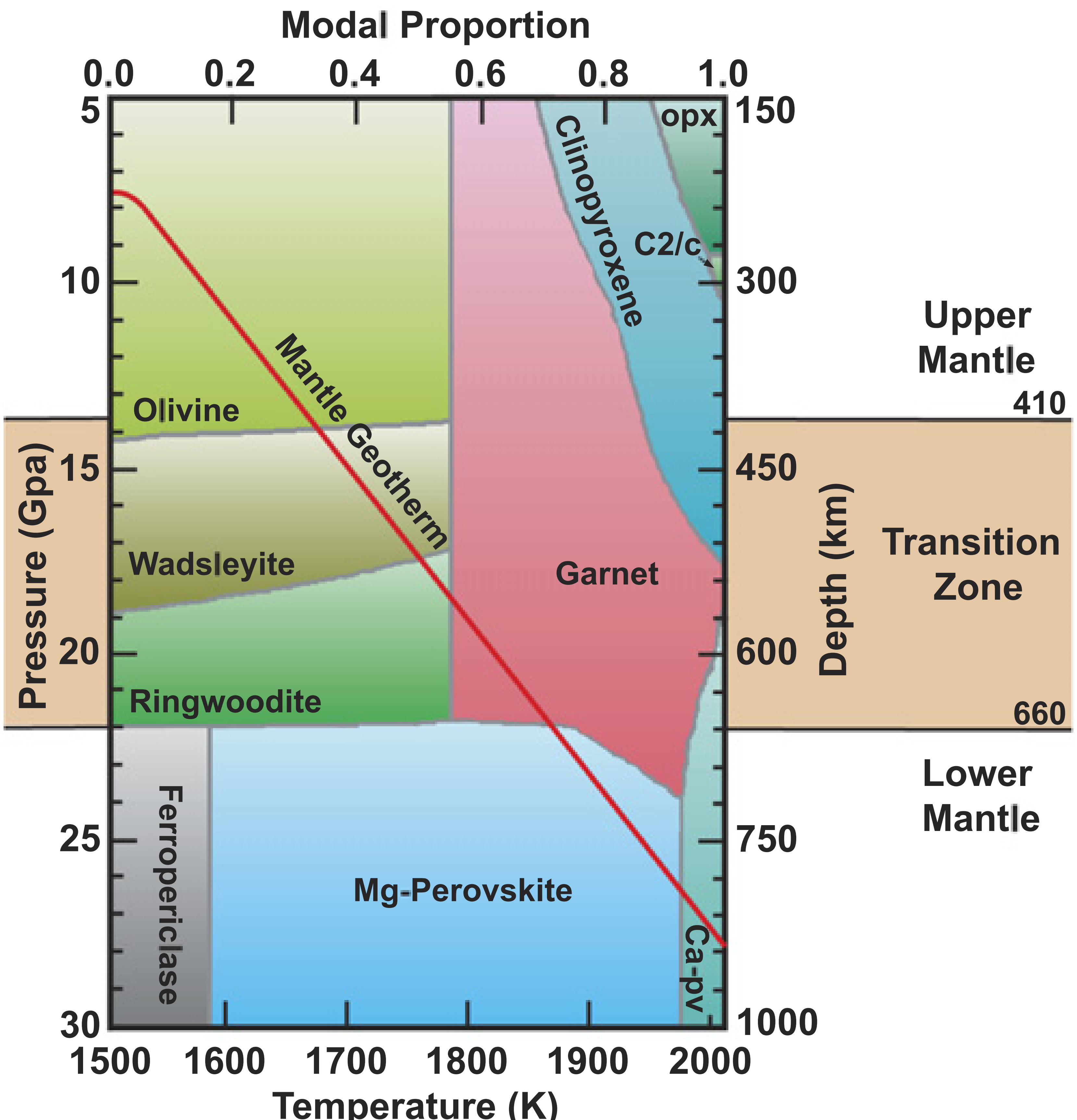
no basal heating

mixed heating

Geotherm: temperature vs. depth

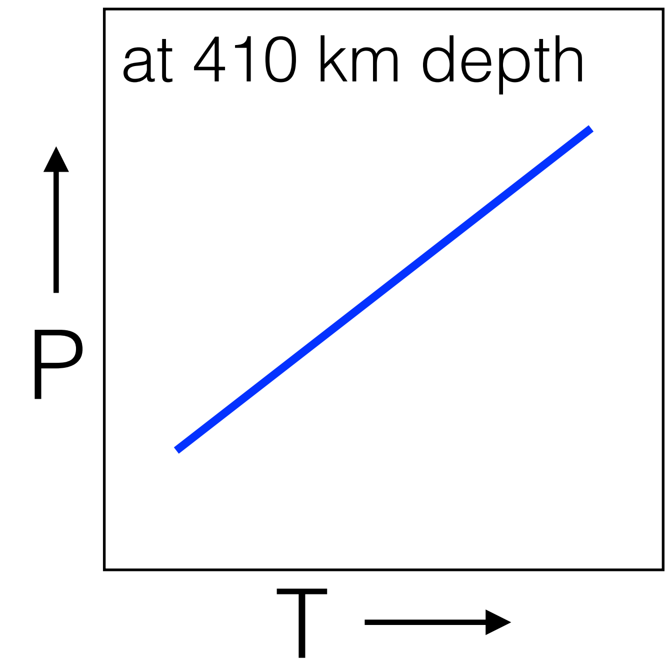


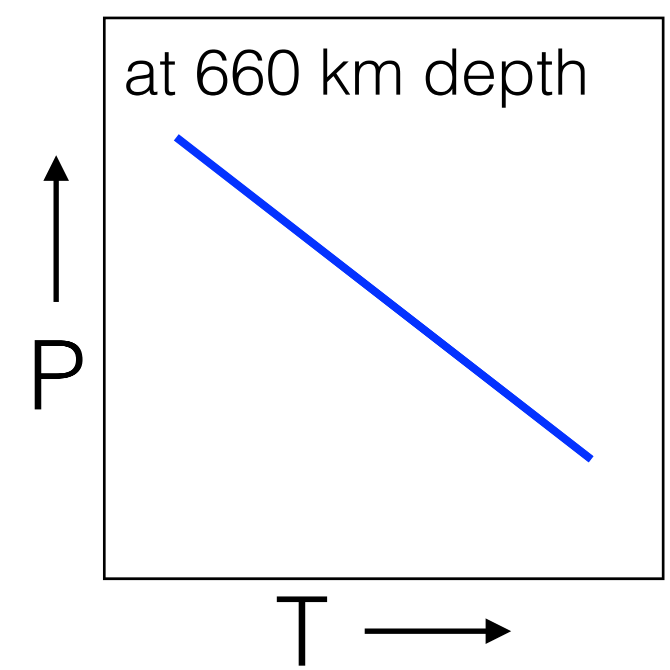
Phase transitions



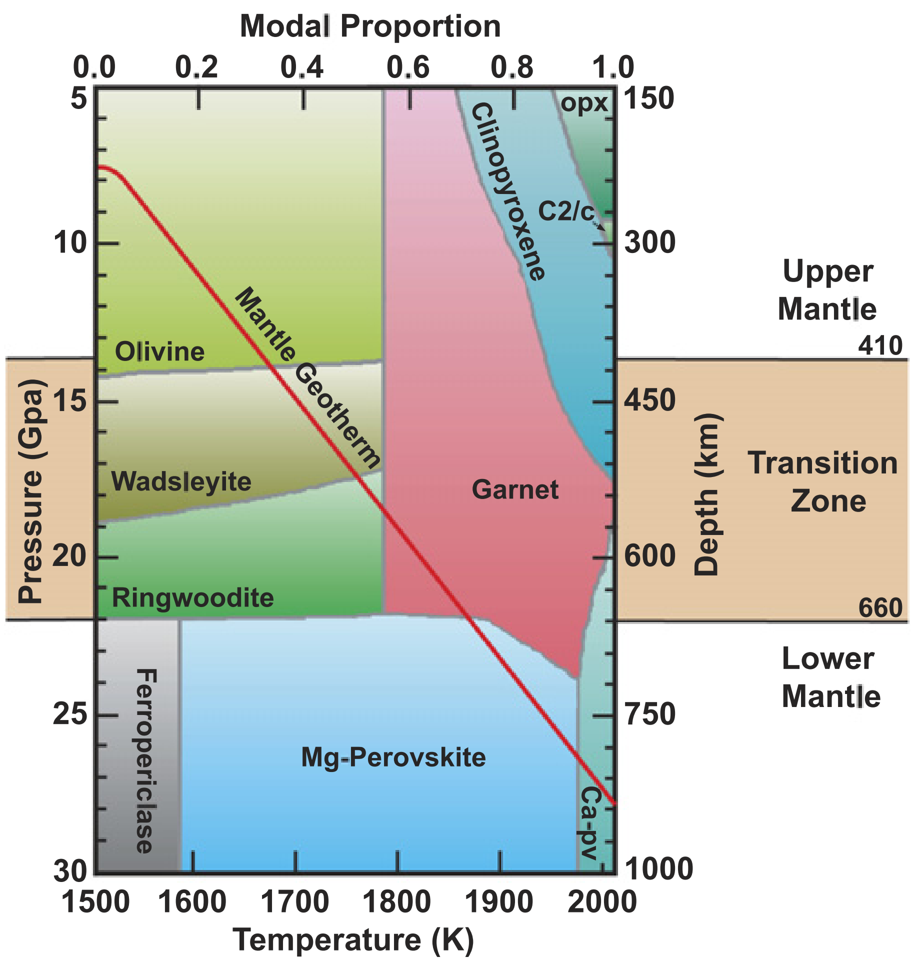
Bellini et al. 2013

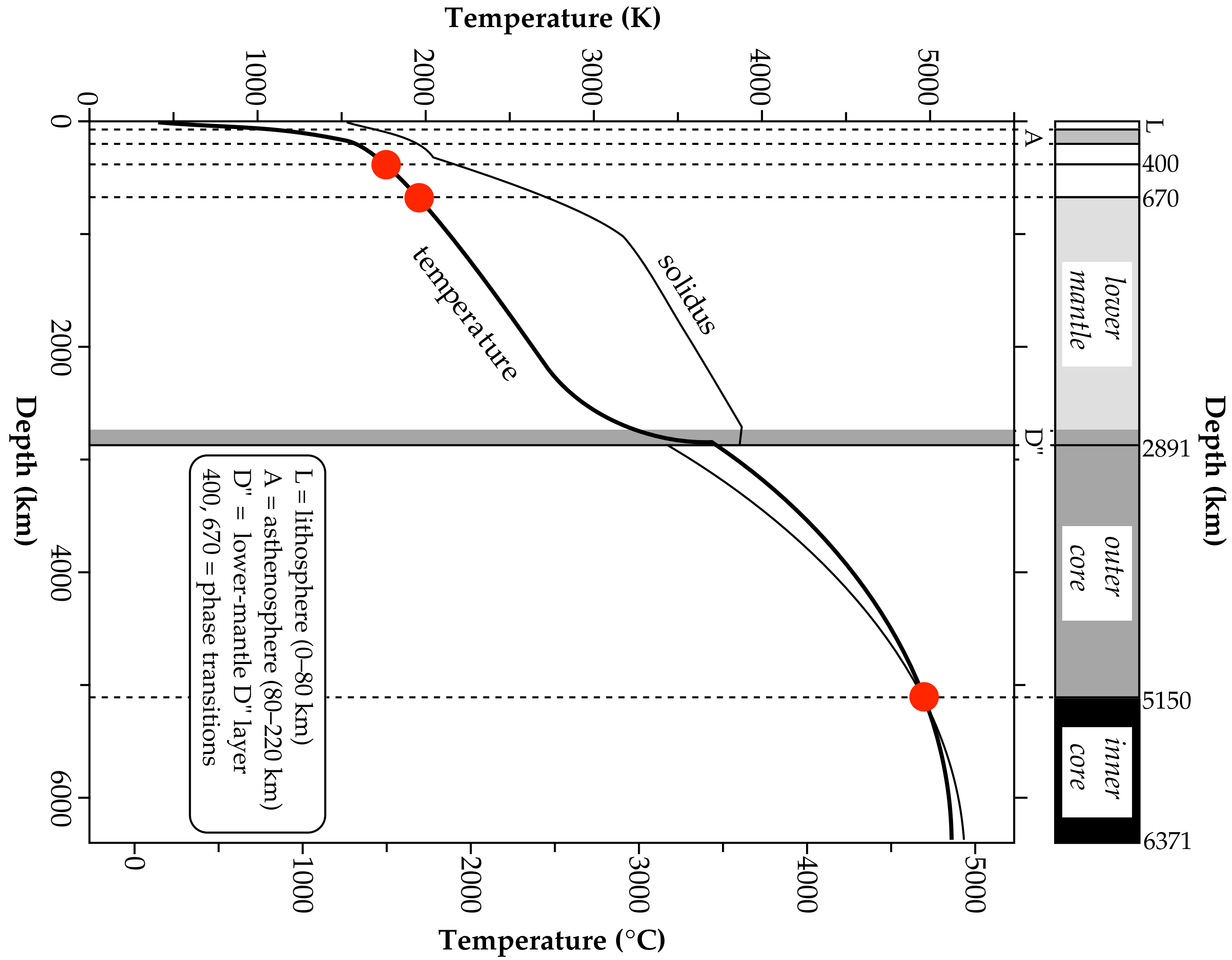
Clapeyron slope (phase equilibrium)

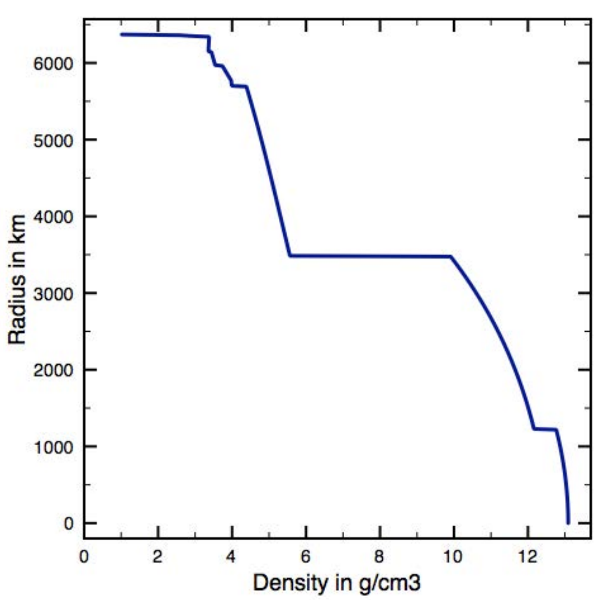


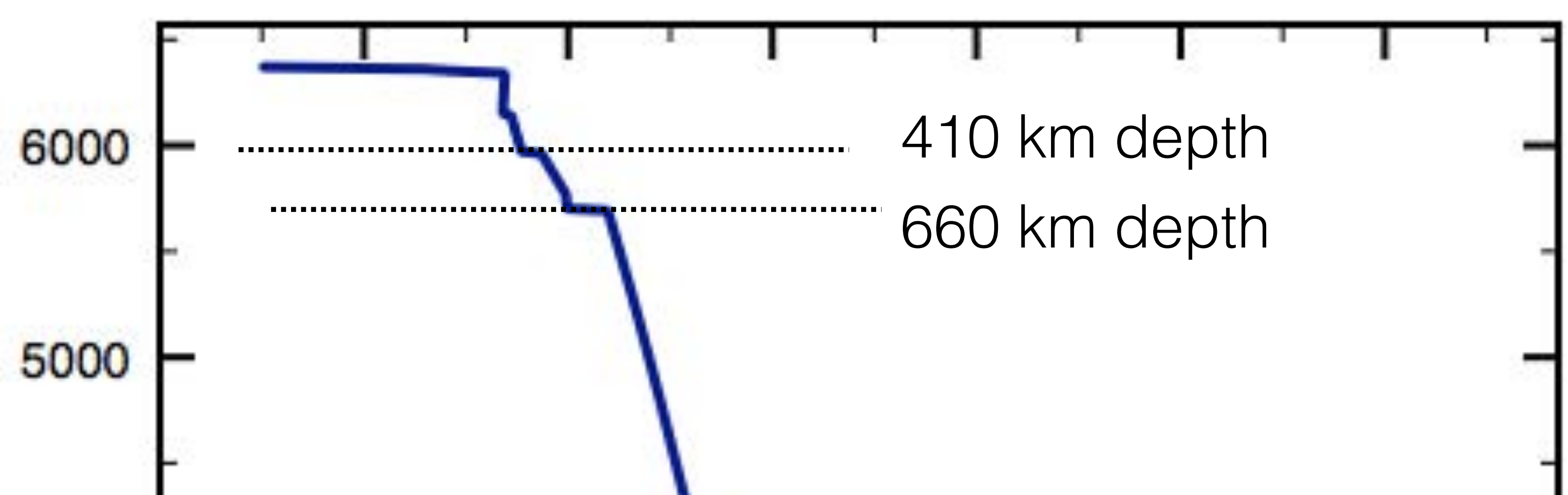


Anchor points for geotherm

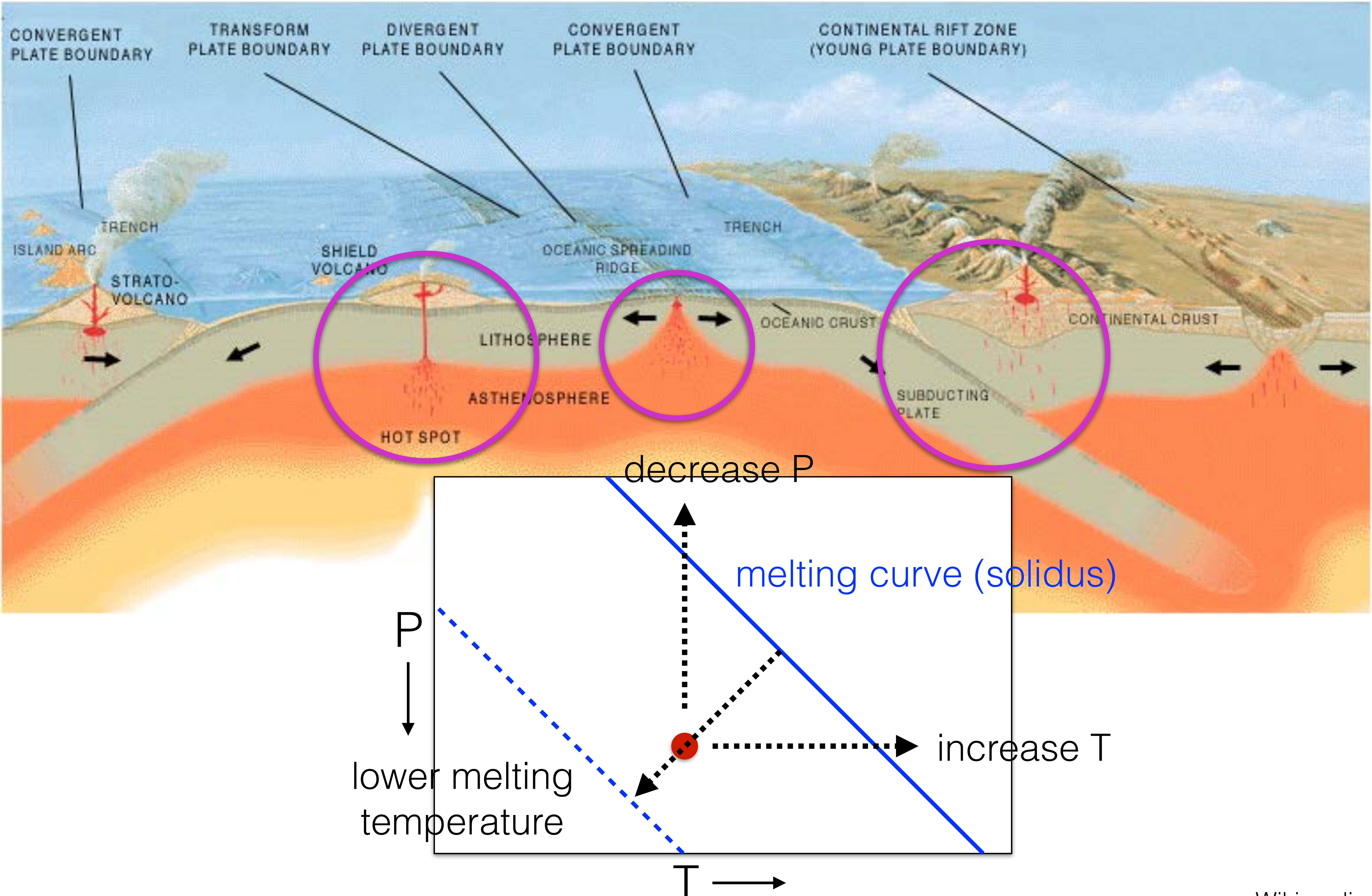






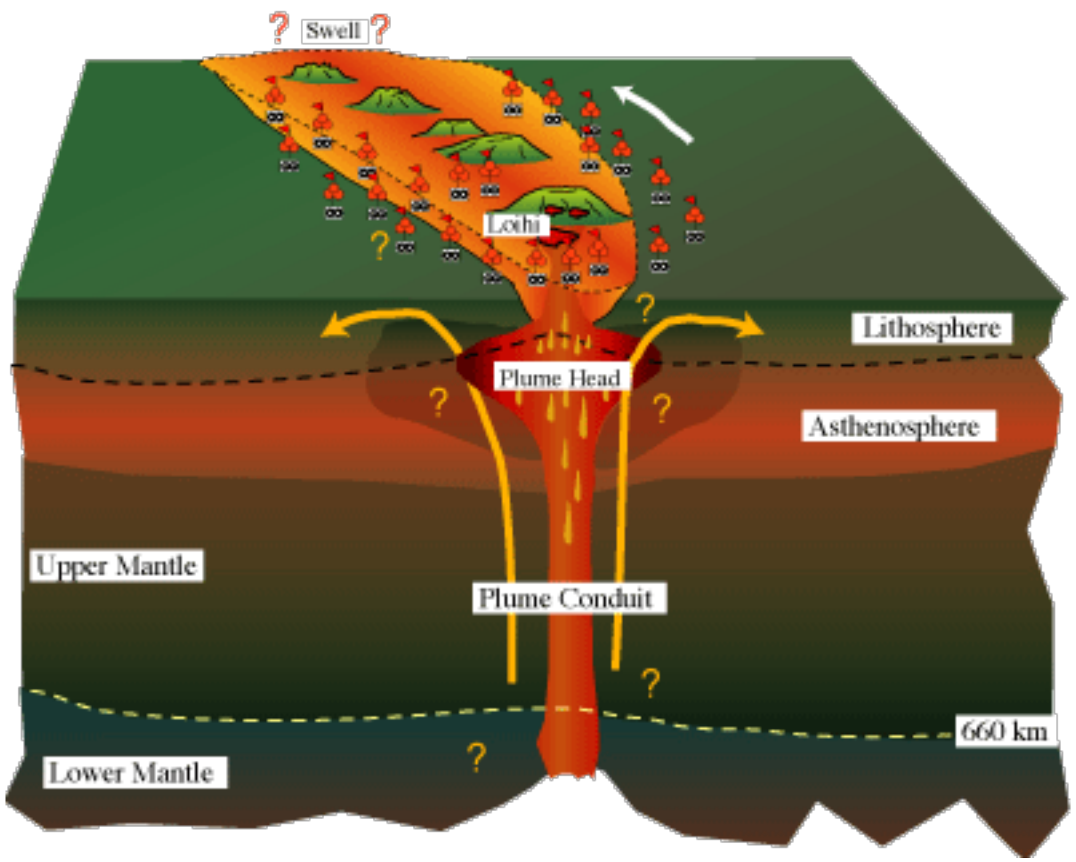


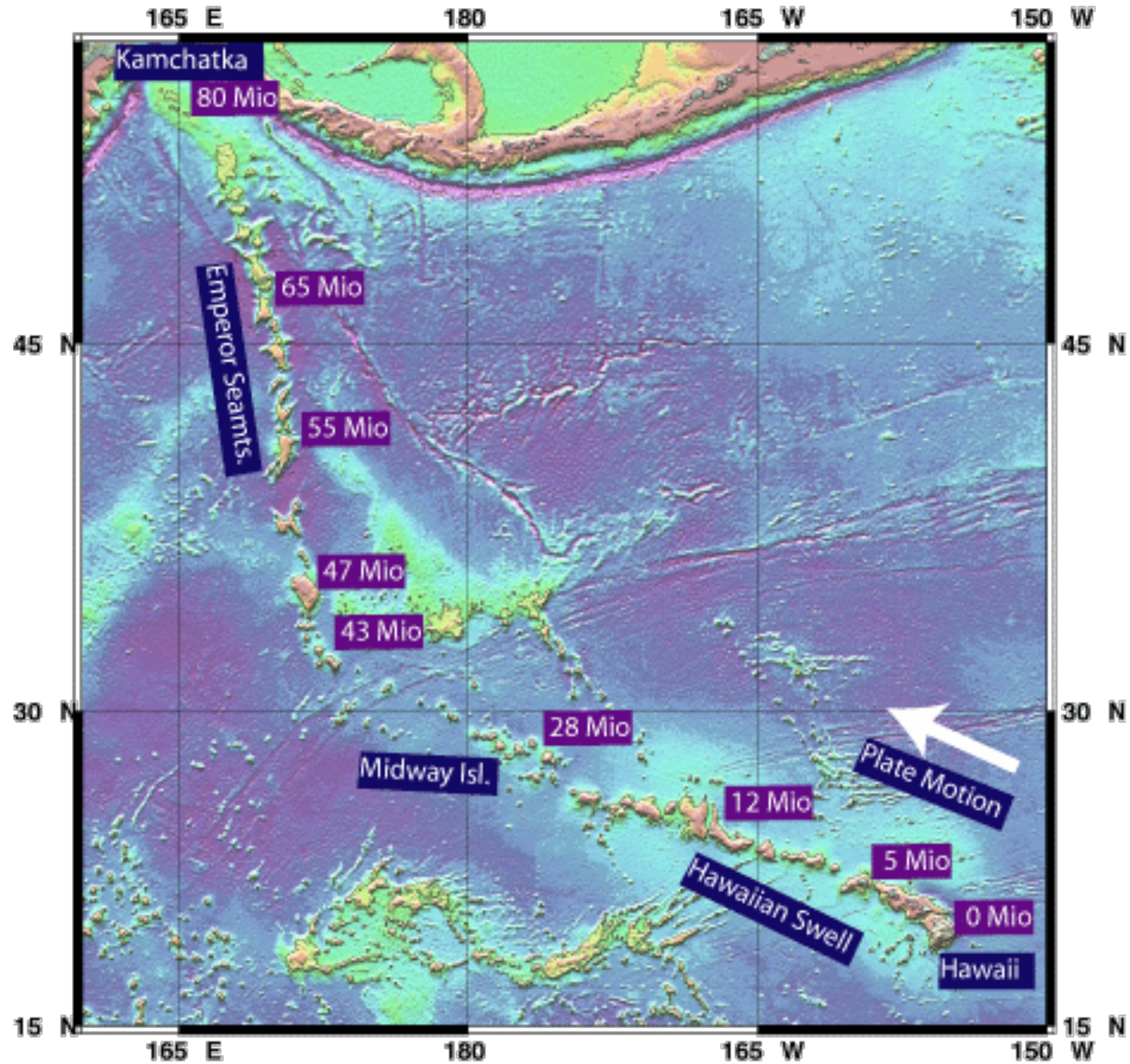
Partial melting in the shallow mantle



Wikimedia

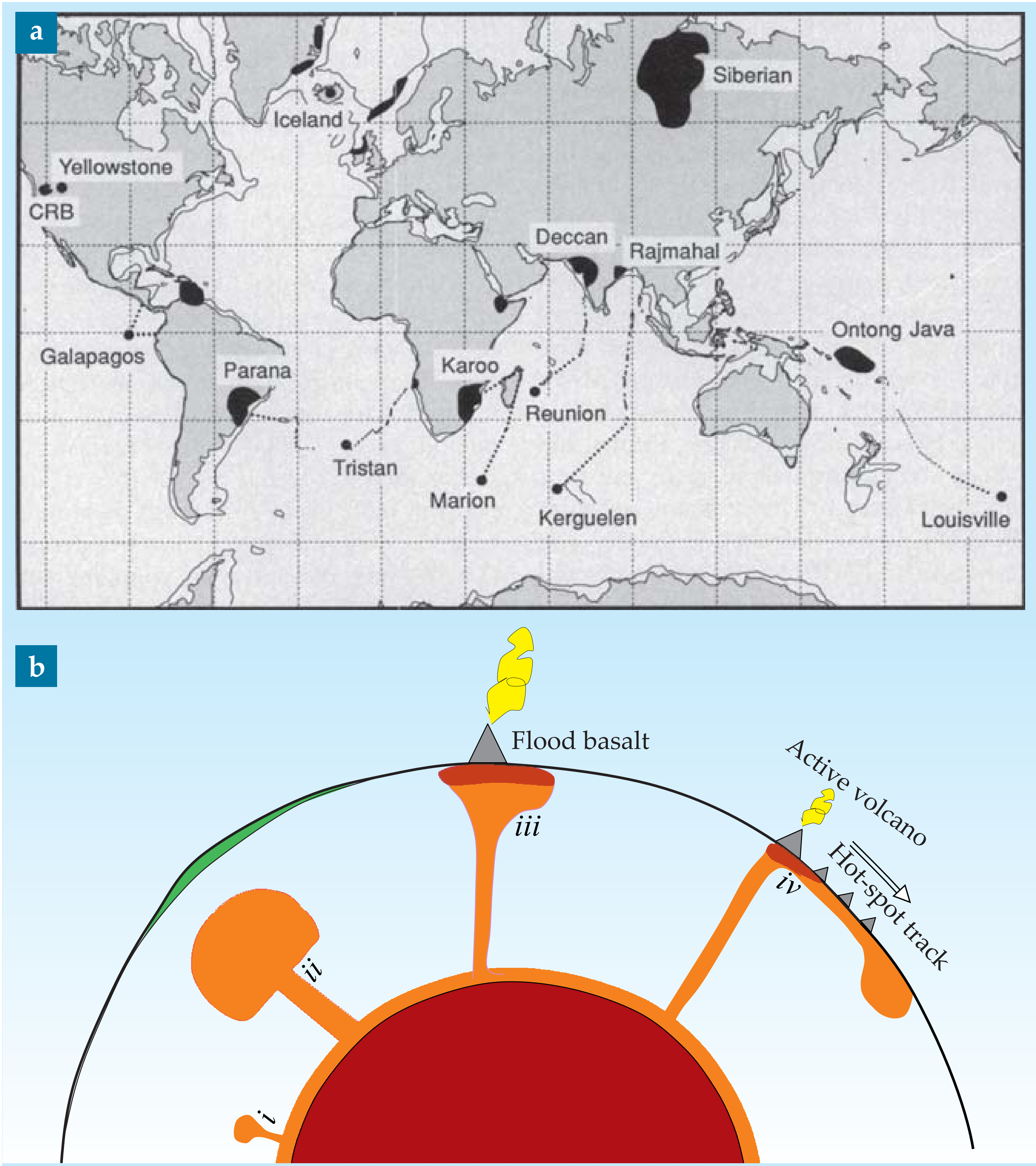
Hot spots: Hawaii

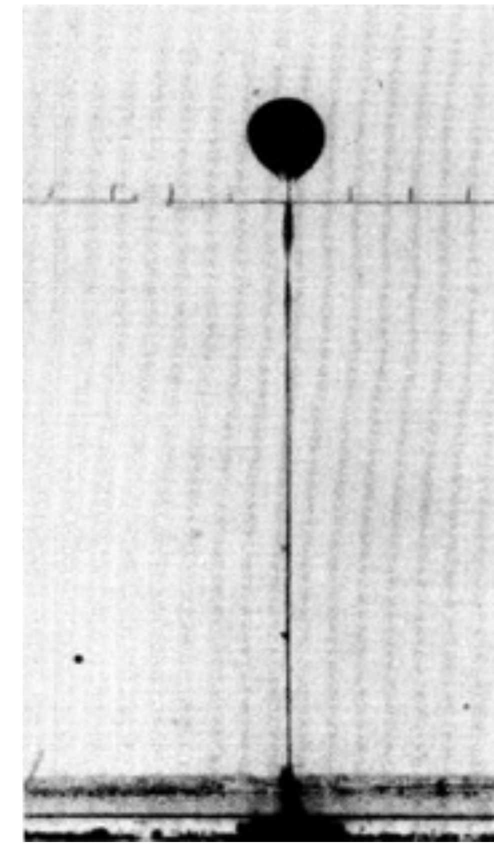




Mantle "plumes"

upwellings of hot material





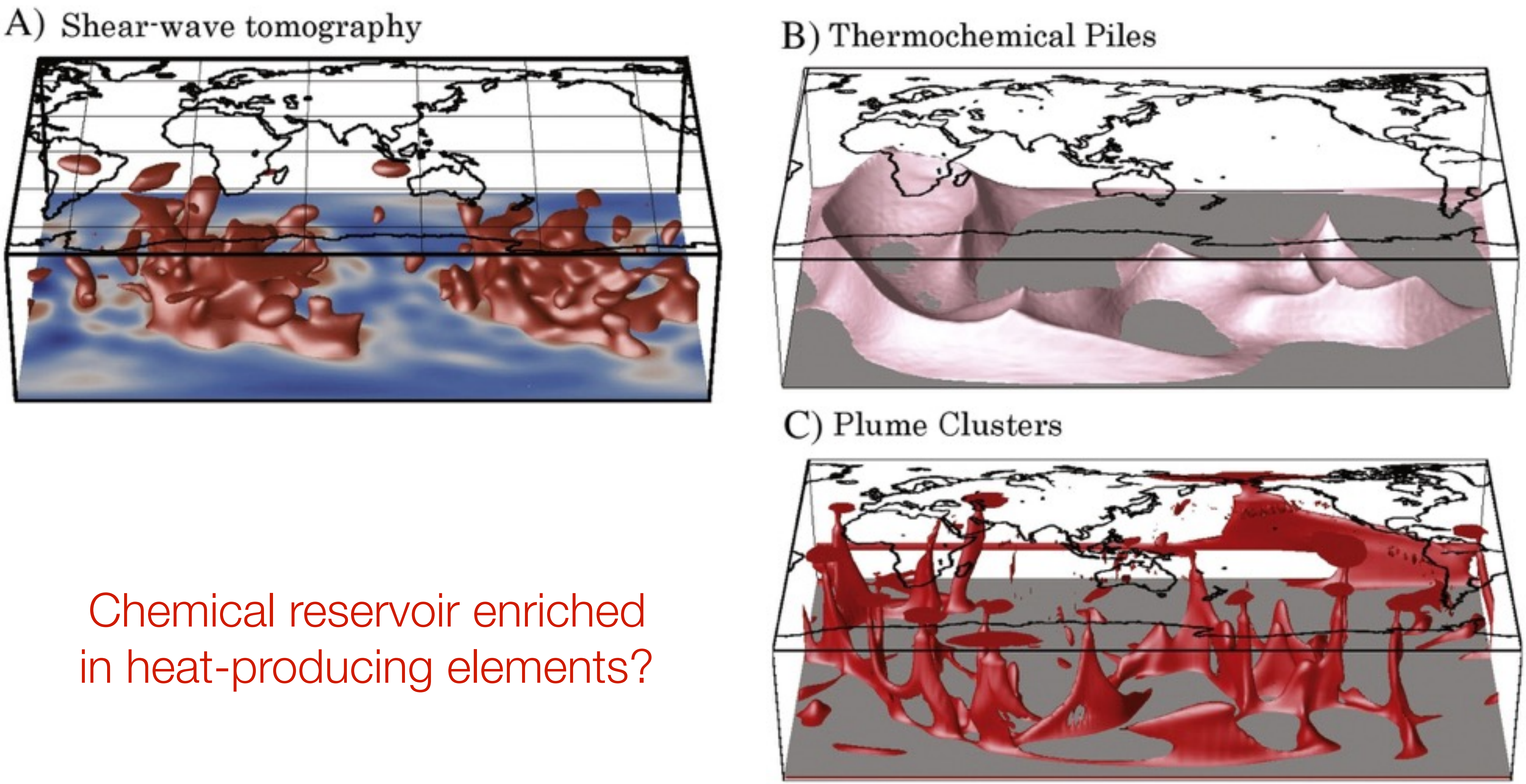
Richards et al. 1987

flood basalts
hot spots

Humphreys & Schmandt 2011

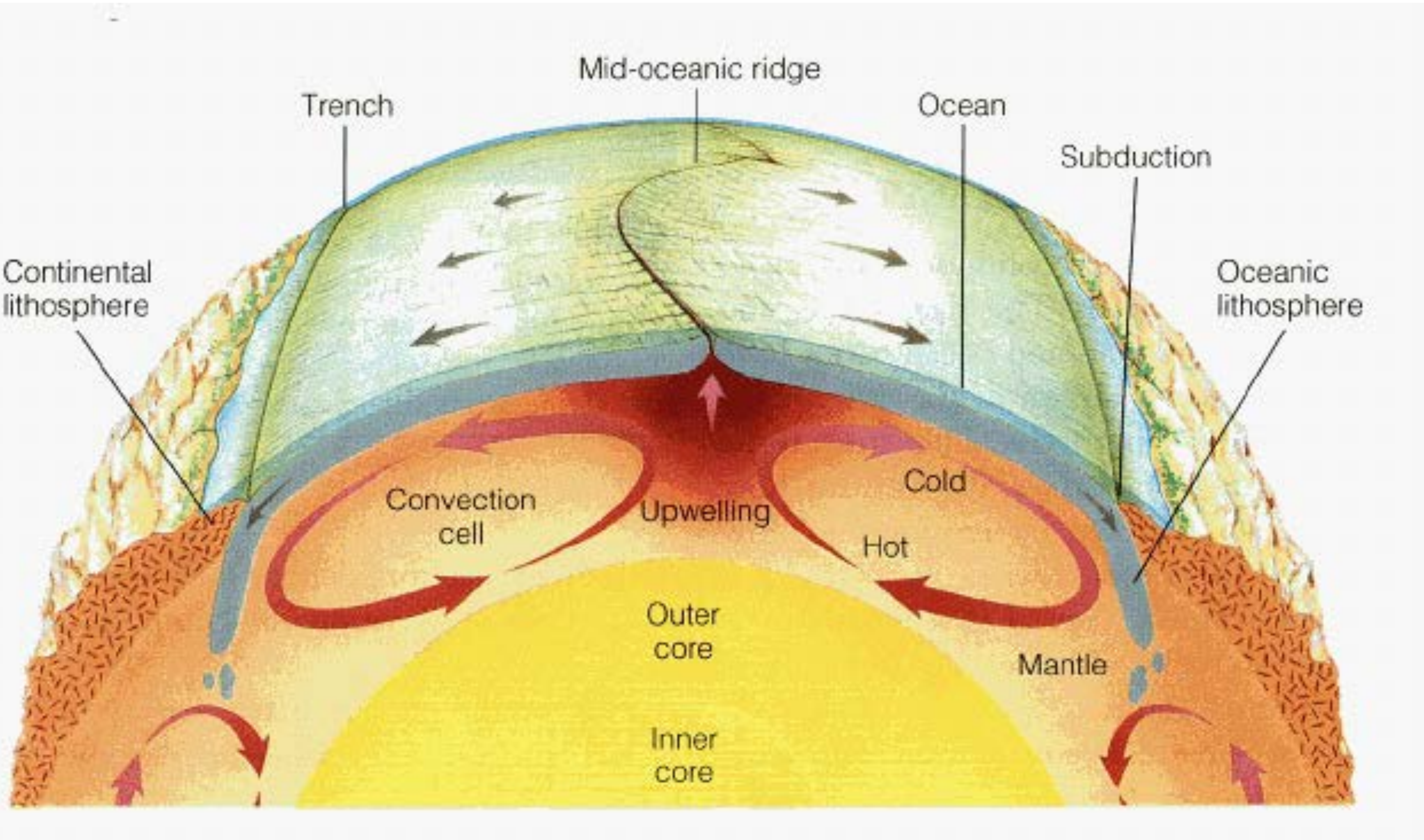
Deep mantle

Layering, chemical reservoirs in the mantle?

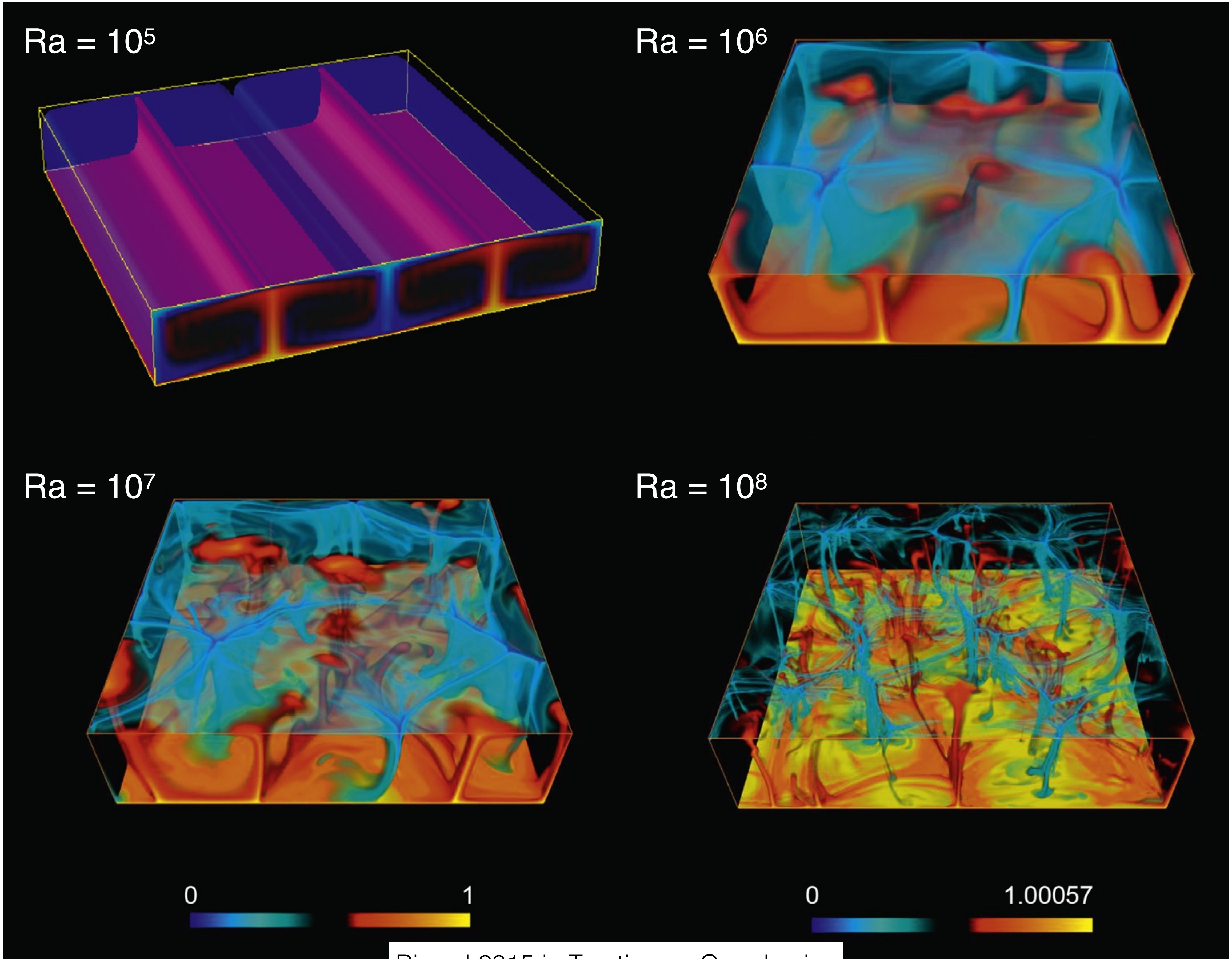


Chemical reservoir enriched
in heat-producing elements?

Lassak et al. 2010

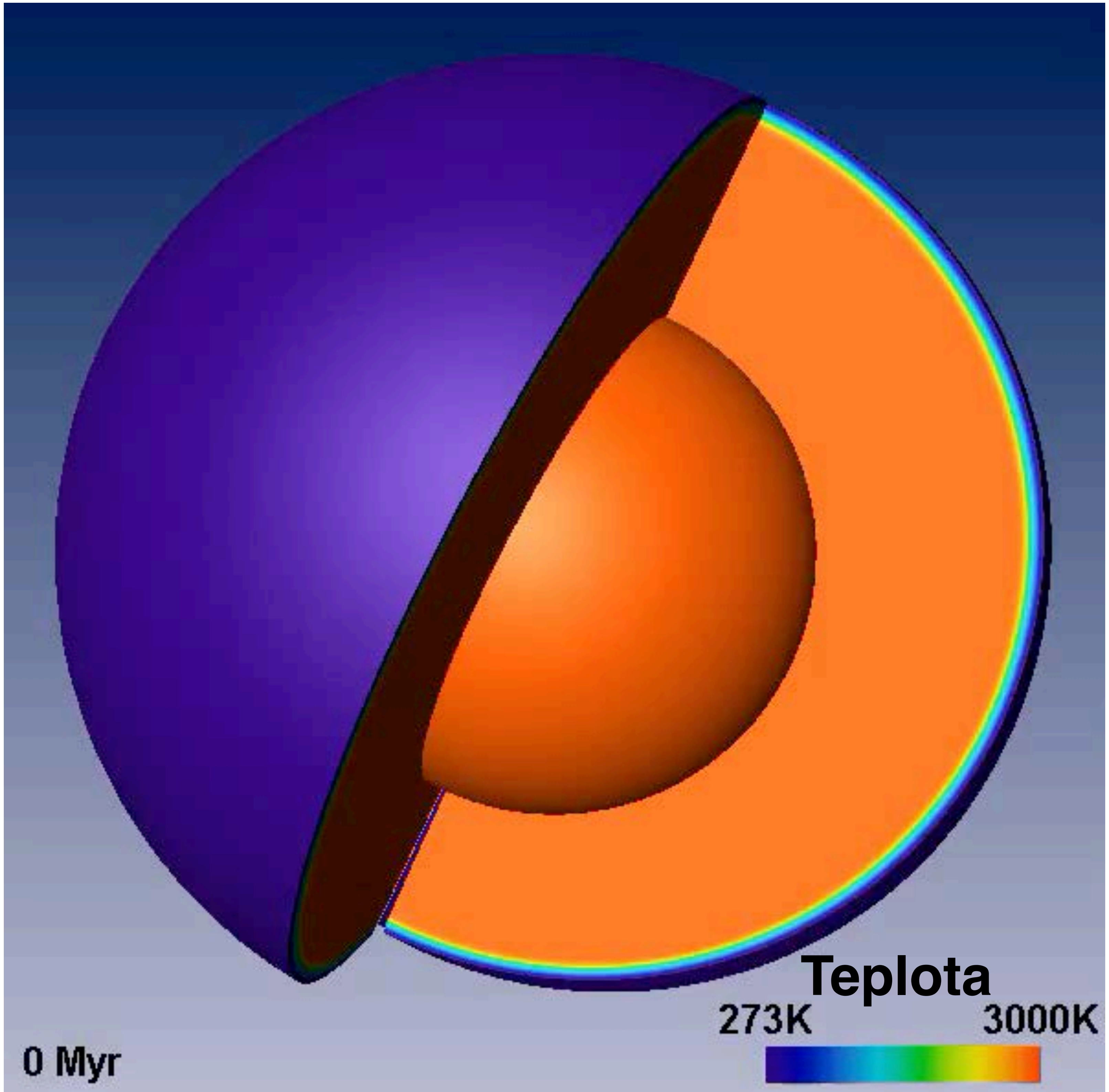

Mid-oceanic ridge
Trench
Ocean
Subduction
Continental lithosphere
Oceanic lithosphere
Convection cell
Upwelling
Cold
Hot
Outer core
Mantle
Inner core

Increasing the Rayleigh number



Ricard 2015 in Treatise on Geophysics

Numerical modeling of mantle convection



from Hana Čížková

Thermal convection:
hot material rises
cold material sinks

Solving equations of conservation laws numerically