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Gran Sasso Science Institute
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Neutrino Physics and Detection Methods



Neutrino Physics and Detection Methods

4 hours of lectures, what will not be covered:

- history of the neutrino (Pauli proposal, Fermi theory, Reines and Cowan first detection) – skip!
- discovery of neutrino flavours (Standard Model lepton flavour structure) – skip!
- discovery of neutrino oscillations – skip!
- particle physicists and geologists (after the first two days) already know about neutrinos 😊

I am assuming you know the above and/or can read up about this at your leisure (*see e.g. 2015 Nobel Prize in Physics; also 1995, 1988, 2002 Nobel Prizes in Physics*).

Neutrino Physics and Detection Methods

what will be covered:

- understanding neutrino oscillations
- neutrino oscillation matter effects
- the important question of Majorana versus Dirac nature of the neutrino – double beta decay
- objectives of the current, global neutrino experimental program
- if time permits (advanced): CP violation in the neutrino sector
- neutrino detection methods for “lower energy” neutrinos, highlighting some of the experiments that used the detection techniques

Why Neutrino Physics?

- helps the geo neutrino hunters know what neutrino physicists are doing
 - What is the motivation for all our efforts building these giant neutrino detectors?
- massive neutrinos: the *only confirmed* physics beyond the Standard Model

Chart of Elementary Particles

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

S

Leptons spin = 1/2

Flavor	Mass GeV/c ²	Electric charge
ν_e electron neutrino	$<1 \times 10^{-8}$	0
e electron	0.000511	-1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	-1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	-1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.003	2/3
d down	0.006	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	175	2/3
b bottom	4.3	-1/3

force carriers
spin = 0, 1, 2, ...

Name	Mass GeV/c ²	Electric charge
g gluon	0	0

Color Charge

A quark carries one of three types of "color charge," also called "color charge." Color charges have nothing to do with the colors of visible light. There are eight possible colors of color charge for quarks. Just as electrons in strong interactions color-charged particles and W and Z bosons have no strong

are involved in color-neutral particles called hadrons. Multiple exchanges of gluons among the quarks and gluons move apart, the energy is eventually converted into additional quarks and antiquarks then combine into types of hadrons have been observed in

neutrons to form nuclei is due to residual strong interactions. It is similar to the residual electromagnetic forces to form molecules. It can also be observed in

Mesons $q\bar{q}$

Mesons are bosonic hadrons. There are about 140 types of mesons.

Name	Quark content	Electric charge	Mass GeV/c ²	Spin
π^+	$u\bar{d}$	+1	0.140	0
π^0	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	0	0.135	0
π^-	$d\bar{u}$	-1	0.135	0
K^+	$u\bar{s}$	+1	0.494	0
K^0	$d\bar{s}$	0	0.498	0
K^-	$s\bar{u}$	-1	0.494	0
K^0_S	$\frac{1}{\sqrt{2}}(d\bar{s} + s\bar{d})$	0	0.498	0
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Better Chart!

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c ²	Electric charge
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e electron	0.000511	-1
ν_M middle neutrino*	$(0.009-0.13)\times 10^{-9}$	0
μ muon	0.106	-1
ν_H heaviest neutrino*	$(0.04-0.14)\times 10^{-9}$	0
τ tau	1.777	-1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

is neutron β (beta) decay.

B^0 and B^+ mesons via a virtual Z boson or a virtual photon.

(hidden) dimensions of space?

with ordinary matter?

predicting more than one type of Higgs?

As of today: Oscillation of 3 massive active neutrinos is clearly the dominant effect:

If neutrinos have mass: $|\nu_l\rangle = \sum U_{li} |\nu_i\rangle$

For 3 Active neutrinos.

Experimentalist Slide

Majorana or Dirac?

$$U_{li} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

(Double β decay only)

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_2/2} & 0 \\ 0 & 0 & e^{-i\alpha_3/2+i\delta} \end{pmatrix}$$

Atmospheric, Accel.

CP Violating Phase

Reactor, Accel.

Solar, Reactor

Majorana CP Phases

where $c_{ij} = \cos \theta_{ij}$, and $s_{ij} = \sin \theta_{ij}$

Range defined for $\Delta m_{12}, \Delta m_{23}$

For two neutrino oscillation in a vacuum: (a valid approximation in many cases)

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right)$$

CP Violating Phase or Majorana Phases: Antimatter/matter asymmetry in Early Universe?

As of today: Oscillation of 3 massive active neutrinos is clearly the dominant effect:

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For 3 Active neutrinos.

Theorist Slide

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Pontecorvo-Maki-Nakagawa-Sakata matrix

(Double β decay only)

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_2/2} & 0 \\ 0 & 0 & e^{-i\alpha_3/2+i\delta} \end{pmatrix}$$

Atmospheric, Accel.

CP Violating Phase

Reactor, Accel.

Solar, Reactor

Majorana CP Phases

where $c_{ij} = \cos \theta_{ij}$, and $s_{ij} = \sin \theta_{ij}$

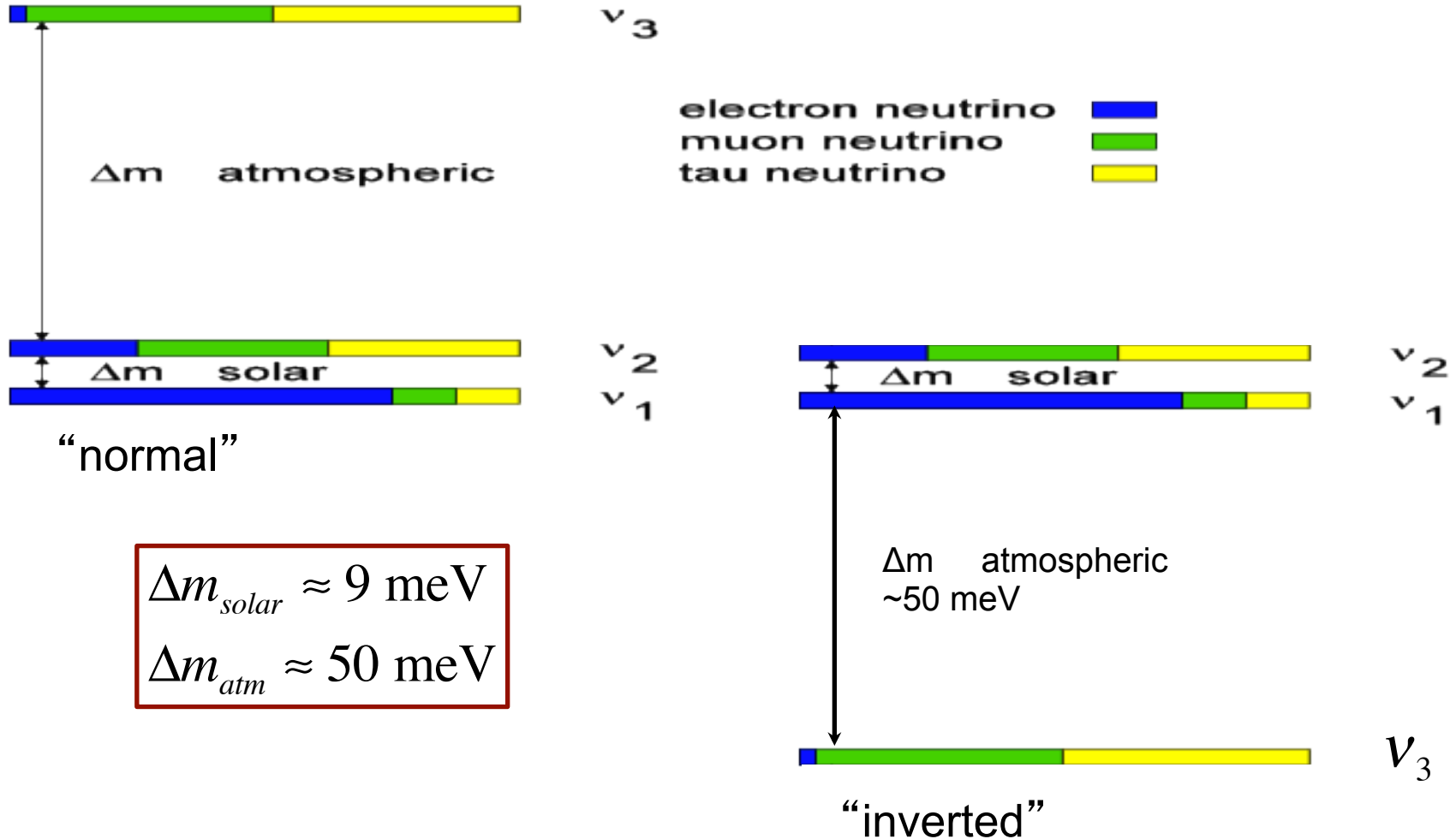
Range defined for $\Delta m_{12}, \Delta m_{23}$

For two neutrino oscillation in a vacuum: (a valid approximation in many cases)

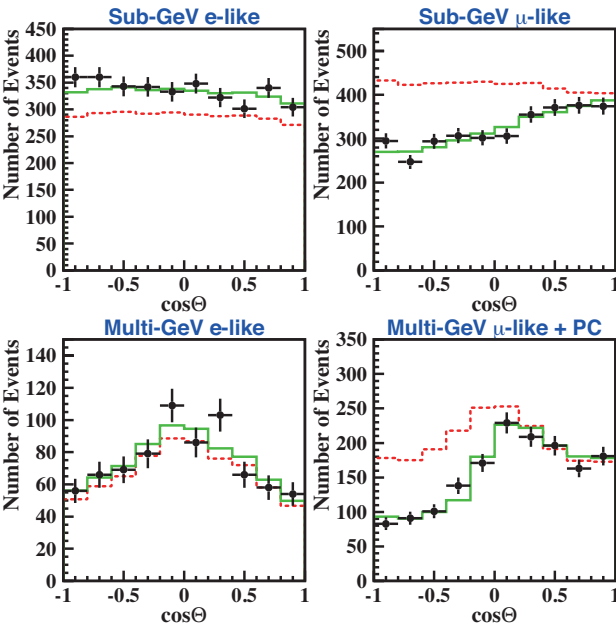
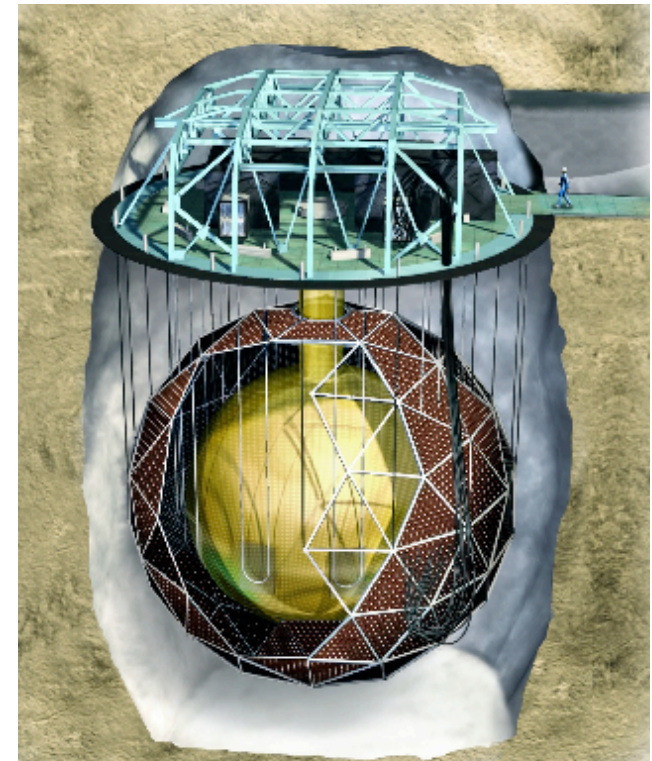
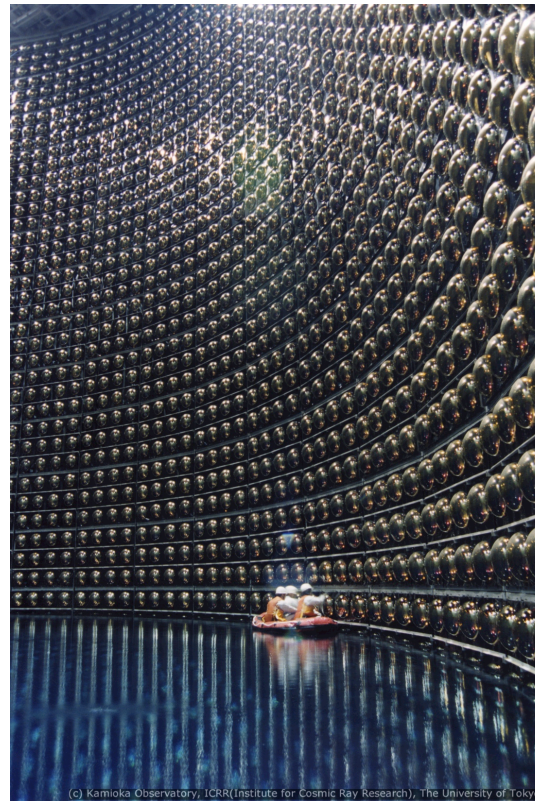
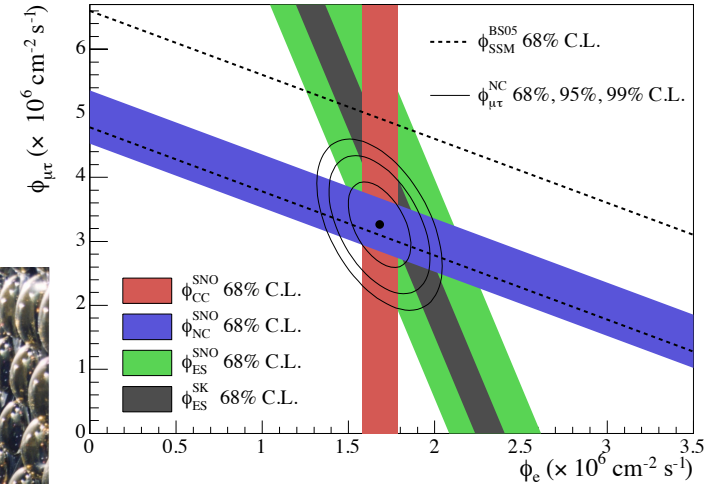
$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right)$$

CP Violating Phase or Majorana Phases: Antimatter/matter asymmetry in Early Universe?

Neutrino Mass Hierarchy



Neutrinos Oscillate *thus they have mass*



- flux of atmospheric muon neutrinos produced by cosmic rays is not up-down symmetric
- solar neutrinos produced as electron neutrinos in the Sun are detected by SNO as other flavours (ν_μ , ν_τ)

To be complete...

- we've also seen the disappearance of reactor antineutrinos due to oscillations at long baselines (~ 180 km) and short baselines (~ 1 km)
- we've also seen the disappearance of accelerator-produced beams of ν_μ and also their appearance downstream as ν_e and ν_τ

We know neutrinos oscillate – they can change flavour as they propagate!

Neutrino Oscillations

- flavour eigenstates and mass eigenstates mix in the lepton sector, like the quarks do

$$\nu_f = \sum_i U_{fi} \nu_i$$

simplified expressions for two-flavour mixing:

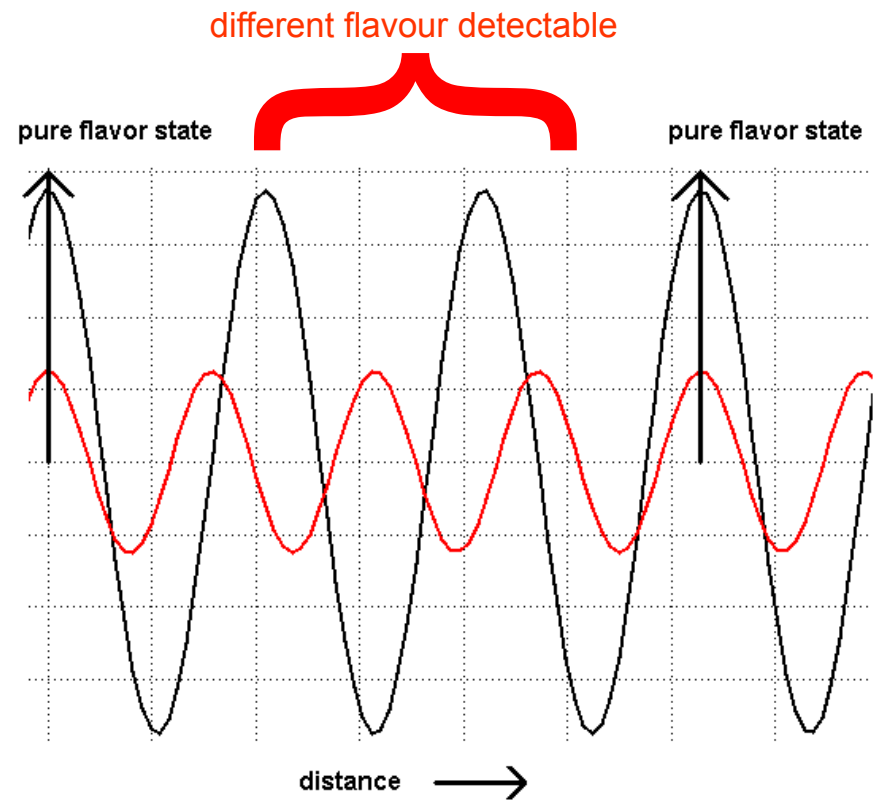
$$\nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta$$

$$\nu_\mu = -\nu_1 \sin \theta + \nu_2 \cos \theta$$

$$P_{e\mu} = \sin^2 2\theta \sin^2 \frac{1.267 \Delta m^2 L}{E}$$

Δm^2 in [eV²], E in [MeV], L in [m]

where $\Delta m^2 = m_2^2 - m_1^2$



Characteristic Oscillation Length

$$P_{e\mu} = \sin^2 2\theta \sin^2 \frac{1.267 \Delta m^2 L}{E}$$

$$\frac{1.267 \Delta m^2 L_{osc}}{E} = \pi$$

$$L_{osc} [\text{m}] = \frac{\pi}{1.267} \frac{E [\text{MeV}]}{\Delta m^2 [\text{eV}^2]}$$

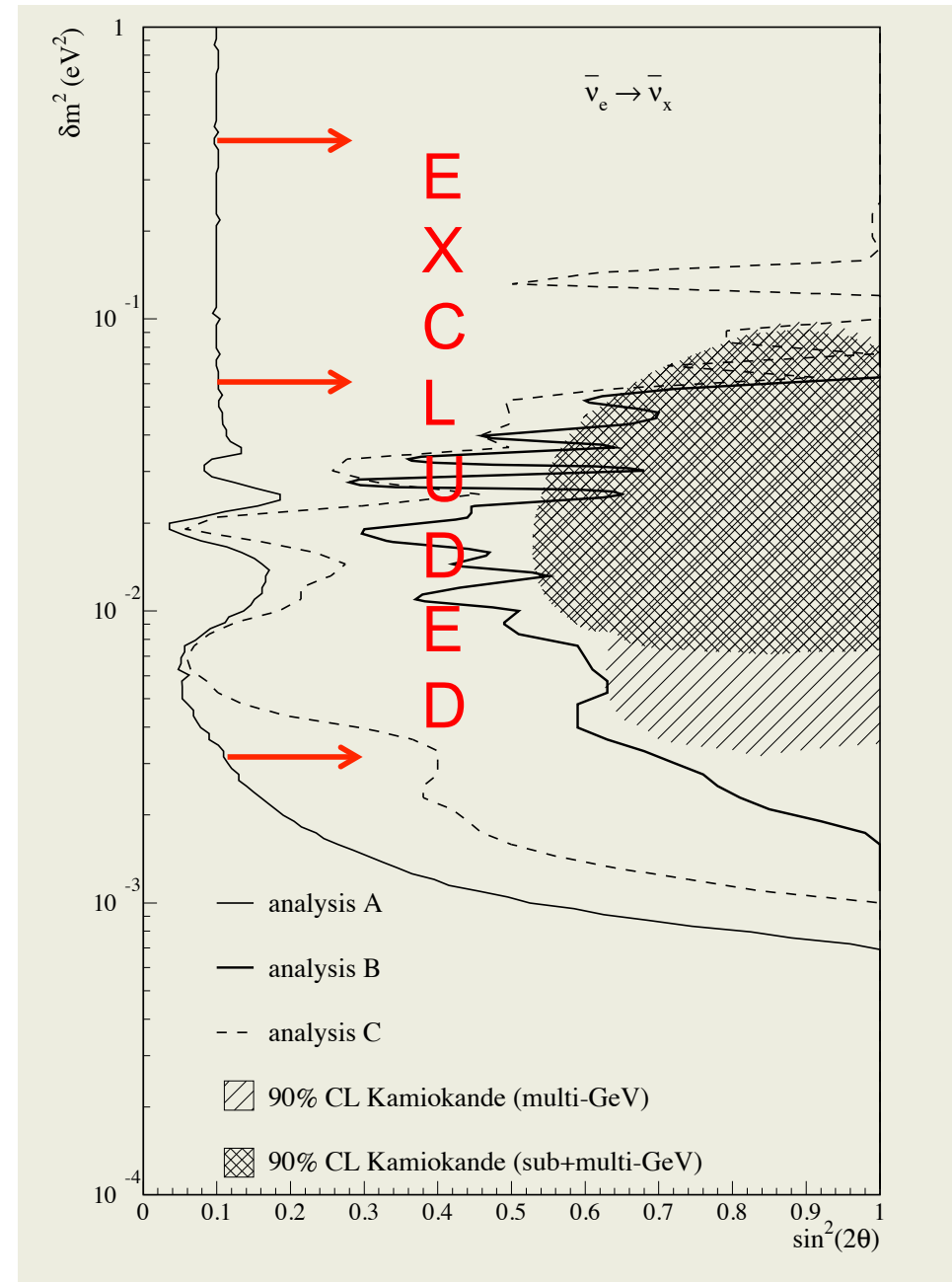
calculate a few of these for yourself: KamLAND reactor neutrinos, T2K/NOvA long baseline GeV neutrinos, Daya Bay reactor neutrinos, JUNO reactor neutrinos

$$E_\nu = 5 \text{ MeV} \\ \Delta m^2 = 7.5\text{e-}5 \text{ eV}^2$$

$$E_\nu = 2 \text{ GeV} \\ \Delta m^2 = 2.4\text{e-}3 \text{ eV}^2$$

Typical 2- ν Oscillation Result

CHOOZ
Reactor $\bar{\nu}_e$
Disappearance



Neutrino Oscillations are Puzzling

- So...an electron neutrino is produced but then propagates with different mass eigenstates(??) and then can change from one flavour to another while propagating(!!)...
- How does it do this?
- How does “the neutrino” have different masses? And it propagates as though it has different masses??? The lighter mass component travels a little bit faster???
- What is the meaning of the “mass of the muon neutrino”?
- How does the flavour change while propagating? [mathematics and quantum mechanics tell us, but can we really understand this?]
- How do I make any sense of this at all?

The following slides will try to help you really understand neutrino oscillations...

Schrödinger's Cat

- Neutrino oscillations are like Schrödinger's Cat™

Hmm...
wait, I get it!

The neutrino wavefunction is simultaneously ν_1 and ν_2 as it propagates!



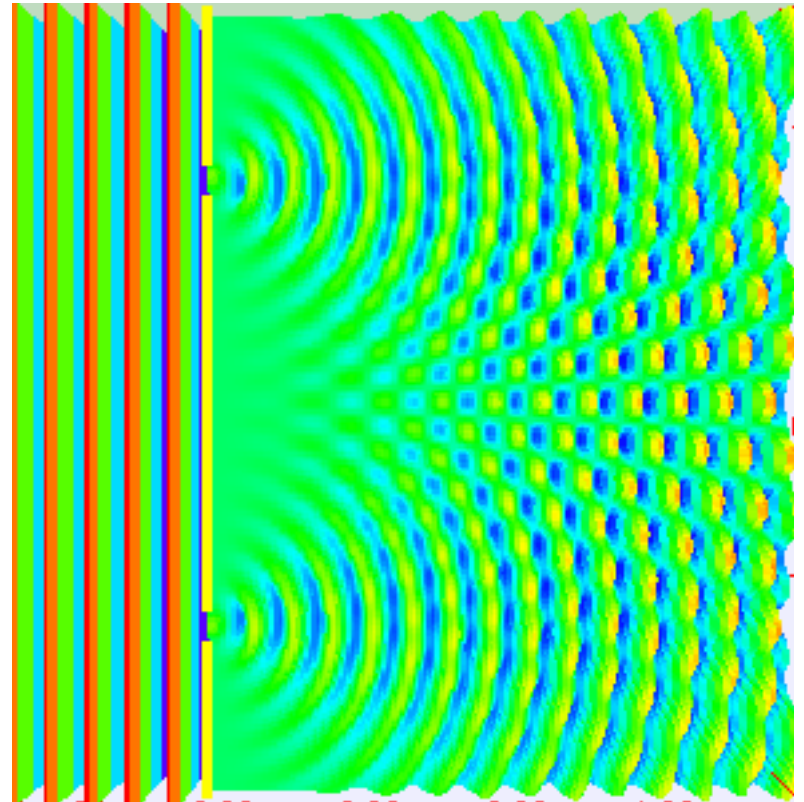
Young's Two-Slit Experiment

- Neutrino oscillations are like the two-slit experiment!

If I measure which neutrino mass eigenstate was produced, I will get a “single-slit pattern”.

If I don't measure which mass eigenstate was emitted in the charged-current reaction, both are involved and I will get a “two-slit” interference pattern.

That's neutrino oscillations!



Quark Mixing

- CKM – Cabibbo-Kobayashi-Maskawa matrix describes quark flavour mixing
 - we think of this slightly differently than we usually do for leptons

from Wikipedia

~~into up quarks ($|V_{ud}|^2$ and $|V_{us}|^2$ respectively)~~. In particle physics parlance, the object that couples to the up quark via charged-current weak interaction is a superposition of down-type quarks, here denoted by d' .^[4] Mathematically this is:

$$d' = V_{ud}d + V_{us}s,$$

or using the Cabibbo angle:

$$d' = \cos \theta_c d + \sin \theta_c s.$$

Charged-Current Interactions with Quarks

- top quarks often decay to bottom quarks, sometimes to strange quarks, very occasionally to down quarks
 - *nobody has a problem with this!*
- bottom quarks decay (undergo charged-current interactions that transform them) into charm quarks or sometimes up quarks
 - *nobody has a problem with this!*

Translate Neutrino Interactions into Quark Language

- muons decay (undergo charged-current interactions) sometimes into ν_1 , sometimes to ν_2 , and sometimes to ν_3
- if we have a ν_2 state propagating, it can undergo a charged-current interaction that could transform it into an electron, muon (or a tau, if energetic enough)

Perfectly analogous!

Why Oscillations?

- If we don't know whether it is a ν_2 state or a ν_1 state that is propagating, we have to consider that it is both, mixed as appropriate for the way the states were produced, coherent if produced that way, and propagating with different phases for the mass eigenstates, interfering with each other.
- The combination ν_2 state and ν_1 state can undergo a charged-current interaction transforming it into an electron, muon, or tau...depending on the coherent superposition of the possibilities for each of the ν_2 state and ν_1 state (which depends on their phases at that instant).
- *It takes some words to say correctly...but, if you understand the above, you've understood neutrino oscillations completely!*

So, the Next Time Somebody Asks You...

- why do neutrinos oscillate?
- what is the mass of the muon neutrino?
- why don't electrons and muons “oscillate”?
- why don't quarks “oscillate”?
 - *or do they?*
- *...you will be able to answer!*

Three-Flavour Neutrino Oscillations (in vacuum, plane-wave model)

- I was going to write this on the chalkboard...
- then, thought I'd LaTeX it up for PowerPoint...
- then, decided, let's just cut and paste from [Giunti](#) and cite him

$$|\nu_k(x, t)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \implies |\nu_\alpha(x, t)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} |\nu_k\rangle$$

$$|\nu_\alpha(x, t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(x, t)} |\nu_\beta\rangle$$

$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle$

Transition Probability

$$P_{\nu_\alpha \rightarrow \nu_\beta}(x, t) = |\langle \nu_\beta | \nu_\alpha(x, t) \rangle|^2 = |\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(x, t)|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right|^2$$

Three-Flavour Oscillations, cont'd

ultrarelativistic neutrinos $\Rightarrow t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$\begin{aligned}
P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) &= \left| \sum_k U_{\alpha k}^* e^{-im_k^2 L/2E} U_{\beta k} \right|^2 \\
&= \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 \quad \leftarrow \text{constant term} \\
&\quad + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \quad \leftarrow \text{oscillating term} \\
&\hspace{15em} \updownarrow \\
&\hspace{15em} \text{coherence}
\end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Neutrinos and Antineutrinos

antineutrinos are described by CP-conjugated fields:

C \Rightarrow Particle \Leftrightarrow Antiparticle

P \Rightarrow Left-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS

ANTINEUTRINOS

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}(L, E) = \delta_{\alpha\beta} + 2\text{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

have to get this exactly right if we are going to go after CP violation in the neutrino sector

PMNS Neutrino Mixing Matrix

$$\nu_f = \sum_i U_{fi} \nu_i$$

Pontecorvo, Maki, Nakagawa, Sakata

$N = 3 \implies$ 3 Mixing Angles 1 Dirac Phase 2 Majorana Phases

standard parameterization (convenient)

$$(c_{ij} \equiv \cos \vartheta_{ij}, \quad s_{ij} \equiv \sin \vartheta_{ij})$$

$$U = R_{23} W_{13} R_{12} D(\lambda)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Majorana phases

atmospheric,
accelerator

reactor,
accelerator

solar,
KamLAND

Being Pedantic – How Many Phases?

3×3 unitary matrix (complex-valued)

9 unitarity equations

= 9 real parameters or 3 angles and 6 phases

- if neutrinos are Dirac fermions, all but one phase can be rotated away in the definition of the fields
- if neutrinos are Majorana fermions, only 3 phases can be absorbed into the definition of the fields

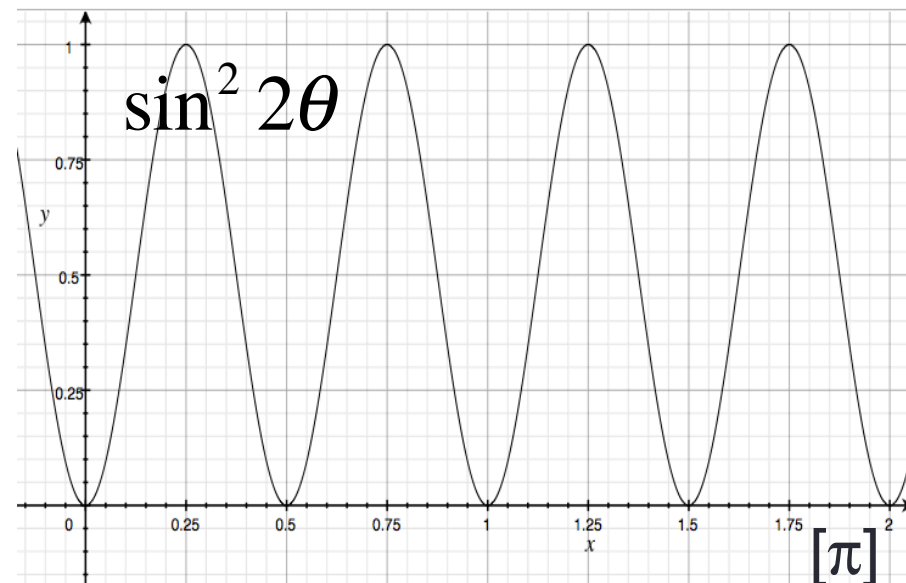
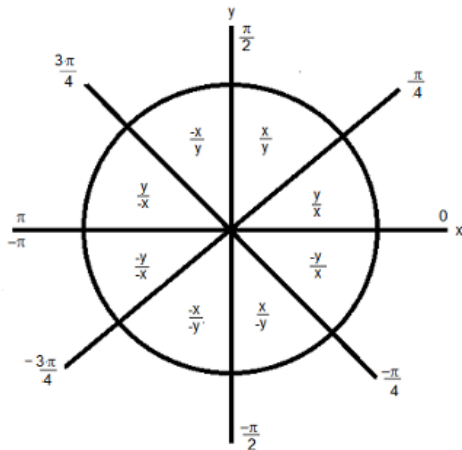
PMNS neutrino mixing matrix, U , therefore has either:

- 3 Majorana phases
- or 1 Dirac phase

Being Pedantic – Octant Degeneracy

- what are the possible values of the PMNS matrix elements, $U_{\alpha k}$?
- construction of the full 3×3 matrix with complex phase is non-trivial...the octant of the angles can (does) matter
- oscillation experiments typically explore $\sin^2 2\theta$, resulting in an octant degeneracy in the $U_{\alpha k}$ rotation angles θ_{12} , θ_{13} , θ_{23}

$$P_{e\mu} = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$



Step Back to 2×2

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- the 2×2 unitary matrix is trivial
- if θ is negative (between π and 2π), $\cos \theta$ stays the same and $\sin \theta \rightarrow -\sin(-\theta)$, so we can map it to the positive angle and the matrix is just the transpose (no effect on oscillations)
- if $\theta > \pi/2$, $\cos \theta \rightarrow -\cos(-\theta)$, $\sin \theta$ stays the same, so we can map it back to the first quadrant and the matrix is just the transpose, multiplied by -1 (no effect)

Angles, Octants, Mass Hierarchy

- if $\theta > \pi/4$, $\cos\theta \rightarrow \sin(\pi/2-\theta) \rightarrow \sin\theta'$
 $\sin\theta \rightarrow \cos(\pi/2-\theta) \rightarrow \cos\theta'$ and then we could map it back to the **first octant** and it would be the same as **flipping the mass hierarchy** with a relative *phase* of $e^{i\pi} = -1$ between them

No effect on 2-flavour, *vacuum* oscillations...

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \sin\theta' & \cos\theta' \\ -\cos\theta' & \sin\theta' \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

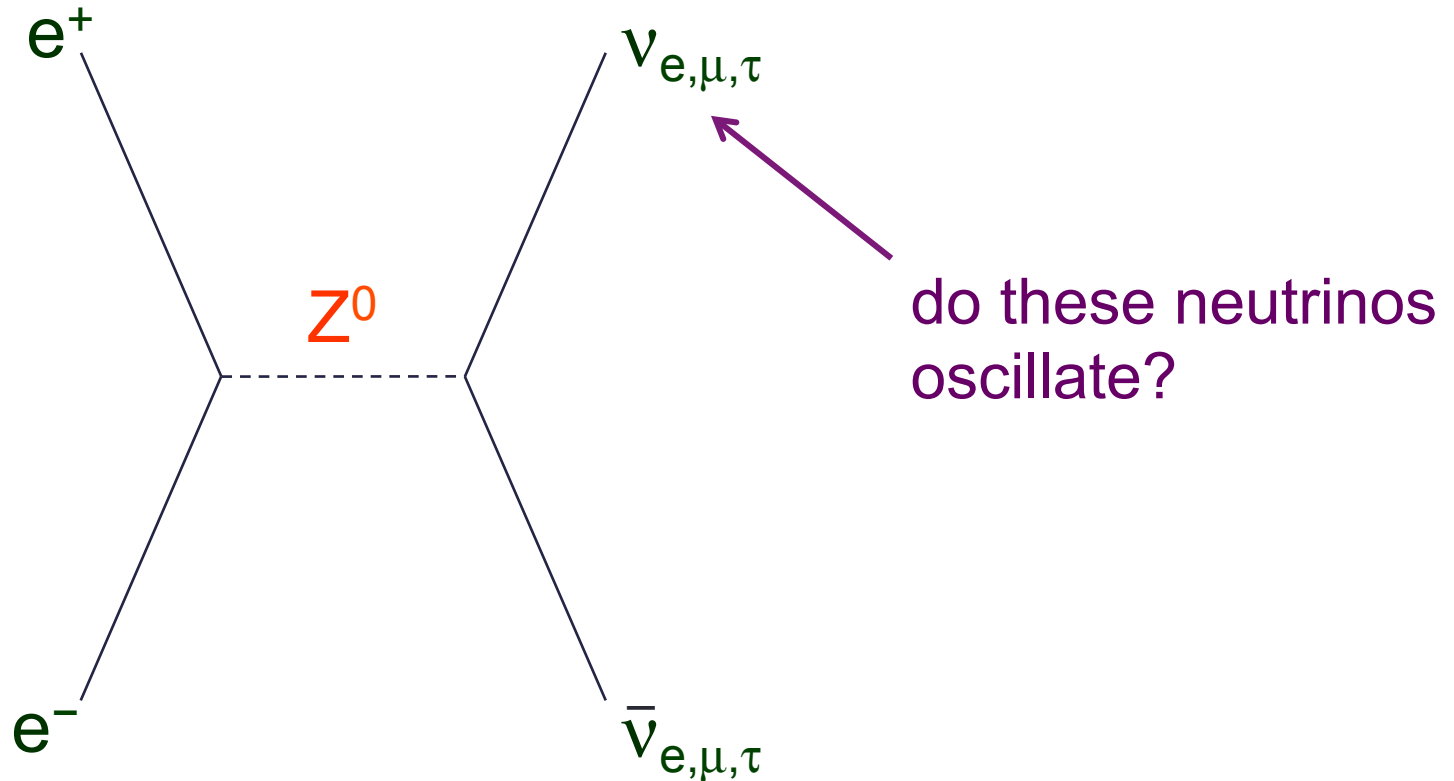
Conclusions

$$P_{e\mu} = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

- for 2-neutrino mixing, the first octant is sufficient for describing vacuum oscillations, without loss of generality
- the second octant is equivalent to flipping the mass hierarchy, which an oscillation experiment *in vacuum* can't determine in any case
 - i.e. the sign of Δm^2 doesn't matter...unless neutrinos propagate in matter! (more on this next)
- once we introduce matter effects, the hierarchy does matter and the second octant is not degenerate with the first
- you hear all the time that the 2-neutrino approximation is a good one (it is, for what we use it for!); but, we live in a 3-neutrino (or more?!) world and the full treatment does matter when we look at more subtle details like CP violation

Neutrino Production by NC

- e.g. supernova neutrinos, thermal production



Think about this for your homework!