



## SUMMER INSTITUTE: USING PARTICLE PHYSICS TO UNDERSTAND AND IMAGE THE EARTH

11-21 July 2016 *Gran Sasso Science Institute*

# Volcano structure and eruption

Giovanni Macedonio

Istituto Nazionale di Geofisica e Vulcanologia  
Osservatorio Vesuviano, Napoli, Italy

L'Aquila, July 11-21, 2016

# Summary

- 1 An example: Mt. St. Helens (Washington, USA)
- 2 Volcano structure
- 3 Eruption processes and modeling (Vesuvius, Etna)
  - Magma chamber and conduit
  - Volcanic eruption columns
  - Ash dispersal
  - Pyroclastic flows
  - Lava flows
- 4 Bibliography

# Mt. St. Helens (Washington, USA) in 1973



(photo National Geographic)

# Mt. St. Helens on April 27, 1980

Look at the strong deformation!



David Johnston USGS 10 km from the crater (died on May 18, 1980)



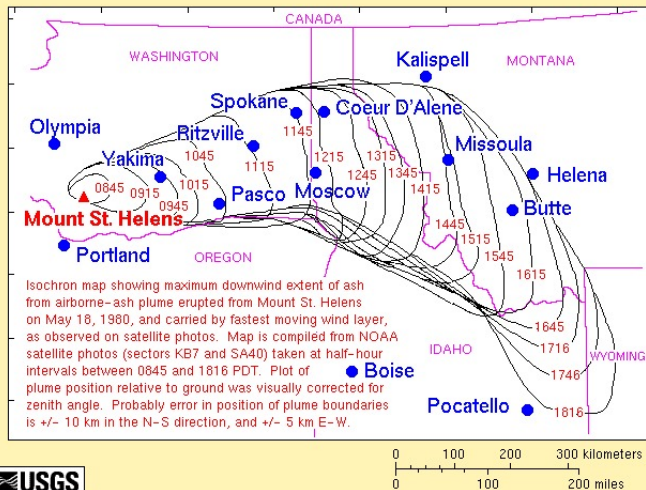
# Mt. St. Helens eruption May 18, 1980

Landslide followed by a lateral blast



# Mt. St. Helens eruption May 18, 1980

## Mount St. Helens May 18, 1980 Ash Plume Path



Topinka, USGS/CVO, 1999, Modified from: Sama-Wojcicki, et al., 1981, IN: USGS Professional Paper 1250

# The eruption of Mt. St. Helens May 18, 1980

Effect of an eruption on the volcanic structure

Before the eruption



After the eruption



## Two years later (May 19, 1982)



(By Lyn Topinka, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=3158771>)

# Types of volcanic eruptions

Merapi, Indonesia, 2010 (ash fallout)



Merapi, 2010 (pyroclastic flows)



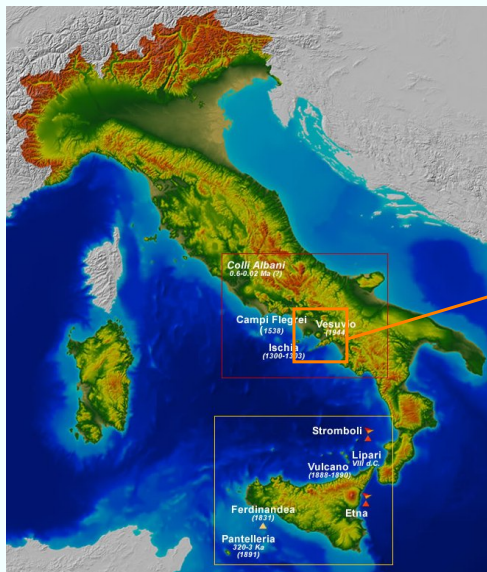
Etna, 2006 (lava flow)



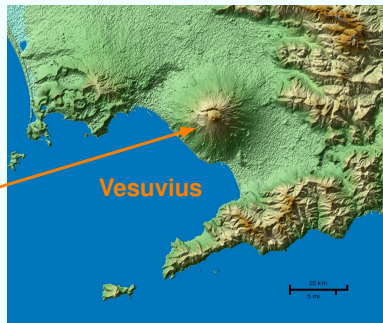
Etna, 2002 (lava flow)



# The structure of a volcano (focus on Vesuvius)



**Vesuvius is an active volcano**



- Last eruption occurred in 1944
- Now the conduit is closed
- Monitored 24/24 by INGV

# Vesuvius and its surroundings

More than 550,000 people live in the “Red Zone”





# Knowledge of the internal structure of a volcano

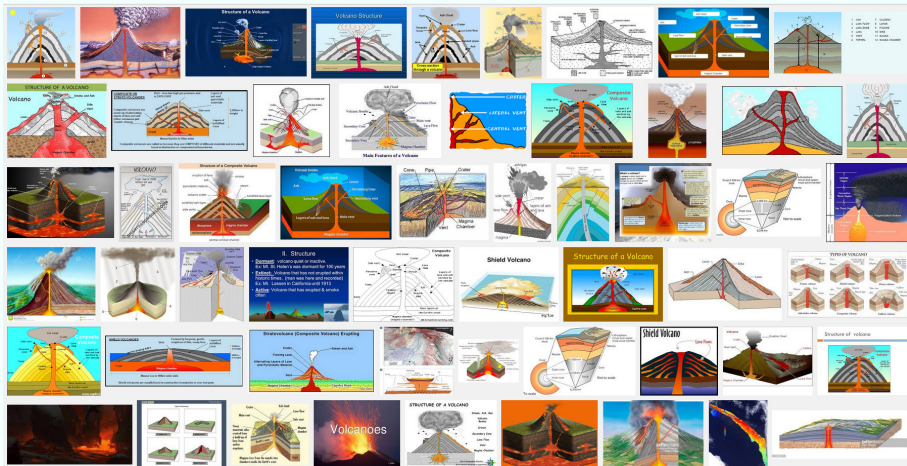
The first studies of Kircher in 1638 (Vesuvius)





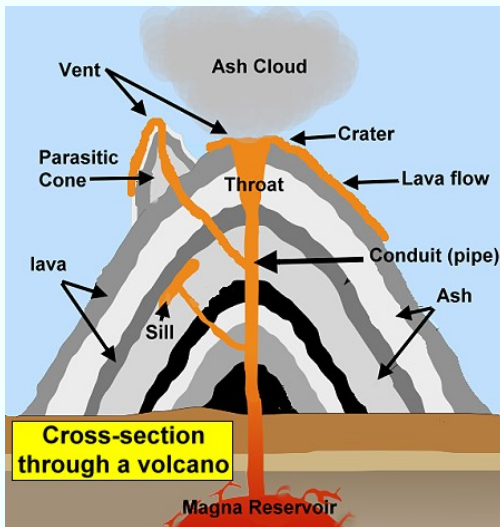
# What is the internal structure of a volcano ?

Images (from google: "internal structure of volcano")



# What is the internal structure of a volcano ?

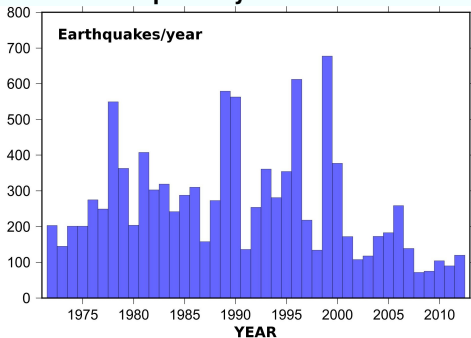
Images (from google: "internal structure of volcano")



# The internal structure of Vesuvius

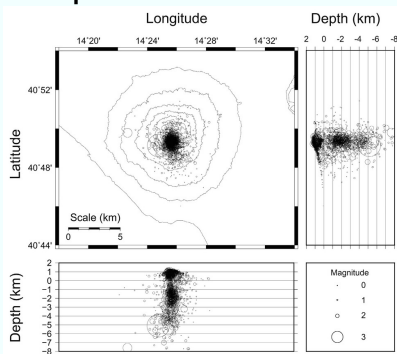
Studies based on the natural seismicity

## Vesuvius earthquakes/year 1971-2012



- Typical magnitude  $M_L < 2.0$
- Largest earthquake  $M_L = 3.6$

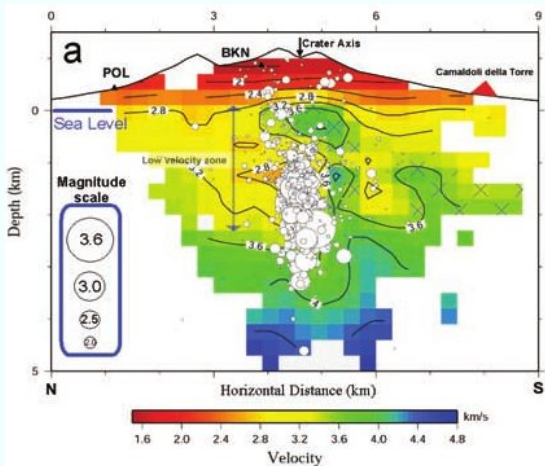
## Earthquakes locations 1999-2012



(from D'Auria et al., Annals of Geophysics, 2013)

# The internal structure of Vesuvius

High resolution passive seismic tomography (300-500 m)



(modified after Scarpa et al., 2002)

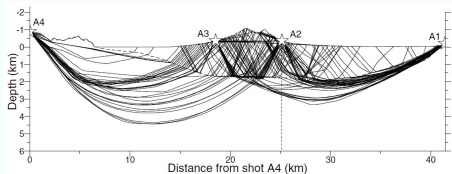
## P-wave velocity structure

- Based on simultaneous inversion of the 3-D velocity structure and the earthquake location
- Data from 2139 earthquakes recorded from minimum 7, up to 19 stations (8600 P-waves, 1900 S-wave readings)
- Magmatic bodies inside the investigated volume were not found

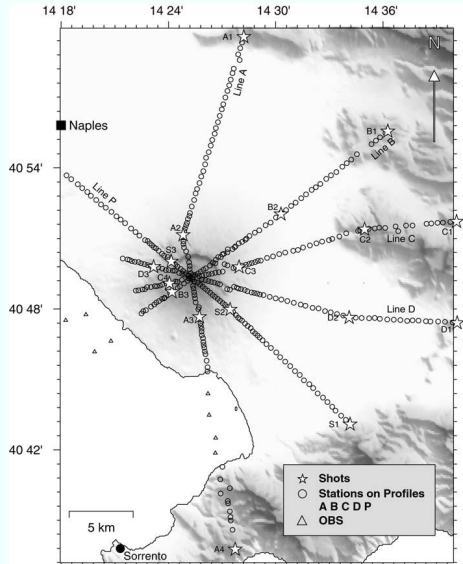
# Active seismic tomography of Vesuvius

## TomoVes-94 and TomoVes-1996 experiments

- Spatial resolution  $\approx 0.5 - 1$  km
- 388 stations, 8 OBS
- P-line, 3 shots (1994); A-D lines, 14 shots (1996)



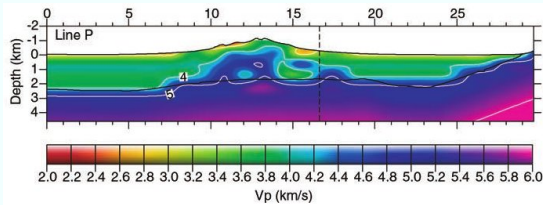
(Di Stefano and Chiarabba, JGR, 2002)



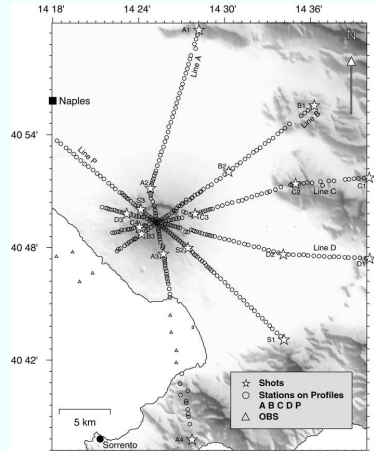
# Active seismic tomography of Vesuvius

## TomoVes-94 and TomoVes-1996 experiments

- The high- $V_p$  body below the volcano is interpreted as a solidified magma body
- No clear low- $V_p$  body exists at shallow depth ( $<2.5\text{km}$ ). No fresh magma?



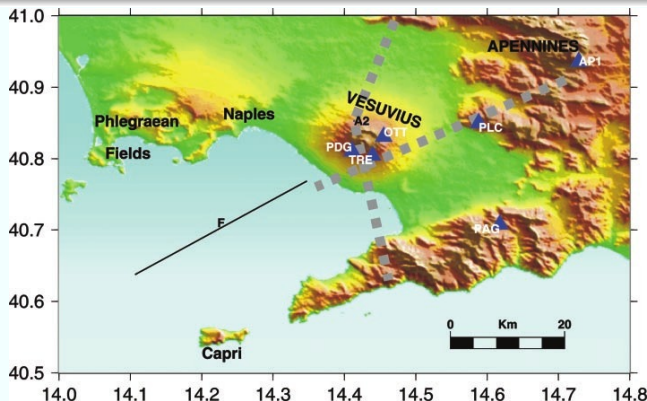
(modified from Di Stefano and Chiarabba, 2002)



# Active seismic tomography of Vesuvius

## The MAREVES-1997 experiment

- 1800 shots off-shore (along line F)
- 25 temporary seismic stations + permanent stations of the Osservatorio Vesuviano network

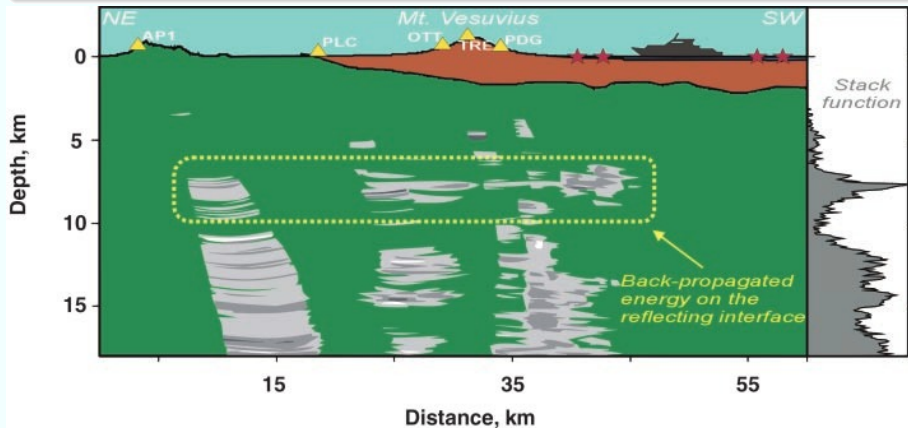


(from Auger et al., 2001)

# Active seismic tomography at Vesuvius

## The MAREVES-1997 experiment

- Found the top of a magmatic body at 8 km depth, at least 400 km<sup>2</sup>
- Thickness of the magmatic body not constrained



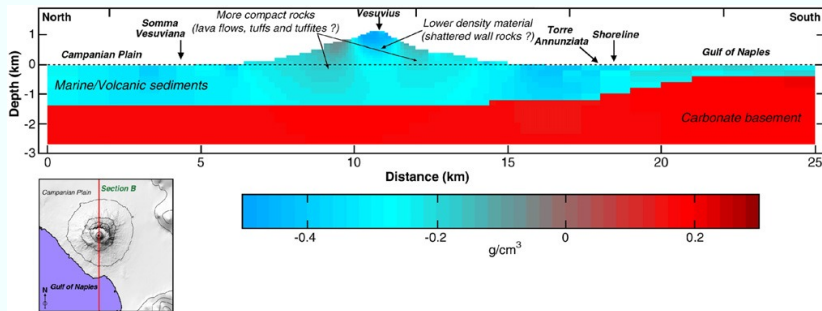
(from Auger et al., 2001)



# The internal structure of Vesuvius

## Gravimetric field inversion

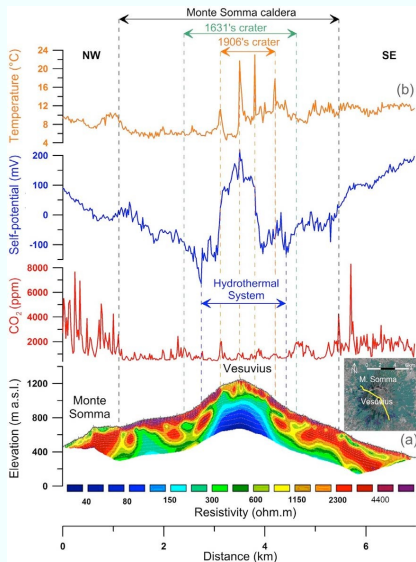
- About 300 m spatial resolution
- Model misfit  $\approx 5\%$



(from Cella et al., JVGR, 2007)

# The internal structure of Vesuvius

## Electric resistivity

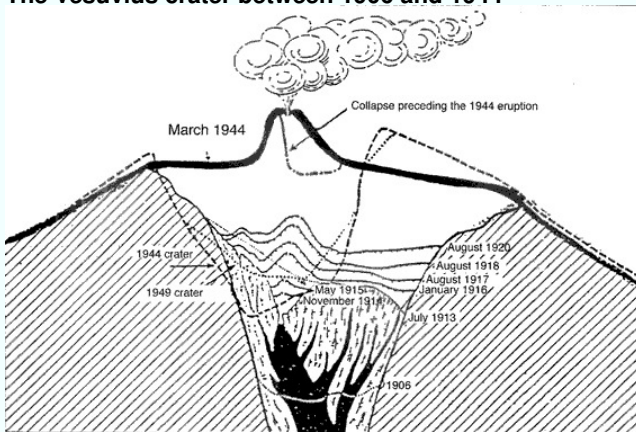


- Work still in progress
- Total profile length 7 km
- Spacing between electrodes 40 cm
- Reach 500 m depth

(Finizola et al., 2014)

# Observations of the upper part of Vesuvius

## The Vesuvius crater between 1906 and 1944



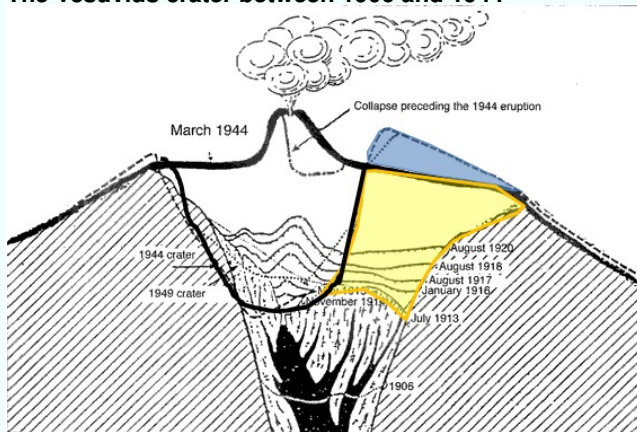
(from Imbò, 1949)

## Vesuvius crater in 1944



# Observations of the upper part of Vesuvius

## The Vesuvius crater between 1906 and 1944



## Vesuvius crater in 1944



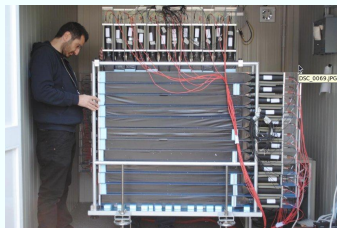
## Vesuvius crater today



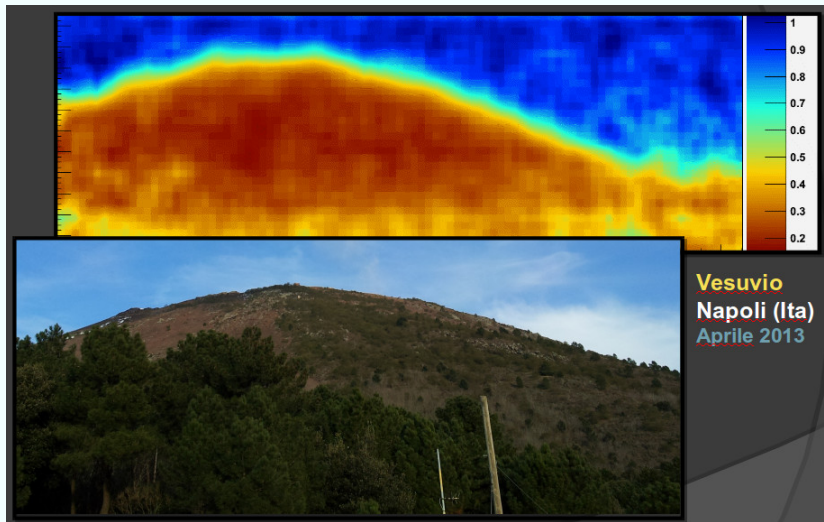
# Muon radiography

The MU-RAY experiment at Vesuvius

Mounting the Japanes detector, 2009-2010



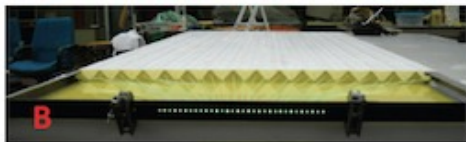
# Results from the MU-RAY experiment



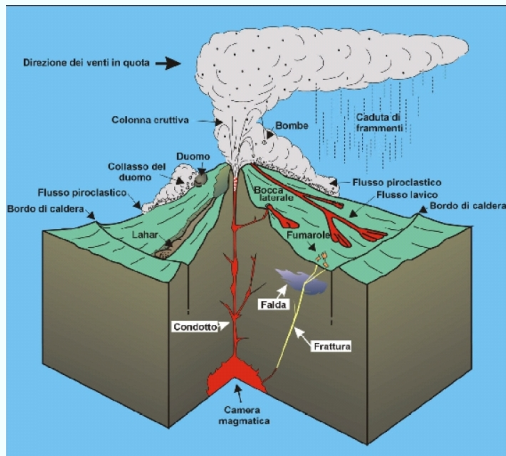
# The MURAVES Project

INFN, INGV, Universities of Napoli and Florence (Italy)

- The prototype under test

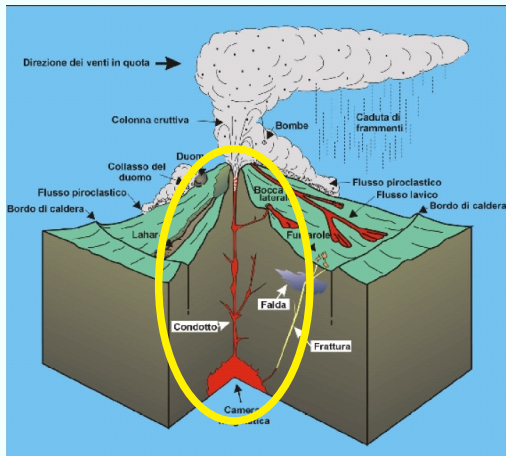


# Volcano structure and eruption processes

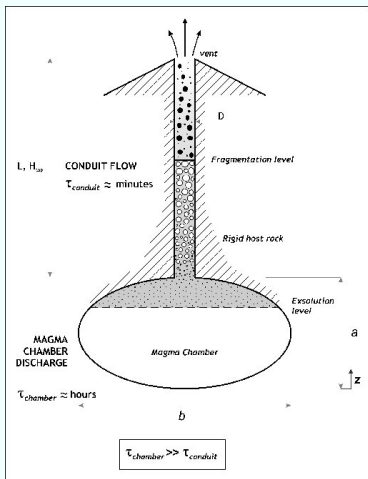




# Magma chamber-conduit system



# Simple magma chamber-conduit model



(Macedonio et al., 2005)

## Magma chamber (hydrostatic)

$$\frac{dP}{dz} = -\rho g$$

## Conduit (steady state, multiphase flow)

$$\frac{d}{dz}(\rho v) = 0 \text{ (mass balance)}$$

$$\rho v \frac{dv}{dz} = -\frac{dP}{dz} - \rho g - \frac{4f}{D} \rho \frac{v^2}{2} \text{ (momentum balance)}$$

## Constitutive equations

$$\text{Solub (H}_2\text{O)} = k\sqrt{P} \quad ; \quad \rho = \rho(P)$$

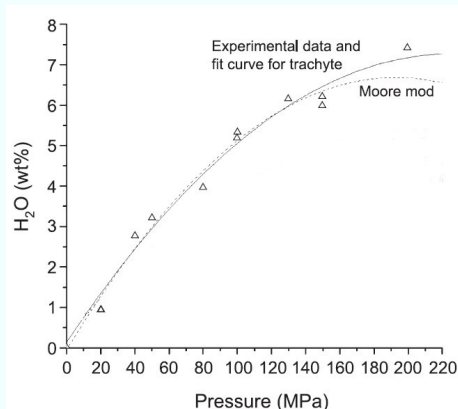
$$f = \frac{16}{Re} \quad \text{non fragmented}$$

$$f \simeq 0 \quad \text{fragmented magma}$$

$$\text{Fragmentation at } \varepsilon_{gas} \geq 0.7$$

# Water solubility in magma

Experimental data (trachyte from Campi Flegrei)



(modified from Di Matteo et al., JVGR, 2004)

## Water solubility in magma

- Depends on pressure, temperature and composition
- Thermodynamic models are available for H<sub>2</sub>O e CO<sub>2</sub> (eg: Papale et al., Chem. Geol., 2006)

## Empirical model

$$\text{Solub (H}_2\text{O)} \approx k \sqrt{P}$$

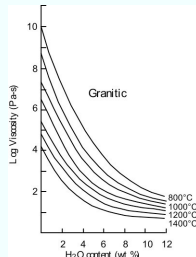
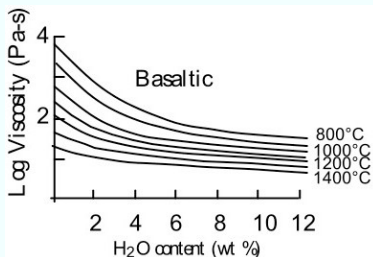
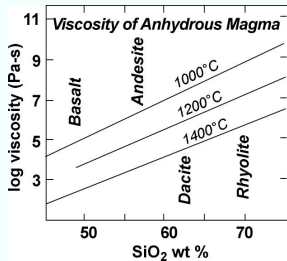
$$(k \approx 4.1 \times 10^{-6}, P \text{ in Pa})$$

# Physical properties of magma

## Viscosity of the liquid phase

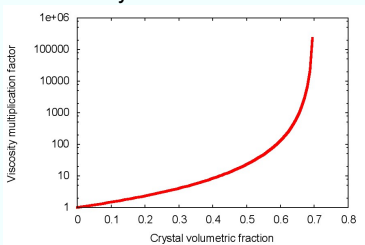
- Important effect of **temperature**
- Important effect of **dissolved H<sub>2</sub>O**
- Vogel-Fulcher-Tamman (VFT) equation (Misiti et al., 2011)

$$\log \mu = a + \frac{b}{(T - c)} + \frac{d}{(T - e)} \cdot \exp \left( g \cdot \frac{w}{T} \right)$$



# Effect of crystals and bubbles on magma viscosity

## Effect of crystals/bubbles



(from the Einstein-Roscoe equation)

## Einstein-Roscoe equation

$$\mu = \frac{\mu_0}{(1 - \phi/\phi_0)^{2.5}}$$

- $\mu_0$  crystals/bubbles free viscosity
- $\phi$  crystals/bubbles volumetric fraction
- $\phi_0$  “critical” volumetric fraction ( $\approx 0.7$ )

At high crystal contents fluid is no more Newtonian (eg: becomes Binghamian)

# Magma fragmentation

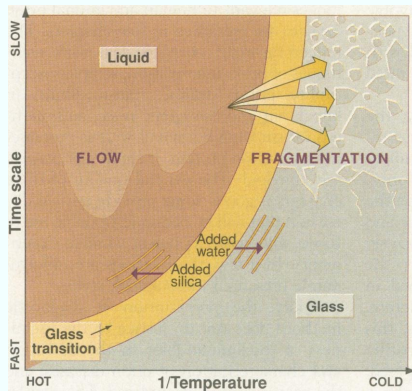
## Empirical model

- Magma fragments when bubble volume reaches a critical value ( $\phi \approx 70\%$ )

## Model based on the glass transition

- Magma fragments at low temperatures and/or rapid decompression

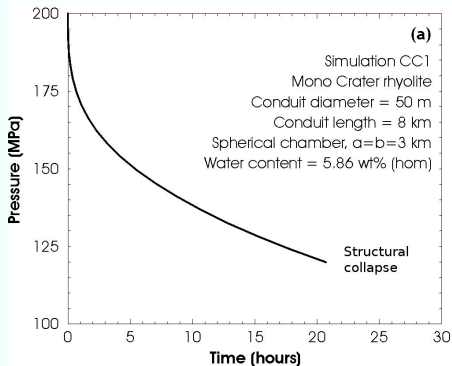
## Glass transition



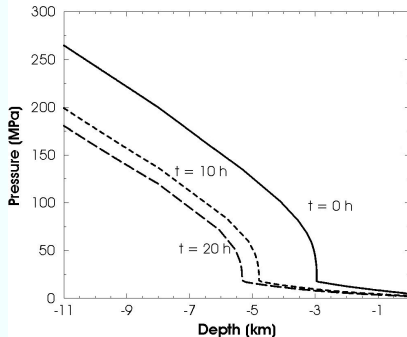
(from Dingwell, 1996)

# Magma chamber-conduit system: model results

Pressure at the chamber top



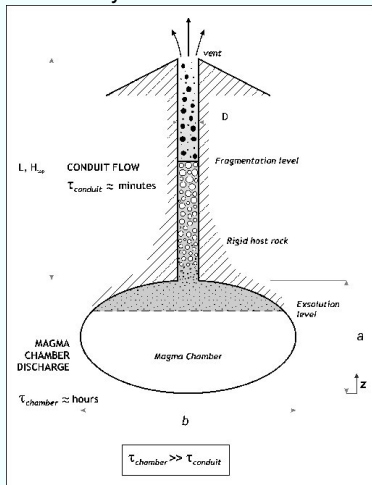
Pressure distribution along  $z$



(modified after Macedonio et al., 2005)

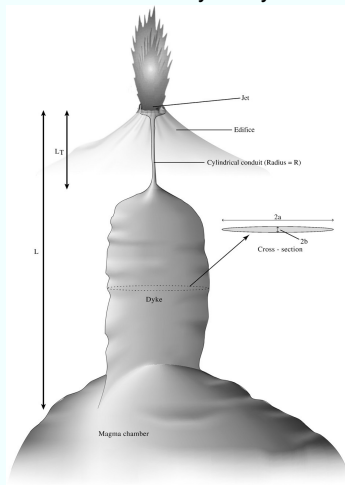
# Magma chamber-conduit model

## Cylindrical conduit



(Macedonio et al., 2005)

## More realistic: dyke+cylinder

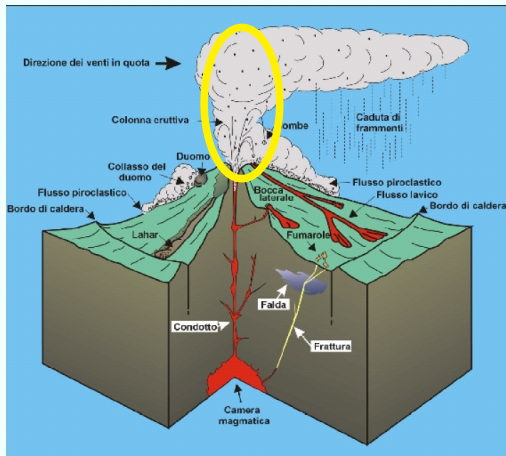


(Costa et al., 2009)



# Plinian eruptions

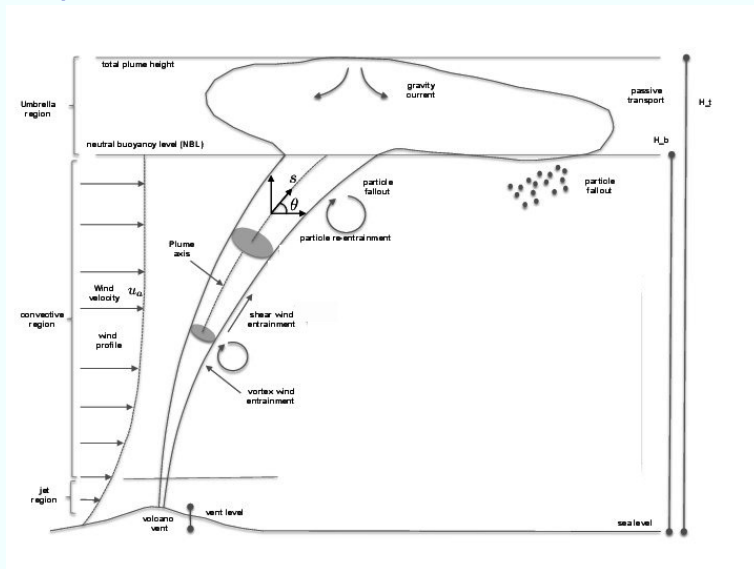
## Eruption column



## Lascar, 1993



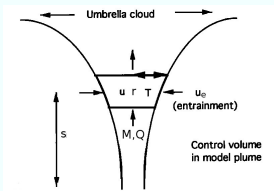
# Eruption column



(modified from Folch et al., 2016)

# Modeling Plinian columns

Model based on the Buoyant Plume Theory (Morton et al., 1956)



(modified from Woods, 1995)

## Simplified model (based on the Buoyant Plume Theory)

$$M = M_p + M_a + M_w \quad (1)$$

$$\frac{dM}{ds} = 2\pi r \rho_a u_e \quad (2)$$

$$\frac{dQ}{ds} = \pi r^2 (\rho_a - \rho) g \sin \theta + u_a \cos \theta (2\pi r \rho_a u_e) \quad (3)$$

$$\frac{dE}{ds} = 2\pi r \rho_a u_e \left( c_a T_a + gz + \frac{1}{2} u_e^2 \right) \quad (4)$$

$$\frac{dM_w}{ds} = 2\pi r \rho_a u_e w_a \quad (5)$$

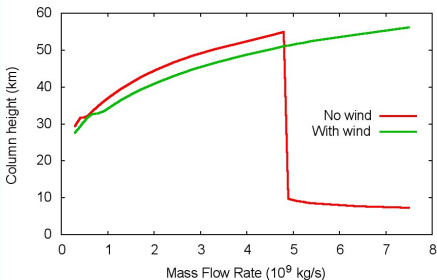
... etc ...

# Plume model results

## Conditions for column collapse

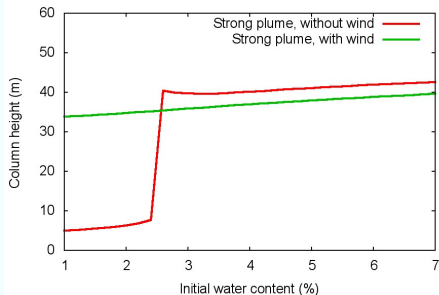
- High mass flow rate
- Low gas content

Effect of the mass flow rate



(FPlume model, Folch et al. (2016))

Effect of the initial water content



(FPlume model, Folch et al. (2016))

**Wind enhances air entrainment and prevents column collapse**

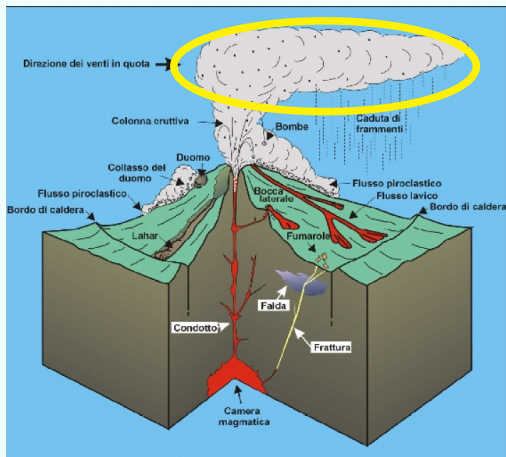
# Plume models

From the IAVCEI Intercomparison exercise

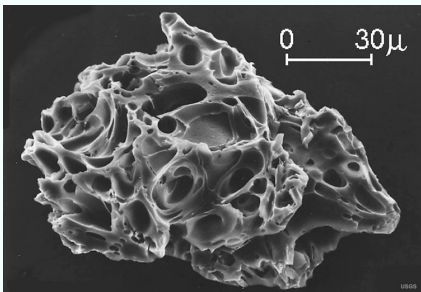
<b>Num.</b>	<b>Name</b>	<b>Reference author</b>	<b>Model type</b>
1	Puffin	M. Bursik	1D
2	Degruyter	C. Bonadonna and W. Degruyter	1D
3	PlumeMoM	M. de'Michieli Vitturi	1D
4	Devenish	B. Devenish	1D
5	FPluMe	A. Folch	1D
6	PPM	F. Girault	1D
7	Plumeria	L. Mastin	1D
8	PlumeRise	M. Woodhouse	1D
9	Cerminara 1D	M. Cerminara	1D
10	ATHAM	M. Herzog	3D
11	SK-3D	Y.J. Suzuki	3D
12	Cerminara 3D	M. Cerminara	3D
13	PDAC	T. Ongaro	3D

# Ash transport and fallout

## Umbrella region

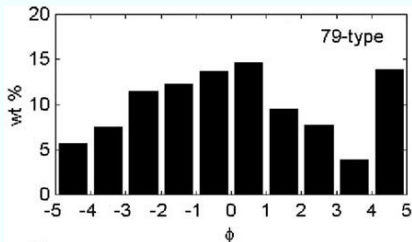


# Volcanic ash and pumice

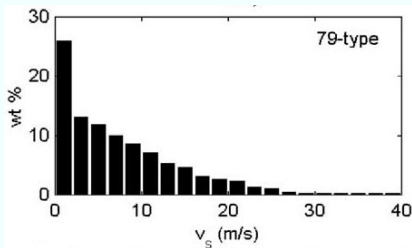


# Dimension and settling velocity of ash particles

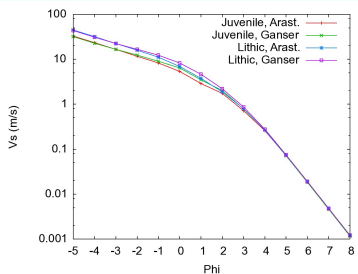
## Particles diameter distribution



## Settling velocity distribution



## Diameter-settling velocity relationship





# Ash dispersal models

## Advection-diffusion equation

$$\frac{\partial C_i}{\partial t} + \nabla(\mathbf{U}C_i) - \frac{\partial(V_s C_i)}{\partial z} = \nabla(K\nabla C_i)$$

- $C_i$  = Ash concentration,  $t$  = time
- $\mathbf{U}$  = Wind
- $V_s$  = Particles settling velocity
- $K$  = Atmospheric diffusion coefficients (vortexes, turbulence)

# Ash dispersal models

## Advection-diffusion equation

$$\boxed{\frac{\partial C_i}{\partial t}}_{\text{Time}} + \nabla(\mathbf{U}C_i) - \frac{\partial(V_s C_i)}{\partial z} = \nabla(K\nabla C_i)$$

- $C_i$  = Ash concentration,  $t$  = time
- $\mathbf{U}$  = Wind
- $V_s$  = Particles settling velocity
- $K$  = Atmospheric diffusion coefficients (vortexes, turbulence)

# Ash dispersal models

## Advection-diffusion equation

$$\boxed{\frac{\partial C_i}{\partial t}} + \boxed{\nabla(\mathbf{U}C_i)} - \frac{\partial(V_s C_i)}{\partial z} = \nabla(K \nabla C_i)$$

Time                  Wind

- $C_i$  = Ash concentration,  $t$  = time
- $\mathbf{U}$  = Wind
- $V_s$  = Particles settling velocity
- $K$  = Atmospheric diffusion coefficients (vortexes, turbulence)

# Ash dispersal models

## Advection-diffusion equation

$$\boxed{\frac{\partial C_i}{\partial t}} + \boxed{\nabla(\mathbf{U}C_i)} - \boxed{\frac{\partial(V_s C_i)}{\partial z}} = \nabla(K\nabla C_i)$$

Time                  Wind                  Fallout

- $C_i$  = Ash concentration,  $t$  = time
- $\mathbf{U}$  = Wind
- $V_s$  = Particles settling velocity
- $K$  = Atmospheric diffusion coefficients (vortexes, turbulence)

# Ash dispersal models

## Advection-diffusion equation

$$\boxed{\frac{\partial C_i}{\partial t}} + \boxed{\nabla(\mathbf{U}C_i)} - \boxed{\frac{\partial(V_s C_i)}{\partial z}} = \boxed{\nabla(K\nabla C_i)}$$

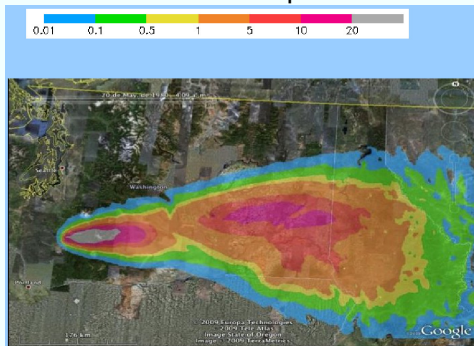
Time                  Wind                  Fallout                  Diffusion

- $C_i$  = Ash concentration,  $t$  = time
- $\mathbf{U}$  = Wind
- $V_s$  = Particles settling velocity
- $K$  = Atmospheric diffusion coefficients (vortexes, turbulence)

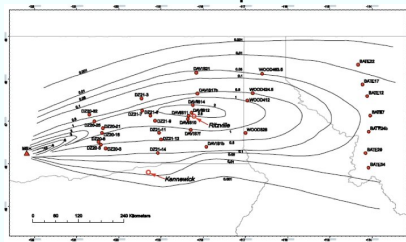
# Example of ash dispersal model (FALL3D)

Example of simulation at Mt. St. Helens

## Simulated deposit



## Observed deposit

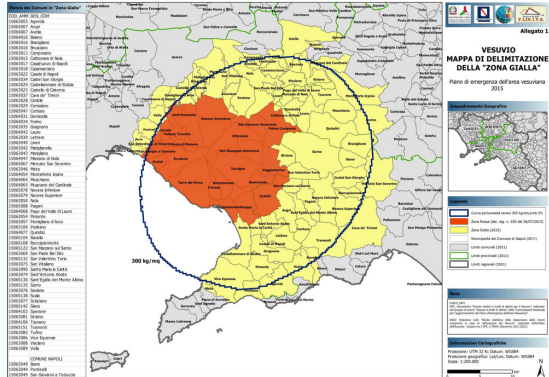
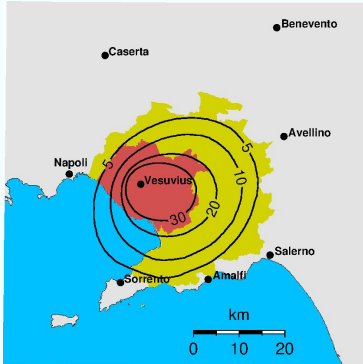


The secondary maximum is due to particle aggregation

# Application for hazard assessment at Vesuvius

Probability of a loading of  
 $300 \text{ kg/m}^2$

Yellow Zone of Vesuvius (Protezione Civile,  
2015)



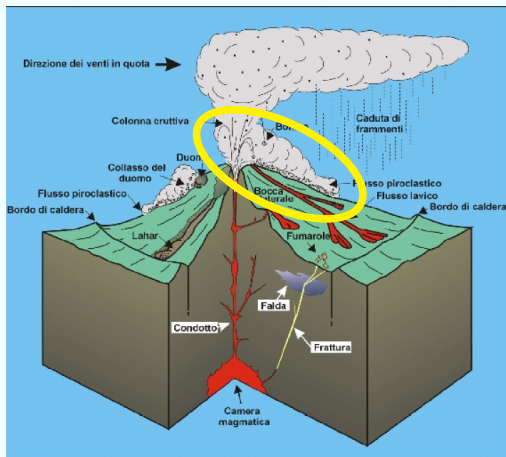
- Simulations of ash deposits based on 20 years of daily wind profiles
- The 5% probability contour line of  $300 \text{ kg/m}^2$  ash loading was selected for delimiting the hazard zone

# Available volcanic ash dispersal models

<b>Num.</b>	<b>Name</b>	<b>Type</b>
1	ASH3D	Eulerian
2	ATHAM	Eulerian
3	FALL3D	Eulerian
4	FLEXPART	Lagrangian
5	HAZMAP	Eulerian/Analytical
6	HYSPLIT	Hybrid
7	JMA-GATM and JMA-RATM	Lagrangian
8	MLDP0	Lagrangian
9	MOCAGE	Eulerian
10	NAME	Hybrid
11	PUFF	Lagrangian
12	TEPHRA2	Eulerian/Analytical
13	VOL-CALPUFF	Hybrid



# Pyroclastic flows



# Pyroclastic flows

Pyroclastic flow (Montserrat 2001)



# Pyroclastic flows (multiphase systems)

## Model equations

(Esposti Ongaro et al., 2007, 2008, 2011; Neri et al., 2003, 2007)

### Mass balance

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g) = 0$$

**Gas:**

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k) = 0 \quad k=1,2,\dots,N \quad \varepsilon_g + \sum_{k=1}^N \varepsilon_k = 1$$

### Momentum balance

**Gas:** 
$$\frac{\partial}{\partial t} \varepsilon_g \rho_g \mathbf{v}_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g \mathbf{v}_g) = -\varepsilon_g \nabla P_g + \nabla \mathbf{T}_g + \varepsilon_g \rho_g \mathbf{g} + \sum_{k=1}^N D_{g,k} (\mathbf{v}_k - \mathbf{v}_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k \mathbf{v}_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k \mathbf{v}_k) = -\varepsilon_k \nabla P_g + \nabla \mathbf{T}_k + \varepsilon_k \rho_k \mathbf{g} - D_{g,k} (\mathbf{v}_k - \mathbf{v}_g) + \sum_{j=1}^N D_{k,j} (\mathbf{v}_j - \mathbf{v}_k)$$
$$k, j = 1, 2, \dots, N$$

### Energy balance

**Gas:** 
$$\frac{\partial}{\partial t} \varepsilon_g \rho_g h_g + \nabla \cdot (\varepsilon_g \rho_g h_g \mathbf{v}_g) = \varepsilon_g \left( \frac{\partial P_g}{\partial t} + \mathbf{v}_g \cdot \nabla P_g \right) + \nabla \cdot (k_{ge} \varepsilon_g \nabla T_g) + \sum_{k=1}^N Q_k (T_k - T_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k h_k + \nabla \cdot (\varepsilon_k \rho_k h_k \mathbf{v}_k) = \nabla \cdot (k_{ke} \varepsilon_k \nabla T_k) - Q_k (T_k - T_g); k=1,2,\dots,N$$

# Pyroclastic flows (multiphase systems)

## Model equations

(Esposti Ongaro et al., 2007, 2008, 2011; Neri et al., 2003, 2007)

### Mass balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g) = 0$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k) = 0 \quad k=1,2,\dots,N \quad \varepsilon_g + \sum_{k=1}^N \varepsilon_k = 1$$

### Momentum balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g \mathbf{v}_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g \mathbf{v}_g) = -\varepsilon_g \nabla P_g + \nabla \mathbf{T}_g + \varepsilon_g \rho_g \mathbf{g} + \sum_{k=1}^N D_{g,k} (\mathbf{v}_k - \mathbf{v}_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k \mathbf{v}_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k \mathbf{v}_k) = -\varepsilon_k \nabla P_g + \nabla \mathbf{T}_k + \varepsilon_k \rho_k \mathbf{g} - D_{g,k} (\mathbf{v}_k - \mathbf{v}_g) + \sum_{j=1}^N D_{k,j} (\mathbf{v}_j - \mathbf{v}_k) \\ k, j = 1, 2, \dots, N$$

### Energy balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g h_g + \nabla \cdot (\varepsilon_g \rho_g h_g \mathbf{v}_g) = \varepsilon_g \left( \frac{\partial P_g}{\partial t} + \mathbf{v}_g \cdot \nabla P_g \right) + \nabla \cdot (k_{ge} \varepsilon_g \nabla T_g) + \sum_{k=1}^N Q_k (T_k - T_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k h_k + \nabla \cdot (\varepsilon_k \rho_k h_k \mathbf{v}_k) = \nabla \cdot (k_{ke} \varepsilon_k \nabla T_k) - Q_k (T_k - T_g); k=1,2,\dots,N$$

# Pyroclastic flows (multiphase systems)

## Model equations

(Esposti Ongaro et al., 2007, 2008, 2011; Neri et al., 2003, 2007)

### Mass balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g) = 0$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k) = 0 \quad k = 1, 2, \dots, N \quad \varepsilon_g + \sum_{k=1}^N \varepsilon_k = 1$$

### Momentum balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g \mathbf{v}_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g \mathbf{v}_g) = -\varepsilon_g \nabla P_g + \nabla \mathbf{T}_g + \varepsilon_g \rho_g \mathbf{g} + \sum_{k=1}^N D_{g,k} (\mathbf{v}_k - \mathbf{v}_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k \mathbf{v}_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k \mathbf{v}_k) = -\varepsilon_k \nabla P_g + \nabla \mathbf{T}_k + \varepsilon_k \rho_k \mathbf{g} - D_{g,k} (\mathbf{v}_k - \mathbf{v}_g) + \sum_{j=1}^N D_{k,j} (\mathbf{v}_j - \mathbf{v}_k)$$

$k, j = 1, 2, \dots, N$

### Energy balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g h_g + \nabla \cdot (\varepsilon_g \rho_g h_g \mathbf{v}_g) = \varepsilon_g \left( \frac{\partial P_g}{\partial t} + \mathbf{v}_g \cdot \nabla P_g \right) + \nabla \cdot (k_{ge} \varepsilon_g \nabla T_g) + \sum_{k=1}^N Q_k (T_k - T_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k h_k + \nabla \cdot (\varepsilon_k \rho_k h_k \mathbf{v}_k) = \nabla \cdot (k_{ke} \varepsilon_k \nabla T_k) - Q_k (T_k - T_g); k = 1, 2, \dots, N$$

# Pyroclastic flows (multiphase systems)

## Model equations

(Esposti Ongaro et al., 2007, 2008, 2011; Neri et al., 2003, 2007)

### Mass balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g) = 0$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k) = 0 \quad k = 1, 2, \dots, N \quad \varepsilon_g + \sum_{k=1}^N \varepsilon_k = 1$$

### Momentum balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g \mathbf{v}_g + \nabla \cdot (\varepsilon_g \rho_g \mathbf{v}_g \mathbf{v}_g) = -\varepsilon_g \nabla P_g + \nabla \mathbf{T}_g + \varepsilon_g \rho_g \mathbf{g} + \sum_{k=1}^N D_{g,k} (\mathbf{v}_k - \mathbf{v}_g)$$

**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k \mathbf{v}_k + \nabla \cdot (\varepsilon_k \rho_k \mathbf{v}_k \mathbf{v}_k) = -\varepsilon_k \nabla P_g + \nabla \mathbf{T}_k + \varepsilon_k \rho_k \mathbf{g} - D_{g,k} (\mathbf{v}_k - \mathbf{v}_g) + \sum_{j=1}^N D_{k,j} (\mathbf{v}_j - \mathbf{v}_k)$$

$k, j = 1, 2, \dots, N$

### Energy balance

**Gas:**

$$\frac{\partial}{\partial t} \varepsilon_g \rho_g h_g + \nabla \cdot (\varepsilon_g \rho_g h_g \mathbf{v}_g) = \varepsilon_g \left( \frac{\partial P_g}{\partial t} + \mathbf{v}_g \cdot \nabla P_g \right) + \nabla \cdot (k_{ge} \varepsilon_g \nabla T_g) + \sum_{k=1}^N Q_k (T_k - T_g)$$

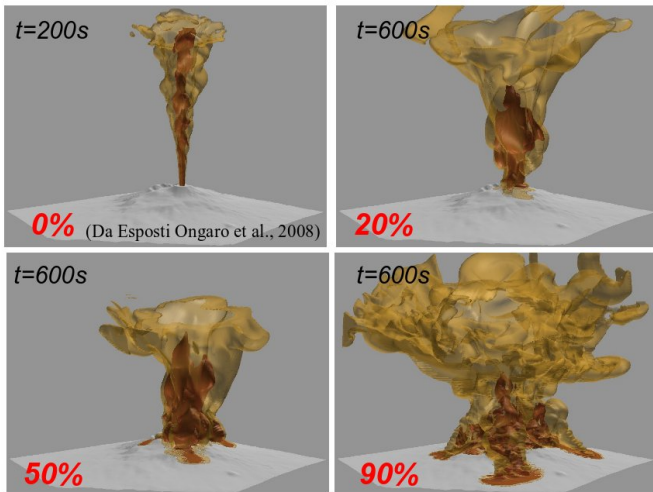
**Particelle:**

$$\frac{\partial}{\partial t} \varepsilon_k \rho_k h_k + \nabla \cdot (\varepsilon_k \rho_k h_k \mathbf{v}_k) = \nabla \cdot (k_{ke} \varepsilon_k \nabla T_k) - Q_k (T_k - T_g); k = 1, 2, \dots, N$$

# Pyroclastic flows

Example of 3D simulation (INGV Pisa)

*Evento Sub-Plinianio ( $5 \times 10^7$  kg/s): **wt% di massa collassata***

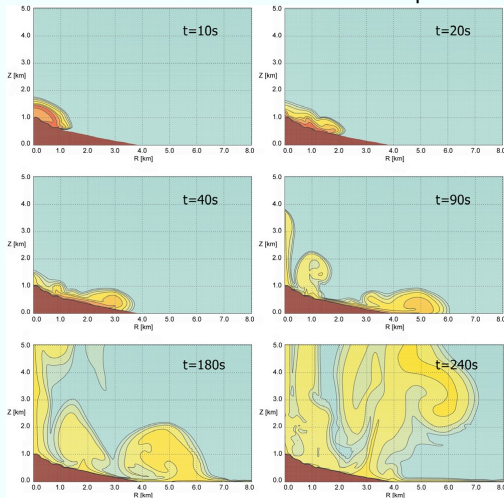


(from Esposti Ongaro et al., 2008)

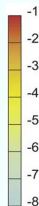
# Example of pyroclastic flows simulation

Montserrat eruption, West Indies, 1997

## Particles concentration in the atmosphere



(simulation C-2D, from Esposti Ongaro et al. (2008))

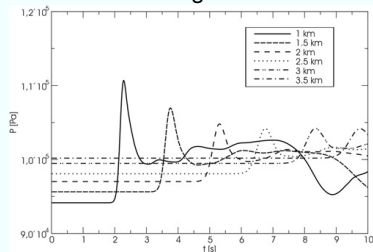


$\log_{10}$  particles volumetric fraction

$d = 50$  (35%) +  $500$  (40%) +  $5000$  (25%)  $\mu m$

Mass =  $60 \times 10^9$  kg

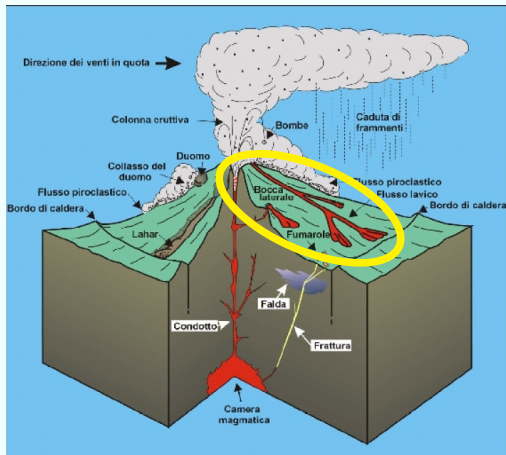
## Pressure at ground level





# Lava flows

## Lava flows



# Lava flows

Etna, 1985



## Principal parameters

- Topography
- Magma rheology
- Mass eruption rate

## Characteristics of lava flows

- Free surface
- Capacity to surmount barriers
- Cooling
- Formation of channels and tunnels

# General model (deterministic)

3D, time-dependent

## Continuity and Navier-Stokes equations

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla(\mu \nabla \mathbf{u}) + \rho \mathbf{g}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

where:

$\mathbf{u}$  = velocity

$\rho$  = density

$\mu$  = viscosity (temperature dependent)

$p$  = pressure

$\mathbf{g}$  = gravity constant

# Depth averaged equations

2D, time-dependent

## Simplifying assumptions of 2D models

- Fluid is approximated as a thin layer
- Vertical velocity component is neglected
- Navier-Stokes and continuity equations are integrated in  $z$  (become 2D equations)

# Depth averaged equations

## Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial(Uh)}{\partial x} + \frac{\partial(Vh)}{\partial y} = 0$$

## Momentum balance equations

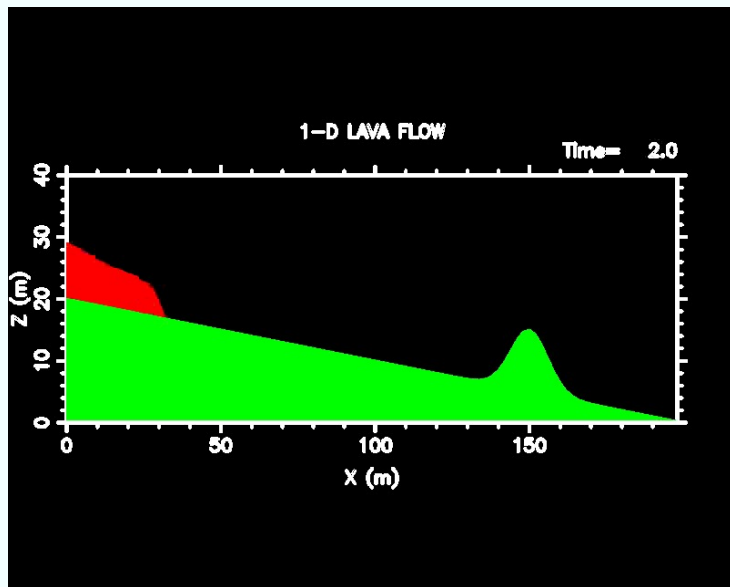
$$\frac{\partial(Uh)}{\partial t} + \frac{\partial(U^2h + gh^2/2)}{\partial x} + \frac{\partial(UVh)}{\partial y} = -gh\frac{\partial H}{\partial x} - \gamma U$$

$$\frac{\partial(Vh)}{\partial t} + \frac{\partial(UVh)}{\partial x} + \frac{\partial(V^2h + gh^2/2)}{\partial y} = -gh\frac{\partial H}{\partial y} - \gamma V$$

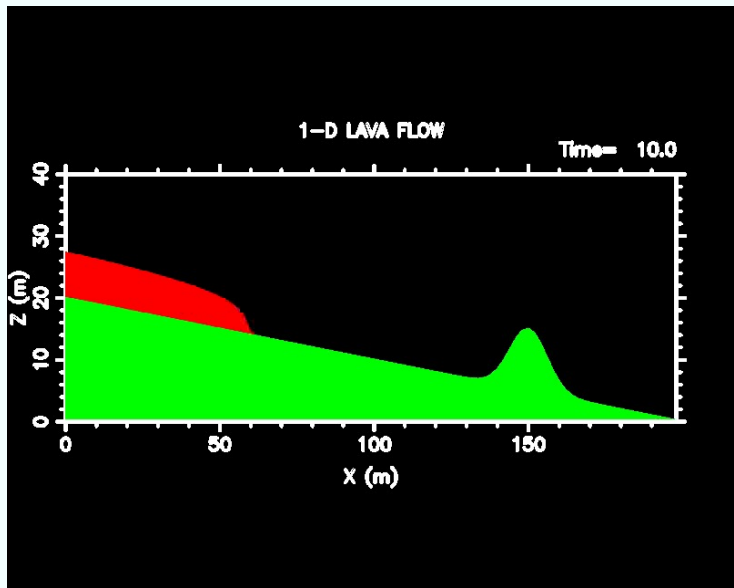
$h$  = fluid thickness;  $H$  = topography;  $U, V$  = x,y components of the velocity;  $\gamma$  = friction coefficient (viscous effects)

NOTE: Topography and viscous effects are “source terms”

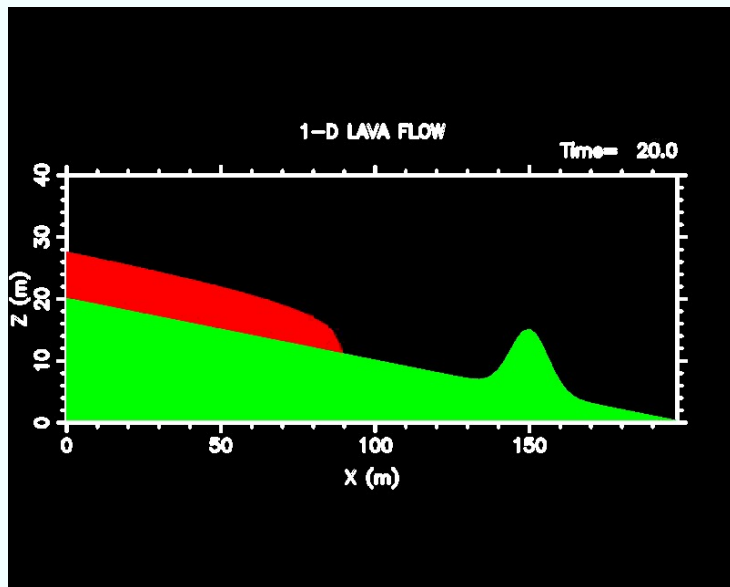
# Simulation 1D



# Simulation 1D

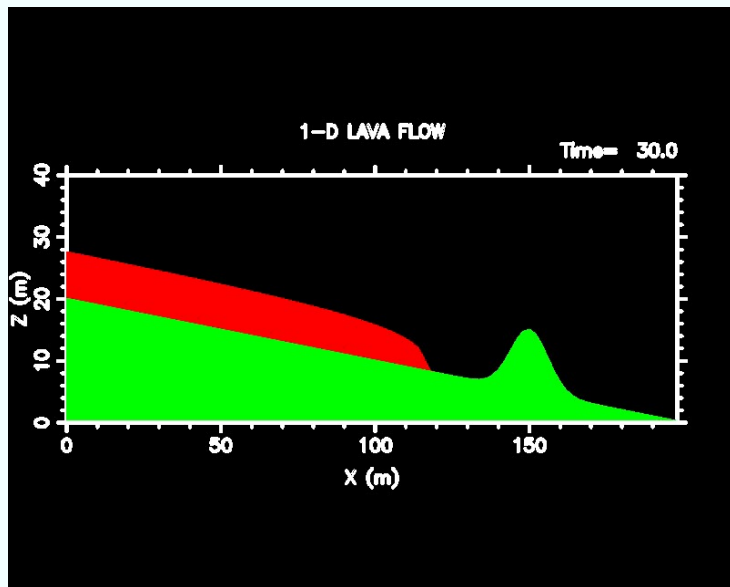


# Simulation 1D

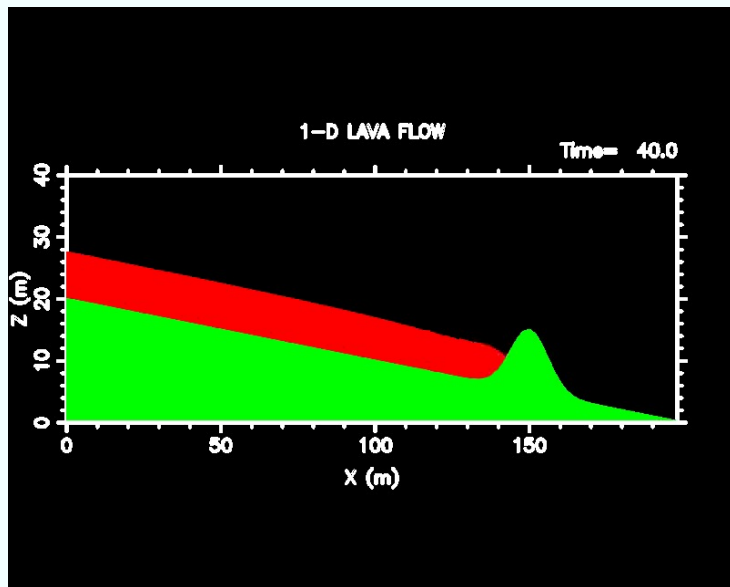




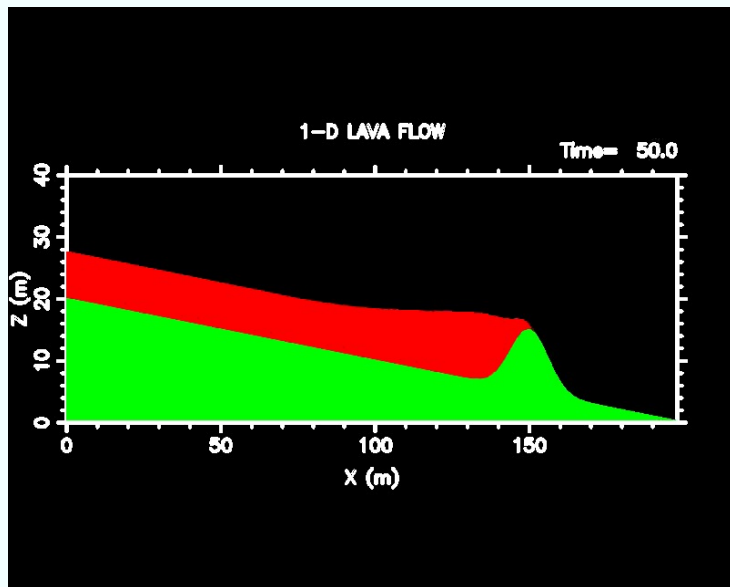
# Simulation 1D



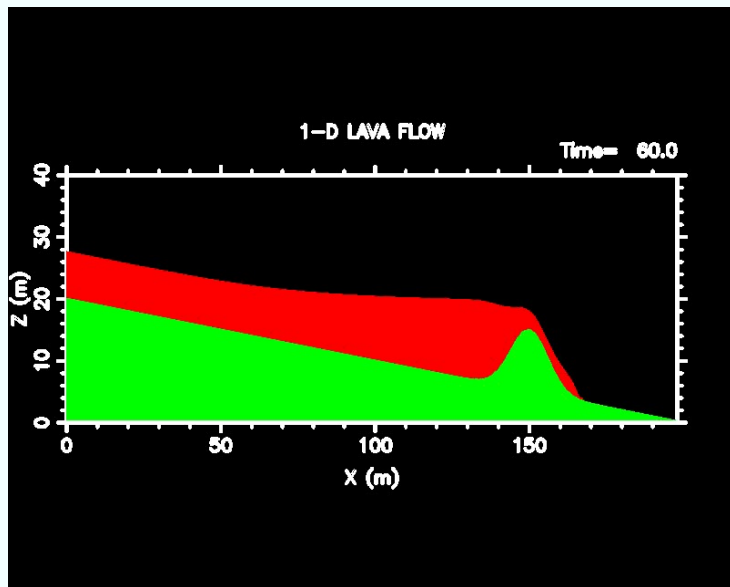
# Simulation 1D



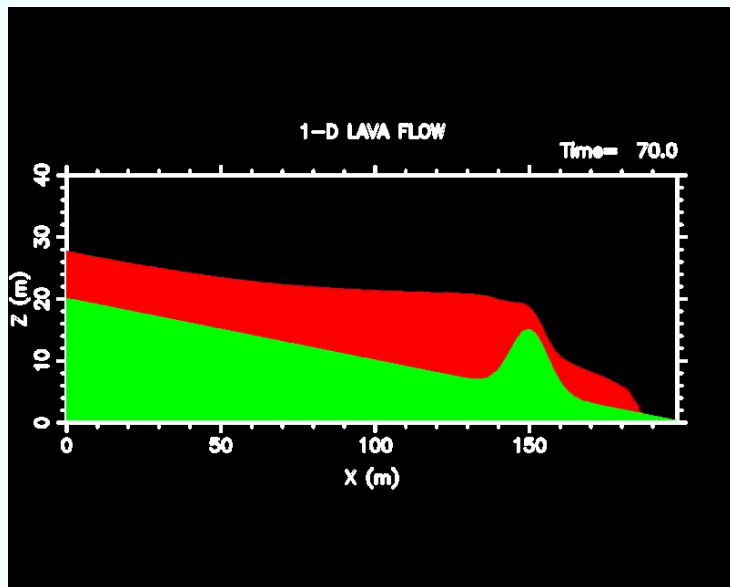
# Simulation 1D



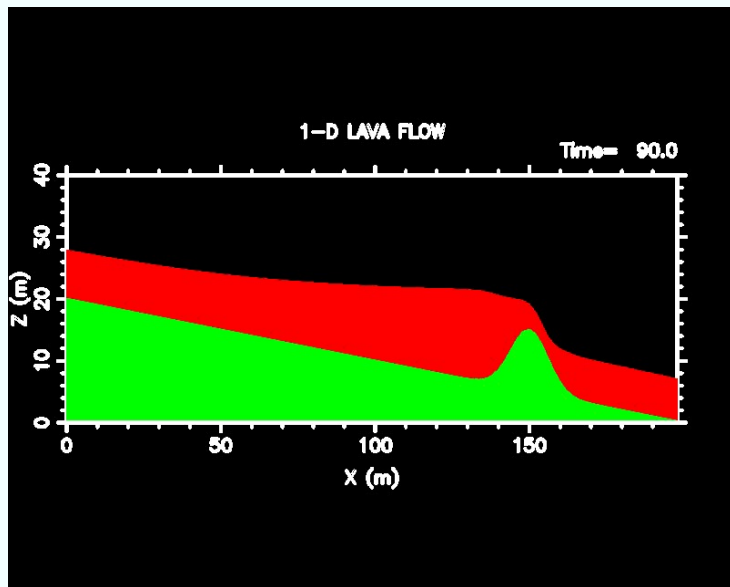
# Simulation 1D



# Simulation 1D

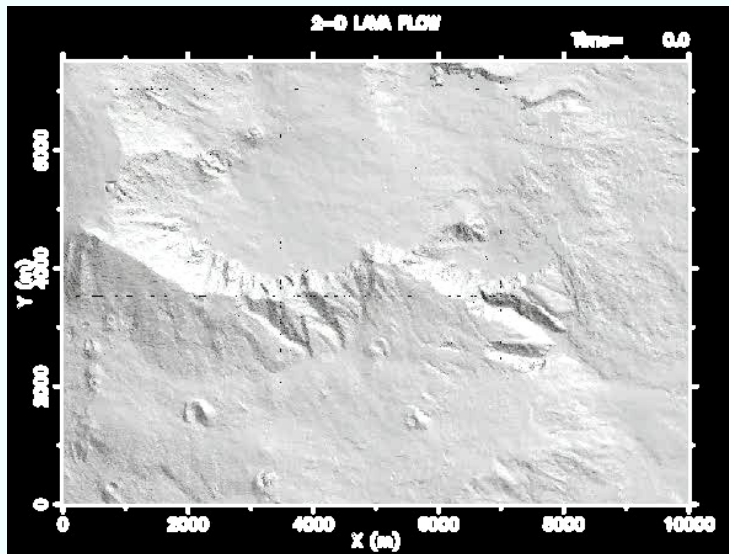


# Simulation 1D



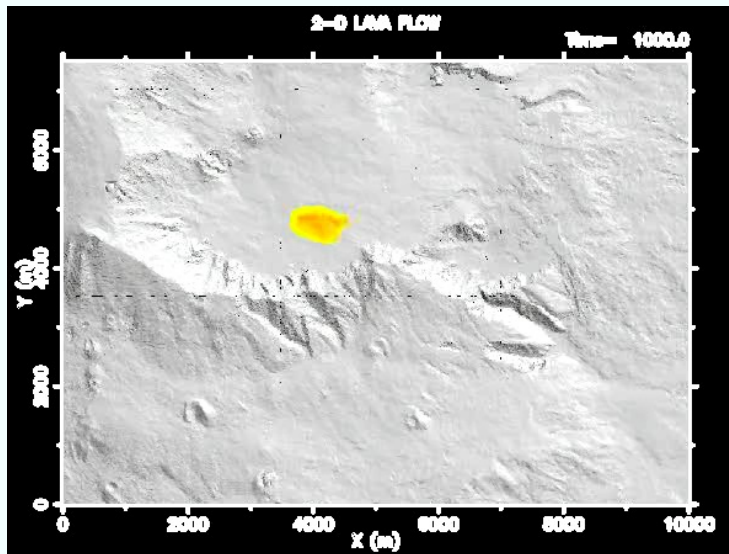
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



# Simulation 2D (Etna 1991)

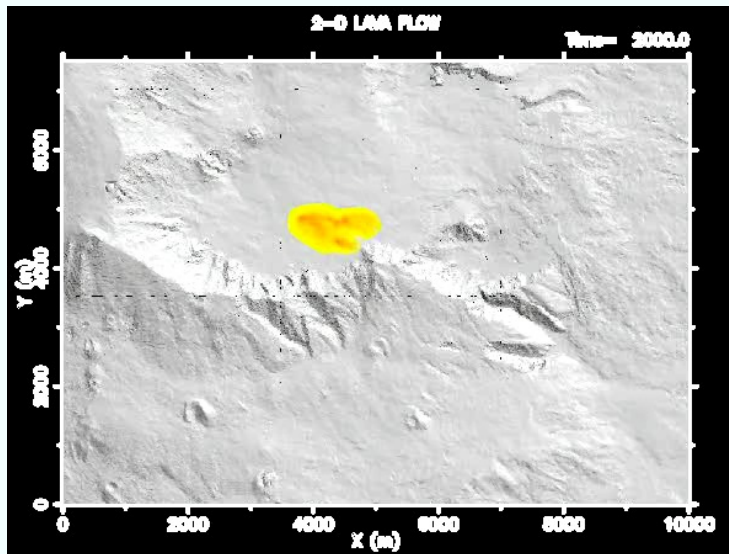
SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)





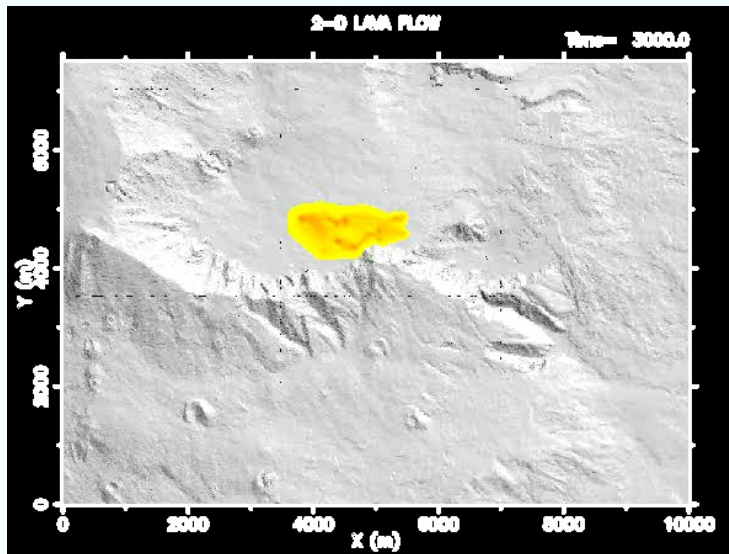
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



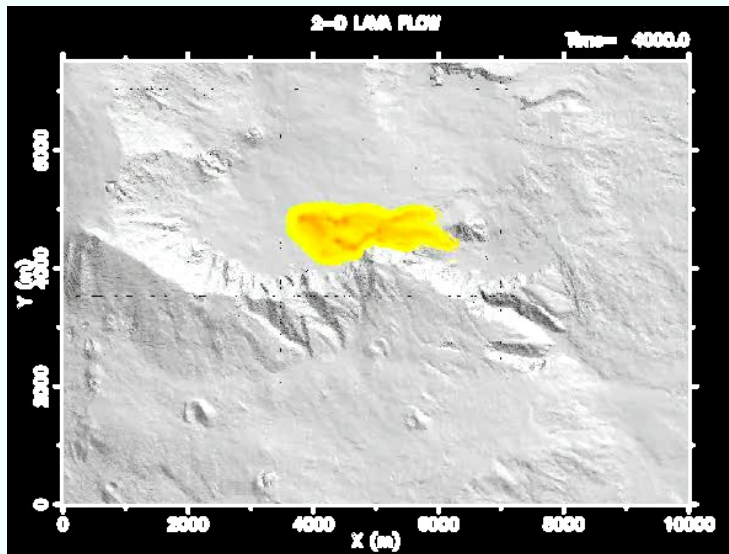
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



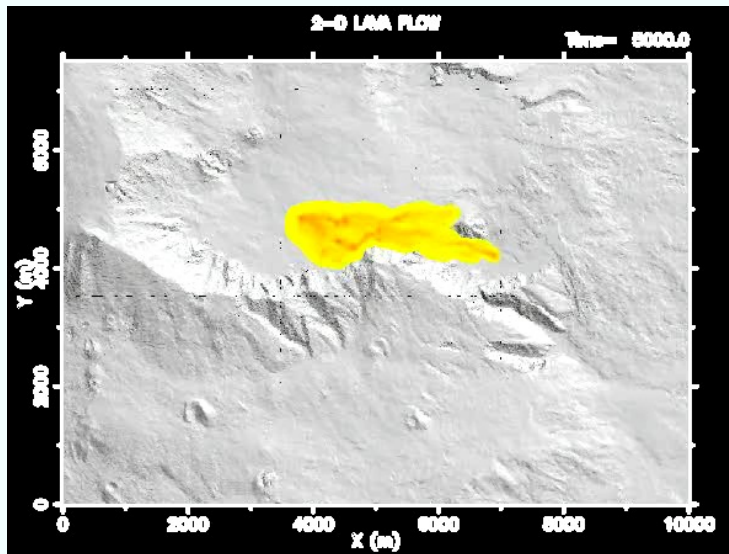
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



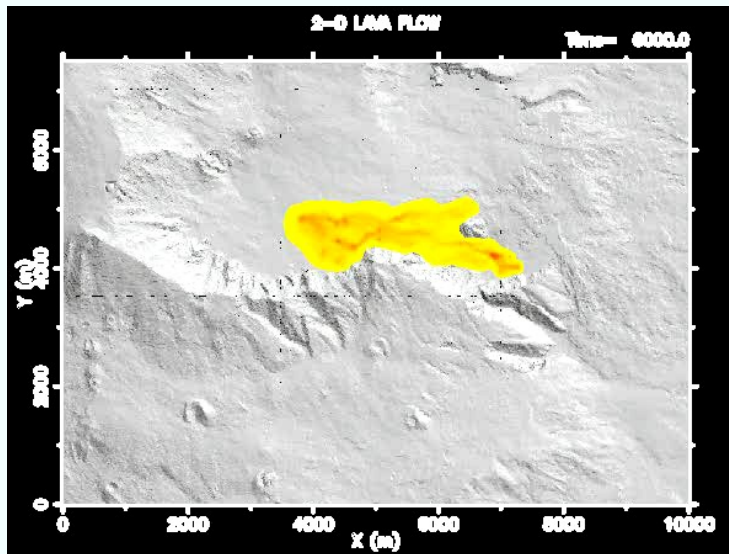
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



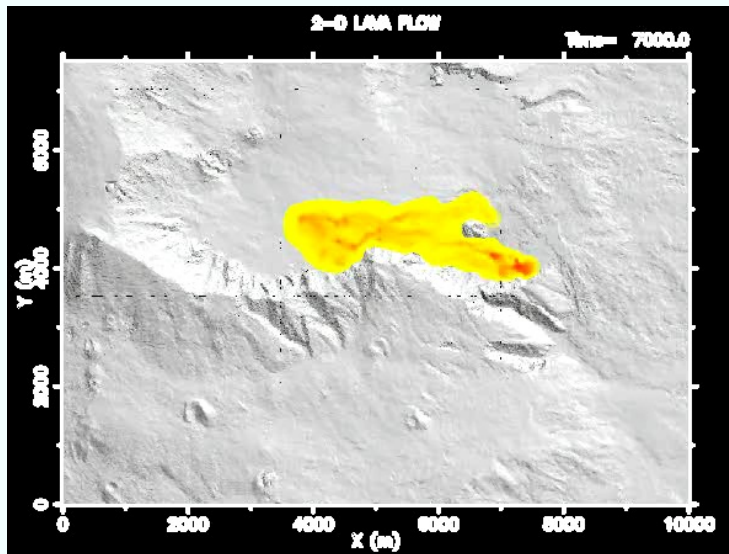
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



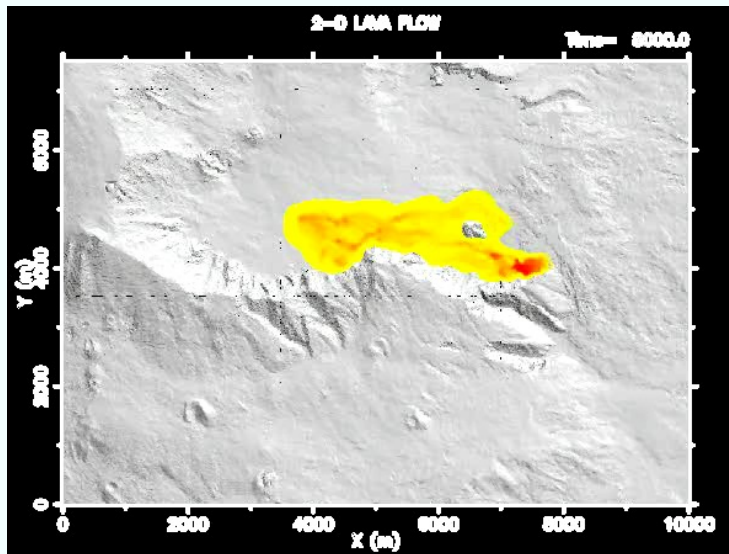
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



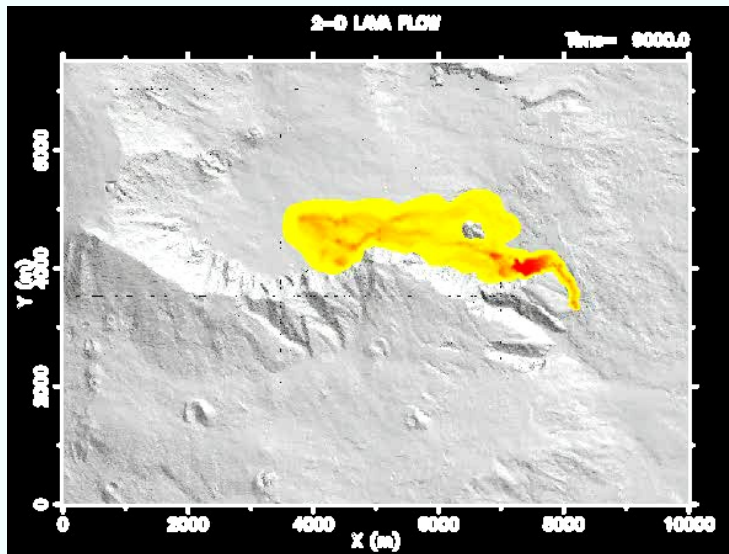
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



# Simulation 2D (Etna 1991)

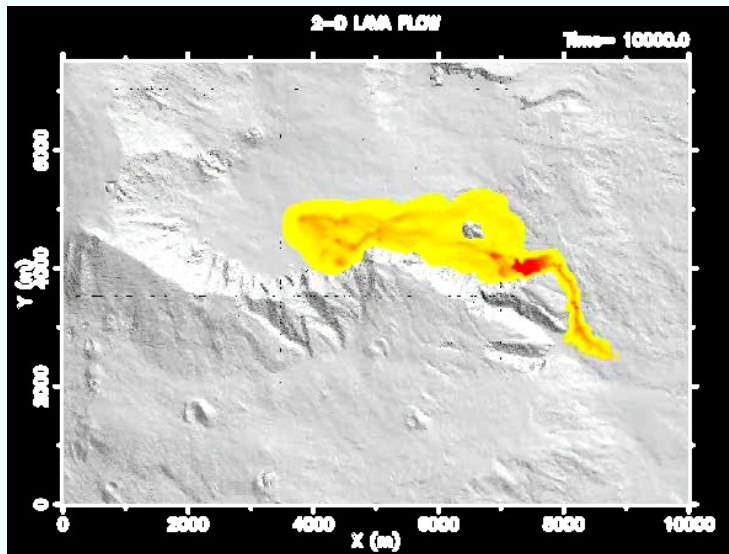
SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)





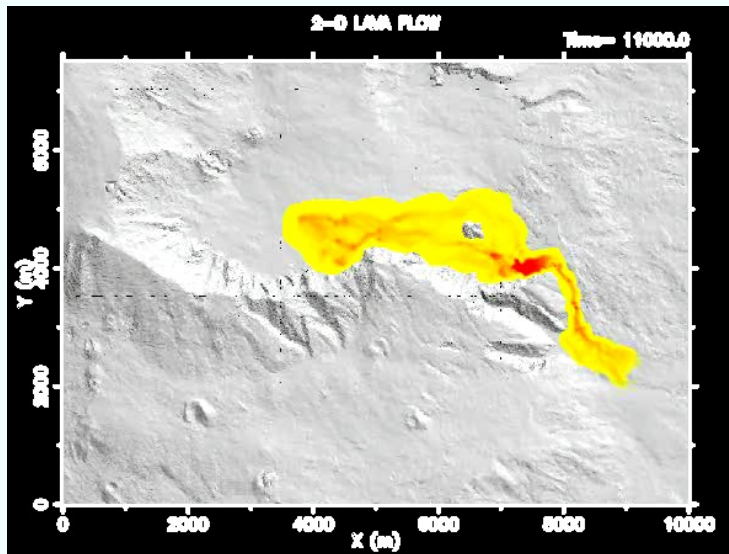
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



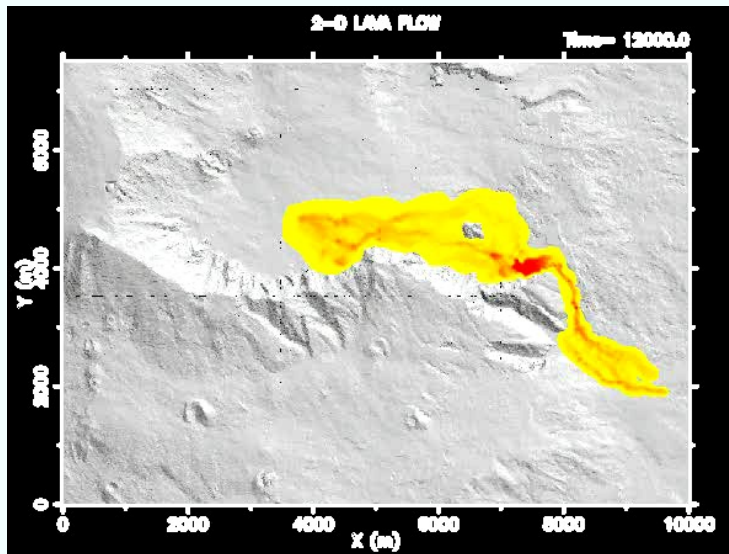
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



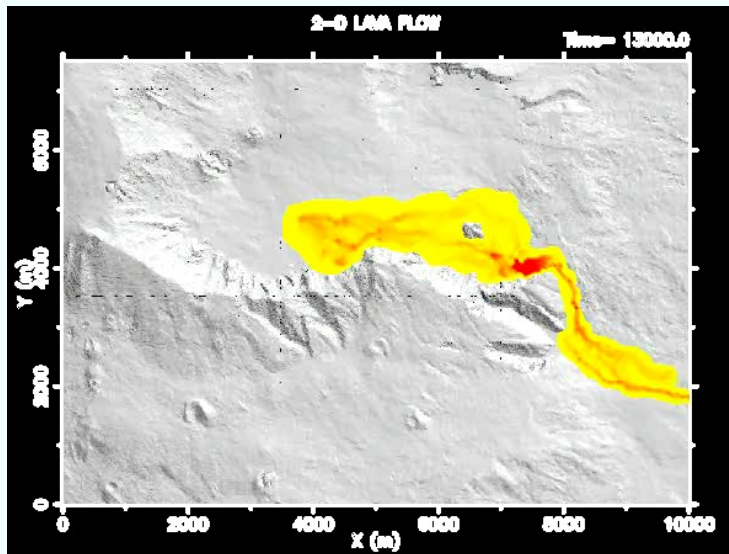
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



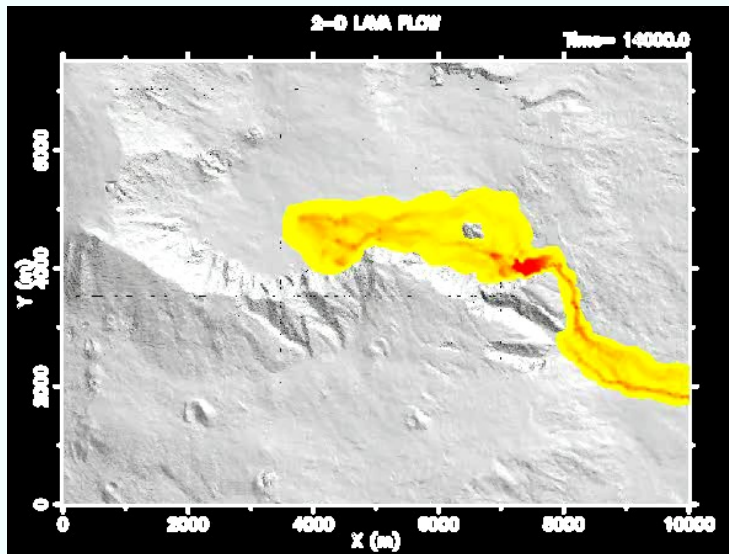
# Simulation 2D (Etna 1991)

SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



# Simulation 2D (Etna 1991)

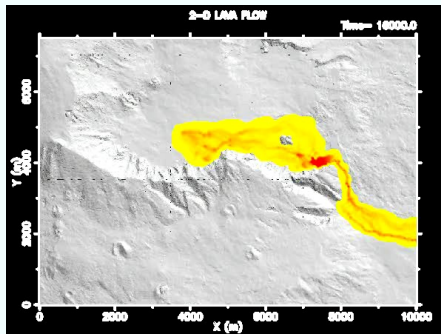
SWELAVA model, INGV Napoli (Costa and Macedonio, 2005)



# Etna 1991

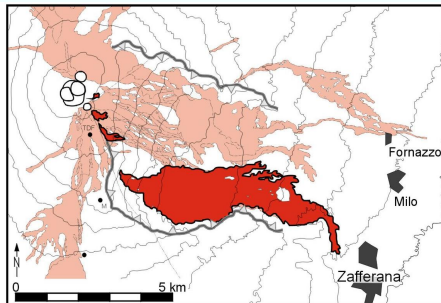
## Comparison with observations

### Simulation



Duration: few CPU minutes

### Observation



Duration: some months

# Main lava flow models

<b>Num.</b>	<b>Name</b>	<b>Model type</b>	<b>Authors</b>
1	SCIARA	Cellular automata	Crisci et al. (1982)
2	Ishihara	Cellular automata	Ishihara et al. (1989)
3	DANIMIX	Cellular automata	Miyamoto & Sasaki (1997)
4	FLOWGO	Steady-state 1D	Harris & Rowland (2001)
5	DOWNFLOW	Probabilistic	Favalli et al. (2005)
6	LavaSIM	Deterministic 3D	Hidaka et al. (2005)
7	SLAG	Deterministic 2D	Macedonio & Costa (2005)
8	MAGFLOW	Cellular Automata	Vicari et al. (2007)
9	VOLCFLOW	Deterministic 2D	Kelfoun (2015)

# Bibliography

- Auger, E., Gasparini, P., Virieux, J., and Zollo, A.: Seismic evidence of an extended magmatic sill under Mt. Vesuvius, *Science*, 294, 1510–1512, 2001.
- Costa, A. and Macedonio, G.: Numerical simulation of lava flows based on depth-averaged equations, *Geophys. Res. Lett.*, 32, L05304, doi:10.1029/2004GL021817, 2005.
- Costa, A., Sparks, R. S. J., Macedonio, G., and Melnik, O.: Effects of wall-rock elasticity on magma flow in dykes during explosive eruptions, *Earth Planet. Sci. Lett.*, 288, 455–462, doi:10.1016/j.epsl.2009.10.006, 2009.
- Di Stefano, R. and Chiarabba, C.: Active source tomography at Mt. Vesuvius: Constraints for the magmatic system, *J. Geophys. Res.*, 107, 2278, doi:10.1029/2001JB000792, 2002.
- Dingwell, D. B.: Volcanic dilemma: Flow or blow ?, *Science*, 273, 1054–1055, doi:10.1126/science.273.5278.1054, 1996.
- Esposti Ongaro, T., Clarke, A. B., Neri, A., Voight, B., and Widiwijayanti, C.: Fluid dynamics of the 1997 Boxing Day volcanic blast on Montserrat, West Indies, *J. Geophys. Res.*, 113, B03211, doi:10.1029/2006JB004898, 2008.
- Esposti Ongaro, T., Neri, A., Menconi, G., de' Michieli Vitturi, M., Marianelli, P., Cavazzoni, C., Erbacci, G., and Baxter, P. J.: Transient 3D numerical simulations of column collapse and pyroclastic density current scenarios at Vesuvius, *J. Volcanol. Geotherm. Res.*, 178, 378–396, doi:10.1016/j.jvolgeores.2008.06.036, 2008.
- Finizola, A., Ricci, T., Poret, M., Delcher, E., Peltier, A., Antoine, R., Bernard, J., Boudoire, G., Brothelande, E., Fanizza, G., Fargier, Y., Gailler, L., Gueguen, E., Gusset, R., Matera, A., Mezon, C., Piscitelli, S., Portal, A., Rizzo, E., Rossi, M., Calamita, G., Bellucci Sessa, E., and Nave, R.: Fluid circulation at Somma-Vesuvius volcanic complex inferred by electrical resistivity tomography, self-potential, temperature and soil degassing, in: MED-SUV 1<sup>th</sup> year meeting, Nicolosi (catania) 7-9 July 2014, *Miscellanea INGV* no. 23, pp. 52–53, INGV, 2014.
- Folch, A., Costa, A., and Macedonio, G.: FALL3D: A computational model for transport and deposition of volcanic ash, *Comput. Geosci.*, 35, 1334–1342, doi:10.1016/j.cageo.2008.08.008, 2009.
- Folch, A., Costa, A., and Macedonio, G.: FPLUME-1.0: An integral volcanic plume model accounting for ash aggregation, *Geosci. Model Dev.*, 9, 431–450, doi:10.5194/gmd-9-431-2016, 2016.
- Macedonio, G., Neri, A., Martí, J., and Folch, A.: Temporal evolution of flow conditions in sustained magmatic explosive eruptions, *J. Volcanol. Geotherm. Res.*, 143, 153–172, doi:10.1016/j.jvolgeores.2004.09.015, 2005.
- Misiti, V., Vetere, F., Freda, C., Scarlato, P., Behrens, H., Mangiacapra, A., and Dingwell, D. B.: A general viscosity model of Campi Flegrei (Italy) melts, *Chem. Geol.*, 290, 50–59, doi:10.1016/j.chemgeo.2011.08.010, 2011.
- Morton, B. R., Taylor, G., and Turner, J. S.: Turbulent gravitational convection from maintained and instantaneous sources, *Proc. Roy. Soc. London, Ser. A*, 234, 1–23, 1956.
- Scarpa, R., Tronca, F., Bianco, F., and Del Pezzo, E.: High resolution velocity structure beneath Mt. Vesuvius from seismic array data, *Geophys. Res. Lett.*, 29, 2040–2043, doi:10.1029/2002GL015576, 2002.
- Woods, A. W.: The dynamics of explosive volcanic eruptions, *Rev. Geophys.*, 33, 495–530, 1995.