



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA



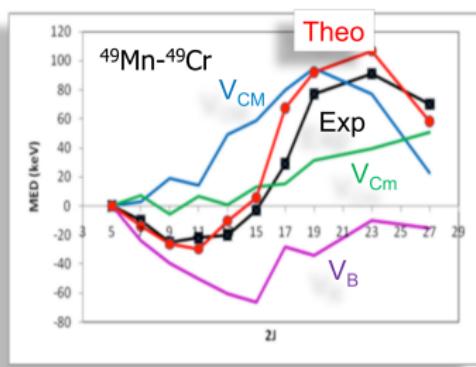
## Isospin Symmetry Breaking in mirror nuclei $^{23}\text{Mg}$ - $^{23}\text{Na}$

Alberto Boso et al.

# Mirror Energy Differences

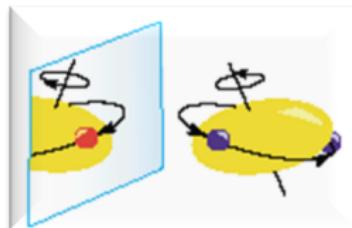
## Mirror Energy Differences (MED)

- $\text{MED}_{J,T} = E_{J,T,T_z=-T}^* - E_{J,T,T_z=T}^*$
- If isospin is a good q.n.: **Electromagnetic origin**
- Very sensitive to nuclear structure



A. P. Zuker et al., PRL 89, 142502 (2002)

M.A. Bentley and S.M. Lenzi, Prog. Part. Nucl. Phys. 59(2007)



## Multipole Term of MED

Protons alignment

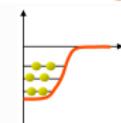


## Monopole Term of MED

Radial effect

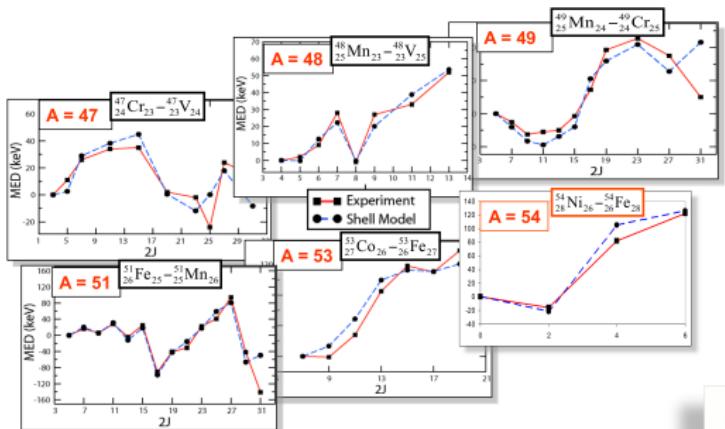


Single particle effects



# ISB TERM OF “NUCLEAR” ORIGIN IS NEEDED!

# Mirror Energy Differences in the $f_{7/2}$ shell

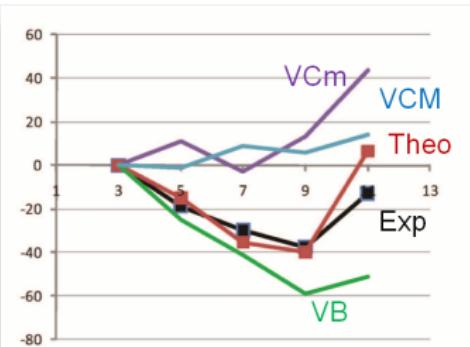


## $f_{7/2}$ shell

- $V_B$  term from  $A=42$
- “Nuclear” ISB term is **needed** in all the  $f_{7/2}$  shell

## Open questions

- Is the  $V_B$  term needed also in the  $sd$  shell??
- Is it a general feature of the effective N-N interaction??



**SYSTEMATIC STUDIES ARE NEEDED!**

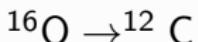
# The Experiment: MED in mirror nuclei $^{23}\text{Mg} - ^{23}\text{Na}$



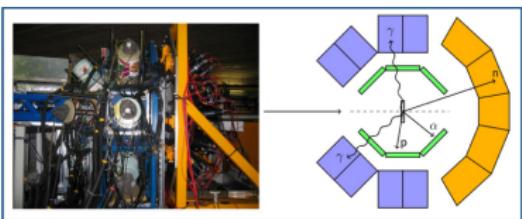
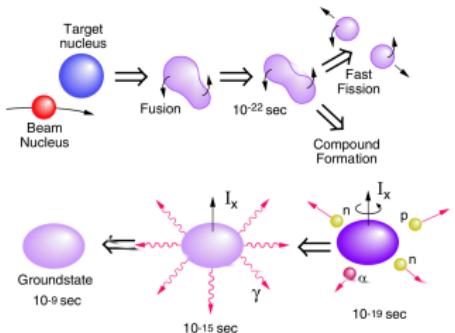
## Goal of the Experiment

Study of the MED in mirror nuclei  
 $^{23}\text{Mg} - ^{23}\text{Na}$  up to high spin

Fusion Evaporation reaction



P25	P26 20 ms (3+)	P27 260 ms 1/2+	P28 270.3 ms 3+	P29 4.149 s 1/2+	P30 2.498 m 1+
Si124 102 ms 0+	Si125 230 ms 5/2+	Si126 2.234 s 0+	Si127 4.16 s 5/2+	Si128 93.23 0+	Si129 4.67 1/2+
ECp 0.47 s	ECp 2.053 s 4+	Al125 7.183 s 5/2+	Al126 7.177 s 5/2+	Al127 100 5/2+	Al128 2.2414 m 3+
ECp 3.857 s 0+	Mg22 11.37 s 3/2+	Mg24 0 0+	Mg25 5/2+	Mg26 0+	Mg27 9.438 m 1/2+
EC 22.49 s 3/2+	Na21 2.6019 y 3+	Na23 100 0+	Na24 14.9596 h 4+	Na25 39.1 s 5/2+	Na26 1.072 s 3+
Ne20 0+	Ne21 3/2+	Ne22 0+	Ne23 37.24 s 5/2+	Ne24 3.38 m 0+	Ne25 602 ms (1/2, 3/2+)
90.48	0.27	9.25	0	$\beta^-$	$\beta^-$



- EXOGAM: 10 Clover of HPGe
- DIAMANT: 80 CsI
- Neutron Wall: 50 scintillators

# The Experiment: Analysis

## Channel Selection

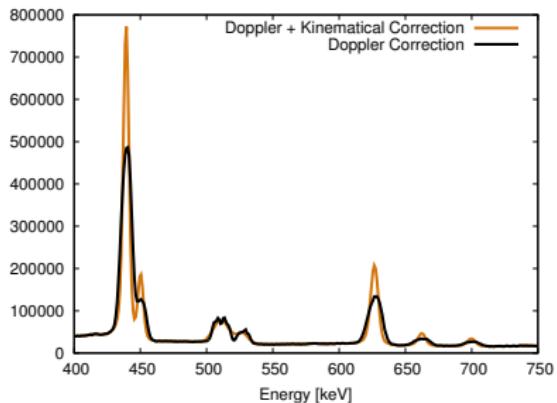
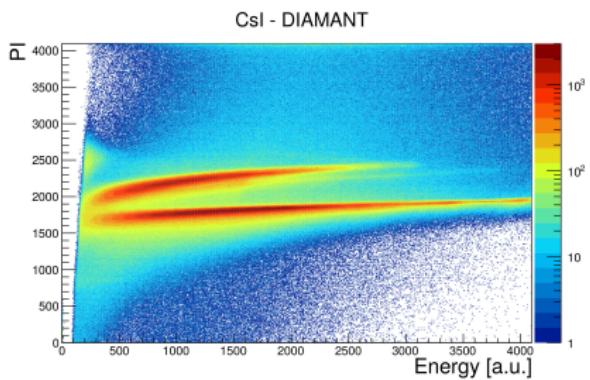
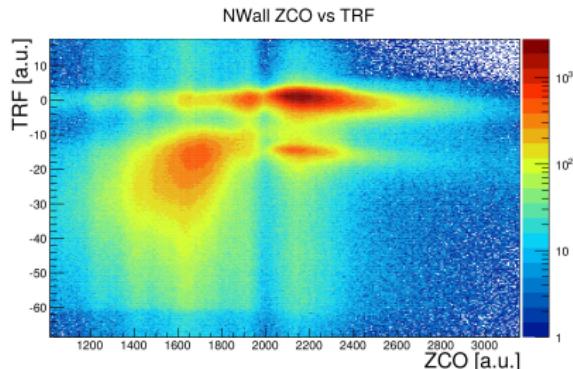
**NWALL:**

$n-\gamma$ : ZCO vs TRF

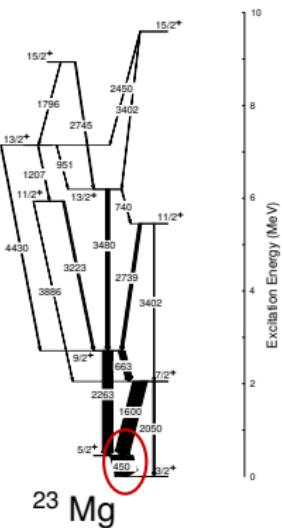
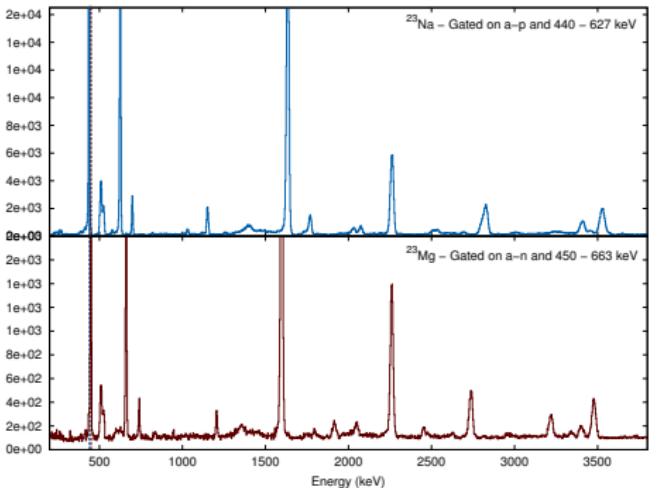
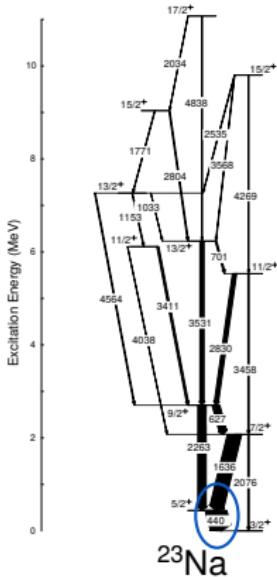
**DIAMANT:**

$\alpha-p$ : PI vs Energy

**Kinematical Correction**



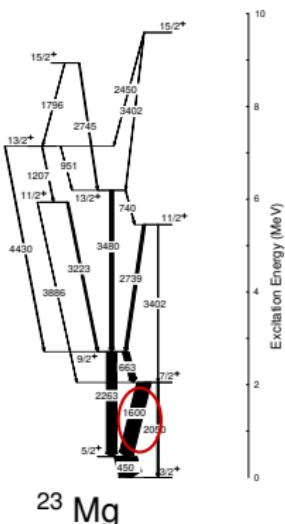
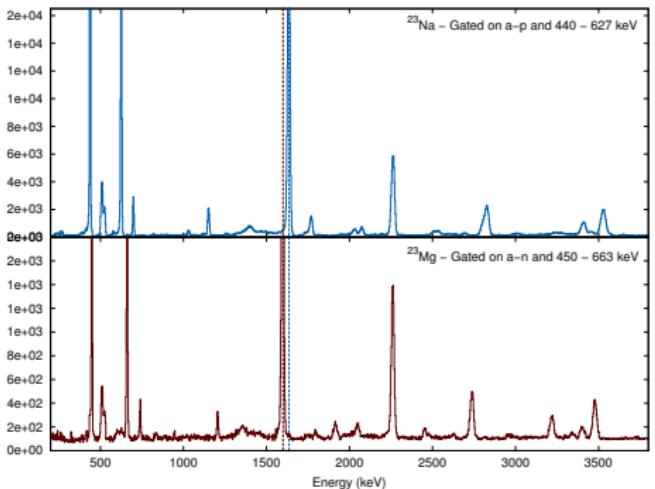
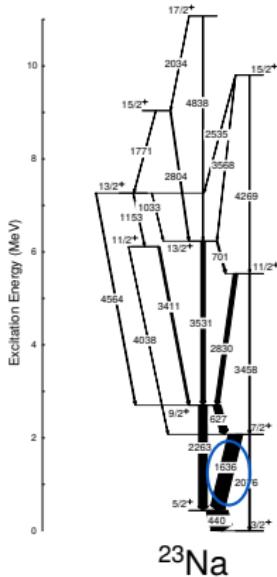
# $^{23}\text{Mg}$ - $^{23}\text{Na}$ Level Scheme



$$^{23}\text{Na}; 440 \text{ keV}: \frac{5}{2}^+ \rightarrow \frac{3}{2}^+$$

$$^{23}\text{Mg}; 450 \text{ keV}: \frac{5}{2}^+ \rightarrow \frac{3}{2}^+$$

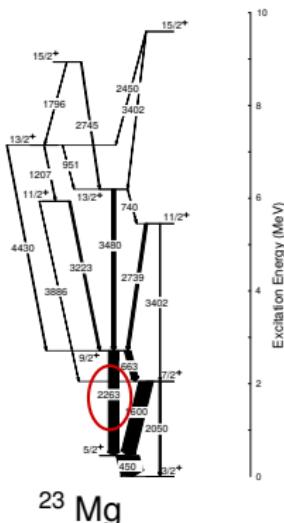
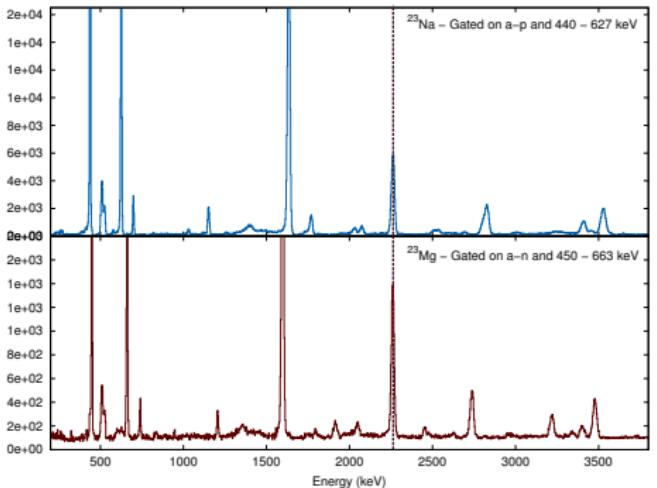
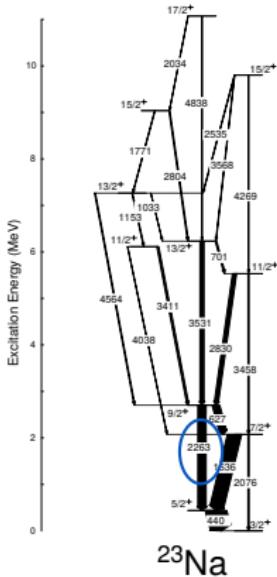
# $^{23}\text{Mg}$ - $^{23}\text{Na}$ Level Scheme



$$^{23}\text{Na}; 1636 \text{ keV}: \frac{7}{2}^+ \rightarrow \frac{5}{2}^+$$

$$^{23}\text{Mg}; 1600 \text{ keV}: \frac{7}{2}^+ \rightarrow \frac{5}{2}^+$$

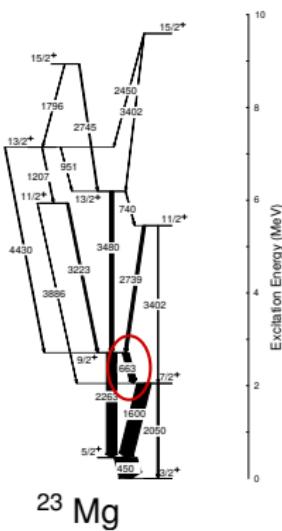
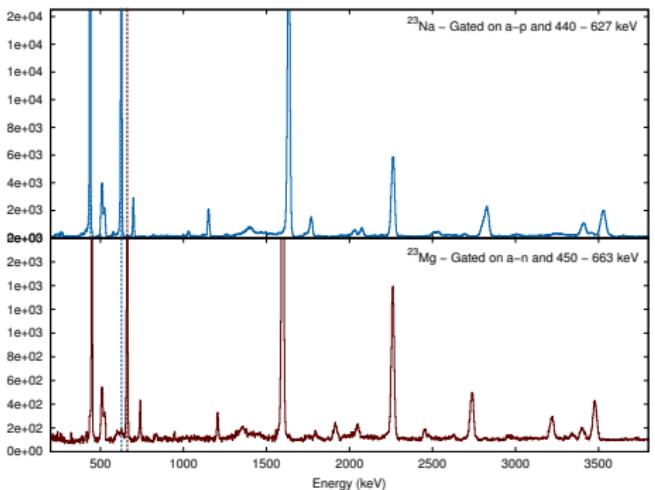
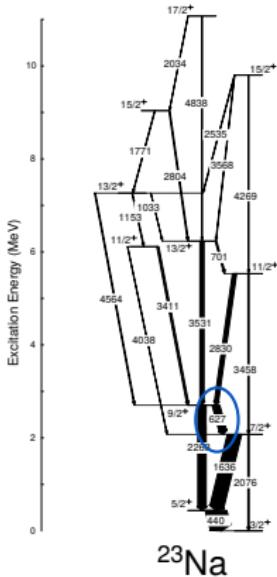
# $^{23}\text{Mg}$ - $^{23}\text{Na}$ Level Scheme



$$^{23}\text{Na}; 2263 \text{ keV}: \frac{7}{2}^+ \rightarrow \frac{5}{2}^+$$

$$^{23}\text{Mg}; 2263 \text{ keV}: \frac{7}{2}^+ \rightarrow \frac{5}{2}^+$$

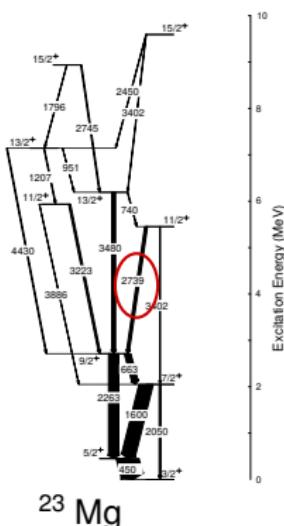
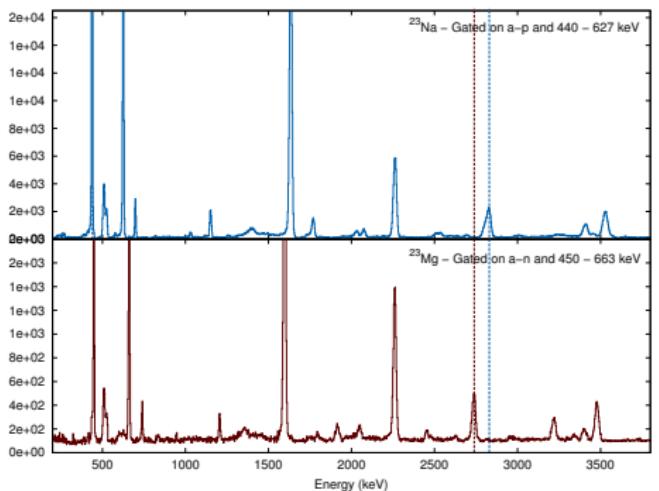
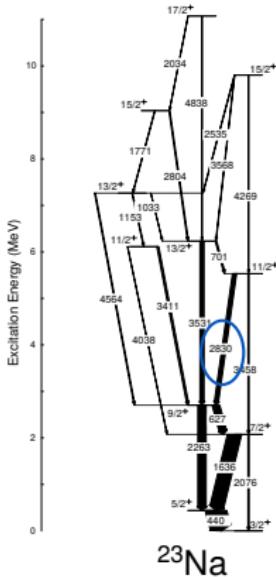
# $^{23}\text{Mg}$ - $^{23}\text{Na}$ Level Scheme



$$^{23}\text{Na}; 627 \text{ keV}: \frac{9}{2}^+ \rightarrow \frac{7}{2}^+$$

$$^{23}\text{Mg}; 663 \text{ keV}: \frac{9}{2}^+ \rightarrow \frac{7}{2}^+$$

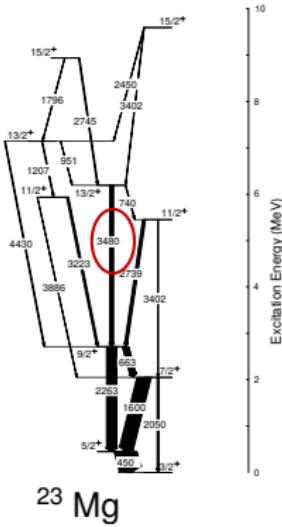
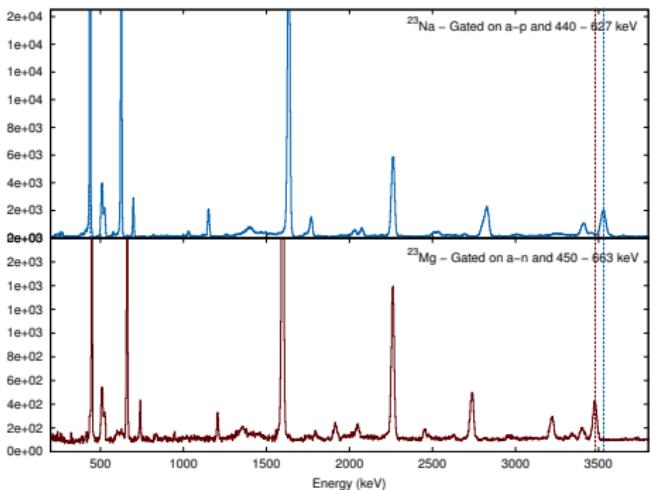
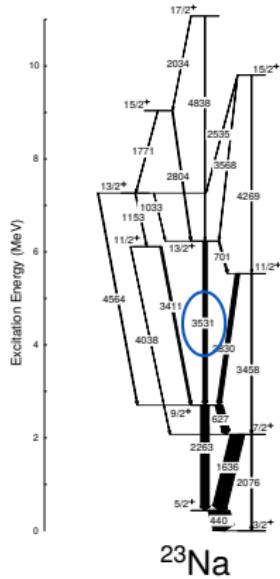
# $^{23}\text{Mg}$ - $^{23}\text{Na}$ Level Scheme



$$^{23}\text{Na}; 2830 \text{ keV}: \frac{11}{2}^+ \rightarrow \frac{9}{2}^+$$

$$^{23}\text{Mg}; 2739 \text{ keV}: \frac{11}{2}^+ \rightarrow \frac{9}{2}^+$$

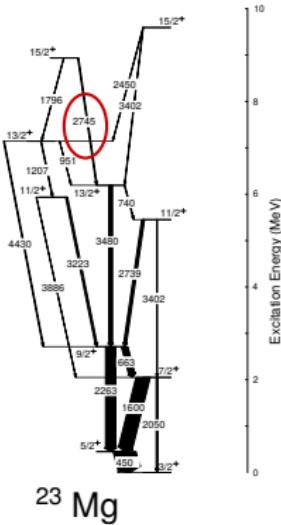
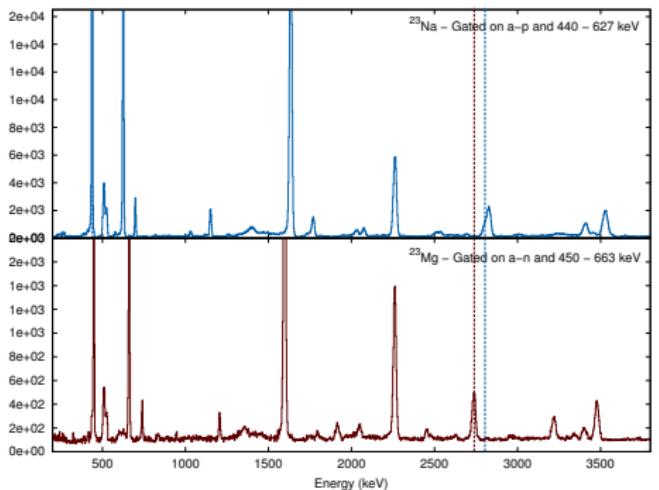
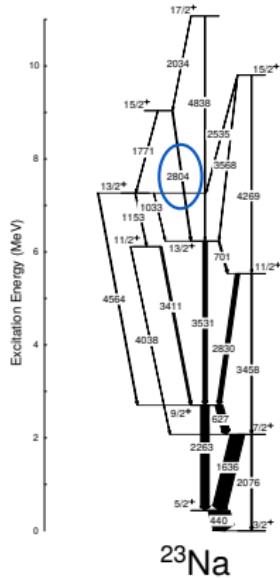
## <sup>23</sup>Mg-<sup>23</sup>Na Level Scheme



$^{23}\text{Na}$ ; 3531 keV:  $\frac{13}{2}^+ \rightarrow \frac{9}{2}^+$

$^{23}\text{Mg}$ ; 3480 keV:  $\frac{13}{2}^+ \rightarrow \frac{9}{2}^+$

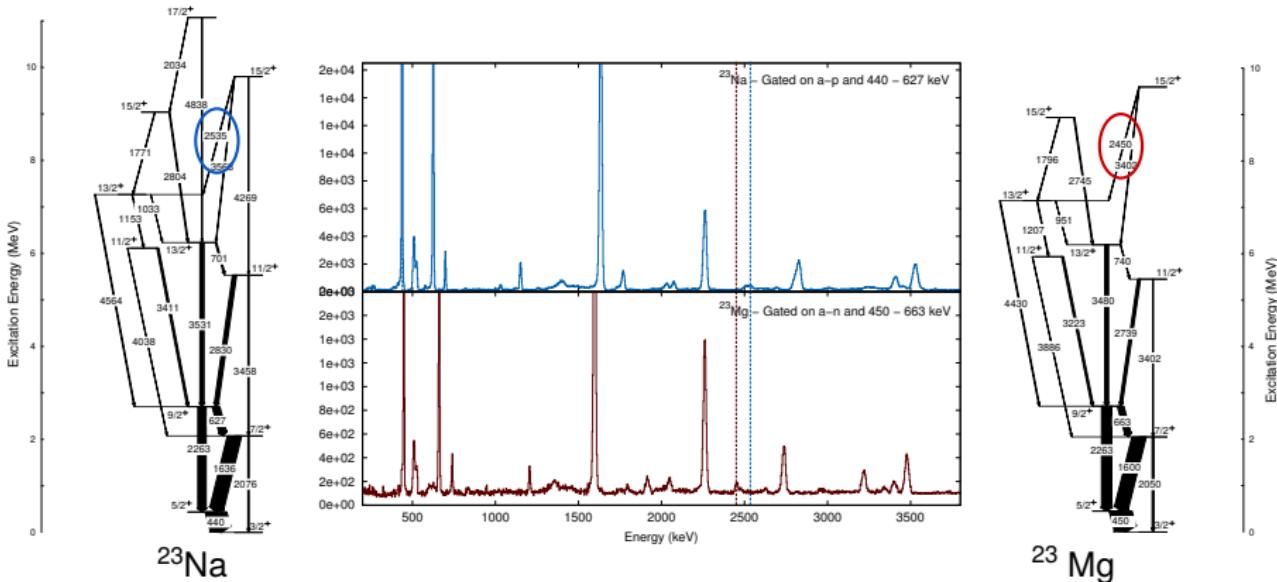
## <sup>23</sup>Mg-<sup>23</sup>Na Level Scheme



**$^{23}\text{Na}$ ; 2804 keV:**  $\frac{15}{2}^+ \rightarrow \frac{13}{2}^+$

$^{23}\text{Mg}$ ; 2745 keV:  $\frac{15}{2}^+ \rightarrow \frac{13}{2}^+$

# $^{23}\text{Mg}$ - $^{23}\text{Na}$ Level Scheme



$$^{23}\text{Na}; 2535 \text{ keV}: \frac{15}{2}^+ \rightarrow \frac{13}{2}^+$$

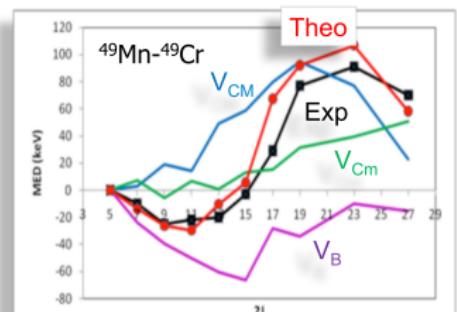
$$^{23}\text{Mg}; 2450 \text{ keV}: \frac{15}{2}^+ \rightarrow \frac{13}{2}^+$$

# Shell Model calculations: the “standard” approach

## MED terms

$f_{7/2}$  prescription:

- Multipole Coulomb Term
- Single particle corrections
- “Nuclear” ISB:  $V_B(J = 2)$
- Radial Term



A. P. Zuker et al., PRL 89, 142502 (2002); M.A. Bentley and S.M. Lenzi, Prog. Part. Nucl. Phys. 59(2007)

# The Radial Term $V_R$

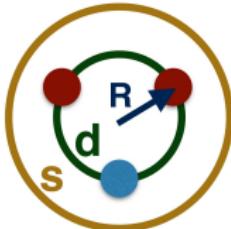
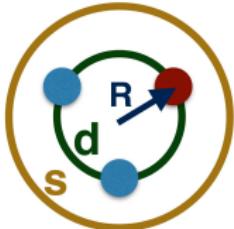
## Coulomb monopole Term

Charged sphere:

$$E_C = \frac{3}{5} \frac{Z(Z-1)e^2}{R_C}$$

It amounts to tens of MeV...  
...but the MED are **normalized** to  
ground state!

Bulk effect **vanishes!!**



## Radius variation with J

Related to **orbitals radii**:

- $l < \rightarrow r >$

**fp shell:** *p* orbitals **bigger** than *f*<sub>7/2</sub>  
**sd shell:** *s*<sub>1/2</sub> **bigger** than *d* orbitals

$$r_s - r_d \sim 1.5 \text{ fm}$$

Sensitive to **difference** in occ numbers

$$V_r \propto \left( \frac{m_\pi(0) + m_\nu(0)}{2} - \frac{m_\pi(J) + m_\nu(J)}{2} \right)$$

Ground State:  
 $V_r = 0$

# The Radial Term $V_R$

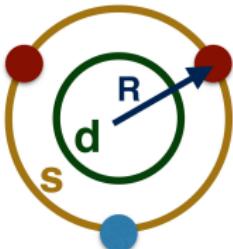
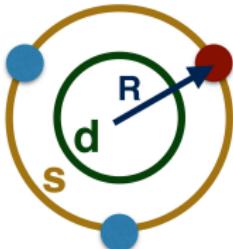
## Coulomb monopole Term

Charged sphere:

$$E_C = \frac{3}{5} \frac{Z(Z-1)e^2}{R_C}$$

It amounts to tens of MeV...  
...but the MED are **normalized** to  
ground state!

Bulk effect **vanishes!!**



## Radius variation with J

Related to **orbitals radii**:

- $J < \rightarrow r >$

**fp shell:** *p* orbitals **bigger** than *f*<sub>7/2</sub>  
**sd shell:** *s*<sub>1/2</sub> **bigger** than *d* orbitals

$$r_s - r_d \sim 1.5 \text{ fm}$$

Sensitive to **difference** in occ numbers

$$V_r \propto \left( \frac{m_\pi(0) + m_\nu(0)}{2} - \frac{m_\pi(J) + m_\nu(J)}{2} \right)$$

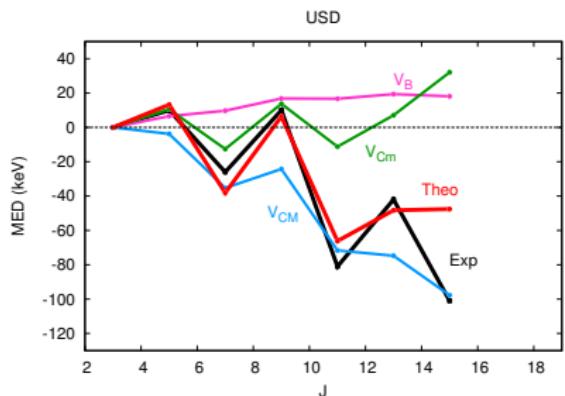
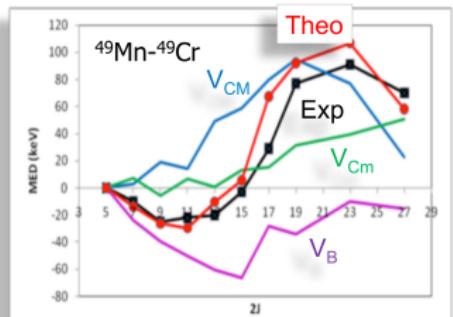
Excited State:  
 $V_r < 0$

# Shell Model calculations: the “standard” approach

## MED terms

$f_{7/2}$  prescription:

- Multipole Coulomb Term
- Single particle corrections
- “Nuclear” ISB:  $V_B(J=2)$
- Radial Term



## MED in $A=23$

- USD interaction
- Coulomb terms reproduce the experimental trend
- $V_B$  term makes things **worse**

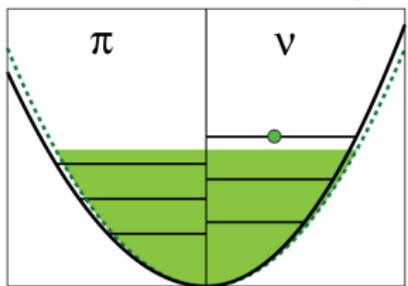
A. P. Zuker et al., PRL 89, 142502 (2002); M.A. Bentley and S.M. Lenzi, Prog. Part. Nucl. Phys. 59(2007)

# Shell Model calculations: alternative approach

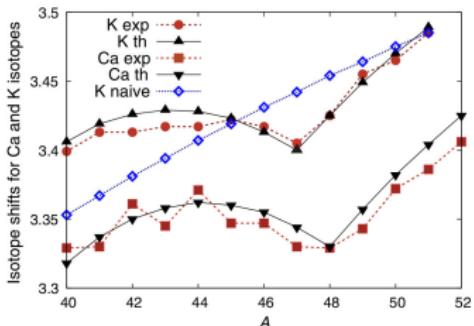
## MED terms

- Realistic NN interaction **N3LO**
- Coulomb interaction: **included**
- EMSO term: **included**
- Nuclear ISB term: **included**
- **Different**  $\hbar\omega$  for protons and neutrons

J. Bonnard et al., PRL 116, 212501(2016)



Extra particle polarizes both fluids



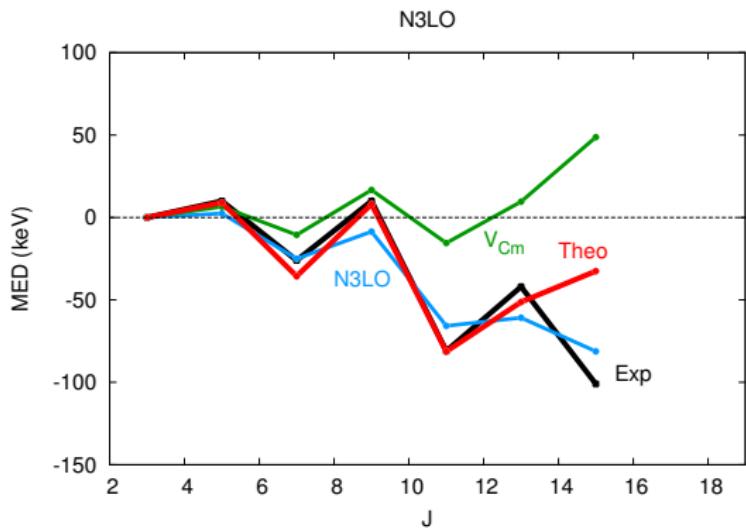
$r_\pi$  increases filling  $p_{3/2}$  with  $\nu$   
IV monopole polarizability

## Potential Wells

- $r_p$  from experiment
- IS:  $r_p(Z > N) = r_n(Z < N)$
- From MDE we get  $r_n(Z > N)$
- $r \rightarrow \hbar\omega$
- **Different**  $\hbar\omega$  for each **nucleus**
- Valid for **ground states**

$V_r$  in MED treated in the usual way

# Application to $^{23}\text{Mg}$ - $^{23}\text{Na}$ mirror nuclei



## Results

- N3LO reproduce the MED trend
- Radial term is **needed**
- $V_r(J = \frac{15}{2}^+)$  needs clarification

## Next Steps

- Different  $\hbar\omega$  for each **state** to account for changes in the radius with  $J$
- MED of **yrare** and **negative** parity states



# The Collaboration

A. Boso<sup>1,2</sup>, S. M. Lenzi<sup>1,2</sup>, F. Recchia<sup>1,2</sup>, J. Bonnard<sup>1,2</sup>, S. Aydin<sup>13</sup>, M. A. Bentley<sup>8</sup>, B. Cederwall<sup>6</sup>, E. Clement<sup>9</sup>, G. de France<sup>9</sup>, A. Di Nitto<sup>5</sup>, A. Dijon<sup>9</sup>, M. Doncel<sup>6</sup>, F. Ghazi-Moradi<sup>6</sup>, A. Gottardo<sup>3</sup>, T. Henry<sup>8</sup>, T. Huyuk<sup>7</sup>, G. Jaworski<sup>12</sup>, P.R. John<sup>1,2</sup>, I. Kuti<sup>4</sup>, B. Melon<sup>10</sup>, D. Mengoni<sup>1,2</sup>, C. Michelagnoli<sup>1,2</sup>, V. Modamio<sup>3</sup>, D.R. Napoli<sup>3</sup>, B.M. Nyakò<sup>4</sup>, J. Nyberg<sup>11</sup>, M. Palacz<sup>12</sup>, J.J. Valiente-Dobon<sup>3</sup>

<sup>1</sup> Dipartimento di Fisica e Astronomia Università degli Studi di Padova, Padova, Italy.

<sup>2</sup> INFN, Sezione di Padova, Padova, Italy.

<sup>3</sup> INFN, Laboratori Nazionali di Legnaro, Legnaro (Padova) Italy.

<sup>4</sup> Institute of Nuclear Research (ATOMKI) of Hung. Acad. Sciences, Debrecen, Hungary

<sup>5</sup> Dipartimento di Scienze Fisiche and INFN, Sezione di Napoli, Napoli, Italy

<sup>6</sup> Department of Physics, Royal Institute of Technology, Stockholm, Sweden

<sup>7</sup> IFIC-CSIC, Valencia, Spain.

<sup>8</sup> University of York, York, United Kingdom.

<sup>9</sup> GANIL, Caen, France.

<sup>10</sup> Dipartimento di Fisica and INFN, Sezione di Firenze, Firenze, Italy

<sup>11</sup> Department of Physics and Astronomy, Uppsala University, Uppsala, Sweden

<sup>12</sup> Heavy Ion Laboratory, Warsaw University, Warszawa, Poland

<sup>13</sup> Aksaray Üniversitesi, Department of Physics, Aksaray, Turkey

## Thank you for your attention !!

# Duflo-Zuker approach

## Proton radii

$$\sqrt{\langle r_\pi^2 \rangle} = \rho_\pi = A^{1/3} \left( \rho_0 - \frac{\zeta}{2} \frac{t}{A^{4/3}} - \frac{v}{2} \left( \frac{t}{A} \right)^2 \right) e^{(g/A)}$$

$$+ \lambda [z(D_\pi - z)/D_\pi^2 \times n(D_\nu - n)/D_\nu^2] A^{-1/3},$$

Related to  $\hbar\omega$  by:

$$\hbar\omega_\pi = \frac{41.47}{\langle r_\pi^2 \rangle} \sum_i z_i (p_i + 3/2) / Z$$

To take into account shell effect

$$\langle r_\pi^2 \rangle = \frac{41.47}{\hbar\omega_\pi} \sum_i z_i (p_i + 3/2 + \delta_i) / Z,$$

where  $\delta \neq 0$  only for  $s_{1/2}$

$$\delta = \begin{cases} \delta_- & \text{if } N \text{ and } Z < 14 \\ \delta_+ & \text{if } N \text{ or } Z \geq 14. \end{cases}$$

J. Bonnard et al., PRL 116, 212501(2016)

