Shape coexistence effects on isospin-symmetry breaking and $\beta$ decay of proton-rich $A\sim 70$ nuclei

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Outline

• complex EXCITED VAMPIR beyond-mean-field model

• shape-coexistence and isospin-symmetry-breaking effects in the A=74 isovector triplet on:
  - Coulomb energy differences (CED)
  - mirror energy differences (MED)
  - triplet energy differences (TED)
  - superallowed Fermi $\beta$-decay of the $Z=\text{N}+2$ isotope $^{74}\text{Sr}$

• shape-coexistence effects on terrestrial and stellar weak interaction rates of $^{74}\text{Sr}$
A~70 proton-rich nuclei manifest exotic structure and dynamics generated by the interplay of

- shape coexistence and shape mixing
- competing $T=0$ and $T=1$ pairing correlations
- isospin-symmetry-breaking interactions

responsible for
drastic changes in structure with number of nucleons, spin, and excitation energy

Challenges for theory

- realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods
- comprehensive understanding of structure phenomena and $\beta$-decay properties
- reliable predictions on stellar weak interaction rates

based on
self-consistent description of experimentally accessible properties
complex VAMPIR model family

• the model space is defined by a finite dimensional set of spherical single particle states

• the effective many-body Hamiltonian is represented as a sum of one- and two-body terms

• the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua

• the HFB transformations are essentially complex and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry (T=1 and T=0 neutron-proton pairing correlations already included at the mean field level)

• the broken symmetries (s=N, Z, I, p) are restored by projection before variation

* The models allow to use rather large model spaces and realistic effective interactions
Beyond-mean-field variational procedure

complex Vampir

$$E^s[F^s_1] = \frac{\langle F^s_1 | \hat{H} \Theta^s_{00} | F^s_1 \rangle}{\langle F^s_1 | \Theta^s_{00} | F^s_1 \rangle}$$

$$|\psi(F^s_1); s M\rangle = \frac{\Theta^s_{M0} | F^s_1 \rangle}{\sqrt{\langle F^s_1 | \Theta^s_{00} | F^s_1 \rangle}}$$

complex Excited Vampir

$$|\psi(F^s_i); s M\rangle = \sum_{j=1}^{i} |\phi(F^s_j)\rangle \alpha^i_j \quad \text{for } i = 1, \ldots, n - 1$$

$$|\phi(F^s_i); s M\rangle = \Theta^s_{M0} | F^s_i \rangle$$

$$|\psi(F^s_n); s M\rangle = \sum_{j=1}^{n-1} |\phi(F^s_j)\rangle \alpha^n_j + |\phi(F^s_n)\rangle \alpha^n_n$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi^{(n)}_{\alpha}; s M\rangle = \sum_{i=1}^{n} |\psi_i; s M\rangle a_i^{(n)} , \quad \alpha = 1, \ldots, n$$
$A \sim 70$ mass region

$^{40}\text{Ca} - \text{core}$

**model space for both: protons and neutrons**

\begin{align*}
1p_{1/2} & \ 1p_{3/2} & \ 0f_{5/2} & \ 0f_{7/2} & \ 1d_{5/2} & \ 0g_{9/2}
\end{align*}

(charge-symmetric basis + Coulomb contributions to the $\pi$-spe from the core)

**renormalized $G$-matrix (Bonn CD potential)**

- pairing properties enhanced by short range Gaussians for:
  
  \[ T = 1 : \ pp \ (-35 \text{ MeV}), \ np \ (-20 \text{ MeV}), \ nn \ (-35 \text{ MeV}) \]
  
  \[ T = 0, \ S = 0 \text{ and } S = 1 \ (-35 \text{ MeV}) \]

- onset of deformation influenced by monopole shifts:

  \[ <0g_{9/2} \ 0f; \ T=0 \ |G| \ 0g_{9/2} \ 0f;T=0> \ (0f_{5/2}, \ 0f_{7/2}) \]

  \[ <1d_{5/2} \ 1p; \ T=0 \ |G| \ 1d_{5/2} \ 1p;T=0> \ (1p_{1/2}, \ 1p_{3/2}) \]

- Coulomb interaction between valence protons added
Isospin-symmetry breaking and shape coexistence effects in the $A=74$ isovector triplet $^{38}\text{Sr}_{36} - ^{37}\text{Rb}_{37} - ^{36}\text{Kr}_{38}$


Coulomb Energy Differences

$$CED_{J,T=1} = E^{*}_{J,T=1,Tz=0} - E^{*}_{J,T=1,Tz=1}$$

spectroscopic quadrupole moments

<table>
<thead>
<tr>
<th>$I(\hbar)$</th>
<th>$^{74}\text{Kr}$ Exp</th>
<th>$^{74}\text{Rb}$</th>
<th>$^{74}\text{Sr}$</th>
</tr>
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<tbody>
<tr>
<td>$2_1^+$</td>
<td>-54</td>
<td>-57</td>
<td>-50</td>
</tr>
<tr>
<td>$2_2^+$</td>
<td>49</td>
<td>53</td>
<td>48</td>
</tr>
<tr>
<td>$4_1^+$</td>
<td>-74</td>
<td>-77</td>
<td>-70</td>
</tr>
<tr>
<td>$4_2^+$</td>
<td>68</td>
<td>72</td>
<td>67</td>
</tr>
<tr>
<td>$6_1^+$</td>
<td>-85</td>
<td>-86</td>
<td>-81</td>
</tr>
<tr>
<td>$6_2^+$</td>
<td>78</td>
<td>81</td>
<td>80</td>
</tr>
</tbody>
</table>

Exp - full symbols
theory - open symbols
Shape mixing manifested in the structure of wave functions \( A = 74 \)

- Significant oblate-prolate mixing decreasing with increasing spin

<table>
<thead>
<tr>
<th>( ^{74}\text{Kr} )</th>
<th>( I(\hbar) )</th>
<th>Prolate content</th>
<th>Oblate content</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^+ )</td>
<td>82(1)(1) %</td>
<td>14(1)(1) %</td>
<td></td>
</tr>
<tr>
<td>( 2^+ )</td>
<td>92(1)(1) %</td>
<td>6 %</td>
<td></td>
</tr>
<tr>
<td>( 4^+ )</td>
<td>95(1)(1) %</td>
<td>3 %</td>
<td></td>
</tr>
<tr>
<td>( 6^+ )</td>
<td>97(1) %</td>
<td>1(1) %</td>
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</table>

<table>
<thead>
<tr>
<th>( ^{74}\text{Rb} )</th>
<th>( I(\hbar) )</th>
<th>Prolate content</th>
<th>Oblate content</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^+ )</td>
<td>85(1) %</td>
<td>12(1) %</td>
<td></td>
</tr>
<tr>
<td>( 2^+ )</td>
<td>94(1) %</td>
<td>4 %</td>
<td></td>
</tr>
<tr>
<td>( 4^+ )</td>
<td>96(1) %</td>
<td>2 %</td>
<td></td>
</tr>
<tr>
<td>( 6^+ )</td>
<td>97(1) %</td>
<td>1 %</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( ^{74}\text{Sr} )</th>
<th>( I(\hbar) )</th>
<th>Prolate content</th>
<th>Oblate content</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^+ )</td>
<td>77(2) %</td>
<td>19(1) %</td>
<td></td>
</tr>
<tr>
<td>( 2^+ )</td>
<td>87(1) %</td>
<td>11 %</td>
<td></td>
</tr>
<tr>
<td>( 4^+ )</td>
<td>90(1) %</td>
<td>8 %</td>
<td></td>
</tr>
<tr>
<td>( 6^+ )</td>
<td>92(1) %</td>
<td>5(1) %</td>
<td></td>
</tr>
</tbody>
</table>

- Maximum oblate-prolate mixing in \( ^{74}\text{Sr} \)
Isospin-symmetry-breaking and shape-mixing effects on
Mirror Energy Differences & Triplet Energy Differences

**Charge-symmetry breaking:**
\[ V^{(1)}_{\text{INC}} = V_{pp} - V_{nn} \]
\[ (V_{nn} \text{ 1\% stronger than } V_{pp}) \]

**MED_J,T=1 = E^*_{J,T=1,Tz=-1} - E^*_{J,T=1,Tz=+1} \]

**Charge-independence breaking:**
\[ V^{(2)}_{\text{INC}} = V_{pp} + V_{nn} - 2 V_{pn} \]
\[ (V_{pn} \text{ 2.5\% stronger than the average of } V_{pp} \text{ and } V_{nn}) \]

\[ TED_{J,T=1} = E^*_{J,T=1,Tz=-1} + E^*_{J,T=1,Tz=+1} - 2E^*_{J,T=1,Tz=0} \]

**A=74:** complex Excited Vampir predictions

*experimental results* (J. Henderson, Phys. Rev. C 90, 051303(R) (2014))
Self-consistent terrestrial and stellar weak interaction rates for $^{74}\text{Sr}$

**Fermi transition probabilities**

\[
B_{if}(F) = \frac{1}{2J_i + 1} \frac{g^2_V}{4\pi} |M_F|^2
\]

\[
M_F \equiv (\xi_f J_f || \hat{1} || \xi_i J_i)
= \delta_{J_i J_f} \sum_{ab} M_F(ab) (\xi_f J_f || [c^+_a \tilde{c}_b]_0 || \xi_i J_i)
\]

\[
M_F(ab) = (a || \hat{1} || b)
\]

**Gamow-Teller transition probabilities**

\[
B_{if}(GT) = \frac{1}{2J_i + 1} \frac{g^2_A}{4\pi} |M_{GT}|^2
\]

\[
M_{GT} \equiv (\xi_f J_f || \hat{\sigma} || \xi_i J_i)
= \sum_{ab} M_{GT}(ab) (\xi_f J_f || [c^+_a \tilde{c}_b]_1 || \xi_i J_i)
\]

\[
M_{GT}(ab) = 1/\sqrt{3}(a || \hat{\sigma} || b)
\]

**Isospin-symmetry-breaking and shape coexistence effects on superallowed Fermi $\beta$-decay**

\[
f t(1 + \delta_R)(1 - \delta_c) = \frac{K}{2G^2_{V}(1 + \Delta^2_R)}
\]

$\delta_c$ – isospin-symmetry-breaking correction


$Q_{EC} = 11.090 \text{ MeV}$

$1\% \leq \delta_c \leq 3\%$

*Nonanalog branches: $0^+_{II}, 0^+_VI \leq 0.8 \%$*
Relevant for astrophysical scenarios on rp-process path may be $2^+_{\text{yrast}}$, $0^+_{\text{exc}}$, $2^+_{\text{sec}}$ decay.

$1\% \leq \delta_c \leq 3.6\%$

Nonanalog branches:
$2^+_{II} \leq 1.3\%$, $2^+_{IV} \leq 0.8\%$
Gamow-Teller $\beta$ decay and shape coexistence for $^{74}\text{Sr}$


Independent chains of variational calculations in parent and daughter nuclei

Large variety of deformations in daughter states revealed by spectroscopic quadrupole moments
Gamow-Teller strength distributions for the decay of $0^+$ and $2^+$ states in $^{74}\text{Sr}$

Specific shape mixing for each parent state influences the strength distributions.
Influence of specific shape mixing in the parent and daughter states on strength distributions

Contributions from $p^{\nu(\pi)}_{1/2}p^{\pi(\nu)}_{3/2}$, $p^{\nu(\pi)}_{3/2}p^{\pi(\nu)}_{3/2}$, $f^{\nu}v_5^{\pi}f_5^{\pi}_{5/2}$, $f^{\nu(\pi)}_{5/2}f^{\pi(\nu)}_{7/2}$, $g^{\nu}_{9/2}g^{\pi}_{9/2}$ matrix elements
(coherent / cancelling effect)
Terrestrial half-lives

\[ \frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0 < E_f < Q_{EC}} f(Z, E_f)[B_{if}(GT) + B_{if}(F)] \]

\[ T_{1/2}^{GT} = 137 \text{ ms} \quad T_{1/2}^{F} = 48 \text{ ms} \quad T_{1/2}^{EXVAM} = 36 \text{ ms} \quad T_{1/2}^{\text{exp}} = 27(8) \text{ ms} \]
Weak interaction rates in X-ray burst astrophysical environment

In the X-ray burst stellar environment at densities ($\sim10^6$ mol/cm$^3$) and temperatures ($\sim10^9$K) typical for the rp-process the contribution of thermally populated low-lying $0^+$ and $2^+$ states may be relevant.


\[ \chi^\alpha = \frac{\ln 2}{K} \sum_i (2J_i + 1) e^{-E_i/(kT)} \sum_i B_{ij} \phi^\alpha_{ij} \]

\[ G(Z, A, T) = \sum_i (2J_i + 1) \exp(-E_i/(kT)) \]

\[ B_{ij} = B_{ij}(F') + B_{ij}(GT) \]

\[ \phi^{\text{ec}}_{ij} = \int_{w_l}^\infty wp(Q_{ij} + w)^2 F(Z, w) S_e(w)(1 - S_\nu(Q_{ij} + w)) dw \]

\[ \phi^{\beta^+}_{ij} = \int_{1}^{Q_{ij}} wp(Q_{ij} - w)^2 F(-Z + 1, w)(1 - S_p(w))(1 - S_\nu(Q_{ij} - w)) dw \]
Stellar rates for $^{74}$Sr: $\beta^+$ - decay

$E^{th}_{0^+_{exc}} = 0.564$ MeV \hspace{1cm} $E_{2^+_\text{yrast}} = 0.471$ MeV \hspace{1cm} $E^{th}_{2^+_\text{sec}} = 0.823$ MeV

\begin{align*}
\lambda_{F}(s^{-1}) & \hspace{1cm} \lambda_{G}(s^{-1}) \hspace{1cm} \lambda(s^{-1}) \\
\text{T(GK)} & \hspace{1cm} \text{T(GK)} & \hspace{1cm} \text{T(GK)} \\
\end{align*}
\( \beta^+ \) and electron capture rates for \(^{74}\text{Sr}\)
Summary

complex EXCITED VAMPIR beyond-mean-field model
  self-consistently describes shape-coexistence effects on
  
  • low-spin structure in $A=74$ isovector triplet
  
  • superallowed Fermi $\beta$-decay of the $Z=N+2$ isotope $^{74}\text{Sr}$
  
  • terrestrial and stellar weak interaction rates under X-ray burst environment for $^{74}\text{Sr}$