

# **Shape coexistence effects on isospin-symmetry breaking and $\beta$ decay of proton-rich A~70 nuclei**

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## *Outline*

- *complex EXCITED VAMPIR beyond-mean-field model*
- **shape-coexistence and isospin-symmetry-breaking effects in the A=74 isovector triplet on:**
  - *Coulomb energy differences (CED)*
  - *mirror energy differences (MED)*
  - *triplet energy differences (TED)*
  - *superallowed Fermi  $\beta$ -decay of the Z=N+2 isotope  $^{74}\text{Sr}$*
- **shape-coexistence effects on terrestrial and stellar weak interaction rates of  $^{74}\text{Sr}$**

*A~70 proton-rich nuclei manifest exotic structure and dynamics generated by the interplay of*

- *shape coexistence and shape mixing*
- *competing T=0 and T=1 pairing correlations*
- *isospin-symmetry-breaking interactions*

*responsible for*

*drastic changes in structure with number of nucleons, spin, and excitation energy*

*Challenges for theory*

- *realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods*
- *comprehensive understanding of structure phenomena and  $\beta$ -decay properties*
- *reliable predictions on stellar weak interaction rates*

*based on*

*self-consistent description of experimentally accessible properties*

## *complex* VAMPIR model family

- the **model space** is defined by a finite dimensional set of **spherical single particle states**
  - the effective many-body Hamiltonian is represented as a sum of **one- and two-body terms**
  - the basic building blocks are **Hartree-Fock-Bogoliubov (HFB) vacua**
  - the HFB transformations are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry  
*( $T=1$  and  $T=0$  neutron-proton pairing correlations already included at the mean field level )*
  - the broken symmetries ( $s=N, Z, I, p$ ) are restored by projection before variation
- \* *The models allow to use rather large model spaces and realistic effective interactions*

## Beyond-mean-field variational procedure

### complex Vampir

$$E^s[F_1^s] = \frac{\langle F_1^s | \hat{H} \hat{\Theta}_{00}^s | F_1^s \rangle}{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}$$

$\hat{\Theta}_{00}^s$  - symmetry projector  
 $|F_1^s\rangle$  - HFB vacuum

$$|\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}_{M0}^s |F_1^s\rangle}{\sqrt{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}}$$

### complex Excited Vampir

$$|\psi(F_i^s); sM\rangle = \sum_{j=1}^i |\phi(F_j^s)\rangle \alpha_j^i \quad \text{for } i = 1, \dots, n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}_{M0}^s |F_i^s\rangle$$

$$|\psi(F_n^s); sM\rangle = \sum_{j=1}^{n-1} |\phi(F_j^s)\rangle \alpha_j^n + |\phi(F_n^s)\rangle \alpha_n^n$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM\rangle = \sum_{i=1}^n |\psi_i; sM\rangle f_{i\alpha}^{(n)}, \quad \alpha = 1, \dots, n$$

## *A ~ 70 mass region*

### *$^{40}\text{Ca}$ - core*

*model space for both: protons and neutrons*

$1p_{1/2} \ 1p_{3/2} \ 0f_{5/2} \ 0f_{7/2} \ 1d_{5/2} \ 0g_{9/2}$

*(charge-symmetric basis + Coulomb contributions to the  $\pi$ -spe from the core)*

*renormalized G-matrix ( Bonn CD potential)*

- *pairing properties enhanced by short range Gaussians for:*

T = 1 : pp (-35 MeV), np (-20 MeV), nn (-35 MeV)

T = 0, S = 0 and S = 1 (-35 MeV)

- *onset of deformation influenced by monopole shifts:*

$\langle 0g_{9/2} \ 0f; T=0 | G | 0g_{9/2} \ 0f; T=0 \rangle \quad (0f_{5/2}, \ 0f_{7/2})$

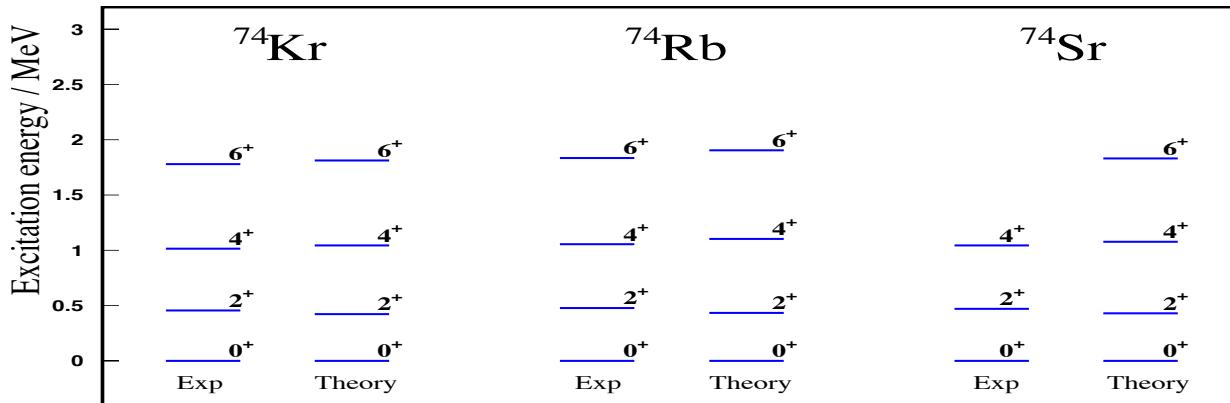
$\langle 1d_{5/2} \ 1p; T=0 | G | 1d_{5/2} \ 1p; T=0 \rangle \quad (1p_{1/2}, \ 1p_{3/2})$

- *Coulomb interaction between valence protons added*

# Isospin-symmetry breaking and shape coexistence effects in the $A=74$ isovector triplet



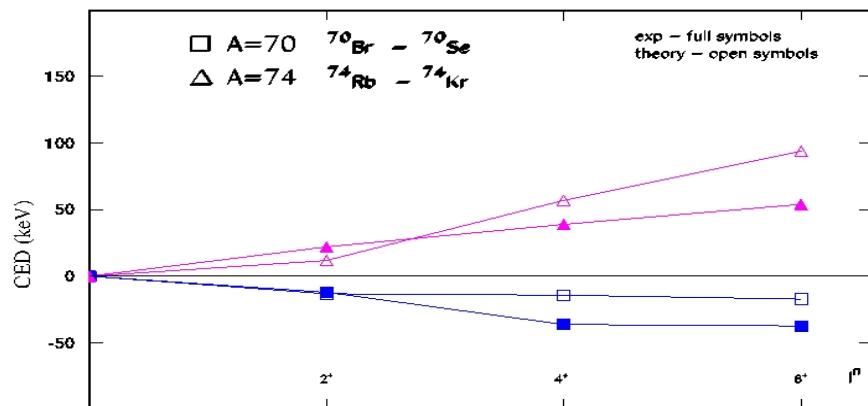
A. Petrovici, Phys. Rev. C 91, 014302 (2015)



## Coulomb Energy Differences

$$CED_{J,T=1} = E^*_{J,T=1,Tz=0} - E^*_{J,T=1,Tz=+1}$$

spectroscopic quadrupole moments



$I(\hbar)$	$^{74}\text{Kr}$	Exp	$^{74}\text{Rb}$	$^{74}\text{Sr}$
$2_1^+$	-54	-53(24)	-57	-50
$2_2^+$	49	24(21)	53	48
$4_1^+$	-74	-80(40)	-77	-70
$4_2^+$	68		72	67
$6_1^+$	-85	-130(50)	-86	-81
$6_2^+$	78		81	80

## *Shape mixing manifested in the structure of wave functions*    $A = 74$

- ♦ significant oblate-prolate mixing decreasing with increasing spin

$^{74}\text{Kr}$     $I(\hbar)$    *Prolate content*   *Oblate content*

$0^+$	$82(1)(1) \%$	$14(1)(1) \%$
$2^+$	$92(1)(1) \%$	$6 \%$
$4^+$	$95(1)(1) \%$	$3 \%$
$6^+$	$97(1) \%$	$1(1) \%$

$^{74}\text{Rb}$     $I(\hbar)$    *Prolate content*   *Oblate content*

$0^+$	$85(1) \%$	$12(1) \%$
$2^+$	$94(1) \%$	$4 \%$
$4^+$	$96(1) \%$	$2 \%$
$6^+$	$97(1) \%$	$1 \%$

$^{74}\text{Sr}$     $I(\hbar)$    *Prolate content*   *Oblate content*

$0^+$	$77(2) \%$	$19(1) \%$
$2^+$	$87(1) \%$	$11 \%$
$4^+$	$90(1) \%$	$8 \%$
$6^+$	$92(1) \%$	$5(1) \%$

- ♦ maximum oblate-prolate mixing in  $^{74}\text{Sr}$

# *Isospin-symmetry-breaking and shape-mixing effects on Mirror Energy Differences & Triplet Energy Differences*

*Charge-symmetry breaking:*

$$V^{(1)}_{INC} = V_{pp} - V_{nn}$$

( $V_{nn}$  1% stronger than  $V_{pp}$ )

$$MED_{J,T=I} = E^*_{J,T=I,Tz=-I} - E^*_{J,T=I,Tz=+I}$$

*Charge-independence breaking:*

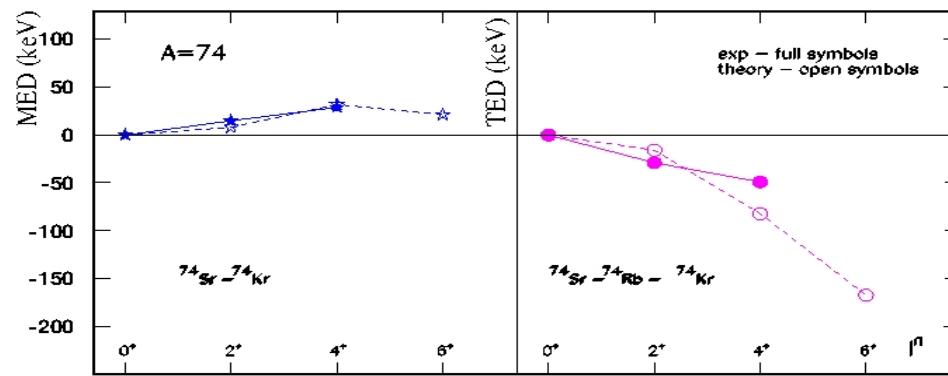
$$V^{(2)}_{INC} = V_{pp} + V_{nn} - 2 V_{pn}$$

( $V_{pn}$  2.5% stronger than the average of  $V_{pp}$  and  $V_{nn}$ )

$$TED_{J,T=I} = E^*_{J,T=I,Tz=-I} + E^*_{J,T=I,Tz=+I} - 2E^*_{J,T=I,Tz=0}$$

**A=74:** complex Excited Vampir predictions

*experimental results* (*J. Henderson, Phys. Rev. C 90, 051303(R) (2014)*)



# *Self-consistent terrestrial and stellar weak interaction rates for $^{74}\text{Sr}$*

## *Fermi transition probabilities*

$$B_{if}(F) = \frac{1}{2J_i + 1} \frac{g_V^2}{4\pi} |M_F|^2$$

$$M_F \equiv (\xi_f J_f || \hat{1} || \xi_i J_i)$$

$$= \delta_{J_i J_f} \sum_{ab} M_F(ab) (\xi_f J_f || [c_a^\dagger \tilde{c}_b]_0 || \xi_i J_i)$$

$$M_F(ab) = (a || \hat{1} || b)$$

## *Gamow-Teller transition probabilities*

$$B_{if}(GT) = \frac{1}{2J_i + 1} \frac{g_A^2}{4\pi} |M_{GT}|^2$$

$$M_{GT} \equiv (\xi_f J_f || \hat{\sigma} || \xi_i J_i)$$

$$= \sum_{ab} M_{GT}(ab) (\xi_f J_f || [c_a^\dagger \tilde{c}_b]_1 || \xi_i J_i)$$

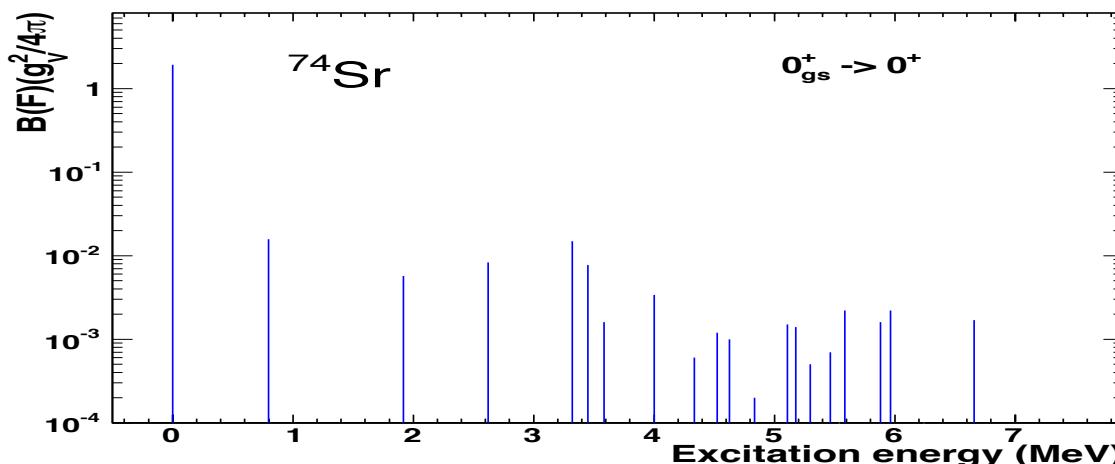
$$M_{GT}(ab) = 1/\sqrt{3}(a || \hat{\sigma} || b)$$

## *Isospin-symmetry-breaking and shape coexistence effects on superallowed Fermi $\beta$ -decay*

$$ft(1 + \delta_R)(1 - \delta_c) = \frac{K}{2G_v^2(1 + \Delta_R^v)}$$

$\delta c$  – isospin-symmetry-breaking correction

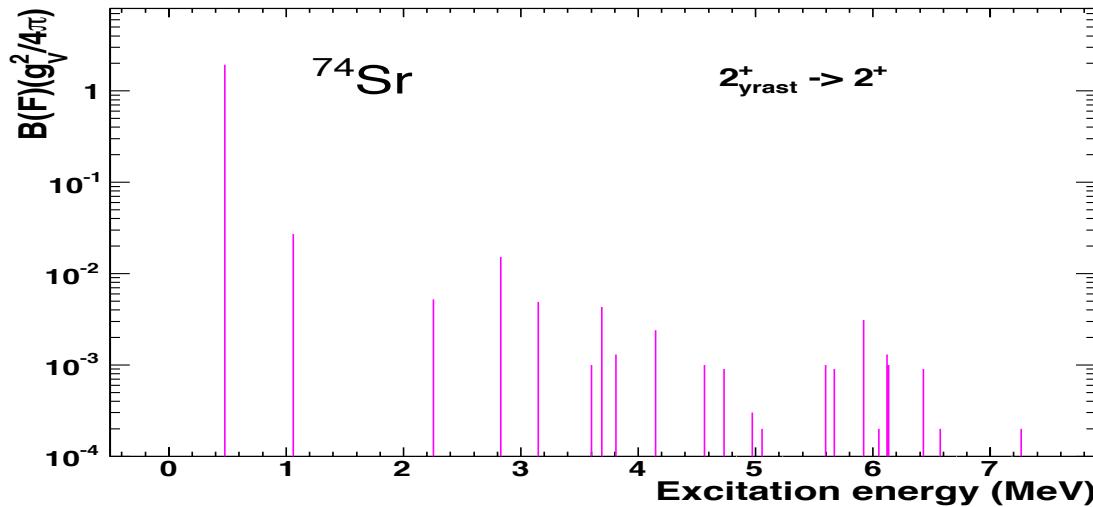
A. Petrovici, J. Phys.: Conf. Series (2016)



$1\% \leq \delta_c \leq 3\%$

Nonanalog branches:  
 $0^+_{II}$ ,  $0^+_{VI} \leq 0.8\%$

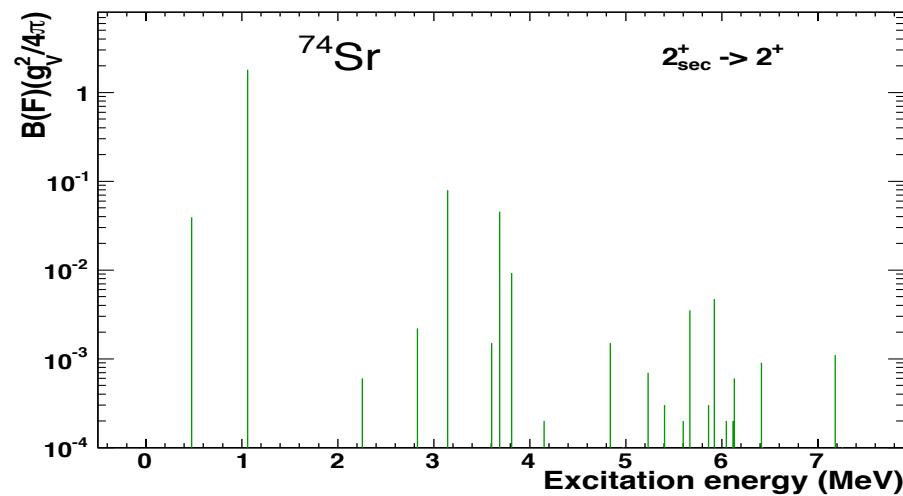
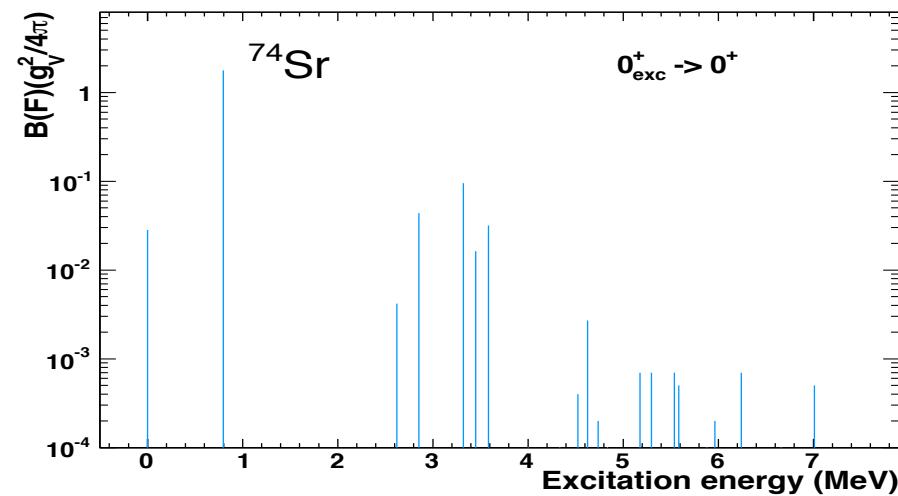
*Relevant for astrophysical scenarios on rp-process path may be  $2^+_{\text{yrast}}$ ,  $0^+_{\text{exc}}$ ,  $2^+_{\text{sec}}$  decay*



$$1\% \leq \delta_c \leq 3.6\%$$

Nonanalog branches:

$$2^+_{II} \leq 1.3\%, \quad 2^+_{IV} \leq 0.8\%$$

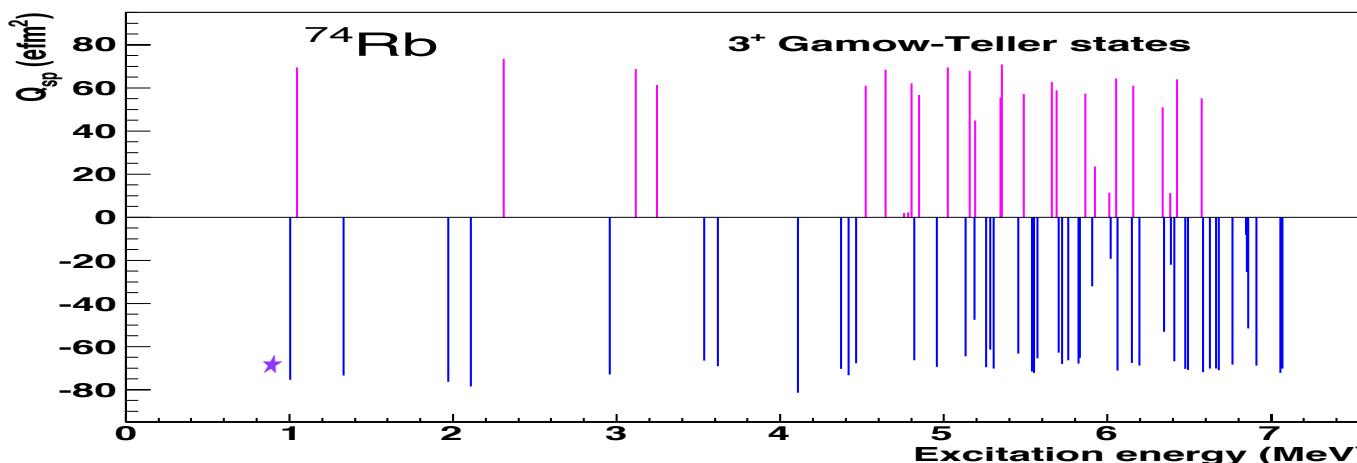
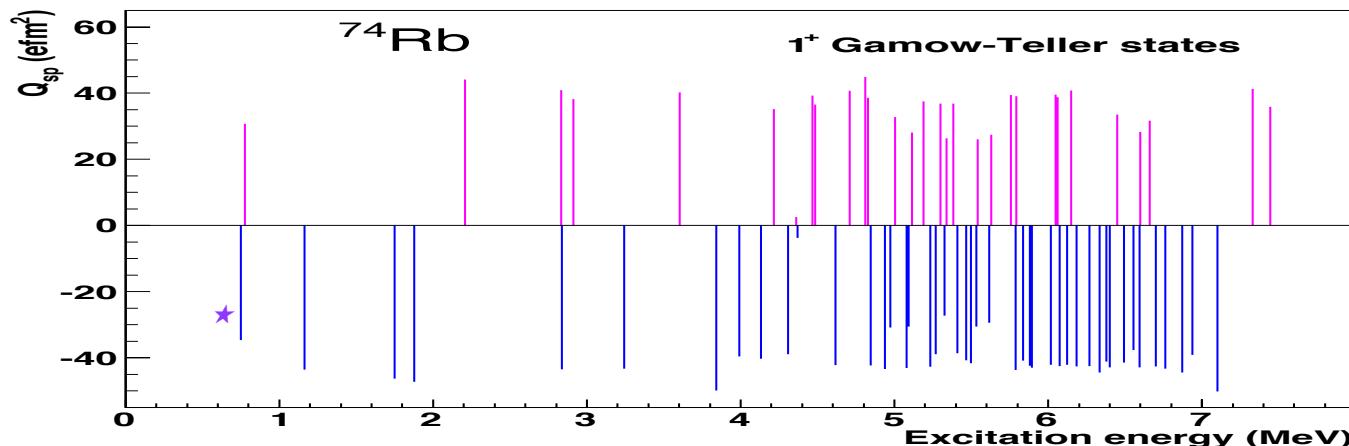


# *Gamow-Teller $\beta$ decay and shape coexistence for $^{74}\text{Sr}$*

*A. Petrovici and O. Andrei, Phys. Rev. C92, 064305 (2015)*

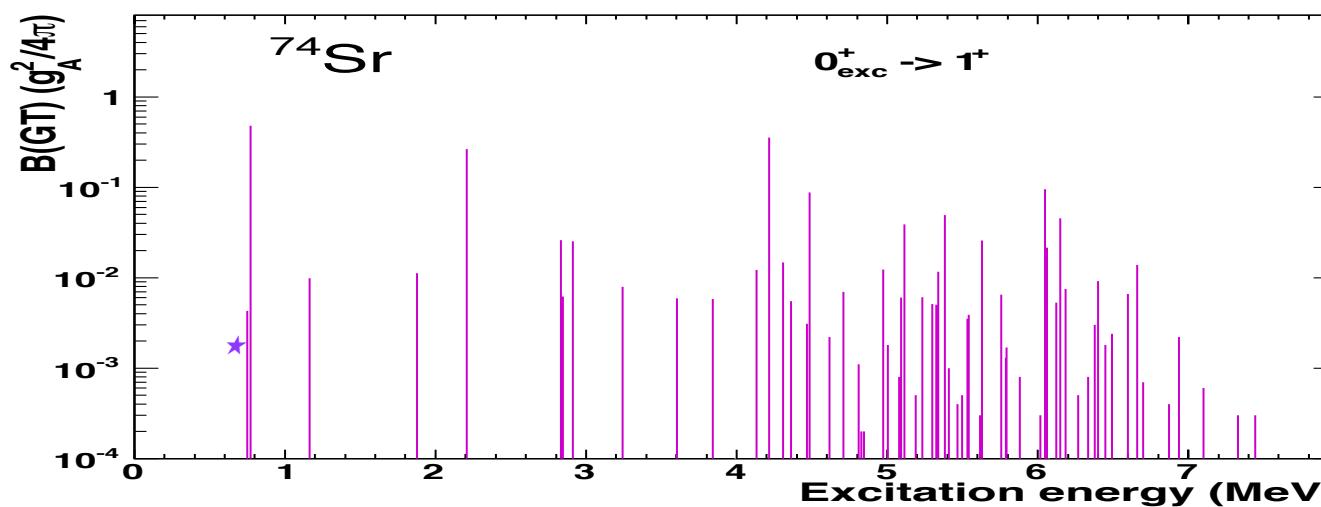
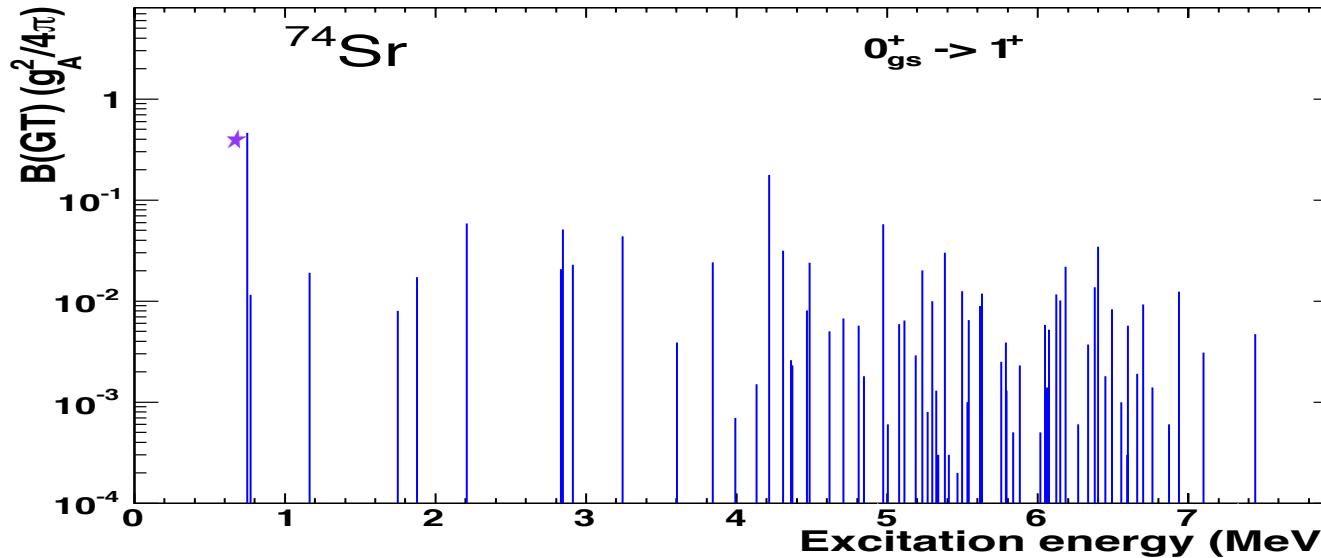
*Independent chains of variational calculations in parent and daughter nuclei*

*Large variety of deformations in daughter states revealed by spectroscopic quadrupole moments*

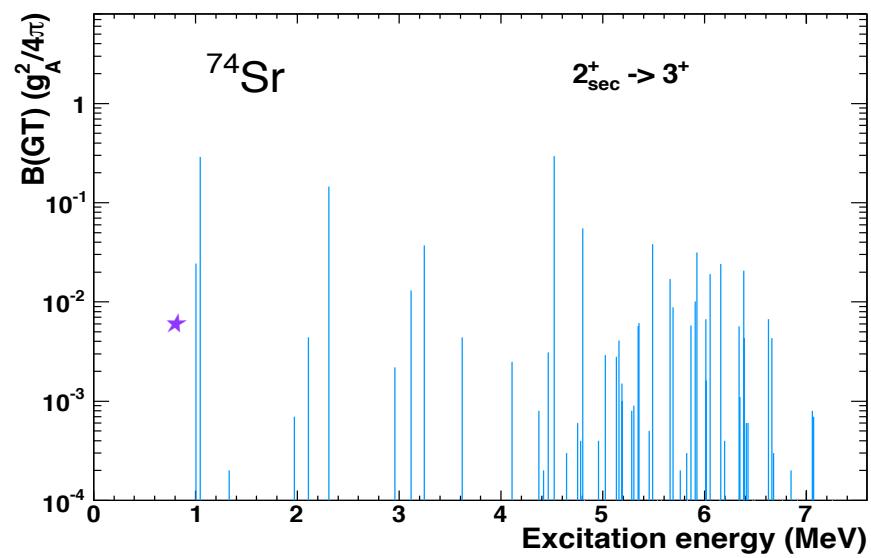
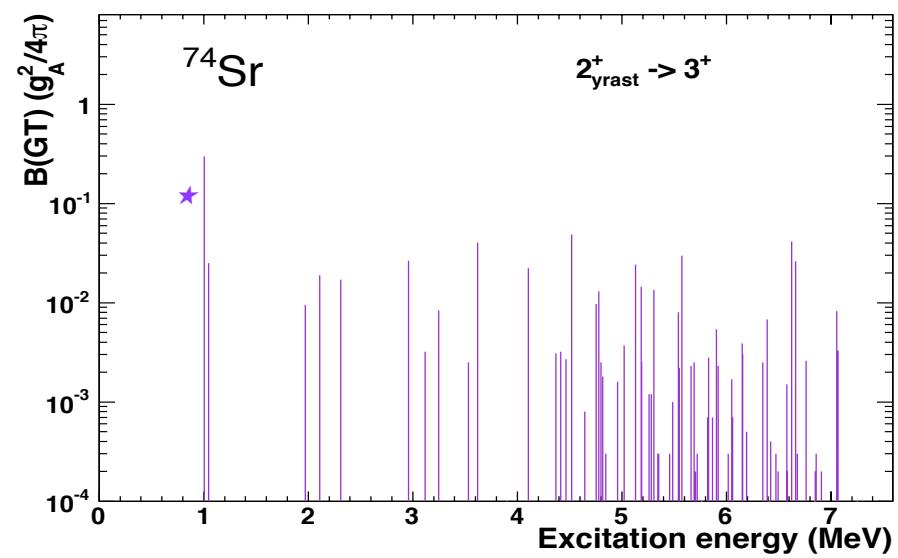
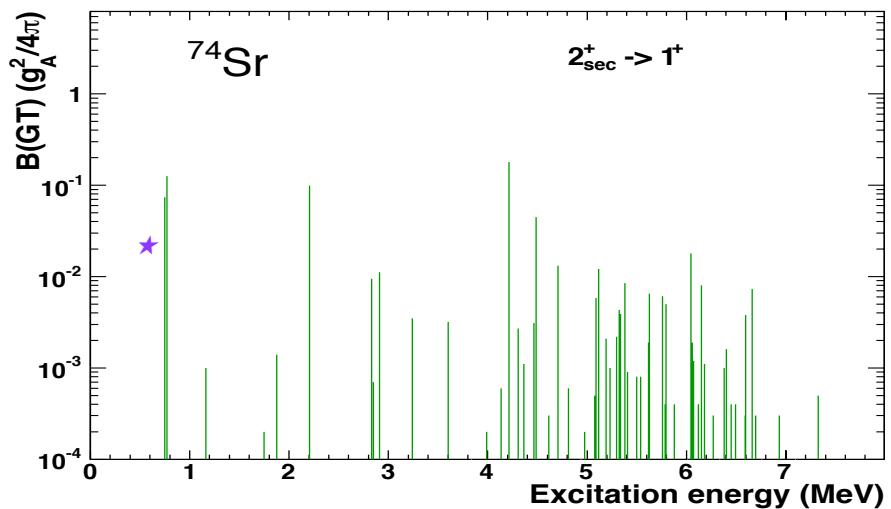
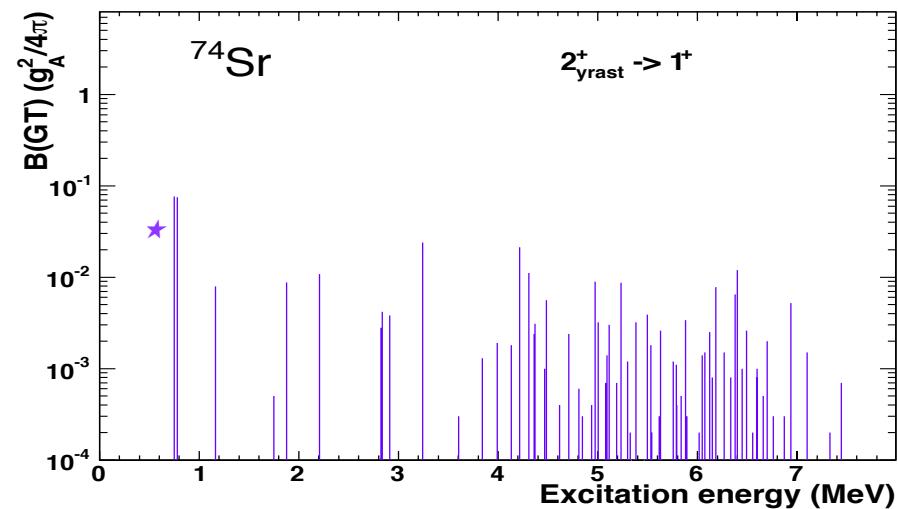


# *Gamow-Teller strength distributions for the decay of $0^+$ and $2^+$ states in $^{74}\text{Sr}$*

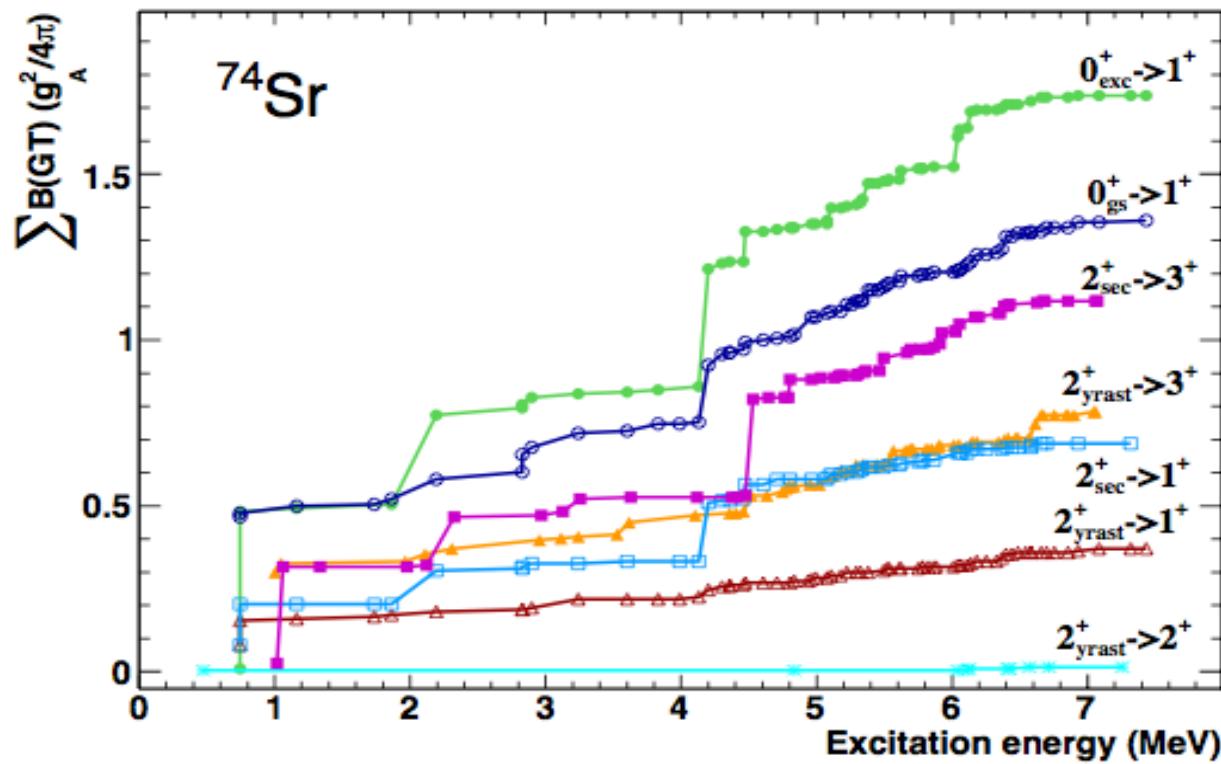
*Specific shape mixing for each parent state influences the strength distributions*



# Influence of specific shape mixing in the parent and daughter states on strength distributions



Contributions from  $p_{1/2}^{\nu(\pi)} p_{3/2}^{\pi(\nu)}$ ,  $p_{3/2}^{\nu} p_{3/2}^{\pi}$ ,  $f_{5/2}^{\nu} f_{5/2}^{\pi}$ ,  $f_{5/2}^{\nu(\pi)} f_{7/2}^{\pi(\nu)}$ ,  $g_{9/2}^{\nu} g_{9/2}^{\pi}$  matrix elements  
(coherent / cancelling effect)



### Terrestrial half-lives

$$\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0 < E_f < Q_{EC}} f(Z, E_f) [B_{if}(GT) + B_{if}(F)]$$

$$T_{1/2}^{\text{GT}} = 137 \text{ ms} \quad T_{1/2}^{\text{F}} = 48 \text{ ms}$$

$$T_{1/2}^{\text{EXVAM}} = 36 \text{ ms}$$

$$T_{1/2}^{\text{exp}} = 27(8) \text{ ms}$$

## **Weak interaction rates in X-ray burst astrophysical environment**

*In the X-ray burst stellar environment at densities ( $\sim 10^6 \text{ mol/cm}^3$ ) and temperatures ( $\sim 10^9 \text{ K}$ ) typical for the rp-process the contribution of thermally populated low-lying  $0^+$  and  $2^+$  states may be relevant.*

(H. Schatz et al., Phys. Rep. 294, 167 (1998))

$$\lambda^\alpha = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1)e^{-E_i/(kT)}}{G(Z, A, T)} \sum_j B_{ij} \phi_{ij}^\alpha$$

$$G(Z, A, T) = \sum_i (2J_i + 1) \exp(-E_i/(kT))$$

$$B_{ij} = B_{ij}(F) + B_{ij}(GT)$$

$$\phi_{ij}^{ec} = \int_{w_l}^{\infty} wp(Q_{ij} + w)^2 F(Z, w) S_e(w) (1 - S_\nu(Q_{ij} + w)) dw$$

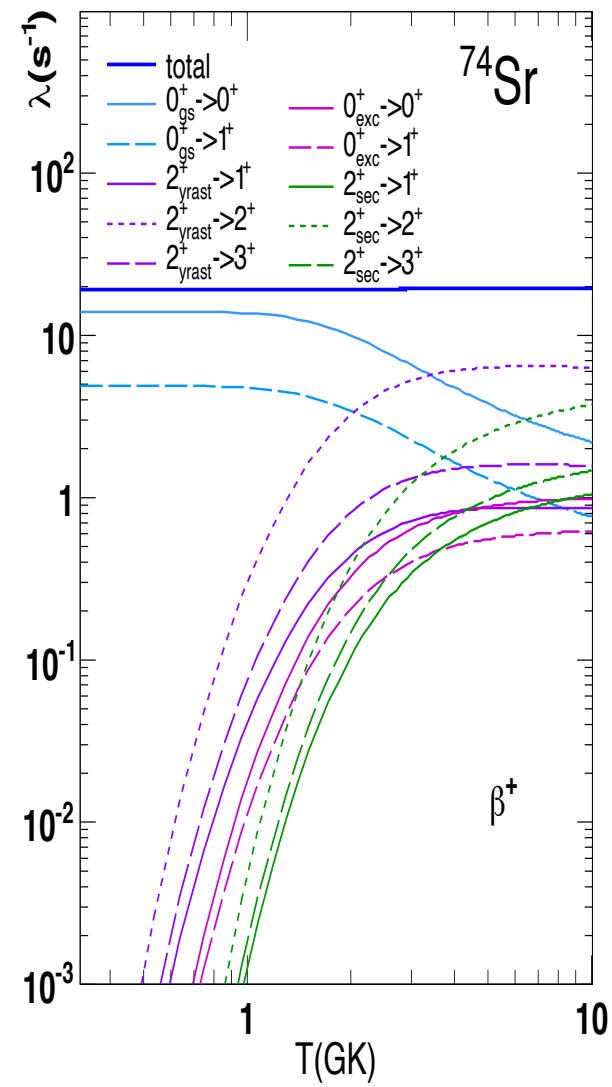
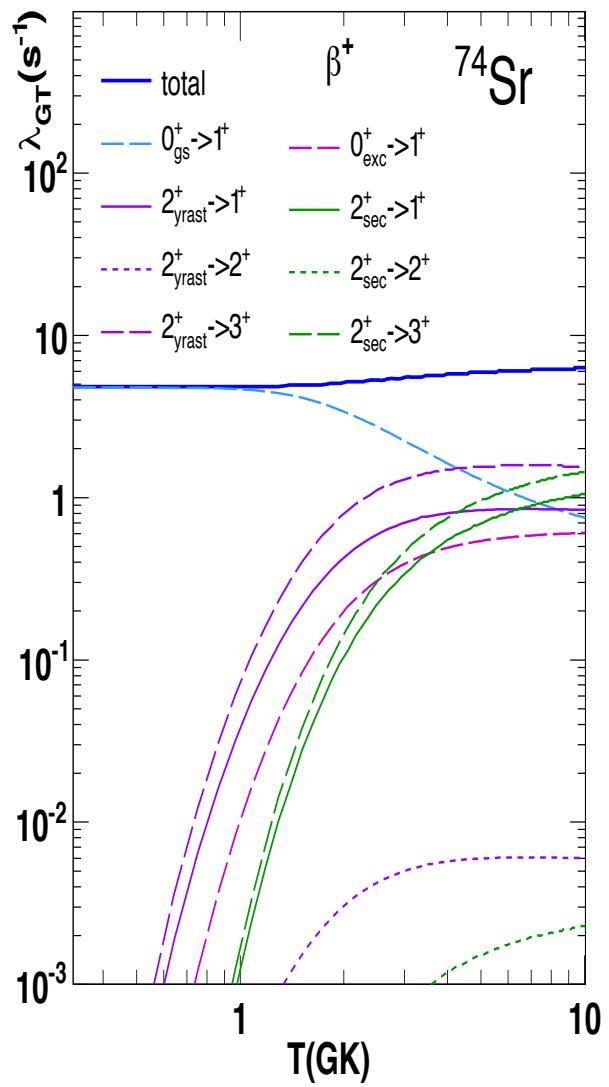
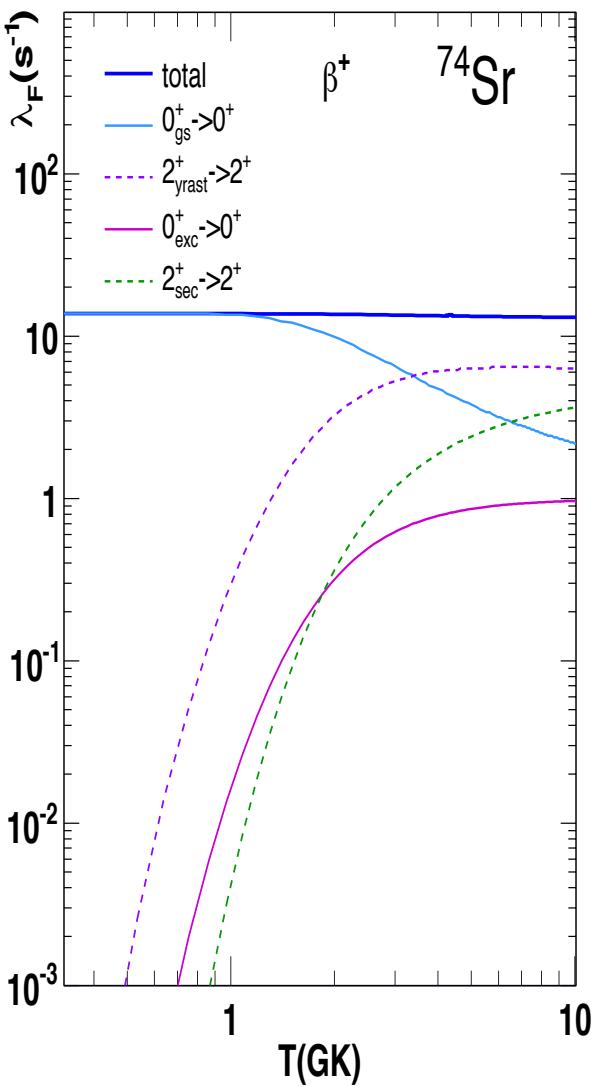
$$\phi_{ij}^{\beta^+} = \int_1^{Q_{ij}} wp(Q_{ij} - w)^2 F(-Z + 1, w) (1 - S_p(w)) (1 - S_\nu(Q_{ij} - w)) dw$$

# *Stellar rates for $^{74}\text{Sr}$ : $\beta^+$ - decay*

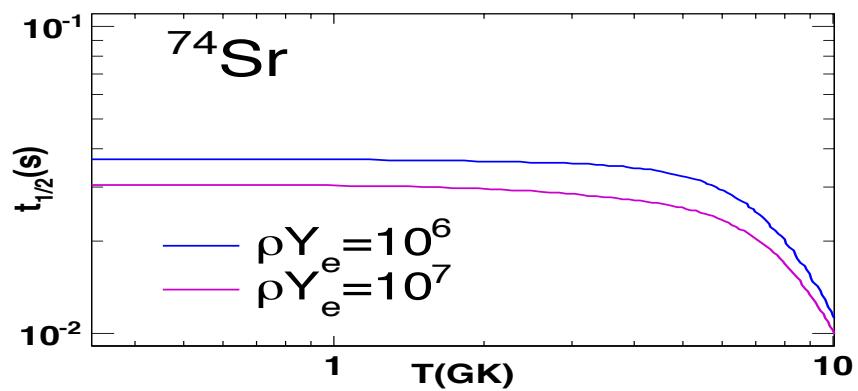
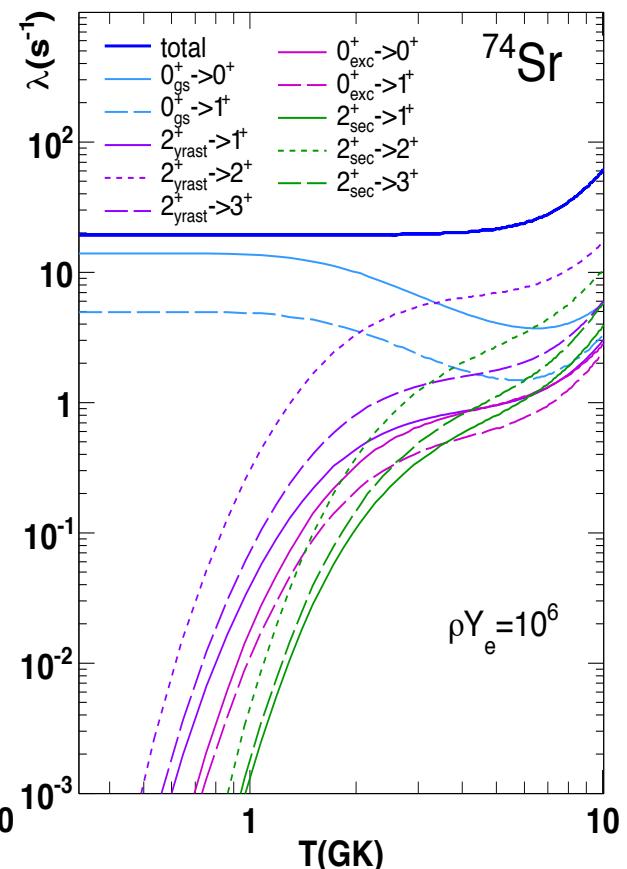
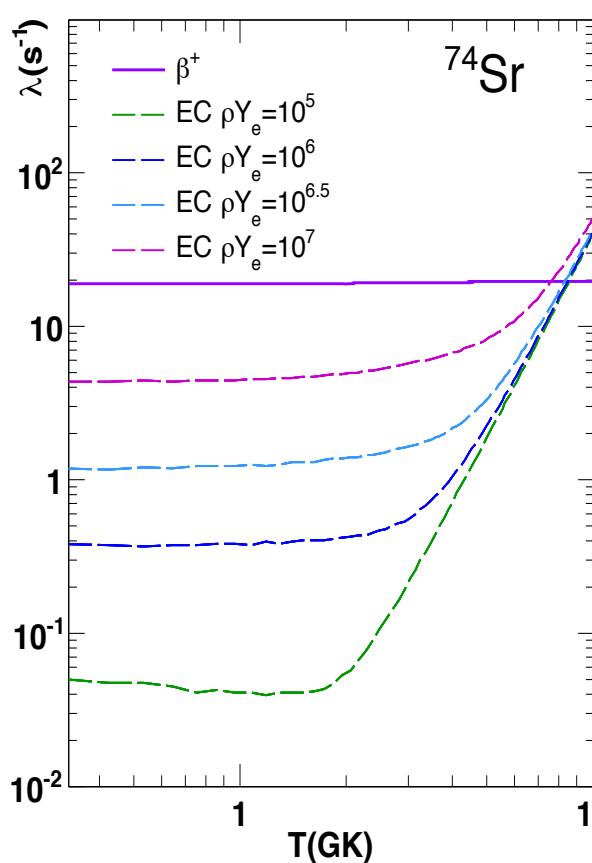
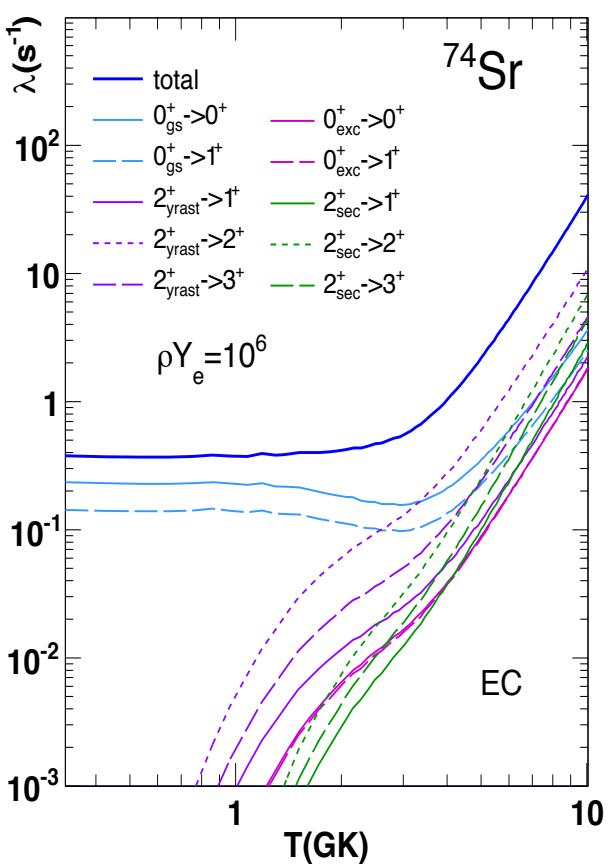
$$E_{0^+_{\text{exc}}}^{\text{th}} = 0.564 \text{ MeV}$$

$$E_{2^+_{\text{yrast}}} = 0.471 \text{ MeV}$$

$$E_{2^+_{\text{sec}}}^{\text{th}} = 0.823 \text{ MeV}$$



# $\beta^+$ and electron capture rates for $^{74}\text{Sr}$



## **Summary**

*complex EXCITED VAMPIR beyond-mean-field model*

*self-consistently describes shape-coexistence effects on*

- *low-spin structure in A=74 isovector triplet*
- *superallowed Fermi  $\beta$ -decay of the Z=N+2 isotope  $^{74}\text{Sr}$*
- *terrestrial and stellar weak interaction rates under X-ray burst environment for  $^{74}\text{Sr}$*