Shell-model applications to gamma-ray strength function and level density

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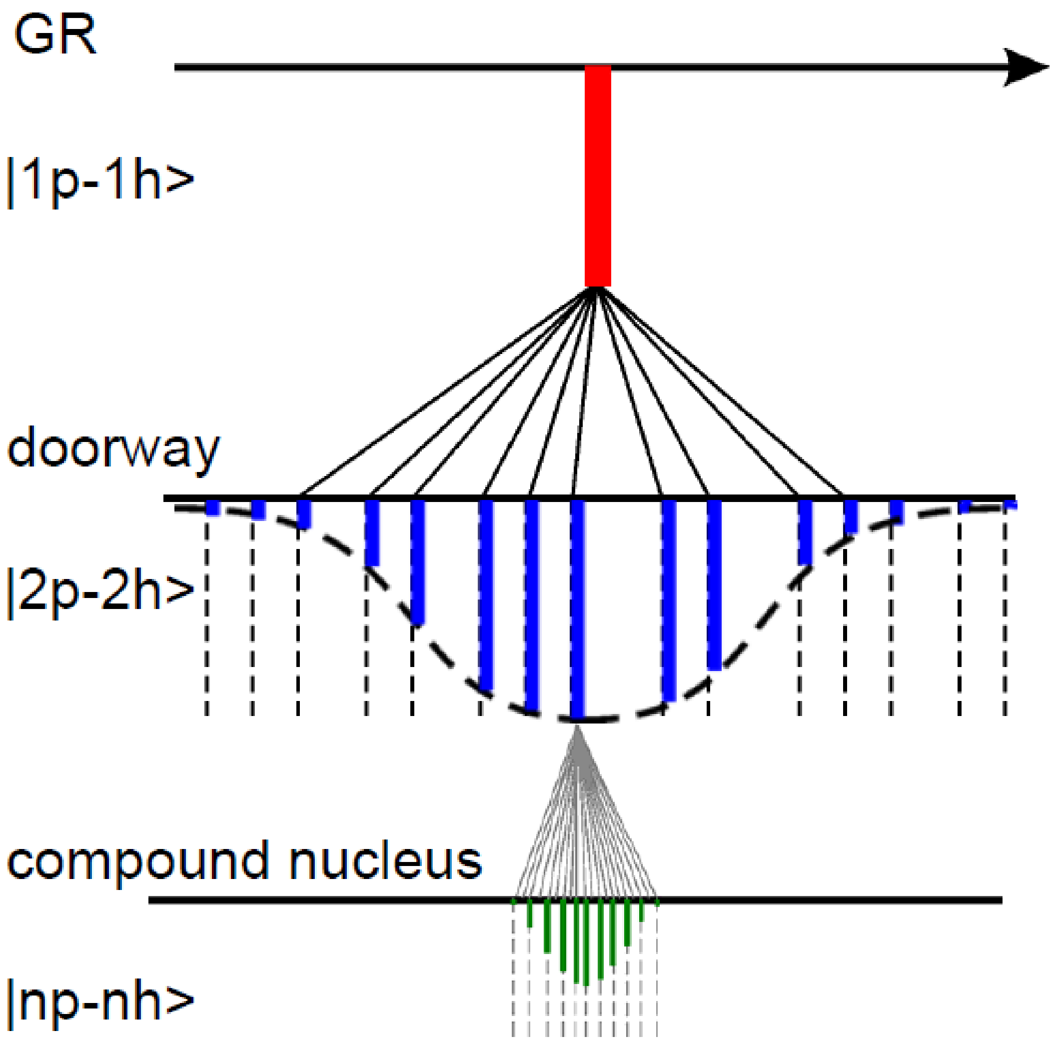
Nuclear structure in the GDR region



Taken from a figure by A. Richter

- Close relation among
  - Fine structure
  - Width of GDR
  - Level density

- Shell model (CI)
  - All the levels in a given model space
  - Transitions between excited states

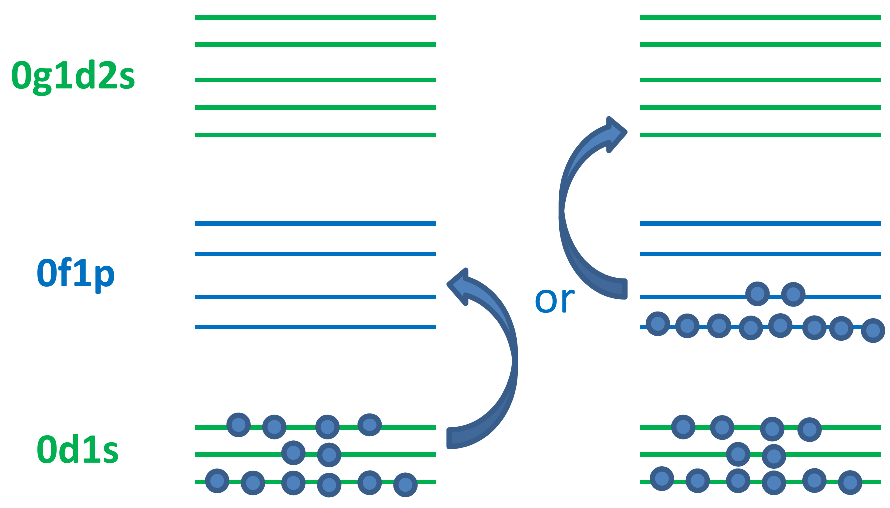
Outline of the present talk

- Studying the gamma-ray strength function and level density on the same footing using large-scale shell-model calculations

1. E1 strength function including transitions among excited levels
    - GDR of Ca isotopes: $1\hbar\omega$ vs. $(1+3)\hbar\omega$
    - Two-phonon state probed with decay probabilities
    - Pygmy dipole resonances
    - Examining the Brink hypothesis
2. Level density
    - New method of calculating level density in the shell model
    - Application to ^{58}Ni: equilibration of positive- and negative-parity levels

Framework

- Objectives
  - *pf*-shell nuclei (e.g. Ca isotopes)
- Valence shell
  - Full *sd*-*pf*-*sdg* shell
  - $0\hbar\omega$ and $1\hbar\omega$ states for the ground and 1^- levels, respectively



- Effective interaction
  - same as the one used for 3^-_1 levels in Ca isotopes (Y. Utsuno et al., PTP Suppl., 2012)
  - USD (*sd*) + GXPF1B (*pf*) + V_{MU} (remaining)
    - Successful in *sd*-*pf* shell calculations including exotic nuclei (e.g. ^{42}Si, ^{44}S)
  - $g_{9/2}$ SPE: fitted to $9/2^+_1$ in ^{51}Ti
    - Reasonable agreement with a recent C^2S data in ^{49}Ca (A. Gade et al., 2016)
- Removal of spurious center-of-mass motion with Lawson method

Lanczos strength function method

Efficient way to avoid calculating all the eigenstates

1. Take an initial vector: $\overrightarrow{u_1} = T(E1)|\mathrm{g.s.}\rangle$
2. Follow the usual Lanczos iterations
   - $H\overrightarrow{u_k} = \beta_{k-1}\overrightarrow{u_{k-1}} + \alpha_k\overrightarrow{u_k} + \beta_k\,\overrightarrow{u_{k+1}}$: defining a normalized vector $\overrightarrow{u_{k+1}}$
3. Calculate the strength function $\sum_\nu B(E1; g.s. \to \nu)\frac{1}{\pi}\frac{\Gamma/2}{(E-E_\nu+E_0)^2+(\Gamma/2)^2}$ by summing up all the eigenstates ν in the Krylov subspace with an appropriate smoothing factor Γ until good convergence is achieved.

Example of convergence with Γ = 1 MeV

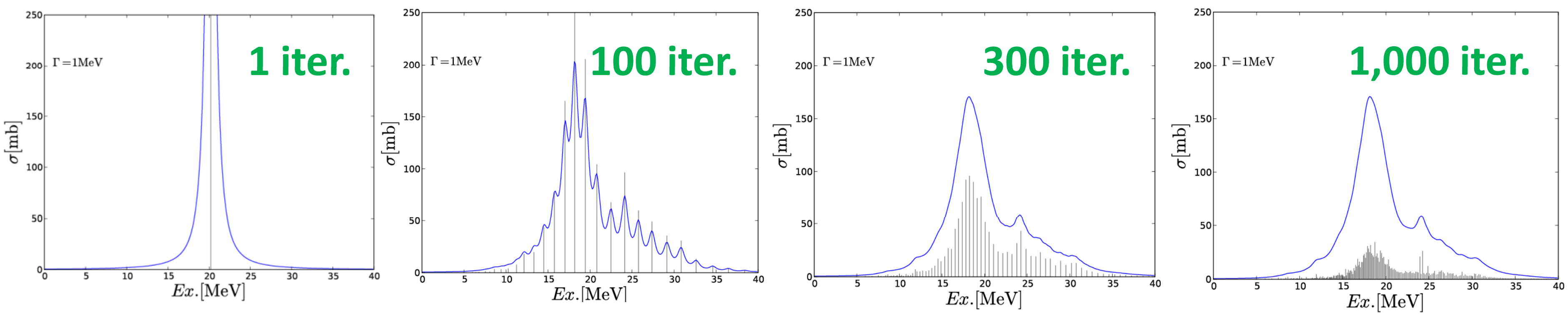
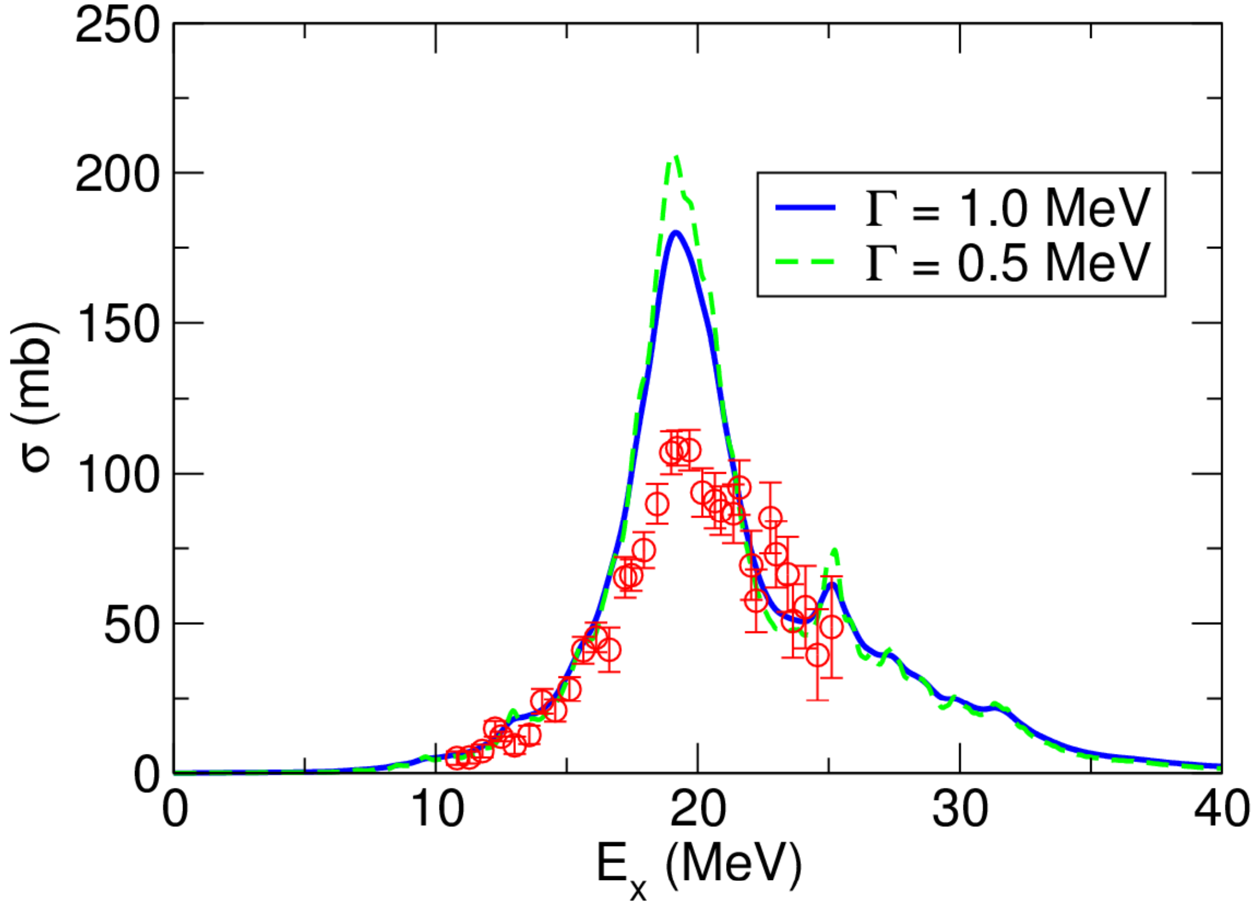
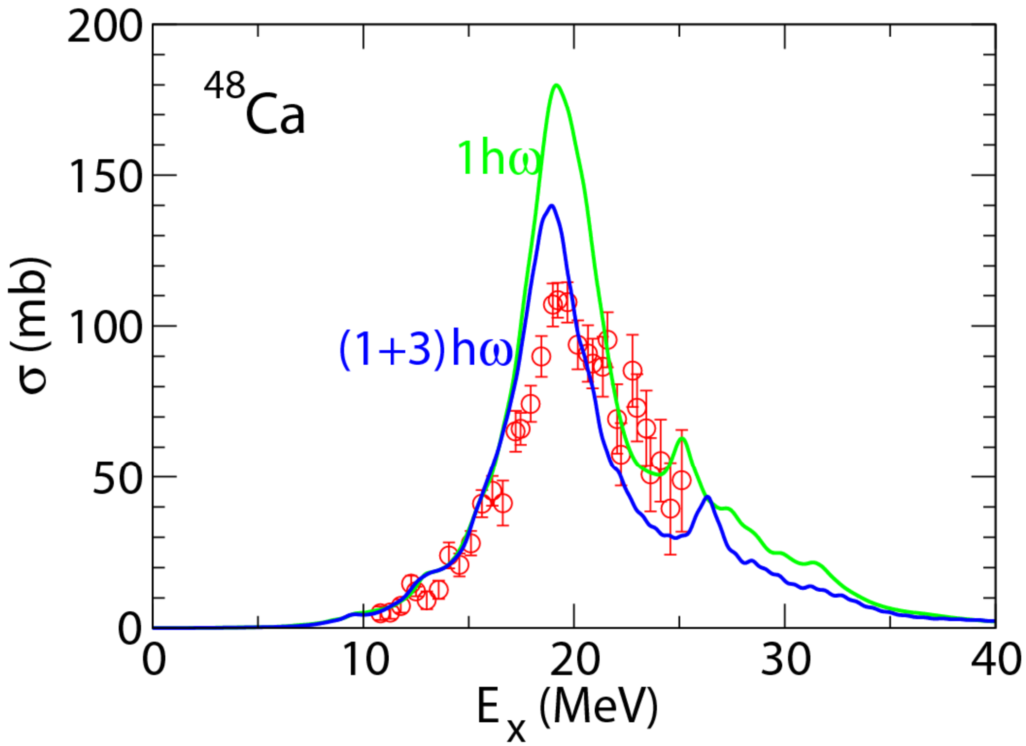


Photo-absorption cross sections for ^{48}Ca

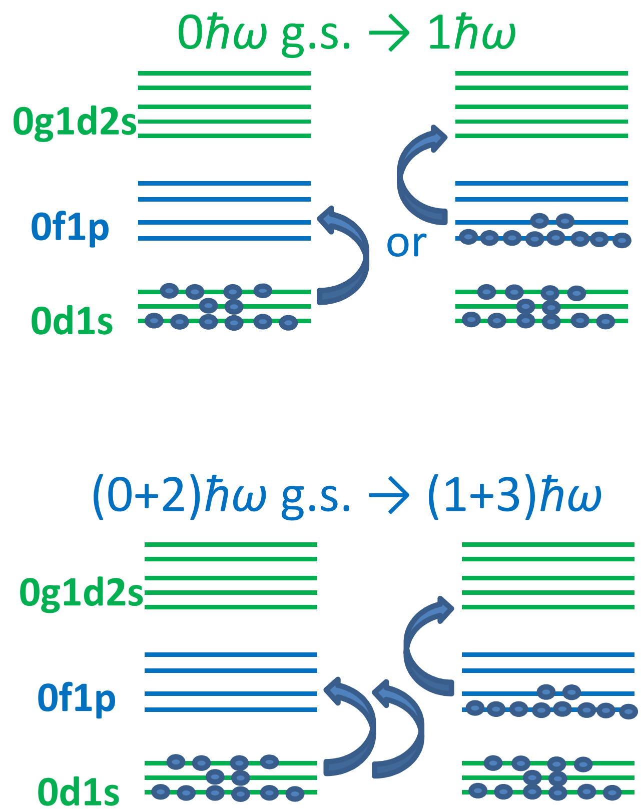


- GDR peak position: good
- GDR peak height: overestimated
- GDR tail: weak dependence on the choice of Γ ➡ fine structure

$E1$ strength function: $1\hbar\omega$ *vs.* $(1+3)\hbar\omega$



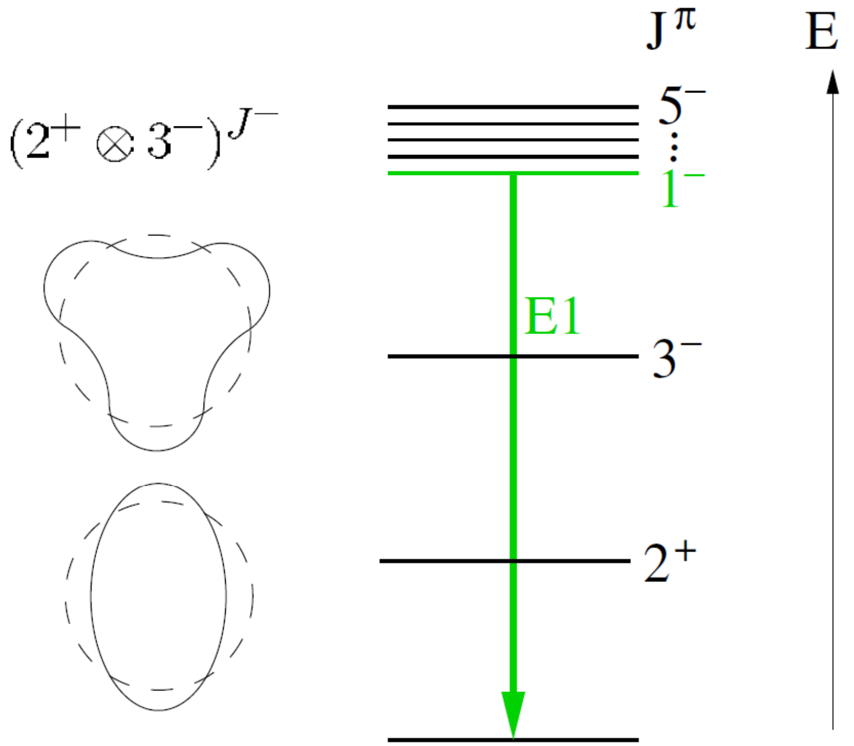
Expt.: G.J. O'Keefe et al., Nucl. Phys. A 469, 239 (1987)



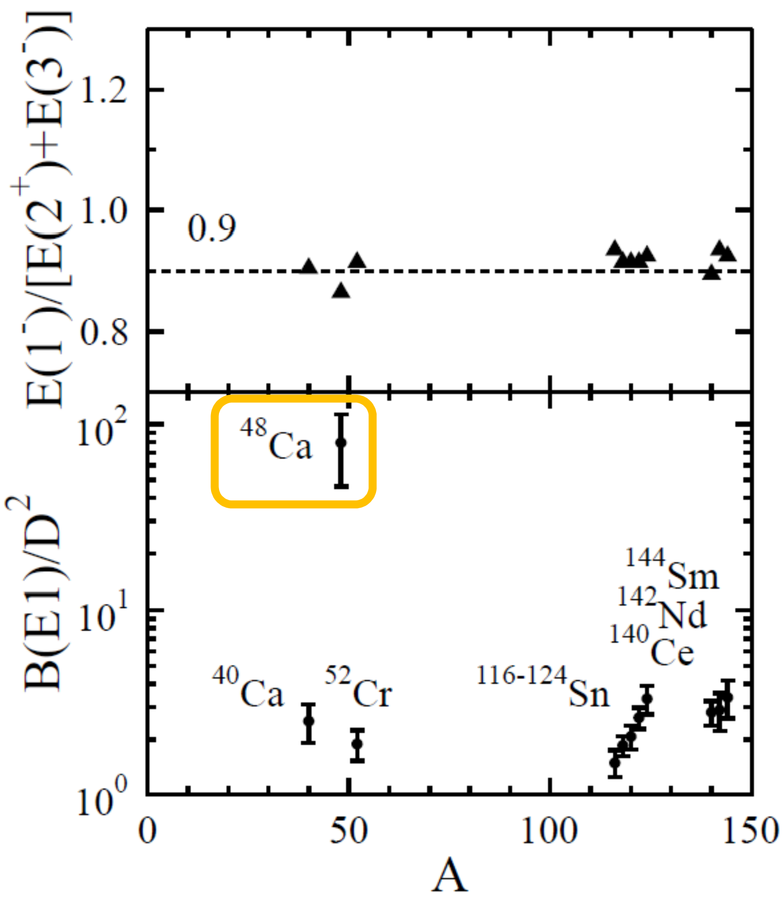
Up to 10^{10} *m*-scheme dimensions in Ca isotopes

- Improved GDR peak height
- Low-energy tail is almost unchanged.
  - The following results are restricted to $1\hbar\omega$, concentrated on the tail region.

$2^+ \otimes 3^-$ two-phonon states



Systematics of two-phonon 1⁻ states

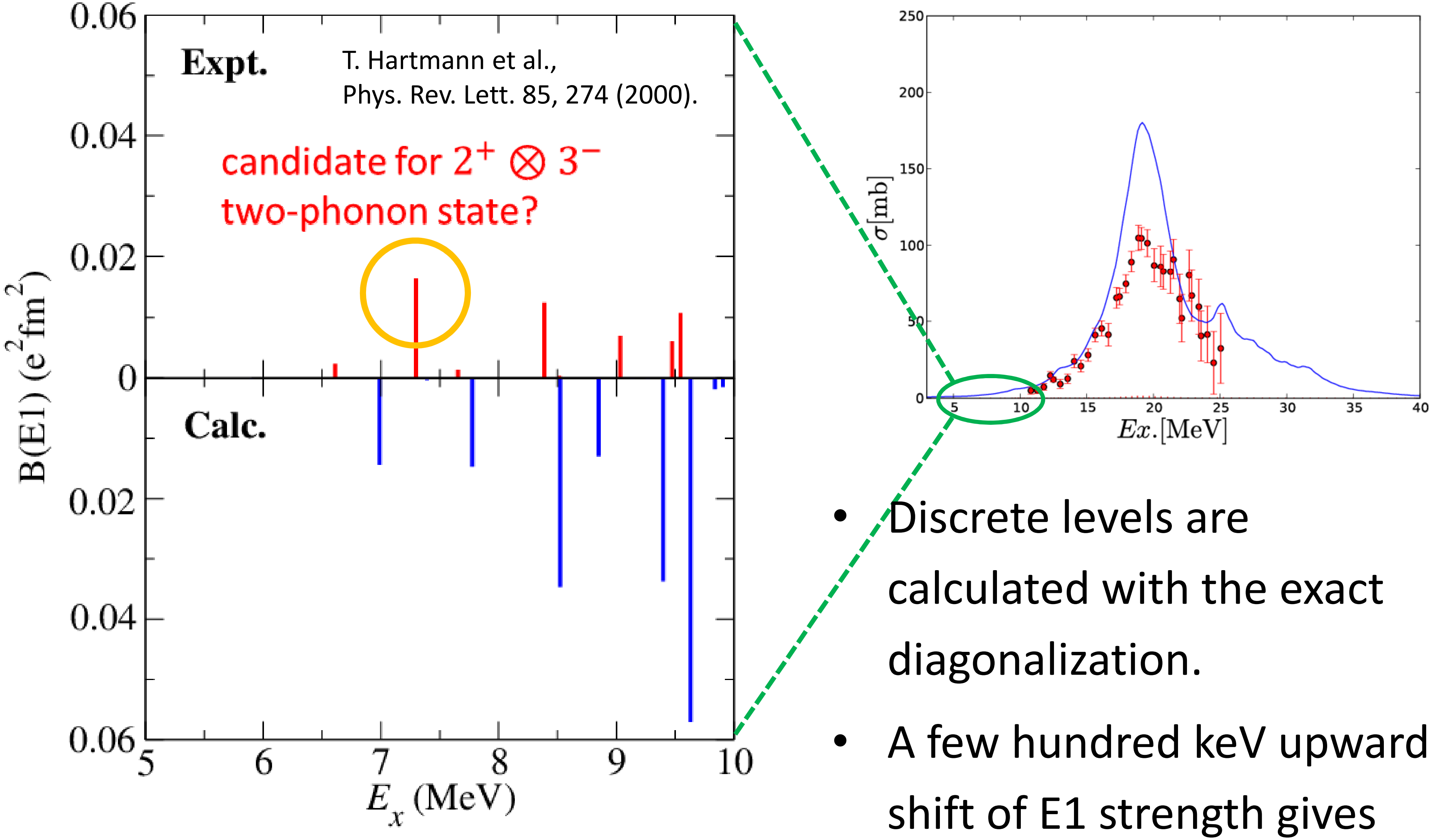


$$D = 5.367 \times 10^{-4}(Z+N)Z\beta_2\beta_3 \text{ (efm)}$$

- Two-phonon states in ^{48}Ca
  - The proposed state has much stronger B(E1) than other isotopes.
  - No E2 or E3 measurement

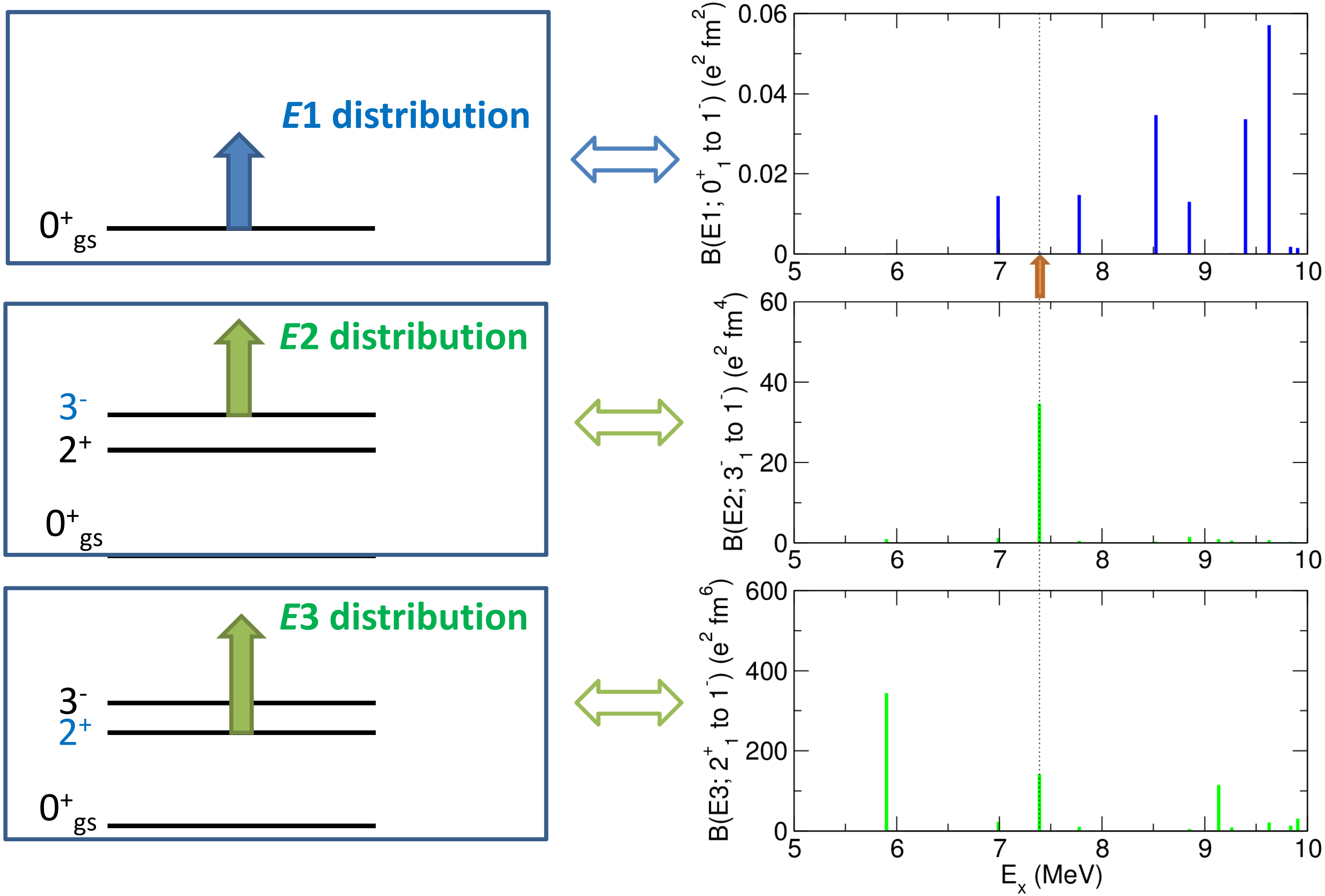
T. Hartmann et al., Phys. Rev. Lett. 85, 274 (2000).

Which 1$^-$ level is the two-phonon state?



- Discrete levels are calculated with the exact diagonalization.
- A few hundred keV upward shift of E1 strength gives excellent agreement.

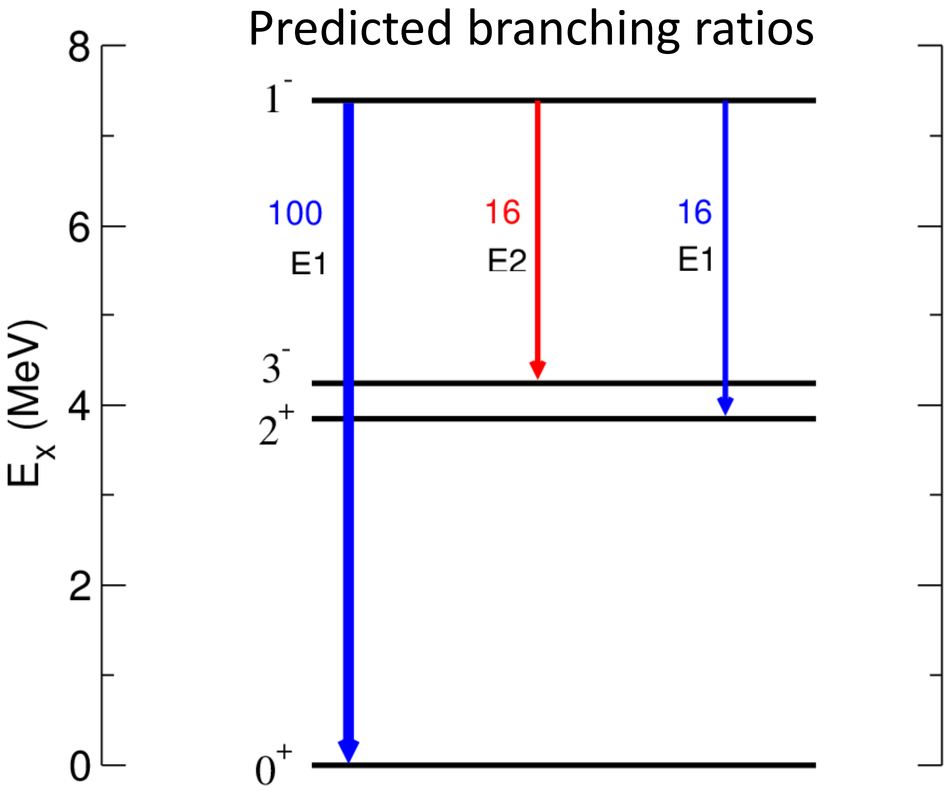
Probing $2^+ \otimes 3^-$ two-phonon character

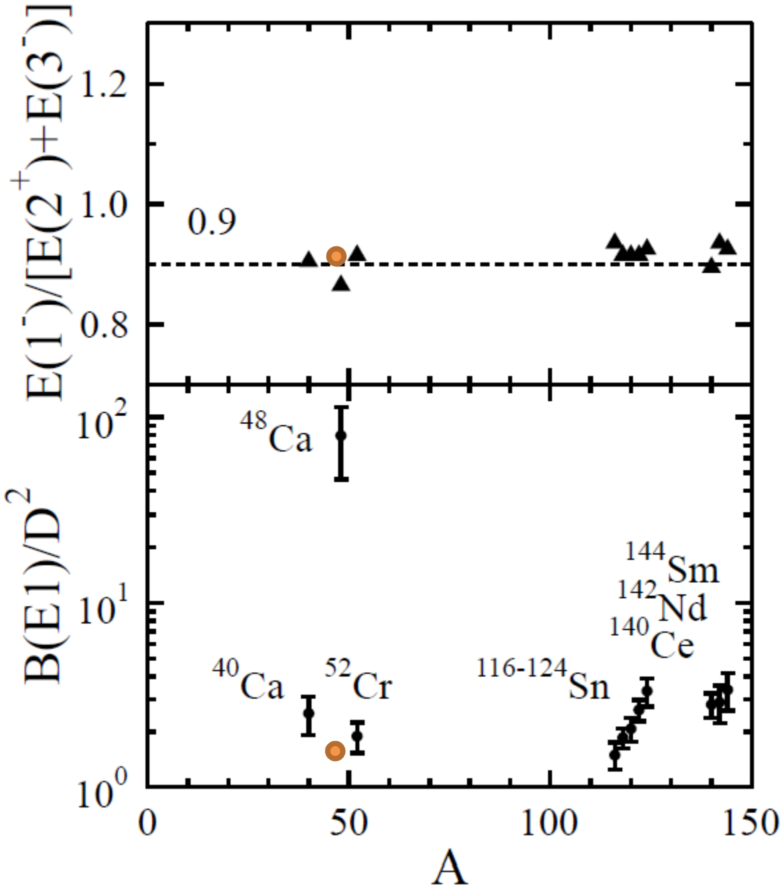


- Two-phonon state: the 1^- level with a very small B(*E*1)

Properties of the 1^-_{2ph} state

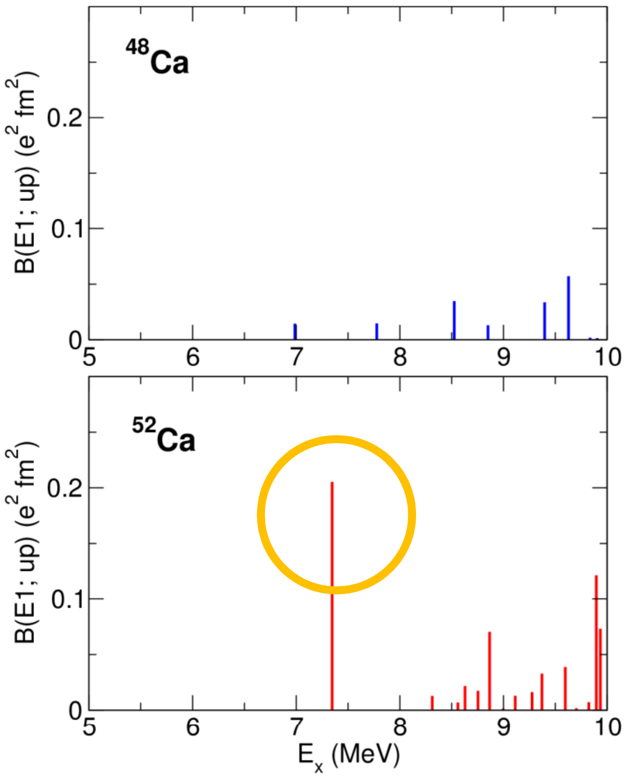
- Agreement with the systematics
  - Position: $\frac{E(1^-_{2\mathrm{ph}})}{[E(2^+)+E(3^-)]} = 0.91$
  - B(E1): $B(E1)/D^2 = 1.6$

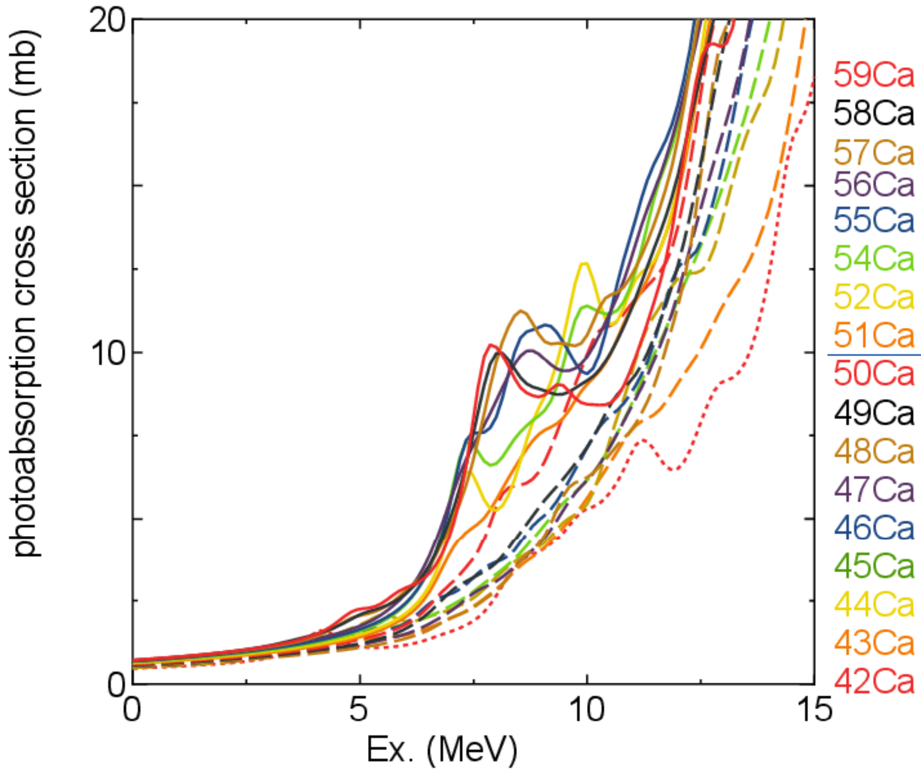




E2 decay competes with E1.

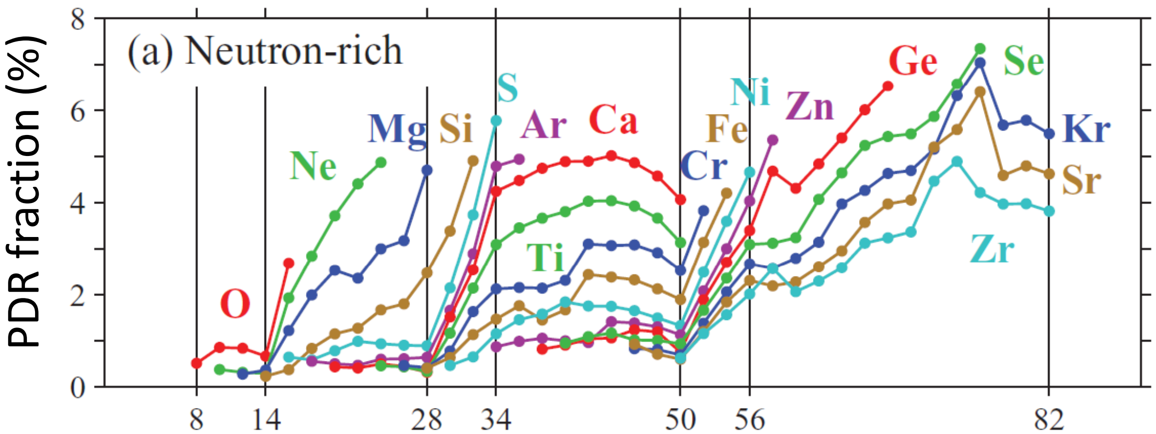
Development of pygmy dipole resonance





solid lines

dashed lines



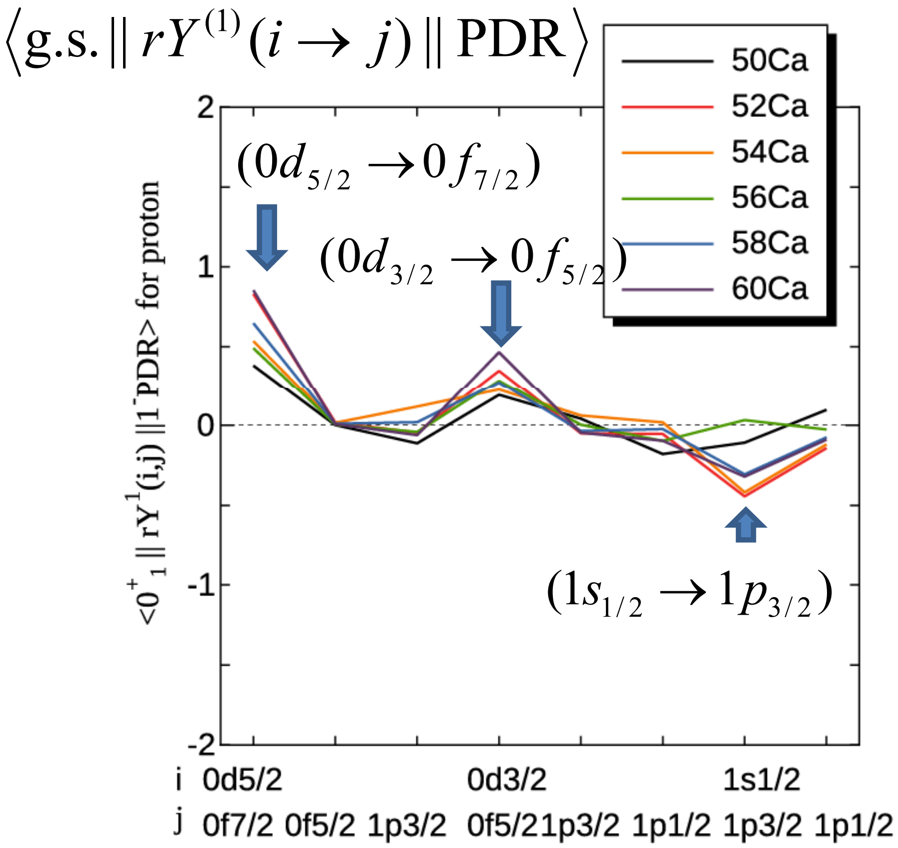
T. Inakura et al., Phys. Rev. C 84, 021302(R) (2011)

pointed out strong correlation with the occupation of the *p* orbitals

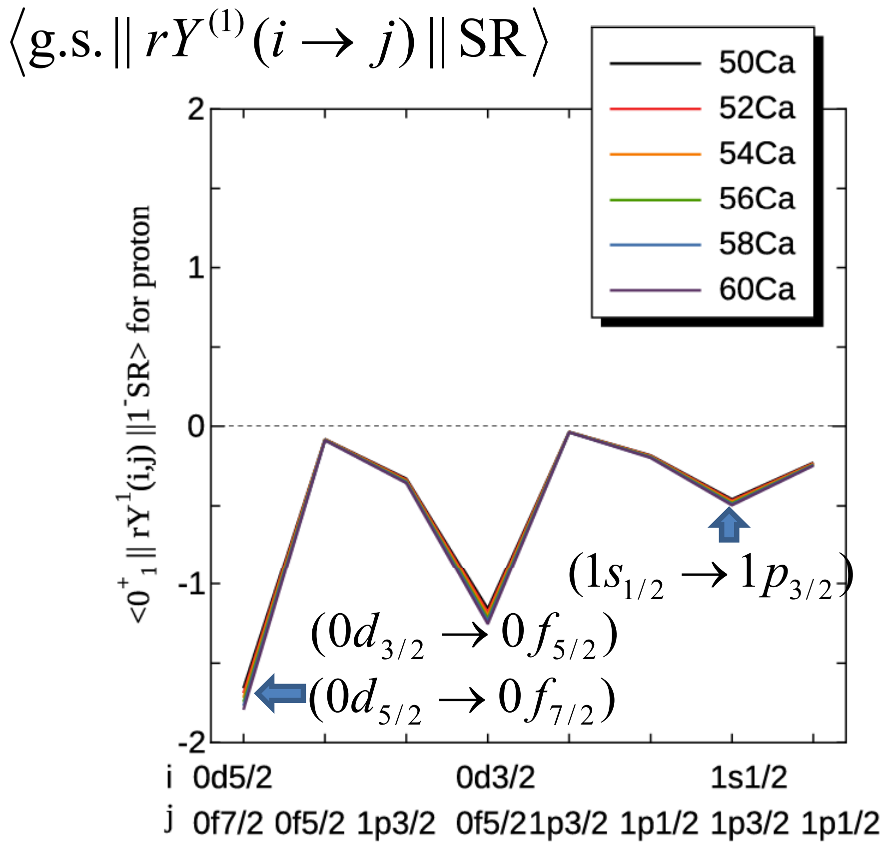
Proton $rY^{(1)}$ matrix elements

- Decomposing the E1 mat. ele. into $\sum_{i,j}\langle g.s.\|rY^{(1)}(i \to j)\|1^{-}\rangle$

PDR state

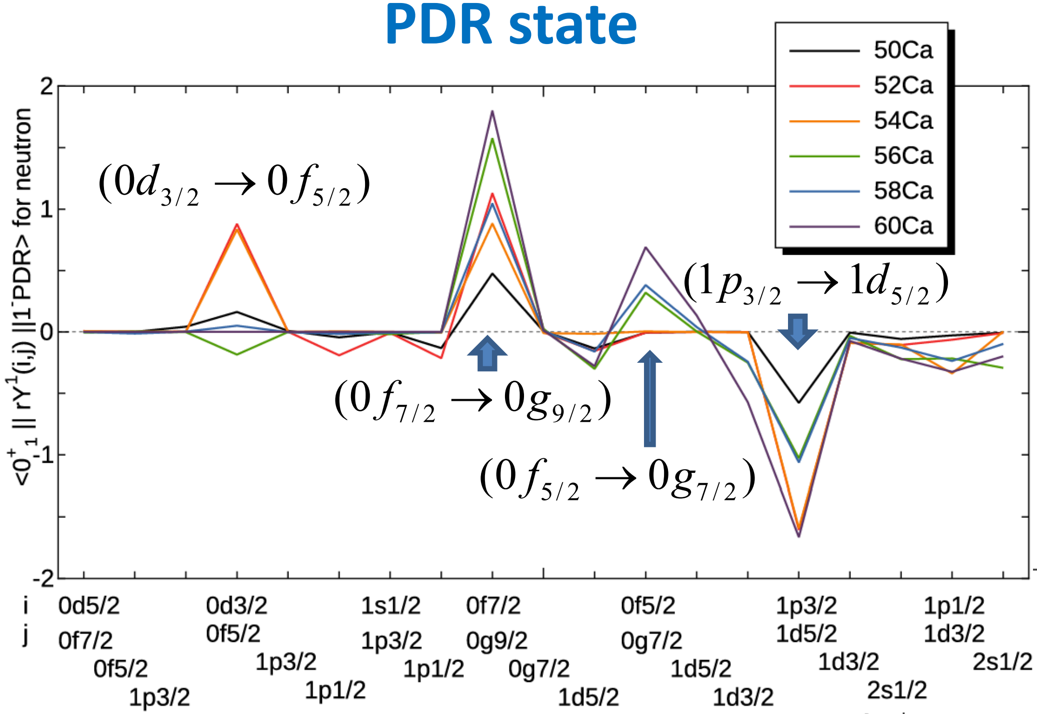


Sum rule state (GDR)



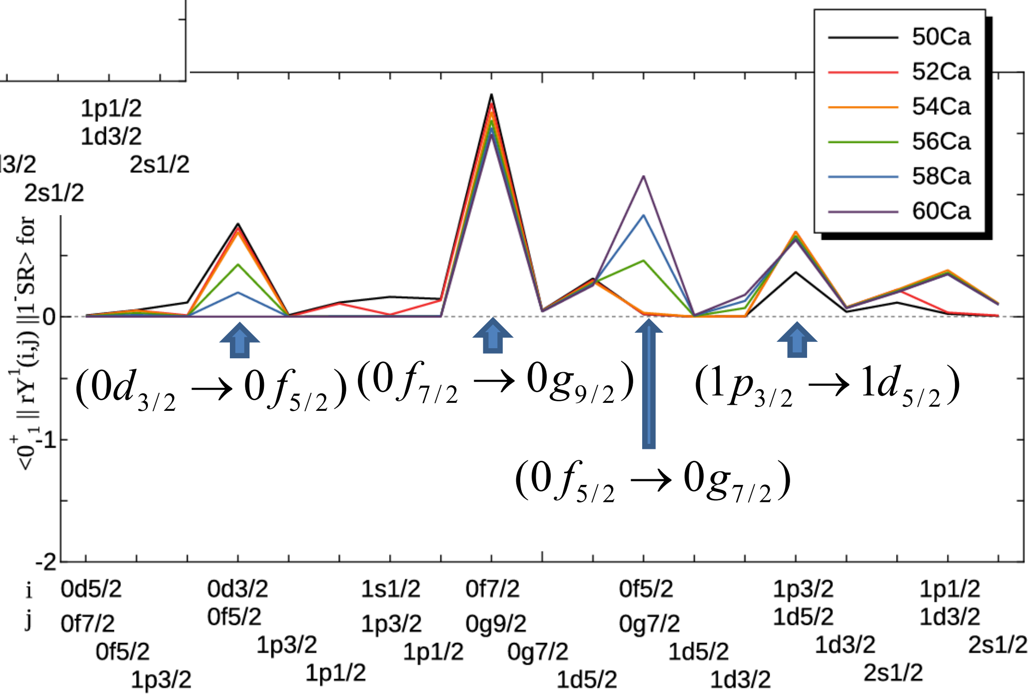
Neutron $rY^{(1)}$ matrix elements

PDR state



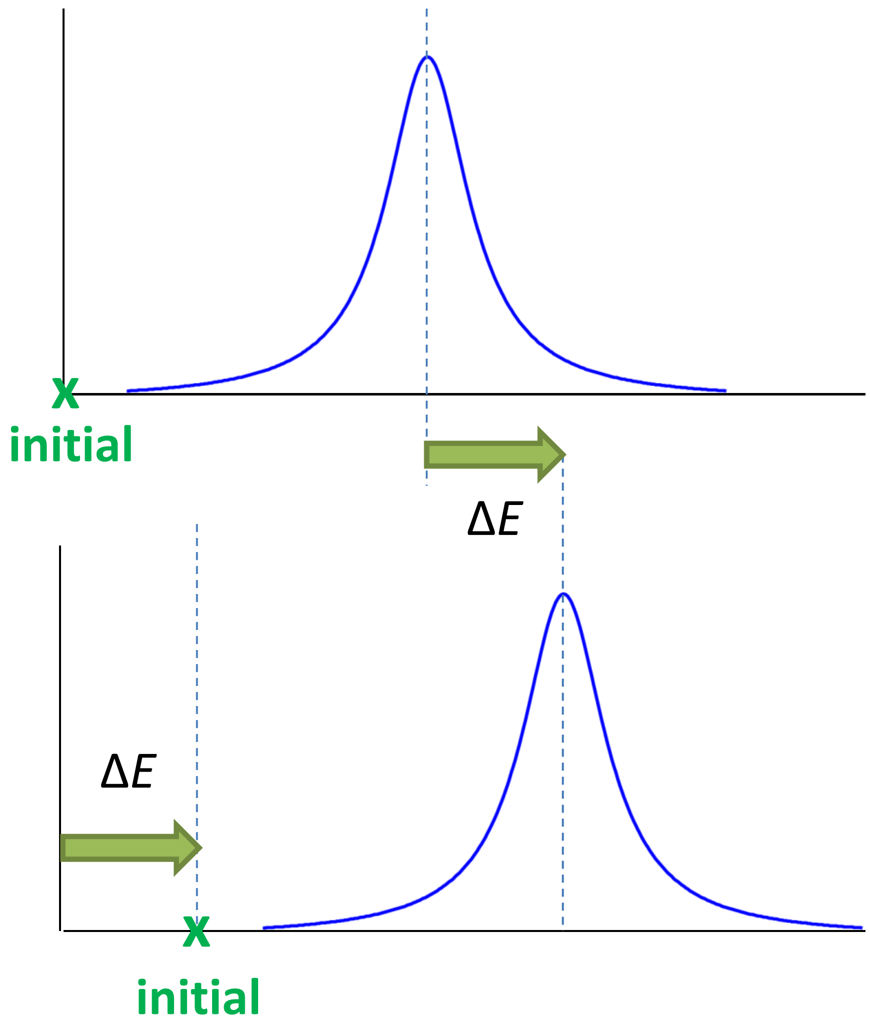
Incoherence of higher-node transition

Sum rule state (GDR)

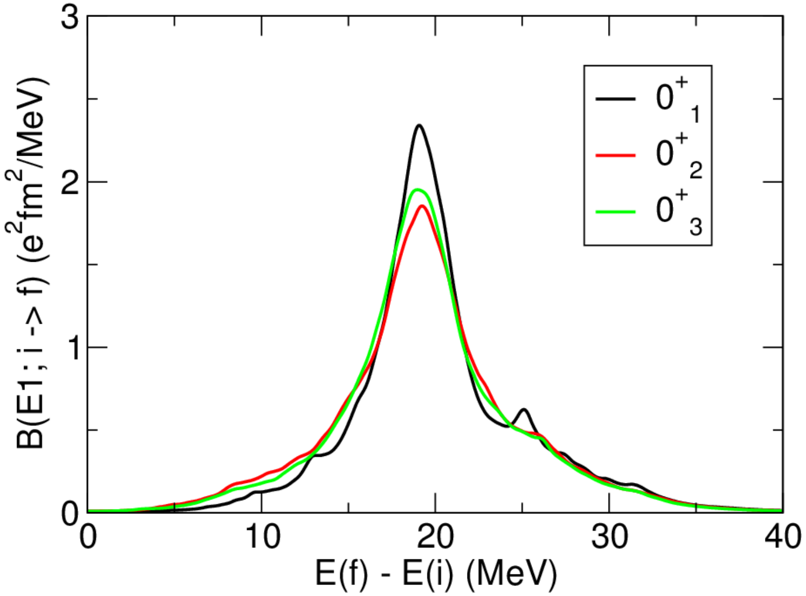


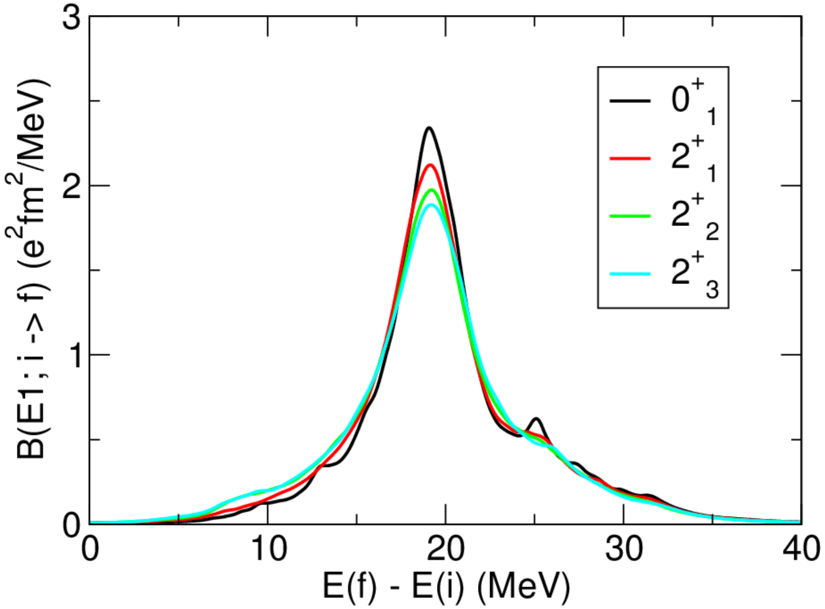
Brink hypothesis

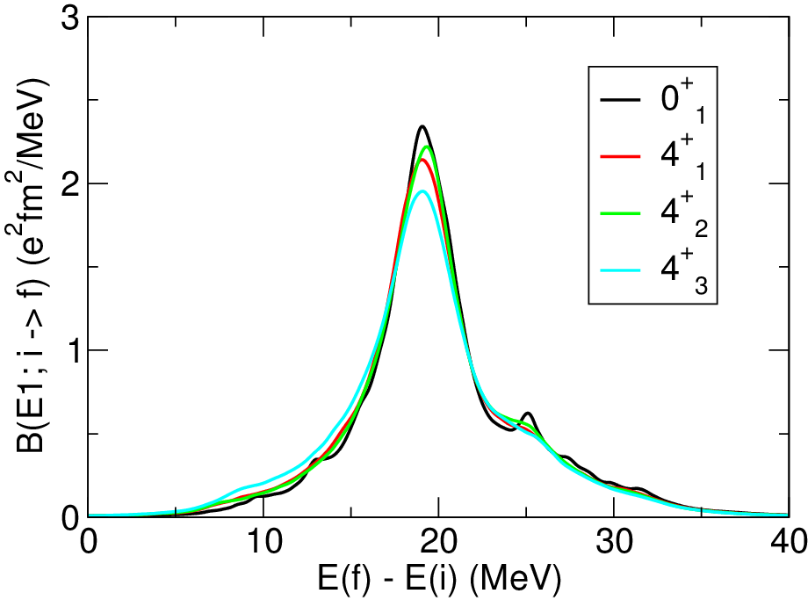
- GDR built on excited states
  - Assumed to be identical with that of the ground state by shifting Ex. of the initial state
    - geometric nature of GDR
  - Practically important to evaluate (*n*, γ) cross sections
- Recent experiments
  - Generalized Brink hypothesis valid in ^{238}Np (M. Guttormsen et al., 2016)
  - Possible deviation from the Brink hypothesis based on $^{89}Y(p, \gamma)^{90}Zr$ (L. Netterdon et al., 2015)



Examining the Brink hypothesis in ^{48}Ca

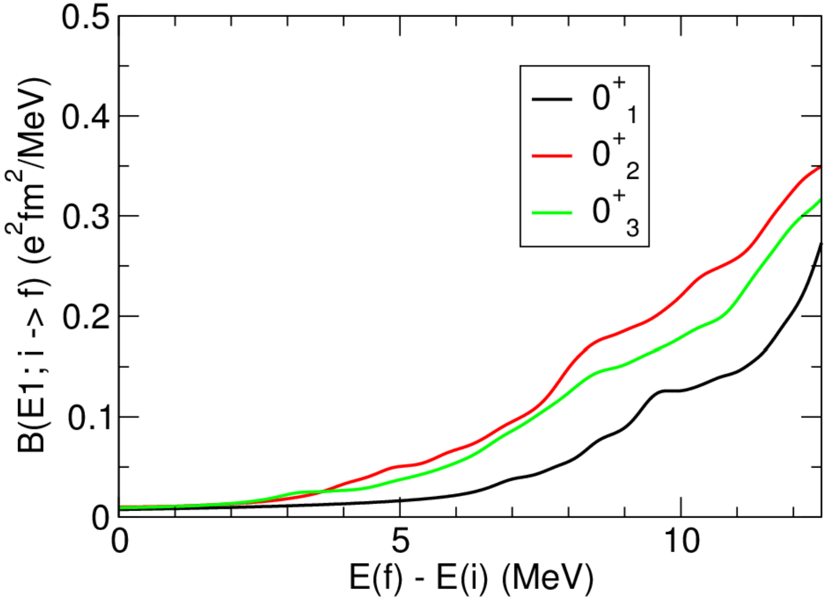


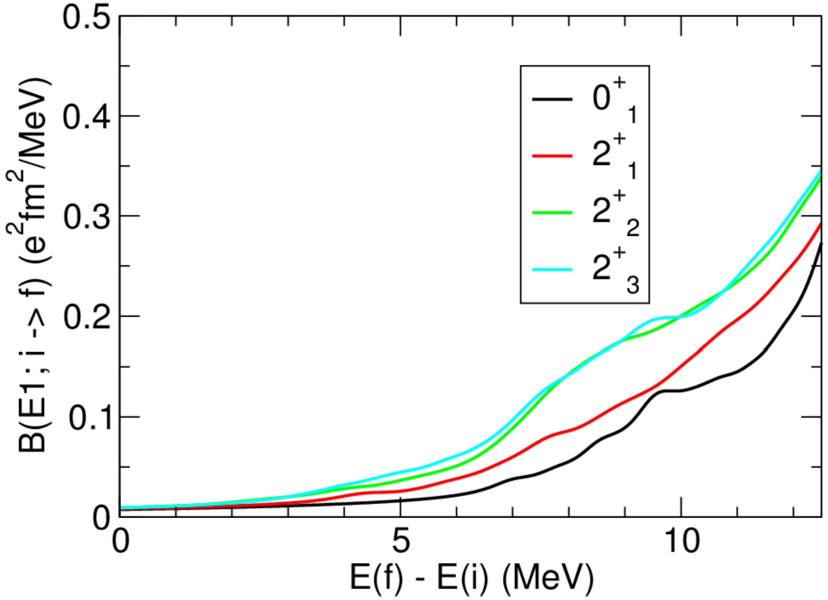


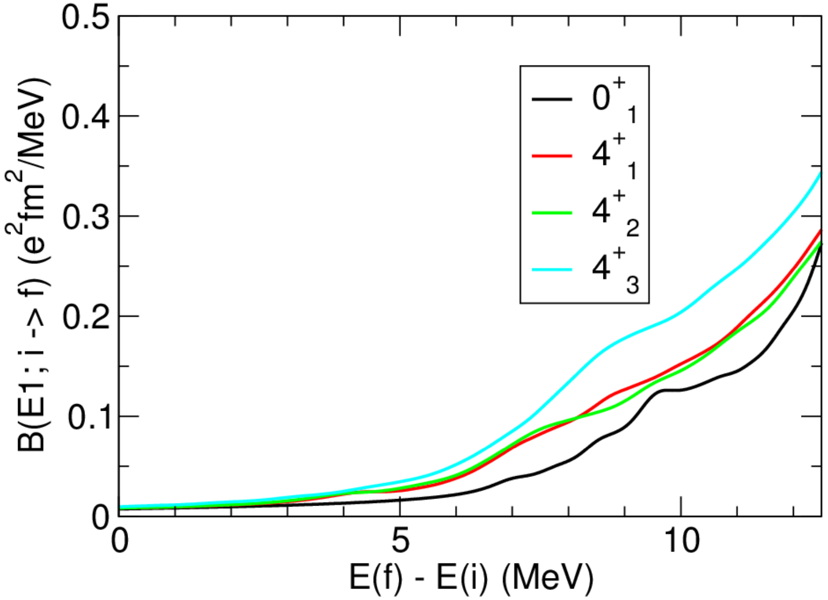


- The "generalized" Brink hypothesis (independent of excitation energy and spin) looks valid as far as the GDR region is concerned.

Brink hypothesis in the tail region: ^{48}Ca

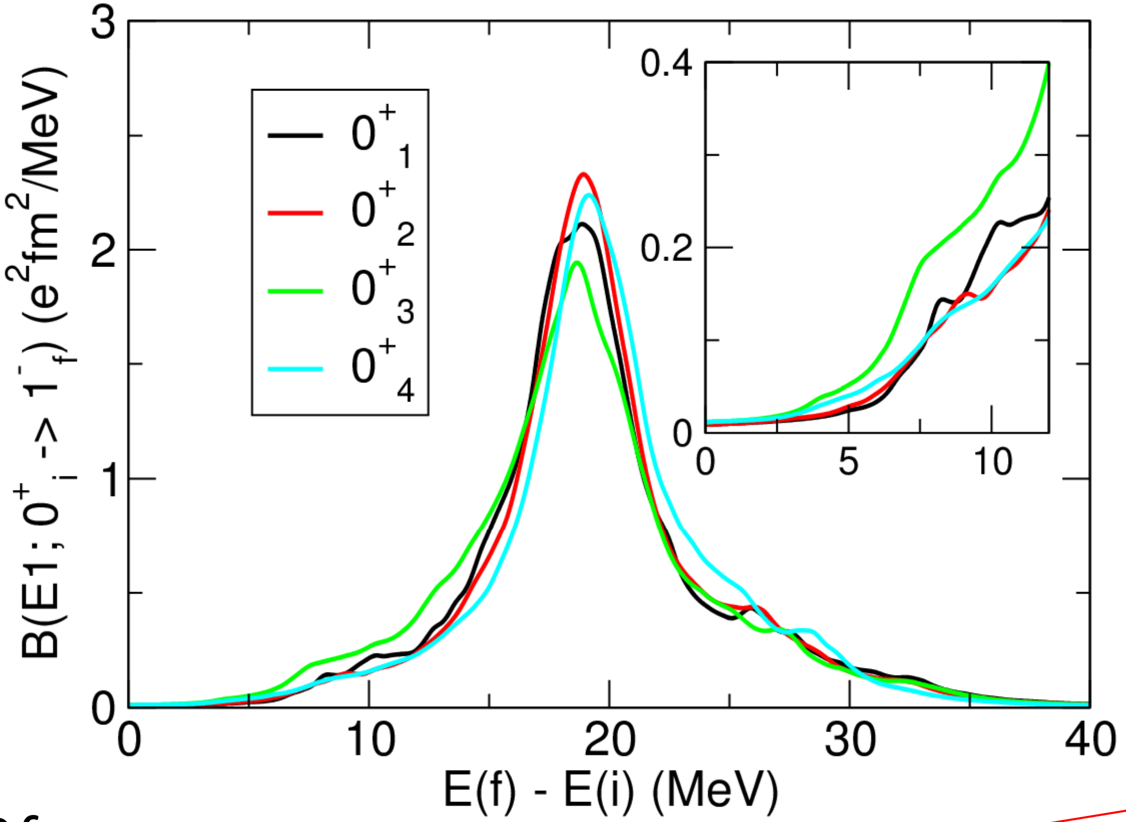






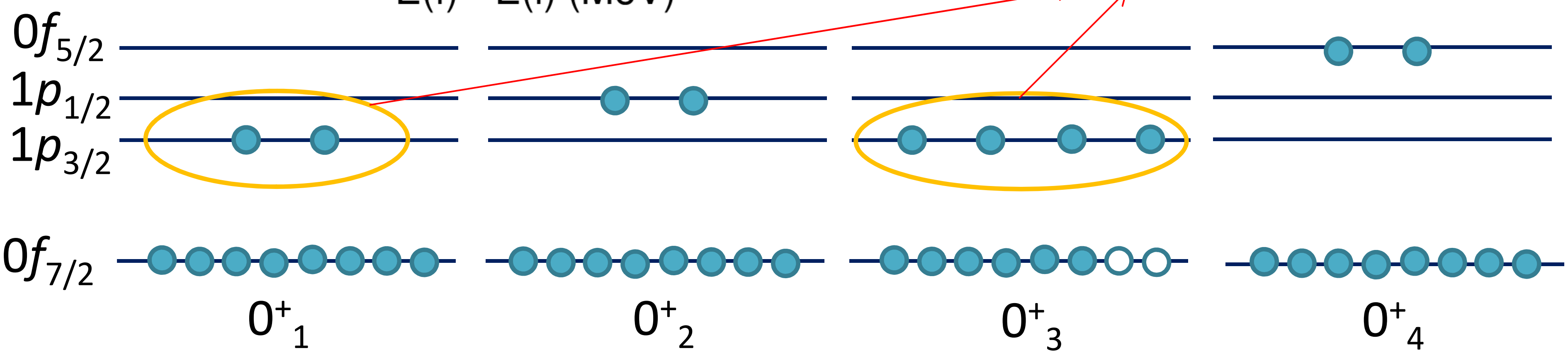
- Excited states have larger E1 strengths in the tail region.
- What dominates the difference? Excitation energy?

Brink hypothesis in the tail region: ^{50}Ca



- Order of E1 strength:
  no Ex. dependence
  - 1st: 0^+_3
  - 2nd: 0^+_1
  - 3rd: 0^+_2 & 0^+_4

configuration that favors pygmy



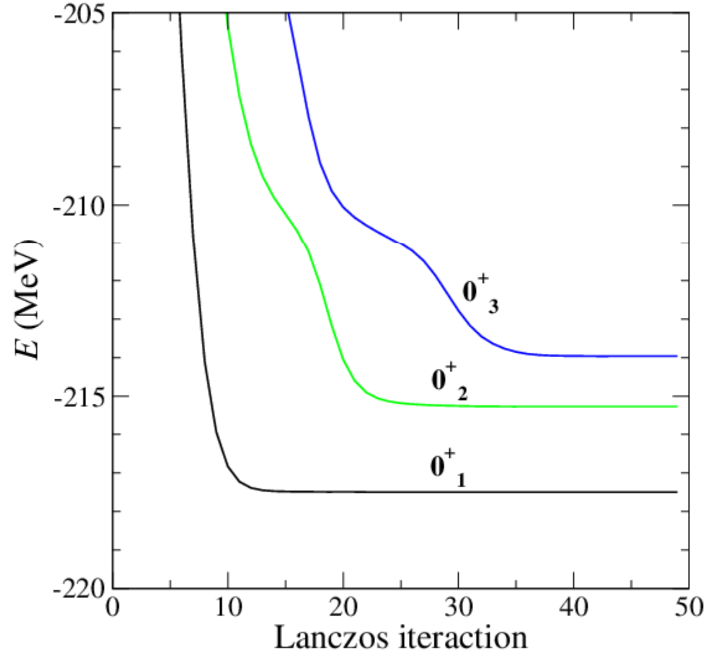
Shell-model calculation for level density

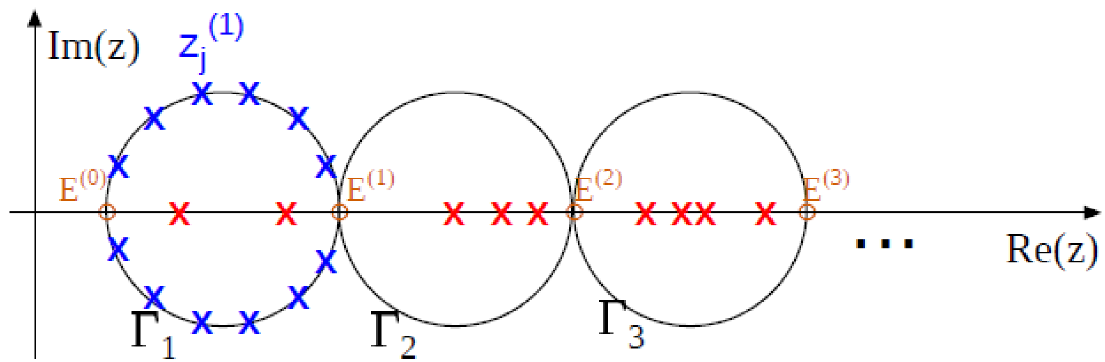
- Direct counting with Lanczos diag.
  - Practically impossible because high-lying levels are very slow to converge
- New method (Shimizu, Futamura, Sakurai)
  - Utilizing contour integral

$$\mu_k = \frac{1}{2\pi i}\oint_{\Gamma_k} dz \sum_i^D (z-\lambda_i)^{-1}$$

$$= \frac{1}{2\pi i}\oint_{\Gamma_k} dz\, \mathrm{tr}(z-H)^{-1}$$

Typical convergence pattern





Stochastic estimate of the trace

- Remaining task: estimating

$\mathrm{tr}(z-H)^{-1}$

$$= \sum_i^{D} \boldsymbol{e}_i^T (z-H)^{-1} \boldsymbol{e}_i$$

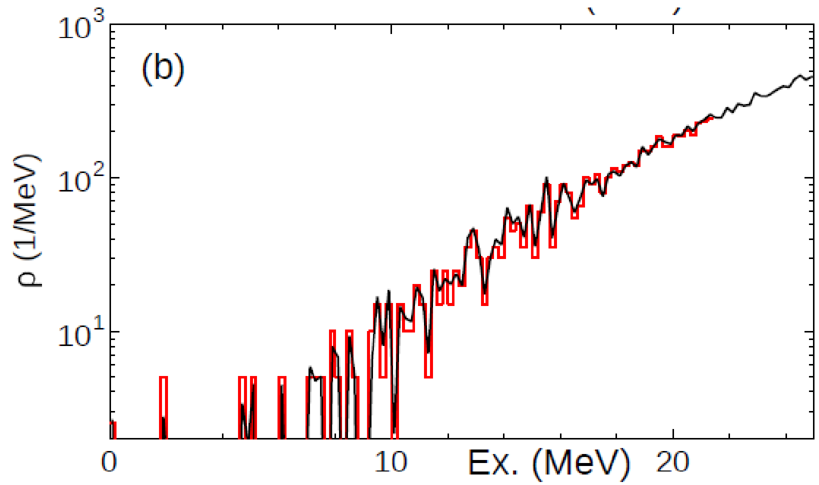
← dimension

- Stochastic sampling

$$\simeq \frac{1}{N_s} \sum_s^{N_s} \boldsymbol{v}_i^T (z-H)^{-1} \boldsymbol{v}_i$$

← # sampling

Exact vs. stochastic estimate (N_s=32) for the level density in ^{28}Si

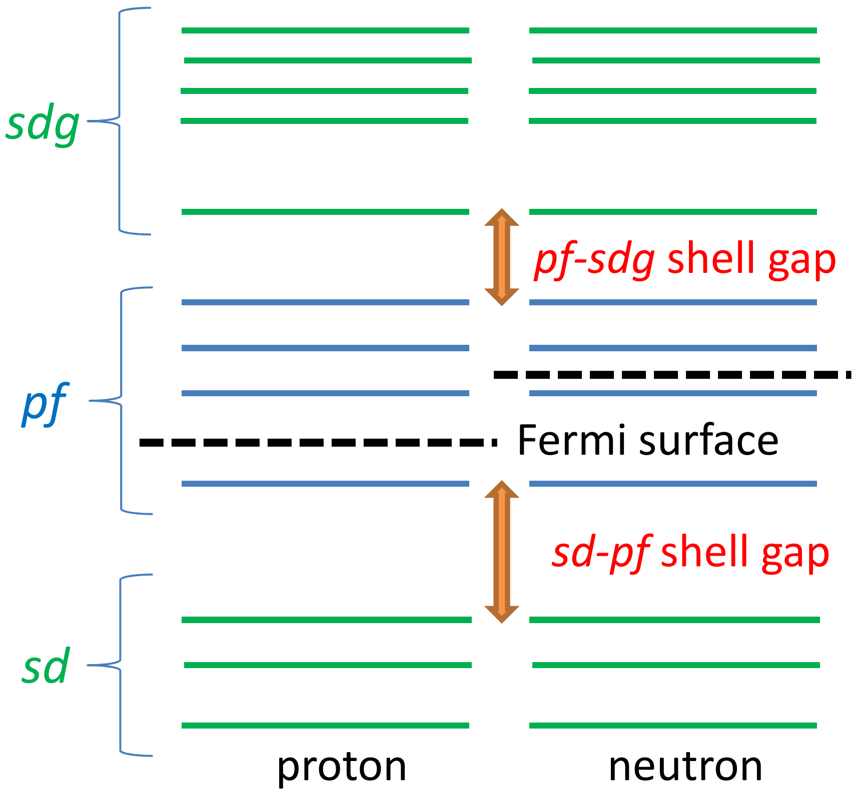


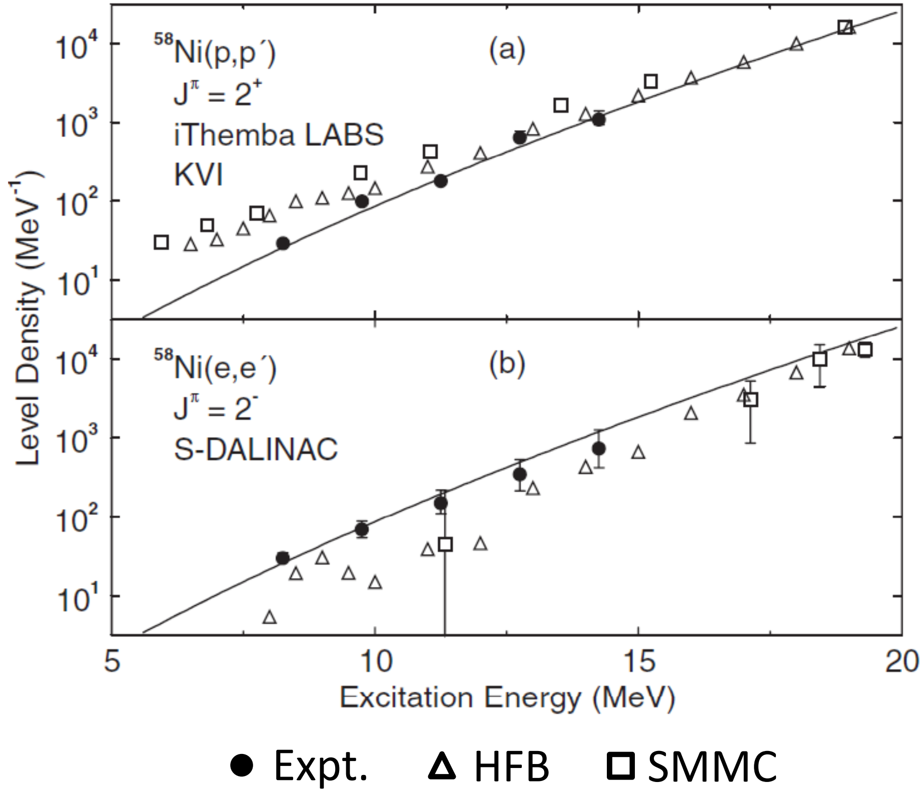
Solve $\boldsymbol{v}_i = (z-H)\boldsymbol{x}_i$ using the conjugate gradient method

When $\boldsymbol{v}_i$'s are chosen to have good quantum numbers (J, π), spin-parity dependent level densities are obtained.

The trace can be excellently estimated with a small number of sampling (known in computational mathematics).

Application: 2^+ and 2^- level density in ^{58}Ni





Y. Kalmykov et al.,Phys. Rev. Lett. 99, 202502 (2007).

- Middle of the *pf* shell: Large energy is needed to excite a nucleon across the major shells. It is thus not easy to reproduce the equilibration of 2^+ and 2^- levels observed recently. Is it possible to simultaneously describe low-lying spectroscopic strengths and the parity equilibration?

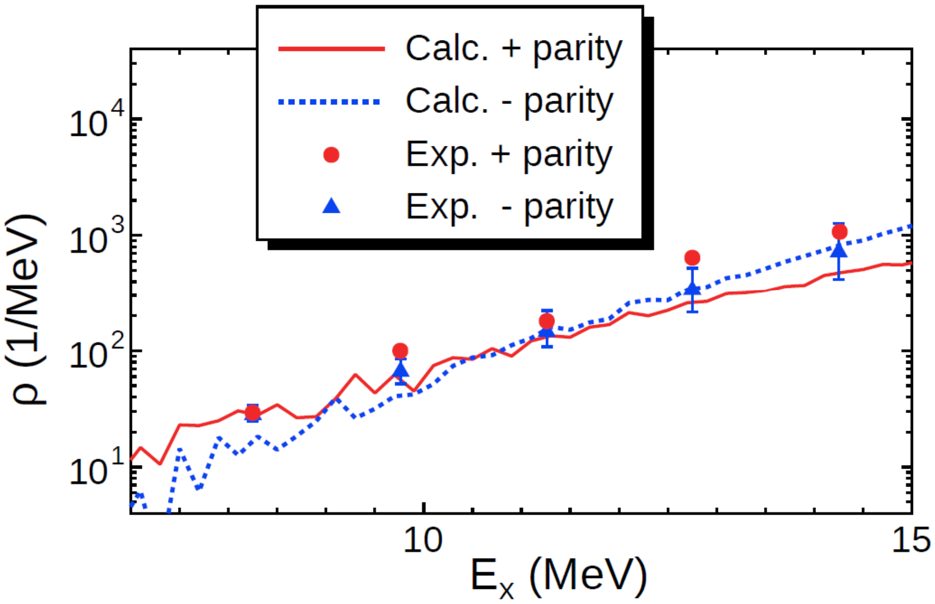
Results of the shell-model calculation

- Very large-scale shell-model calculation
  - $0\hbar\omega$ and $1\hbar\omega$ calc.: *M*-scheme dimension = 1.5×10^{10} for $1\hbar\omega$
  - SPEs of the *sdg* orbitals are fine-tuned to obtain good spectroscopic strengths.

Spectroscopic strengths

Nucl.	J^π	E_x (MeV) Cal.	E_x (MeV) Exp.	j	C^2S Cal.	C^2S Exp.
^{57}Co	$7/2_1^-$	0	0	$\pi 0f_{7/2}^{-1}$	5.28	4.27, 5.53
-1*p*	$1/2_1^+$	3.037	2.981	$\pi 1s_{1/2}^{-1}$	0.98	1.05, 1.31
	$3/2_1^+$	3.565	3.560	$\pi 0d_{3/2}^{-1}$	1.70	1.50, 2.33
^{57}Ni	$3/2_1^-$	0	0	$\nu 1p_{3/2}^{-1}$	1.14	1.04, 1.25, 0.96
-1*n*	$1/2_1^+$	5.581	5.580	$\nu 1s_{1/2}^{-1}$	0.51	0.62, 1.08
	$3/2_1^+$	5.579	4.372	$\nu 0d_{3/2}^{-1}$	0.29	0.01
	$3/2_2^+$	6.093	6.027	$\nu 0d_{3/2}^{-1}$	0.22	0.66, 0.54
^{59}Cu	$3/2_1^-$	0	0	$\pi 1p_{3/2}^{+1}$	0.53	0.46, 0.49, 0.25
+1*p*	$9/2_1^+$	3.139	3.023	$\pi 0g_{9/2}^{+1}$	0.26	0.24, 0.32, 0.27
^{59}Ni	$3/2_1^-$	0	0	$\nu 1p_{3/2}^{+1}$	0.51	0.82, 0.33
+1*n*	$9/2_1^+$	3.053	3.054	$\nu 0g_{9/2}^{+1}$	0.63	0.84, 0.39
	$5/2_1^+$	4.088	3.544	$\nu 1d_{5/2}^{+1}$	0.04	0.03
	$5/2_2^+$	4.595	4.506	$\nu 1d_{5/2}^{+1}$	0.30	0.23, 0.14
	$1/2_1^+$	4.399	5.149	$\nu 2s_{1/2}^{+1}$	0.00	0.09
	$1/2_2^+$	5.492	5.569	$\nu 2s_{1/2}^{+1}$	0.18	0.02
	$1/2_3^+$	5.589	5.692	$\nu 2s_{1/2}^{+1}$	0.02	0.13

2^+ and 2^- level densities in ^{58}Ni



N. Shimizu et al., Phys. Lett. B 753, 13 (2016).

Summary

- Large-scale shell-model calculations are carried out to study gamma-ray strength function and level density on the same footing.
  - Coupling to non-collective levels is automatically included in the shell-model framework, thus leading to good GDR tail properties.
  - Transitions between excited levels can be obtained.
  - Some topics in Ca isotopes
    - Two-phonon state
    - Pygmy dipole resonances
    - Examining the Brink hypothesis
      - good for the GDR region, not necessarily perfect for the tail region due to configuration dependence in PDR
  - Level density
    - New method which enables large-scale calculations
    - ^{58}Ni: equilibration of positive- and negative-parity levels