Shell-model applications to gamma-ray strength function and level density

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Nuclear structure in the GDR region

- Close relation among:
  - Fine structure
  - Width of GDR
  - Level density

- Shell model (CI)
  - All the levels in a given model space
  - Transitions between excited states

Taken from a figure by A. Richter
Outline of the present talk

• Studying the gamma-ray strength function and level density on the same footing using large-scale shell-model calculations

  1. E1 strength function including transitions among excited levels
     • GDR of Ca isotopes: \( 1\hbar \omega \) vs. \( (1+3)\hbar \omega \)
     • Two-phonon state probed with decay probabilities
     • Pygmy dipole resonances
     • Examining the Brink hypothesis

  2. Level density
     • New method of calculating level density in the shell model
     • Application to \(^{58}\text{Ni}\): equilibration of positive- and negative-parity levels
Framework

• Objectives
  – \textit{pf}-shell nuclei (e.g. Ca isotopes)

• Valence shell
  – Full \textit{sd-pf-sdg} shell
  – $0\hbar\omega$ and $1\hbar\omega$ states for the ground and 1$^-$ levels, respectively

• Effective interaction
  – same as the one used for 3$^-_1$ levels in Ca isotopes (Y. Utsuno et al., PTP Suppl., 2012)
  – USD (\textit{sd}) + GXPF1B (\textit{pf}) + $V_{MU}$ (remaining)
    • Successful in \textit{sd-pf} shell calculations including exotic nuclei (e.g. $^{42}\text{Si}$, $^{44}\text{S}$)
  – $g_{9/2}$ SPE: fitted to 9/2$^+_1$ in $^{51}\text{Ti}$
    • Reasonable agreement with a recent $C^2S$ data in $^{49}\text{Ca}$ (A. Gade et al., 2016)

• Removal of spurious center-of-mass motion with Lawson method
Lanczos strength function method

Efficient way to avoid calculating all the eigenstates

1. Take an initial vector: \( \overrightarrow{u_1} = T(E1)|\text{g.s.}\) 
2. Follow the usual Lanczos iterations 
\[ H\overrightarrow{u_k} = \beta_{k-1}\overrightarrow{u_{k-1}} + \alpha_k\overrightarrow{u_k} + \beta_k\overrightarrow{u_{k+1}} : \text{defining a normalized vector } \overrightarrow{u_{k+1}} \]
3. Calculate the strength function \( \sum_v B(E1; \text{g.s.} \rightarrow v) \frac{1}{\pi(E-E_v+E_0)^2+(\Gamma/2)^2} \) by summing up all the eigenstates \( v \) in the Krylov subspace with an appropriate smoothing factor \( \Gamma \) until good convergence is achieved.

Example of convergence with \( \Gamma = 1 \text{ MeV} \)
Photo-absorption cross sections for $^{48}$Ca

- GDR peak position: good
- GDR peak height: overestimated
- GDR tail: *weak dependence on the choice of* $\Gamma$  

fine structure
**E1 strength function: 1ℏω vs. (1+3)ℏω**

- Improved GDR peak height
- Low-energy tail is almost unchanged.
  - The following results are restricted to 1ℏω, concentrated on the tail region.

**Graphical Representation**

- **48Ca**
- **σ (mb)** vs. **E_x (MeV)**

- **0ℏω g.s. → 1ℏω**
  - 0g1d2s
  - 0f1p
  - 0d1s

- **(0+2)ℏω g.s. → (1+3)ℏω**
  - 0g1d2s
  - 0f1p
  - 0d1s

Up to $10^{10}$ m-scheme dimensions in Ca isotopes
Two-phonon states in $^{48}\text{Ca}$

- The proposed state has much stronger $B(E1)$ than other isotopes.
- No $E2$ or $E3$ measurement

$$D = 5.367 \times 10^{-4}(Z + N)Z\beta_2\beta_3 \text{ (efm)}$$

Which $1^-$ level is the two-phonon state?

- Discrete levels are calculated with the exact diagonalization.
- A few hundred keV upward shift of E1 strength gives excellent agreement.


Candidate for $2^+ \otimes 3^-$ two-phonon state?
Probing $2^+ \otimes 3^-$ two-phonon character

- Two-phonon state: the $1^-$ level with a very small $B(E1)$
Properties of the $1^{-}_{2\text{ph}}$ state

- Agreement with the systematics
  - Position: $\frac{E(1^{-}_{2\text{ph}})}{[E(2^+) + E(3^-)]} = 0.91$
  - $B(E1): \frac{B(E1)}{D^2} = 1.6$

E2 decay competes with E1.
Development of pygmy dipole resonance


pointed out strong correlation with the occupation of the $p$ orbitals
Proton $rY^{(1)}$ matrix elements

- Decomposing the E1 mat. ele. into $\sum_{i,j} \langle g\text{. s.} | rY^{(1)}(i \rightarrow j) | 1^- \rangle$

**PDR state**

$$\langle g\text{. s.} | rY^{(1)}(i \rightarrow j) | PDR \rangle$$

- $\langle g\text{. s.} | rY^{(1)}(i \rightarrow j) | PDR \rangle$

- $(0d_{5/2} \rightarrow 0f_{7/2})$
- $(0d_{3/2} \rightarrow 0f_{5/2})$
- $(1s_{1/2} \rightarrow 1p_{3/2})$

**Sum rule state (GDR)**

$$\langle g\text{. s.} | rY^{(1)}(i \rightarrow j) | SR \rangle$$

- $\langle g\text{. s.} | rY^{(1)}(i \rightarrow j) | SR \rangle$

- $(0d_{3/2} \rightarrow 0f_{5/2})$
- $(0d_{5/2} \rightarrow 0f_{7/2})$
- $(1s_{1/2} \rightarrow 1p_{3/2})$
Neutron $rY^{(1)}$ matrix elements

**PDR state**

Incoherence of higher-node transition

**Sum rule state (GDR)**

![Graph showing various transitions and coherences in neutron matrix elements](image)
Brink hypothesis

• GDR built on excited states
  – Assumed to be identical with that of the ground state by shifting Ex. of the initial state
    • geometric nature of GDR
  – Practically important to evaluate $(n, \gamma)$ cross sections

• Recent experiments
  – Generalized Brink hypothesis valid in $^{238}\text{Np}$ (M. Guttormsen et al., 2016)
  – Possible deviation from the Brink hypothesis based on $^{89}\text{Y}(\rho, \gamma)^{90}\text{Zr}$ (L. Netterdon et al., 2015)
Examining the Brink hypothesis in $^{48}$Ca

- The “generalized” Brink hypothesis (independent of excitation energy and spin) looks valid as far as the GDR region is concerned.
Brink hypothesis in the tail region: $^{48}$Ca

- Excited states have larger E1 strengths in the tail region.
- What dominates the difference? Excitation energy?
Brink hypothesis in the tail region: $^{50}$Ca

- Order of E1 strength:
  - no Ex. dependence
  - 1$^{\text{st}}$: $0^+_3$
  - 2$^{\text{nd}}$: $0^+_1$
  - 3$^{\text{rd}}$: $0^+_2$ & $0^+_4$

Configuration that favors pygmy
Shell-model calculation for level density

- **Direct counting** with Lanczos diag.
  - Practically impossible because high-lying levels are very slow to converge
- **New method** (Shimizu, Futamura, Sakurai)
  - Utilizing contour integral

\[
\mu_k = \frac{1}{2\pi i} \oint_{\Gamma_k} dz \sum_i^D (z - \lambda_i)^{-1} \\
= \frac{1}{2\pi i} \oint_{\Gamma_k} dz \text{tr} (z - H)^{-1}
\]
Stochastic estimate of the trace

- Remaining task: estimating 
  \[ \text{tr}(z - H)^{-1} \]
  \[ = \sum_i e_i^T (z - H)^{-1} e_i \]
- Stochastic sampling
  \[ \approx \frac{1}{N_s} \sum_{s}^{N_s} \nu_i^T (z - H)^{-1} \nu_i \]

Solve \( \nu_i = (z - H)x_i \) using the conjugate gradient method

When \( \nu_i \)'s are chosen to have good quantum numbers \( (J, \pi) \), spin-parity dependent level densities are obtained.

The trace can be excellently estimated with a small number of sampling (known in computational mathematics).
Application: $2^+$ and $2^-$ level density in $^{58}\text{Ni}$

- **Middle of the pf shell**: Large energy is needed to excite a nucleon across the major shells. It is thus not easy to reproduce the equilibration of $2^+$ and $2^-$ levels observed recently. Is it possible to simultaneously describe low-lying spectroscopic strengths and the parity equilibration?

Results of the shell-model calculation

- Very large-scale shell-model calculation
  
  - $0\hbar \omega$ and $1\hbar \omega$ calc.: $M$-scheme dimension = $1.5 \times 10^{10}$ for $1\hbar \omega$
  
  - SPEs of the $sdg$ orbitals are fine-tuned to obtain good spectroscopic strengths.

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<th>$J^p$</th>
<th>$E_x$ (MeV)</th>
<th>$C^2S$</th>
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Spectroscopic strengths

$2^+$ and $2^-$ level densities in $^{58}$Ni

Summary

• Large-scale shell-model calculations are carried out to study gamma-ray strength function and level density on the same footing.
  – Coupling to non-collective levels is automatically included in the shell-model framework, thus leading to good GDR tail properties.
  – Transitions between excited levels can be obtained.
  – Some topics in Ca isotopes
    • Two-phonon state
    • Pygmy dipole resonances
    • Examining the Brink hypothesis
      – good for the GDR region, not necessarily perfect for the tail region due to configuration dependence in PDR
  – Level density
    • New method which enables large-scale calculations
    • $^{58}$Ni: equilibration of positive- and negative-parity levels