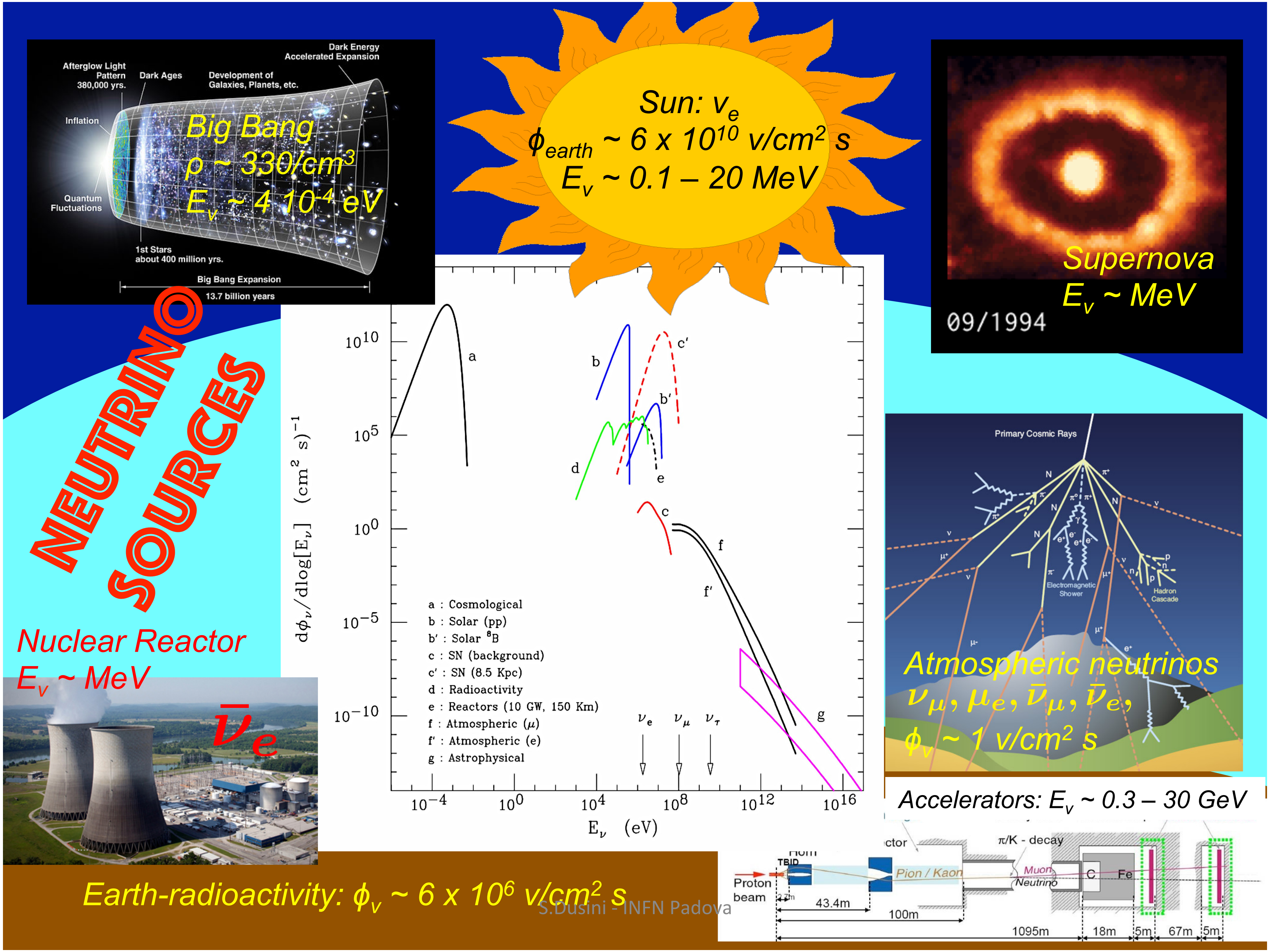
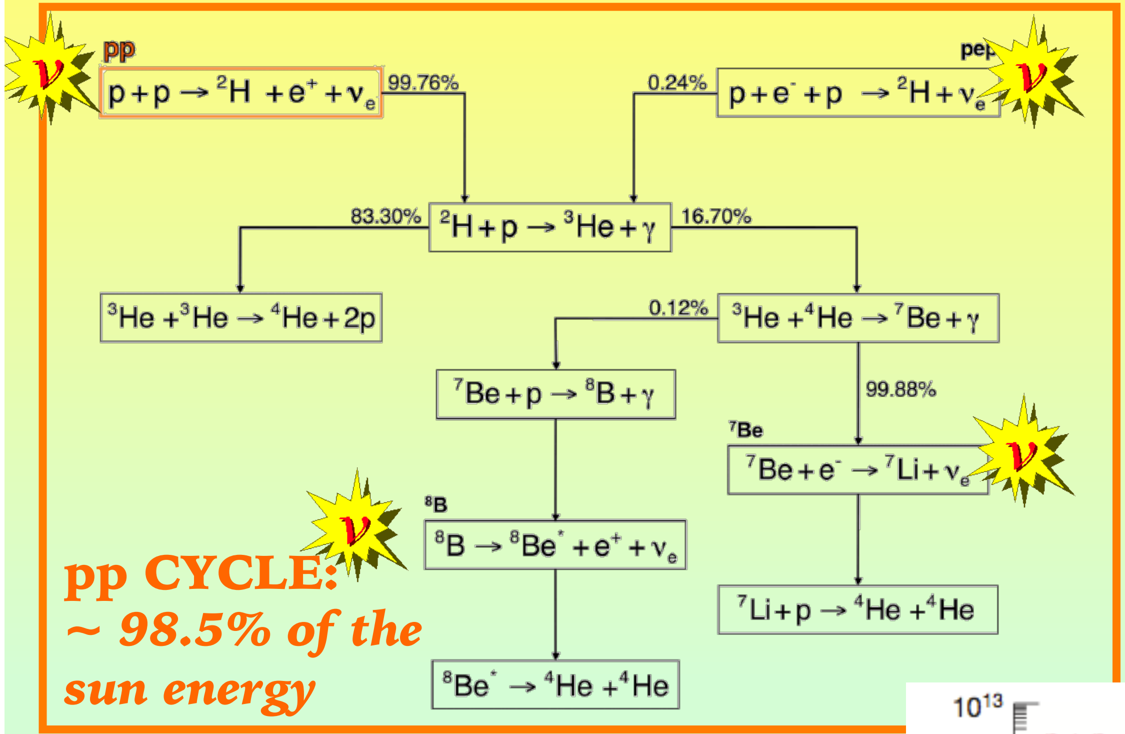
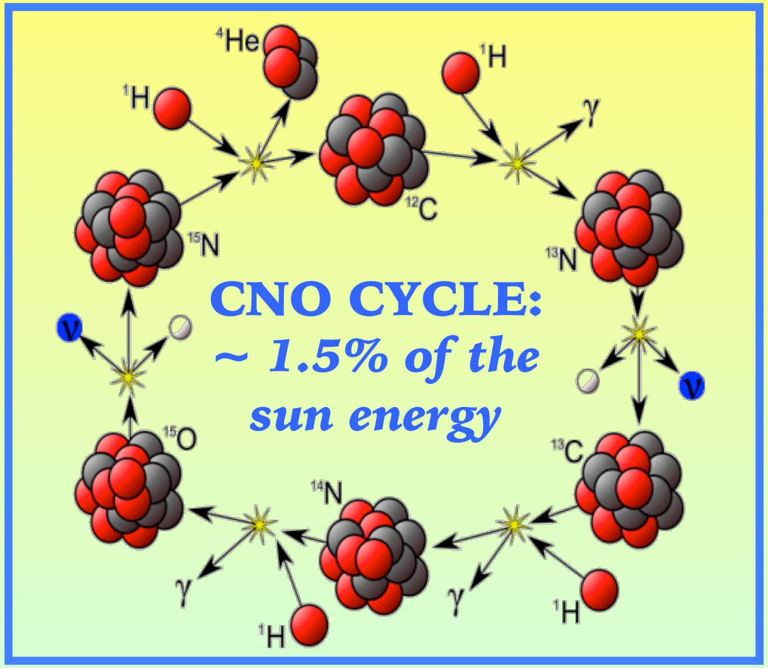
Solar, ***atmospheric*** *and* **REACTOR** *neutrinos…* *what we can learn without a beam*

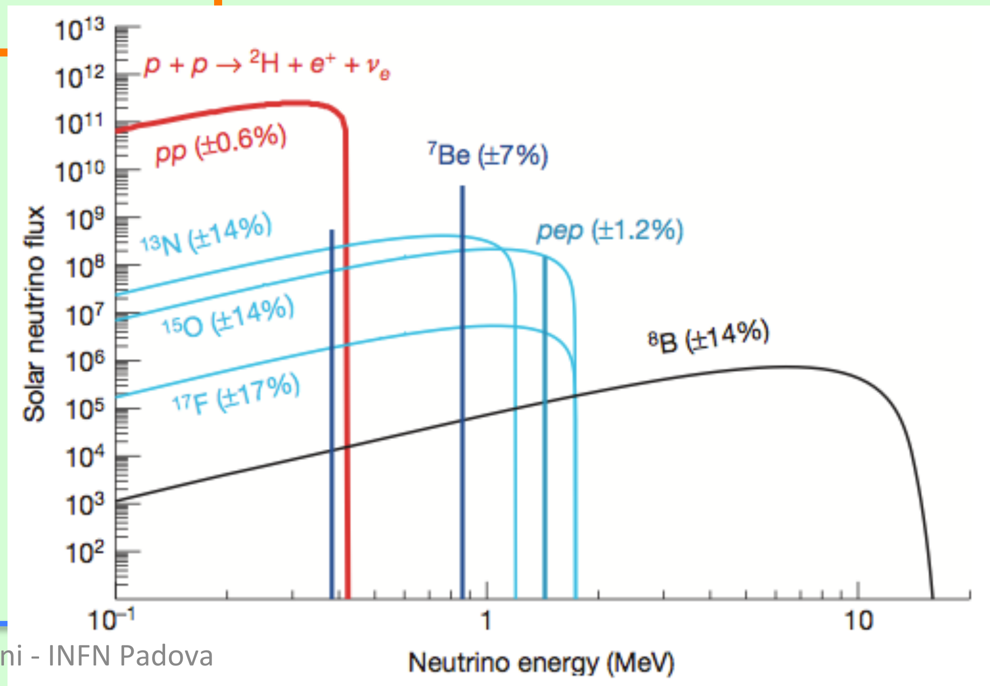
Stefano Dusini
INFN Padova


NEUTRINO SOURCES
Big Bang
ρ ~ 330/cm³
E_ν ~ 4 10⁻⁴ eV
Afterglow Light Pattern 380,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Dark Energy Accelerated Expansion
Inflation
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years
Sun: ν_e
φ_earth ~ 6 x 10¹⁰ ν/cm² s
E_ν ~ 0.1 – 20 MeV
Supernova
E_ν ~ MeV
09/1994
a : Cosmological
b : Solar (pp)
b′ : Solar ⁸B
c : SN (background)
c′ : SN (8.5 Kpc)
d : Radioactivity
e : Reactors (10 GW, 150 Km)
f : Atmospheric (μ)
f′ : Atmospheric (e)
g : Astrophysical
dφ_ν/dlog[E_ν] (cm² s)⁻¹
E_ν (eV)
Nuclear Reactor
E_ν ~ MeV
ν̄_e
Primary Cosmic Rays
Electromagnetic Shower
Hadron Cascade
Atmospheric neutrinos
ν_μ, μ_e, ν̄_μ, ν̄_e,
φ_ν ~ 1 ν/cm² s
Accelerators: E_ν ~ 0.3 – 30 GeV
Proton beam
Pion / Kaon
π/K - decay
Muon
Neutrino
43.4m
100m
1095m
18m
5m
67m
5m
Earth-radioactivity: φ_ν ~ 6 x 10⁶ ν/cm² s
S.Dusini - INFN Padova

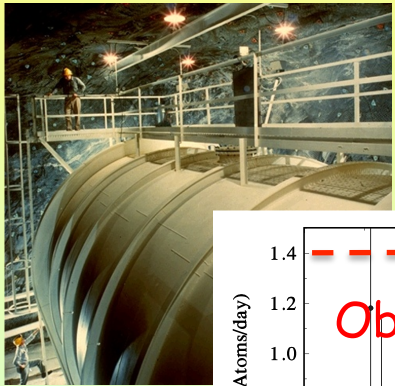




SOLAR NEUTRINOS

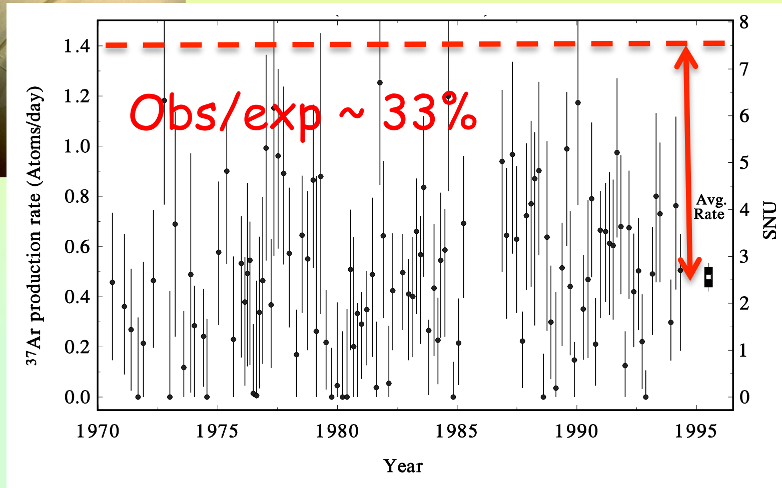


SOLAR NEUTRINOS: *first hint toward* **neutrino oscillation**



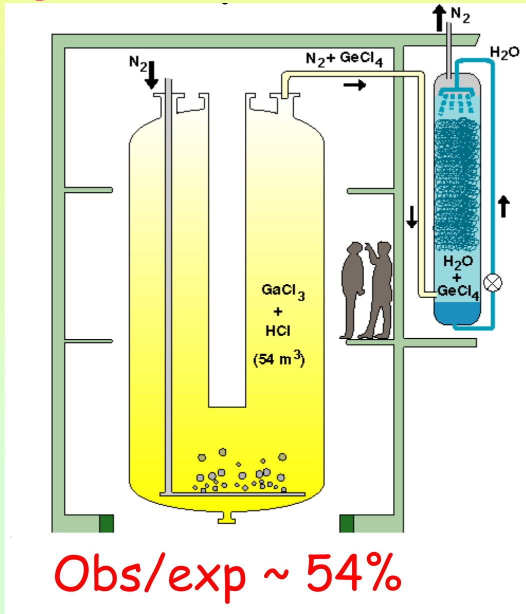
Homestake
(R. Davis & coll.)

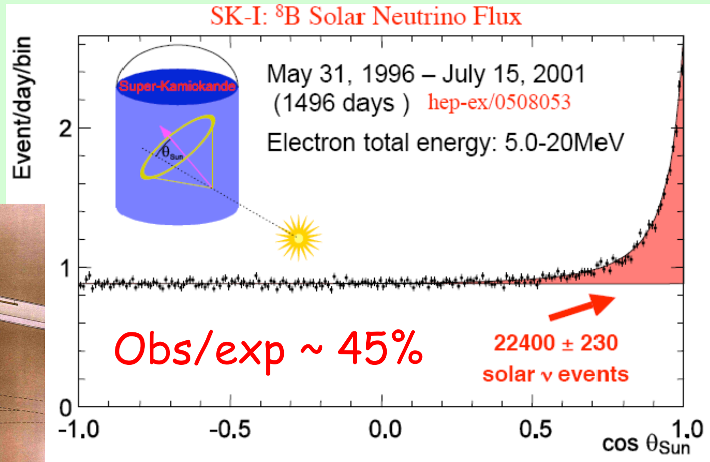
$\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$

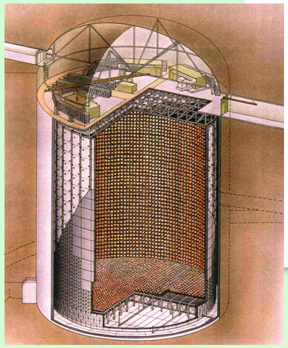


GALLEX/GNO @ LNGS

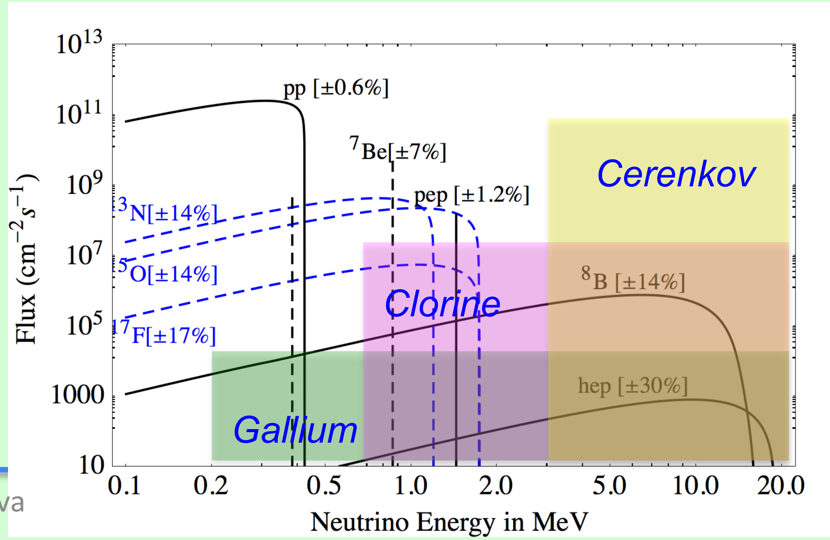
$\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$







Super-Kamiokande



SNO: Heavy water *Cerenkov detector*

CC: $\nu_e + d \rightarrow e^- + p + p$ $(\nu_e + n \rightarrow e^- + p)$

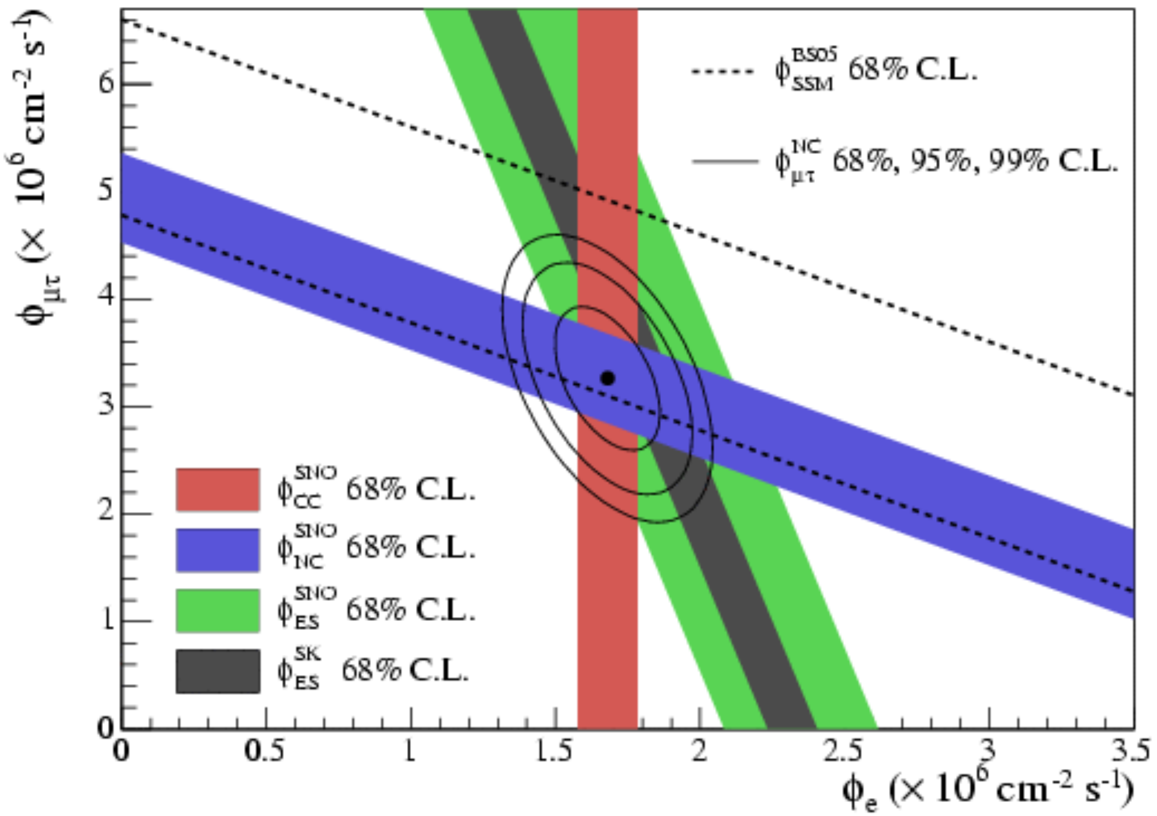
NC: $\nu_{e\mu\tau} + d \rightarrow \nu_{e\mu\tau} + n + p$ (n capture on H, NaCl)

ES: $\nu_{e\mu\tau} + e^- \rightarrow \nu_{e\mu\tau} + e^-$

ν_e *survival probability*

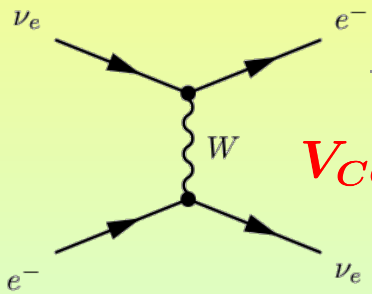
$$P_{SNO} = \frac{\Phi_{CC}}{\Phi_{NC}} = 0.34$$

$\nu_e \rightarrow \nu_\mu, \nu_\tau$ *transformation*



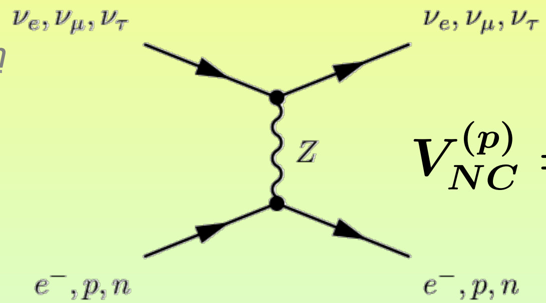
Neutrino propagation in MATTER

Coherent forward scattering of neutrinos



electron density of the medium

$$V_{CC} = \sqrt{2}G_F N_e$$



$$V_{NC}^{(p)} = -V_{NC}^{(e^-)}$$

$\psi_{\alpha\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle$ *Transition amplitude*

$$A_{CC} = 2EV_{CC} = 2\sqrt{2}EG_F N_e$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\theta + A_{CC} & \Delta m^2 \sin 2\theta \\ -\Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - A_{CC}\end{pmatrix}\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix}$$

Effective mixing matrix in matter $\quad U_M^T \mathcal{H}_F U_M = \frac{1}{4E}\mathrm{diag}(-\Delta m_M^2, \Delta_M^2)$

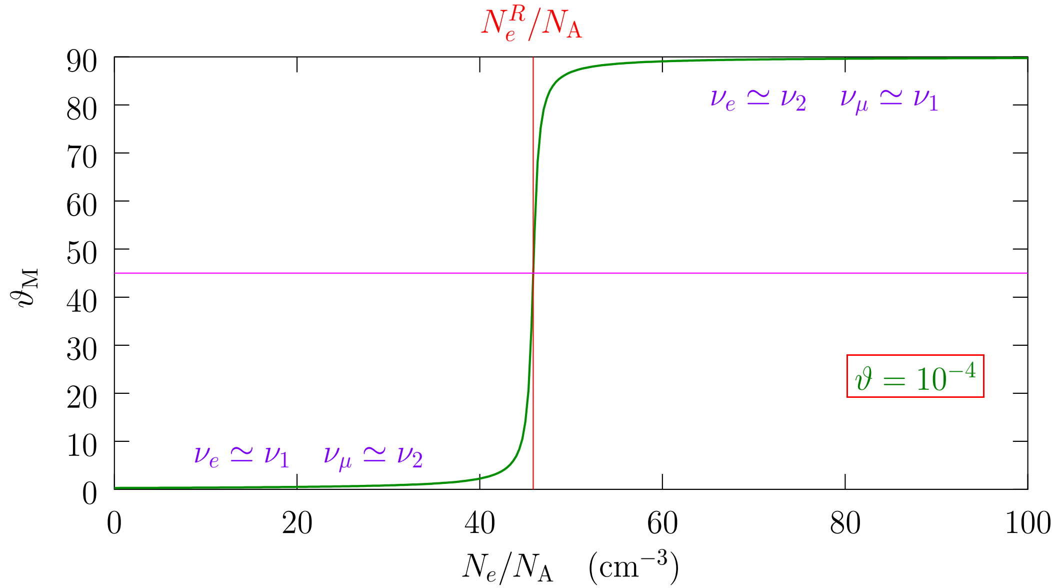
Effective squared-mass difference

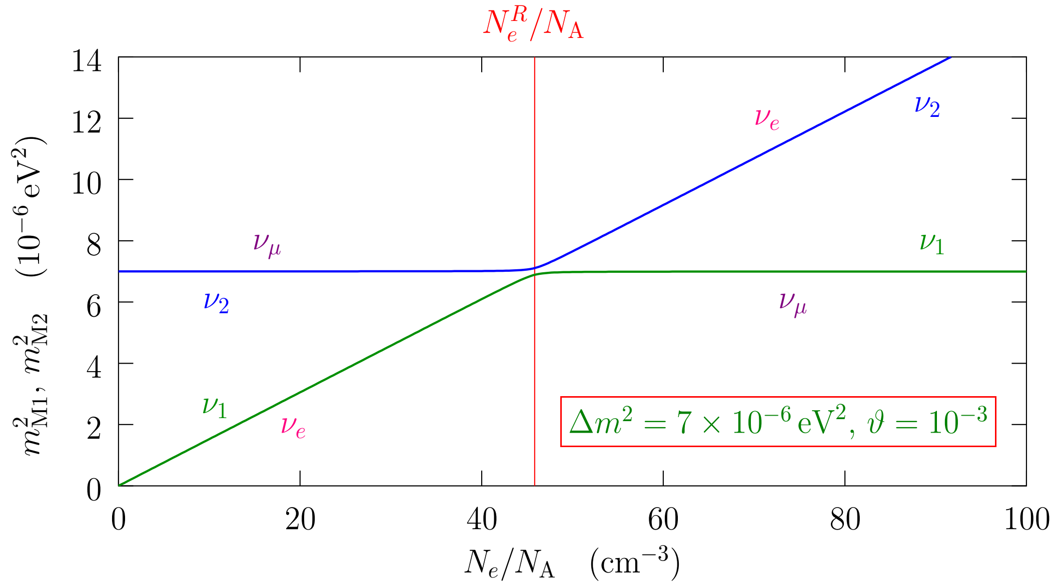
$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\theta - A_{CC})^2 + (\Delta m^2 \sin 2\theta)^2}$$

Effective mixing angle

$$\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}}$$

MSW Effect (Resonant Transitions in Matter)





$$\nu_e = \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2$$

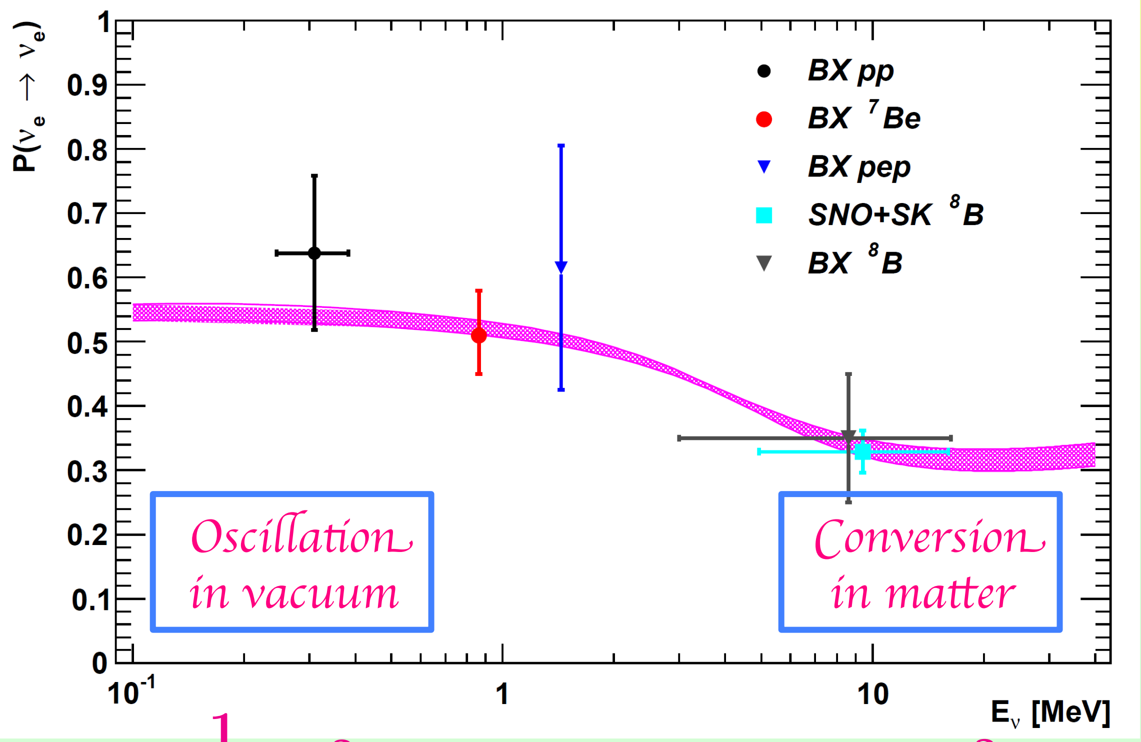
$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

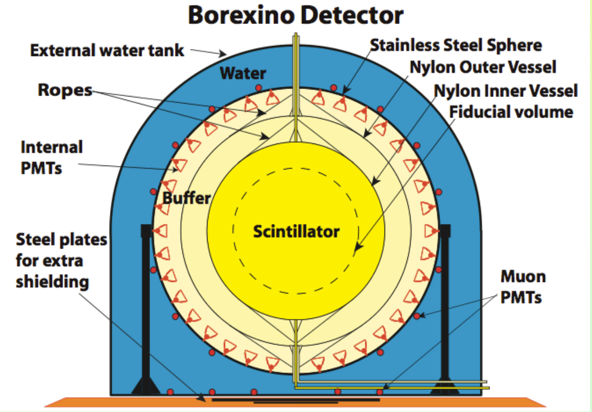
$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

Surface of the sun ← Centre of the sun

Adiabatic conversion

*Observation of **pp neutrinos** by Borexino (@LNGS)*
Confirmed the adiabatic flavour neutrino conversion in the Sun





$$P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

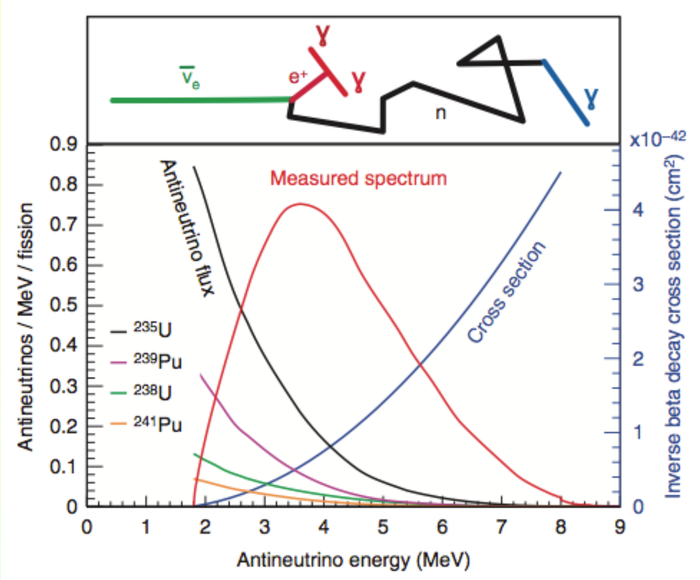
$$P_{ee} = \sin^2\theta_{12} + \cos 2\theta_{12}\cos^2\theta_m^0 + f_{reg}$$

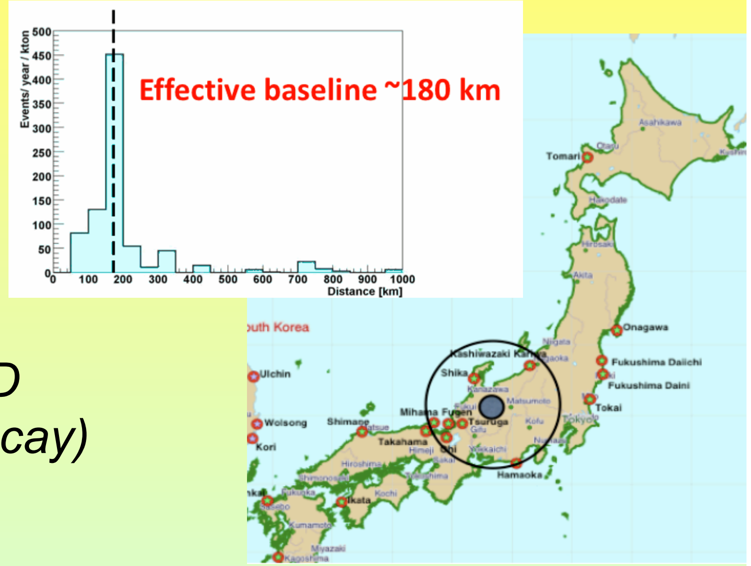
non-oscillatory transition ($E_\nu \to$ inf) ~ 0.31

Residual oscillation: ~ 0.015

Earth regeneration: ~ 0.015

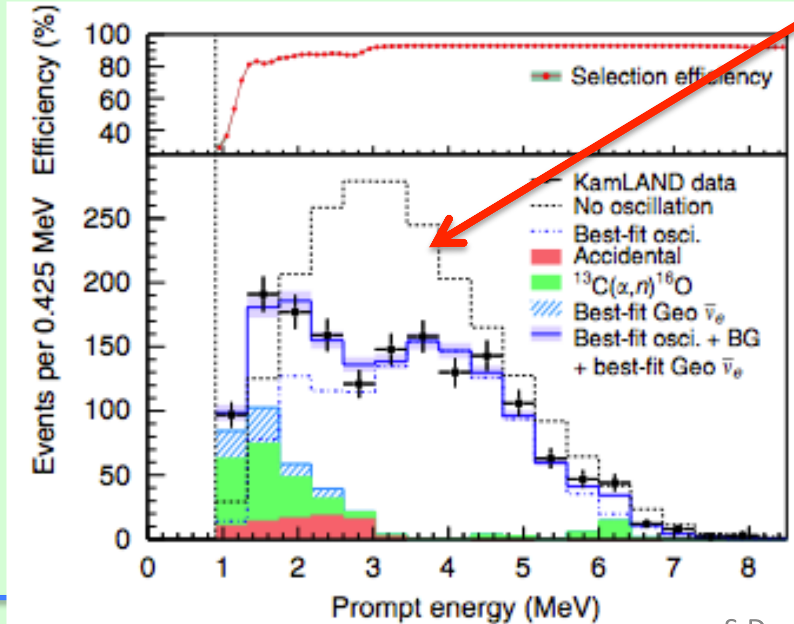
Reactor neutrinos: **KamLAND**

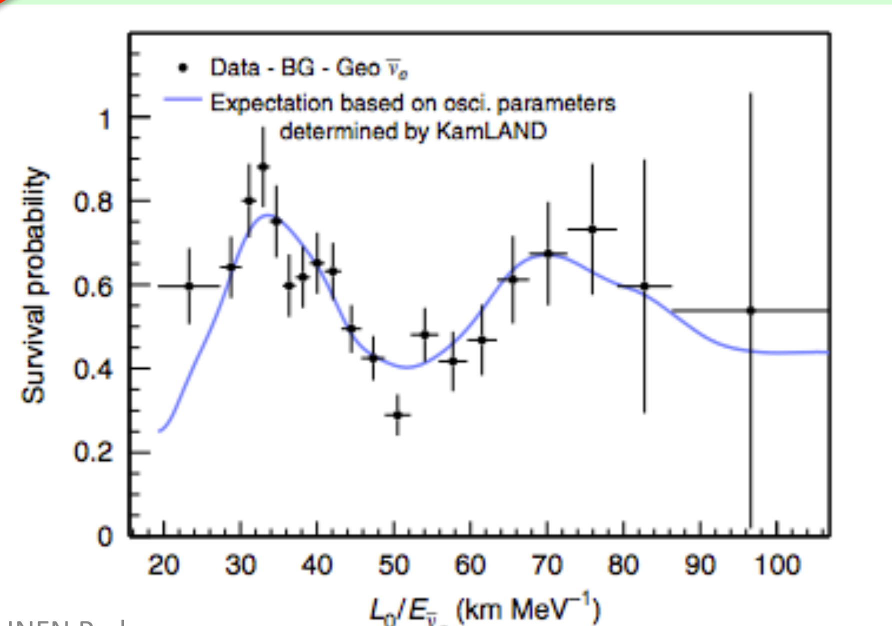




Anti-neutrino IBD (inverse beta decay)

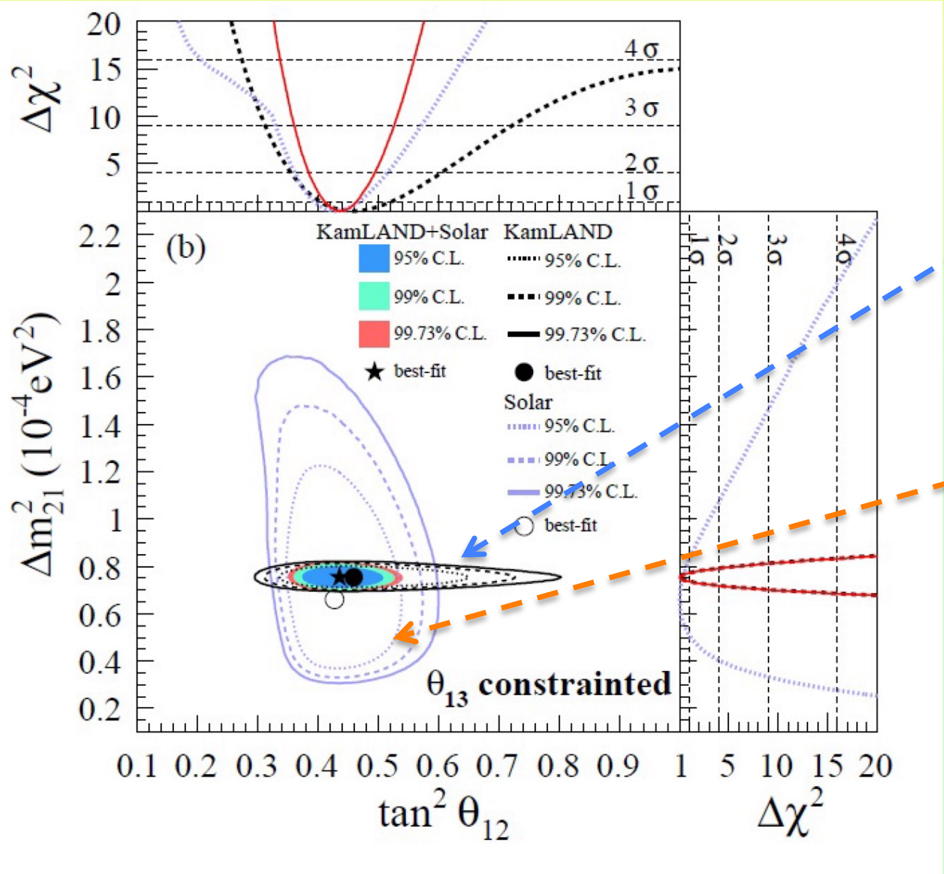
$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \sin^2 2\theta \sin^2 \left(\frac{1.267 \Delta m^2 [\mathrm{eV}^2] L[\mathrm{m}]}{E[\mathrm{MeV}]} \right)$$





SOLAR NEUTRINOS + KamLAND

Disappearance probability is invariant under CP $P(\nu_e \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$



Vacuum oscillation

of anti-neutrinos consistent with parameters of LMA **MSW SOLUTION**

$$\Delta m^2_{12} = (7.53 \pm 0.18) \times 10^{-5} eV^2$$

2.3% precision

$$\tan^2\theta_{12} = 0.436^{+0.029}_{-0.025}$$

6.6% precision

Global fit to SOLAR NEUTRINO DATA + KamLAND

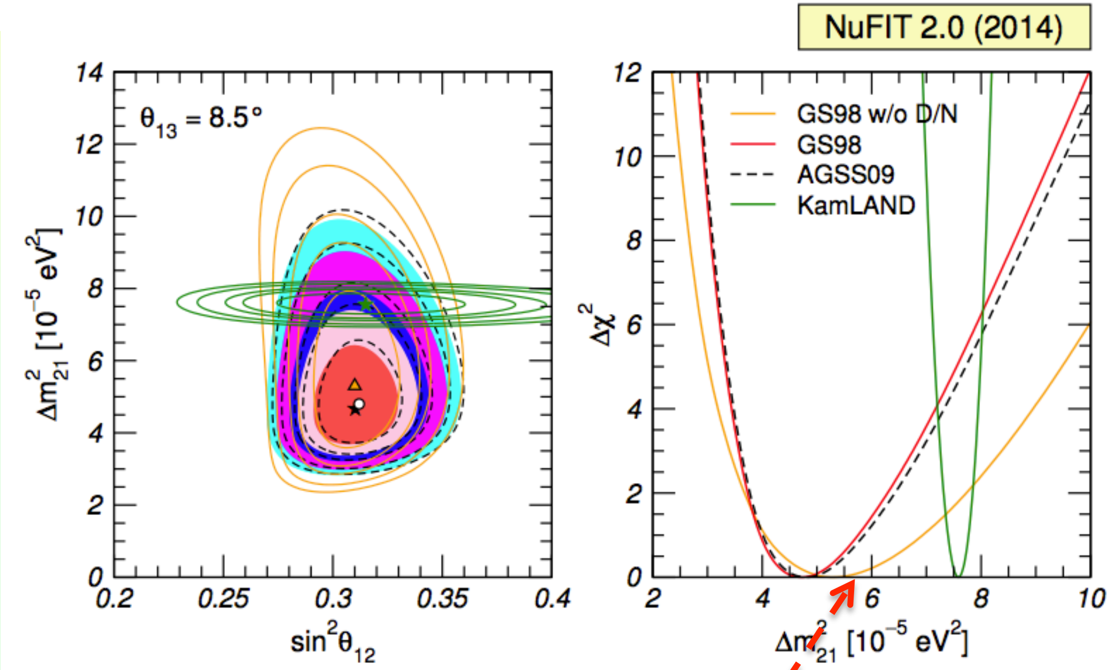
Test of three flavour structure ($\theta_{13} \neq 0$)

$$P_{ee}^{3\nu} = \sin^4\theta_{13} + \cos^4\theta_{13} P_{ee}^{2\nu}(\Delta m^2_{21}, \theta_{12})$$

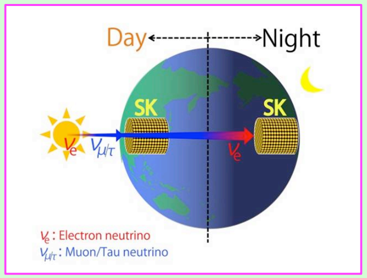
$$P_{ee}^{2\nu,\mathrm{sun}} \simeq 1 - \frac{1}{2}\sin^2(2\theta_{12})$$ ← low E

$$P_{ee}^{2\nu,\mathrm{sun}} \simeq \sin^2(\theta_{12})$$ ← high E

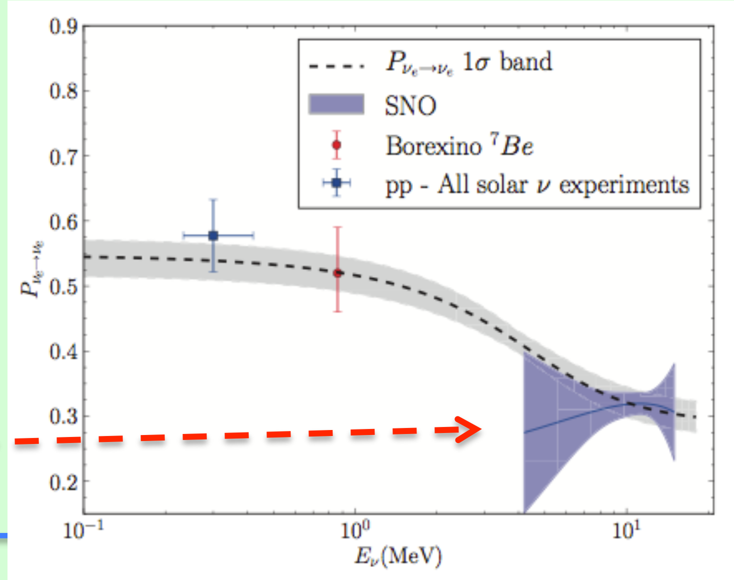
$$P_{ee}^{2\nu,\mathrm{kam}} = 1 - \frac{1}{2}\sin^2(2\theta_{12})\sin^2\frac{\Delta m^2_{21} L}{2E_\nu}.$$



Small tension (~2σ) in Δm^2 between solar and KamLAND data.



- *Indication for Day/Night asymmetry in SK*
- *no up- turn seen in ^8B spectrum so far in solar data*



Transition region

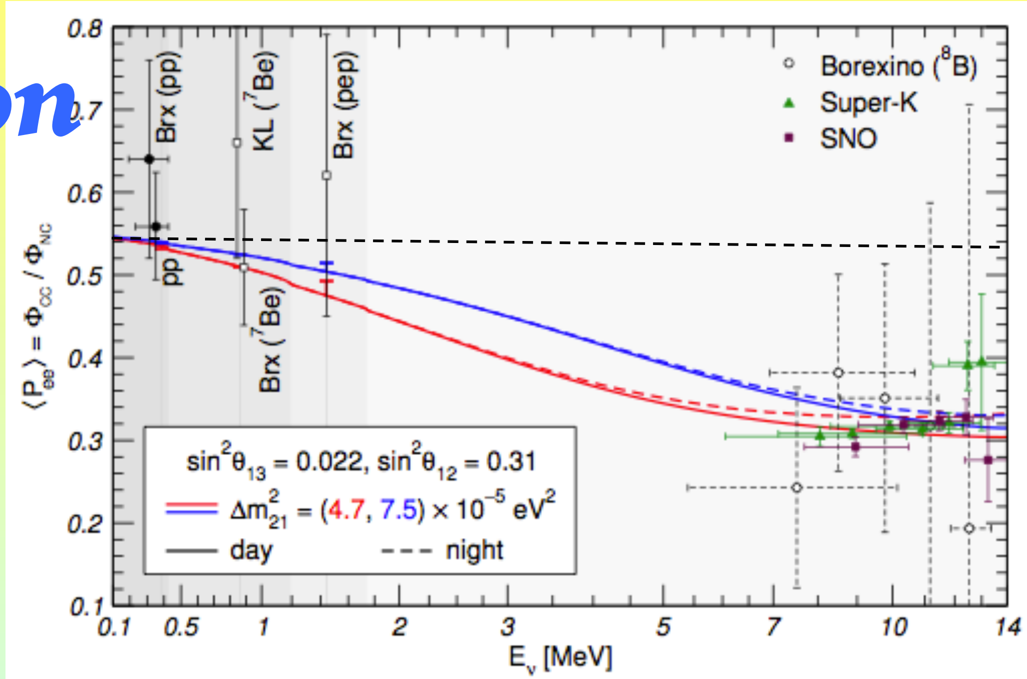
Important to test of
flavour conversion in the sun

- *up-turn*
- *Day/Night asymmetry @ up-turn*

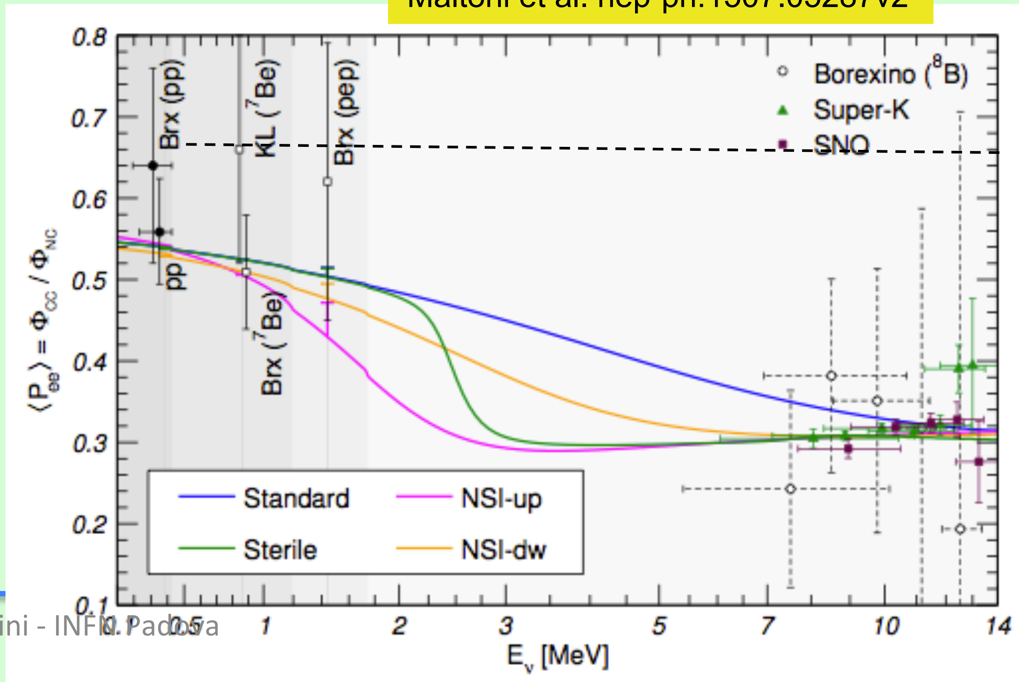
new physics effects

Extra sterile neutrino with
$Dm^2_{01} = 1.2 \times 10^{-5}\ eV^2$, *and*
$sin^2\ 2a = 0.005$

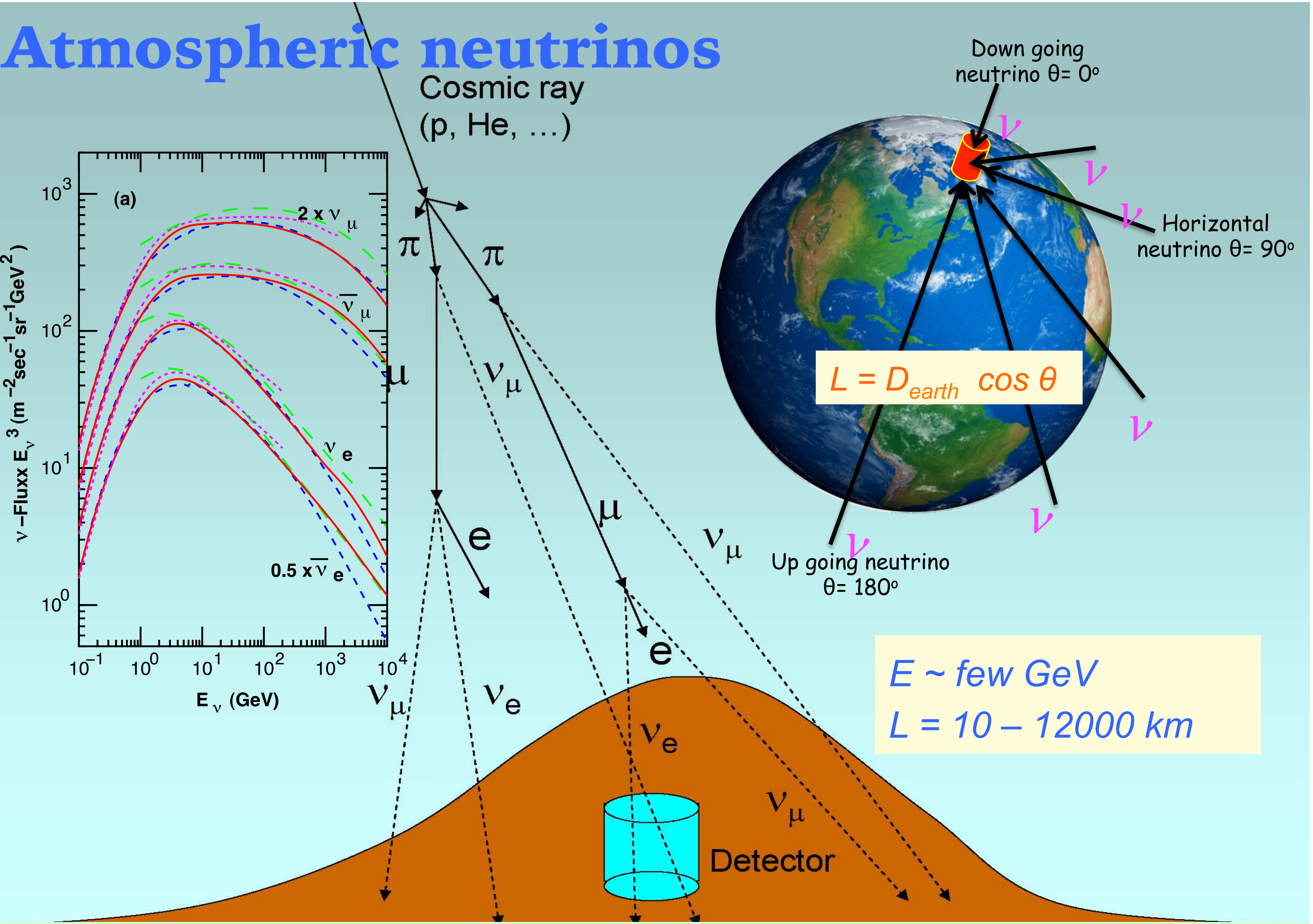
Non-standard interactions with
$e^u_D = -0.22$, $e^u_N = -0.30$
$e^d_D = -0.12$, $e^d_N = -0.16$



Maltoni et al. hep-ph:1507.05287v2



Atmospheric neutrinos

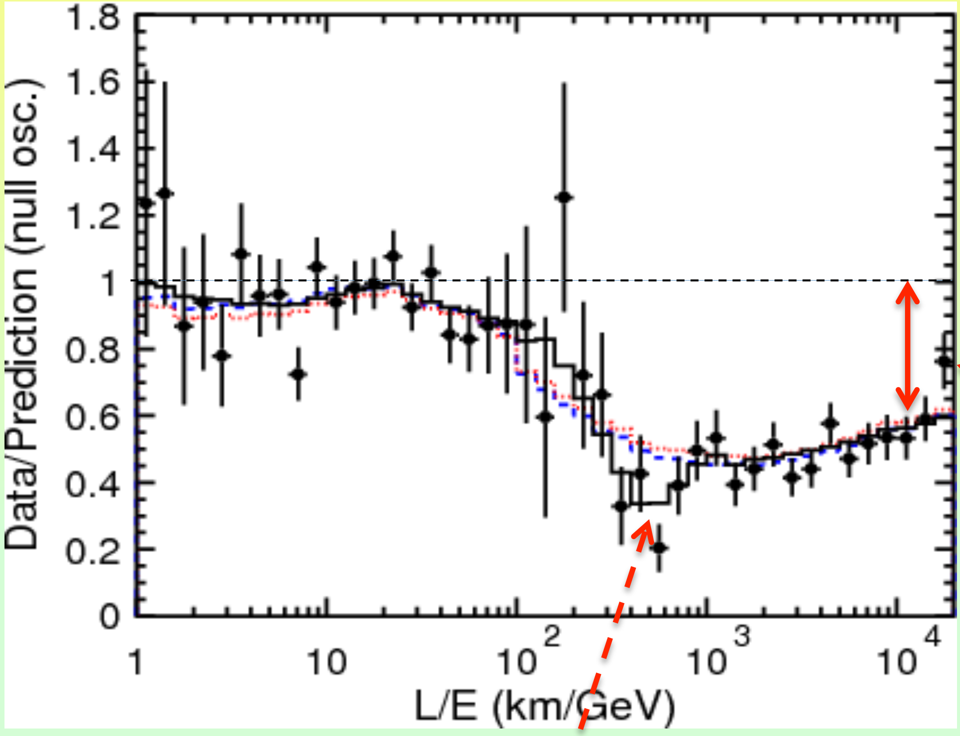


$L = D_{earth}\ \cos\theta$

E ~ few GeV

L = 10 – 12000 km

SuperKamiokande



$\nu_\mu, \bar{\nu}_\mu$ *disappearance*

Deficit of the μ – like events

Determines
mixing angle θ_{23}

dotted: decoherence,
dashed: neutrino decay

No oscillation effect

The first oscillation minimum - determines Δm_{32}^2

Averaged oscillations

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{atm} \sin^2 \left(\frac{\Delta m^2_{atm} L}{4E} \right)$$

$\Delta m_{32}^2 =$ 2.5×10^{-3} eV^2
$\sin^2\theta_{23} =$ $0.45 – 0.55$

Atmospheric neutrinos and mass hierarchy

Like solar neutrinos are affected by the interaction with matter, atmospheric neutrinos crossing the earth can undergo adiabatic flavour conversion induced by (1,3) mixing.

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}^M \sin^2\left(\Delta^M \frac{L}{4E}\right)$$

$$\Delta^M \simeq \sqrt{\left(\Delta m^2_{31}\cos 2\theta_{13} - A\right)^2 - \left(\Delta m^2_{31}\sin 2\theta_{13}\right)^2}$$

$$\sin^2 2\theta_{13}^M \simeq \frac{\Delta^2_{31}\sin 2\theta_{13}}{\Delta^M}$$

$$A = \pm 2\sqrt{2}G_F n_e E_\nu$$

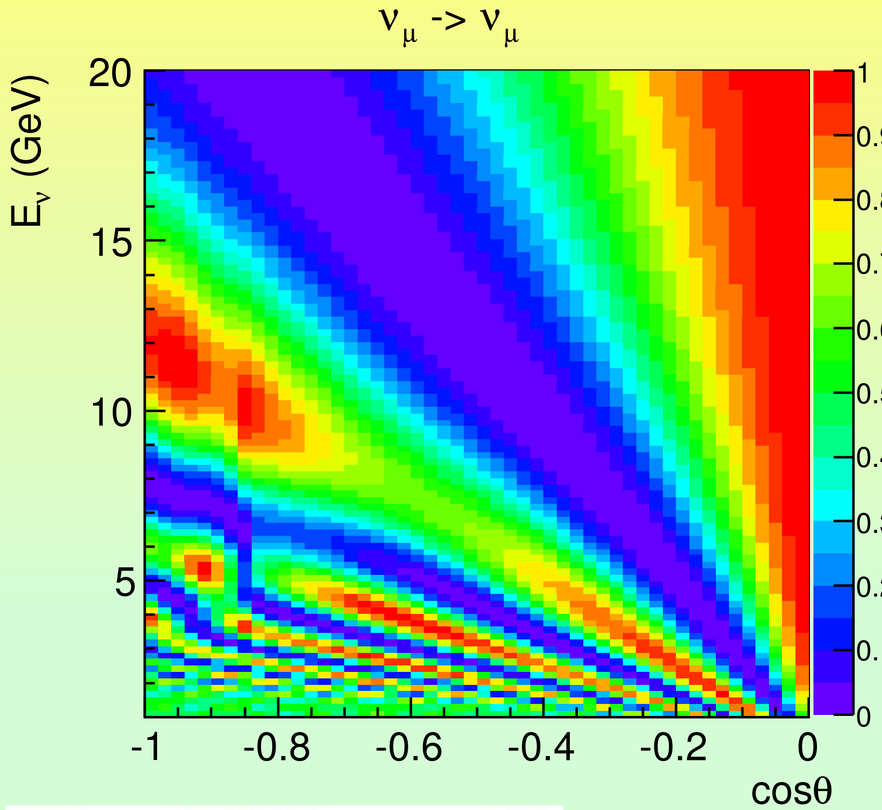
Like for solar neutrinos "matter effects" are sensitive to the sign of Δm^2_{31}

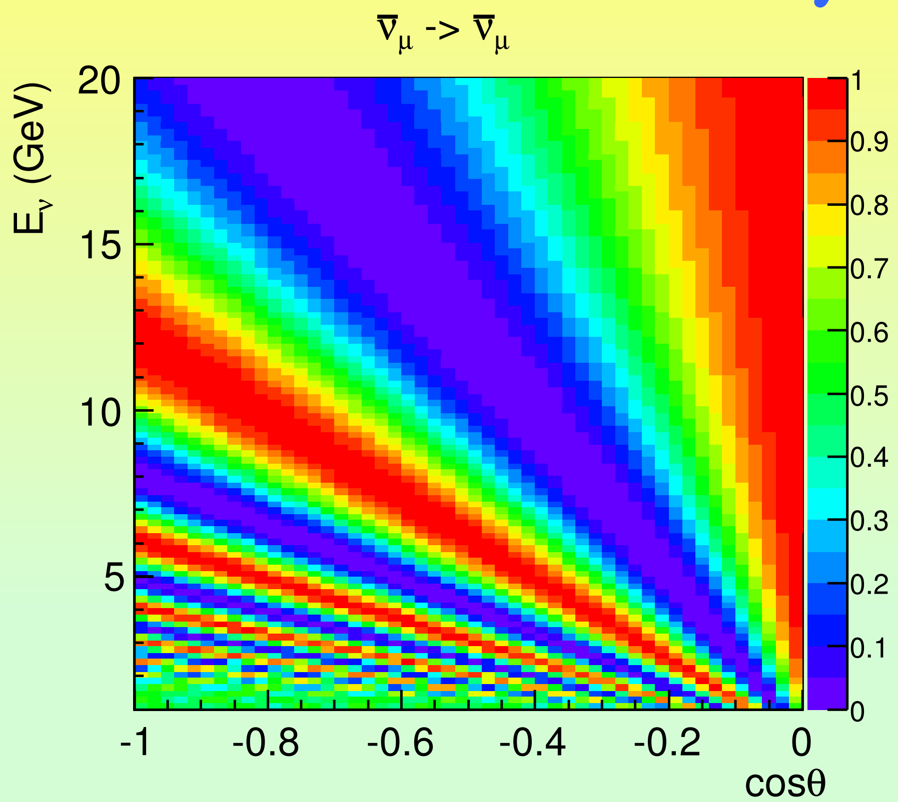
$$A = \Delta m^2_{31}\cos 2\theta_{13}$$

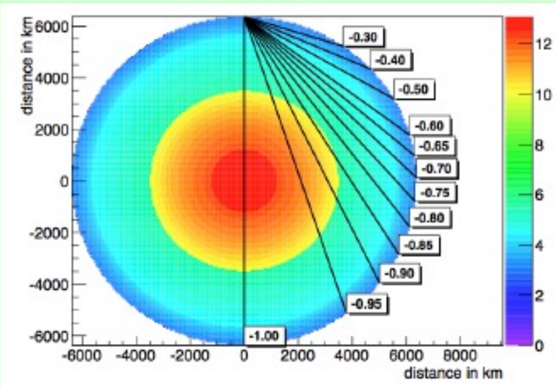
Either neutrinos or anti-neutrinos cross the resonance depending of the sign of Δm^2_{31}

Resonance for E_ν= [5, 8] GeV

Atmospheric neutrinos and mass hierarchy





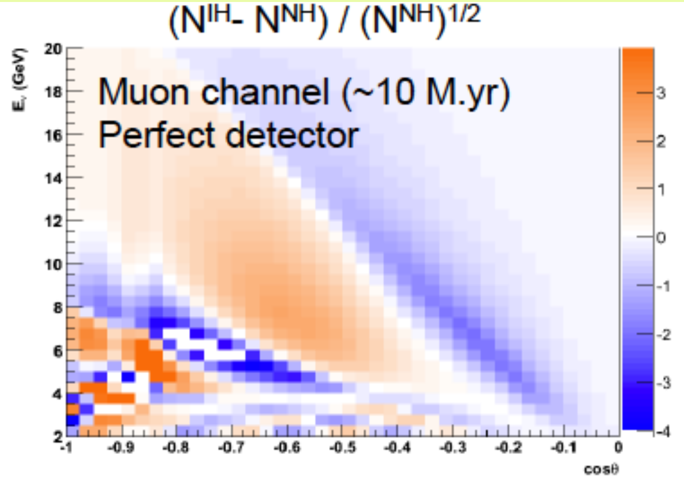


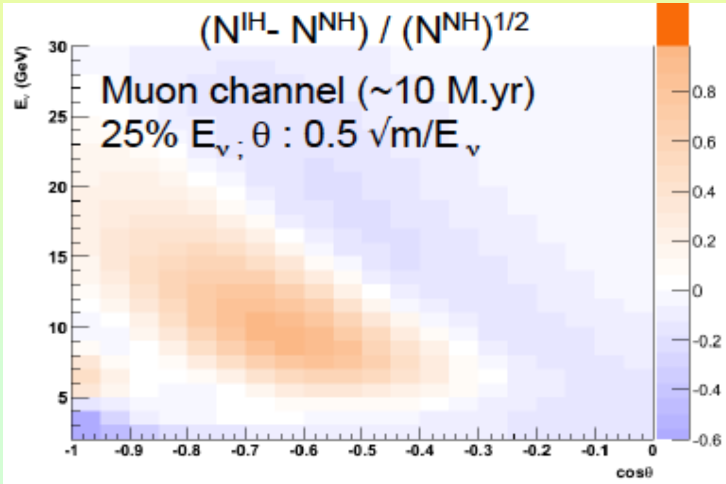
Measurement possible if:

- *the detector is capable to select neutrinos from anti-neutrinos (magnetize iron calorimeter)*
- *good knowledge of neutrino and anti-neutrino fluxes exploiting σ(ν) ≠ σ(anti-ν)*
- *Large mass*

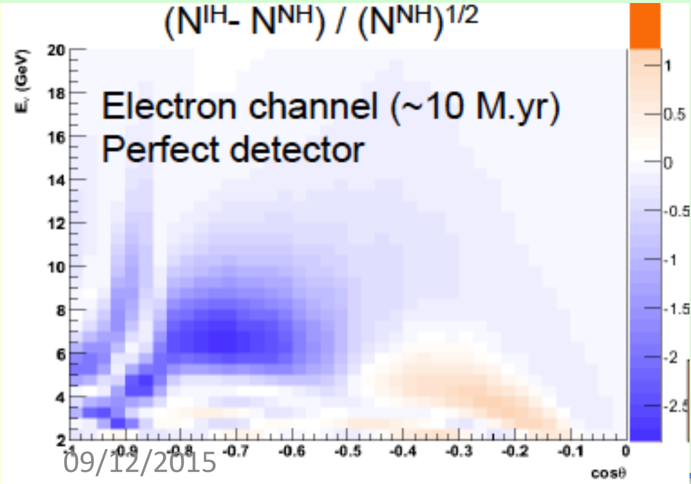
Smeared distributions

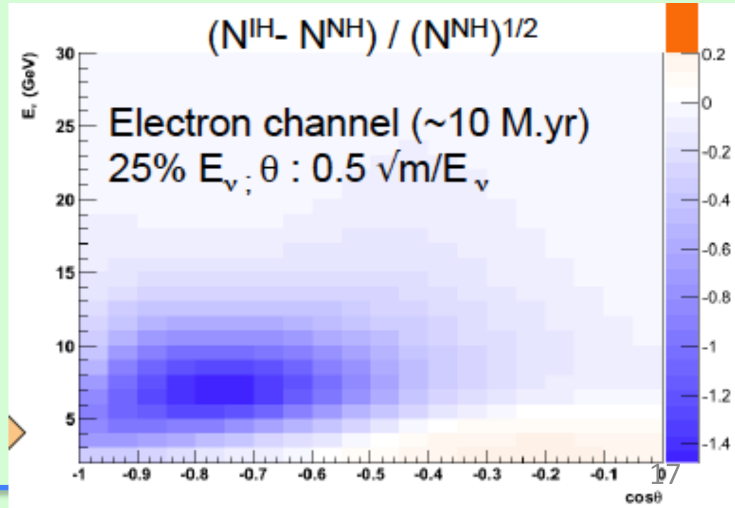
Muon- and electron-channels contribute to net hierarchy asymmetry.
Electron channel more robust against detector resolution effects:





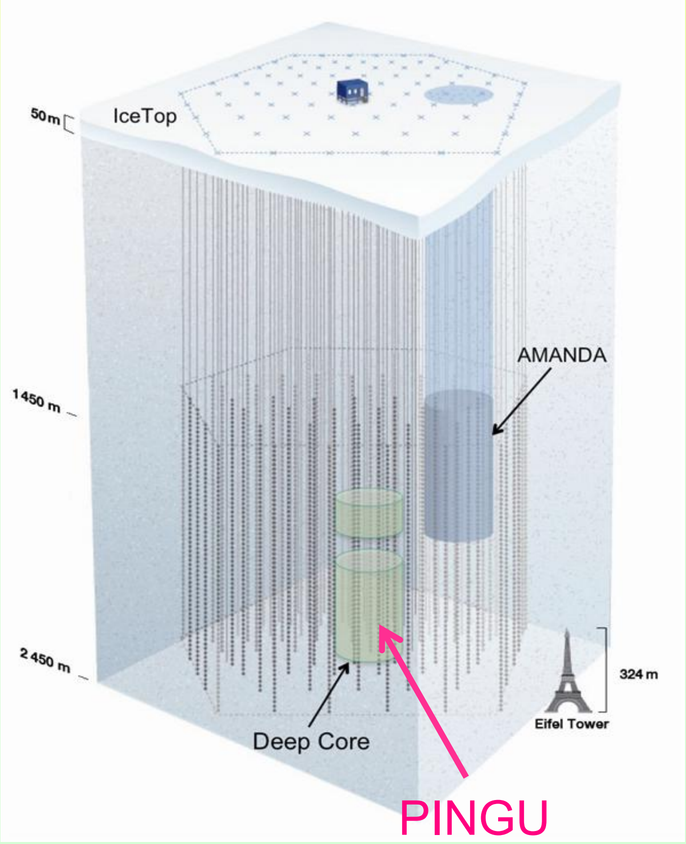
E, θ smearing (kinematics + detector resolution)





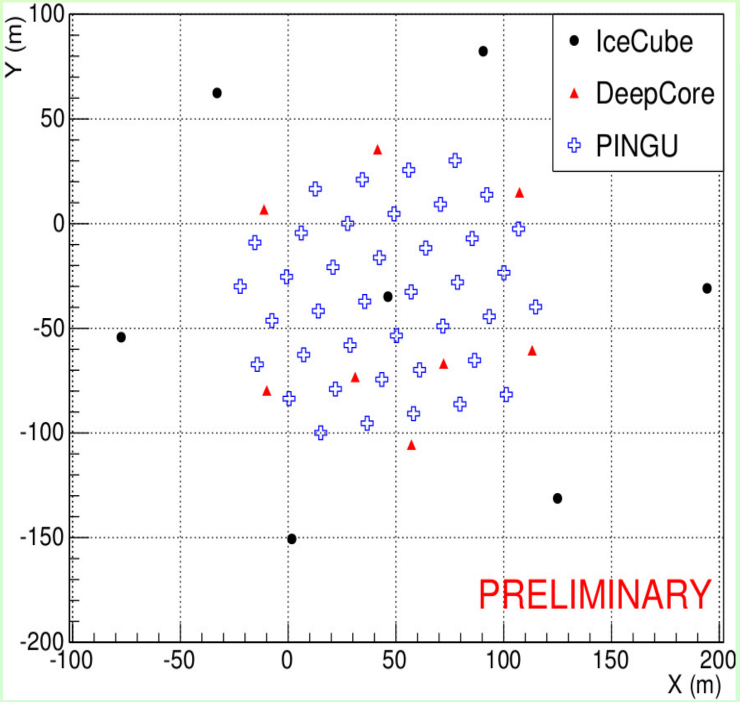
PINGU

Precision IceCube
Next Generation
Upgrade



Letter of Intent PINGU- arXiV:1401.2046

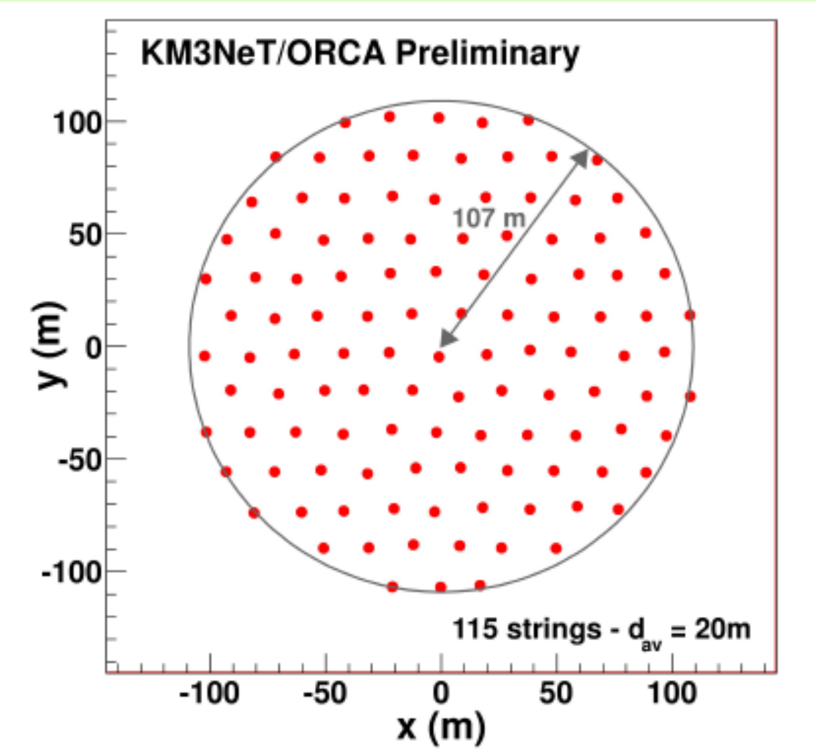
40 strings
96 DOM's per string

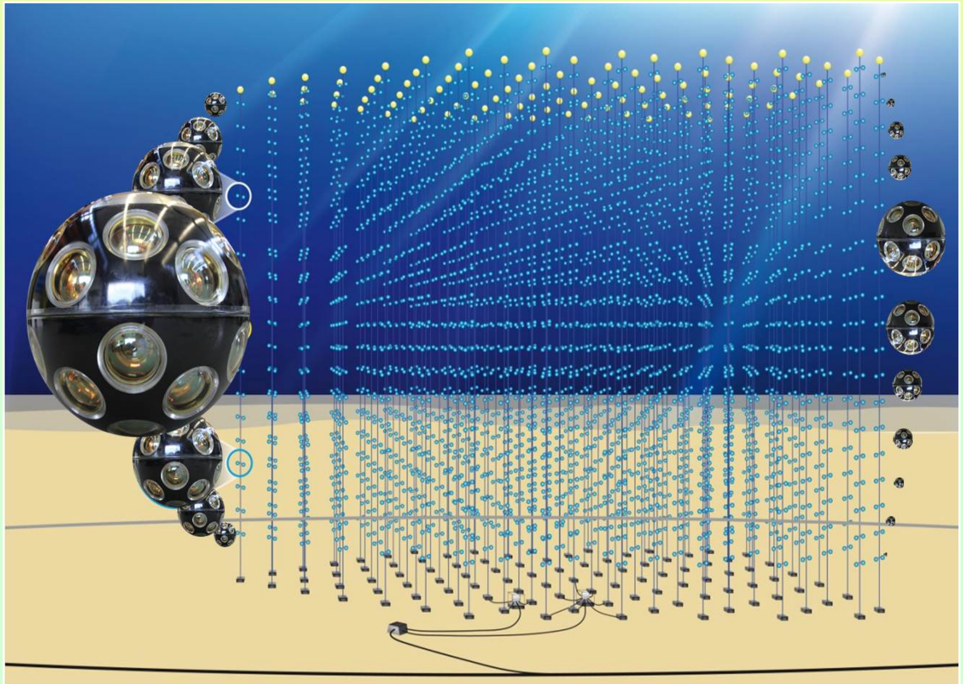


Km^3-ORCA

Oscillation Research with
Cosmics in the Abyss

115 lines, 20m spaced,
18 DOMs/line, 6m spaced
Instrumented volume ~3.8 Mt,
2070 optical module.



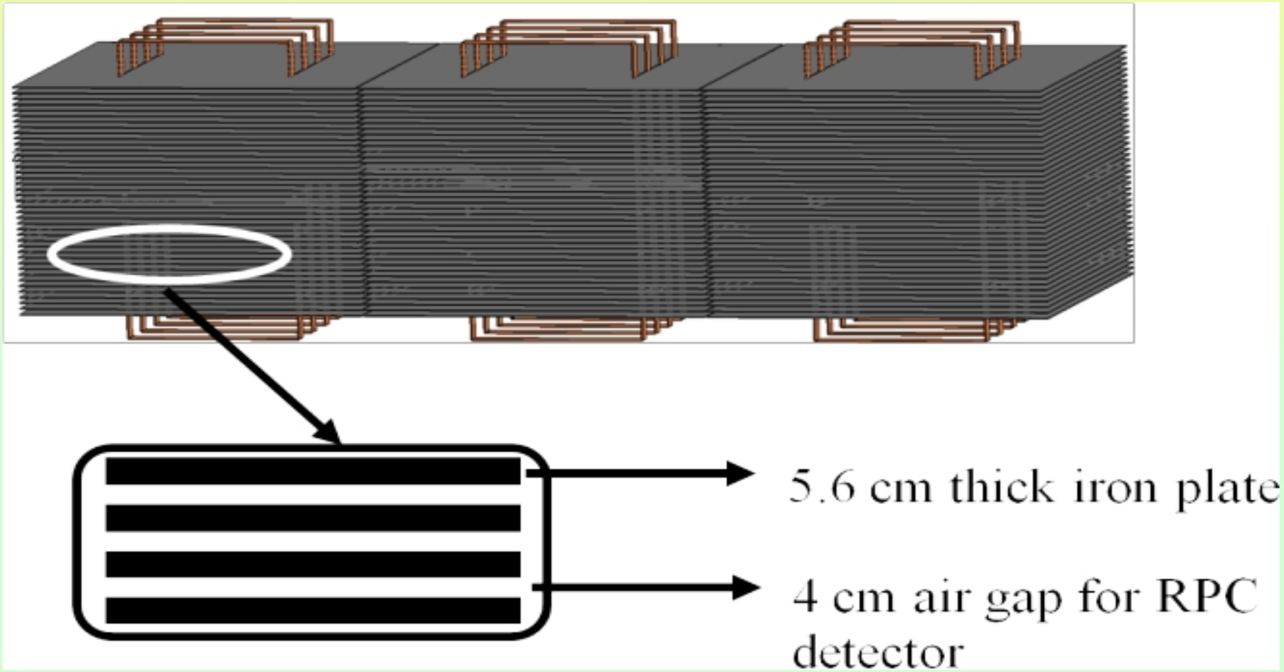


- 31 3" PMTs
- Digital photon counting
- Directional information
- Wide angle view

INO-ICAL

ICAL Collaboration (Ahmed Shakeel et al.) arXiv: 1505.07380 [physics.ins-det]

The 50 kt magnetized iron calorimeter (ICAL) detector at the India-based Neutrino Observatory (INO)



Resistive plate chambers

Energy and direction of the muons; energy of multi-GeV hadrons; charge of muon

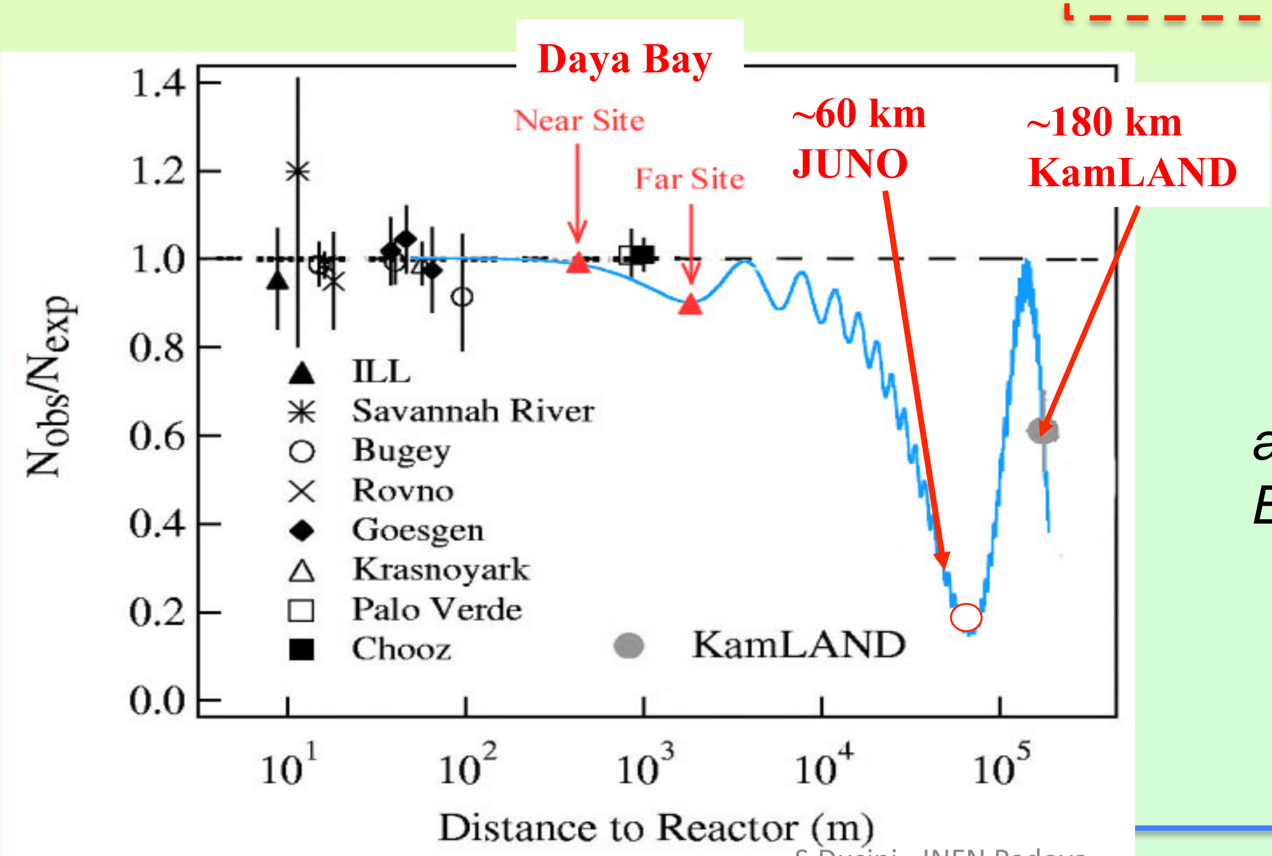
The energy and zenith angle dependence of the atmospheric neutrinos in the multi-GeV range.

Reactor neutrinos

Fast oscillation

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \left(\cos^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{31} L}{4E} + \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{32} L}{4E} \right)$$
$$- \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21} L}{4e}$$

Slow oscillation



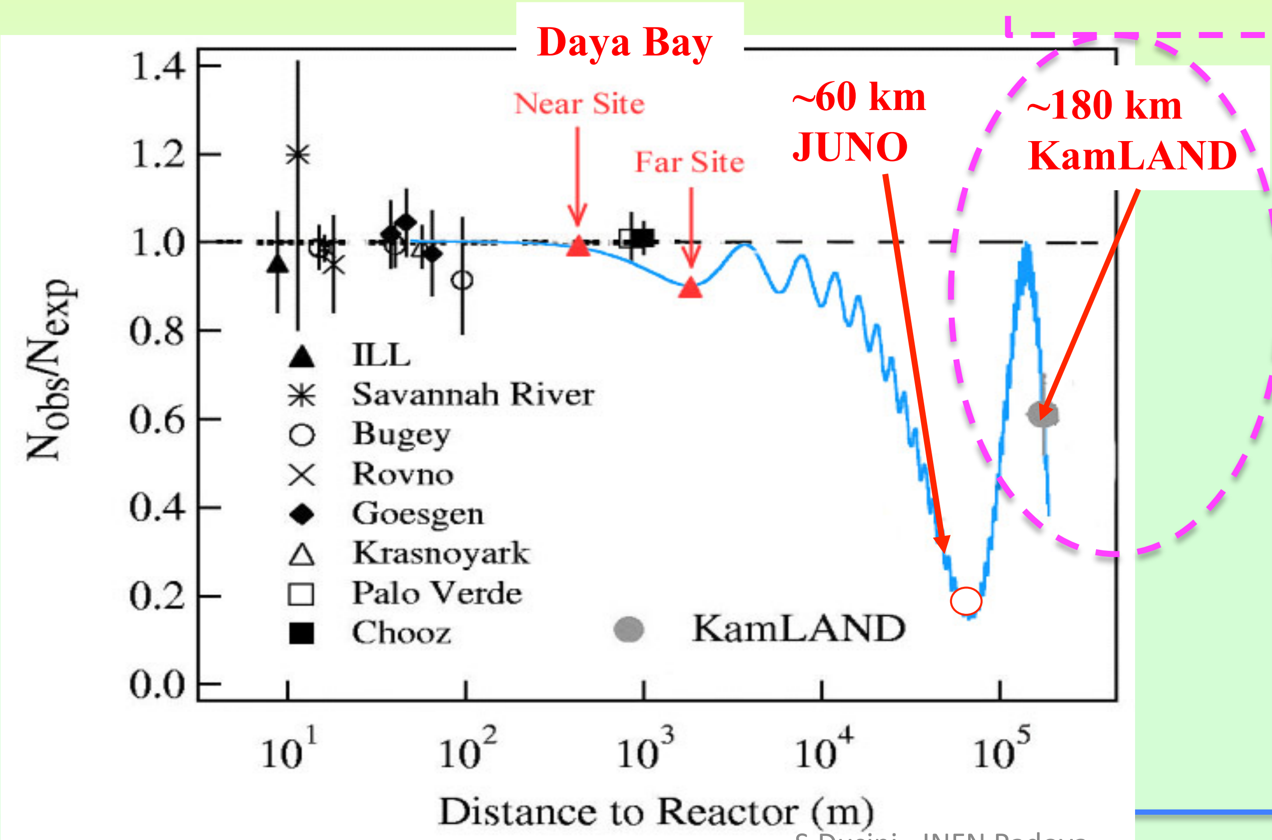
anti-ν_e disappearance
E_ν ~ 4 MeV

Reactor neutrinos

anti-ν_e disappearance
$E_\nu \sim 4$ MeV

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13}\left(\cos^2 2\theta_{12}\sin^2\frac{\Delta m^2_{31}L}{4E} + \sin^2 2\theta_{12}\sin^2\frac{\Delta m^2_{32}L}{4E}\right)$$

$$-\cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\frac{\Delta m_{21}L}{4e}$$



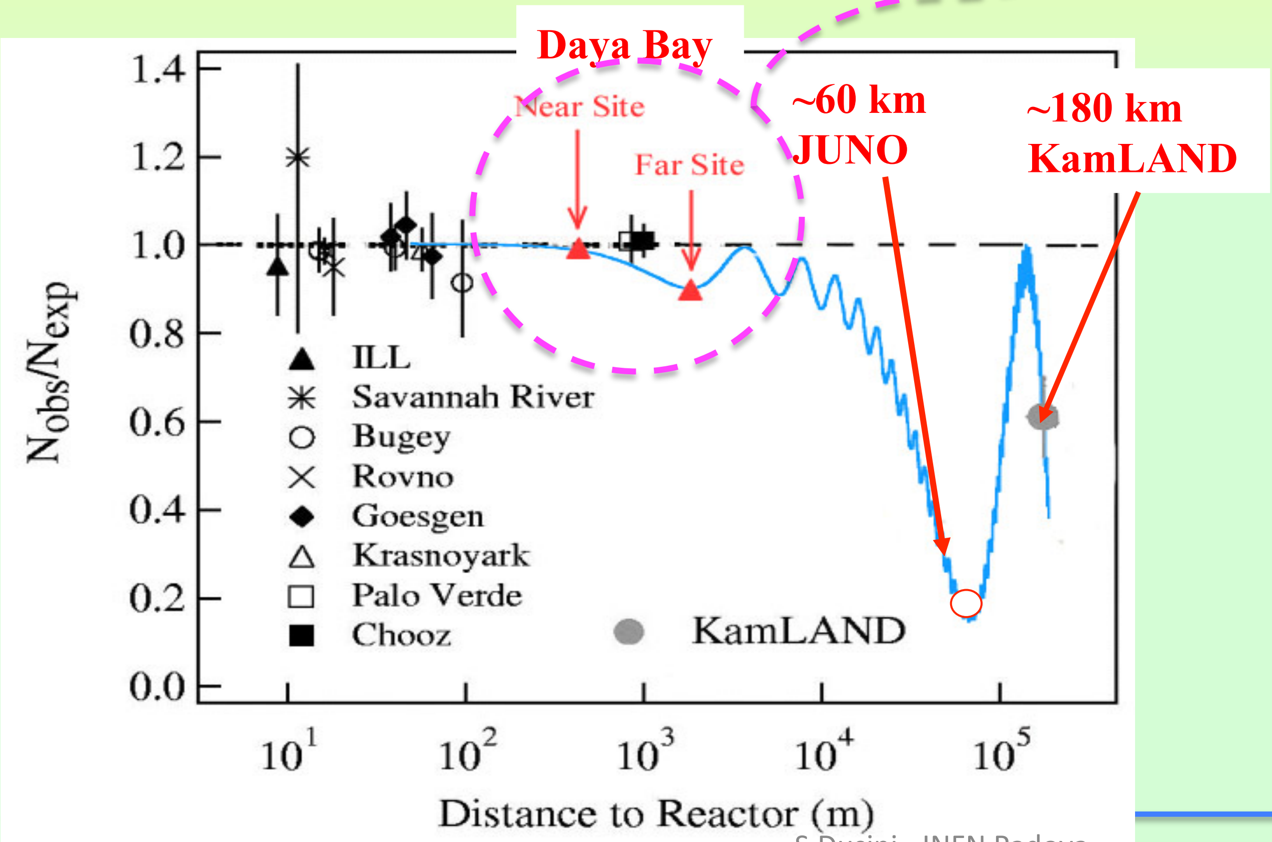
L ~ 180 km

Sensitive to Δm^2_{21} and θ_{12}

Reactor neutrinos

anti-ν_e disappearance
E_ν ~ 4 MeV

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13}\left(\cos^2 2\theta_{12}\sin^2\frac{\Delta m^2_{31}L}{4E} + \sin^2 2\theta_{12}\sin^2\frac{\Delta m^2_{32}L}{4E}\right) - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\frac{\Delta m_{21}L}{4e}$$



L ~ 1.6 km

Sensitive to Δm^2_{21} and θ_{12}

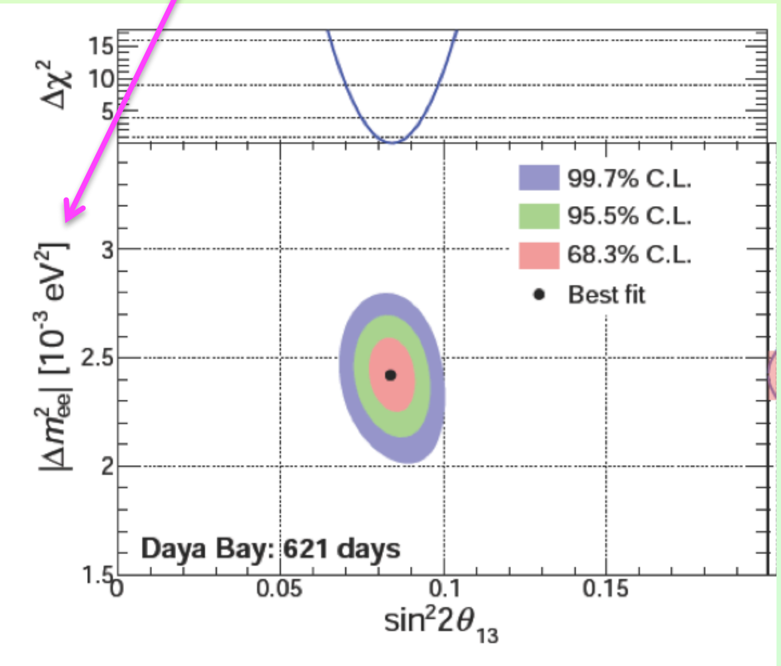
θ_{13} mesurment

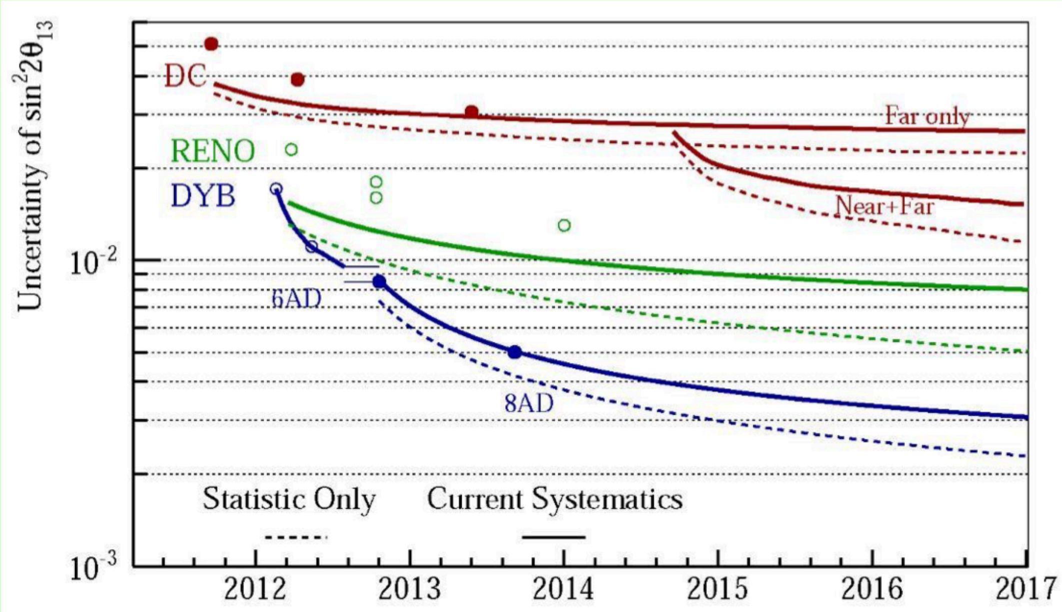
$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \left(\cos^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{31} L}{4E} + \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{32} L}{4E} \right)$$

$$\sin^2 \frac{\Delta m^2_{ee} L}{4E}$$

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$
$$\theta_{13} = 8.4° \pm 0.3°$$

< 6%



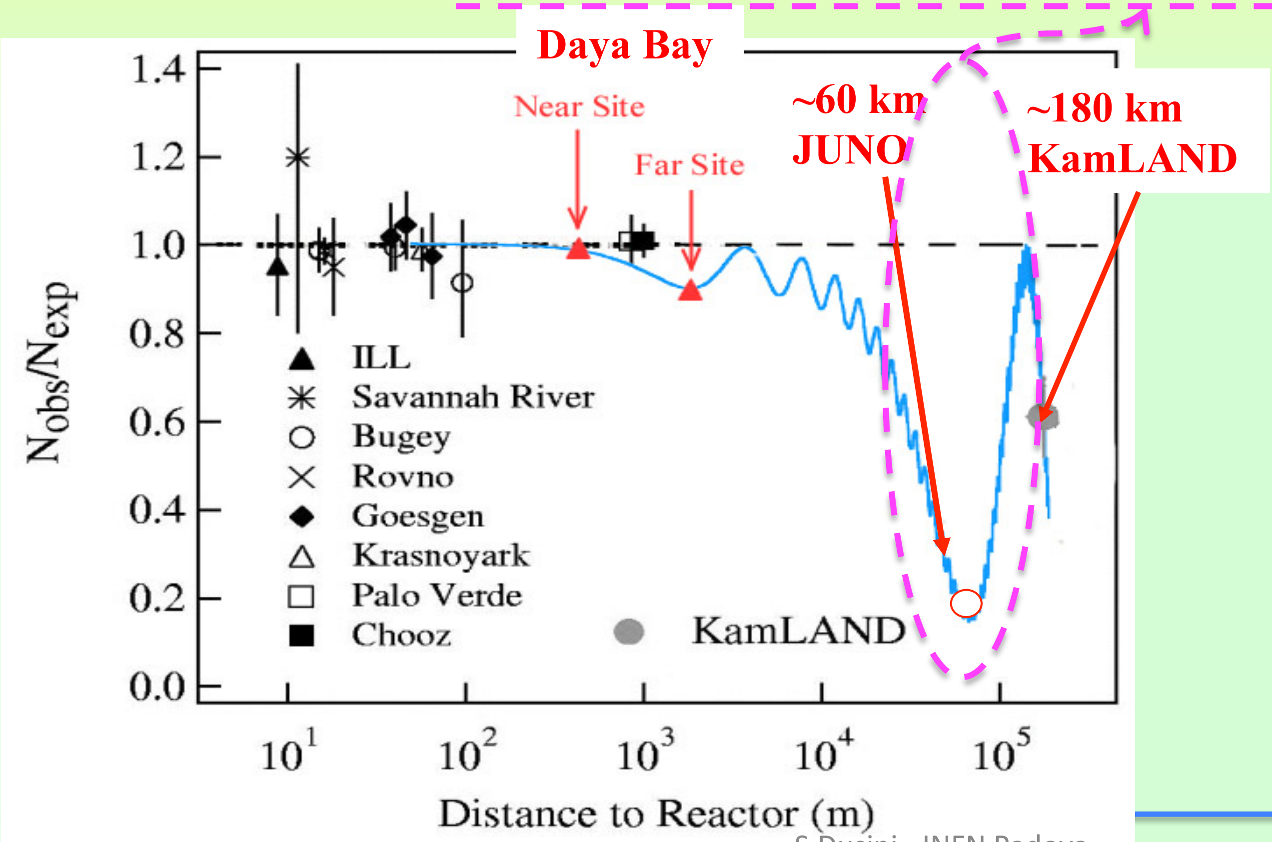


Important for CP and unitarily checks

Reactor neutrinos

anti-ν_e disappearance
$E_\nu \sim 4$ MeV

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13}\left(\cos^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{31} L}{4E} + \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{32} L}{4E}\right) - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21} L}{4e}$$



L ~ 53 km

Sensitive to mass hierarchy

Measure hierarchy in vacuum: JUNO

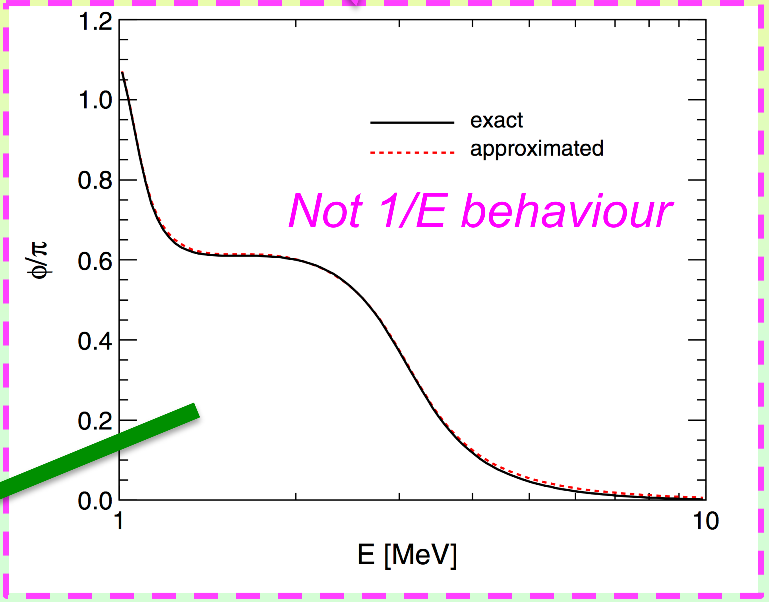
(+) Normal, (-) inverted hierarchy

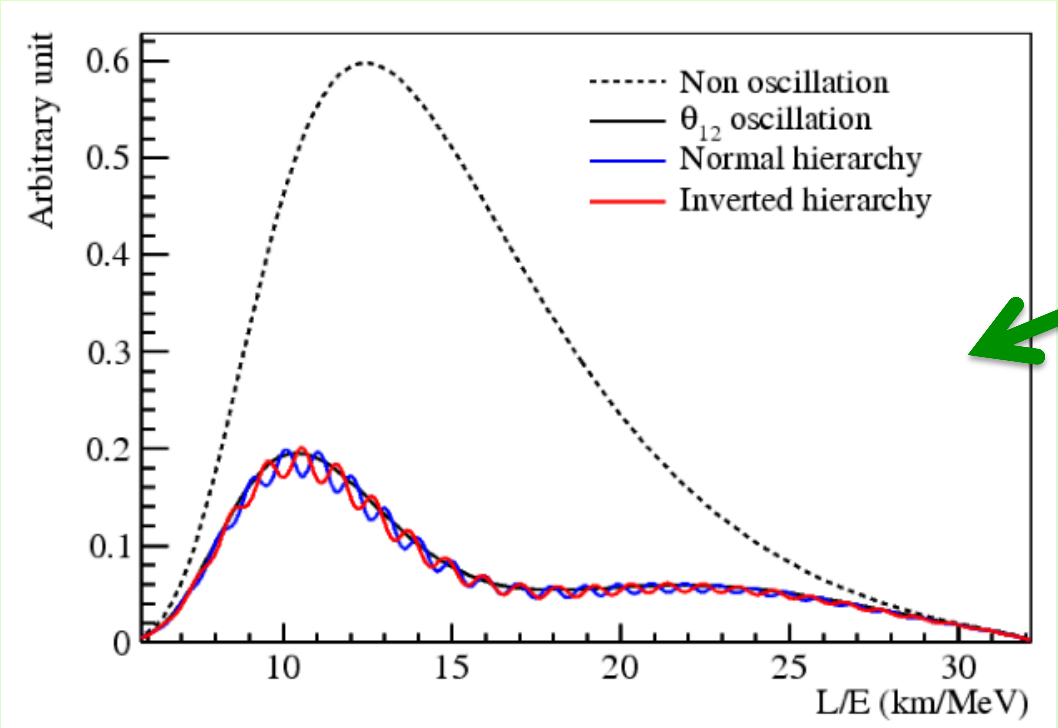
$$P^{3\nu}_{VAC} = c^4_{13} P^{2\nu}_{VAC} + s^4_{13} + 2 s^2_{13} c^2_{13} \sqrt{P^{2\nu}_{VAC}} \cos(2\Delta_{ee} \pm \varphi)$$

$c_{ab} = \cos\theta_{ab}$

$s_{ab} = \sin\theta_{ab}$

$$\Delta_{ee} = \frac{L}{4E}\left[\Delta m^2 \pm (c^2_{12} - s^2_{12})\delta m^2\right]$$



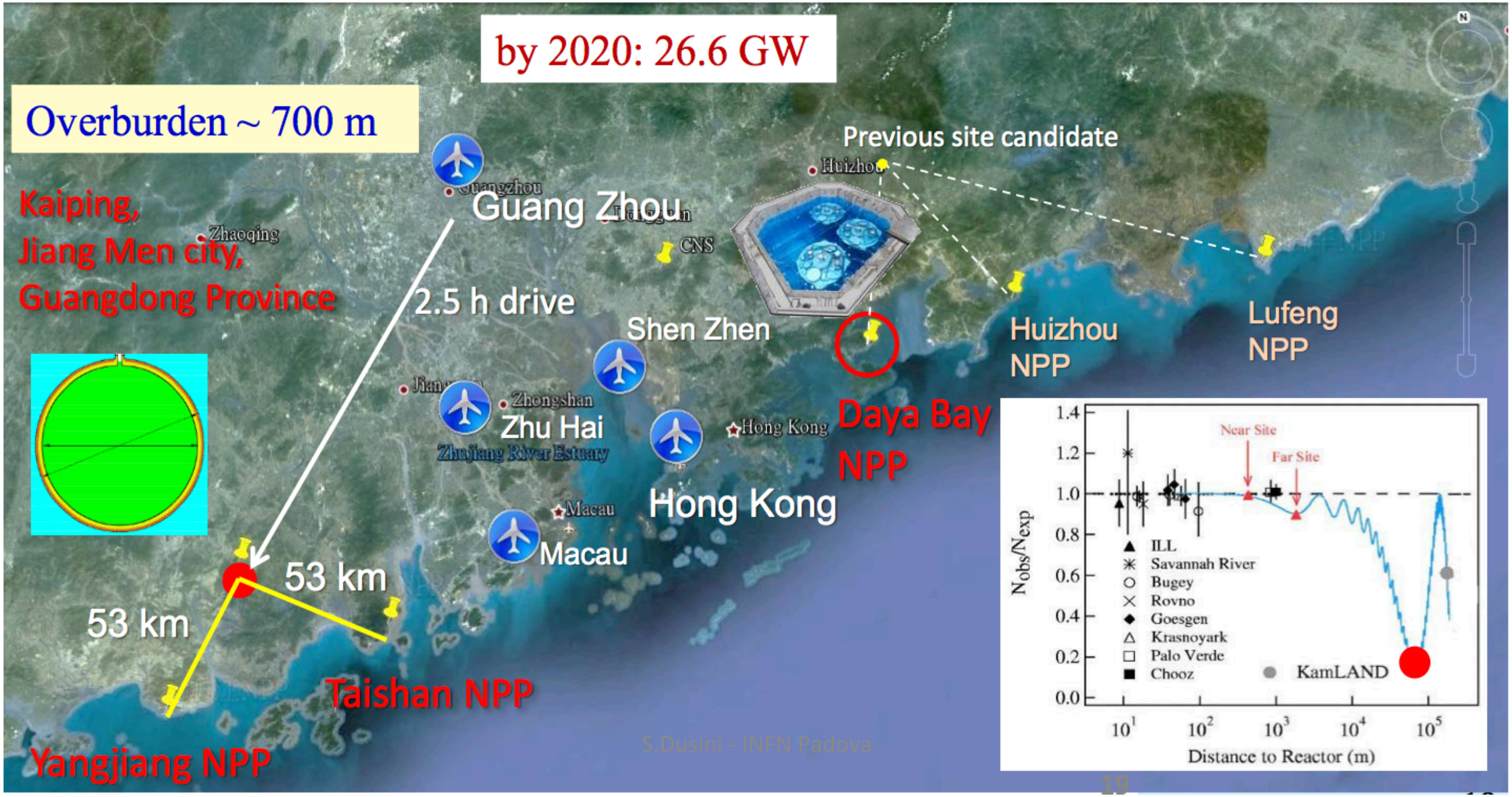


Non L/E phases which determine an anticipation or retardation of oscillation phases depending on the mass hierarchy

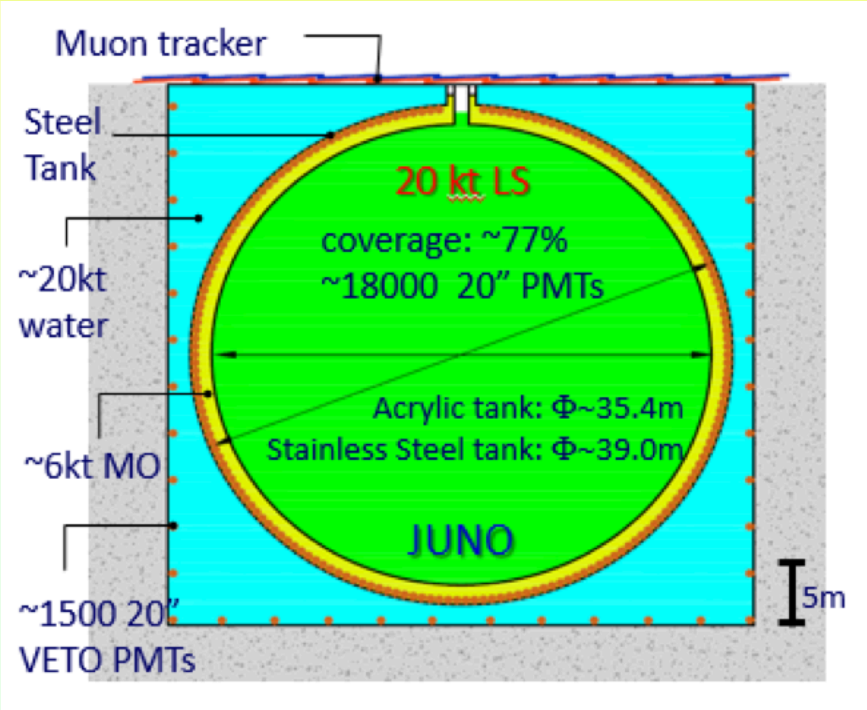
Maximal sensitivity @ minimum of Δm^2_{21} oscillation

JUNO for Mass Hierarchy

NPP	Daya Bay	Huizhou	Lufeng	Yangjiang	Taishan
Status	Operational	Planned	Planned	Under construction	Under construction
Power	17.4 GW	17.4 GW	17.4 GW	17.4 GW	18.4 GW



JUNO detector



Large mass liquid scintillator

→ *statistics*

Extreme high energy resolution 3% @ 1MeV

→ *reconstruct the "wiggles"*

Energy non-linearity < 1%

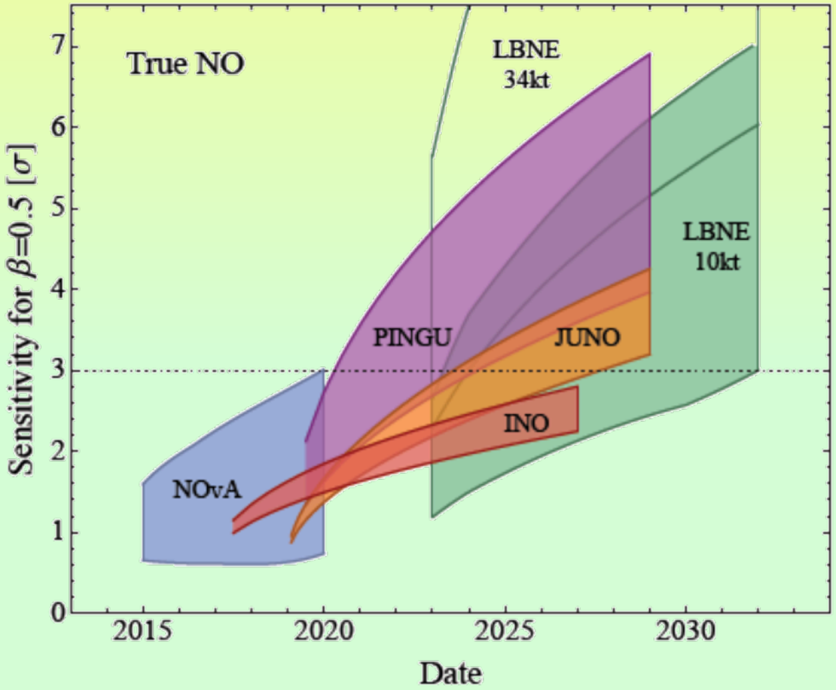
→ *not to fake MH effect*

- ✓ 77% Photocathode coverage
- ✓ LS attenuation length ~ 20 m
- ✓ PMT QE ~ 35%

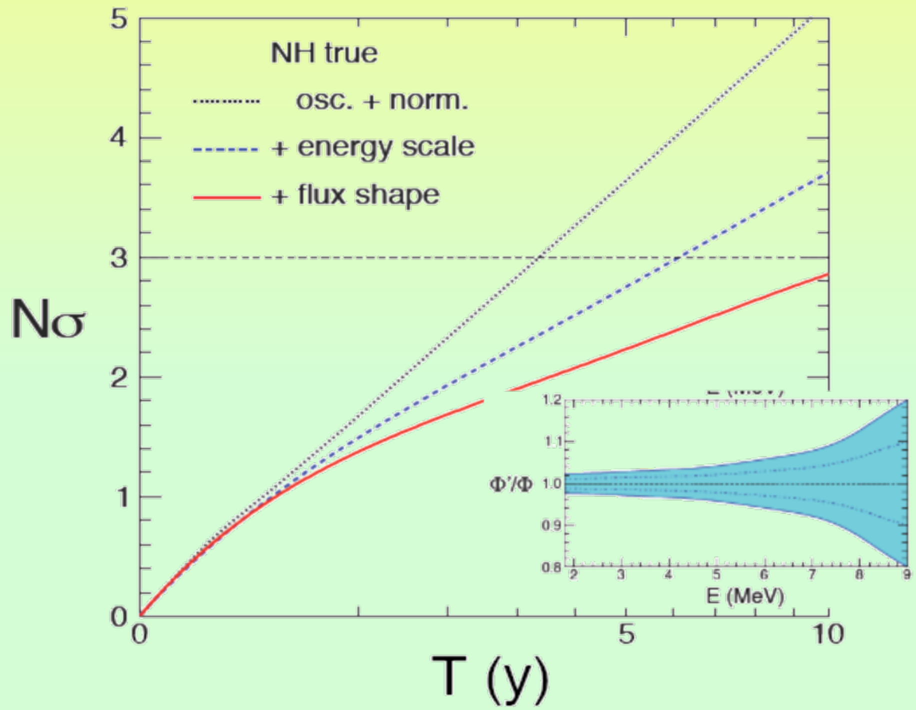
	KamLAND	BOREXINO	JUNO
LS mass	1 kt	0.5 kt	20 kt
Energy Resolution	$6\%/\sqrt{E}$	$5\%/\sqrt{E}$	$3\%/\sqrt{E}$
Light yield	250 p.e./MeV	511 p.e./MeV	1200 p.e./MeV

Mass hierarchy sensitivity curves

Mass hierarchy is a difficult measurement which require a proper treatment of the systematics



M. Blennow et al., JHEP 1403 (2014) 028



A.Marrone @ TAUP 2015

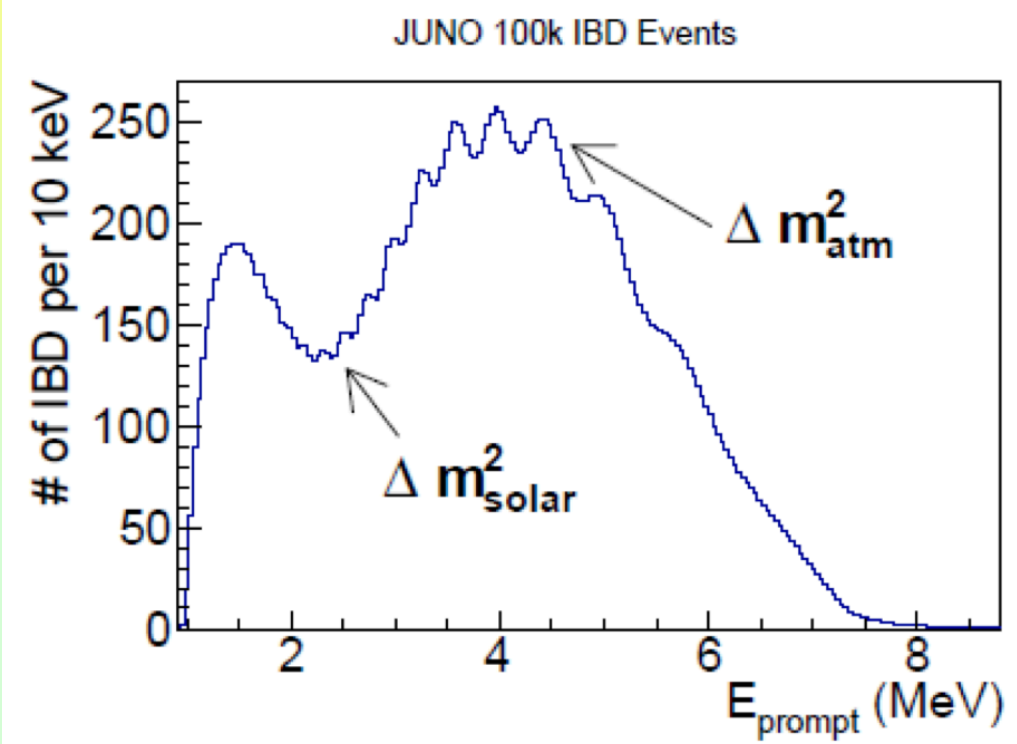
Size of the bands:
NOvA, LBNE: CP δ
INO PINGU: θ_{23}
JUNO: σ_E = 3 – 3.5% /√E

Precision measurements

Large anti-ν_e statistics
~ 15 k events/year

Unprecedented precision in measurement of "solar" oscillation parameters.

Important for CP @ LBL experiments

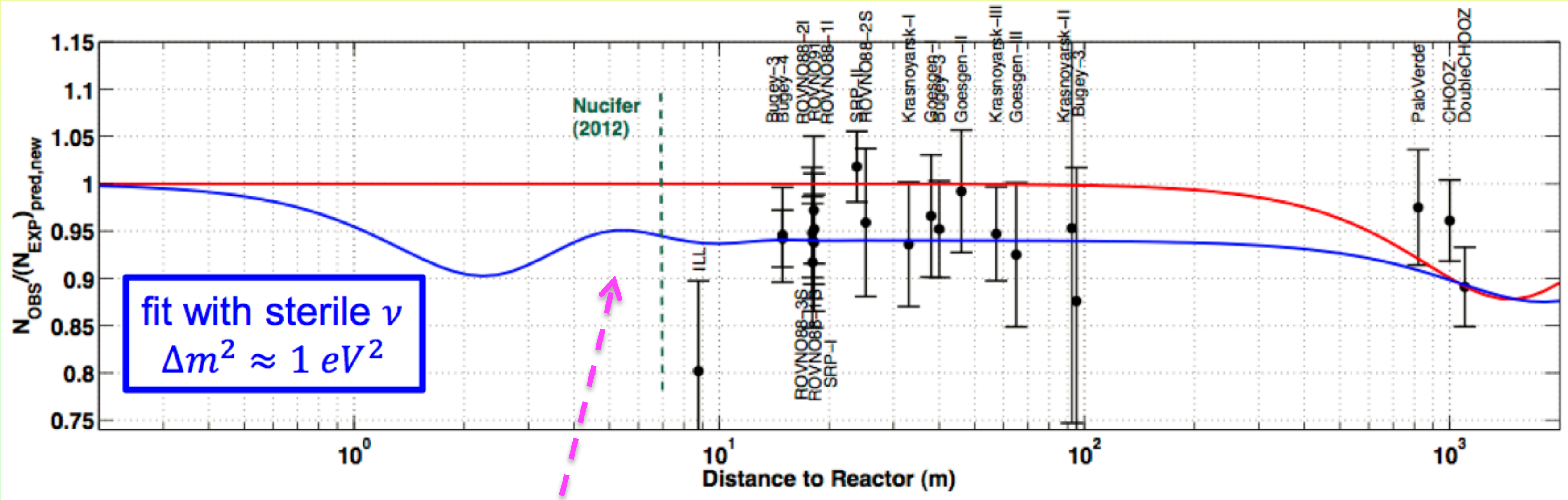


	Stat. only	Stat. + sys.
$\sin^2\theta_{12}$	0.54 %	0.67 %
Δm^2_{21}	0.24 %	0.59 %
Δm^2_{ee}	0.27 %	0.44 %

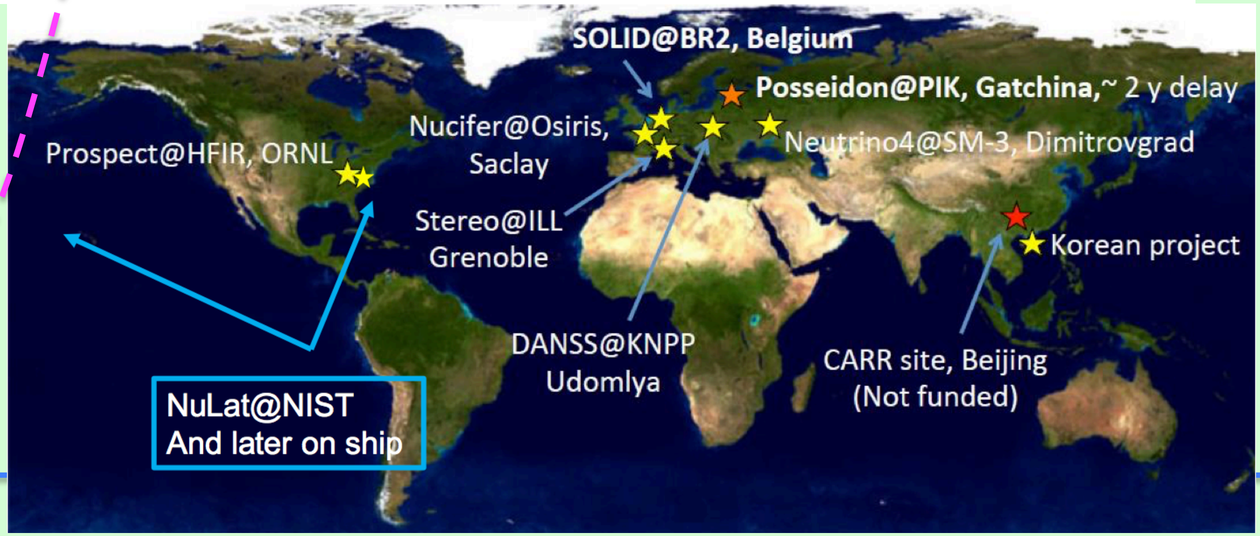
Probing the unitarily of U_{PMNS} to 1% level

Reactor neutrinos: Sterile neutrinos

Recent calculation of anti-ν_e flux from reactor are few % higher the observed eV sterile neutrino?



- *Tensions with beam and atmospheric data*
- *Trigger large activity on very-SBL reactor experiments*



Conclusions

Neutrino natural beam *are a great opportunity to study neutrino properties*
….great probe for astrophysical, cosmological and geological studies

With the measurement of θ_{13} neutrino physics is entering in the ***precision era****.*

Solar Neutrinos:
- *Tension on Δm^2_{21}*
- *Up-turn in the matter-vacuum transition region*
- *Day-Night asymmetry*
- *New physics? sterile neutrinos?, non standard interactions?*

Mass Hierarchy:
- *θ_{13} large has open the opportunity for MH and CP measurement*
- *Vacuum Oscillation vs. Matter Effects: two complementary approaches → redundancy and consistency checks*

Backup slides

Lepton Mixing

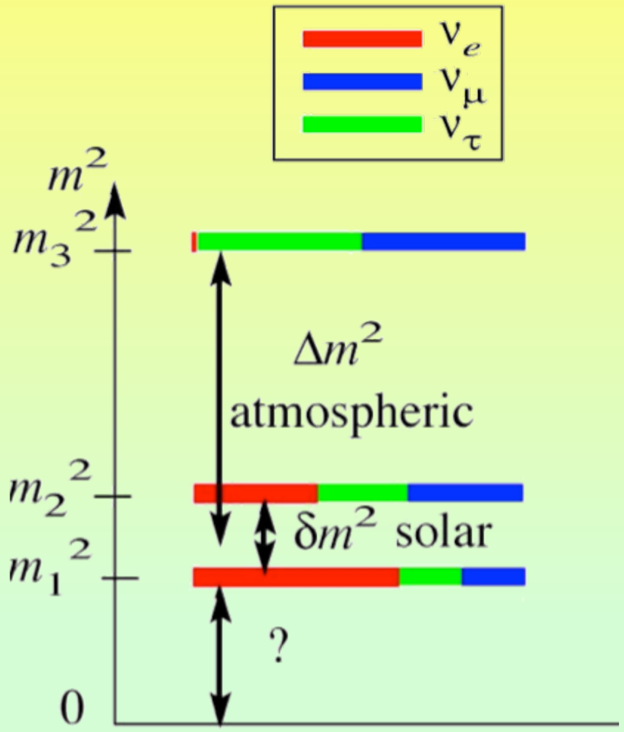
$$\sin^2\theta_{12} = 0.304 \pm 0.014$$

$$\sin^2\theta_{23} = 0.514^{+0.055}_{-0.018}$$

$$\sin^2\theta_{13} = 0.0219 \pm 0.0012$$

$$\Delta m^2_{12} = (7.53 \pm 0.18) \times 10^{-5} eV^2$$

$$\Delta m^2_{23} = (2.42 \pm 0.06) \times 10^{-3} eV^2$$



"Atmospheric" term

"Reactor" term

"Solar" term

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Detecting Reactor Antineutrino

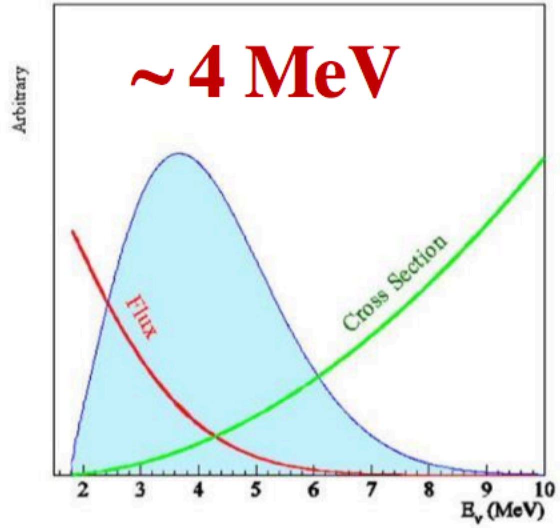
- ν-e scattering
- Inverse beta decay (IBD)

Prompt signal

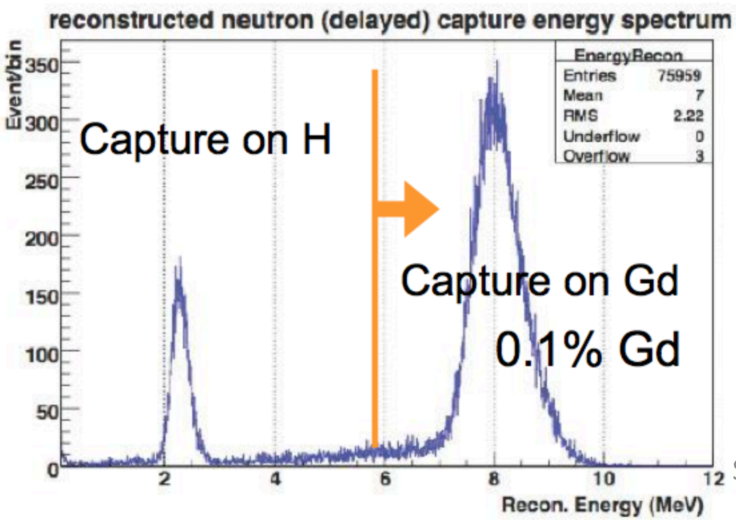
$$e^+ + e^- \rightarrow 2\gamma$$

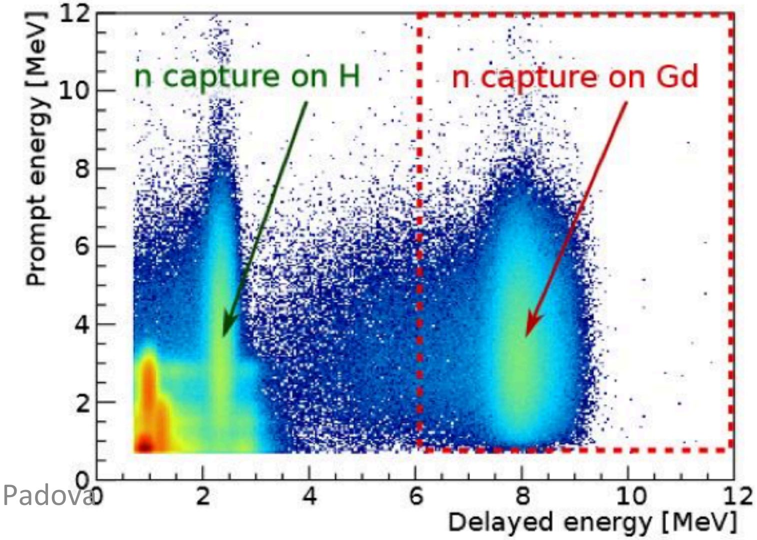
$$\bar{\nu}_e + p \rightarrow e^+ + n$$

Delayed signal, Capture on H (2.2 MeV, ~180μs) or Gd (8 MeV, ~30μs)

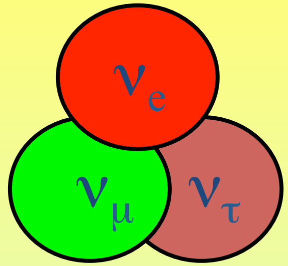


$$E(\bar{\nu}_e) = E_{\mathrm{prompt}} + Q - m_e$$
$$\sim E_{\mathrm{prompt}} + 0.8\ \mathrm{MeV}$$

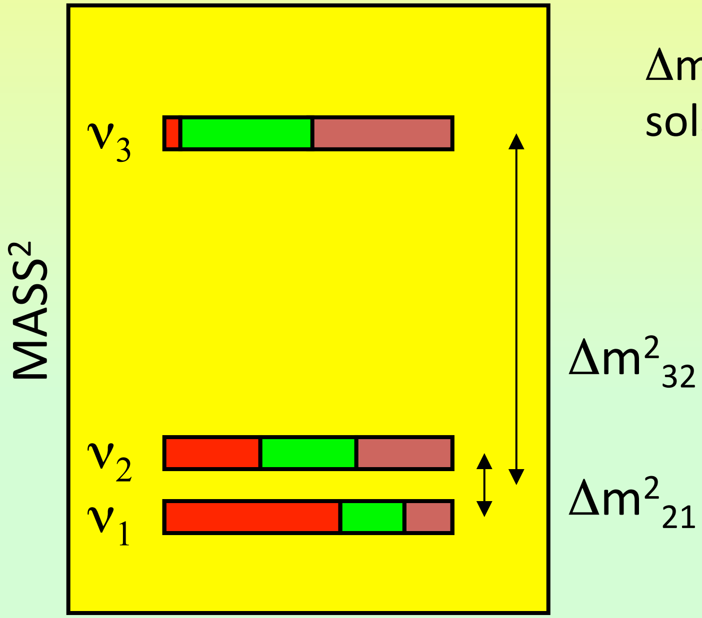




Neutrino mass ordering



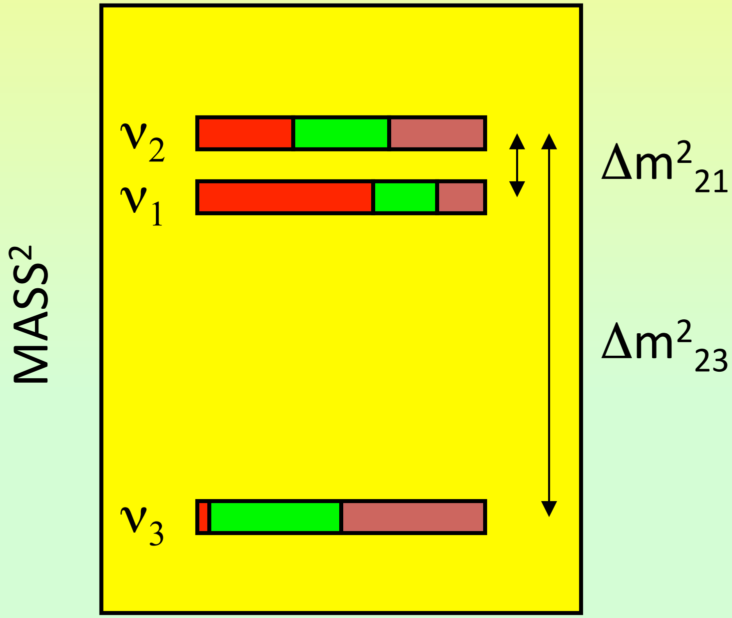
Δm^2_{21} fixed by solar neutrinos



Normal mass hierarchy

$\Sigma m > m_h$

$|\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$

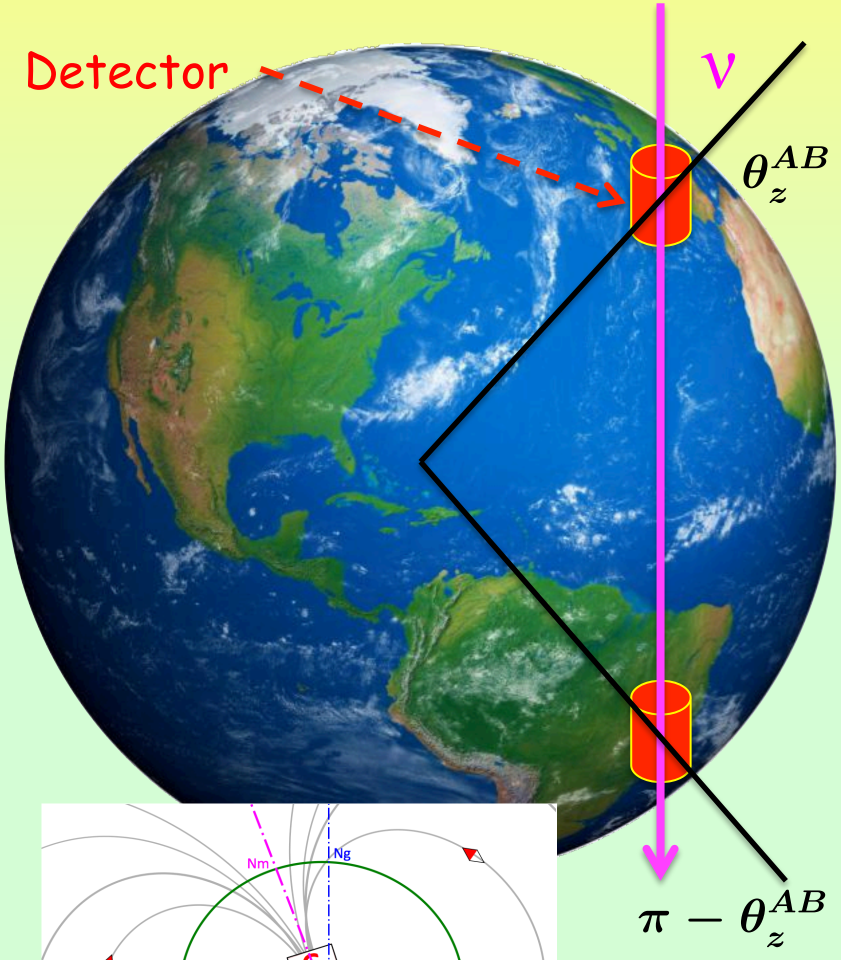


Inverted mass hierarchy

$\Sigma m > 2\, m_h$

$|\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$

The almost isotopic flux of primary cosmic ray imply that the neutrino flux at any location for Ev > 2GeV is up-down symmetric

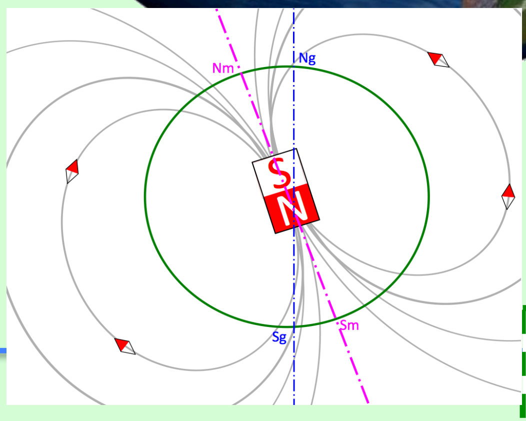


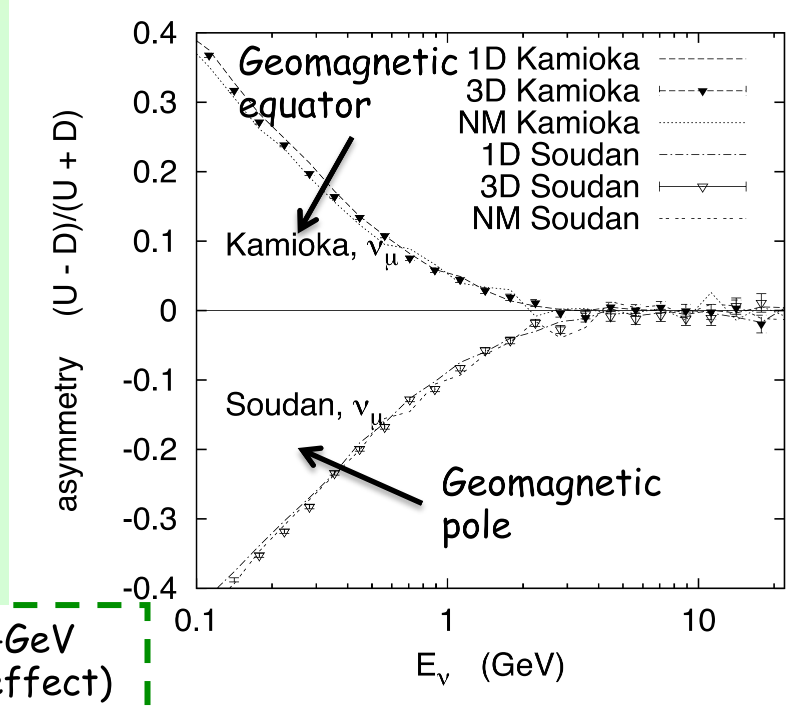
$$\phi_\nu^{(A)}(\theta_z^{AB}) = \phi_\nu^{(B)}(\pi - \theta_z^{AB})$$

Since the production of neutrino in the atmosphere is uniform

$$\phi_\nu^{(A)}(\theta_z^{AB}) = \phi_\nu^{(B)}(\theta_z^{AB})$$

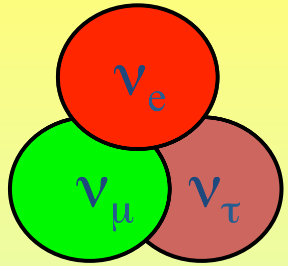
which imply $\phi_\nu^{(A)}(\theta_z^{AB}) = \phi_\nu^{(A)}(\pi - \theta_z^{AB})$

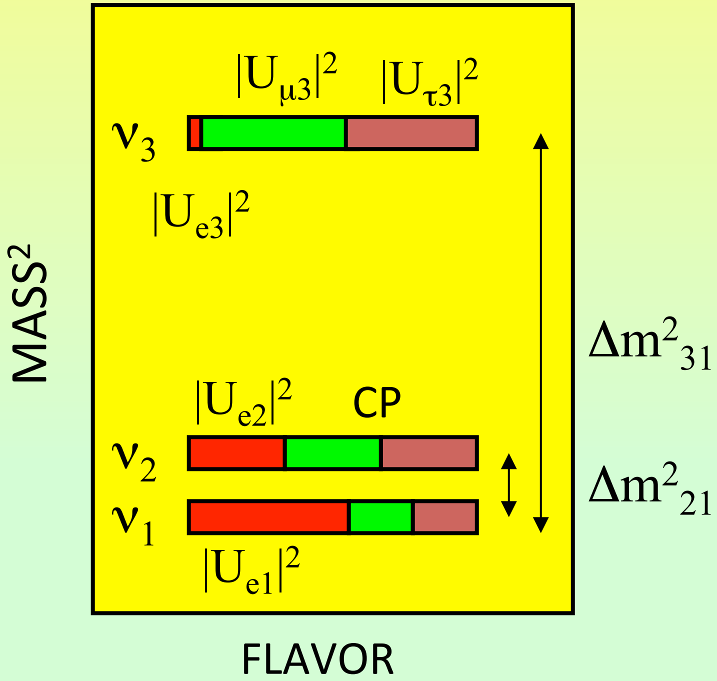




Up/down asymmetry @sub-GeV
(due to geomagnetic field effect)

LEPTON MIXING





Mixing parameters

$\tan^2\theta_{12} = |U_{e2}|^2 \,/\, |U_{e1}|^2$ ~ 1/2

$\sin^2\theta_{13} = |U_{e3}|^2$ = 0.022

$\tan^2\theta_{23} = |U_{\mu 3}|^2 \,/\, |U_{\tau 3}|^2$ ~ 1.0

Normal mass hierarchy

$\Delta m^2_{ij} = m^2_i - m^2_j$

$\Delta m^2_{32} = 2.5 \times 10^{-3}\ eV^2$

$\Delta m^2_{21} = 7.5 \times 10^{-5}\ eV^2$

Mixing matrix:

$\nu_f = U_{PMNS}\, \nu_{mass}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

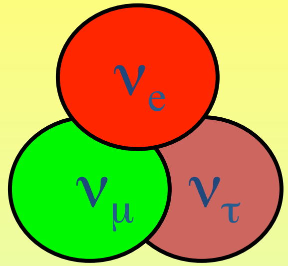
Standard parametrization

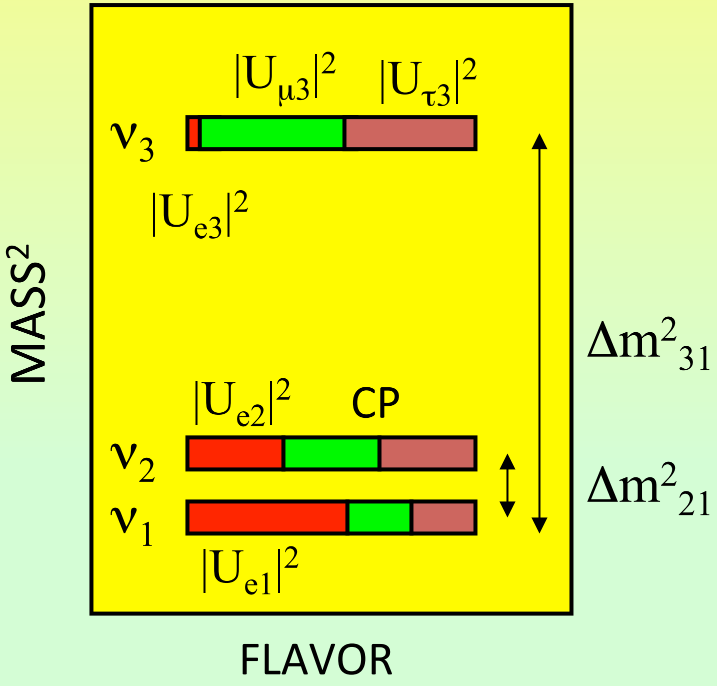
$U_{PMNS} = U_{23} I_\delta\, U_{13} I_{-\delta}\, U_{12}$

$I_\delta = \mathrm{diag}\,(1, 1, e^{i\delta})$

CP-phase

LEPTON MIXING





Mixing parameters

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$\sin^2\theta_{13} = |U_{e3}|^2$ = 0.022

$\tan^2\theta_{23} = |U_{\mu 3}|^2 / |U_{\tau 3}|^2$ ~ 1.0

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"Atmospheric" term *"Reactor" term* *"Solar" term*

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$