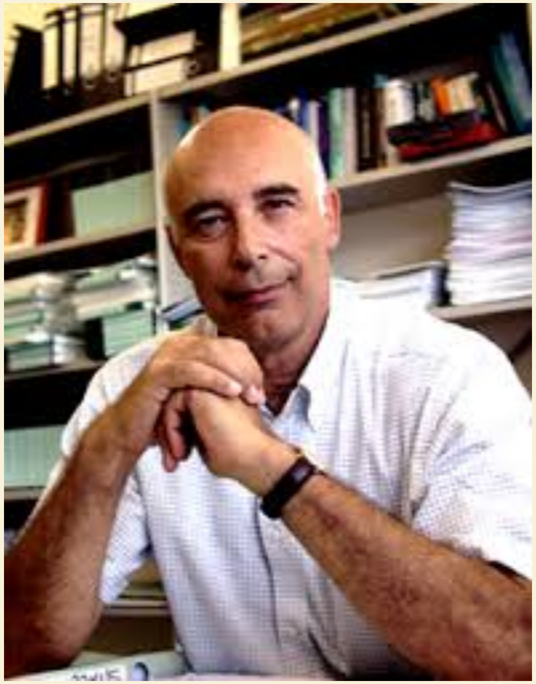
Neutrino masses and mixing angles: a tribute to Guido Altarelli

Padova, December 9th 2015

XVIII Roma Tre Topical Seminar on Subnuclear Physics:
Neutrinos (in memoria di Guido)

Ferruccio Feruglio
Universita' di Padova



Guido Altarelli [1941-2015]:
a true giant of particle physics.
His contributions to physics span all subjects, from strong to electroweak interactions, from neutrinos to theories beyond the Standard Model.

His best known contribution is the derivation of the QCD evolution equations for parton densities (1977) known as the Altarelli-Parisi or DGLAP equations.

Here:
following an historical path, I will describe his contribution to the field of neutrino masses and mixing angles

- member of the Polish Academy of Sciences
- 2011 Julius Wess Award
- 2012 J. J. Sakurai Prize for Theoretical Particle Physics [APS]
- 2015 High Energy and Particle Physics Prize - EPS HEPP Prize

Guido "principles" about neutrinos

a new insight into the flavour puzzle?

[Parodi, Rudeau, Stocchi 9802289]

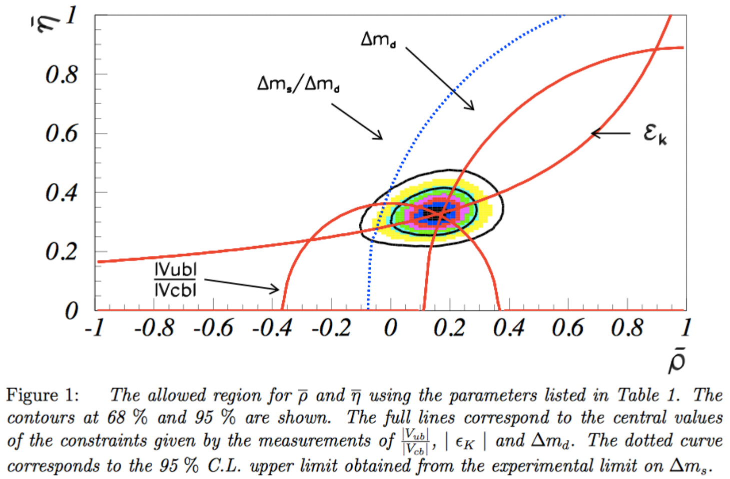
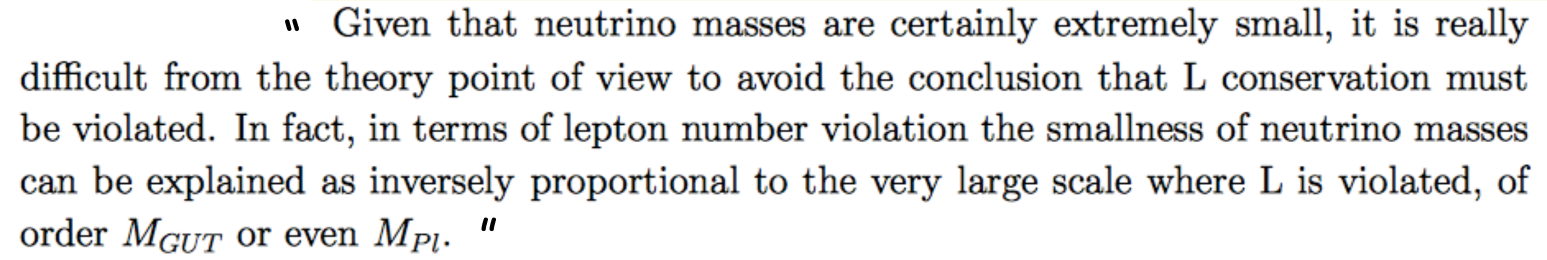


Figure 1: *The allowed region for $\bar{\rho}$ and $\bar{\eta}$ using the parameters listed in Table 1. The contours at 68 % and 95 % are shown. The full lines correspond to the central values of the constraints given by the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, $|\epsilon_K|$ and Δm_d. The dotted curve corresponds to the 95 % C.L. upper limit obtained from the experimental limit on Δm_s.*

Quark sector reasonably well-known at the time, but baseline model for quark masses and mixing angles missing.

neutrino masses and large ϑ_{23} were interesting new inputs

violation of L at a large scale M



" Given that neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the conclusion that L conservation must be violated. In fact, in terms of lepton number violation the smallness of neutrino masses can be explained as inversely proportional to the very large scale where L is violated, of order M_{GUT} or even M_{Pl}. "

$$m_\nu \approx \sqrt{\Delta m^2_{atm}} \approx \frac{(\text{EWscale})^2}{M} \Rightarrow M \approx 10^{15}\ GeV$$

"the most impressive numerology that comes out from neutrinos"

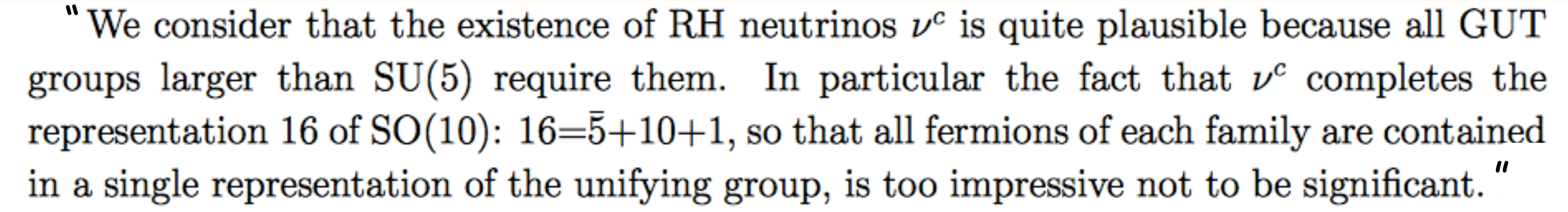
[GA, Neutrino 2004, Paris]

neutrino masses and GUTs

$$m_\nu \approx \frac{(\text{EWscale})^2}{M}$$

very plausible that this arises from the see-saw mechanism

the simplest realization (type I) needs a right-handed neutrino ν^c



"We consider that the existence of RH neutrinos ν^c is quite plausible because all GUT groups larger than SU(5) require them. In particular the fact that ν^c completes the representation 16 of SO(10): $16=\bar{5}+10+1$, so that all fermions of each family are contained in a single representation of the unifying group, is too impressive not to be significant."

"GUTs are the most attractive conjecture for the large scale picture of particle physics. GUT is not the SM, is beyond the SM, but is the most standard physics beyond the SM. Most of us think that there should be something like a GUT."

[GA, Neutrino 2004, Paris]

- $m_\nu = -m_D^{\nu\,T} M^{-1} m_D^\nu$ ➡ neutrino masses potentially related to the other charged fermion masses in a GUT

  $\boxed{m_e, m_u, m_d}$

- "another big plus of neutrinos is the elegant picture of baryogenesis through leptogenesis (after LEP has disfavoured BG ath the weak scale)"

1998 - the work starts: textures

$$m_\nu = U m_{diag} U^T$$

in the flavour basis

$$U_{fi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

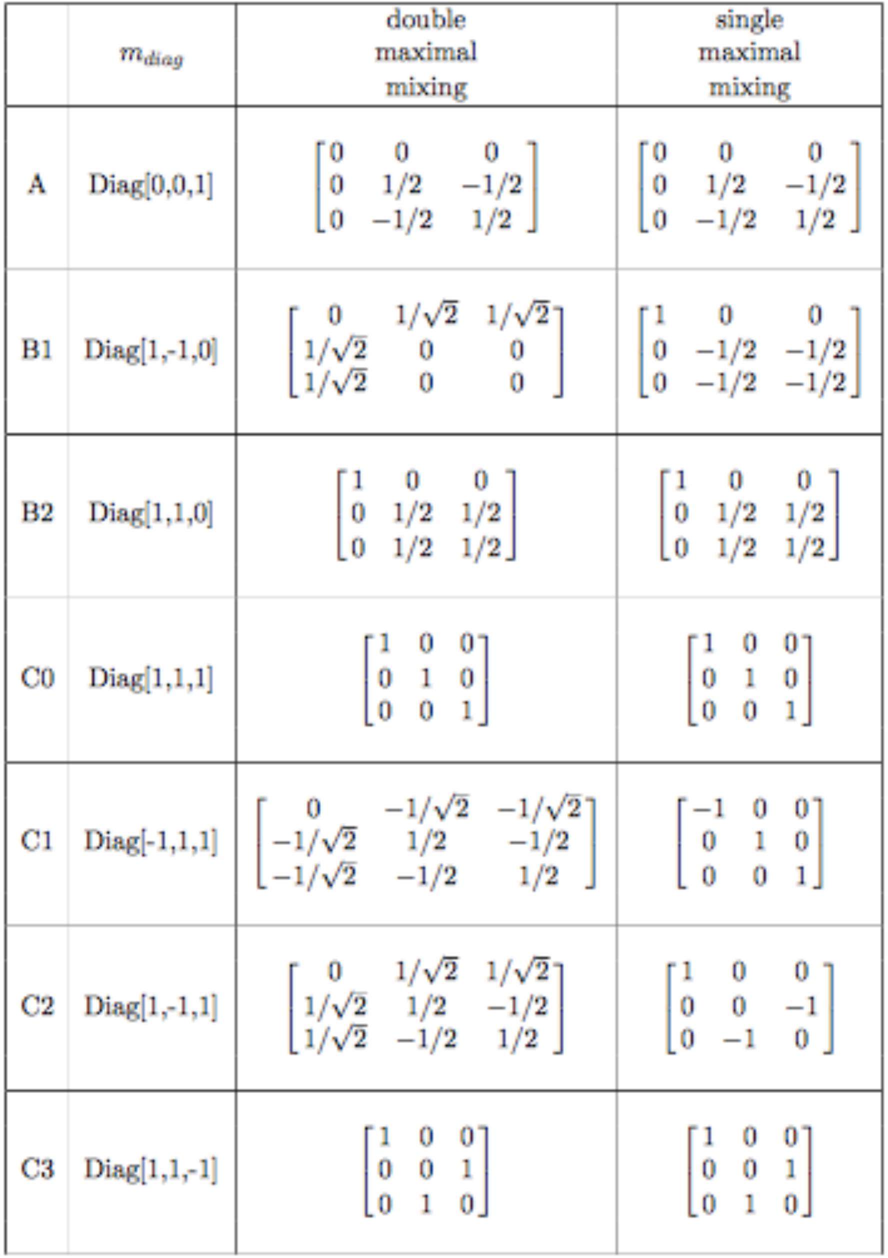
neglecting Δm^2_{sol} and ϑ_{13} and taking $\vartheta_{12}=\pi/4$ or 0

if see-saw, degeneracy need conspiracy between m_D^ν and M.
m_ν is quadratic in m_D^ν, any hierachy in m_D^ν gets amplified in m_ν

A=NH

B=IH

C=degenerate



	m_{diag}	double maximal mixing	single maximal mixing
A	Diag[0,0,1]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$
B1	Diag[1,-1,0]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$
B2	Diag[1,1,0]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$
C0	Diag[1,1,1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
C1	Diag[-1,1,1]	$\begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
C2	Diag[1,-1,1]	$\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
C3	Diag[1,1,-1]	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Guido's favorite texture

$$m_\nu \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix} m$$

large mixing requires degenerate states?

$$m_3 = (1+x^2)m \qquad m_{1,2} = 0$$

here x=O(1) implies large mixing and det[23]=0 guarantees the large splitting needed by atm ν

$$\Delta m^2_{atm} = m^2(1+x^2)^2 \qquad \sin^2 2\vartheta_{23} = \frac{4x^2}{(1+x^2)^2}$$

$$\vartheta_{13} = 0$$

$\Delta m^2_{sol} = 0$ $\qquad$ ϑ_{12} undetermined

$$\sin^2 2\vartheta_{23} \geq 0.9 \quad [2000]$$

$$0.7 \leq |x| \leq 1.4$$

- compatible with MSW SA, LA LOW and VO

- when embedded in SU(5), compatible with small quark mixing angles

assumptions
-- minimal SU(5) field content (3 light neutrinos)
-- Dirac masses of u,d,e, ν dominated by third generation [LO]

$$\bar{5} = (l, d^c)$$

$$10 = (q, u^c, e^c)$$

$$\Phi_5 = (\Phi_D, \Phi_T)$$

$$\bar{\Phi}_5 = (\bar{\Phi}_D, \bar{\Phi}_T)$$

fermion masses in minimal SU(5)

$$10\, y_u 10\, \Phi_5 \xrightarrow{10} y_u^{diag}$$

$$\bar{5}\,\frac{w}{M}\,\bar{5}\,\Phi_5\Phi_5 \xrightarrow{\bar{5}} m_\nu^{diag}$$

$$\bar{5}\, y_d 10\, \bar{\Phi}_5 \longrightarrow y_e = y_d^T$$

$$(d^c\ l)\, y_d \begin{pmatrix} q \\ e^c \end{pmatrix} \bar{\Phi}_5$$

must be corrected for 1st and 2nd generations, but OK at the LO

contains both V_{CKM} and U_{PMNS}

LEFT q mixing <-> RIGHT e mixing
RIGHT q mixing <-> LEFT e mixing

$V_{CKM} \approx 1$ -> small LEFT quark mixing

RIGHT quark mixing completely free
[not measurable in weak interactions]

non-hermitian y_d

$$y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & 0 & 1 \end{pmatrix}$$

[Hagiwara, Okamura '98;
Berezhiani, Rossi '98
Altarelli, F. '98]

$$\Longrightarrow \left[\begin{array}{l} y_d^+ y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1+x^2 \end{pmatrix} \\ \\ y_d y_d^+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix} \end{array} \right.$$

$$V_{CKM} = 1 \times U(\vartheta_C)$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times U(\vartheta_{12})$$

- for a long time prejudice was in favour of hermitian textures $y_{u,d}$ because they were predictive:
  -- Gatto Sartori Tonin relation
  -- Fritzsch textures

$$\sin\vartheta_C \approx \sqrt{\frac{m_d}{m_s}}$$

- well-compatible with the see-saw and very stable versus M

$$\bar{5}\,\frac{w}{M}\,\bar{5}\,\Phi_5\Phi_5 \qquad \text{from} \qquad 1\,y_\nu\,\bar{5}\,\Phi_5 + 1\,M\,1$$

assuming

$$y_\nu \approx y_u \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \quad m_\nu = y_\nu^T M^{-1} y_\nu\, v_u^2 \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{v_u^2}{M_{33}}$$

whatever M is!
[$M_{33} \neq 0$]

- LO picture can be translated into a more realistic model by replacing the zeros with small quantities

  $U(1)_{FN}$ abelian flavour symmetry,
  spontaneously broken by $\lambda = \langle\vartheta\rangle/\Lambda < 1$

  -- fix mass relations of 1st and 2nd generation
  -- address DT splitting problem
  -- check gauge coupling unification, p-decay,…

[Altarelli,F 9812475; Altarelli, F, Masina 0007254]

[MSW SA/LA, LOW, VO]

flavor puzzle made simpler in SU(5) ?

suppose that y_u, y_e, y_ν and M/Λ are anarchical matrices [O(1) matrix elements] and that the observed hierarchy is due to some sort of wave function renormalization of matter multiplets

$$\begin{array}{ccc} 10 & \rightarrow & F_{10}\,10 \\ \bar{5} & \rightarrow & F_{\bar{5}}\,\bar{5} \end{array}$$

$$F_X = \begin{pmatrix} \varepsilon'_X & 0 & 0 \\ 0 & \varepsilon_X & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$1 \geq \varepsilon_X \geq \varepsilon'_X$$

F_X can arise from $U(1)_{FN}$ symmetries, a 5th Extra Dimension, Partial Compositness

large mixing in lepton sector suggests

$$F_{\bar{5}} \approx \mathrm{diag}(\varepsilon'_5, 1, 1)$$

hierarchy mostly due to F_{10}

$$F_{10} \approx \mathrm{diag}(\varepsilon'_{10}, \varepsilon_{10}, 1)$$

$$\Upsilon_u = F_{10}\, y_u\, F_{10} \qquad \Upsilon_d = F_{\bar{5}}\, y_d\, F_{10} \qquad \Upsilon_e = F_{10}\, y_d^T\, F_{\bar{5}}$$

in the extreme case $\varepsilon'_5 = 1$ [ANARCHY] [Hall, Murayama, Weiner 1999
De Gouvea, Murayama 1204.1249]

➡

$$m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$$

$$V_{ub} \approx V_{us} \times V_{cb}$$

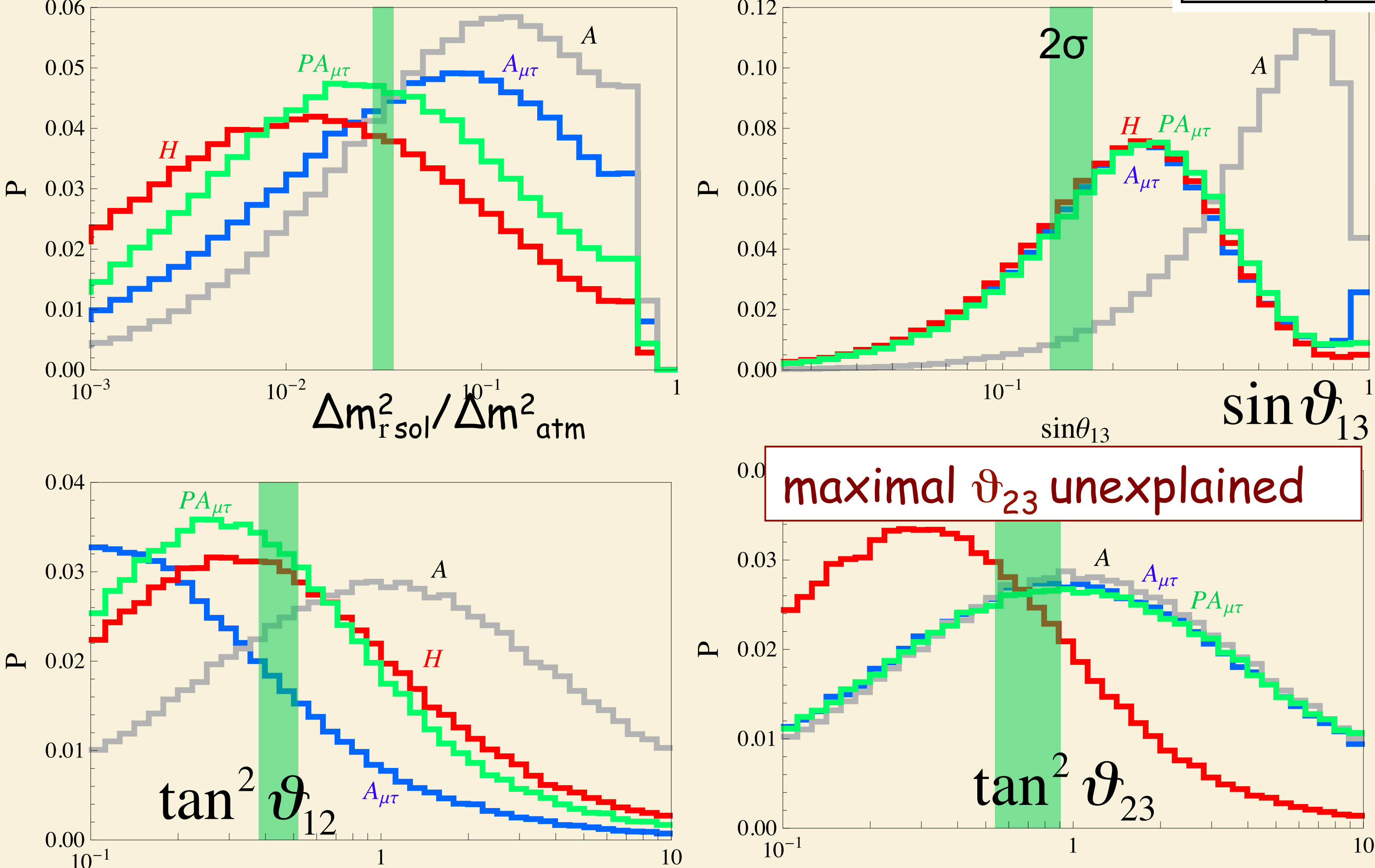
approximately true

but Guido was not an extremist!

	$FN(\bar{5})$	λ
A	(0,0,0)	
$A_{\mu\tau}$	(1,0,0)	0.25
$PA_{\mu\tau}$	(2,0,0)	0.35
H	(2,1,0)	0.45

$$F(\bar{5}_i) = \lambda^{FN(\bar{5}_i)}$$

[Buchmuller, Domcke, Schmitz, 1111.387;
Altarelli,F,Masina, Merlo 1207.0587;
Bergstrom, Meloni, Merlo, 1403.4528]



maximal ϑ_{23} unexplained

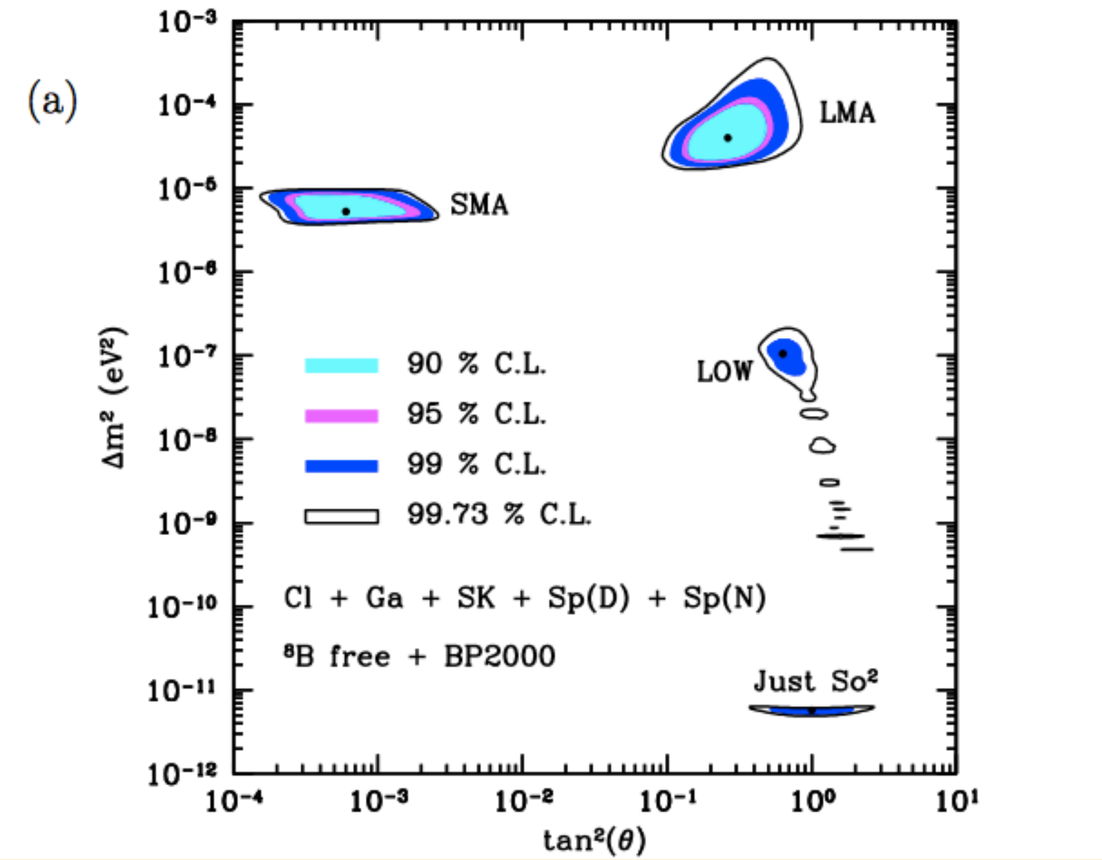
$$\sin^2\vartheta_{13} \approx \frac{\Delta m^2_{12}}{\Delta m^2_{13}}$$

NH favoured

large number
of independent
O(1) parameters

difficult
to go
beyond
order-of-
magnitude
predictions

Solar Neutrino Solutions < 2002



[Bahcall, Krastev, Smirnov 2001]

2002: the solar ν problem is solved

- by 2002 the MSW SA solution was ruled out by the large SK statistics [E-spectrum, time variation]

- **Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory**

  (Dated: 19 April 2002)

$$\begin{array}{llll} \nu_e + \mathrm{d} & \rightarrow & \mathrm{p}+\mathrm{p}+\mathrm{e}^- & (\mathrm{CC}), \\ \nu_x + \mathrm{d} & \rightarrow & \mathrm{p}+\mathrm{n}+\nu_x & (\mathrm{NC}), \\ \nu_x + \mathrm{e}^- & \rightarrow & \nu_x + e^- & (\mathrm{ES}). \end{array}$$

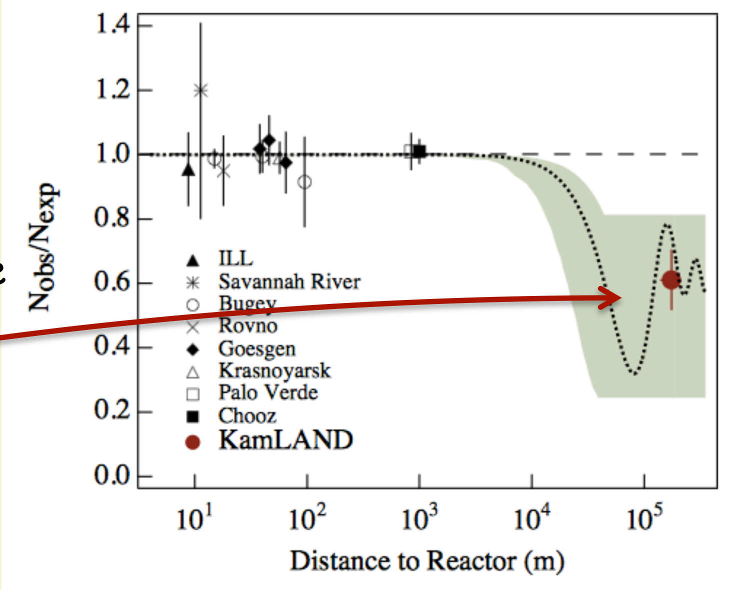
$$\begin{aligned} \phi_e &= 1.76^{+0.05}_{-0.05}(\mathrm{stat.})^{+0.09}_{-0.09}\ (\mathrm{syst.}) \\ \phi_{\mu\tau} &= 3.41^{+0.45}_{-0.45}(\mathrm{stat.})^{+0.48}_{-0.45}\ (\mathrm{syst.}) \end{aligned}$$

[MSW LA solution favoured, maximal ϑ_{12} mixing excluded]

- **First Results from KamLAND: Evidence for Reactor Anti-Neutrino Disappearance**

  (Dated: December 9, 2002)

KamLAND experiment exploits the low-energy electron anti-neutrinos (E≈3 MeV) produced by Japanese and Korean reactors at an average distance of L≈180 Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2



MSW LA finally determined

$$\sin^2 2\theta = 0.833 \text{ and } \Delta m^2 = 5.5 \times 10^{-5}\ \mathrm{eV}^2$$

Tri-BiMaximal Mixing [TBM]

- MSW LA → $\sin^2\vartheta_{12} = 0.32^{+0.05}_{-0.06}$ [Bahcall, Gonzalez-Garcia, Pena-Garay 0212147]

$$(|U_{l\nu}|^2) = \begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix} \end{array}$$

[Harrison, Perkins, Scott 0202074]

assuming ϑ_{13} negligible

so "symmetric" and soon derived from A_4 discrete symmetry

Ma, Rajasekaran 0106291, Babu, Ma, Valle 0206292; Hirsch, Romao, Skadauge, Valle, Villanova del Moral 0312244, Ma 0404199, 0409075]

- A_4 was the upgrade of the μ−τ parity symmetry [Grimus, Lavoura 0110041, 0305046] in the flavour basis, require m_ν invariant under U

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$U^2=1$

$$m_\nu = \begin{pmatrix} x & y & y \\ y & w & z \\ y & z & w \end{pmatrix} \Leftrightarrow \begin{array}{l} \vartheta_{13} = 0 \\ \vartheta_{23} = \dfrac{\pi}{4} \end{array}$$

ϑ_{12} undetermined

TBM is obtained when $x + y = w + z$

now m_ν invariant also under S

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$U^2 = S^2 = 1 \quad [S,U] = 0$$

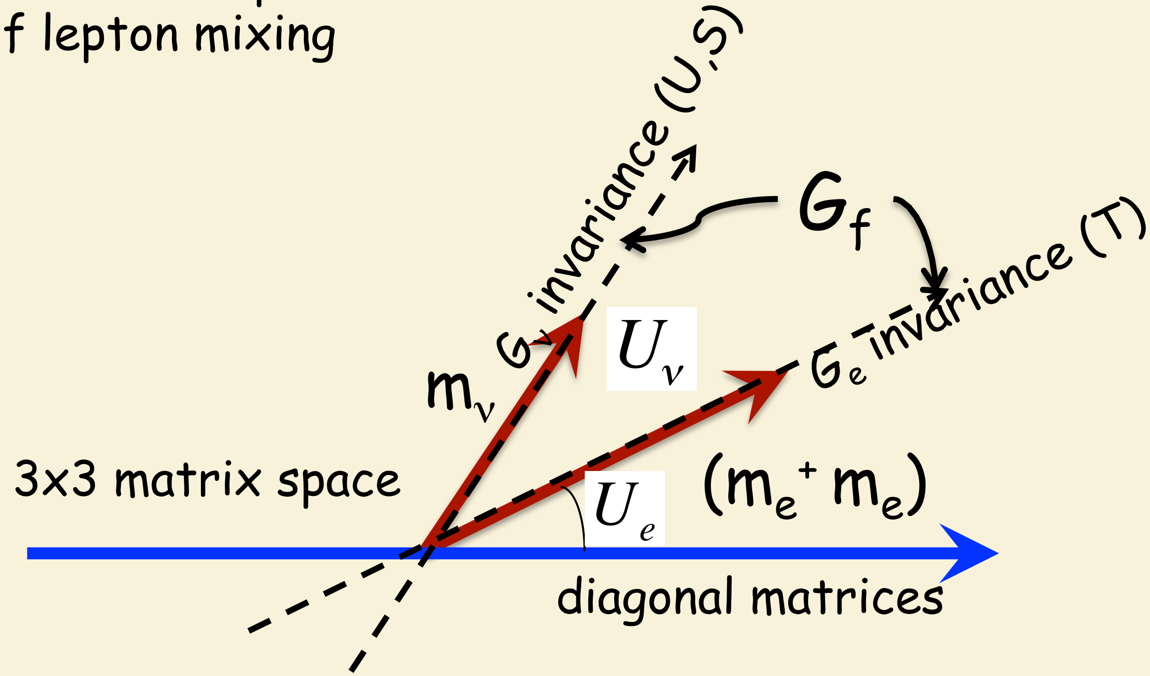
$Z_2 \times Z_2$ the most general symmetry of m_ν if neutrinos are Majorana the flavour basis can be guaranteed if $(m_e^+ m_e)$ is invariant under

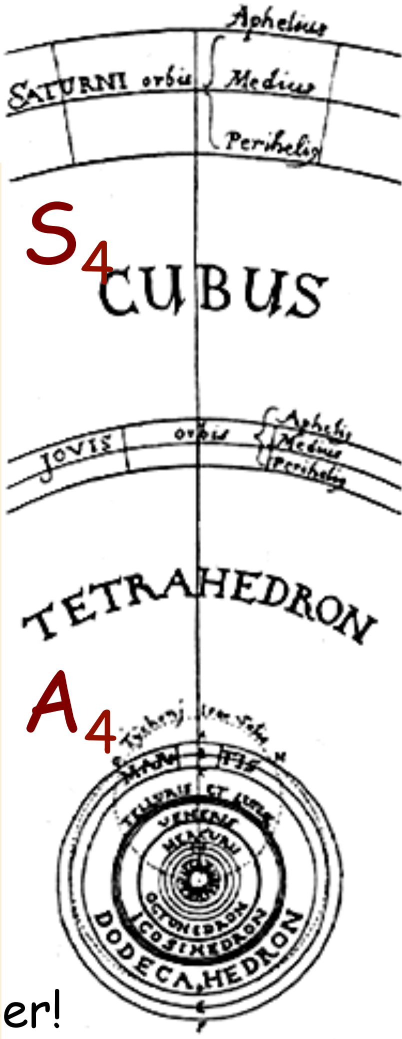
$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{i\frac{2\pi}{3}}$$

[Lam 0708.3665 + 0804.2622]

- (S,T) generate A_4 (U can arise as an accidental symmetry)
  (S,T,U) generate S_4
  …
  …

geometrical picture of lepton mixing





[Kepler 1596 Mysterium Cosmographicum]

very unfortunate Kepler's paper!

Tri-BiMaximal Mixing from A_4

[AF 0504165, 0512103]

we built a model with a number of nice features…

- desired breaking – $G_\nu = \{U,S\}$ $G_e = \{T\}$ – achieved dynamically
  G_ν and G_e selected by the minimum of the energy density of the theory

  vacuum alignment at LO $\langle\varphi_T\rangle = (1\,0\,0)\,V_T$ $\langle\varphi_S\rangle = (1\,1\,1)\,V_S$

- LO lepton mixing angles – TBM – completely determined by the breaking
  -- no ad-hoc relations among parameters required
  -- formalism totally basis independent

- μ−τ parity symmetry naturally incorporated: U generator arises as an accidental symmetry

- charged lepton mass hierarchy explained by $U(1)_{FN}$ (-> Z_4 in a more minimal version) [Altarelli, Meloni 0905.0620]

- study of NLO corrections induced by higher-dimensional operators,…

$$U_{PMNS} = U_{TB} + O(\varepsilon)$$

$$\varepsilon = \frac{V_T}{\Lambda}, \frac{V_S}{\Lambda}$$

expected size of ε fixed
by the agreement $\vartheta_{12}^{TB} \approx \vartheta_{12}^{EXP}$ → $0.01 < \varepsilon < 0.05$

and some alarming predictions…

ϑ_{23} nearly maximal still compatible with data

$\vartheta_{13} < 0.05$ wrong!

me: very much excited about this neat prediction!

Guido:

" Special models are those where some symmetry or dynamical feature assures in a natural way the near vanishing of θ_{13} and/or of $\theta_{23} - \pi/4$. Normal models are conceptually more economical and much simpler to construct. We expect that experiment will eventually find that θ_{13} is not too small and that θ_{23} is sizably not maximal." [Altarelli, 2005]

2011/2012 breakthrough: $\vartheta_{13} \neq 0$

- from LBL experiments searching for $\nu_\mu \to \nu_e$ conversion

T2K: muon neutrino beam produced
at JPARC [Tokai]
E=0.6 GeV and sent to
SK 295 Km apart [1106.2822]

MINOS: muon neutrino beam produced
at Fermilab [E=3 GeV] sent to
Soudan Lab 735 Km apart [1108.0015]

$$P(\nu_\mu \to \nu_e) = \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

both experiments favor
$\sin^2\vartheta_{13} \sim$ few %

- from SBL reactor experiments searching for anti-ν_e disappearance

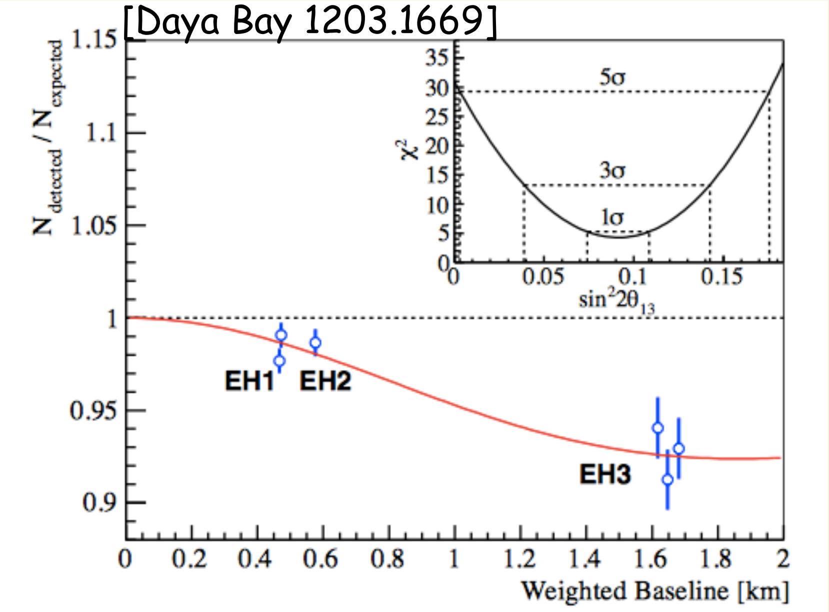
Double Chooz (far detector):

Daya Bay (near + far detectors):

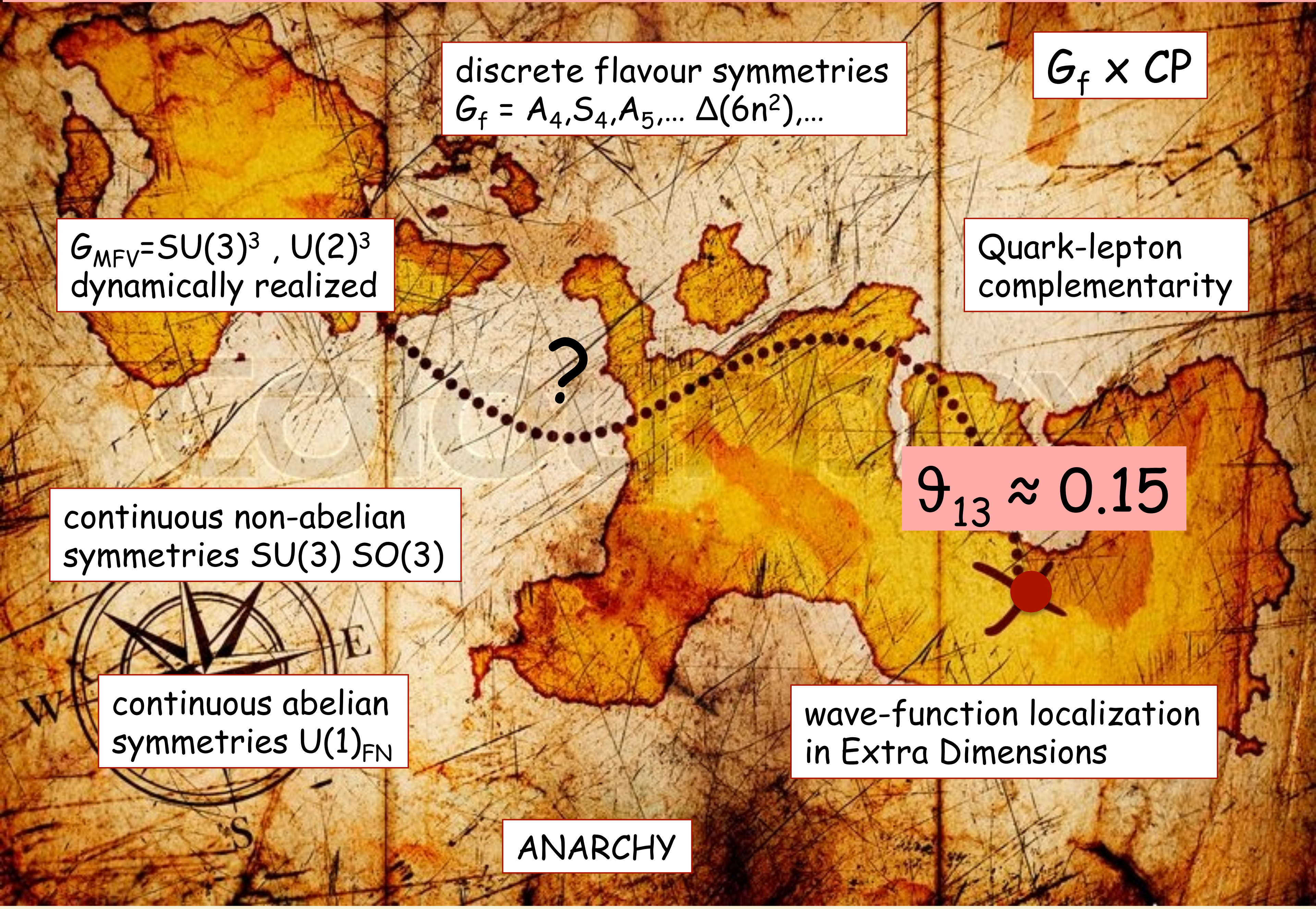
RENO (near + far detectors):

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

DC: $\sin^2\vartheta_{13} = 0.022 \pm 0.013$
DB: $\sin^2\vartheta_{13} = 0.024 \pm 0.004$
R: $\sin^2\vartheta_{13} = 0.029 \pm 0.006$


[Daya Bay 1203.1669]

Which Direction ?



discrete flavour symmetries
$G_f = A_4, S_4, A_5, \ldots \Delta(6n^2), \ldots$

$G_f \times CP$

$G_{MFV} = SU(3)^3$, $U(2)^3$
dynamically realized

Quark-lepton
complementarity

$\vartheta_{13} \approx 0.15$

continuous non-abelian
symmetries SU(3) SO(3)

continuous abelian
symmetries $U(1)_{FN}$

wave-function localization
in Extra Dimensions

ANARCHY

Unfortunately $\vartheta_{13} \approx 0.15$ does not indicate any precise direction in the chart of possible models

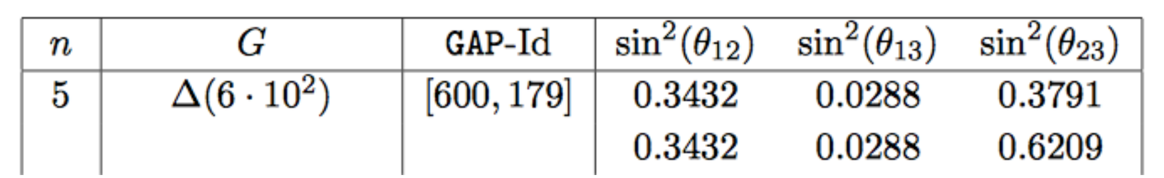
$\vartheta_{13} \approx 0.15$ rad and the hint for non maximal ϑ_{23} have strengthened the case for anarchy, and for variants based on $U(1)_{FN}$ abelian continuous symmetries, Extra Dimensions,…

But discrete symmetries can also easily cope with $\vartheta_{13} \approx 0.15$

-- add "large" corrections $O(\vartheta_{13}) \approx 0.15$ to TBM pattern

-- change discrete group G_f and try to fit lepton mixing

F.F., C. Hagedorn, R. de A.Toroop
hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen1, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Hagedorn, Meroni, Vitale 1307.5308]



n	G	GAP-Id	$\sin^2(\theta_{12})$	$\sin^2(\theta_{13})$	$\sin^2(\theta_{23})$
5	$\Delta(6 \cdot 10^2)$	[600, 179]	0.3432	0.0288	0.3791
			0.3432	0.0288	0.6209

complete classification of $|U_{PMNS}|$ from any finite group available now!

[Fonseca, Grimus 1405.3678]

-- change LO pattern

$$U^0_{PMNS} = U_{BM}$$

[G. Altarelli, F.F., L. Merlo
and E. Stamou hep-ph/1205.4670;
Altarelli, Machado, Meloni 1504.05514]

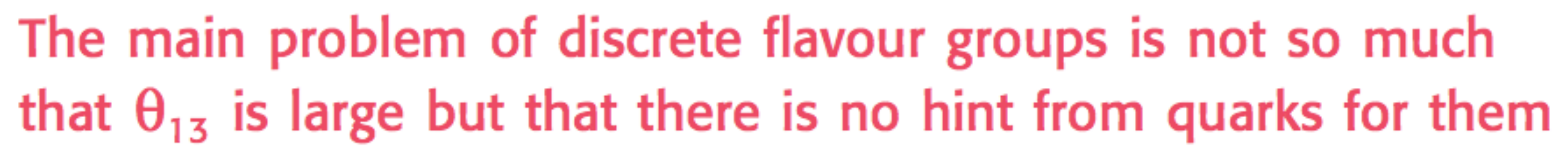
-- include CP in the SB pattern

[F. F, C. Hagedorn and R. Ziegler 1211.5560, 1303.7178
Ding,King,Luhn,Stuart 1303.6180 Ding, King, Stuart 1307.4212]

-- relax symmetry requirements

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, Hernandez,Smirnov 1204.0445]

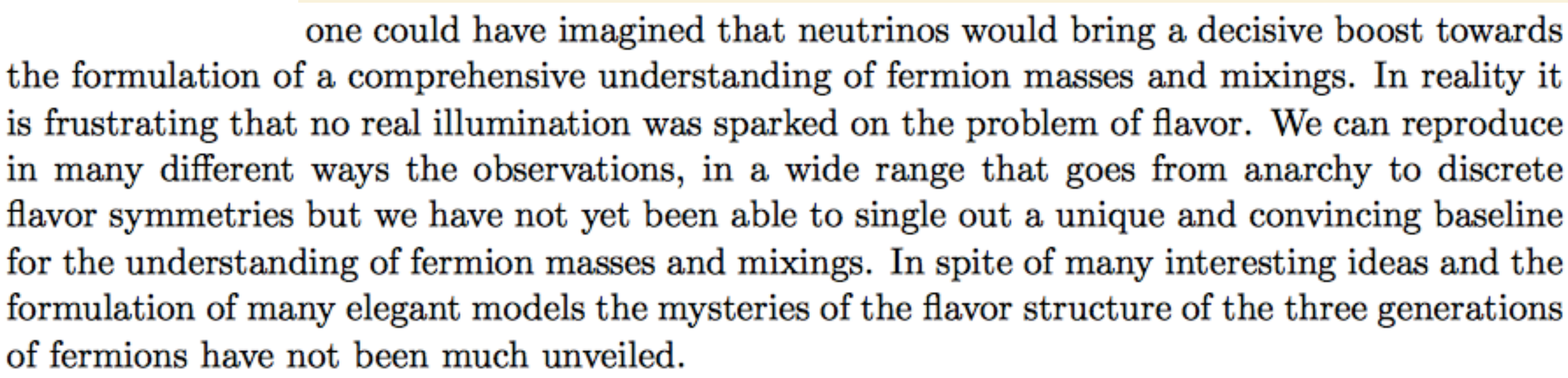
Conclusion



The main problem of discrete flavour groups is not so much that θ_{13} is large but that there is no hint from quarks for them

[Guido, Corfu 2014]

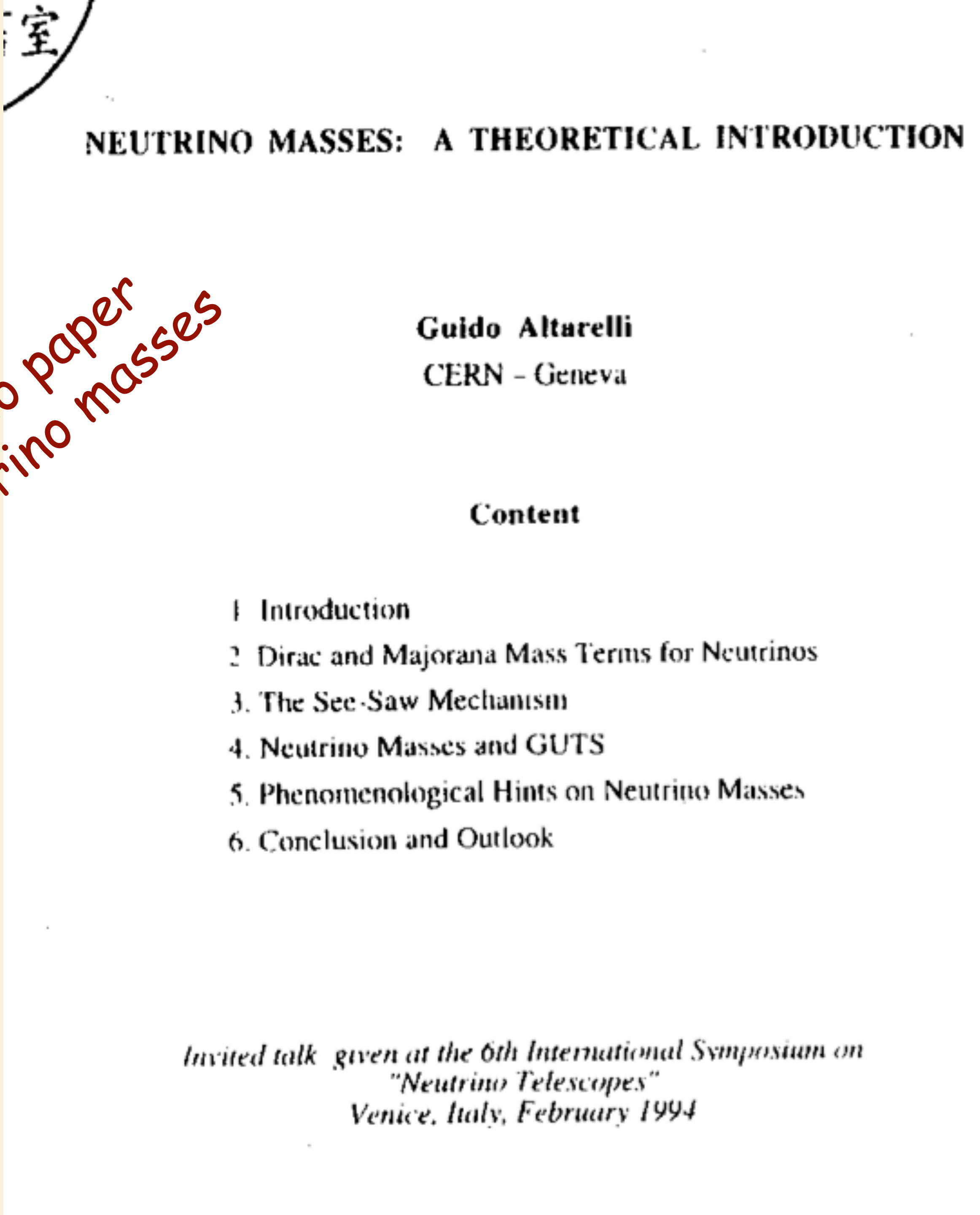
- no clear role in the quark sector
  large hierarchies and small mixing angles seem not require discrete groups
- extension to GUTs possible (many existence proofs) but rather complicated
  quark mass ratios and quark mixing angles from small parameters ≠ ε
  [$U(1)_{FN}$, Extra Dimensions,…]



one could have imagined that neutrinos would bring a decisive boost towards the formulation of a comprehensive understanding of fermion masses and mixings. In reality it is frustrating that no real illumination was sparked on the problem of flavor. We can reproduce in many different ways the observations, in a wide range that goes from anarchy to discrete flavor symmetries but we have not yet been able to single out a unique and convincing baseline for the understanding of fermion masses and mixings. In spite of many interesting ideas and the formulation of many elegant models the mysteries of the flavor structure of the three generations of fermions have not been much unveiled.

[Guido Altarelli, "Status of Neutrino Mass and Mixing" 1404.3859]

I will miss you a lot, Guido !



NEUTRINO MASSES: A THEORETICAL INTRODUCTION

Guido Altarelli
CERN - Geneva

Content

1 Introduction
2 Dirac and Majorana Mass Terms for Neutrinos
3. The See-Saw Mechanism
4. Neutrino Masses and GUTS
5. Phenomenological Hints on Neutrino Masses
6. Conclusion and Outlook

Invited talk given at the 6th International Symposium on
"Neutrino Telescopes"
Venice, Italy, February 1994

1st Guido paper
on neutrino masses

Backup slides

Plan of the talk

1969 – 1997:
-- neutrino timeline

1998 – 2005:
-- struggling with textures
-- abelian flavour symmetries
-- GUTs

2005 -2011:
-- discrete flavour symmetries

2011 -2013:
-- new directions

1998: convincing evidence of neutrino oscillations [SuperKamiokande]

2002: solar neutrino problem solved [SNO CC and NC, Kamland]

2011: T2K, Minos, Daya Bay, RENO measure ϑ_{13}

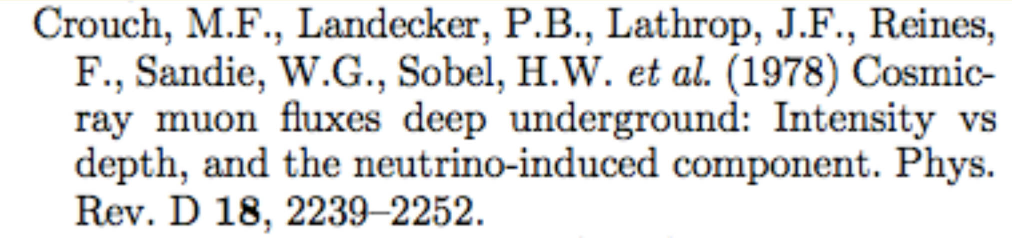
Solar Neutrino Timeline

1969	1st detection of solar neutrinos by R. Davis at the Homestake mine $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ solar ν problem starts, no other solar ν experiments for 20 yr!
1969	solution in terms of ν_e -> ν_μ oscillations by Gribov and Pontecorvo
1974	GUT proposed by Georgi and Glashow
1977	see-saw mechanism for neutrino masses [Minkowski, Gell-Mann, Ramond, Slanski and Yanagida]
1978 1986	Wolfenstein, Mikheyev, Smirnov (MSW effect) sizeable solar ν_e conversion possible with small mixing angle
1987	detection of neutrinos from SN1987A by Kamiokande, IMB, Baksan. Kamiokande lower the E threshold below solar ν energies ~ 10 MeV
1989	$N_\nu = 3$ from LEP
90s	SAGE, GALLEX, GNO $\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge$ confirm the solar ν problem in the low-energy region of ν spectrum
1994	$m_{\nu_e} < 2.2\ eV$ [Troitsk]

Atmospheric Neutrino Timeline

1978 first measurement of

$$\Phi_{th}(\nu_\mu)/\Phi_{\exp}(\nu_\mu) = 1.6 \pm 0.4$$


Crouch, M.F., Landecker, P.B., Lathrop, J.F., Reines, F., Sandie, W.G., Sobel, H.W. *et al.* (1978) Cosmic-ray muon fluxes deep underground: Intensity vs depth, and the neutrino-induced component. Phys. Rev. D **18**, 2239–2252.

80s several proton decay experiments started M = 100 – 3000 tons
atmospheric ν, serious background for p-decay searches, are carefully studied
Kamiokande, IMB, Soudan

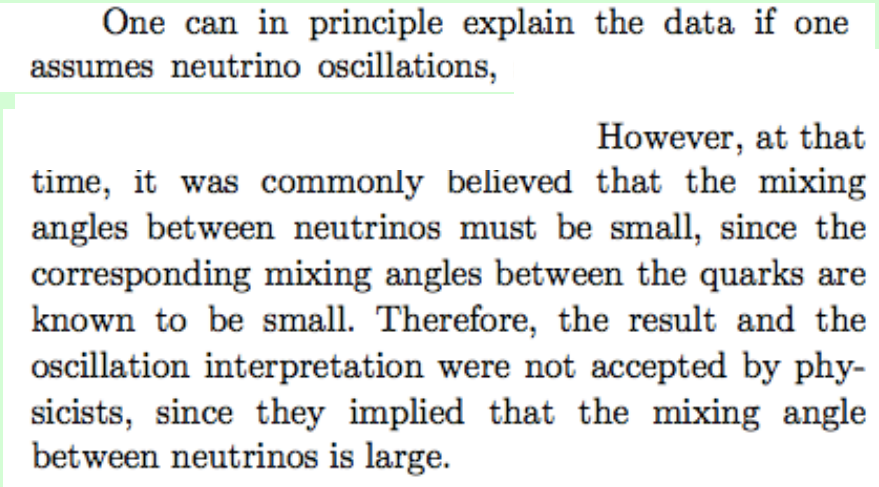
$$R = (\mu/e)_{data}/(\mu/e)_{MC} \approx 0.6$$

atmospheric ν problem

Prejudices < 1997

solar ν problem:
several solutions possible
-- SSM not correct
-- resonant spin-flavour precession of ν
-- FCNC solution
-- MSW SA attractive

atmospheric ν problem:
it will fade away since it requires
a large mixing angle


One can in principle explain the data if one assumes neutrino oscillations,

However, at that time, it was commonly believed that the mixing angles between neutrinos must be small, since the corresponding mixing angles between the quarks are known to be small. Therefore, the result and the oscillation interpretation were not accepted by physicists, since they implied that the mixing angle between neutrinos is large.

[T. Kajita 2010]

1997 - 1998 turnpoint

1997 solar sound speed from helioseismology compared with predictions of SSM (test T-profile in solar interior)

SSM reliable

Bahcall, Pinsonneault, Sarbani Basu, Christensen-Dalsgaard
Phys.Rev.Lett. 78 (1997) 171

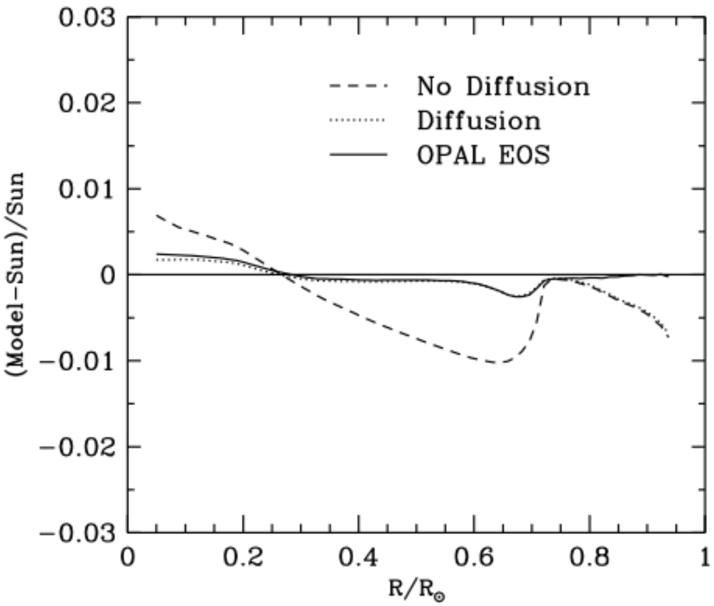


FIG. 1. Comparison of sound speeds predicted by different standard solar models with the sound speeds measured by helioseismology. There are no free parameters in the models;

1996 Superkamiokande starts, atmospheric ν data shown at Neutrino '98

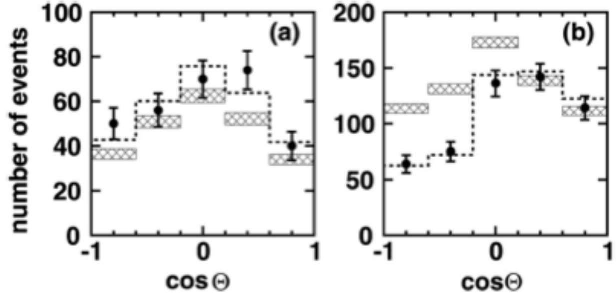


Fig. 14. Zenith angle distributions for multi-GeV atmospheric neutrino events reported at the Nuetrino'98 conference based on 535 days exposure of the Super-Kamiokande detector. The left and right panels show the distributions for e-like and μ-like events, respectively. Θ shows the zenith angle, and $\cos\Theta = 1$ and -1 represent events whose direction is vertically downward-going and upward-going, respectively.

- -- zenith angular distributions of atmospheric ν
- -- oscillation solution becomes compelling
- -- determination of

$$(\Delta m^2_{atm}, \sin^2 2\vartheta_{23})$$

≈1 -> maximal mixing

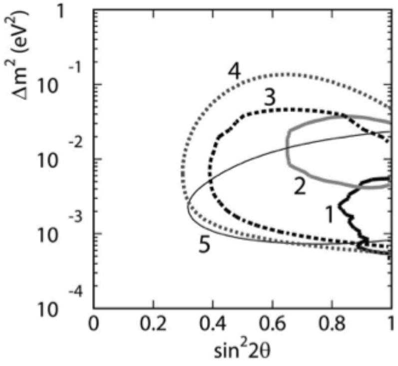


Fig. 15. Allowed parameter regions of $\nu_\mu \to \nu_\tau$ oscillations from Super-Kamiokande and Kamiokande shown at the Neutrino'98 conference.[7] Contours are obtained based on; (1) contained events from Super-Kamiokande, (2) contained events from Kamiokande, (3) upward through-going events from Super-Kamiokande, (4) upward through-going events from Kamiokande and (5) stop/through ratio analysis for upward-going muons from Super-Kamiokande.

Conclusion

From the theoretical side, for ν masses and mixings
we do not have so far a compelling theoretical picture
and many possibilities are still open.

Actually, also for quarks and charged leptons
we do not have a theory of flavour that explains the observed
spectrum, mixings and CP violation.

Yet in spite of impressive progress important
experimental open questions remain:
Absolute scale of m^2? Inverse or normal hierarchy?
CP violation? Flavour symmetry? Sterile ν's? DM?..

Thus ν's are interesting because they can provide new clues
on the flavour problem

[Guido, Corfu 2014]

anything special from data, requiring a symmetry?

1 ϑ_{23} maximal ?

2 $\delta_{CP} = -\pi/2$?

3 U_{PMNS} close to TB (BM,...) ?

3 examples from
a longer list...

1 today most precise single determination of ϑ_{23} is from T2K ($P_{\mu\mu}$)

[1403.1532]

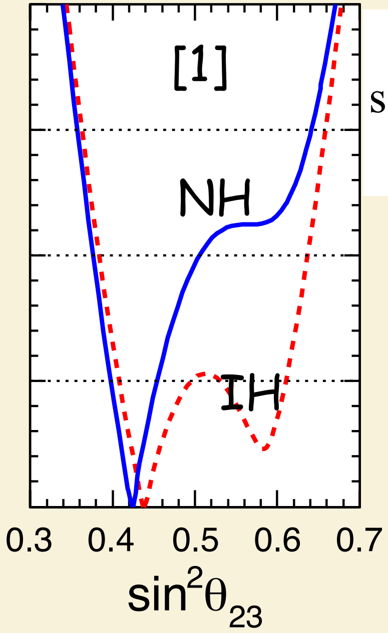
$$\sin^2\vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & \text{(NH)} \\ 0.511^{+0.055}_{-0.055} & \text{(IH)} \end{cases}$$

well compatible with
ϑ_{23} maximal

global fits hint at ϑ_{23} non-maximal
main effect: interplay between
SBL reactor experiments (P_{ee}) and
LBL experiments searching ($P_{\mu e}$)

$$P_{ee} = 1 - \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

$$P_{\mu e} = \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$



$$\sin^2\vartheta_{23} = \begin{cases} 0.567^{+0.032}_{-0.128} & \text{(NH)} \\ 0.573^{+0.025}_{-0.043} & \text{(IH)} \end{cases}$$

[2]

global fit:
[1] Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo 1312.2878
[2] Forero, Tortola, Valle 1405.7540

a small change of P_{ee} and/or $P_{\mu e}$ within about 1σ can bring back ϑ_{23} to maximal

- difficult to improve ϑ_{23} from $P_{\mu\mu}$

$$\delta\vartheta_{23} \approx \sqrt{\delta P_{\mu\mu}} / 2 \qquad \delta P_{\mu\mu} \approx 0.01 \quad \Rightarrow \quad \delta\vartheta_{23} \approx 0.05 \text{ rad } (2.9^0)$$

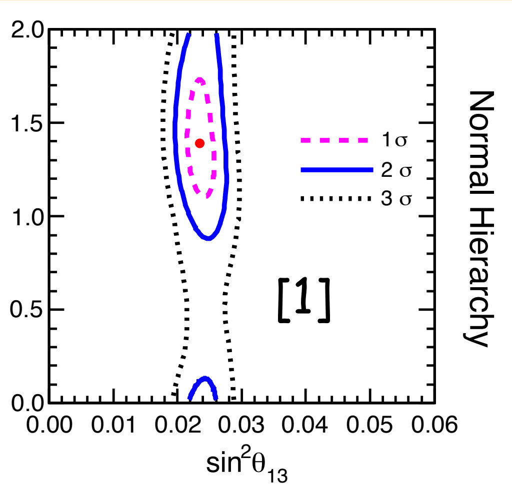
ϑ_{23} nearly maximal would be a crucial piece of information

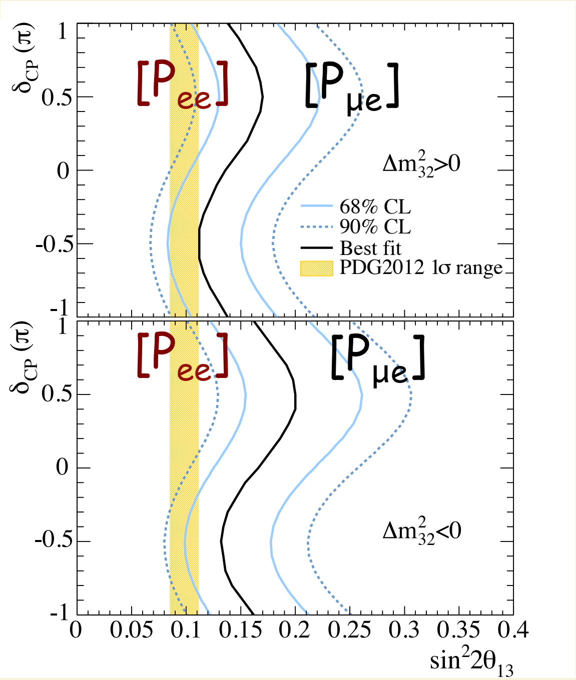
- ϑ_{23} cannot be made maximal by RGE evolution [barring tuning of b.c. and/or thresold corrections]

- when a flavour symmetry is present, ϑ_{23} is determined entirely by breaking effects [no maximal ϑ_{23} from an exact symmetry]

broken abelian symmetries do not work [not a theorem but no counterexamples] ⇒ we are left with broken non-abelian symmetries

2 $\delta_{CP} = -\pi/2$?





[T2K: 1311.4750 and 1311.4114]

1 add large corrections $O(\vartheta_{13}) \approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of FC/CPV if NP is at the TeV scale

2 relax symmetry requirements

[Hernandez,Smirnov 1204.0445]

G_e as before

$G_\nu = Z_2$

2 predictions:
2 combinations of $\vartheta^0_{12} \quad \vartheta^0_{23} \quad \vartheta^0_{13} \quad \delta_{CP}$

two deformations of TB, called Trimaximal [TM] mixing

TM$_1$

$$U^0 = U_{TB} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & e^{i\delta}\sin\alpha \\ 0 & -e^{-i\delta}\sin\alpha & \cos\alpha \end{pmatrix}$$

TM$_2$

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

leads to testable sum rules

$$\sin^2\vartheta_{12} = \frac{1}{3} - \frac{2}{3}\sin^2\vartheta_{13} + O(\sin^4\vartheta_{13})$$

$$\sin^2\vartheta_{12} = \frac{1}{3} + \frac{1}{3}\sin^2\vartheta_{13} + O(\sin^4\vartheta_{13})$$

$$\sin^2\vartheta_{23} = \frac{1}{2} - \sqrt{2}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670] deviation from TB is linear in α for $\sin^2\theta_{23}$, whereas is quadratic for $\sin^2\theta_{12}$, the best measured angle

sum rules can be tested by measuring δ_{CP} and improving on $\sin^2\vartheta_{23}$

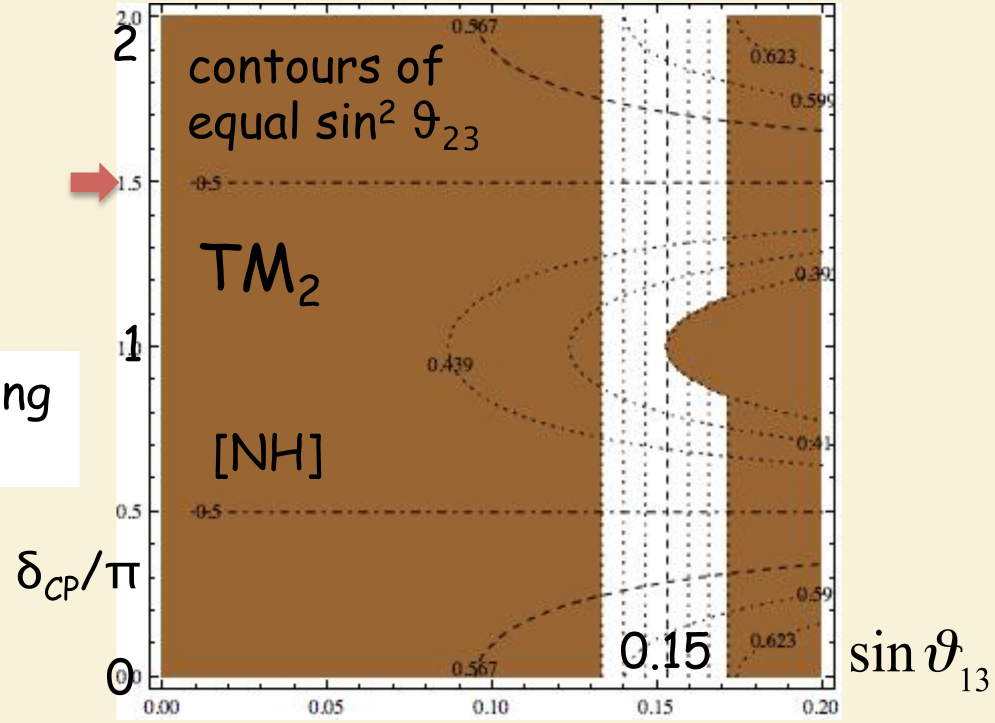
3 change discrete group G_f

- solutions exist
  special forms of TM_2

G_f	$\Delta(96)$	$\Delta(384)$	$\Delta(600)$
α	$\pm\pi/12$	$\pm\pi/24$	$\pm\pi/15$
$\sin^2\vartheta_{13}^0$	0.045	0.011	0.029

$\delta^0 = 0, \pi$ (no CP violation) and
α "quantized" by group theory

complete classification of $|U_{PMNS}|$ from any finite group available now!



$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

F.F., C. Hagedorn, R. de A.Toroop
hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen1, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Hagedorn, Meroni, Vitale 1307.5308]

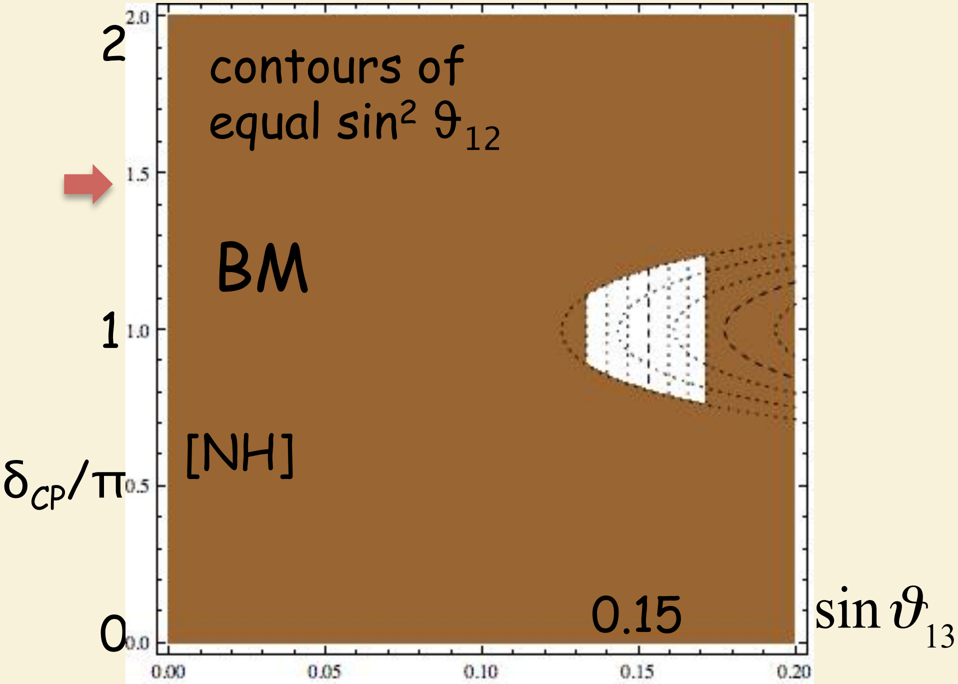
[Fonseca, Grimus 1405.3678]

4 change LO pattern

$$U^0_{PMNS} = U_{BM}$$

corrected by U^e_{12}

$$\sin^2\vartheta_{12} = \frac{1}{2} + \sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$



5 include CP in the SB pattern

$$\begin{tikzcd}
 & G_{CP} = G_f \rtimes CP \arrow[dl] \arrow[dr] & \\
G_e & & G_\nu = Z_2 \times CP
\end{tikzcd}$$

[F. F, C. Hagedorn and R. Ziegler 1211.5560, 1303.7178 Ding,King,Luhn,Stuart 1303.6180 Ding, King, Stuart 1307.4212]

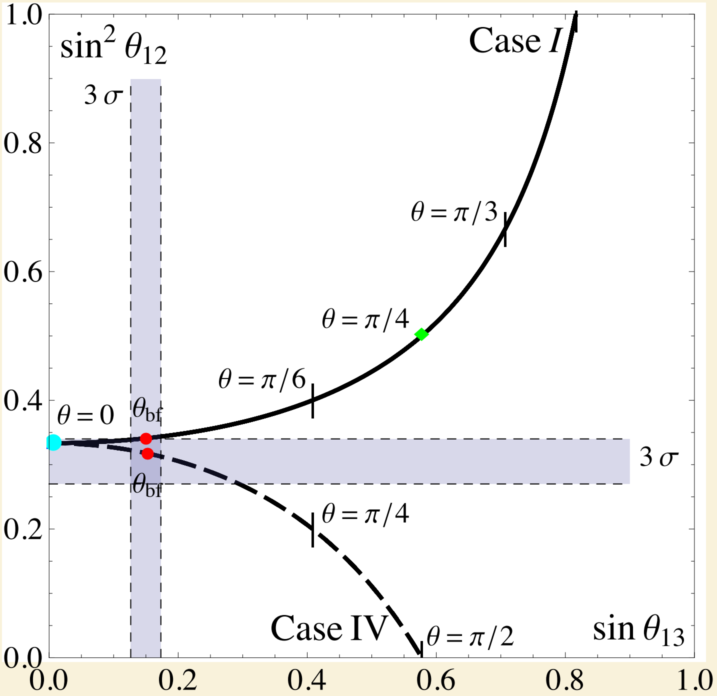
mixing angles and CP violating phases

$$(\vartheta^0_{12}, \vartheta^0_{23}, \vartheta^0_{13}, \delta^0, \alpha^0, \beta^0)$$

predicted in terms of a single real parameter $0 \leq \vartheta \leq \pi$

2 examples with $G_f = S_4$ $G_e = Z_3$

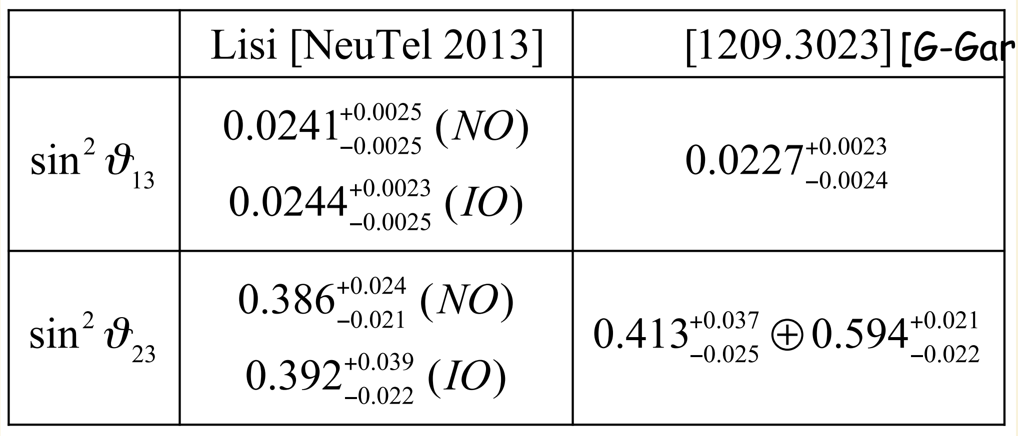
$$\sin^2\vartheta^0_{23} = \frac{1}{2} \qquad |\sin\delta^0| = 1 \qquad \sin\alpha^0 = 0 \qquad \sin\beta^0 = 0$$



2011/2012 breakthrough

-- LBL experiments searching for $\nu_\mu \to \nu_e$ conversion
-- SBL reactor experiments searching for anti-ν_e disappearance

[see Fogli's talk]



	Lisi [NeuTel 2013]	[1209.3023] [G-Garcia, Maltoni, Salvado, Schwetz]
$\sin^2\vartheta_{13}$	$0.0241^{+0.0025}_{-0.0025}\ (NO)$ $0.0244^{+0.0023}_{-0.0025}\ (IO)$	$0.0227^{+0.0023}_{-0.0024}$
$\sin^2\vartheta_{23}$	$0.386^{+0.024}_{-0.021}\ (NO)$ $0.392^{+0.039}_{-0.022}\ (IO)$	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$

10σ away from 0

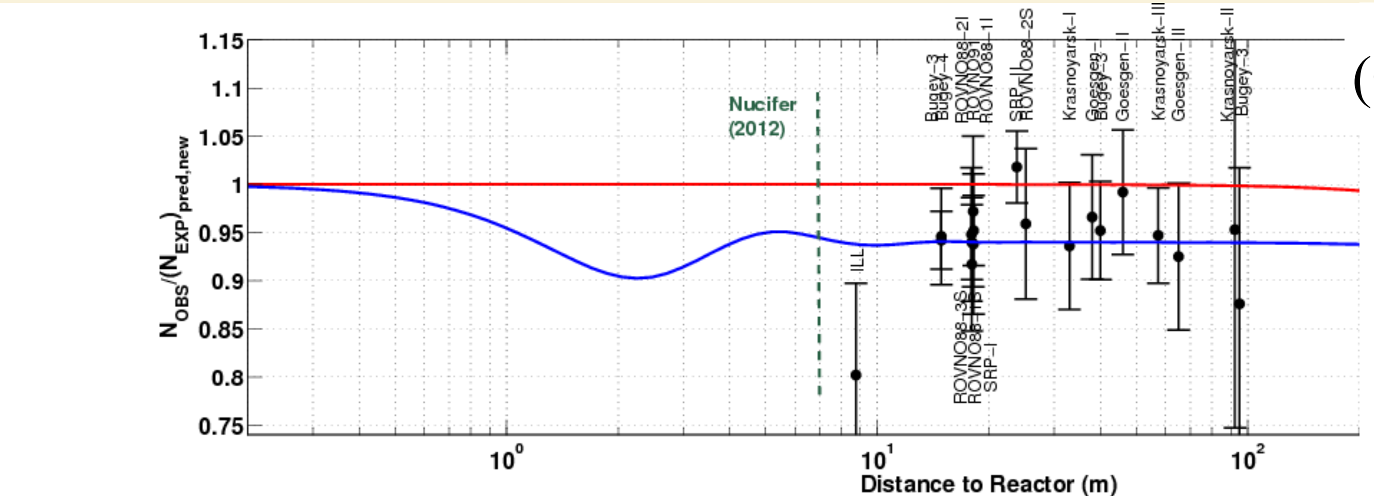
impact on flavor symmetry (part 3)

hint for non maximal ϑ_{23}

sterile neutrinos coming back

1 reactor anomaly (anti-ν_e disappearance)

re-evaluation of reactor anti-ν_e flux: new estimate 3.5% higher than old one



$$(\Phi_{\exp} - \Phi_{th})/\Phi_{th} \approx -6\%$$

[th. uncertainty?]

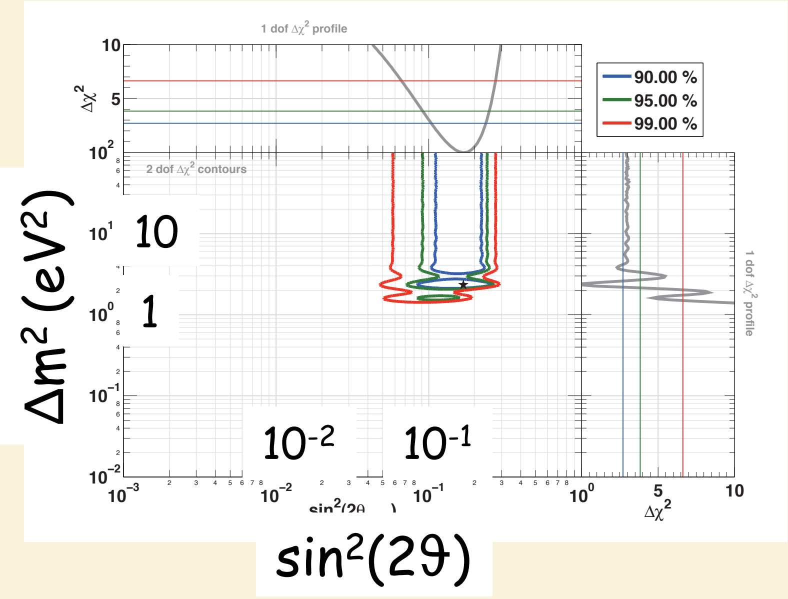
very SBL L ≲ 100 m

$$\vartheta_{es} \approx 0.2$$

$$\Delta m^2 \approx m_s^2 \geq 1\, eV^2$$

supported by the Gallium anomaly

ν_e flux measured from high intensity radioactive sources in Gallex, Sage exp



$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

[error on σ or on Ge extraction efficiency]

most recent cosmological limits

[depending on assumed cosmological model, data set included,…]

relativistic degrees of freedom at recombination epoch

$$N_{eff} = 3.30 \pm 0.27$$

[Planck, WMAP, BAO, high multiple CMB data]

fully thermalized non relativistic ν

$$N_{eff} < 3.80 \quad (95\%\, CL)$$

$$m_s < 0.42\, eV \quad (95\%\, CL)$$

2 long-standing claim

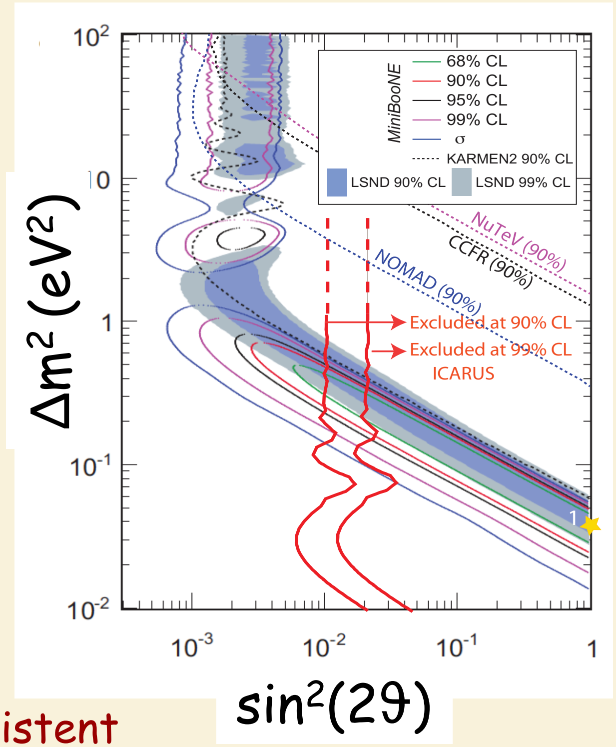
evidence for ν_μ -> ν_e appearance in accelerator experiments

exp		$E(MeV)$	$L(m)$	
$LSND$	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$10 \div 50$	30	3.8σ
$MiniBoone$	$\nu_\mu \rightarrow \nu_e$ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$300 \div 3000$	541	3.8σ [signal from low-energy region]

parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

$$\Delta m^2 \approx 0.5\, eV^2$$

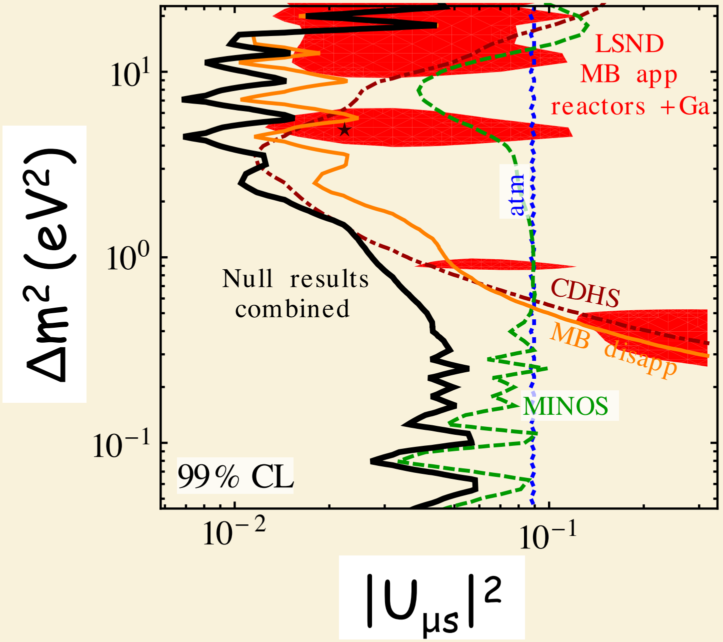


3 interpretation in 3+1 scheme: inconsistent (more than 1s disfavored by cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \Rightarrow \vartheta_{\mu s} \approx 0.2$$

predicted suppression in ν_μ disappearance experiments: undetected

by ignoring LSND/Miniboone data the reactor anomaly can be accommodated by $m_s \geq 1$ eV and $\vartheta_{es} \approx 0.2$
[not suitable for WDM, more on this later]



A_4 as a leftover of Poincare symmetry in D>4

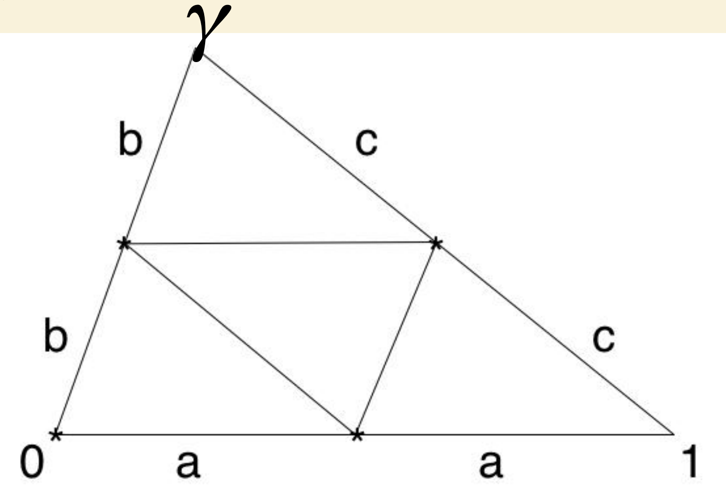
[AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

⟷

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x …

a discrete subgroup of the (D-4) euclidean group = translations x rotations
can survive in specific geometries



Example: D=6

2 dimensions
compactified on T^2/Z_2

$$z \rightarrow z+1$$
$$z \rightarrow z+\gamma$$
$$z \rightarrow -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron
invariant under

$$S: \quad z \rightarrow z+\frac{1}{2}$$ [translation]

$$T: \quad z \rightarrow \gamma^2 z$$ [rotation by 120^0]

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)]