A MODEL FOR SPHEROIDAL GALAXIES WITH PREVALENCE OF RADIAL COMPONENT IN THE VELOCITY DISTRIBUTION OF STARS

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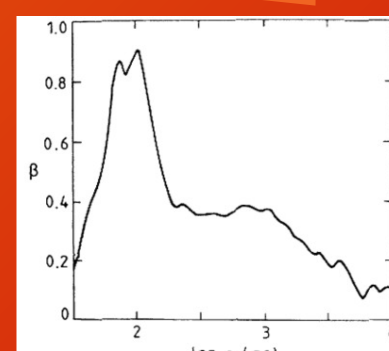
ANISOTROPY REQUESTS

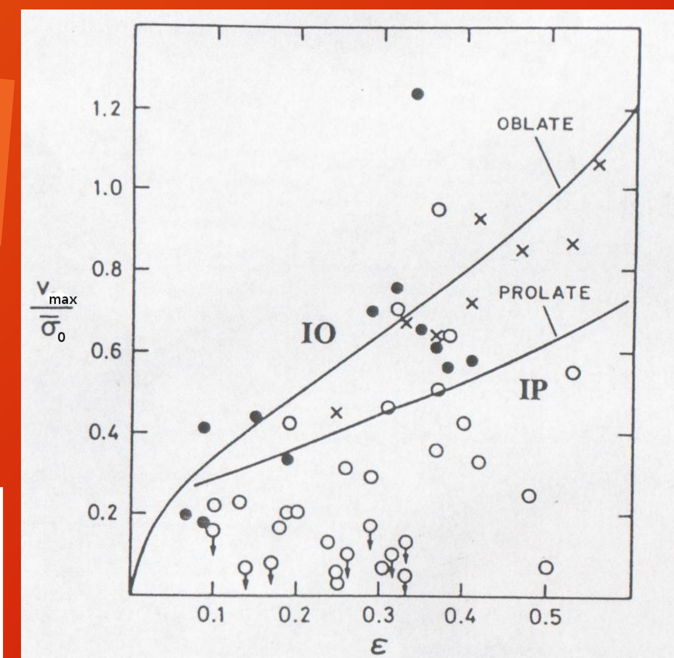
Anisotropy means: $\langle v_r^2 \rangle \neq \langle v_t^2 \rangle$ In this case: $\langle v_r^2 \rangle > \langle v_t^2 \rangle$

v_r = radial velocity v_t = tangential velocity

Anisotropy occurs in different stellar clusters:

- In giant elliptical galaxies
  (Bynney & Tremaine, 1987;
  Sheffler & Elsässer, 1988)





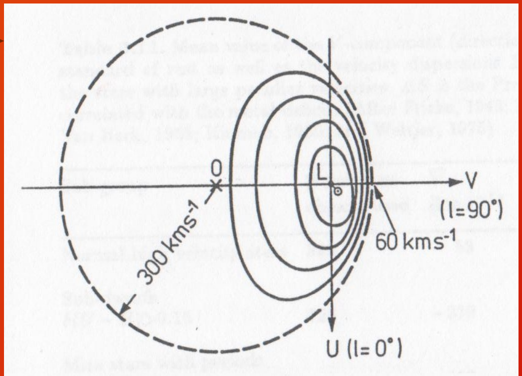
- In globular clusters (GCs)

(Michie, 1961; Roueff et al., 1997; van Leeuwen et al., 2000)

- In spheroidal galaxies and bulges (Hernquist, 1990)

- In high velocity stars in our Galaxy

(Sheffler & Elsässer, 1988; Binney & Mihalas, 1981)



- In dark matter halos: in spiral and elliptical galaxies, in galaxy clusters

(Navarro et al., 2010; Host et al.,2009 and reference therein)

In some of these studies there is the evidence of a prevalence of radial velocity (especially in certain regions of the specific system, as in the intermediate zone) in the velocity distribution of stars

Previous models

Anisotropy in the velocity distribution implies a distribution function (DF) dependent by the angular momentum L, as well as the energy E: $f(E, L)$

- Osipkov and Merrit (Binney & Tremaine, 1987) studied anisotropic system with the following DF

$$f(Q) = \frac{M}{2\pi^3 (GMr_J)^{\frac{3}{2}}} \left[F_-(\sqrt{2\tilde{Q}}) - \sqrt{2} F_-\left(\sqrt{\tilde{Q}}\right) - \sqrt{2}\left(1 + \frac{r_J^2}{2r_a^2}\right) F_+\left(\sqrt{\tilde{Q}}\right) + \left(1 + \frac{r_J^2}{r_a^2}\right) F_+\left(\sqrt{2\tilde{Q}}\right)\right]$$

where

$$Q \equiv \mathcal{E} - \frac{L^2}{2r_a^2}$$

$$\tilde{Q} \equiv (Qr_J/GM).$$

$$F_\pm(x) \equiv e^{\mp x^2} \int_0^x e^{\pm x'^2} dx'$$

- Michie (Michie, 1961) studied the structure and evolution of spherical Gcs, using DF:

$$f_M(\mathcal{E}, L) = \begin{cases} \rho_1 (2\pi\sigma^2)^{-3/2} e^{-L^2/(2r_a^2\sigma^2)} \left[e^{\mathcal{E}/\sigma^2} - 1\right], & \mathcal{E} > 0, \\ 0, & \mathcal{E} \leq 0. \end{cases}$$

where

$$\Psi \equiv -\Phi + \Phi_0$$

$$\mathcal{E} \equiv -E + \Phi_0 = \Psi - \tfrac{1}{2}v^2.$$

Both found systems with an isotropic velocity distribution at the center and nearly radial in the outer regions.

Michie found agreement with the data of GC 47 Tucanae.

- **BKMV-model** (Bisnovatyi-Kogan & Merafina & Vaccarelli, 2009)**:** prevalence of tangential motion over the radial one $\langle v_t^2 \rangle > \langle v_r^2 \rangle$ with DF:

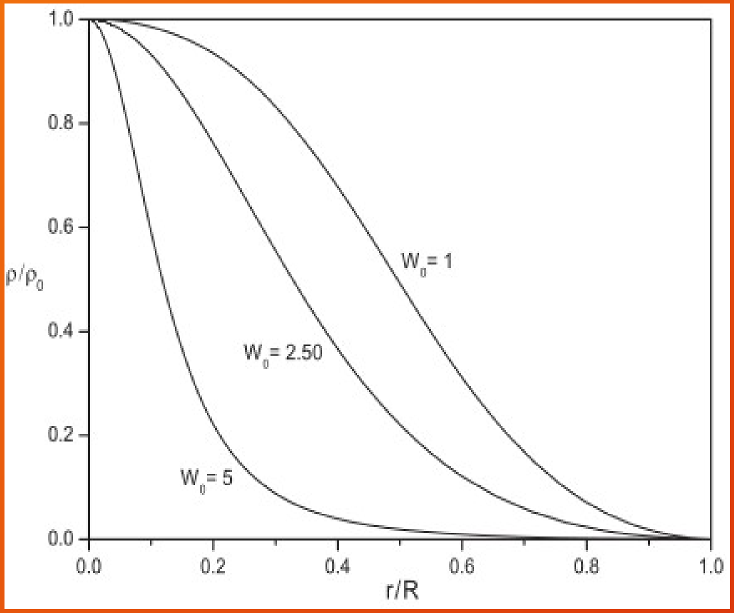
$$f = A\left(1 + \frac{L^2}{L_c^2}\right) e^{\frac{-E}{T}}, \quad E \leqslant E_c$$
$$f = 0, \qquad E > E_c$$

$L_c = m\sigma r_a$ $\quad \sigma^2 = \frac{2T}{m}$ $\quad r_a$: *anisotropy radius*

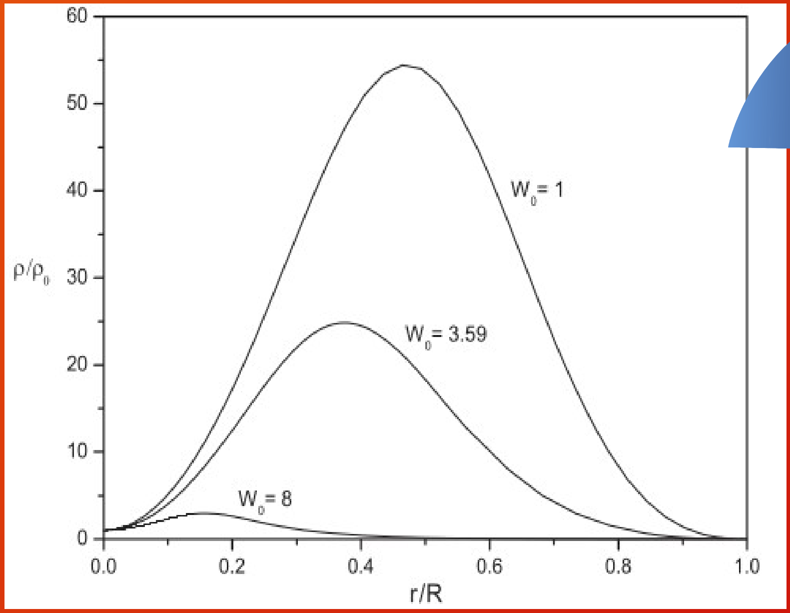
The introduction of a cut-off in energy on the DF is the correct method to limit the system and it is related to the gravitational potential of a massive neighbour systems.

Density profiles

Small anisotropy:

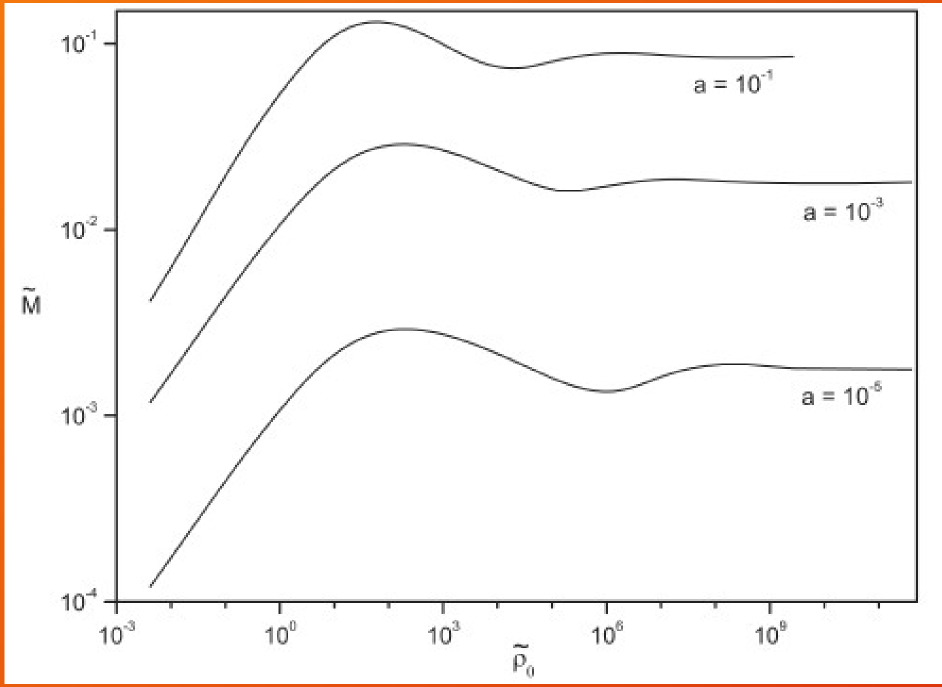


High anisotropy:



Hollow region

Mass profiles $M(\rho_0)$



Anisotropy increases

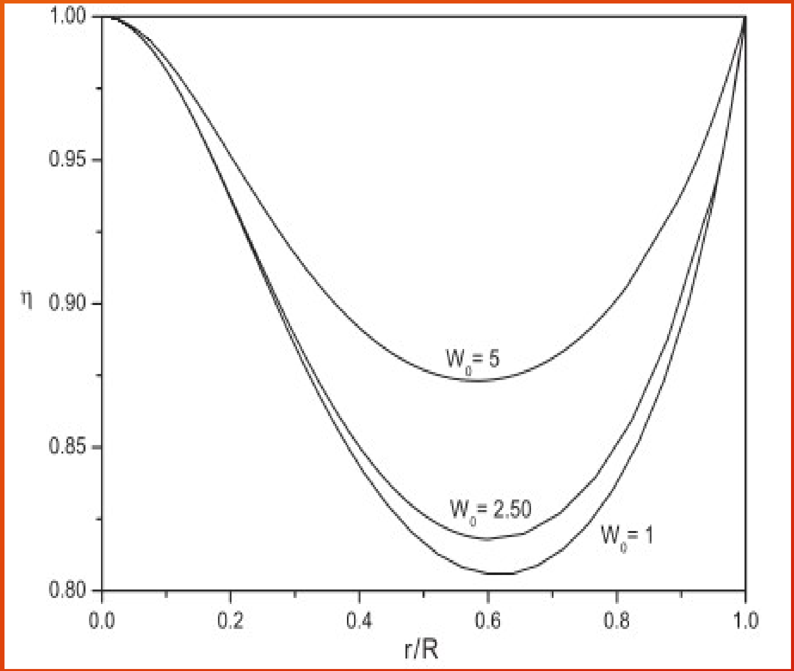
η profiles

Define η as

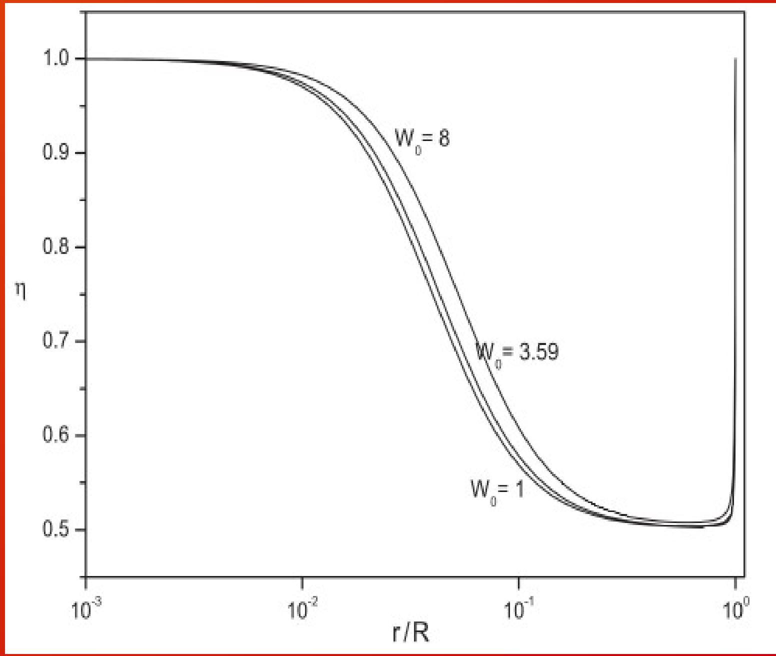
$$\eta = \frac{2\langle v_r^2 \rangle}{\langle v_t^2 \rangle} = \frac{P_{rr}}{P_t}$$

Quantifies the level of anisotropy

Small anisotropy



High anisotropy



Model Development

We use the following anisotropic DF:

$$f = A\left(1+\frac{L^2}{L_c^2}\right)^{-l} e^{-\frac{E}{T}}, E \leqslant E_c$$
$$f = 0, \qquad E > E_c$$

Using useful variables $\varepsilon = \frac{p^2}{2m},\ x = \frac{\varepsilon}{T},\ W = \frac{\varepsilon_c}{T},\ \varepsilon_c = m(\phi_R - \phi),\ p_r = p\cos(\theta),\ \ p_t = p\sin(\theta)$

we can write the distribution function as

$$f = B e^{W-x} \sum_{k=1}^{l+1} \binom{l}{k-1} (-1)^{k-1} \frac{\left(\frac{r}{r_a}\right)^{2(k-1)} x^{k-1} \left(\sin(\theta)\right)^{2(k-1)}}{\left[1+\left(\frac{r}{r_a}\right)^2 x \sin^2(\theta)\right]^{k-1}}$$

From this DF we can derive the expressions of the thermodynamic quantities.

In the DF we set the parameter l=1

Number density $n=\int f\, d^3\mathbf{p}=\int f\, p_t\, dp_r\, dp_t\, d\varphi$

(n ⟷ ρ)

Substituting the DF, one obtains

$$n=B\pi(m\sigma)^3\sum_{k=1}^{2}\binom{1}{k-1}(-1)^{k-1}\left(\frac{r}{r_a}\right)^{2(k-1)}\int_0^W e^{W-x}x^{k-1/2}dx\int_0^\pi \frac{(\sin(\theta))^{2k-1}}{\left[1+\left(\frac{r}{r_a}\right)^2 x\sin^2(\theta)\right]^{k-1}}d\theta$$

Integrating the angular side,
we obtain:

$$n=2\pi B(m\sigma)^3\left(\frac{r}{r_a}\right)^{-1}\int_0^W e^{W-x}\frac{\ln\left(\sqrt{1+\left(\frac{r}{r_a}\right)^2 x}+\left(\frac{r}{r_a}\right)\sqrt{x}\right)}{\sqrt{1+\left(\frac{r}{r_a}\right)^2 x}}dx$$

Radial pressure

$$P_{rr}=\int f\, p_r \frac{d\varepsilon}{dp_r} d^3\boldsymbol{p}=\int f\, p_r \frac{d\varepsilon}{dp_r} p_t\, dp_r\, dp_t\, d\varphi$$

Substituting the DF, one obtains

$$P_{rr}=\frac{\pi B(m\sigma)^5}{m}\sum_{k=1}^{2}\begin{pmatrix}1\\k-1\end{pmatrix}(-1)^{k-1}\left(\frac{r}{r_a}\right)^{2(k-1)}\int_0^W \mathrm{e}^{W-x}x^{k+1/2}\,dx\int_0^{\pi}\frac{(\sin(\theta))^{2k-1}\cos^2(\theta)}{\left(1+\left(\frac{r}{r_a}\right)^2 x\sin^2(\theta)\right)^{k-1}}\,d\theta$$

Integrating the angular side,
we obtain:

$$P_{rr}=\frac{2\pi B(m\sigma)^5}{m}\left(\frac{r}{r_a}\right)^{-2}\left\{\left(\frac{r}{r_a}\right)^{-1}\int_0^W \mathrm{e}^{W-x}\sqrt{1+\left(\frac{r}{r_a}\right)^2 x}\,\ln\left(\sqrt{1+\left(\frac{r}{r_a}\right)^2 x}+\left(\frac{r}{r_a}\right)\sqrt{x}\right)dx-\int_0^{\pi}\mathrm{e}^{W-x}x^{1/2}\,dx\right\}$$

Tangential pressure

$$P_t = \frac{1}{2}\int f\, p_t \frac{d\varepsilon}{dp_t} d^3\boldsymbol{p} = \frac{1}{2}\int f\, p_t \frac{d\varepsilon}{dp_t} p_t\, dp_r\, dp_t\, d\varphi$$

Substituting the DF, one obtains

$$P_t = \frac{\pi B (m\sigma)^5}{2m} \sum_{k=1}^{2} \binom{1}{k-1} (-1)^{k-1} \left(\frac{r}{r_a}\right)^{2(k-1)} \int_0^W e^{W-x} x^{k+\frac{1}{2}} dx \int_0^{\pi} \frac{(\sin(\theta))^{2k+1}}{\left(1+\left(\frac{r}{r_a}\right)^2 x \sin^2(\theta)\right)^{k-1}} d\theta$$

Integrating the angular side,
we obtain:

$$P_t = \frac{\pi B (m\sigma)^5}{m} \left(\frac{r}{r_a}\right)^{-2} \left\{ \int_0^W e^{W-x} x^{1/2} dx - \left(\frac{r}{r_a}\right)^{-1} \int_0^W e^{W-x} \frac{\ln\left(\sqrt{1+\left(\frac{r}{r_a}\right)^2 x} + \left(\frac{r}{r_a}\right)\sqrt{x}\right)}{\sqrt{1+\left(\frac{r}{r_a}\right)^2 x}} dx \right\}$$

Equilibrium equations

1) ***<u>Mass equation</u>***: $$\frac{dM_r}{dr} = 4\pi\rho r^2$$

The equation for the radial pressure can be derived from the "**collisionless Boltzmann equation"**

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\Phi\cdot\frac{\partial f}{\partial \mathbf{v}} = 0$$

which written in spherical coordinates take the following form

$$0 = \frac{\partial f}{\partial t} + v_r\frac{\partial f}{\partial r} + \frac{v_\theta}{r}\frac{\partial f}{\partial \theta} + \frac{v_\varphi}{r\sin(\theta)}\frac{\partial f}{\partial \varphi} + \left(\frac{v_\theta^2 + v_\varphi^2}{r} - \frac{\partial \Phi}{\partial r}\right)\frac{\partial f}{\partial v_r} +$$

$$+\frac{1}{r}\left(v_\varphi^2\cot(\theta) - v_r v_\theta - \frac{\partial \Phi}{\partial \theta}\right)\frac{\partial f}{\partial v_\theta} - \frac{1}{r}\left[v_\varphi\left(v_r + v_\theta\cot(\theta)\right) + \frac{1}{\sin(\theta)}\frac{\partial \Phi}{\partial \varphi}\right]\frac{\partial f}{\partial v_\varphi}$$

Multiplying by v_r, integrating over all the velocities, doing some simplifications and mathematical manipulations, we arrive at the "**Jeans equation**"

$$\frac{d\left(\nu\overline{v_r^2}\right)}{dr}+\frac{\nu}{r}\left[2\overline{v_r^2}-\left(\overline{v_\theta^2}+\overline{v_\varphi^2}\right)\right]=-\nu\frac{d\Phi}{dr}$$

From the pressure tensor, we can write: $P_{rr}=\rho\overline{v_r^2}$ and $P_t=\rho\frac{\overline{v_t^2}}{2}$

The Newton's theorem give us that:

$$\boldsymbol{F}(r)=-\frac{d\Phi}{dr}\hat{r}=-\frac{GM_r}{r^2}\hat{r}$$

$$\nabla^2\Phi=4\pi G\rho$$

From these relations, we derive the expression of:

2) ***Radial pressure equation***:

$$\frac{dP_{rr}}{dr}=-\frac{2}{r}\left(P_{rr}-P_t\right)-\frac{GM_r\rho}{r^2}$$

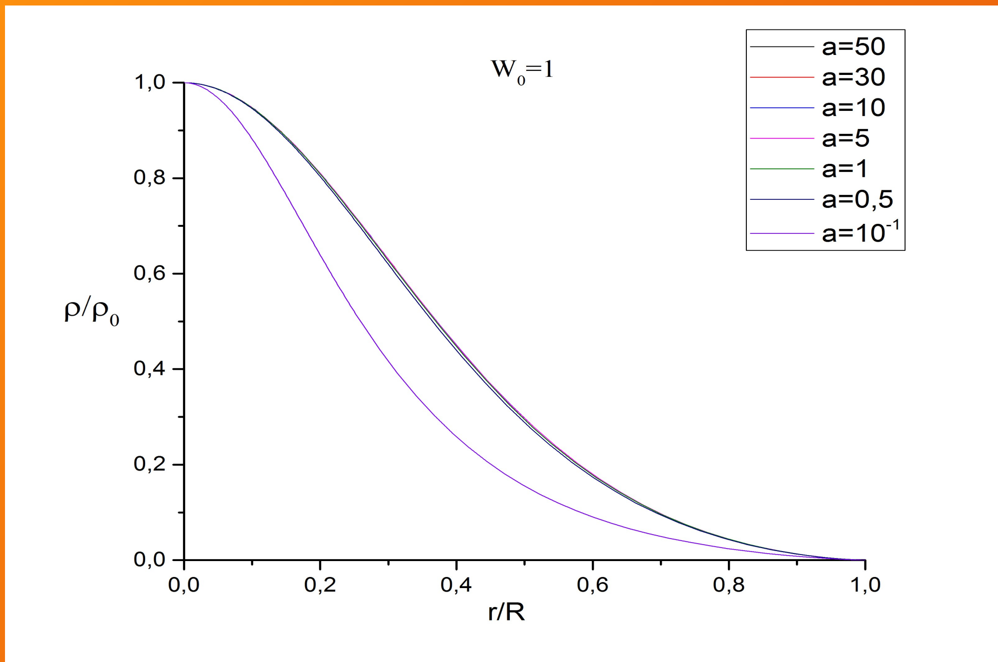
That, using the variables introduced before and the **Poisson's equation**, can be written as

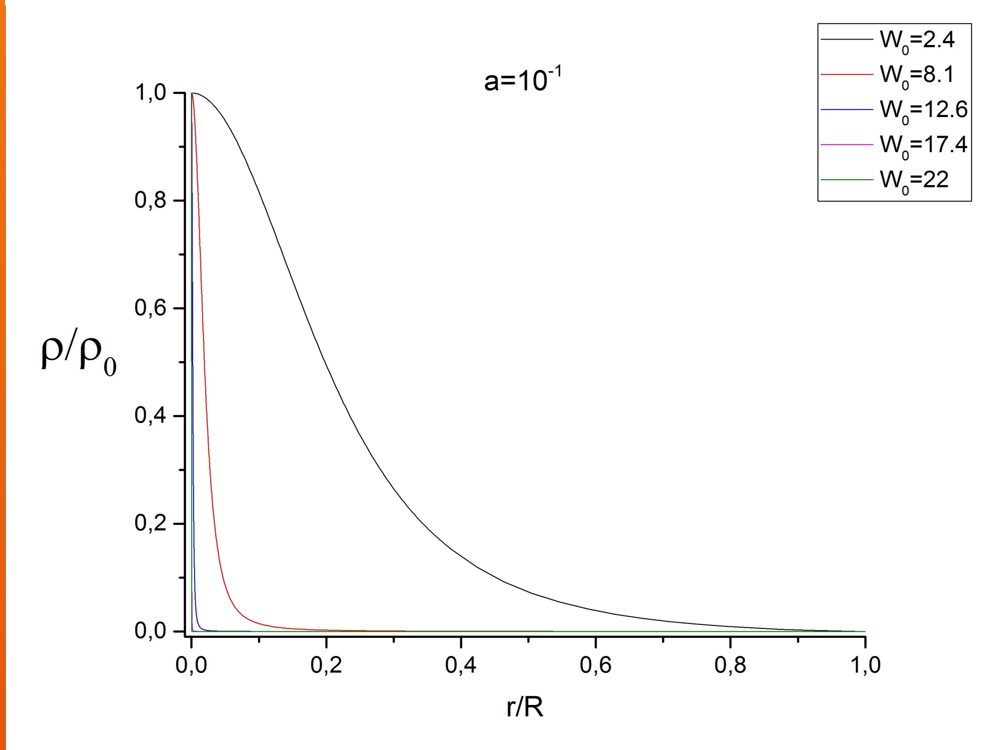
$$\frac{d^2W}{dr^2}+\frac{2}{r}\frac{dW}{dr}=-\frac{8\pi G\rho}{\sigma^2}$$

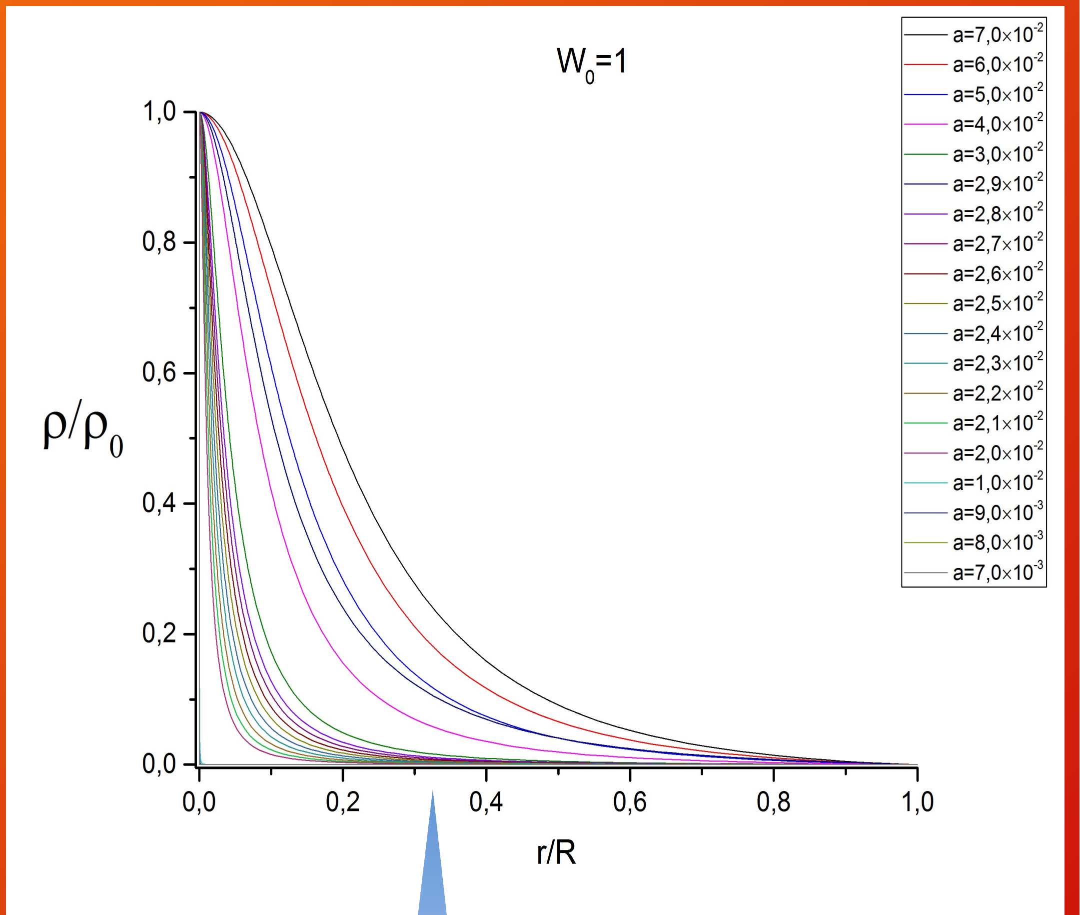
$$W(0)=W_0,\ W'(0)=0$$

Results

Number density



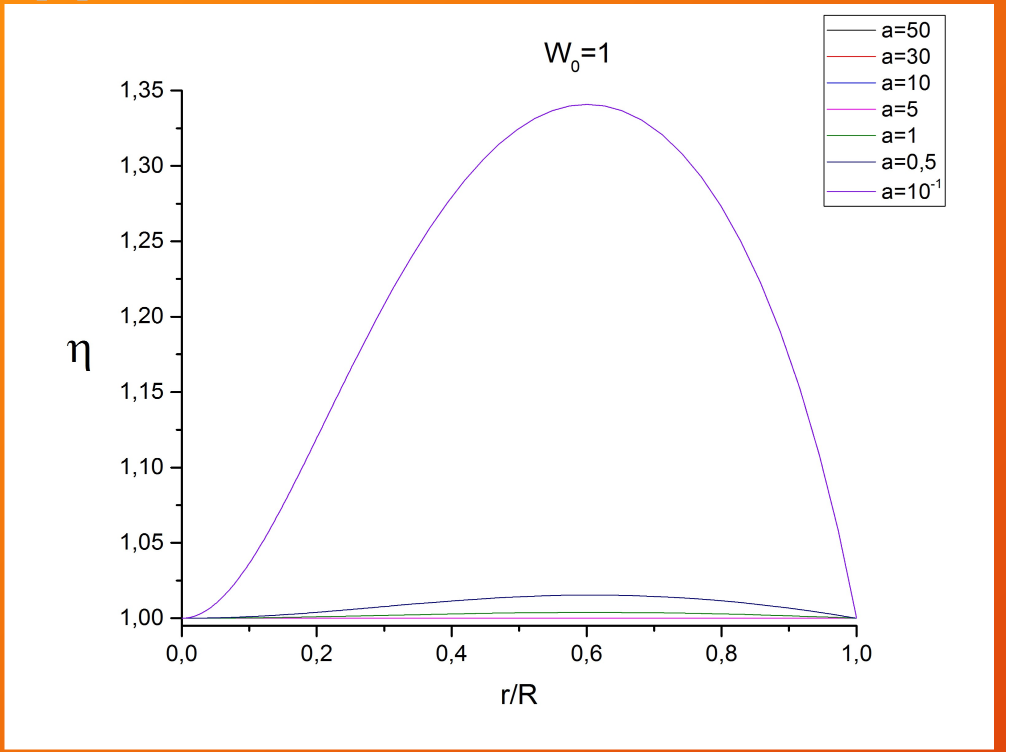


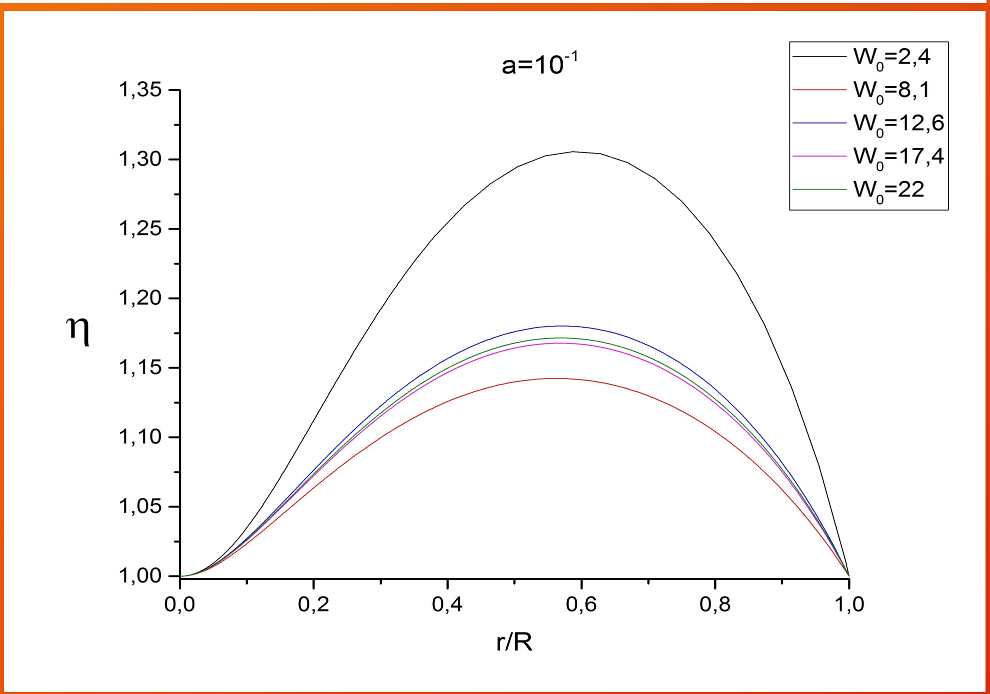


For $7\times10^{-3} \leqslant a \leqslant 7\times10^{-2}$ there is a "transition zone" through high anisotropy regime

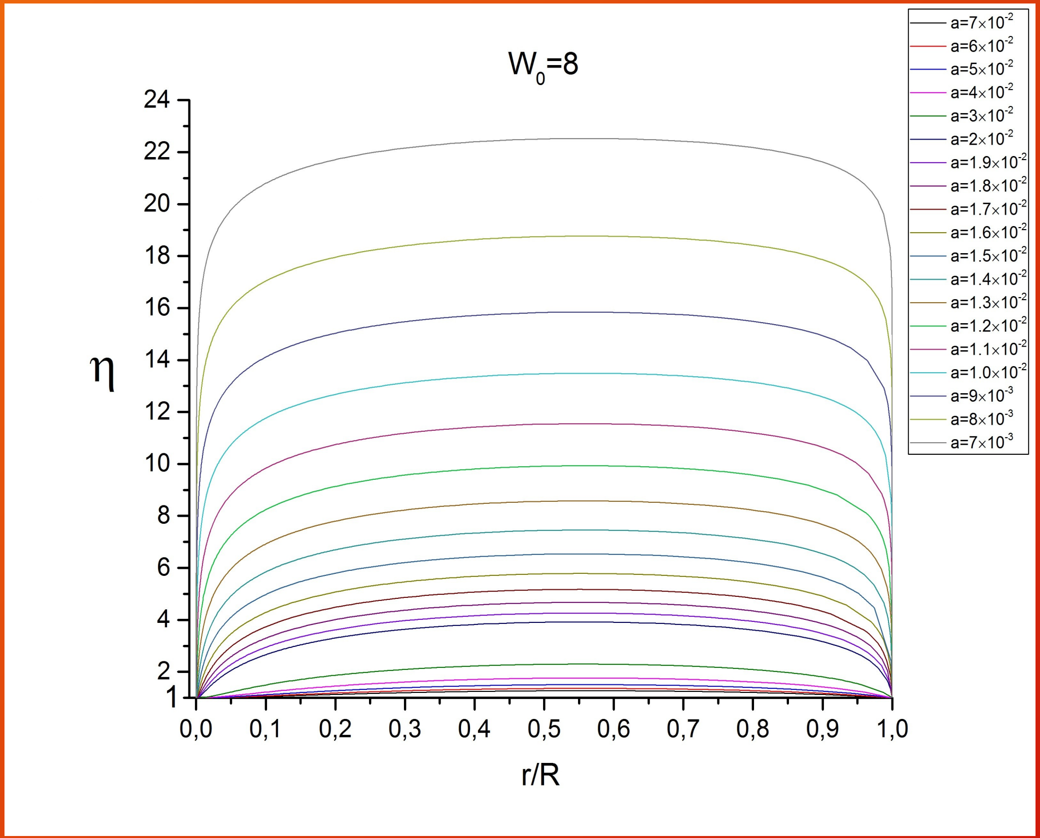
ρ** becomes as a **δ−function

η profiles





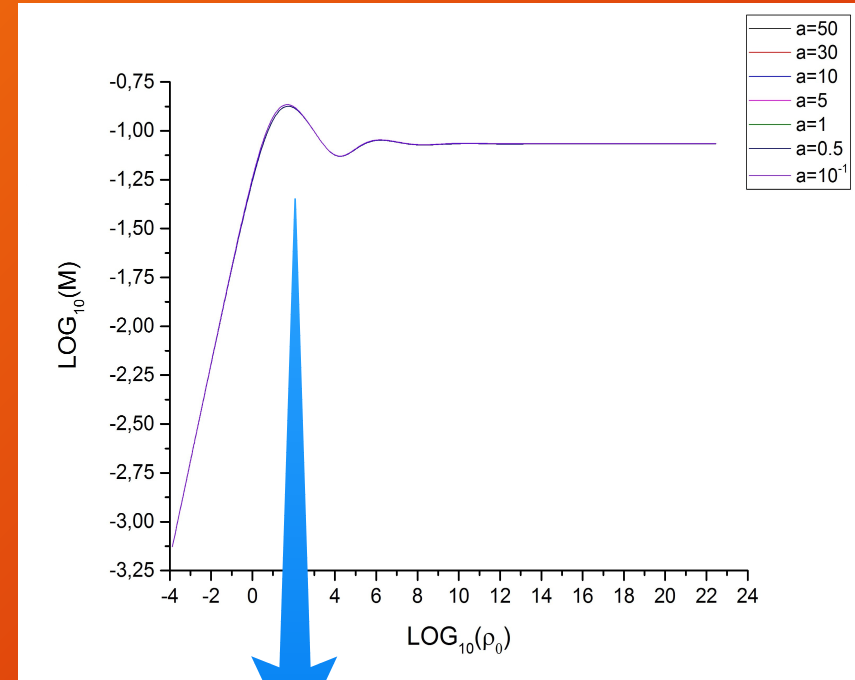
"transition zone"



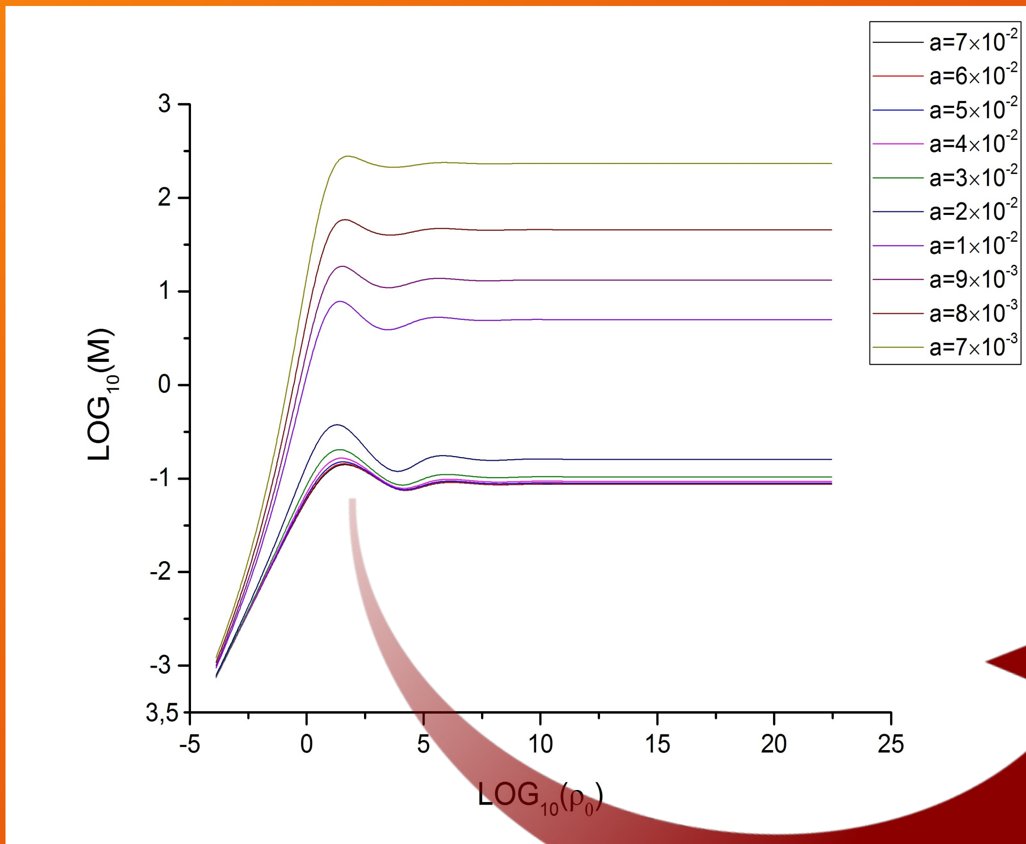
Decreasing *a* involves an increase of η through all the system

Predomincance of radial motion and then more radial orbits

$M(\rho_0)$ profiles



Anisotropy increases

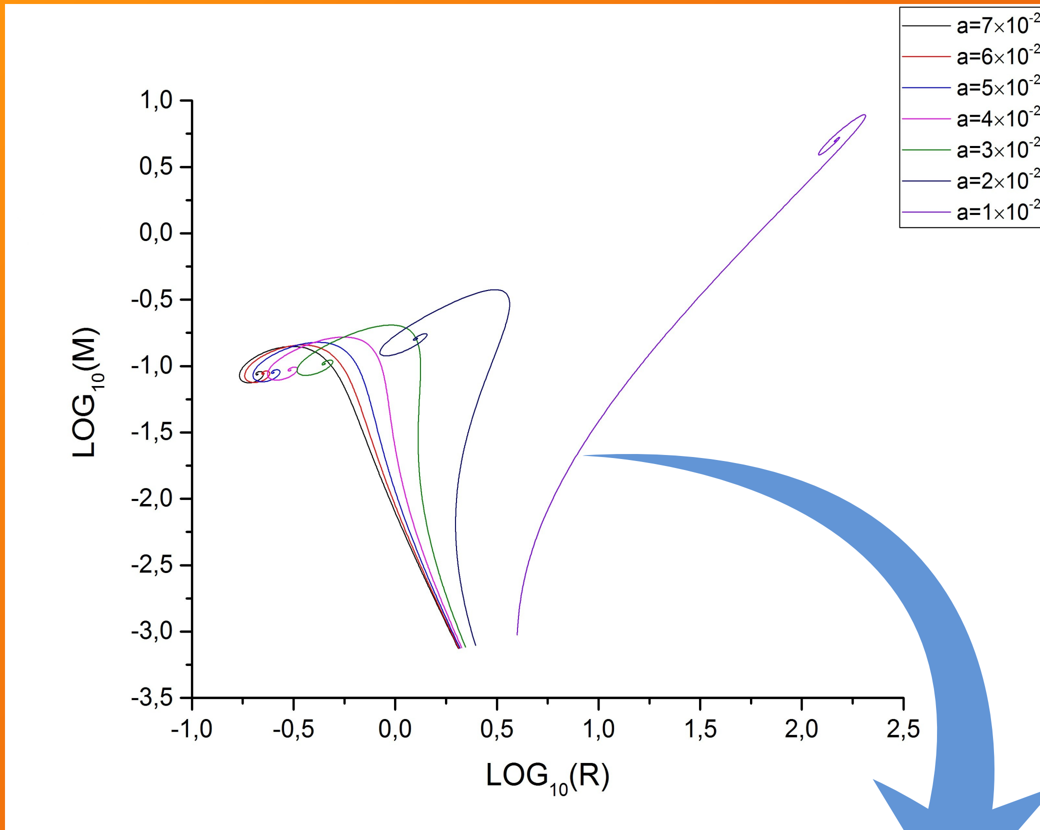


Thermodynamical instabilities

$$C = C(W_0) = cost.$$

For smaller values of *a*, M grow like a **scaling law:**
and appears only a single maximum

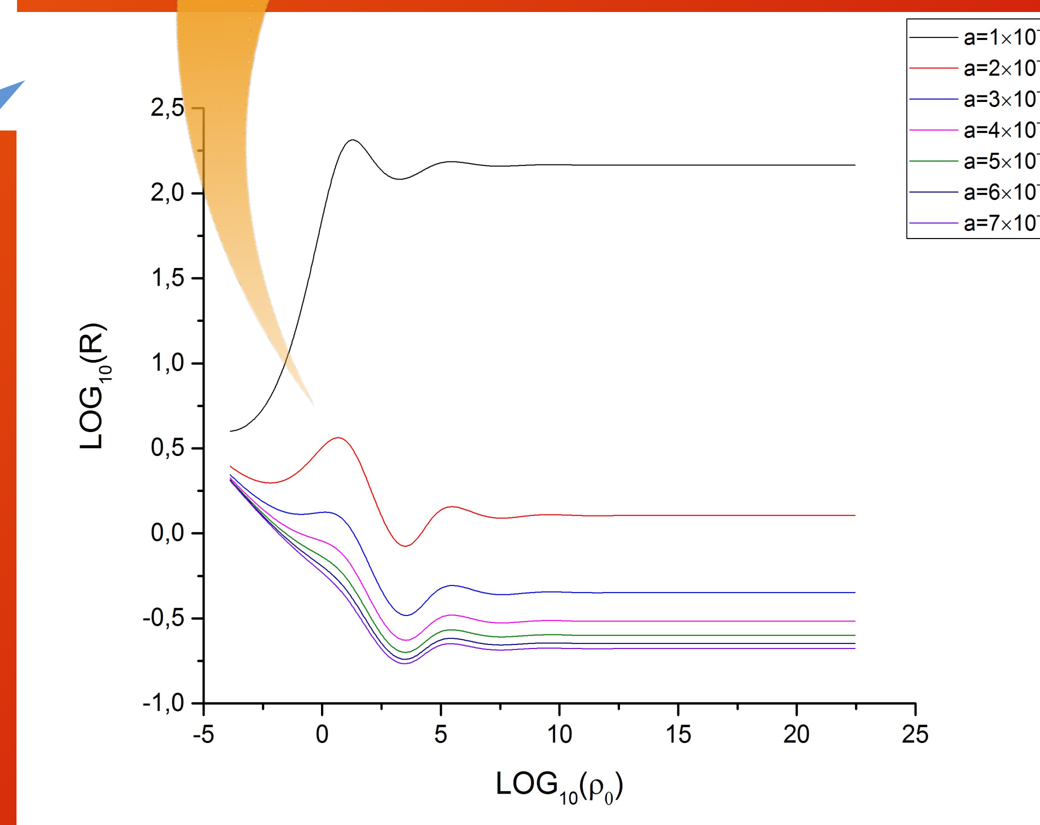
$$M = 10^{\left(\frac{C}{a}\right)}$$



$M(R)$ *proflies*

Thermodynamical instabilities

$R(\rho_0)$ *profiles*



Varying W_0 there are oscillations of M(R) until, in high anisotropy regime, $M \propto R$

In the high anisotropy regime, R follow a scaling law:

$$C = C(W_0) = cost.$$

$$R = 10^{\left(\frac{C}{a}\right)}$$

From the observational data and the obtained values of η , until now, the range of values of interest of the anisotropy parameter “*a*” *is*

$$10^{-2} \leqslant a \leqslant 10^{-1}$$

- “*a*” is a very discriminating parameter
- *For* very small values of “*a*” there is a radicalization of the profiles

THANKS

FOR

THE ATTENTION