One-loop predictions

for the pion VFF

in resonance chiral theory

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[Forthcoming; in collaboration with A. Pich and I. Rosell]

Outline:

- **Framework for the $\pi\pi$-VFF:**

  Resonance chiral theory and $1/N_C$ expansion

- **ImF(s)** at $s\to\infty$

- **Full F(s)** at $s\to\infty$

- **F(s) at low energies**

- **Numerical estimates** and conclusions

$\pi\pi$-VFF,

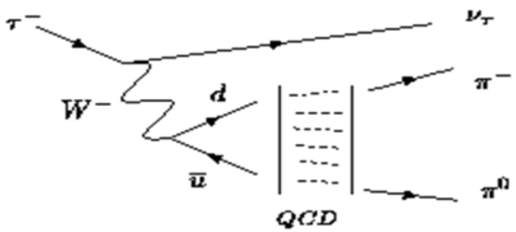
RχT

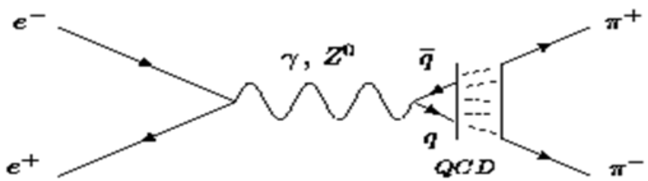
and the $1/N_C$ expansion

The amplitude:
ππ vector form-factor

Hadronic interaction theory ?

Resonance loops ?





$$\langle\, \pi^+\pi^- | \frac{1}{2}\bar{u}\gamma^\mu u - \frac{1}{2}\bar{d}\gamma^\mu d|0\,\rangle \;=\; (p_{\pi^+} - p_{\pi^-})^\mu \;\; F(q^2)$$

$$\text{with } \; q = p_{\pi^+} + p_{\pi^-}$$

- Very good measurements
- Well dominated by the vector

→Test for our hadronic theory

Ingredients of

a chiral theory for resonances (RχT)

[Ecker et al.'89] … [Cirigliano et al.'06] …

- Large N_C → $U(n_f)$ multiplets
- $1/N_C$ supression of meson loops
- Goldstones from SχSB (π, K, η_8, η_1)
- **SRA:** First resonance multiplets (V, A, S, P)
- **Chiral symmetry** invariance
- Just $O(p^2)$ operators ? ←*High energy conditions* [Trnka,SC'09]

  ? ←*Field Redefinitions* [Xiao,SC'07]

No restriction on the number of R fields: $\mathcal{L}_{R\chi T} = \mathcal{L}^{GB} + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$

$$\mathcal{L}^{GB}[\pi] = \frac{F^2}{4}\langle u^\mu u_\mu + \chi_+ \rangle$$

$\mathcal{L}^{GB}$

[Weinberg'79]

[Gasser & Leutwyler'84]

[Gasser & Leutwyler'85]

The only relevant ones at large N_C (tree-level LO)

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu] \rangle$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu} \rangle$$

$$\mathcal{L}_S = c_d \langle S u_\mu u^\mu \rangle$$

$\sum_R \mathcal{L}_R$

[Ecker et al.'89]

[Cirigliano et al.'06]

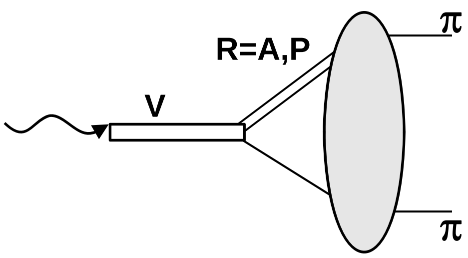
[Pich,Rosell & SC'08]

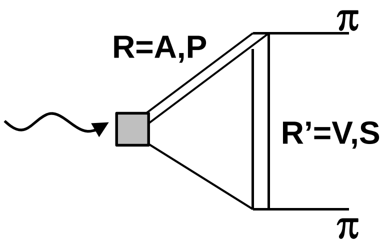
$$\mathcal{L}_{VA} = i\lambda_2^{VA}\langle\, [V^{\mu\nu}, A_{\nu\alpha}]h^\alpha_\mu\,\rangle + i\lambda_3^{VA}\langle\, [\nabla^\mu V_{\mu\nu}, A^{\nu\alpha}]u_\alpha\,\rangle$$
$$+ i\lambda_4^{VA}\langle\, [\nabla_\alpha V_{\mu\nu}, A^{\alpha\nu}]u^\mu\,\rangle + i\lambda_5^{VA}\langle\, [\nabla_\alpha V_{\mu\nu}, A^{\mu\nu}]u^\alpha\,\rangle$$

$$\mathcal{L}_{SA} = \lambda_1^{SA}\langle\, \{\nabla_\mu S, A^{\mu\nu}\}u_\nu\,\rangle$$

$$\mathcal{L}_{PV} = i\lambda_1^{PV}\langle\, [\nabla^\mu P, V_{\mu\nu}]u^\nu\,\rangle$$

$$\mathcal{L}_{SP} = \lambda_1^{SP}\langle\, u_\alpha\{\nabla^\alpha S, P\}\,\rangle$$





$$\sum_{R,R'}\mathcal{L}_{RR'} + \sum_{R,R',R''}\mathcal{L}_{RR'R''} + \dots$$

NEGLECTED

→We will neglect RR' absorptive cuts

We must build the RχT that best mimics QCD

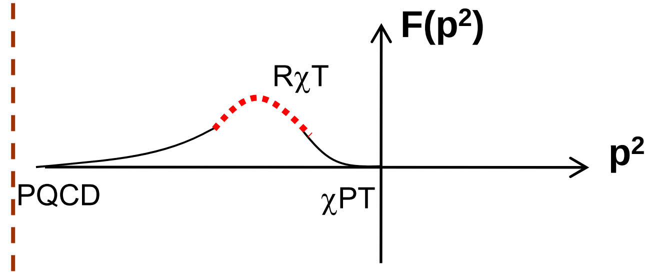
[Weinberg'79]

- **AT LONG-DISTANCE: Chiral symmetry invariance** [Gasser,Leutwyler'84,'85]

  Ensures the right low-energy QCD structure (χPT),

  [Catà,Peris'02]
  [Harada,Yamawaki'03]
  [Rosell,Pich,SC'04 '06 '08] …

  even at the loop level !

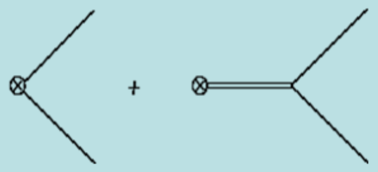


- **AT SHORT-DISTANCE:**

  Demand to the theory the high-energy power behaviour prescribed by QCD

  [Bordsky,Lepage'79]

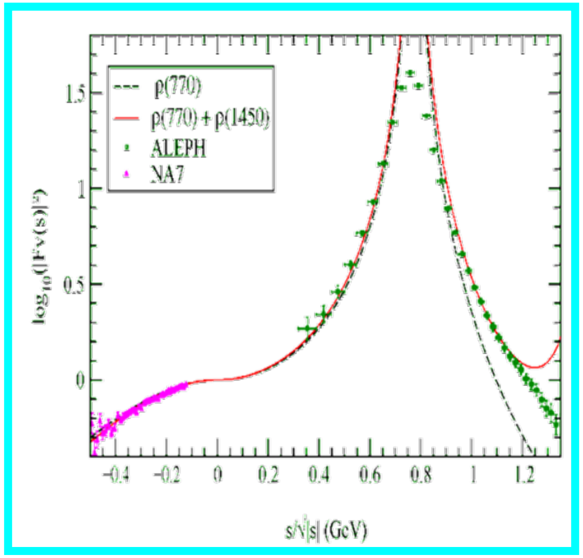
LO in $1/N_C$



QCD

Short Dist.

$$F(q^2) = 1 + \underbrace{\frac{F_V G_V}{F^2}}_{F_V G_V = F^2} \frac{q^2}{M_V^2 - q^2} \quad \xrightarrow{q^2\to\infty} \quad 0$$



•This can be also understood from a Padé app. Point of view:

→ [0/1] Pade Type (M_V fixed) [Masujan,Peris,SC'08]

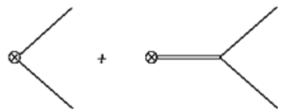
•It leads to the low-energy large-N_C expression $\mathcal{F}(s) = 1 + \frac{2L_9\, s}{F^2} + \mathcal{O}(s^2)$

with the LEC prediction $L_9^{N_C\to\infty} = \frac{F_V G_V}{2M_V^2} = \frac{F^2}{2M_V^2} \simeq 6.8\cdot 10^{-3}$ [Ecker *et al.*'89]

to be compared to the experimental numbers for **$10^3\cdot L_9(M_\rho)$** :

6.9±0.7	[$O(p^4)\chi$PT; Gasser,Leutwyler'85],	**5.93±0.43**	[$O(p^6)\chi$PT; Bijnens,Talavera'02],
7.04±0.23	[τ-RχT; Pich,SC'03],	**7.2**	[largeN_C; Kaiser'05],
6.54±0.15	[$O(p^4)$ τ-SR;Gonzalez-Alonso et al.'09]	**5.50±0.40**	[$O(p^6)$ τ-SR;Gonzalez-Alonso et al.'09]

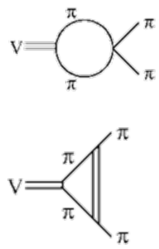
Developing RχT beyond large N_C



$L^{LO}_{R\chi T}$

RχT at LO

[Ecker *et al.*'89], …



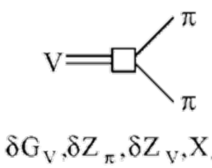
One Loop Diagrams ⟶ NLO Contributions

[Catà,Peris'02]

[Rosell, Pich,SC'04]

[Rosell *et al.*'05]

However, Loops=UV Divergences!!



$L^{NLO}_{R\chi T}$

New NLO pieces (NLO couplings)?

[Rosell,Pich,SC'04,'06]

[Ruíz-Femenía *et al.*'05,'06]

[SC'07]

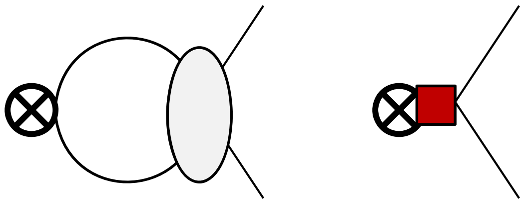
[Xiao,SC'07]

[Trnka,SC'09]

Removable through EoM

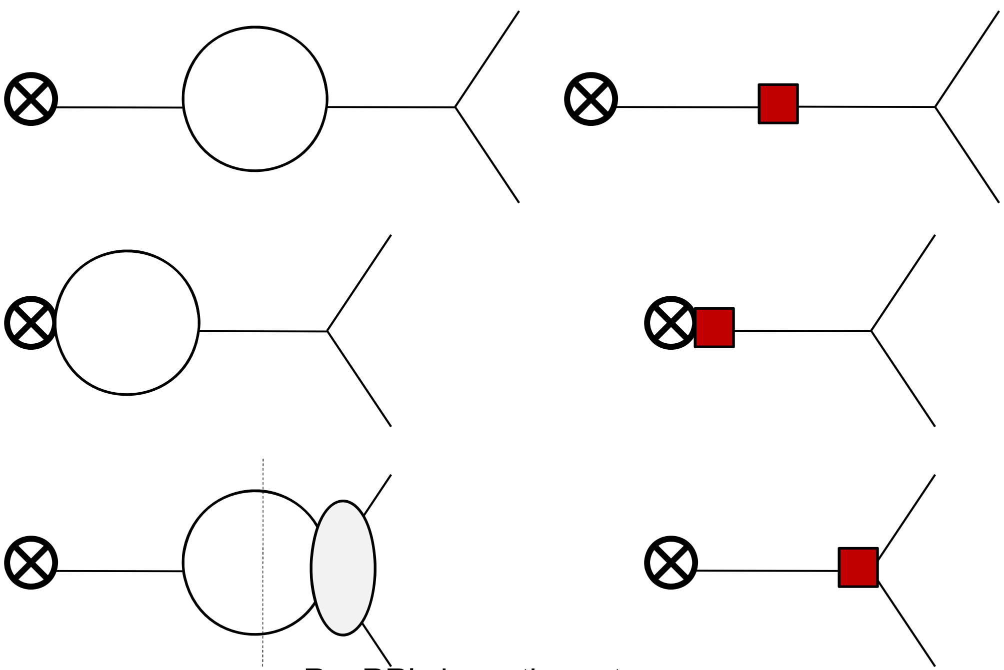
and with proper short-distance

UV divergences → NLO lagrangian



- Only Goldstones counterterms

$$\mathcal{L}_{\rm NLO}^{\rm GB} = -i\tilde{L}_9 \langle f_+^{\mu\nu} u_\mu u_\nu \rangle$$



ππ, Rπ, RR’ absorptive cuts

- Counterterms with resonance fields

$$\mathcal{L}_{\rm NLO}^{V} = X_Z \langle V_{\lambda\nu} \nabla^\lambda \nabla_\rho \nabla^2 V^{\rho\nu} \rangle$$

$$+ X_F \langle V_{\mu\nu} \nabla^2 f_+^{\mu\nu} \rangle$$

$$+ 2i X_G \langle V_{\mu\nu} \nabla^2 [u^\mu, u^\nu] \rangle$$

However,

Redundant operators (Proportional to EoM) [Rosell,Pich,SC'04]

$$\nabla^\mu u_\mu = \frac{i}{2}\chi_- + \ldots$$

$$\nabla^\mu\nabla_\rho \mathbf{V}^{\rho\nu} - \nabla^\nu\nabla_\rho \mathbf{V}^{\rho\mu} = -\mathbf{M_V^2}\mathbf{V}^{\mu\nu} - \frac{\mathbf{F_V}}{\sqrt{2}}\mathbf{f}_+^{\mu\nu} - \frac{\mathbf{iG_V}}{\sqrt{2}}[\mathbf{u}^\mu, \mathbf{u}^\nu] + \ldots$$

→ $\mathcal{L}_V^{NLO}$ operators removable through meson field redefinitions ξ

Instead of the original set of couplings,

the amplitude depends

only on effective combinations

$$\begin{aligned}
&X_{Z,F,G} \xrightarrow{\xi} 0,\\
&\tilde{L}_9 \xrightarrow{\xi} \tilde{L}_9 + (\sqrt{2}X_F G_V + 2\sqrt{2}F_V X_G - X_Z F_V G_V),\\
&F_V \xrightarrow{\xi} F_V + \left(2X_Z F_V M_V^2 - 2\sqrt{2}X_F M_V^2\right),\\
&G_V \xrightarrow{\xi} G_V + \left(2X_Z G_V M_V^2 - 4\sqrt{2}X_G M_V^2\right),\\
&M_V^2 \xrightarrow{\xi} M_V^2 + 2X_Z M_V^4.
\end{aligned}$$

•VFF up to NLO in $1/N_C$

$$\mathcal{F}(s) \;=\; 1 + \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s}$$ ← LO tree-level

$$+\frac{2\widetilde{L}_9 s}{F^2} - 2X_Z\frac{F_V G_V}{F^2}\frac{s^3}{(M_V^2-s)^2} - 4\sqrt{2}\frac{F_V X_G}{F^2}\frac{s^2}{M_V^2-s} - 2\sqrt{2}\frac{X_F G_V}{F^2}\frac{s^2}{M_V^2-s}$$ ← NLO tree-level

$$+\mathcal{F}(s)_{1PI} + \frac{F_V G_V}{F^2}\frac{s\,\Sigma(s)}{(M_V^2-s)^2} + \frac{F_V \Gamma(s)}{F^2}\frac{s}{M_V^2-s} + \frac{\Phi(s) G_V}{F^2}\frac{s}{M_V^2-s}$$ ← 1-loop

[F_V, G_V , X_Z , X_F … renormalize every single Vertex-Function]

•VFF up to NLO in $1/N_C$ (AFTER EoM SIMPLIFICATION)

$$\mathcal{F}(s) \;=\; 1 + \frac{F_V^{\rm eff} G_V^{\rm eff}}{F^2}\frac{s}{M_V^2 - s} + \frac{2\widetilde{L}_9^{\rm eff} s}{F^2} + \mathcal{F}(s)^{1-\ell oop}$$

MEANING: ***$F_V G_V$, M_V^2 and L_9 are able to make F(s) finite***

•From now on, we will always refer to the simplified lagrangian → "eff" superscript assumed

Step 1)

VFF spectral function:

ImF(s) at $s \to \infty$

•The full VFF must the behaviour F(s)→0 when s→ ∞

•Similarly, for its spectral function ImF(s)→0 when s→ ∞

•We will demand this behaviour for every two-meson cut

$\mathrm{Im}F(s)|_{m1,m2} \to 0$ when s→ ∞

•The spectral function shows the generic form:

$$\mathbf{Im}\mathcal{F}(\mathbf{s}) \;=\; \mathbf{s}\left(\alpha_{\mathbf{1}}^{(\mathbf{p})} + \alpha_{\mathbf{1}}^{(\ell)}\ \ln\frac{-\mathbf{s}}{\mathbf{M^2}}\right) \;+\; \left(\alpha_{\mathbf{0}}^{(\mathbf{p})} + \alpha_{\mathbf{0}}^{(\ell)}\ \ln\frac{-\mathbf{s}}{\mathbf{M^2}}\right) \;+\; \ldots$$

which requires the constraints $\boxed{\alpha_{\mathbf{1}}^{(\mathbf{p})} = \alpha_{\mathbf{1}}^{(\ell)} = \alpha_{\mathbf{0}}^{(\mathbf{p})} = \alpha_{\mathbf{0}}^{(\ell)} = \mathbf{0}}$

[Notice that at large Nc, $\alpha_k^{(p)}=\alpha_k^{(\ell)}=0$ trivially]

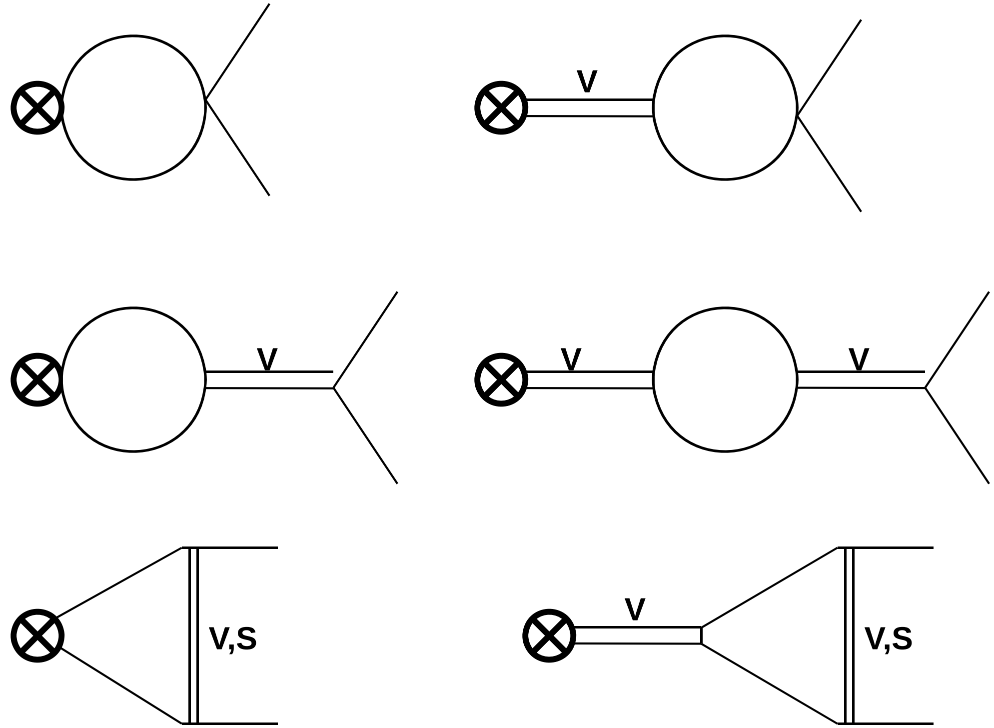
$\pi\pi$ cut:

$$\mathbf{F_V G_V = F^2}$$ [Ecker et al.'89]

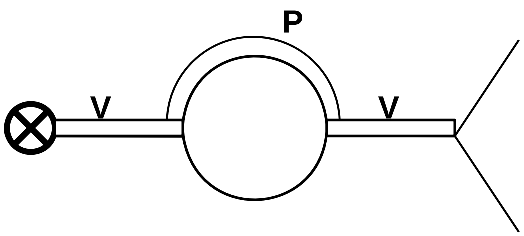
$$\mathbf{3G_V^2 + 2c_d^2 = F^2}$$ [Guo,Zheng,SC'07]

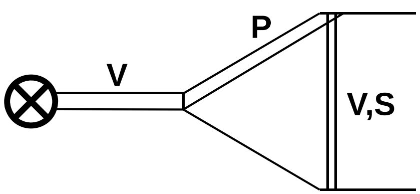
$$\left(\mathbf{0 \leq c_d^2 \leq F^2/2}, \quad \mathbf{0 \leq G_V^2 \leq F^2/3} \right)$$

(Everything fixed in terms of M_V,M_S,G_V)



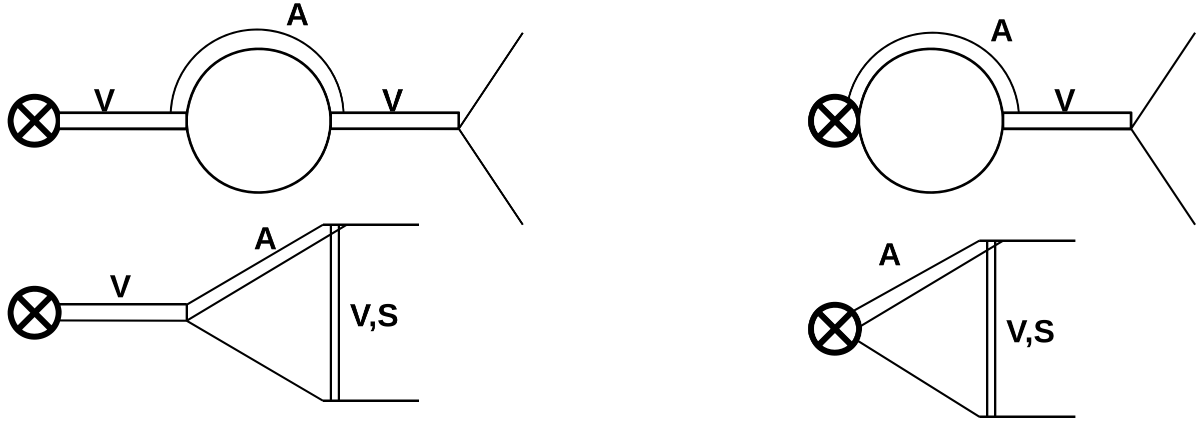
Pπ cut:





$$\lambda_1^{PV} = 0 \quad \rightarrow \quad F(s)|_{P\pi} = 0 \ \text{ trivially}$$

$A\pi$ cut:



Complicate system → Various solutions (6)

➔(1st) Trivial solution: $\lambda_3^{VA} - 2\lambda_2^{VA} = 0$

$$2\lambda_2^{VA} - 2\lambda_3^{VA} + \lambda_4^{VA} + 2\lambda_5^{VA} = 0$$

$$\lambda_1^{SA} = 0$$

$$\Rightarrow \quad \mathrm{Im}\mathcal{F}(s)|_{A\pi} = 0$$

➔ (2nd) $A\pi$-VFF constraint solution

[Pich, Rosell,SC'08]

$$\lambda_3^{VA} - 2\lambda_2^{VA} = 0$$

$$2\lambda_2^{VA} - 2\lambda_3^{VA} + \lambda_4^{VA} + 2\lambda_5^{VA} = F_A/F_V$$

$$\lambda_1^{SA} = -\frac{F_A G_V \left(M_A^2 - 4M_V^2\right)}{3\sqrt{2}M_A^2 c_d F_V}$$

(All λ^{VA} comb. fixed)

$$\Rightarrow \quad \mathrm{Im}\mathcal{F}(s)|_{A\pi} \neq 0$$

Step 2)

Full VFF asymptotic behaviour:

$F(s)$ at $s\to\infty$

•After constraining ImF(s), the VFF has the structure:

$$\mathcal{F}(s) \;=\; 1 + \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s} + \frac{2\widetilde{L}_9 s}{F^2} \;+\; \mathcal{F}(s)^{1-\ell oop}$$

[Unique decomposition from dispersion relations] [Pich,Rosell,SC'08]

with the 1-loop structure $\mathcal{F}(s)^{1-\ell oop} = \overline{\mathcal{F}}(s)^{1-\ell oop} + \frac{2s}{F^2}\hat{\delta}_1 + \hat{\delta}_0 \frac{s}{M_V^2 - s} + \hat{\delta}_{-1}\frac{s}{(M_V^2 - s)^2}$

$$\mathcal{F}(s) \;=\; 1 + \left(\frac{F_V G_V}{F^2} + \hat{\delta}_0\right)\frac{s}{M_V^2 - s} + \frac{2s}{F^2}\left(\widetilde{L}_9 + \hat{\delta}_1\right) \;+\; \overline{\mathcal{F}}(s)^{1-\ell oop}$$

[logs and logs² part]

•At high-energies, this results $\overline{\mathcal{F}}(s)^{1-\ell oop} \stackrel{s\to\infty}{=} \delta_0 + \mathcal{O}\left(s^{-1}\right)$

$$\mathcal{F}(s) \;=\; \frac{2s}{F^2}\left(\widetilde{L}_9 + \hat{\delta}_1\right) + \left(1 - \frac{F_V G_V}{F^2} - \hat{\delta}_0 + \delta_0\right) \;+\; \mathcal{O}\left(\frac{1}{s}\right)$$

➔This leads to the the NLO constraints:

$$\widetilde{L}_9 + \hat{\delta}_1 = 0$$

$$\frac{F_V G_V}{F^2} + \hat{\delta}_0 = 1 + \delta_0$$

[Not really physical here; renorm.-scheme choice]

to be compared with their large-N_C values $\widetilde{L}_9 = 0\,, \quad \frac{F_V G_V}{F^2} = 1$

• $\pi\pi$ contribution:

After using the high-energy $\mathrm{ImF}(s)|_{\pi\pi}$ constraints $\rightarrow$ $\delta_0^{\pi\pi} \approx 0.23$

$$\rightarrow \frac{F^2}{2M_V^2}\delta_0^{\pi\pi} \approx 1.5 \cdot 10^{-3}$$

• $P\pi$ contribution:

From the $\mathrm{ImF}(s)|_{P\pi}$ constraints $\rightarrow$ $\delta_0^{P\pi} = 0$

• $A\pi$ contribution:

After using the high-energy $\mathrm{ImF}(s)|_{A\pi}$ constraints

$\rightarrow$ Complicate expression, but $\rightarrow$ $\delta_0^{A\pi} \approx 0.14$

$$\rightarrow \frac{F^2}{2M_V^2}\delta_0^{A\pi} \approx 1.0 \cdot 10^{-3}$$

Low-energy expansion

Low-energy predictions

•At s→0, the RChT expression shows the structure,

$$\mathcal{F}(s) \stackrel{s\to 0}{=} \frac{2s}{F^2}\,\xi_{L_9} + \frac{s}{F^2}\,\frac{G_9}{16\pi^2}\left(\frac{5}{3} - \ln\frac{-s}{\mu^2}\right) + \mathcal{O}(s^2)$$

$$\mathcal{F}(s) \;=\; 1 \;+\; \frac{2s}{F^2}\left[(\widetilde{L}_9 + \hat{\delta}_1) + \frac{F^2}{2M_V^2}\left(\frac{F_V G_V}{F^2} + \hat{\delta}_0\right) + \xi_{L_9}\right] + \frac{s}{F^2}\,\frac{G_9}{16\pi^2}\left(\frac{5}{3} - \ln\frac{-s}{\mu^2}\right) \;+\; \mathcal{O}(s^2)$$

with the log coefficient $G_9 = \Gamma_9 = \frac{1}{4}$, matching ChPT

Chiral symmetry

•This has the same form as ChPT ,

$$\mathcal{F}(s) \;=\; 1 \;+\; \frac{2L_9(\mu_\chi)\, s}{F^2} + \frac{s}{F^2}\,\frac{\Gamma_9}{16\pi^2}\left(\frac{5}{3} - \ln\frac{-s}{\mu_\chi^2}\right) \;+\; \mathcal{O}(s^2)$$

•This leads to the prediction for the ChPT LECs,

$$\mathbf{L_9}(\mu_\chi) \;=\; \frac{\mathbf{F^2}}{\mathbf{2M_V^2}}\left(\frac{\mathbf{F_V G_V}}{\mathbf{F^2}} + \hat{\hat{\delta}}_{\mathbf{0}}\right) \;+\; (\widetilde{\mathbf{L}}_{\mathbf{9}} + \hat{\hat{\delta}}_{\mathbf{1}}) \;+\; \xi_{\mathbf{L_9}} \;+\; \frac{\mathbf{\Gamma_9}}{\mathbf{32\pi^2}} \ln\frac{\mu^2}{\mu_\chi^2}$$

For instance, with only the ππ loops considered,

$$\xi_{\mathbf{L_9}} = \frac{\mathbf{c_d^2}\log\left(\frac{\mathbf{M_S^2}}{\mu^2}\right)}{\mathbf{64\pi^2F^2}} - \frac{\mathbf{11c_d^2}}{\mathbf{384\pi^2F^2}} + \frac{\mathbf{F_VG_V^3}\log\left(\frac{\mathbf{M_V^2}}{\mu^2}\right)}{\mathbf{64\pi^2F^4}} - \frac{\mathbf{5F_VG_V^3}}{\mathbf{192\pi^2F^4}} + \frac{\mathbf{G_V^2}\log\left(\frac{\mathbf{M_V^2}}{\mu^2}\right)}{\mathbf{128\pi^2F^2}} + \frac{\mathbf{25G_V^2}}{\mathbf{768\pi^2F^2}}$$

•Using the high-energy constraints up to NLO→ $\mathbf{L_9}(\mu) \;=\; \frac{\mathbf{F^2}}{\mathbf{2M_V^2}}\left(\mathbf{1} \;+\; \delta_{\mathbf{0}}\right) \;+\; \xi_{\mathbf{L_9}}$

<u>**TO NOTICE:**</u> Exact recovery of the μ running dependence

•$\pi\pi$ contribution:

After using the high-energy $\mathrm{Im}F(s)|_{\pi\pi}$ constraints $\rightarrow \xi_{L9}^{\pi\pi} \approx -1.6 \cdot 10^{-3}$

•$P\pi$ contribution:

From the $\mathrm{Im}F(s)|_{P\pi}$ constraints $\rightarrow \xi_{L9}^{P\pi} = 0$

•$A\pi$ contribution:

After using the high-energy $\mathrm{Im}F(s)|_{A\pi}$ constraints

$\rightarrow$ Complicate expression, but $\xi_{L9}^{A\pi} \approx -0.1 \cdot 10^{-3}$

Numerical determinations

(PRELIMINARY)

•**Inputs:** M_V=0.76⇔0.78 GeV, M_S=0.98⇔1.2 GeV, F=89 MeV, G_V = F/√3⇔40 MeV,

[Aπ channel→] M_A= 1.23⇔ 1.00 GeV, F_A = 123⇔89 MeV

•At LO in $1/N_C$: $$\mathbf{L_9^{N_C\to\infty}} = \mathbf{6.7\cdot 10^{-3}}$$

•NLO with ππ: $$\mathbf{L_9(\mu_0)} = \mathbf{(6.6\pm 0.4)\cdot 10^{-3}}, \qquad \mathbf{with}\,\mu_0 = \mathbf{0.77\,GeV}$$

(=NLO with ππ + Pπ)

•NLO with ππ + Pπ + Aπ : $$\mathbf{L_9(\mu_0)} = \mathbf{(7.5\pm 0.5)\cdot 10^{-3}}, \qquad \mathbf{with}\,\mu_0 = \mathbf{0.77\,GeV}$$

PRELIMINARY

Conclusions

and

PROSPECTS

- Clear theoretical organization of the NLO computation:

  - Loop contribution
  - Relevant physical couplings

- Succesful test of the $1/N_C$ expansion in RChT

- Very slight problem with the $A\pi$ channel ("high" range of L_9^{exp} values)

- **Next step:**

  - Detailed uncertainty estimate
  - Extraction of the VFF $O(p^6)$ LECs
  - Analysis of experimental VFF data

$$\frac{\mathcal{F}(t)}{t} = \frac{D(t)}{\left(M_V^2 - t\right)^2},$$

$$\frac{\mathcal{F}(s)}{s} = \frac{1}{2\pi i}\oint \mathrm{d}t\, \frac{\mathcal{F}(t)}{t\,(t-s)}.$$

$$\frac{1}{s}\,\mathcal{F}(s) = \frac{1}{s} + \sum_{m_1,m_2} \frac{1}{s}\,\overline{\mathcal{F}}(s)|_{m_1,m_2} - \frac{\mathrm{Re}D'(M_V^2)}{M_V^2 - s} + \frac{\mathrm{Re}D(M_V^2)}{\left(M_V^2 - s\right)^2},$$

$$\overline{\mathcal{F}}(s)|_{m_1,m_2} = \lim_{\epsilon\to 0}\left[\frac{s}{\pi}\int_0^{M_V^2-\epsilon}\mathrm{d}t\,\frac{\mathrm{Im}\mathcal{F}(t)|_{m_1,m_2}}{t\,(t\,-\,s)} + \frac{s}{\pi}\int_{M_V^2+\epsilon}^{\infty}\mathrm{d}t\,\frac{\mathrm{Im}\mathcal{F}(t)|_{m_1,m_2}}{t\,(t\,-\,s)} \right.$$
$$\left. -\frac{2s}{\pi\epsilon}\lim_{t\to M_V^2}\left\{(M_V^2 - t)^2\,\frac{\mathrm{Im}\mathcal{F}(t)|_{m_1,m_2}}{t\,(t\,-\,s)}\right\}\right].$$

$$\mathcal{F}(t) = 1 + \sum_{m_1,m_2}\overline{\mathcal{F}}(t)|_{m_1,m_2} - \frac{s\,\mathrm{Re}D'(M_V^2)}{M_V^2 - t}$$
$$= 1 + \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s} + \sum_{m_1,m_2}\mathcal{F}(t)|_{m_1,m_2},$$

•What about extra tadpole terms?

They all are real rational functions of the form of

the L_9 (local) and $F_V G_V$ (pole) terms, i.e.,

$$\mathcal{F}(s)^{\text{tad.}} = \frac{2\,s}{F^2}\,\delta_1^{\text{tad.}} - \frac{\delta_0^{\text{tad.}}\; s}{M_V^2 - s}$$

→HOWEVER,

These couplings (or their combination with possible tadpole)

will be fully fixed later

through high-energy constraints

•Likewise, we will consider the on-shell mass scheme ➔ M_V=770 MeV

• **Inputs:** M_V=0.77 GeV , M_S=0.98 GeV , M_A= 0.95 ⇔ 1.3 GeV

F=89 MeV , G_V = 45 ⇔ F/√3

• At LO in 1/N_C: $$\mathbf{L_9^{N_C\to\infty}} = \mathbf{6.7\cdot 10^{-3}}$$

• NLO with ππ: $$\mathbf{L_9(\mu_0)} = \mathbf{6.6\cdot 10^{-3}}, \qquad \mathbf{with}\, \mu_0 = \mathbf{0.77\, GeV}$$

(=NLO with ππ + Pπ)

• NLO with ππ + Pπ + Aπ : $$\mathbf{L_9(\mu_0)} = \mathbf{7.5\cdot 10^{-3}}, \qquad \mathbf{with}\, \mu_0 = \mathbf{0.77\, GeV}$$

•$\pi\pi$ contribution:

After using the high-energy $\mathrm{Im}F(s)|_{\pi\pi}$ constraints → $\delta_1^{\pi\pi} = \delta_0^{\pi\pi} = 0$

•$P\pi$ contribution:

Similarly, from the $\mathrm{Im}F(s)|_{P\pi}$ constraints → $\delta_1^{P\pi} = \delta_0^{P\pi} = 0$

•$A\pi$ contribution:

After using the high-energy $\mathrm{Im}F(s)|_{A\pi}$ constraints

→ Complicate expression, but $\delta_1^{A\pi}$, $\delta_0^{\pi\pi} \neq 0$

Vector Form Factor to Aπ (Figure D.2)

$$\langle\, A^0_{I=1}(p_A,\varepsilon)\pi^-(p_\pi)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \frac{i\sqrt{2}}{M_A}\Big\{(q\varepsilon^*\, p_A^\mu \;-\; qp_A\,\varepsilon^{*\mu})\,\mathcal{F}^v_{A\pi}(q^2) + (q\varepsilon^*\, p_\pi^\mu \;-\; qp_\pi\varepsilon^{*\mu})\,\mathcal{G}^v_{A\pi}(q^2)\Big\}\,,$$

$$\mathcal{F}^v_{A\pi}(q^2) \;=\; \frac{F_A}{F} + \frac{F_V}{F}\frac{M_A^2-q^2}{M_V^2-q^2}\Big[-2\lambda_2^{VA}+2\lambda_3^{VA}-\lambda_4^{VA}-2\lambda_5^{VA}\Big]\,,$$

$$\mathcal{G}^v_{A\pi}(q^2) \;=\; \frac{2F_V}{F}\frac{M_A^2}{M_V^2-q^2}\Big[-2\lambda_2^{VA}+\lambda_3^{VA}\Big]\,,$$