Holographic hadron phenomenology

A bottom-up approach

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Outline

- QCD @ Strong Coupling
- AdS/CFT and AdS/QCD
- Bottom-up approach: Soft Wall model(s?)
- Implementation of a global symmetry: Chiral Symmetry Breaking and Scalar Mesons
- Calculation of correlation functions: Scalar Glueballs
- Static QQbar Potential
- Finite Temperature Effects
- Pros, Cons
- Conclusions and Perspectives

QCD @ Strong Coupling

Low energy → perturbation theory fails → how to evaluate non-perturbative observables (masses, decay constants, correlation functions....)?

QCD

Lattice simulations
Numerical evaluations of the partition function with supercomputers

Schwinger-Dyson Equations
Solve for the correlation function nonperturbatively

Effective Field Theories
Selection of the degrees of freedom to be described

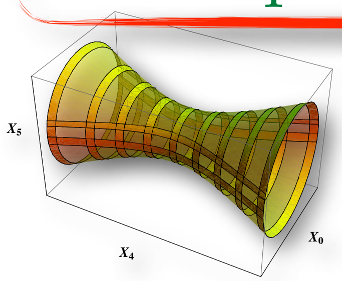
Holographic QCD

can we map the non-perturbative regime of QCD into something (perturbative) else?

AdS/CFT and AdS/QCD

AdS/CFT

IIB superstring theory



$AdS_5 \times S^5$

SUGRA limit

$$\begin{cases} g_s \to 0 \\ g_s N \to \infty \end{cases}$$

$$g_s = g_{\rm YM}^2$$
$$R^4 = 4\pi g_s N \alpha'^2$$

SU(N) $\mathcal{N} = 4$ SYM

$\mathcal{M}_4$ (boundary of AdS_5)

Large N + NP regime

$$\begin{cases} N \to \infty \\ \lambda = g_{\rm YM} N \to \infty \end{cases}$$

field $\phi(x, z)$

$m_5^2 = (\Delta - p)(\Delta + p - 4)$

operator $\mathcal{O}(x)$

dimension Δ

$$Z_S\left[\phi_0(x)\right] = \left\langle e^{\int_{\partial AdS_5} \phi_0 \mathcal{O}} \right\rangle_{\rm CFT}$$

AdS/QCD

Semiclassical Field Theory

AdS_5 - like space

??

QCD (or at least NP regime)

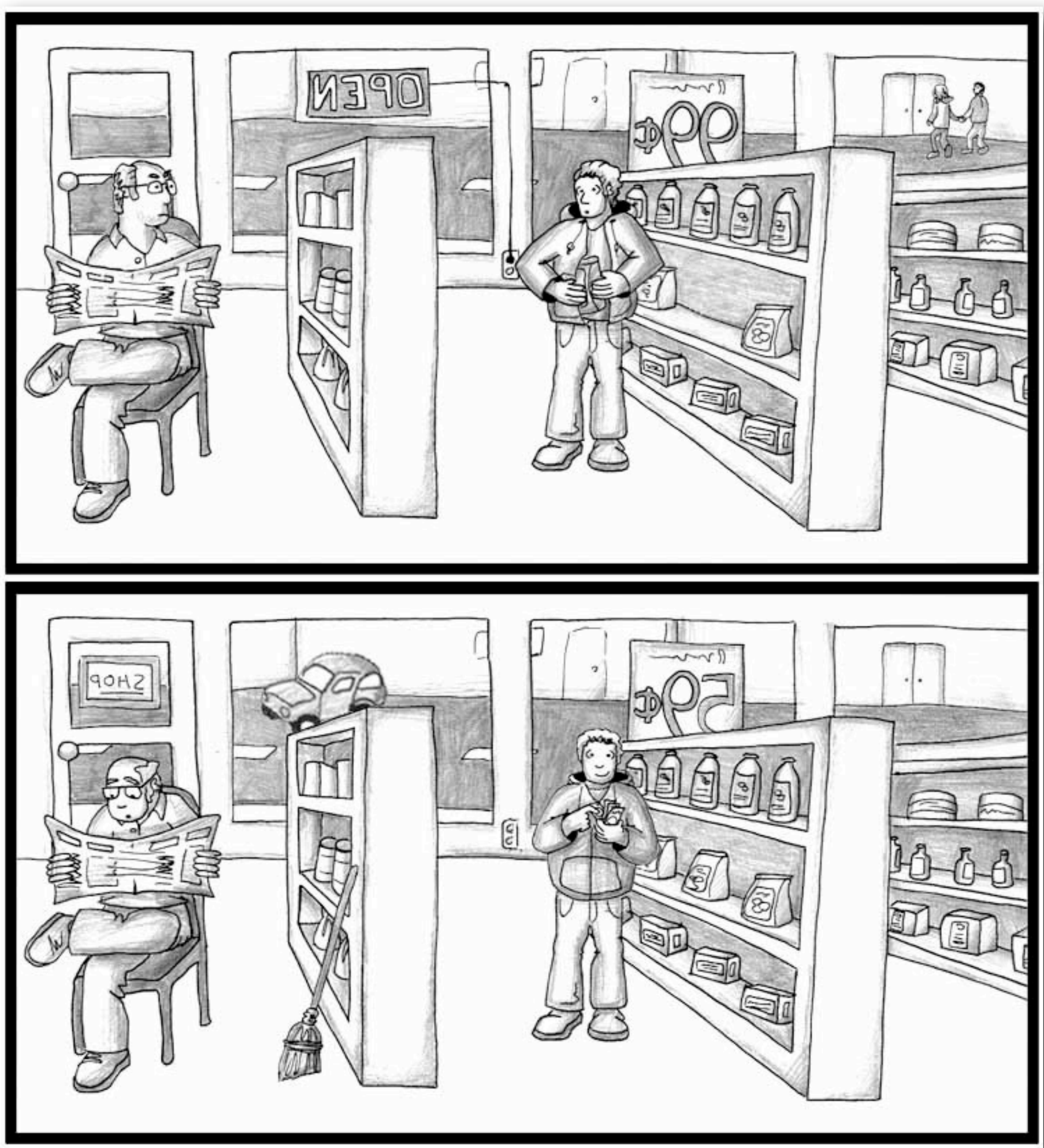
$\mathcal{M}_4$ (boundary of AdS_5)

AdS/QCD @ Work

study the NP regime of QCD through semiclassical approach

Find the differences!

Medium



Easy

$\mathcal{N} = 4$ SYM

- adjoint fermions
- conformally invariant
- maximally supersymmetric
- ...

QCD

- fundamental fermions
- not conformal (....)
- not supersymmetric
- ...

AdS/QCD: how?

about conformal symmetry in QCD:

asymptotic freedom → conformal symmetry in the UV

Possible IR fixed point → conformal symmetry in the IR

What do we need?

- avoid SUSY
- conformal symmetry breaking (with asymptotic UV restoration)

What to do?

Way A (top-down)

string theory

break conf. inv. ↓ break SUSY

QCD-like theory

Way B (bottom-up)

construct a model (no strings!)

NO SUSY

asymptotic conformal inv.

↑

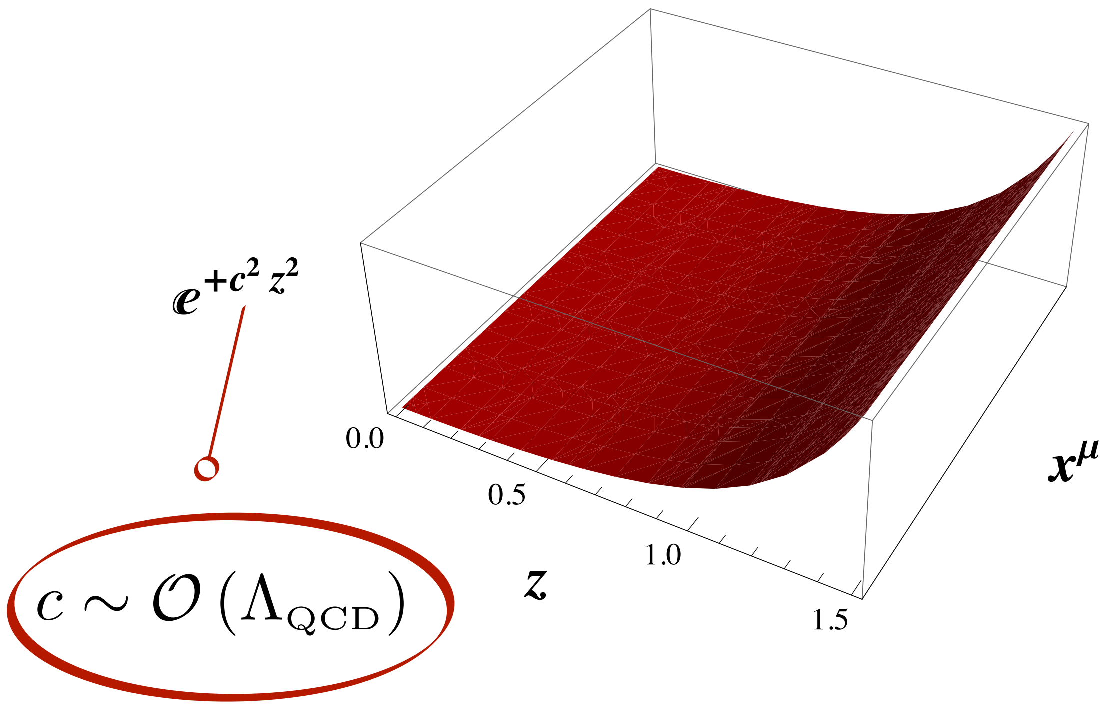
QCD

bottom-up AdS/QCD: Soft-Wall model(s)

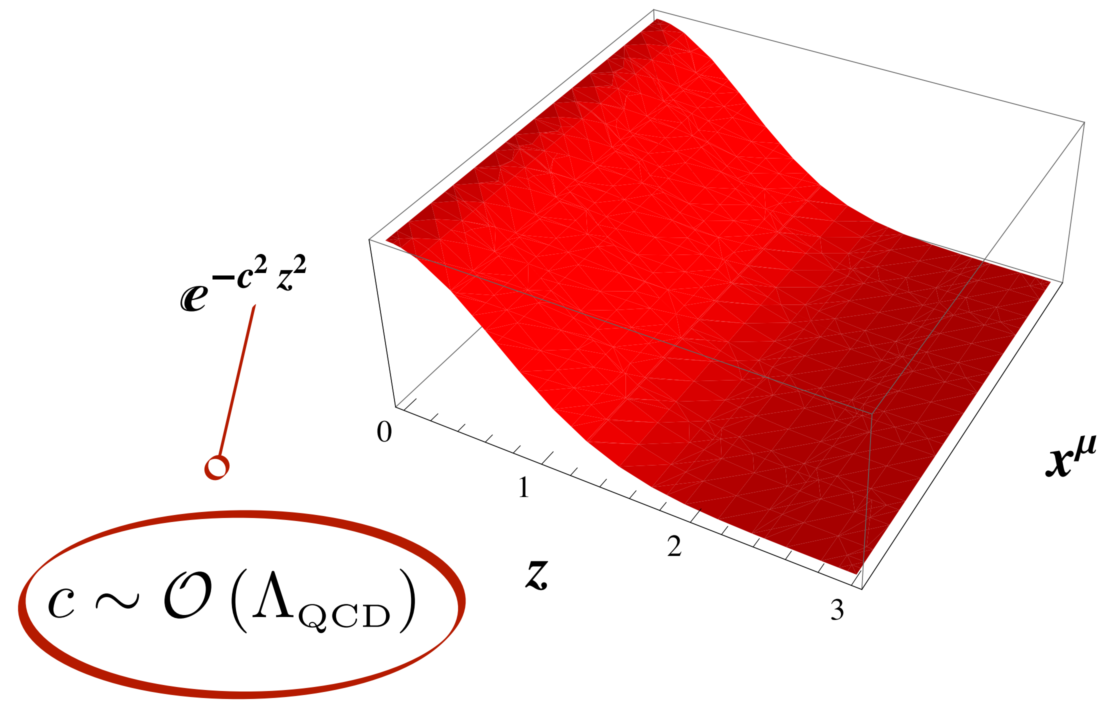
natively without SUSY

introduction of a mass scale ⟷ conformal symmetry breaking

Soft-Wall - (SW-)



Soft-Wall + (SW+)



Regge trajectories

$$g_{\mu\nu} = \frac{R^2}{z^2}\,\eta_{\mu\nu}$$

$$c = m_\rho/2 \simeq 0.388 \text{ GeV}$$

Topics covered: chiral symmetry breaking, hadron spectra, decay constants, form factors, condensates, structure functions, deep inelastic scattering, heavy-quark potential, finite temperature, ...

Chiral Symmetry Breaking and Scalar Mesons

action

$$S = \int d^5x\, e^{-\phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} \left(F_L^2 + F_R^2\right) \right\}$$

field content

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z) X^{\alpha\beta}$	0	3	-3

tachyon

$$X(x,z) = \left[X_0(z) + S^a(x,z) T^a\right] e^{i 2\pi^a t^a}$$

$\langle \bar{q} q \rangle$

scalar mesons

pseudoscalars

symmetry

4-dim

$SU(n_f)_L \otimes SU(n_f)_R$ global symmetry

$A^a_{R\mu}$: $SU(n_f)_R$ gauge field
$A^a_{L\mu}$: $SU(n_f)_L$ gauge field

t^a = gen. of $SU(n_f)$
$T^a = \{\mathbf{1}/\sqrt{n_f}, t^a\}$

5-dim

$SU(n_f)_L \otimes SU(n_f)_R$ local symmetry

$$SU(n_f)_L \otimes SU(n_f)_R \xrightarrow[X_0(z)]{} SU(n_f)_V$$

chiral symmetry breaking

$$\{A^a_{L\mu}, A^a_{R\mu}\} \to \{V_\mu, A_\mu\}$$

equations of motion

$$\partial_z \left[\frac{e^{-\phi(z)}}{z} \partial_z V_\mu \right] - q^2 \frac{e^{-\phi(z)}}{z} V_\mu = 0$$

$$\partial_z \left[\frac{e^{-\phi(z)}}{z} \partial_z A_\mu \right] - q^2 \frac{e^{-\phi(z)}}{z} A_\mu - \frac{g_5^2 X_0(z)^2 e^{-\phi(z)}}{z^3} A_\mu = 0$$

Scalar Glueballs

AdS
- $W(x,z)$
- $m_5^2 R^2 = 0$

dilaton

duality

QCD
- $\mathcal{O}_G(x) = \beta(\alpha_s) G^a_{\mu\nu} G^{a\,\mu\nu}$
- $\Delta = 4$

action

$$S_5^{eff} = -\frac{1}{2k}\int d^5x\, e^{-\phi(z)} \sqrt{g}\, g^{MN} \left(\partial_M W(x,z)\right)\left(\partial_N W(x,z)\right)$$

equation of motion

$$\partial_M \left[\sqrt{g}\, e^{-\phi}\, g^{MN} \partial_N W(x,z)\right] = 0$$

$$W(x,z) = \int d^4x'\, K(x-x',z) W_0(x') \xrightarrow{\text{Fourier}} \tilde{W}(q,z) = \tilde{K}(q,z)\tilde{W}_0(q)$$

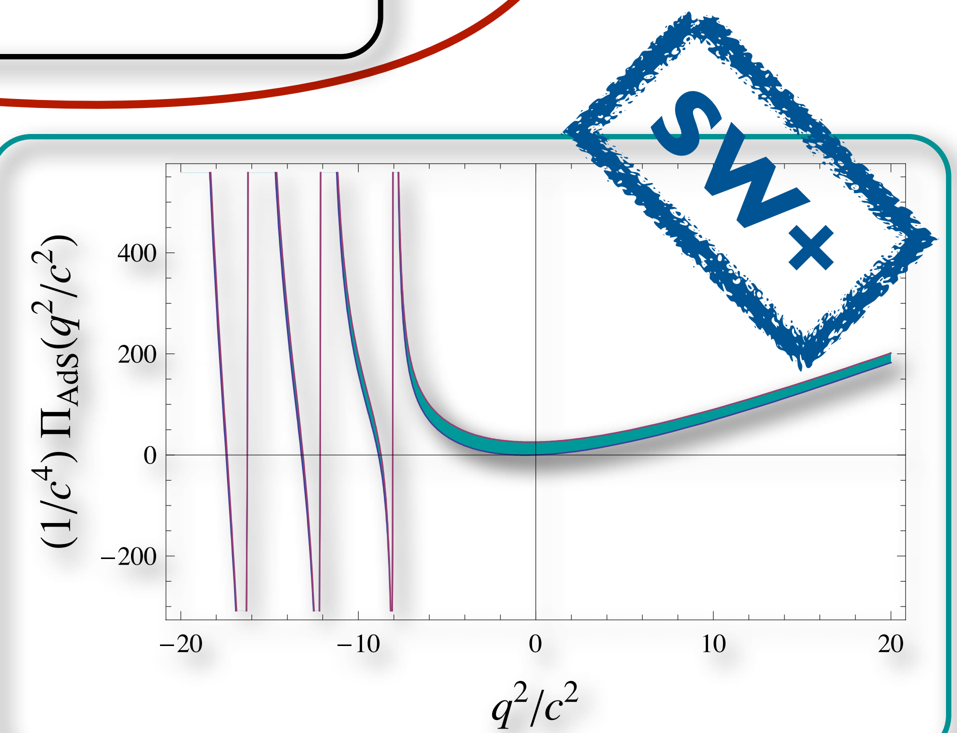
solve (analytically) for the bulk to boundary propagator

we are looking for $\Pi_{\rm QCD}(q^2) = i\int d^4x\, e^{iq\cdot x} \langle 0| T\left[\mathcal{O}(x)\mathcal{O}(0)\right] |0\rangle$

divide the action twice by the source

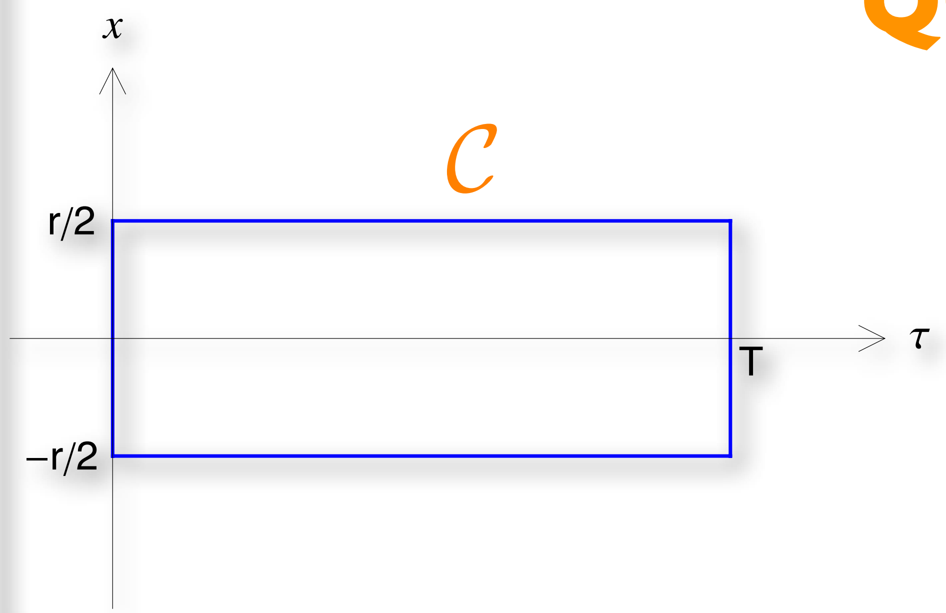
$$\Pi_{AdS}(q^2) = -\frac{R^3}{k} \tilde{K}(q,z) \frac{e^{-c^2z^2}}{z^3} \partial_z \tilde{K}(q,z)\Big|_{z\to 0}$$

$$\Pi_{AdS}(q^2) = -\frac{R^3}{8k} q^2 (q^2 + 4c^2) \left[\ln(c^2/\nu^2) + \psi\left(2 + q^2/4c^2\right) + \gamma_e - 3\right]$$



Static QQbar Potential

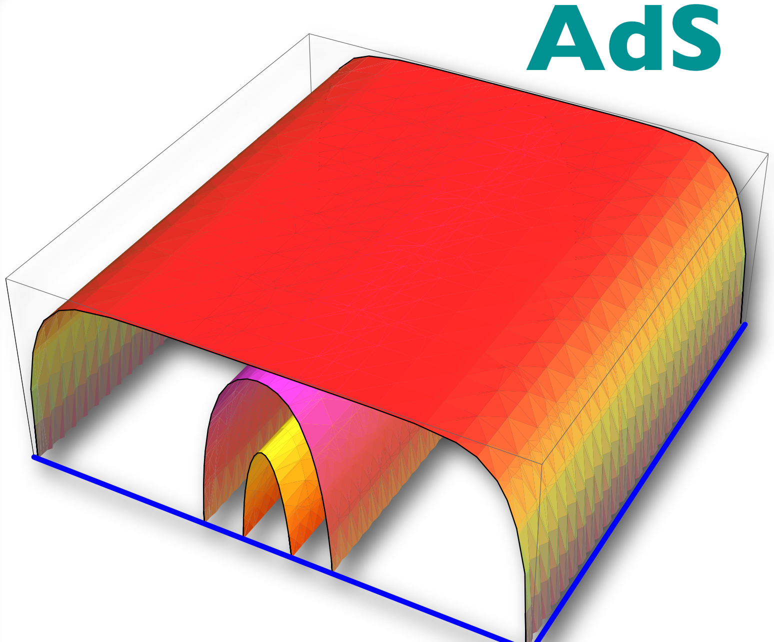
QCD



$$W(\mathcal{C}) = \mathrm{Tr}\left[\mathcal{P}\exp\oint_{\mathcal{C}} dx^\mu A_\mu\right]$$

$$V = \lim_{T\to\infty}\frac{\langle W(\mathcal{C})\rangle}{T}$$

AdS



$$\xi^0 = \tau$$

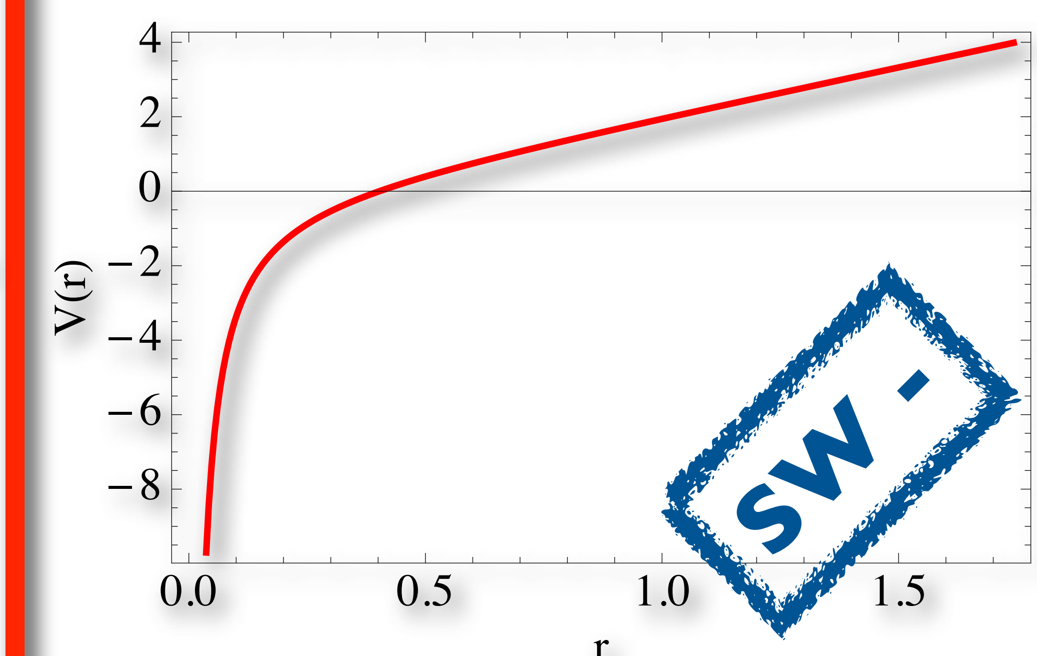
$$\xi^1 = x$$

$$S_{\rm NG} = \frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{\det\gamma}$$

$$\gamma_{ab} = g_{\mu\nu}\,(\partial_a X^\mu)(\partial_b X^\nu)$$

$$g_{\mu\nu} = \frac{e^{c^2 z^2}}{z^2}\delta_{\mu\nu}$$

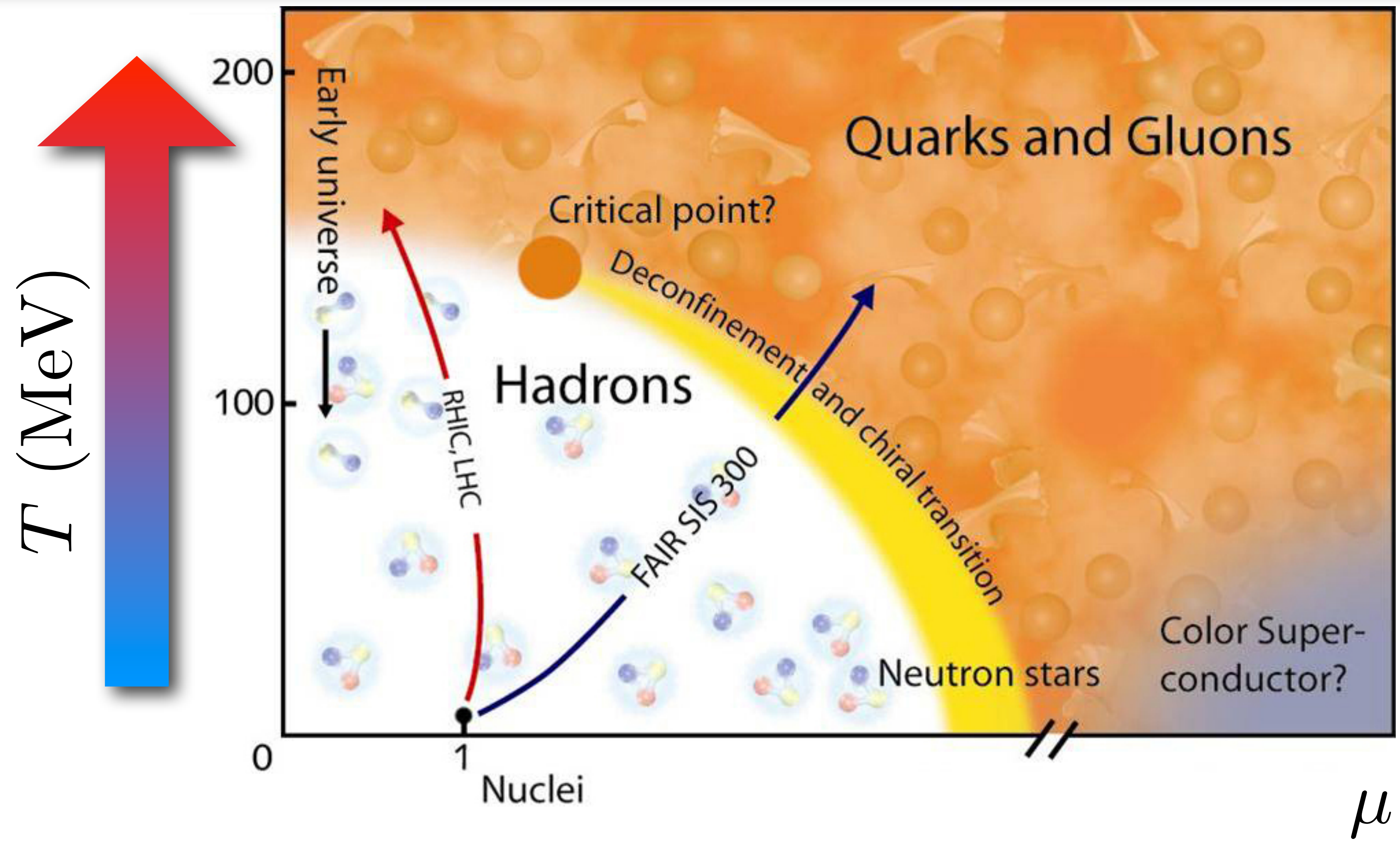
$$\langle W(\mathcal{C})\rangle = e^{-S_{\rm NG}}$$



Finite T Effects

QCD phase diagram

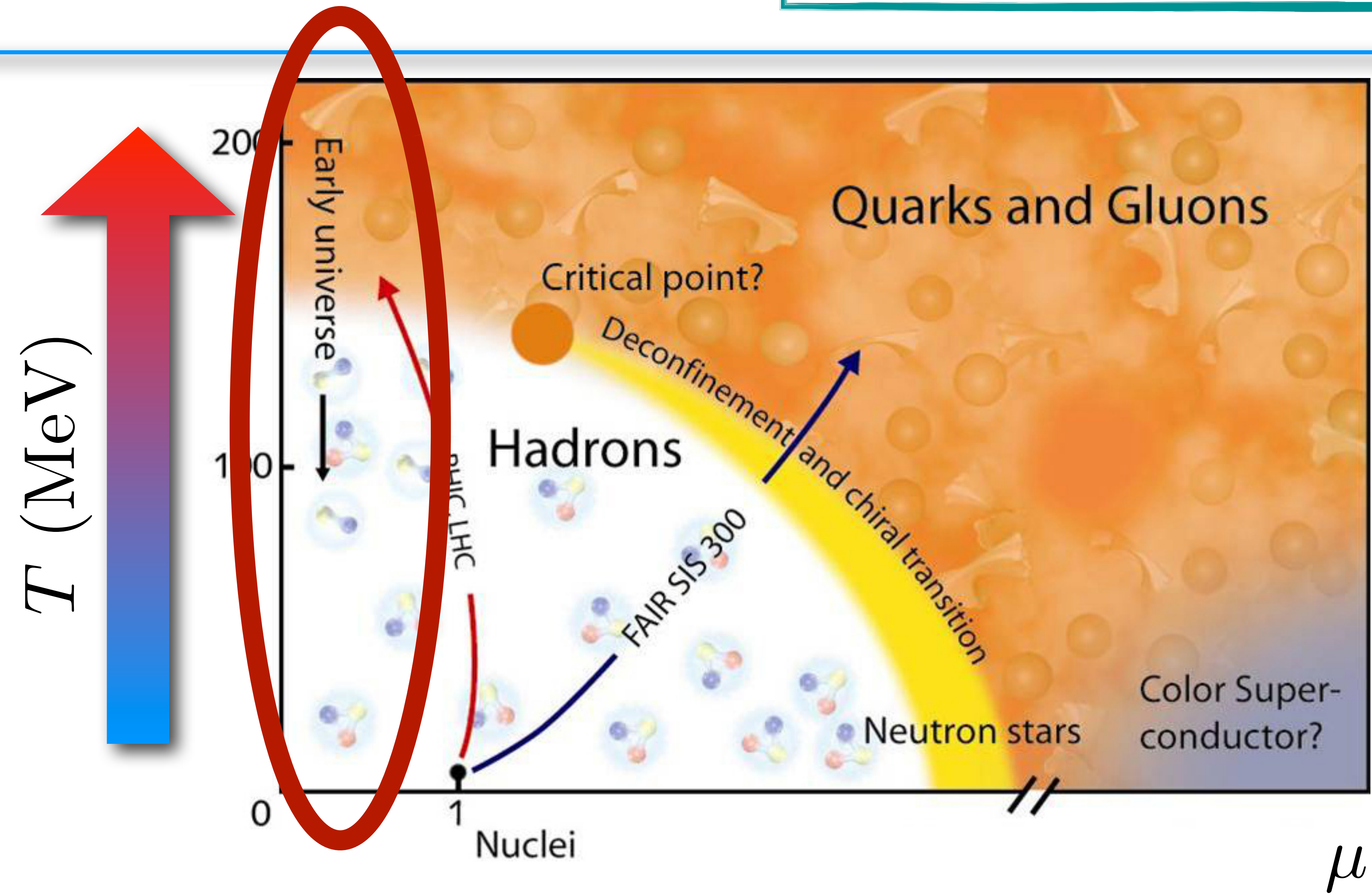
strong dynamics → nontrivial vacuum structure



- Chiral symmetry restoration (vanishing of the chiral condensate)
- Deconfinement transition: non-zero free-quark density (I order? Crossover?)
- Formation of Quark-Gluon Plasma (hot-dense medium)

QCD phase diagram

strong dynamics → nontrivial vacuum structure



- Chiral symmetry restoration (vanishing of the chiral condensate)
- Deconfinement transition: non-zero free-quark density (I order? Crossover?)
- Formation of Quark-Gluon Plasma (hot-dense medium)

Hadrons In Hot Medium

Under investigation

Experiments

- RHIC (Brookhaven)
- LHC (ALICE, hopefully)
- ...

Theory

- Lattice QCD
- Effective models
- ....

What if we increase T?

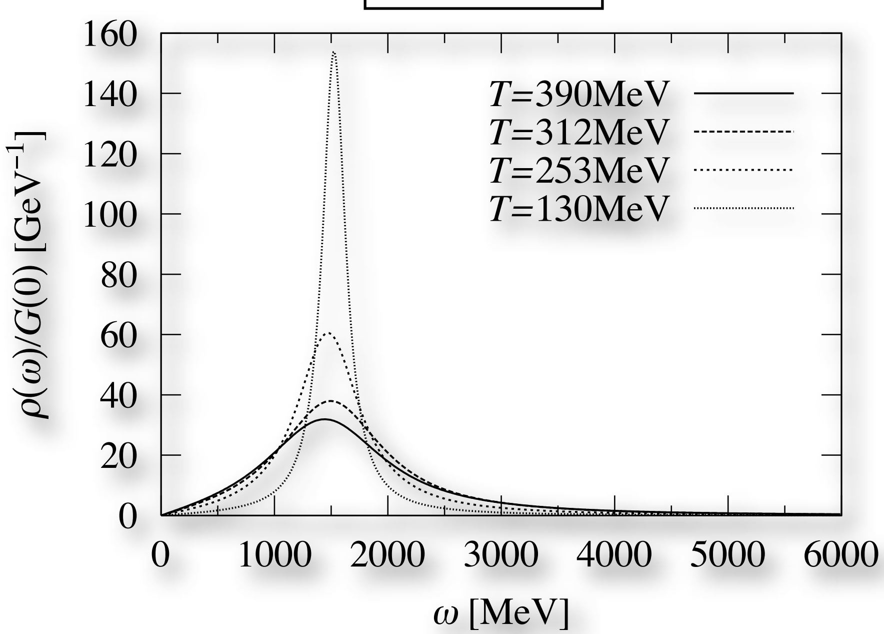
- Chiral symmetry restoration
- Deconfinement transition
- Hadron dissociation (?)

→ quark-gluon plasma

hadrons interact with medium

- Masses decrease
- Widths increase
- Survive after deconfinement

lattice



What happens on the AdS side?

Finite T

- **Thermal AdS**

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\bar{x}^2 - dz^2\right)$$

$$0 \leqslant t \leqslant 1/T$$

- **AdS/Black-Hole**

$$ds^2 = \frac{R^2}{z^2}\left(\xi(z)dt^2 - d\bar{x}^2 - \frac{dz^2}{\xi(z)}\right)$$

$$\xi(z) = 1 - \frac{z^4}{z_h^4} = 1 - (\pi T z)^4$$

Soft-Wall - (SW-)	Soft-Wall + (SW+)
AdS/Black-Hole is stable	Thermal AdS $T < T_{\mathrm{HP}}$
NO Thermal AdS	Confined Phase
	$T = T_{\mathrm{HP}}$
	Deconfined Phase
Deconfinement: QQbar Potential ??	AdS/Black-Hole $T > T_{\mathrm{HP}}$

Scalar Glueball (AdS/BH)

SW ±

Low temperature: narrow peaks

increasing temperature

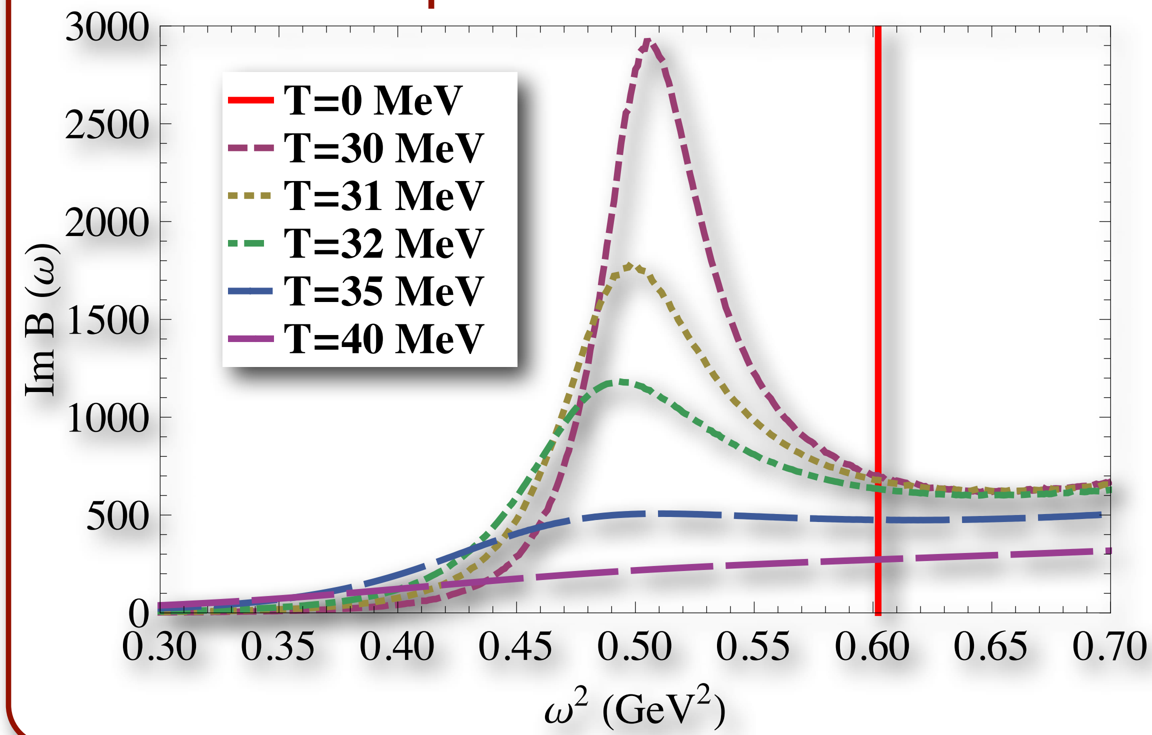
increasing widths

decreasing masses

Hadrons melt into the medium

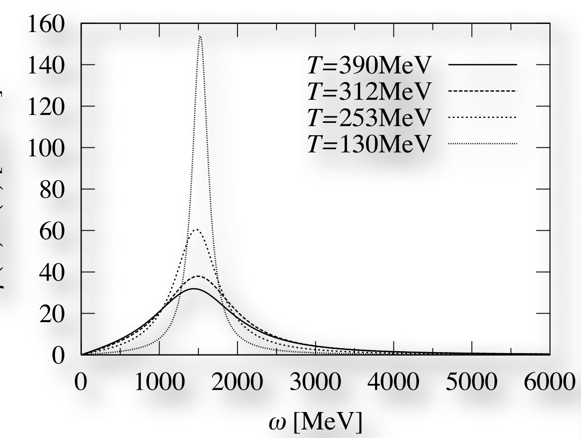
same kind of behaviour as in lattice
different scale of temperature

spectral function

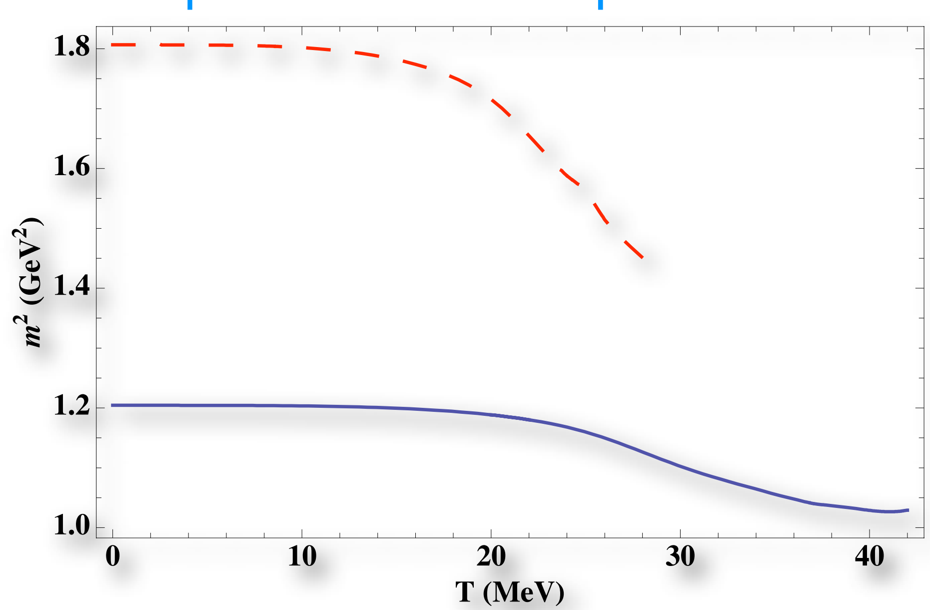


$$\rho(x) = \frac{a\, m\, \Gamma\, x^b}{(x-m^2)^2 + m^2\Gamma^2}$$

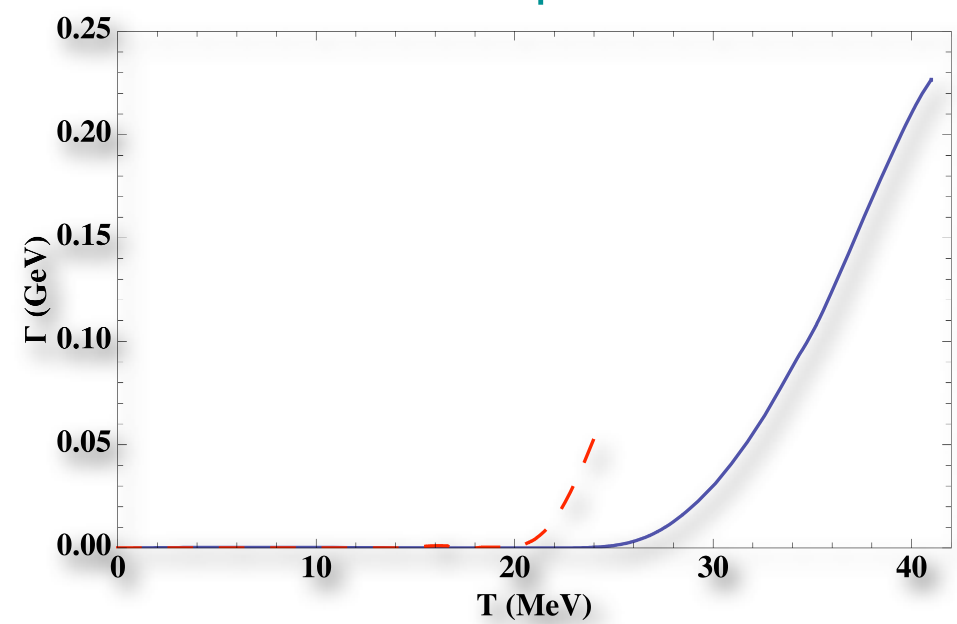
Breit-Wigner



squared mass vs temperature



width vs temperature



✓ qualitatively

✗ quantitatively

Scalar Glueball (ThAdS + AdS/BH)

- $T < T_{\rm HP} \rightarrow$ thermal AdS
- $T > T_{\rm HP} \rightarrow$ AdS/Black-Hole

<u>Thermal AdS</u>

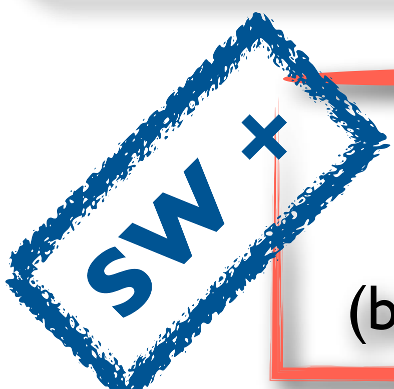
Equations, boundary conditions, masses and widths <u>unaffected</u> by increasing temperature

$T_{\rm HP} \sim 191$ MeV

First-order HP phase transition (deconfinement)

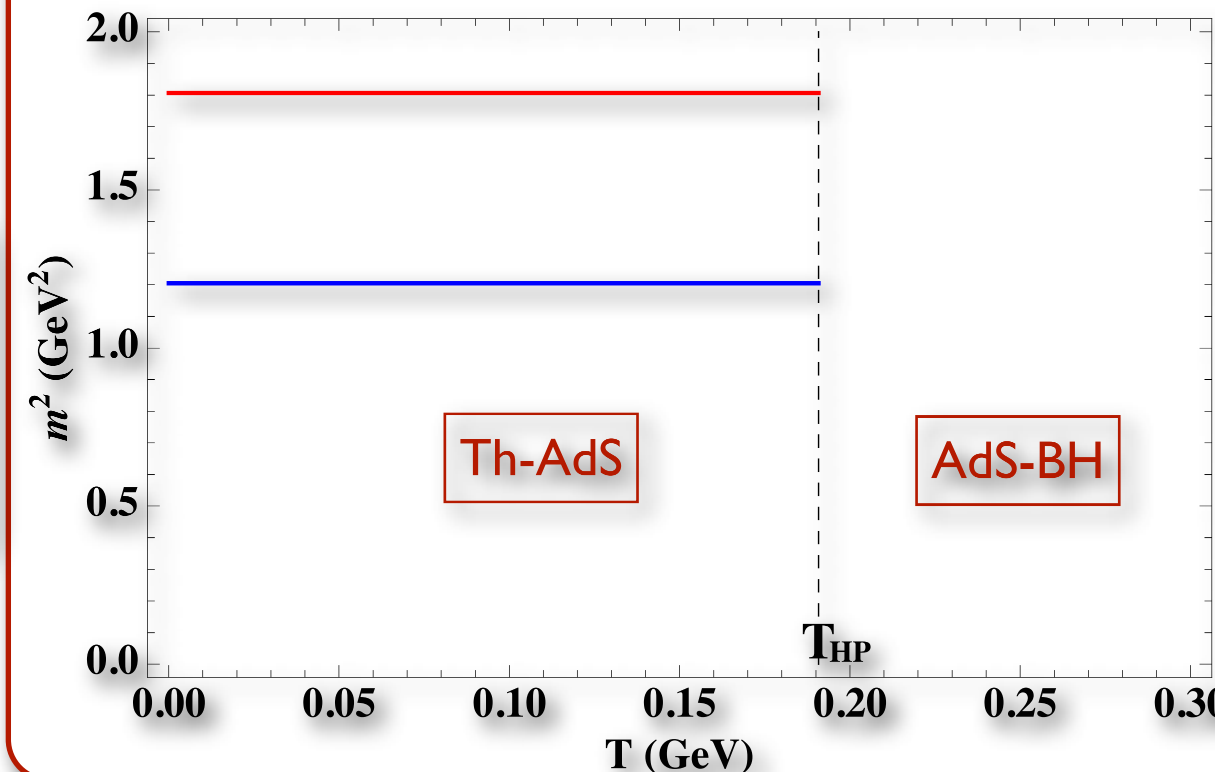
<u>AdS/BH</u>

Same picture as previous slides <u>but</u> for $T>T_{\rm HP}$ hadrons have already melted

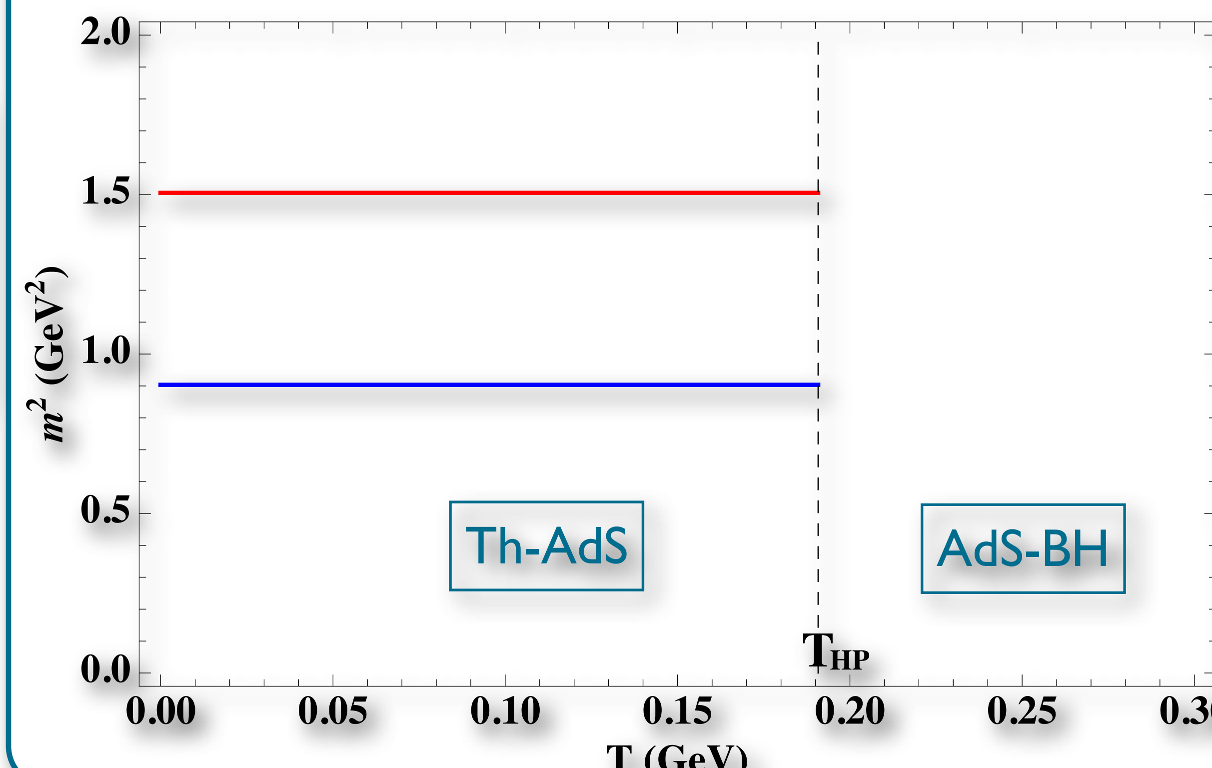


✓ scale of temperature
✗ qualitative behaviour
(but in agreement with Large N)

scalar glueballs



scalar mesons



Pros And Cons

Soft-Wall - (SW-)	Soft-Wall + (SW+)
$T = 0$	
m_q and $\sigma = \langle \bar{q}q \rangle$ independent	$m_q \propto \sigma = \langle \bar{q}q \rangle$
Linear Confinement	NO Linear Confinement
Regge Trajectories	Regge Trajectories
$m_{G_0} = m_\rho = 776$ MeV	$m_{G_0} \sim 1.1$ GeV
$m_{G_{0^+}} < m_{G_{0^-}}$	$m_{G_{0^+}} > m_{G_{0^-}}$
$m_{S_0} \sim 549$ MeV	$m_{S_0} \sim 950$ MeV
$g_{S\pi\pi} \sim \mathcal{O}(100\text{ MeV})$	$g_{S\pi\pi} \sim \mathcal{O}(10\text{ MeV})$
dim 2 condensate	dim 2 condensate
$T > 0$	
Spectral functions compatible with LQCD	Spectral functions compatible with LQCD
Wrong Scale	Wrong Scale (but rigth for large N)
Dissociation independent on deconfinement	Dissociation coincident with deconfinement

Conclusions And Perspectives

Holographic QCD: ($\sim$) new approach to the non-perturbative regime of strong interactions

Soft-Wall model(s)

5-dim models: semiclassical field theory in curved spacetime

- Both SW± catch some key features of QCD
- Relatively simple models
- Both SW± have pros and cons
- At present SW - seems to a little better working than SW+
- The whole approach seems very promising, but new efforts have to be put in the game

Thank you for your attention

Need Anything Else?

Conformal transformations

conformal group in $d+1$-dim $\sim SO(2,d)$ → $g_{\alpha\beta} \to f(x^\mu)\, g_{\alpha\beta}$

$$\dim\left[SO(2,d)\right] = \frac{1}{2}\,(d+1)\,(d+2)$$

	Transformation	Generator
Translation	$x'^\mu = x^\mu + a^\mu$	$P_\mu = -i\partial_\mu$
Rotation	$x'^\mu = M^\mu{}_\nu x^\nu$	$L_{\mu\nu} = i(x_\mu\partial_\nu - x_\nu\partial_\mu)$
Dilation	$x'^\mu = a x^\mu$	$D = -ix^\mu\partial_\mu$
SCT	$x'^\mu = \frac{x^\mu - b^\mu x^2}{1-2b\cdot x+b^2x^2}$	$K_\mu = -i(2x_\mu x^\nu\partial_\nu - x^2\partial_\mu)$

Translation + Rotation → **Poincaré group**

Dilation → **scale invariance** → **no mass scales**

algebra

$$\begin{aligned}
[D, P_\mu] &= iP_\mu \\
[D, K_\mu] &= -iK_\mu \\
[K_\mu, P_\nu] &= 2i(\eta_{\mu\nu}D - L_{\mu\nu}) \\
[K_\rho, L_{\mu\nu}] &= i(\eta_{\rho\mu}K_\nu - \eta_{\rho\nu}K_\mu) \\
[P_\rho, L_{\mu\nu}] &= i(\eta_{\rho\mu}P_\nu - \eta_{\rho\nu}P_\mu) \\
[L_{\mu\nu}, L_{\rho\sigma}] &= i(\eta_{\nu\rho}L_{\mu\sigma} + \eta_{\mu\sigma}L_{\nu\rho} - \eta_{\nu\sigma}L_{\mu\rho} - \eta_{\mu\rho}L_{\nu\sigma})
\end{aligned}$$

Conformal invariance in QFT

what does conformal invariance mean in a QFT?

necessary (but not sufficient) condition : scale invariance → no mass scales → no dimensionful parameters → no particles?? no S matrix??

what about the renormalization group?

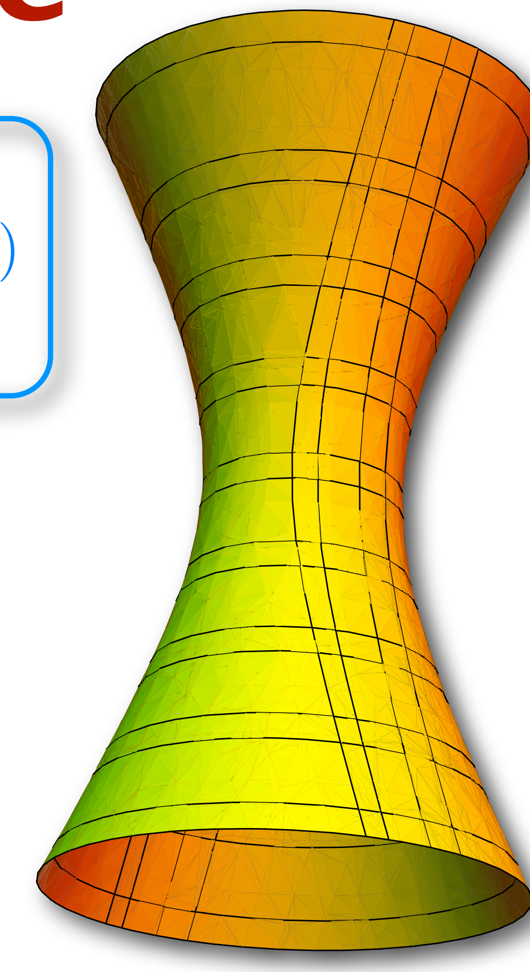
even if the Lagrangian is conformally invariant

regularization scheme → UV cutoff → mass scale → breaking of conformal invariance

any QFT, to be conformal, must be FINITE and with vanishing beta-function

even if → can show conformal behaviour near a fixed point

Anti-de Sitter spacetime



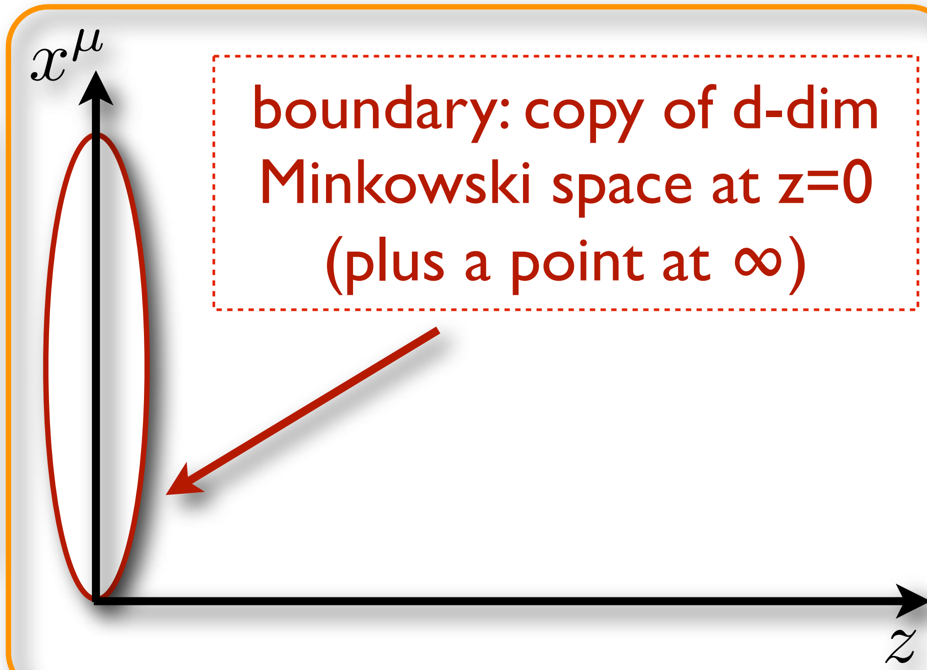
AdS_{d+1} solution of the (d+1)-dim Einstein equations with negative cosmological constant $S \propto \int d^{d+1}x \sqrt{|g|}\,(\mathcal{R} + \Lambda)$

hyperboloid in a (d+2)-dim pseudo-Euclidean spacetime "with two times"

$y^2 = (y^0)^2 + (y^{d+1})^2 - \sum_{i=1}^{d} (y^i)^2 \quad \eta_{ab} = \mathrm{diag}(+,-,-,\ldots,-,+) \quad y^2 = R^2 + \mathrm{const}$

CFT on d-dim Minkowski spacetime
and
QFT on (d+1)-dim AdS spacetime

HAVE THE SAME INVARIANCE GROUP



metric in Poincaré coordinates

$ds_5^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$

isometry group $SO(2,d)$ same as d-dim conformal group
acts on the boundary z=0 (d-dim Minkowski spacetime) as the conformal group

AdS/CFT correspondence

problem

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{CFT} = \frac{\delta^2}{\delta\varphi_0(x)\delta\varphi_0(y)}\Big\langle \ldots \Big\rangle_{CFT}\Big|_{\varphi=0} = \;??$$

AdS/CFT correspondence

SUGRA

CFT

$$e^{iS_5^{eff}[\varphi]} \sim \mathcal{Z}[\varphi_0] = \left\langle \exp \int_{\partial AdS_5} d^4x\, \varphi_0(x)\mathcal{O}(x) \right\rangle_{CFT}$$

problem

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{CFT} = \frac{\delta^2}{\delta\varphi_0(x)\delta\varphi_0(y)} \left\langle \ldots \right\rangle_{CFT}\Big|_{\varphi=0} = \;??$$

help

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{CFT} = \frac{\delta^2}{\delta\varphi_0(x)\delta\varphi_0(y)} e^{iS_5^{eff}[\varphi]}\Big|_{\varphi=0} = i\,\frac{\delta^2 S_5^{eff}[\varphi]}{\delta\varphi_0(x)\delta\varphi_0(y)} \neq \;??$$

it would be fine if we could find something similar for non-conformal theories
(or even better for QCD.....)

Scalar Glueball in AdS/BH soft wall scenario

Deconfined phase: hadrons interacting with a supercooled quark gluon plasma

$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)}\right)$$

$$f(z) = 1 - \frac{z^4}{z_h^4} \qquad 0 < z < z_h$$

AdS

- $W(x,z)$
- $m_5^2 R^2 = 0$

duality ⟷

QCD

- $\mathcal{O}_G(x) = \beta(\alpha_s) G^a_{\mu\nu} G^{a\,\mu\nu}$
- $\Delta = 4$

action

$$S_5^{eff} = -\frac{1}{2k}\int d^5x\, e^{-\phi(z)}\sqrt{g}\, g^{MN}\left(\partial_M W(x,z)\right)\left(\partial_N W(x,z)\right)$$

equation of motion

$$\partial_M\left[\sqrt{g}\, e^{-\phi}\, g^{MN}\partial_N W(x,z)\right] = 0$$

$$W(x,z) = \int d^4x'\, K(x-x',z) W_0(x') \xrightarrow{\text{Fourier}} \tilde{W}(q,z) = \tilde{K}(q,z)\tilde{W}_0(q)$$

$$\tilde{K}''(q,z) - \frac{4 - f(z) + 2c^2 z^2 f(z)}{z f(z)}\tilde{K}'(q,z) + \left(\frac{q_0^2}{f(z)^2} - \frac{\bar{q}^2}{f(z)}\right)\tilde{K}(q,z) = 0$$

Scalar Glueball in AdS/BH soft wall scenario

$$\tilde{K}''(q,z) - \frac{4 - f(z) + 2c^2 z^2 f(z)}{z f(z)} \tilde{K}'(q,z) + \left(\frac{q_0^2}{f(z)^2} - \frac{\bar{q}^2}{f(z)} \right) \tilde{K}(q,z) = 0$$

boundary conditions

$$\begin{cases} \tilde{K}(q,0) = 1 \\ \tilde{K}(q,u) \xrightarrow[u \to 1]{} (1-u)^{-i z_h \sqrt{q_0^2}/4} \end{cases}$$

change of coordinates

$$u = \frac{z}{z_h} \qquad 0 \leqslant u \leqslant 1$$

glueball rest frame

$$\bar{q} = 0$$

$$K(q_0^2, u) \xrightarrow[u \to 0]{} A(q_0^2) \left(1 + \frac{q_0^2 z_h^2}{4} u^2 + \ldots \right) + B(q_0^2) \left(\frac{c^4 z_h^4}{2} u^4 + \ldots \right)$$

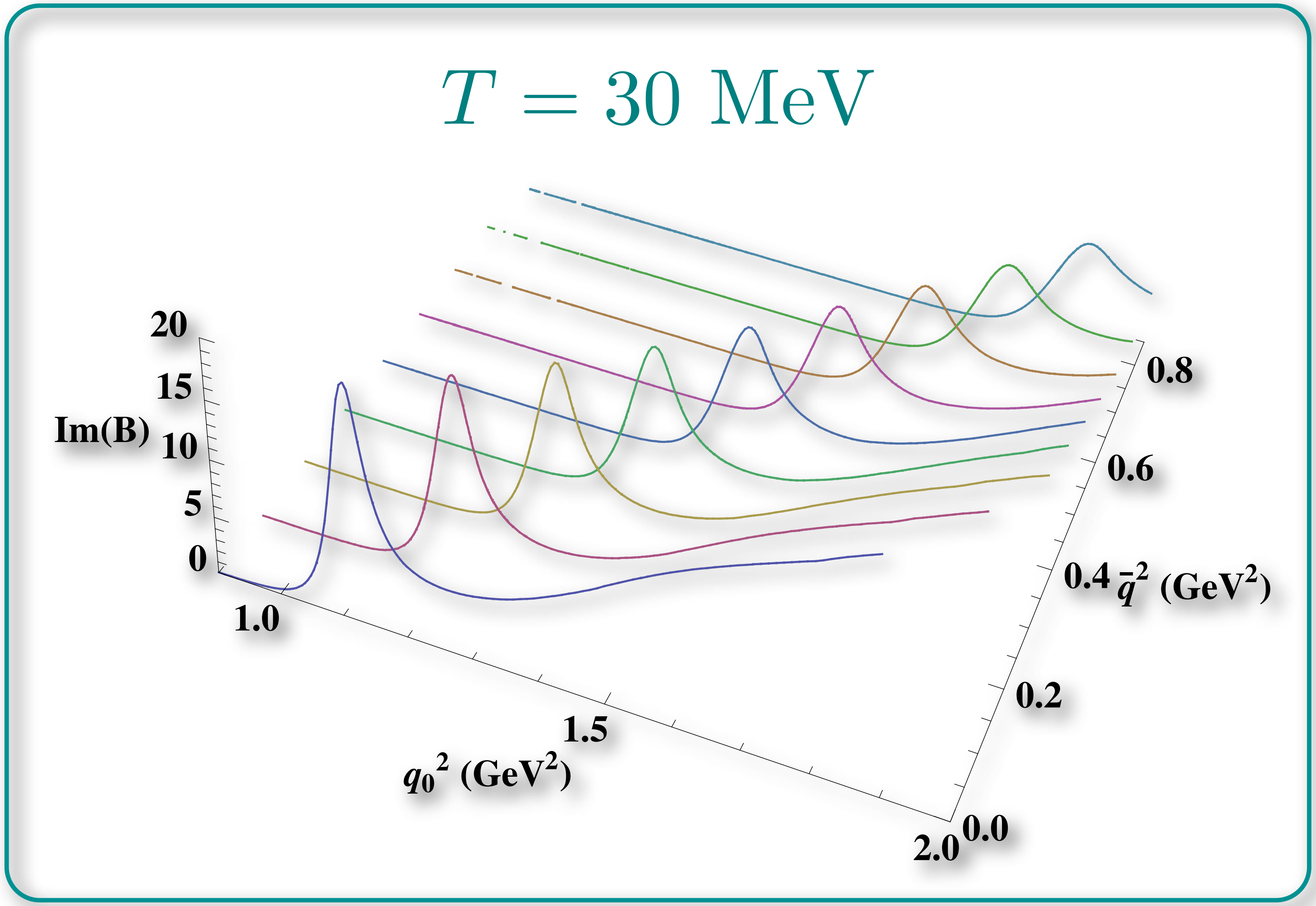
retarded Green's function

$$\Pi(q_0^2) = \frac{\delta^2 S}{\delta W_0 \delta W_0} \bigg|_{W_0 = 0} = \frac{R^3}{2k} \frac{f(u)}{z_h^4 u^3} e^{-\phi} \tilde{K}(q,u) \partial_u \tilde{K}(q,u) \bigg|_{u=0}$$

spectral function

$$\rho\left(q_0^2\right) = \Im\left[\Pi\left(q_0^2\right)\right] \propto \Im\left[B(q_0^2)\right]$$

Scalar Glueball in AdS/BH soft wall scenario: $\bar{q} \neq 0$



increasing the three-momentum → masses increase, widths increase