Muonic hydrogen at finite temperature. A toy model for Heavy Quarkonium dissociation

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16th of June, 2010

Based on the work done with Joan Soto (in preparation).

Outline

- Introduction

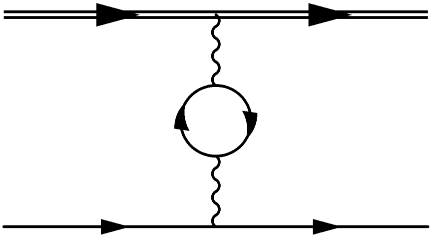
Motivation

Why do we study Muonic Hydrogen (and Hydrogen) at finite temperature?

- J/ψ suppression in QGP not fully understood in QCD.
- QED bound states are very similar to heavy quarkonium.
- It may be suitable to experimental studies.

Analogy with HQ

Dissociation is mainly due to this diagram.



In Hydrogen atom this diagram is only relevant at $T \sim m_e$, but as soon as this mechanism is activated Hydrogen dissociates. In HQ an analogous diagram with gluons and light quarks is relevant even at very low temperature. Muonic Hydrogen ($m_\mu \gg 1/r \sim m_e$) is closer to HQ in the sense that it can survive temperatures where the electrons are thermalized.

EFT's

Effective Field Theories are very useful for problems with different scales. In Muonic Hydrogen at finite temperature we have a lot of them.

- Muon mass, m_μ, hard scale.
- Typical radius and trimomentum, $m_\mu\alpha$,soft scale. The electron mass is of this scale when $n \sim 1$.
- Binding energy, $m_\mu\alpha^2$,ultrasoft scale.
- Temperature T.
- also eT (Debye mass) and e^2T.

NRQED

This is the EFT one obtains after integrating out m_μ.

$$\mathcal{L} = \psi^+ (iD^0 + \frac{\vec{D}^2}{2m_\mu} + \frac{\vec{D}^4}{8m_\mu^3} + c_F e \frac{\vec{\sigma}\vec{B}}{2m_\mu} + c_D e \frac{|\vec{\nabla}\vec{E}|}{8m_\mu^2} + i c_S e \frac{\vec{\sigma}(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})}{8m_\mu^2})\psi + \\ + N^+ iD^0 N - \frac{1}{4} d_1 F_{\mu\nu} F^{\mu\nu} + \frac{d_2}{m^2} F_{\mu\nu} D^2 F^{\mu\nu}$$

[W. E. Caswell and G. P. Lepage, Phys. Lett. B 167, 437 (1986)]

pNRQED

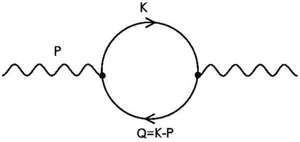
This is the EFT one obtains if the $1/r$ ($r = |\vec{x}_1 - \vec{x}_2|$) scale is also integrated out.

$$L_{pNRQED} = \int d^3\vec{x}(\psi^\dagger\{iD^0 + \frac{\vec{D}^2}{2m_\mu} + \frac{\vec{D}^4}{8m_\mu^3}\}\psi + N^\dagger iD^0 N - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}) +$$
$$+ \int d^3\vec{x}_1\, d^3\vec{x}_2 N^\dagger N(t,\vec{x}_2)(\frac{Z\alpha}{|\vec{x}_1-\vec{x}_2|} + \frac{Ze^2}{m_\mu^2}(-\frac{c_D}{8} + 4d_2)\delta^3(\vec{x}_1 - \vec{x}_2) +$$
$$+ic_S\frac{Z\alpha}{4m_\mu^2}\vec{\sigma}(\frac{\vec{x}_1-\vec{x}_2}{|\vec{x}_1-\vec{x}_2|^3} \times \vec{\nabla}))\psi^\dagger\psi(t,\vec{x}_1)$$

With this Lagrangian we can easily get the fine-structure shift and the Lamb shift (first correction due to quantization of electromagnetic field, of order $m\alpha^5$). [A. Pineda and J. Soto,Nucl. Phys. Proc. Suppl. 64, 428 (1998)]

Hard Thermal Loops

In a gauge theory at finite temperature.



If $T \gg p$, by power counting $\Pi(p, T) \sim e^2 T^2$. What happens if $p \leq eT$?. A resummation is needed (M.LeBellac, Thermal Field Theory), or in the EFT language one needs to integrate out T scale.

$$\mathcal{L}_{YM,HTL} = \mathcal{L}_{YM} + \frac{1}{2} m_D^2 \int \frac{d\Omega_v}{4\pi} Tr \left[\left(\frac{1}{v\mathcal{D}} v^\alpha F_{\alpha\mu} \right) \left(\frac{1}{v\mathcal{D}} v^\beta F^\mu_\beta \right) \right]$$

where v^μ is a light-like vector
[Braaten and Pisarski, Nucl. Phys. B 339, 310 (1990)]

- Muonic Hydrogen with $m_e = 0$.

Muonic Hydrogen with $m_e = 0$

- Academic case, but closer to Heavy Quarkonium.
- Debye screening and other effects due to vacuum polarization will be active at any temperature.

$1/r \gg E \gtrsim T$

The starting point can be pNRQED at $T = 0$. If one wants a precision of $m_\mu \alpha^5$ there is no need to include vacuum polarization effects, hence we are in the same situation as in Hydrogen atom.

[Further details in Escobedo and Soto, Phys. Rev. A 78,032520]

$1/r \gg E \gg T$

This is a particular case of $1/r \gg E \gtrsim T$ for illustration purposes

$$\delta\Gamma_n \sim 0$$
$$\delta E_n \sim \frac{\alpha\langle r^2\rangle_n T^4}{E_n}$$

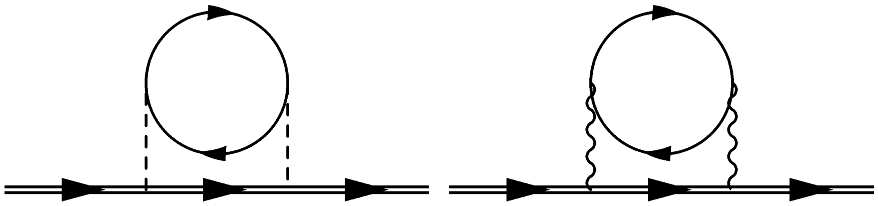
$1/r \gg T \gg E$

One can integrate out T as an intermediated step.
$pNRQED \rightarrow pNRQED_T$.
This leads to a modification of $pNRQED$ potential.

$$\delta V_T^{(LO)} = \frac{\alpha\pi T^2}{3m_\mu} - \frac{4\alpha^2}{3m_\mu^2}\delta^3(\mathbf{r})\left(\frac{1}{\epsilon} + \log\left(\frac{\mu}{T}\right) + \frac{5}{6}\right)$$

Computations are done in dimensional regularization with $\epsilon = (4-D)/2$.

$1/r \gg T \gg E$

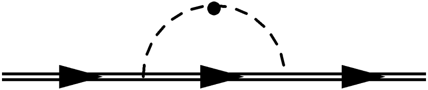


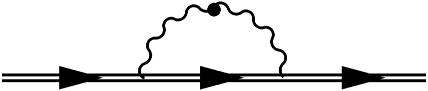
Part of this computation was already done for Heavy Quarkonium [Brambilla, Ghiglieri, Vairo and Petreczky. Phys. Rev. D78, 014017 (2008)].

$$\delta V_T^{(NLO)} = -\frac{3\alpha}{2\pi}\zeta(3)\,T m_D^2 r^2 +$$
$$+\frac{i\alpha T m_D^2}{6} r^2 \left(\frac{1}{\epsilon} + \gamma + \log\pi - \log\frac{T^2}{\mu^2} + \frac{2}{3} - 4\log(2) - 2\frac{\zeta'(2)}{\zeta(2)}\right)$$

$1/r \gg T \gg E$. Calculation in $pNRQED_T$

- $E \gg eT$





$1/r \gg T \gg E$. Calculation in $pNRQED_T$

- $E \gg eT$

$$\delta E_n^E =$$

$$= \frac{2\alpha}{3\pi}\sum_m |\langle n|\mathbf{v}|m\rangle|^2 (E_n - E_m)\left(\frac{1}{\epsilon} + \log\left(\frac{\mu}{|E_n - E_m|}\right) + \frac{5}{6} - \gamma + \log(2\pi)\right)$$

$$-\frac{\alpha\pi T m_D^2}{3}\langle r^2\rangle_n,$$

$$\delta\Gamma_n^E =$$

$$= \frac{4\alpha^3}{3\beta n^2} + \sum_m \frac{\alpha T m_D^2}{3}|\langle n|\mathbf{r}|m\rangle|^2\left(\frac{1}{\epsilon} - 2\log\frac{|E_n - E_m|}{\mu} + \frac{5}{3} - \gamma + \log(\pi/4)\right)$$

Total results for $1/r \gg T \gg E \gg eT$

$$\delta E_n = \frac{\alpha\pi T^2}{3m_\mu} - \frac{3\alpha}{2\pi}\zeta(3) T m_D^2 \langle r^2\rangle_n - \frac{\alpha\pi T m_D^2}{3}\langle r\rangle_n +$$
$$+\frac{2\alpha}{3\pi}\sum_m |\langle n|\mathbf{v}|m\rangle|^2 \left(\log\left(\frac{T}{|E_n-E_m|}\right) - \gamma + \log(2\pi)\right)$$

$$\delta\Gamma_n = \frac{4\alpha^3}{3\beta n^2} +$$
$$+\sum_m \frac{\alpha T m_D^2}{3}|\langle n|\mathbf{r}|m\rangle|^2 \left(1 + 2\log\left(\frac{T}{|E_n-E_m|}\right) - 2\gamma + 2\log(2) + 2\frac{2\zeta'(2)}{\zeta(2)}\right)$$

$1/r \gg T \gg E$. Calculation in $pNRQED_T$

- $eT \gg E$
  The diagrams are the same as in the $E \gg eT$ case

$$\delta E_n^{eT} = \frac{\alpha m_D^3}{6}\langle r^2\rangle_n,$$

$$\delta\Gamma_n^{eT} = \frac{\alpha T m_D^2}{3}\langle r^2\rangle_n\left(\frac{1}{\epsilon} - \gamma + \log\pi + \log\frac{\mu^2}{m_D^2} + \frac{5}{3}\right).$$

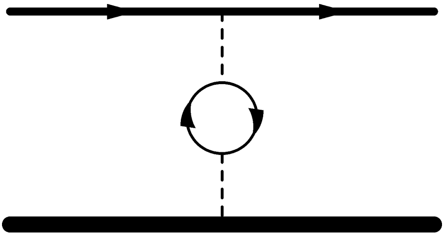
Total results for $1/r \gg T \gg eT \gg E$

$$\delta E_n = \frac{\alpha\pi T^2}{3m_\mu} + \frac{\alpha m_D^3 \langle r^2\rangle_n}{6}$$

$$\delta\Gamma_n = \frac{\alpha T m_D^2}{3}\langle r^2\rangle_n \left(1 - 2\gamma + 2\log\left(\frac{T}{m_D}\right) + 4\log(2) + \frac{2\zeta'(2)}{\zeta(2)}\right)$$

$1/r \sim T$

- In this case the starting point is NRQED at $T = 0$.
- One has to integrate out $1/r$ and T at the same time.
- The resulting EFT would be pNRQED with a T-dependent potential and HTL in the photon and electron sector.
- To our knowledge the analogous situation has not been studied in QCD. This computations is relevant for the light quark sector in the HQ case.



Potential

$$\delta V = -\frac{\alpha m_D^2 r}{4} - \frac{3\alpha}{2\pi}\zeta(3) T m_D^2 r^2 +$$
$$+\frac{\alpha m_D^2}{4\pi^2 T^2 r}\int_0^\infty \frac{du}{u(e^u+1)}(-4 - 4\rho^2 u^2 + (\rho^2 u^2 + 4)\cos(\rho u) +$$
$$+\rho u \sin(\rho u) + (6\rho u + \rho^3 u^3) Si(\rho u)) +$$
$$+\frac{i\alpha m_D^2 T r^2}{6}(\frac{1}{\epsilon} + \gamma + \log\pi + \log(r\mu)^2 - 1) +$$
$$-\frac{i3\alpha m_D^2}{2\pi^2 T}(\frac{1}{2} - \log(rT) - \log\pi) +$$
$$+\frac{i3\alpha m_D^2}{\pi^2 T^2 r}\int_0^\infty \frac{du}{u^4}\sin(\rho)(Li_2(-e^u) + u\log(1+e^u) + \frac{\pi^2}{12} - \frac{u^2}{4}),$$

Where $\rho = 2rT$ and

$$Si(x) = \int_0^x \frac{\sin t}{t}\,dt$$

Comments on the potential

- For $rT \ll 1$, one recovers the contributions from scale T of the case $1/r \gg T \gg E$.
- For $rT \gg 1$ the real part tends to the expansion of the Yukawa potential for small r, the imaginary part goes like $r^2 \log(r)$.
- There are also contributions of the same magnitude from the ultrasoft region which coincide with those of the case $1/r \gg T \gg eT \gg E$.
- In Heavy Quarkonium we expect the additional gluon contribution to be qualitatively similar.

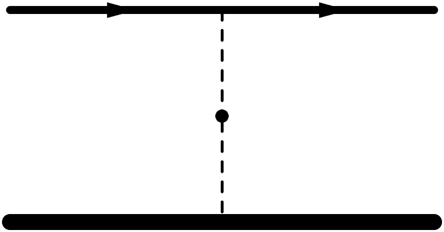
$T \gg 1/r$

The starting point can be NRQED at $T = 0$. First integrate out T scale:

- HTL in electron and photon sector
- NRQED for the muons gets a T-dependent mass redefinition and a correction of c_D (Wilson coefficient related with the Darwin term in atomic physics).

$eT \sim 1/r$

Integrate out eT and $1/r$ at the same time.



$$V(r,T) = -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D + i\alpha T \phi(m_D r)$$

with

$$\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[\frac{sin(zx)}{zx} - 1\right]$$

Exactly the same functional form in HQ and almost the same in Hydrogen atom. For $m_D r \ll 1$ we recover the case $T \gg 1/r$.[Laine, Philipsen, Romatschke and Tassler, JHEP **0703**,054 (2007)]

Dissociation temperature is $T_d \sim m_\mu \alpha^{2/3}/(\log\alpha)^{1/3}$.

$eT \gg 1/r$

- $T \gg T_d$, Hence no bound state.

- Muonic Hydrogen with actual m_e

Muonic Hydrogen with actual m_e

- The actual system that we find in nature.
- Relevant for the role of charm mass in bottomonium.
- Another scale is introduced in the problem.
- For lower lying states $(n = 1, 2)$ $m_e \sim 1/r$.
- For higher lying states $(n \geq 3)$ $m_e \gg 1/r$.

$m_e \sim 1/r \gg T$

- The thermal bath does not have electrons nor positrons.
- The only thermalized particle is the photon.
- In this situation the only difference between Hydrogen and Muonic Hydrogen is the mass of its constituents.

$m_e \sim 1/r \sim T$

- Mass dependent HTL (quoted in our Hydrogen atom paper). [Escobedo and Soto, Phys. Rev. A 78,032520]
- Mass dependent potential.

$$
\begin{aligned}
\delta V = & -\frac{4\alpha^2 f(m_e\beta)m_e^2 r}{\pi} - \frac{2\alpha^2}{\pi r}\int_0^\infty \frac{du}{\sqrt{u^2+1}(e^{\beta m_e\sqrt{u^2+1}}+1)}(1-\cos(2m_eru)-2m_eruSi(2m_eru)) + \\
& + \frac{\alpha^2}{3\pi r}\int_0^\infty \frac{du\sqrt{u^2+1}}{u^2(e^{\beta m_e\sqrt{u^2+1}}+1)}(2-12m_e^2r^2u^2+(4m_e^2r^2u^2-2)\cos(2m_eru) + \\
& + 2m_eru\sin(2m_eru)+8m_e^3r^3u^3Si(2m_eru)) - \frac{\alpha}{\pi}Tm_D^2(\beta m_e)^3\int_0^\infty \frac{x\sqrt{x^2+1}}{e^{\beta m_e\sqrt{x^2+1}}+1} + \\
& + \frac{i8\alpha^2T^3g(m_e\beta)r^2}{3\pi}\left(\frac{1}{\epsilon}+\gamma+\log\pi+\log(r\mu)^2-1\right) - \\
& - \frac{i4\alpha^2T}{\pi(e^{\beta m_e}+1)}\left(\frac{1}{2}-\log(rT)-\log 2-(e^{\beta m_e}+1)\int_0^\infty \frac{du}{u(e^{\beta m_e\sqrt{u^2+1}}+1)}+\int_0^\infty \frac{due^{-\beta m_e u}}{u}\right) + \\
& + \frac{i32\alpha^2}{\pi r}\int_0^\infty \frac{du}{u^4}\sin(Tru)\left(L_{i2}(-e^{u\sqrt{1/4+(\beta m_e)^2/u^2}})+u\sqrt{\frac{1}{4}+\frac{(\beta m_e)^2}{u^2}}\log(1+e^{u\sqrt{1/4+(\beta m_e)^2/u^2}}) + \right. \\
& \left. + \frac{\pi^2}{6}-\frac{u^2}{8}-\frac{(\beta m_e)^2}{2}-g(m_e\beta)+\frac{u^2}{8(e^{\beta m_e}+1)}\right) + \\
& + \frac{i64\pi\alpha^2T}{(2\pi)^2}\int_0^\infty \frac{du}{u^3}\left(\frac{Sinc(m_eru)-1}{e^{\beta m_e\sqrt{1+u^2/4}}+1}-\frac{Sinc(m_eru)-e^{-\beta^3m_e^3u^3}}{e^{\beta m_e}+1}\right)+\frac{i4\alpha^2m_e^2Tr^2}{3\pi(e^{\beta m_e}+1)}\left(\frac{1}{\epsilon}-1+\gamma+2\log(r\mu)+\log\pi\right) \\
& - \frac{i16\alpha^2m_e^2}{3\pi T(e^{\beta m_e}+1)}\Gamma(-2/3),
\end{aligned}
$$

$m_e \sim 1/r \sim T$, Cross-checks

- For $m_e \to 0$ we recover the $m_e = 0$ case.
- For $m_e \gg T, 1/r$ everything is exponentially suppressed.
- In the limit of $rT \gg 1$ and $m_e \sim T$ we recover results computed in Hydrogen atom for $m_e \sim T \gg 1/r \gg eT$.
- For $rT \ll 1$ and $m_e \sim T$ the results are proportional to r^2, as expected from *pNRQED*.

$m_e \sim 1/r \sim T$, pNRQED contribution

$$\delta E_n^{eT} = \frac{\alpha m_D^3}{6} \langle r^2 \rangle_n$$

$$\delta \Gamma_n^{eT} = \frac{16\alpha^2 T^3 \langle r^2 \rangle_n}{3\pi} \left(\frac{1}{\epsilon} - \gamma + \log(\pi) + \log\left(\frac{\mu^2}{m_D^2}\right) + \frac{5}{3} \right) + \\ + \frac{8\alpha^2 m_e^2 T \langle r^2 \rangle_n}{3\pi^2 (e^{m_e/T}+1)} \left(\frac{1}{\epsilon} - 2\log\left(\frac{m_D}{\mu}\right) - \frac{5}{3} + \log(4\pi) - \gamma - 2\log(2) \right)$$

$T \gg 1/r$

- Qualitatively similar to the $m_e = 0$ case.
- Quantitative small corrections suppressed by m_e/T.

- Results

Dissociation temperature for Muonic Hydrogen

The real part of the potential and the imaginary part of the potential are of the same magnitude when

$$p \sim (16\alpha)^{1/3}[g(m_e/T)]^{1/3}T \equiv m_d$$

The typical momentum transfer is

$$p \sim \frac{m_\mu \alpha}{n^2}$$

We define the dissociation temperature with the temperature in which this two scales are the same
[Escobedo and Soto, Phys. Rev. A 78, 032520]

Dissociation temperature for Muonic Hydrogen

n	T_d (MeV)	m_D (MeV)	m_d (MeV)
1	1.7	0.16	0.77
2	0.41	0.036	0.19
3	0.19	0.012	0.086
4	0.13	0.0056	0.048
5	0.10	0.0030	0.031

Table: Dissociation temperature for the lower lying states of muonic hydrogen

Application to bottomonium. The effect of finite charm mass

m_c (MeV)	T_d (MeV)
∞	480
5000	480
2500	460
1200	440
0	420

Table: Dissociation temperature for Upsilon (1S) for different values of the charm mass

Conclusions

- The EFT techniques have proven useful to deal with a non-trivial system with a lot of different energy scales.
- The Muonic Hydrogen have a number of common features with Heavy Quarkonium.
- An experimental program for Muonic Hydrogen at finite temperature (if technically possible) may provide useful insights for Heavy Quarkonium dissociation.
- The dissociation temperature of $\Upsilon(1S)$ is rather sensitive to the charm quark mass.