QCD&Work 2010: Beppe Nardulli Memorial Workshop

International Workshop on Quantum Chromodynamics – Theory and Experiment

Mixing Xi-Xi` effects and static properties of heavy Xi's

T. M. Aliev, A. Ozpineci and V. Zamiralov

Middle East Technical University, Ankara, Turkey and

Institute of Nuclear Physics, M. V. Lomonosov MSU, Moscow, Russia

Introduction

During the last few years, very intriguing observations appeared in charm and beauty baryon spectroscopy (Particle Data Group, 2008).

It would be interesting to discuss how successfully various models describe the masses of the observed baryons. The central problem of all studies on this subject is establishing the structure of new baryons within the quark model, in particular, of the cascade baryons with new flavors (see Ono_1978 - Ebert-Faustov-Galkin_2005 and references therein).

On the other hand, many characteristics of the baryons are successfully determined in the framework of QCD sum rules method {Ioffe_1981} (see Colangelo-Khodjamirian_2001 about this method).

The main issue in the applications of sum rules is the choice of an interpolating current with the same quantum numbers of the corresponding baryon.

In the present work, we discuss some interesting points changing considerably the usual way of constructing mass formulae for the heavy baryons either within the quark model or in the QCD sum rules method.

The plan of the report is as follows.

In section II, we discuss the problem of mixing of the baryon wave functions in the quark model Ono_1978.

Then the obtained results are used to construct wave functions of the baryons which are used to calculate masses and magnetic moments of the baryons.

In the last section, we apply the same method to construct interpolating QCD currents and modify the sum rules.

In conclusion, we discuss our results

Mixing of the cascade baryons Ξ_c , Ξ_c` in the quark model

Let us consider standard non-relativistic quark model (NRQM) wave functions of the charm cascade baryons Ξ_c 's of the SU(4) 20`- plet, having in their content three different quarks **u,d,s,c**.

Upon reduction to SU(3) these baryons occur in the sum of sextet and triplet representations of SU(3) ($20`_4=8_3+6_3+3_3+3_3$). One can choose the wave functions of the Λ- like baryons Ξ_c with the quark content (usc) as

$$\sqrt{6}B^4_{[24]}=\sqrt{6}\Xi_c([us]c)=-c_1\, u_1\, s_2-\, u_1\, c_1\, s_2+s_1\, c_1\, u_2+c_1s_1u_2$$

$$\sqrt{6}B^3_{[23]}=\sqrt{6}\Xi_c([uc]s)=+s_1\, u_1\, c_2+u_1\, s_1\, c_2-s_1\, c_1\, u_2-c_1s_1u_2$$

$$\sqrt{6}B^1_{[21]}=\sqrt{6}\Xi_c([sc]u)=-s_1\, u_1\, c_2-u_1\, s_1\, c_2+s_1\, u_1\, s_2+u_1c_1s_2$$

Here $q_{1,\,2}$ means spin (up, down), respectively

Since the sum of these states is equal to zero, only two of the states are linearly independent. But any two of them are not orthogonal to each other, so we construct to every state three possible orthogonal combinations, corresponding to Σ- like baryons:

$-\sqrt{6}(B^1_{[21]} - B^3_{[23]}) = 2\sqrt{3}\Xi`_c(\{us\}c) =$

$= 2s_1 u_1 c_2 + 2u_1 s_1 c_2 - c_1 u_1 s_2 - u_1 c_1 s_2 - s_1 c_1 u_2 - c_1 s_1 u_2$

$-\sqrt{6}(B^4_{[24]} - B^1_{[21]}) = 2\sqrt{3}\Xi`_c(\{uc\}s) =$

$= 2c_1 u_1 s_2 + 2u_1 c_1 s_2 - s_1 u_1 c_2 - u_1 s_1 c_2 - s_1 c_1 u_2 - c_1 s_1 u_2$

$\sqrt{6}(B^4_{[24]} - B^3_{[23]}) = 2\sqrt{3}\Xi`_c(\{sc\}u) =$

$= 2c_1 s_1 u_2 + 2s_1 c_1 u_2 - s_1 u_1 c_2 - u_1 s_1 c_2 - s_1 u_1 s_2 - u_1 c_1 s_2$

In principle, one can choose any pair of charm cascade baryons Ξ_c, $\Xi`_c$.

Rotating the 1st lines by 60^0 one obtains the states of the 2nd line,

while rotating them by 120^0 one obtains the states of the 3rd line.

Upon reduction of the SU(4) 20'- plet to the SU(3) multiplets

along the values of charm, C=0,1,2, one can see that

Λ_c^+([ud]c), Ξ_c^+([us]c) and Ξ_c^0([ds]c) form the anti-triplet while

Σ_c^+({ud}c), $\Xi_c^{+`}$({us}c) and $\Xi_c^{0`}$({ds}c) enter the SU(3) sextet.

However there is no reason to expect that the experimentally observed charm cascade baryons Ξ_c , Ξ_c` should belong to the pure anti-triplet or sextet states. Upon choosing another pair of the states $\Xi_c$ , $\Xi_c$` these anti-triplet and sextet states mix similar to mixing of the initial pure states of the octet ω_0 state and unitary singlet φ_0 state yielding the observed vector nonet mesons ω and φ.

As an example, let us choose a pair of charm cascade baryons Ξ_c , Ξ_c ` with the quark content (usc).

Formally, masses of the Ξ_c , Ξ_c` are defined as

M_{ Ξ_c }= < Ξ_c | m | Ξ_c >,   M_{ Ξ_c`}=< $\Xi_c$ `| m | Ξ_c` >

where `m` is a mass operator in NRQM {Ono_1978}.

Using group theory, masses of all (usc) cascade baryons

are related to each other as

$M(\Xi_c`(\{us\}c)) + 3M(\Xi_c([us]c)) = 2M(\Xi_c`(\{uc\}s)) + 2M(\Xi_c`(\{cs\}u))$,

$M(\Xi_c([us]c)) + 3M(\Xi_c`(\{us\}c)) = 2M(\Xi_c([uc]s)) + 2M(\Xi_c([cs]u))$,

Analogous relations exist for other choices of the cascade pair:

$M(\Xi_c`(\{uc\}s)) + 3M(\Xi_c([uc]s)) = 2M(\Xi_c`(\{us\}c)) + 2M(\Xi_c`(\{cs\}u))$,

$M(\Xi_c([uc]s)) + 3M(\Xi_c`(\{uc\}s)) = 2M(\Xi_c([us]c)) + 2M(\Xi_c([cs]u))$,

and

$M(\Xi_c`(\{cs\}u)) + 3M(\Xi_c([cs]u)) = 2M(\Xi_c`(\{uc\}s)) + 2M(\Xi_c`(\{us\}c))$,

$M(\Xi_c([cs]u)) + 3M(\Xi_c`(\{cs\}u)) = 2M(\Xi_c([uc]s)) + 2M(\Xi_c([us]c))$, In general, the off-diagonal terms are not zero.

Upon using group-theory relations

non-diagonal mass terms of the pair Ξ_c ({uc}s) and Ξ_c \` ({uc}s) could be written in terms of the other states as

$$\sqrt{3}M\,(\Xi_c`\,(\{us\}c)\Xi_c\,([us]c)) = M\,(\,\Xi\,`\,(\{uc\}s)) - M\,(\,\Xi\,`\,(\{cs\}u)) =$$

$$= -M\,(\,\Xi_c\,([uc]s)) + M\,(\,\Xi_c\,([cs]u)),$$

and similarly for the other two pairs of the heavy cascade baryons:

$$\sqrt{3}M\,(\Xi_c`\,(\{uc\}s)\Xi_c\,([uc]s)) = -M\,(\,\Xi_c`\,(\{us\}c)) + M\,(\,\Xi_c`\,(\{cs\}u)) =$$

$$= M\,(\,\Xi_c\,([us]c)) - M\,(\,\Xi_c\,([cs]u)),$$

and

$$\sqrt{3}M\,(\Xi_c`\,(\{cs\}u),\Xi_c\,([cs]u)) = M\,(\,\Xi_c`\,(\{us\}c)) - M\,(\,\Xi_c`\,(\{uc\}s)) =$$

$$= -M\,(\,\Xi_c\,([us]c)) + M\,(\,\Xi_c\,([cs]u)),$$

It is rather obvious that only for some particular choice of parameters the non-diagonal mass terms are equal to zero.

For example, isotopic invariance leads to the vanishing of these non-diagonal mass terms for the baryons Λ([ud]h) and Σ({ud}h), h=s,c,b.

But what masses should we choose then to compare them with experiment?

It is clear that the states of definite mass and their masses can be

obtained by diagonalizing the mass matrix of the chosen model Ono_1978

$$M\{\Xi_c\} = \begin{pmatrix} M(\Xi_c`) & M(\Xi_c\,\Xi_c`) \\ M(\Xi_c`\,\Xi_c) & M(\Xi_c) \end{pmatrix}$$

The corresponding secular equation yields the physical masses to be:

$$2M(\Xi_c^{\ 1,2}) = (M(\Xi_c) + M(\Xi_c`) \pm [(M(\Xi_c) - M(\Xi_c`))^2 + 4\, M\_(\Xi_c\,\Xi_c`)^2]^{1/2},$$

where the off-diagonal elements are assumed to be equal,

i.e. $M(\Xi_c\,\Xi_c`) = M(\Xi_c`\Xi_c)$.

(Maybe the first time it was written by Jerold Franklin PR D. Recently this formula

was written also by Lipkin et al. arXiv:0706.2163 (2007)

The values of these masses do not depend on which pair of Ξ_c , $\Xi_c`$

is chosen upon the mode of grouping quarks in "diquarks",

since the sum $(M(\Xi_c) + M\_(\Xi_c`)$ as well as the

square root are invariant under rotations in the flavor space by 60^0 and 120^0.

To obtain the representation of the observable

particles, the Ξ_c , Ξ_c\` should be rotated by some angle α

$\Xi_c`_\alpha = \Xi_c` \cos\alpha + \Xi_c \sin\alpha$

$\Xi_{c\ \alpha} = - \Xi_c` \sin\alpha + \Xi_c \cos\alpha$.

Requiring that the off-diagonal elements of the mass matrix for these

newly defined states to be zero, the rotation angle α should be chosen as:

$\tan 2\alpha = 2 < \Xi_c` \mid m \mid \Xi_c > / [M(\Xi_c`) - M(\Xi_c)]$

It would be natural to define these diagonalized states as the physical ones.

In this case a quark structure of the baryon (in the given model!)

would be a superposition of the states Ξ_c and Ξ_c\`.

Quark model for masses of new baryons

Let us apply the approach presented in the previous section to heavy baryons within the model of Ono_1978.

Already more than 20 years ago masses of heavy baryons with new quantum numbers were calculated within quark models (cf., e.g., Ono_1978 and Verma_1988).

It is of interest to note that the predictions on the masses of heavy baryons of these models are in surprisingly good agreement with the modern data.

In the report the mass operator of Ono_1978 is used due to its simplicity and clearness:

$M_B = m_0 + \sum_{i=1}^{3} m_i + \chi \sum_{i>j} \mathbf{S}_i \mathbf{S}_j \; \boldsymbol{/}_{(} m_i \, m_{j)}$,

where m_0 is an overall constant, $m_0 = 77$ MeV, and $\chi = 22.05\ 10^{-3}$ GeV^3;

S_q is the spin operator of the quark q.

Quark masses are taken from Ono_1978:

$m_u = m_d = 336$ MeV, $m_s = 510$ MeV, $m_c = 1680$ MeV, $m_b = 5000$ MeV.

Charm and beauty cascade hyperons

Masses of charm and beauty Ξ and Ω baryons were calculated in Ono_1978 for particular quark combinations of baryons.

We have performed calculations for all the possible quark combinations in baryons with the same formula and put results into the Table 1.

As an example, the analysis for baryons with two different heavy quarks forming SU(3) triplets $\Xi`_{cb}^{+,0}$, $\Omega`_{cb}^{0}$ and $\Xi_{cb}^{+,0}$, Ω_{cb}^{0} is presented below.

We would write in some detail calculations for the pair $\Xi_{cb}^{+,0}$, $\Xi`_{cb}^{+,0}$.

(1) We begin with the quark content proposed in Ono_1978, i.e.

let **b** be a single quark while the pair **(uc)** in (anti)symmetric state form a diquark

For the diagonal elements of the mass matrix, we get

$M_{\Xi cb}([cu]b) = m_0+m_u+m_c+m_b+\chi\,(-3/4)\,(1/m_u\, m_c) =$

$= 7093+22.05(-3/4)\ 2.976^2/5 = 7093 - 29.3 = 7063.7$ MeV; (7064 Ono_1978)

$M_{\Xi`cb}(\{cu\}b)=m_0+m_u+m_c+m_b+(\chi/4)\,(-2/m_u\, m_b\ - 2/m_c\, m_b +1/m_u\, m_c)=$

$=7093-6.5625-1.3125+9.7656=7093+1.891 = 7094.9$ MeV (7095 Ono_1978)

$[M_{\Xi cb}([cu]b) + M_{\Xi`cb}(\{cu\}b)]/2= 7079.3;$

$[M_{\Xi cb}([cu]b) - M_{\Xi`cb}(\{cu\}b)]/2= -31.2;$

and the non diagonal matrix elements are:

$\sqrt{3} < \Xi_{cb}` \mid m \mid \Xi_{cb} > = (3\chi/4)\,(1/m_u\, m_b\ - 1/m_c\, m_b) \sim 7.9$ MeV

Resolving the secular equation

$x_{1,2}$=7079.3± (1/2)[31.2^2+(3/4)7.9^2]$^{1/2}$ = 7079.3 ± 16.4,

one obtains the mass eigenvalues as

x_1=7095.7 MeV, x_2=7063.5 MeV.

In order to go from the initial states $\Xi_{cb}^{+,0}$, $\Xi`_{cb}^{+,0}$ to those with the masses $x_{1,2}$

one should rotate them at the angle α=16.3^0/2=8.15^0, tan 2α=0.2924.

(2) Now, let the quarks **c** and **b** form a diquark while the light quark **u**

is the single one. In this case

$M_{\Xi cb}([cb]u)=m_0+m_u+m_c+m_b+\chi\,(-3/4)\,(1/m_b\, m_c)=$

=7093+22.05(− 3/4) 2.796/25 = 7091 MeV,

$M_{\Xi` cb}(\{cb\}u)=m_0+m_u+m_c+m_b+(\chi/4)\,(-2/m_u\, m_c\ -2/m_u\, m_b+1/m_b\, m_c)=$

=7093−19.53−6.5625+0.65625=7093-25.43625 = 7067.6 MeV.

[$M_{\Xi cb}([cb]u) + M_{\Xi` cb}(\{cb\}u)$]/2 = 7079.3;

[$M_{\Xi cb}([cb]u) - M_{\Xi` cb}(\{cb\}u)$]/2 = 23.4 .

The non-diagonal mass matrix element is now

$\sqrt{3} < \Xi_{cb}`(\{cb\}u) \mid m \mid \Xi_{cb}([cb]u)> = (3\chi/4)\,(1/m_c\, m_u - 1/m_b\, m_u) \sim 19.4$ MeV

wherefrom the eigenvalues are obtained as follows

x_1=7095.2 MeV, x_2=7062.8 MeV.

In order to go from the initial states $\Xi_{cb}^{+,0}$, $\Xi`_{cb}^{+,0}$ to those with the masses $x_{1,2}$

one should rotate them at the angle α=136.25^0/2=68.13^0, tan 2α= − 0.9573.

(3) Now quarks `u` and `b` form the diquark while charm quark `c` is single.

Then, the masses of the cascade baryons are given by:

$M_{\Xi cb}([ub]c)=m_0+m_u+m_c+m_b+\chi\ (-3/4)\ (1/m_b\ mu\ )=$

$=7093+22.05(-3/4)\ 1/5\ \ 2.796=7093-9.9=7083.1$ MeV;

$M_{\Xi`cb}(\{ub\}c)=m_0+m_u+m_c+m_b+(\chi/4)\ (-\ 2/m_u\ m_c\ \ -2/m_b\ m_c\ +1/m_b\ m_u\ )=$

$=7093-27.7=7075.3$ MeV;

$[M_{\Xi cb}([ub]c)+M_{\Xi`cb}(\{ub\}c)]/2=7079.2$;

$[M_{\Xi cb}([ub]c)-M_{\Xi`cb}(\{ub\}c)]/2\ =7.7$.

The non-diagonal mass matrix element being

$\sqrt{3} < \ \Xi_{cb}`(\{ub\}c)\ |\ m\ |\ \Xi_{cb}\ ([ub]c)> =\ (3\chi/4)\ (1/m_u\ m_c\ \ -\ 1/m_b\ m_c\ ) \sim 27.7$ MeV

wherefrom the eigenvalues are obtained as follows

$x_1=7095.7$ MeV, $x_2=7062.9$ MeV.

In order to go from the initial states $\Xi_{cb}^{+,0}$, $\Xi`_{cb}^{+,0}$ to those with the masses $x_{1,2}$

one should rotate them at the angle $\alpha=256.5^0/2=128.25^0$, tan $2\alpha= 4.1539$.

Similar calculations are done also for other charm and beauty baryons and the

results are presented in Table 1.

Table 1. Baryon masses (in MeV) and mixing angles in the Ono model

Baryon	$\{q_1q_2\}q_3$	$[q_1q_2]q_3$	$\sqrt{3}<\Xi`\|m\|\Xi>$	x_1	x_2	α
Ξ_c((us)c)	2603	2504	9.9	2603	2503	3.3^0
Ξ_c((us)c)	2523.8	2558	69.3	2603	2503	63.25^0
Ξ_c((us)c)	2534	2573.3	79.2	2603	2503	123.3^0
Ξ_b((us)b)	5945	5824	3.3	5945	5824	0.9^0
Ξ_b((us)b)	5852.6	5916.4	89.1	5944	5824	60.8^0

Ξ_b((us)b)	5855.9	5913.1	92.4	5945	5824	120.9^0
Ξ_{cb}((uc)b)	7094.9	7963.7	7.9	7095.7	7063.5	8.15^0
Ξ_{cb}((cb)u)	7067.6	7091.0	19.4	7095.2	7062.8	68.13^0
Ξ_{cb}((ub)c)	7075.3	7083.1	27.7	7095.9	7068.9	128.25^0
Ω_{cb}((sc)b)	7267.9	7247.2	4.62	7268.24	7246.86	7.22^0
Ω_{cb}((cb)s)	7250.0	7265.0	13.2	7268.24	7246.86	67.27^0
Ω_{cb}((sb)c)	7254.7	7260.4	17.82	7268.23	7246.87	127.26^0

Magnetic moments of the double-flavored baryons

From our analysis, it follows that the predictions on the mass of the physically observed baryons does not depend on the particular quark construction of the baryons in the given model.

However the wave function of the baryon with the diagonal mass matrix as well as all the characteristics of the baryon such as magnetic moments, weak decay constants, strong couplings and etc. depend upon these particular construction through the mixing angle α.

As an application of our approach, let us consider the magnetic moments of the double-flavored baryons Ξ_{cb}^{+} and $\Xi`_{cb}^{+}$.

Magnetic moment of the baryon with the quark content Ξ_{cb}^{+} ([cb]u), which is used often in the modern works (cf., e.g., Faustov_2005), in the NRQM is equal to the u-quark magneton μ_u, while that of $\Xi`_{cb}^{+}$({cb}u) is equal to $(2\mu_c+2\mu_b-\mu_u)/3$.

However baryons of this quark content in the model of Ono_1978 have large non-diagonal mass terms, so one should choose as the wave function of the baryon a linear combination of baryons Ξ_{cb}^{+} and $\Xi`_{cb}^{+}$ with the mixing angle $\alpha=68.13^0$ (cf. Table 1).

Then for the magnetic moments we have:

$<\Xi_{cb}^{+\,\alpha} \mid \mu \mid \Xi_{cb}^{+\,\alpha}> =$

$=(2\mu_c+2\mu_b-\mu_u)/3 \sin^2\alpha+\mu_u \cos^2\alpha - 2/\sqrt{3} \sin\alpha \cos\alpha \, (\mu_c - \mu_b) =$

$= (1- 4/3 \sin^2\alpha) \, \mu_u = - 0.148 \, \mu_u$;

$<\Xi`_{cb}^{+\,\alpha} \mid \mu \mid \Xi`_{cb}^{+\,\alpha}> =$

$=(2\mu_c+2\mu_b-\mu_u)/3 \cos^2\alpha+\mu_u \sin^2\alpha + 2/\sqrt{3} \sin\alpha \cos\alpha \, (\mu_c - \mu_b) =$

$= (-1/3 + 4/3 \sin^2\alpha) \, \mu_u = 0.815 \, \mu_u$.

where we have neglected μ_c and μ_b in comparison to μ_u .

These predictions do not depend on the particular choice of the baryon pair Ξ_{cb}^{+} and $\Xi`_{cb}^{+}$ provided one takes the mixing angle α corresponding to the given combination, and differ considerably from the predictions given by the "pure" states $\Xi_{cb}^{+}$ ({cb}u) and $\Xi`_{cb}^{+}$ ({cb}u) (cf. Table 2).

Table 2. Magnetic moments of double-flavored baryons

in NRQM for "pure" and mixed states (with index α)

Baryon	NRQM	NRQMα
Ξ_{cb}([cb]u)	μ_u	$-0.148\mu_u$
Ξ_{cb}({cb}u)	$-(1/3)\ \mu_u$	$0.815\mu_u$
Ξ_{cb}([cu]b)	$\mu_b \sim 0$	$-0.148\mu_u$
Ξ_{cb}({cu}b)	$(2/3)\ \mu_u$	$0.815\mu_u$
Ξ_{cb}([ub]c)	$\mu_c \sim 0$	$-0.148\mu_u$
Ξ_{cb}({ub}c)	$(2/3)\ \mu_u$	$0.815\mu_u$

Mixing of the states Ξ_{cb} , Ξ_{cb}` in QCD sum rules

The problem of the Ξ_{cb} , Ξ_{cb}` mixing can also be analyzed within the QCD sum rules framework. The main difference in this case is that, rather than working with the mass matrix, one deals with correlation functions of interpolating currents, i.e. the role of mass matrix elements is played by correlators {SVZ_1978}{Ioffe_1981}

$$\Pi^{\Xi cb\,,\,\Xi cb\,`} = i \int d^4 x\, e^{ipx} \langle 0|\, T\, \{\eta^{\Xi cb\,,\,\Xi cb\,`}(x),\, \eta^{\Xi cb\,,\,\Xi cb\,`}(0)\}\, |0\rangle,$$

which are calculated,

(1) in QCD, using OPE, and

(2) by inserting a complete set of physical states.

Performing Borel transformation to suppress high--excited states and equating both expansions one obtains QCD sum rules Ioffe_1981.

In this case, mixing causes the non-diagonal correlation functions to have non-zero values.

Prior to performing calculations of the physical properties, such as the magnetic moments, meson couplings etc., of the baryons, one should make sure that the corresponding interpolating currents have zero non-diagonal correlators.

To find the combination of $\eta^{\Xi cb}$ and $\eta^{\Xi cb`}$ that have zero non-diagonal correlator, consider the following interpolating currents that are obtained from them after a rotation by α:

$\eta_\alpha` = \eta^{\Xi cb`} \cos \alpha + \eta^{\Xi cb`} \sin \alpha, \qquad \eta_\alpha = - \eta^{\Xi cb`} \sin \alpha + \eta^{\Xi cb`} \cos \alpha,$

where the mixing angle α should be chosen such that:

$\Pi_\alpha^{\Xi cb\,,\,\Xi cb`} = i \int d^4 x\, e^{ipx} <0|\, T\, \{\eta_\alpha (x), \eta_\alpha^{`} (0)\}\, |0> = 0$

goes to zero.

Let us study this problem on a simplified toy-model using the QCD mass sum rules Zhu_1997 written for the octet hyperons Σ and Λ. Generalization to our case seems not to be without problems but is sufficient for our purposes.

Omitting the vacuum expectation values of the quarks c and b and neglecting mass of the u quark in the QCD sum rules, we construct following Zhu_1997 the QCD mass sum rule for the baryon Ξ_{cb}`({ub}c)

$\Pi_\alpha^{\ \Xi cb\,`}(M^2) = (M^6/8L^{4/9})E_2 + (b\ M^2/\ 32\ L^{4/9}) +(\ M^2/\ 4\ L^{4/9})\ a_u\ m_b\ E_0 - (3m_0^{\ 2}/\ 48$

$L^{26/27})a_u m_b = \beta_{\Xi`}^{\ 2}\ e^{-(M\ \Xi`2\ /\ M2)}$,

where a_q , b and $a_q\ m_0^{\ 2}$ are defined as Ioffe_1981:

$a_q = -\ (2\pi)^2 <qq>$, $b=< g_c G^2>$, $L=\ln(M^2/\Lambda^2)/\ln(\mu^2/\ \Lambda^2)$,

$a_q\ m_0^{\ 2}=(2\pi)^2 < g_c\, q\ (\sigma\ G)\, q>$, q=u,d,s,

μ being renormalization point, while G is a gluon field

with the coupling g_c to quarks.

The corresponding sum rule for the Λ- like baryon Ξ_{cb} ([ub]c) reads:

$(M^6/\ 8\ L^{4/9})E_2 + (b\ M^2/\ 32\ L^{4/9}) +(\ M^2/\ 12\ L^{4/9})\ a_u\ (2m_c - m_b)\ E_0 - (3m_0^{\ 2}/\ 48\ L^{26/27})\ a_u$

$(2m_c - 4m_b)\ \ = \beta_{\Xi}^{\ 2}\ e^{-(M\ \Xi 2\ /\ M2)}$,

The non-diagonal correlation function after Borel transformation can also be

written as:

$\Pi_\alpha^{\ \ \Xi cb\ \ \Xi cb\,`}\ (M^2) = [(\ M^2/\ 6\ L^{4/9})\ E_0 - m_0^{\ 2}/\ 24\ L^{26/27}]\ a_u\ m_c$

Requiring that $\Pi_\alpha^{\ \ \Xi cb\ ,\ \Xi cb`} = 0$ leads to the value of the mixing angle given by:

$\tan 2\alpha_{(ub)c} = \sqrt{3}m_{c\,/}\,(2m_b - m_c) \sim 0.342$.

For the other pair of heavy baryons Ξ_{cb}\` ({uc}b) and Ξ_{cb} ([uc]b)

one should just change c → b, wherefrom

$\tan 2\alpha_{(uc)b} = \sqrt{3}m_b / (m_b - 2m_c) \sim 5.18$.

Finally for baryons Ξ_{cb}\` ({cb}u) and Ξ_{cb} ([cb]u) one obtains

$\tan 2\alpha_{(cb)u} = \sqrt{3}(m_c - m_b) / (mc + mb) \sim -0.872$.

These formulae are transformed from one into the other

by shifting the angle α by 60^0 and 120^0.

At m_c=1650 MeV, m_b =5 GeV one obtains

$\alpha_{(ub)c} \sim 9.5^0$, $\alpha_{(cb)u} \sim 69.5^0$, $\alpha_{(uc)b} \sim 129.5^0$.

It is of interest to note that mass relations of QCD in this approximation

yield somewhat unexpected result that the minimal mixing angle

favors the diquark pair (ub) while others lead to large mixing angles.

Calculation of the magnetic moments of the double-flavored baryons

in the quark model with the mixing angles from the QCD toy-model

yield practically the same results as the quark model of Ono_1978.

Conclusion

We have tried to show the importance of mixing

of heavy cascade baryons Ξ_{cb} , Ξ_{cb}` in analysis of their characteristics.

As an example, the non-relativistic quark model of Ono_1978 is used.

The same approach is applied to the interpolating currents

of these baryons in the framework of the QCD sum rules

which is shown on the example of simplified mass QCD sum rules.

The main conclusion is that in any given model of heavy baryons,

one should first find the quark configuration leading to vanishing diagonal

matrix elements. After finding the physical states,

one should perform calculations of the various characteristics of these baryons.

In the case of the QCD sum rules where there is no mass formulae

in the common sense of the word, the problem of the truthful

combination of interpolating currents deserves further study.

Table 1. Baryon masses (in MeV) and mixing angles in the Ono model

Baryon	$\{q_1q_2\}q_3$	$[q_1q_2]q_3$	$\sqrt{3}<\Xi`\|m\|\Xi>$	x_1	x_2	α
Ξ_c((us)c)	2603	2504	9.9	2603	2503	3.3^0
Ξ_c((us)c)	2523.8	2558	69.3	2603	2503	63.25^0
Ξ_c((us)c)	2534	2573.3	79.2	2603	2503	123.3^0
Ξ_b((us)b)	5945	5824	3.3	5945	5824	0.9^0
Ξ_b((us)b)	5852.6	5916.4	89.1	5944	5824	60.8^0

$\Xi_b((us)b)$	5855.9	5913.1	92.4	5945	5824	120.9^0
$\Xi_{cb}((uc)b)$	7094.9	7963.7	7.9	7095.7	7063.5	8.15^0
$\Xi_{cb}((cb)u)$	7067.6	7091.0	19.4	7095.2	7062.8	68.13^0
$\Xi_{cb}((ub)c)$	7075.3	7083.1	27.7	7095.9	7068.9	128.25^0
$\Omega_{cb}((sc)b)$	7267.9	7247.2	4.62	7268.24	7246.86	7.22^0
$\Omega_{cb}((cb)s)$	7250.0	7265.0	13.2	7268.24	7246.86	67.27^0
$\Omega_{cb}((sb)c)$	7254.7	7260.4	17.82	7268.23	7246.87	127.26^0

Table 2. Magnetic moments of double-flavored baryons

in NRQM for "pure" and mixed states (with index α)

Baryon	NRQM	NRQMα
Ξ_{cb}([cb]u)	μ_u	$-0.148\mu_u$
Ξ_{cb}({cb}u)	$-(1/3)\ \mu_u$	$0.815\mu_u$
Ξ_{cb}([cu]b)	$\mu_b \sim 0$	$-0.148\mu_u$

$\Xi_{cb}(\{cu\}b)$	$(2/3)\ \mu_u$	$0.815\mu_u$
$\Xi_{cb}([ub]c)$	$\mu_c \sim 0$	$-0.148\mu_u$
$\Xi_{cb}(\{ub\}c)$	$(2/3)\ \mu_u$	$0.815\mu_u$

S.Okubo, Phys. Rev. D11, 3261-3269 (1975).

A.De Rujula, H.Georgi, S.Glashow, Phys. Rev. D 12, 147-162 (1975).

XPDG08} C.Amsler et al.(Particle Data Group), Phys. Lett. B 667, 1 (2008),

R. Mizuk, arXiv:0712.0310 (2007)

Seiji Ono,"States of hadrons in a five-quark model", Phys. Rev. D 17, 888 (1978).

C.P.Singh, "Masses of the charmed and b-quark hadrons in a quark model",

Phys. Rev. D 24, 2481 (1981).

D.B.Lichtenberg et al., Zeitschr. fur Phys. C 19, 19 (1983); Phys. Rev. Lett. 48,

1653 (1982).

S.Capstick and N.Isgur,Phys. Rev.D 34, 2809 (1986).

L.H.Chan, Phys. Rev. D31, 642 (1985).

W.Y.P.Hwang and D.B.Lichtenberg, Phys. Rev. D35, 3526 (1987).

R.Roncaglia, D.B.Lichtenberg, E.Predazzi, "Predicting the masses of baryons

containing one or two heavy quarks", Phys. Rev. D52, 1722 (1995).

L.Gelmi, V.S.Zamiralov, Moscow State UniversityPhysics bulletin, 26, 27 (1985).

L.Gelmi, V.S.Zamiralov, S.N.Lepshokov, Moscow State University Physics

bulletin, 28, 70 (1987).

R.C.Verma and S.Srivastava, Phys. Rev. D38, 1623 (1988).

D.Ebert, R.N.Faustov and V.O.Galkin,"Masses of heavy baryons in the relativistic quark model", Phys. Rev. D 72, 034026 (2005).

Armand Faessler, Th. Gutsche, M.A.Ivanov,J.C.Korner, V.E.Liubovitskij, D.Nicmorus, K.Pumsa-ard, "Magnetic moments of heavy baryons in the relativistic three-quark model", Phys. Rev. D73, 094013 (2006).

B.Silvestre-Brac, F.Brau, and C.Semay,J.Phys. G29, 2685 (2003).

N.Mathur, R.Lewis, and R.Woloshyn,"Charmed and bottom baryons frome lattice nonrelativistic QCD",Phys. Rev. D66, 014502 (2002).

B.L.Ioffe, Nucl. Phys. B188, 317 (1981);

V.M.Belyev, B.L.Ioffe, JETP 83, 876 (1982) (in Russian);

V.M.Belyev, B.L.Ioffe, JETP 84, 1236 (1983) (in Russian).

P. Colangelo and A. Khodjamirian, At the Frontier of Particle Physics / Handbook of QCD, ed. by M. Shifman (World Scientific, Singapore, 2001). Vol. 3, p1495

M. Karliner, B. Kere--Zur, H. J. Lipkin, and J. L. Rosner, arXiv: 0706.2163 (2007).

A.Ozpineci, S.B.Yakovlev, and V.S.Zamiralov, Mod. Phys. Lett. A20, 243 (2005).

S.-L. Zhu, W.-Y. P. Hwang, and Z.S.Yang, Phys. Rev. D 56, 7273 (1997).