Bulk viscosity for high amplitude oscillations

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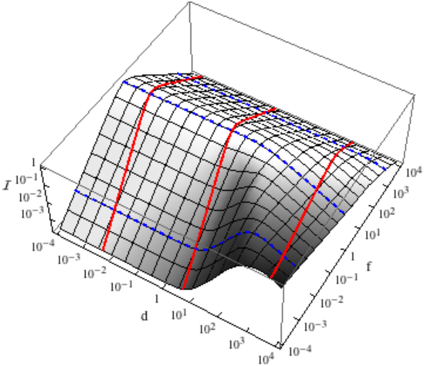
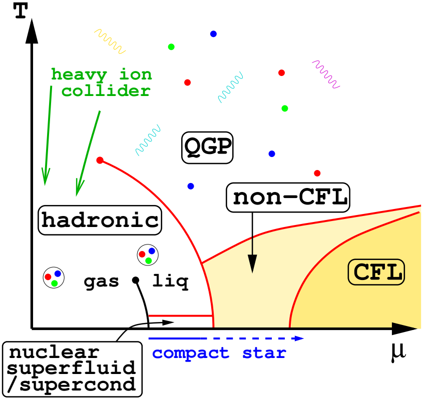
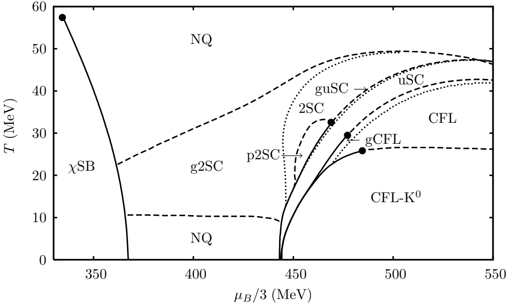


M. Alford, S. Mahmoodifar,
K. Schwenzer, arXiv:1005.3769

QCD phase diagram



NJL model, uniform phases only



Warringa, hep-ph/0606063

But there are also non-uniform phases, such as the crystalline ("LOFF" / "FFLO") phase. (Alford, Bowers, Rajagopal, hep-ph/0008208)

Signatures of color superconductivity in compact stars

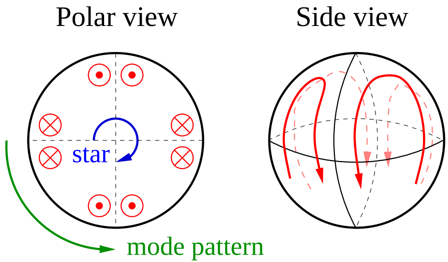
Pairing energy { affects Equation of state. Effects may be subtle.
(Alford, Braby, Paris, Reddy, `nucl-th/0411016`,
Kurkela, Romatschke, Vuorinen, `arXiv:0912.1856`)

Gaps in quark spectra and Goldstone bosons { affect Transport properties:
emissivity, heat capacity, viscosity (shear, bulk),
conductivity (electrical, thermal)...

1. Gravitational waves: r-mode instability, shear and bulk viscosity
2. Glitches and crystalline ("LOFF") pairing
3. Cooling by neutrino emission, neutrino pulse at birth

r-modes and gravitational spin-down

An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



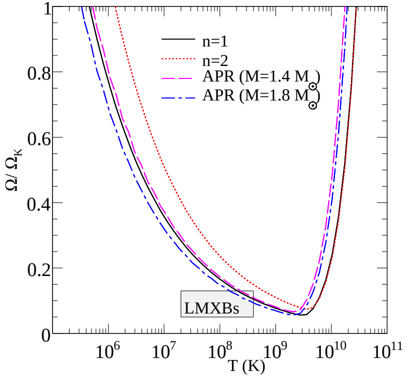
The unstable *r*-mode can spin the star down very quickly, exactly how fast depends on the amplitude at which it saturates (Andersson `gr-qc/9706075`; Friedman and Morsink `gr-qc/9706073`; Lindblom `astro-ph/0101136`).

So if we see a star spinning quickly, we can infer that the interior viscosity must be high enough to damp the *r*-modes.

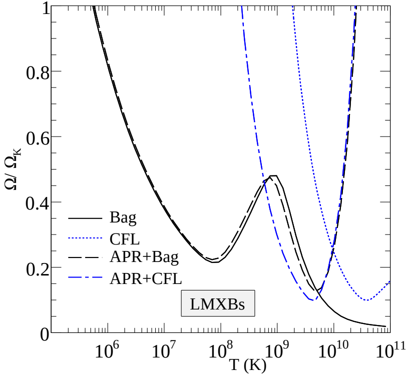
Constraints from r-modes: current results

Regions above curves are forbidden ⇐ viscosity is too low to damp *r*-modes.

Nuclear matter



Quark matter



(Jaikumar, Rupak, Steiner `arXiv:0806.1005`)

But r-modes grow exponentially, so large-amplitude effects must be included.

What is bulk viscosity?

Energy consumed in a compression cycle:

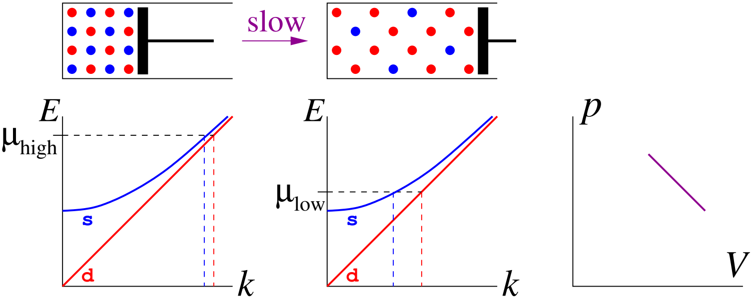
$$V(t) = \bar{V} + \delta V \sin(\omega t)$$
$$p(t) = \bar{p} + \delta p \sin(\omega t + \phi)$$

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\zeta}{\tau} \int_0^\tau (\mathrm{div}\, \vec{v})^2 dt = \frac{\zeta}{2} \omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}} \int_0^\tau p(t) \frac{dV}{dt} dt$$

- Bulk viscosity arises from re-equilibration processes.
- If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to τ, then pressure gets out of phase with volume.
- The driving force then does net work in each cycle.
- There is an exact analogy with V and Q in an R-C circuit.

Bulk viscosity and pressure phase lag

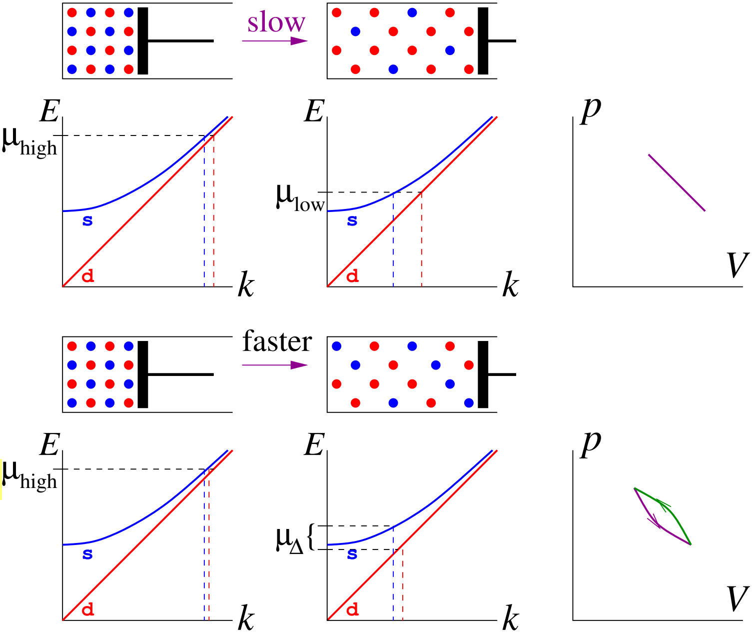
$\mu_d = \mu_s$



Bulk viscosity and pressure phase lag

$\mu_d = \mu_s$

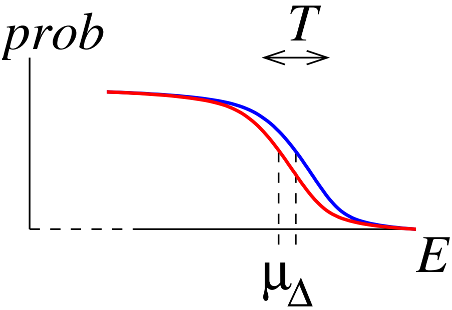
$\mu_\Delta = \mu_d - \mu_s$



Subthermal vs Suprathermal

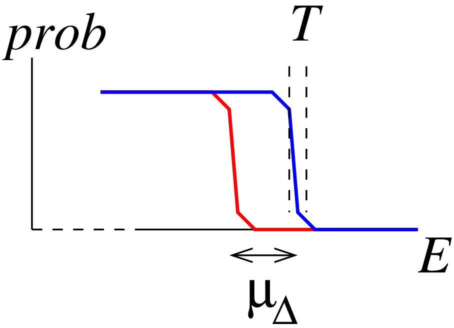
Subthermal

$\mu_\Delta \ll T \ll \mu$



Suprathermal

$T \ll \mu_\Delta \ll \mu$



Madsen, Phys. Rev. D46,3290 (1992); Reisenegger, Bonacic, `astro-ph/0303454`

Calculating bulk viscosity

- Compression at freq ω, so conserved charges oscillate as $n(t) = n_{\rm avg} + \delta n \sin(\omega t)$
- One quantity "Δ" goes out of equilibrium (eg $S - D$ in quark matter). In equilibrium, $\mu_\Delta = 0$.
- EoS is characterized by susceptibilities B,C.

$$\zeta = -\frac{1}{\pi}\frac{\bar{n}}{\delta n}\frac{C}{B}\int_0^\tau \mu_\Delta(t)\cos(\omega t)dt$$

Bulk visc arises from component of μ_Δ that lags behind the forcing oscillation by a phase of 90°; $\mu_\Delta(t)$ is given by

$$\frac{d\mu_\Delta}{dt} = \underbrace{C\omega\frac{\delta n}{n_{\rm avg}}\cos(\omega t)}_{\text{forcing osc.}} - \underbrace{\Gamma(\mu_\Delta, T)}_{\text{equilibration}}$$

Re-express this in dimensionless variables:

Computing departure from equilibrium

- Define dimensionless time (i.e. phase) $\varphi = \omega t$
- Define dimensionless departure from equilibrium $\bar{\mu}_\Delta = \mu_\Delta / T$
- Driving coeff $d = \dfrac{C}{T}\dfrac{\delta n}{n_{\text{avg}}}$
- Equilibration rate: $\Gamma(\mu_\Delta, T) = \tilde{\Gamma}\, T^\kappa\, \gamma(\mu_\Delta/T)$.

  Equilibration coeff $f = \dfrac{B}{\omega}\tilde{\Gamma}\, T^\kappa$.

$$\boxed{\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\gamma(\bar{\mu}_\Delta)}$$

Dependence on density, EoS, driving amplitude, and temperature is contained in d and f. Dependence of equilibration rate on $\bar{\mu}_\Delta$ is contained in $\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta + \chi_1 {\bar{\mu}_\Delta}^3 + \cdots$ **[series terminates]**.

Suprathermal and subthermal bulk viscosity

Subthermal: assume $\bar{\mu}_\Delta \ll 1$ (i.e. $\mu_\Delta \ll T$), so $\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta$,

$$\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\,\bar{\mu}_\Delta$$

$$\bar{\mu}_\Delta(\varphi) = -\frac{f\,d}{1+f^2}\cos\varphi + \frac{d}{1+f^2}\sin\varphi$$

$$\zeta_{\rm sub} = \frac{C^2}{B\omega}\frac{f}{1+f^2} = \frac{C^2}{B}\frac{\gamma_{\rm eff}}{\omega^2+{\gamma_{\rm eff}}^2} \qquad (\gamma_{\rm eff} \equiv B\tilde{\Gamma}T^\kappa)$$

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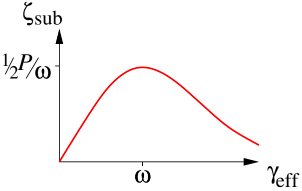
Suprathermal: allow $\bar{\mu}_\Delta \gtrsim 1$ (always assuming $\delta n \ll n_{\rm avg}$),

$$\frac{d\bar{\mu}_\Delta}{d\varphi} = d\cos(\varphi) - f\bar{\mu}_\Delta\left(1 + \chi_1\frac{\bar{\mu}_\Delta{}^2}{T^2} + \cdots + \chi_N\frac{\bar{\mu}_\Delta{}^{2N}}{T^{2N}}\right)$$

Now there are nonlinear effects; $\bar{\mu}_\Delta(\varphi)$ may not be harmonic.

The subthermal bulk viscosity

$$\zeta_{\rm sub}(\omega, T) = P \frac{\gamma_{\rm eff}(T)}{\gamma_{\rm eff}(T)^2 + \omega^2}$$



- $\zeta_{\rm sub}$ is *independent of driving amplitude*.
- Prefactor $P = C^2/B$ is a combination of susceptibilities.
- $\gamma_{\rm eff}$ is the effective rate/particle of the re-equilibration process.

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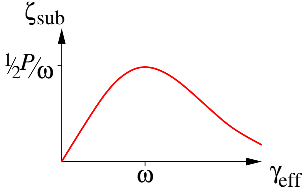

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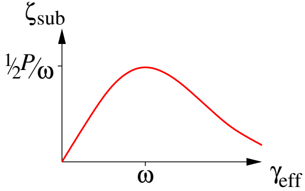

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- As $\gamma_{\rm eff} \to \infty$, $\zeta \to 0$. Infinitely fast equilibration.
- In phases where Fermi surface modes dominate equilibration (nuclear, unpaired quark matter, 2SC) P is constant for $T \ll \mu_q$, and subthermal bulk viscosity peaks when $\gamma_{\rm eff}(T) = \omega$.
- In phases where bosons dominate equilibration (CFL, CFL-K0), $P(T)$ washes out the peak.

The general bulk viscosity

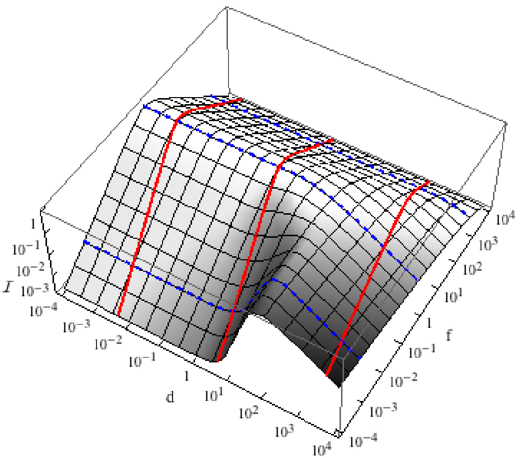
To include the suprathermal regime, we have to solve the diffeq for $\bar{\mu}_\Delta(\varphi)$ numerically.

For a given form of matter, we can summarize dependence on driving amplitude and temperature in dimensionless function $\mathcal{I}(d, f)$,

$$\zeta = \frac{C^2}{2\omega B}\mathcal{I}(d, f)$$

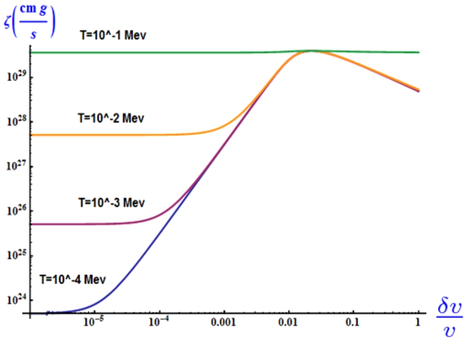
This could then be used to calculate damping time of r-modes.

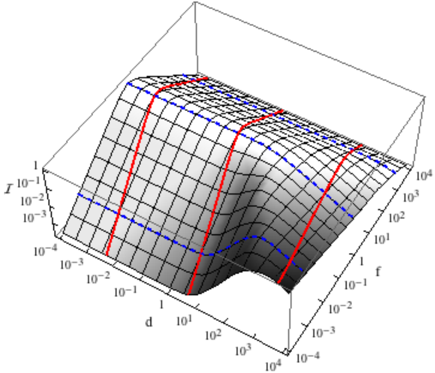
Nuclear matter, modified Urca



Typical amplitude dependence

Unpaired quark matter: $\gamma(\bar{\mu}_\Delta) = \bar{\mu}_\Delta + \chi_1 \bar{\mu}_\Delta{}^3$, can do analytic approx.

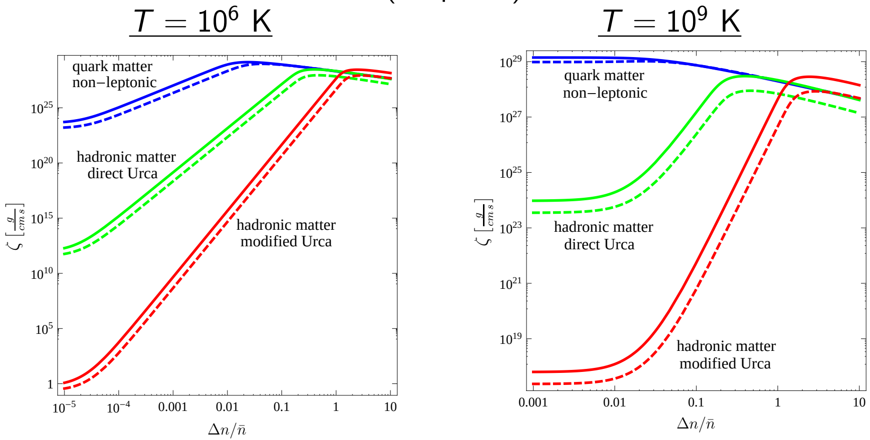




- At low T, bulk visc rises rapidly with amplitude when entering the suprathermal region: could set saturation amplitude for r-modes.
- Maximum value is same for all temperatures/amplitudes.
- If amplitude gets too high, bulk viscosity drops again.

Results for bulk viscosity

(ms pulsar)



- Bulk visc rises very steeply in suprathermal regime
- Max reached at $\delta n/n \sim 0.1$; max value indp of temperature
- Suprathermal enhancement is greater at low T and for matter where ζ goes as higher power of μ_Δ.

Future directions

Transport:

- Extend to other phases of quark matter, eg CFL-K0
- Superfluid nuclear matter: suprathermal leptonic viscosity
- Hyperonic nuclear matter (see Reisenegger)
- Investigate effect of multiple equilibrating quantities

Astrophysics:

- Use our results in calculations of r-mode damping times, obtain saturation amplitude of r-mode and trajectory in (T, Ω) space (requires a cooling model)
- Complications with r-modes: layered stars, role of crust, etc
- Apply to other modes, eg pulsations, f-modes (which emit grav waves)