Astrophysics of dense quark matter in compact stars

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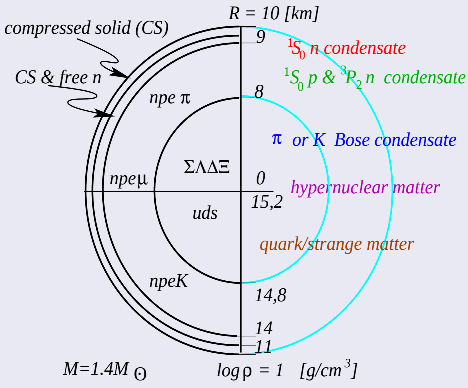
Astrophysical observations vs theory of dense matter

- Astrophysics provides unique ways to learn about the properties of dense matter
- Dense matter may be trapped by gravitational well of a compact star for very long periods of time (compared to terrestrial experiments). However, the experiment comes “as it is” and cannot be manipulated.
- Need to develop methods and tools to extract information

Potential avenues:

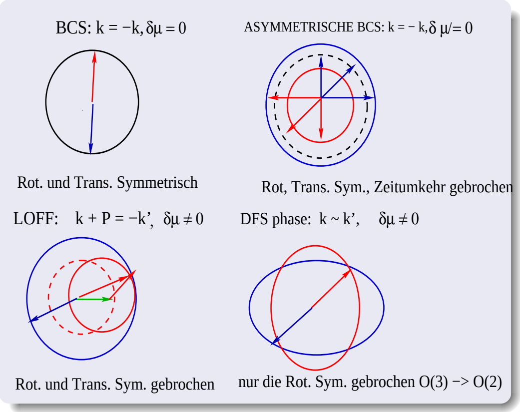
- Gravitational radiation from strained superfluid
- Neutrino cooling of neutron stars with quark cores
- Rotational anomalies (glitches, timing noise, etc).



Stressed pairing

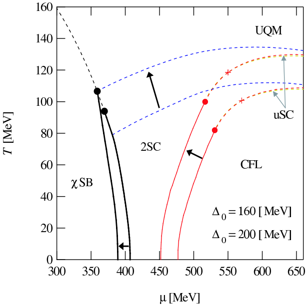
Initially isospin symmetric matter acquires d-quark excess via the inverse β-decay $e + u \rightarrow d + \nu$, this implies shift in the Fermi spheres of by amount $\mu_e = \mu_d - \mu_u$:

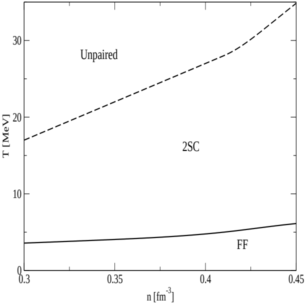
- Standard BCS requires the numbers to be equal, coherence is optimal among the fermions bound in a Cooper pair
- Asymmetric BCS, shifted Fermi surfaces, coherence is destroyed
- LOFF phase, Finite momentum of the condensate, restores coherence. Simplest ansatz $\Delta(\vec{r}) = \Delta \exp(i\vec{r} \cdot \vec{q})$.



Astrophysical observations vs theory of dense matter

The phase diagram

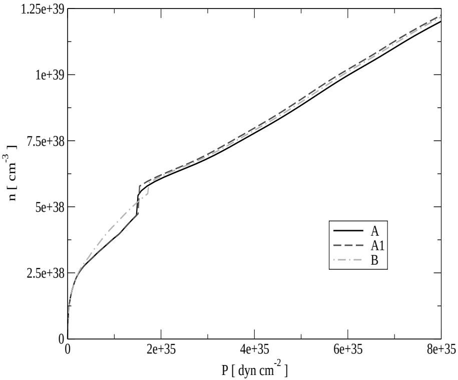




Abuki, Kunihiro, hep-ph/0509172

Huang, Sedrakian, arXiv:1006.0147; $350 \lesssim \mu \lesssim 550$ MeV

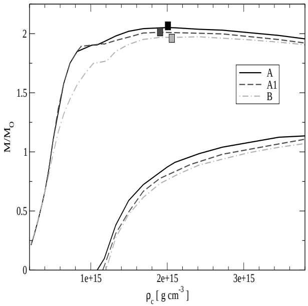
Equations of state



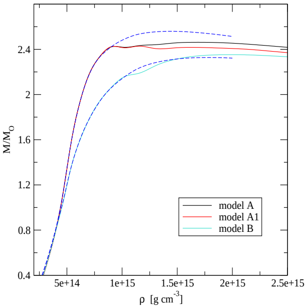
- The nuclear equation of state is taken from covariant BHF theory with two parameterizations (both stiff)
- The two quark equation of states differ by pressure normalization in the vacuum (slight vertical shift)

Stellar configurations

Mass vs central density



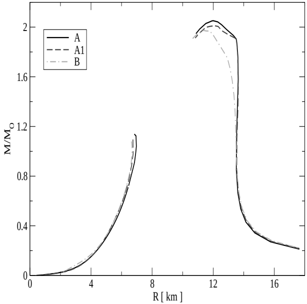
non-rotating objects

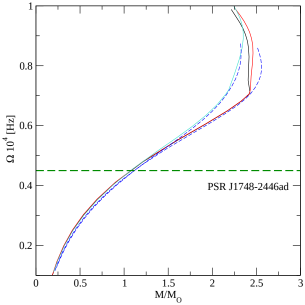


objects rotating at Keplerian frequency

Stellar configurations

M-R relation



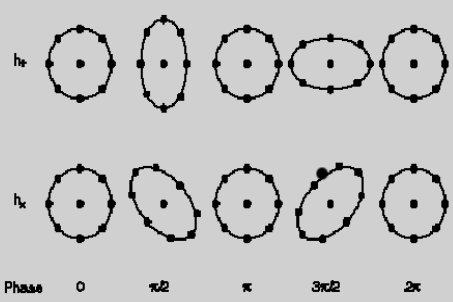


non-rotating objects

objects rotating at Keplerian frequency

Gravitation radiation

Two independent polarization of GW; perturbations of metric $h_{ij} = h_{+}e^{+}_{ij} + h_{X}e^{X}_{ij}$.

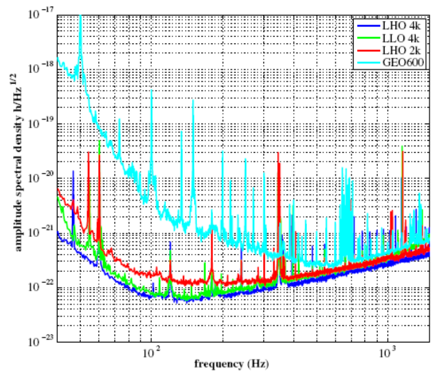


Weak field limit, linearized GR equations, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $h_{\mu\nu}$ perturbation

$$\Box\bar{h}_{\mu\nu} = 0, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h \tag{1}$$

Gravitation radiation

LIGO is sensitive to GW emitting by rotating NS, which is at 2Ω, e.g. Crab pulsar $\Omega \simeq 30$ Hz.



Gravitation radiation

Given a deformation the characteristic strain amplitude:

$$h_0 = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz}\nu^2}{r}, \tag{2}$$

$\epsilon = (I_{xx} - I_{yy})/I_{zz}$ is the equatorial ellipticity. Strain amplitude can be expressed in terms of the $m = 2$ mass quadrupole moment as

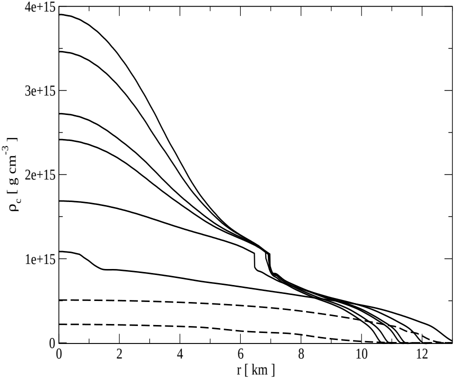
$$h_0 = \frac{16\pi^2 G}{c^4}\left(\frac{32\pi}{15}\right)^{1/2}\frac{Q_{22}\nu^2}{r}, \tag{3}$$

Quadrupole moment

$$Q_{22} = \int_0^{R_{\rm core}} \frac{dr^3}{g(r)}\left[\frac{3}{2}(4-U)t_{rr} + \frac{1}{3}(6-U)t_\Lambda + \sqrt{\frac{3}{2}}\left(8 - 3U + \frac{1}{3}U^2 - \frac{r}{3}\frac{dU}{dr}\right)t_{r\perp}\right], \tag{4}$$

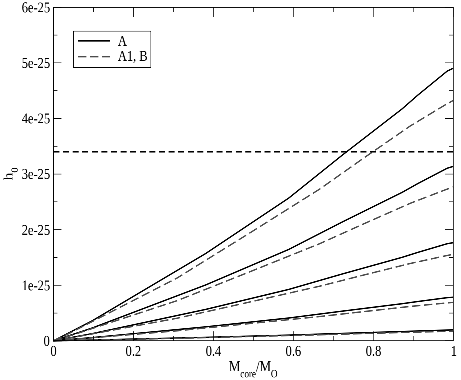
where $U = 2 + d\ln g(r)/d\ln r$ and t_{rr}, t_Λ and $t_{r\perp}$ are the coefficients of the expansion of the shear stress tensor in spherical harmonics.

Internal structure



- L.-M. Lin, Phys. Rev. D **76**, 081502(R) (2007, *incompressible models without nuclear crusts*
- B. Knippel, A. Sedrakian, Phys. Rev. D **79**, 083007 (2009), *microscopic equations of state*

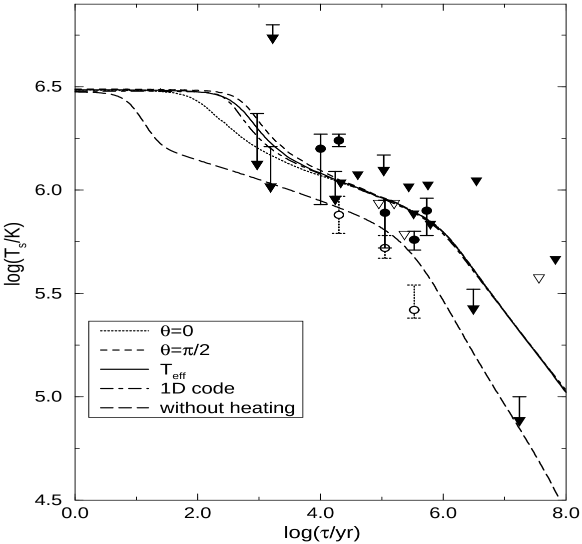
Strain amplitudes



GW strain amplitudes for breaking strain 10^{-4}, Gaps from 10 to 50 MeV.
Dashed line Crab pulsars' upper limit from S5 run

h_0 can pin down the product $\sigma\Delta^2$, currently $\bar{\sigma}_{\max}\Delta^2 \sim 0.25$ MeV^2 (under the assumptions of the present model).

Neutrinos from quark matter



Transport of thermal energy

$$\frac{d}{dr}\left(Te^{\Phi}\right) = \frac{-3\kappa\rho}{16\sigma T^3}\frac{L_\gamma e^{\Phi}}{4\pi r^2\sqrt{1-\frac{2Gm}{rc^2}}} \tag{5}$$

Energy balance equation

$$\frac{d}{dr}\left(Le^{2\Phi}\right) = \frac{-4\pi r^2}{\sqrt{1-\frac{2Gm}{rc^2}}} n e^{\Phi} T \frac{ds}{dt}. \tag{6}$$

L is the total luminosity (neutrino + photon)

$$\frac{dT'}{dt} = -\frac{Le^{2\Phi_c}}{\int\limits_0^{R_c} n c_v dV_p}. \tag{7}$$

The gradients of neutrino luminosity

$$\frac{d}{dr}\left(L_\nu e^{2\Phi}\right) = \frac{4\pi r^2}{\sqrt{1-\frac{2Gm}{rc^2}}} n e^{2\Phi} q_\nu, \quad L_\nu e^{2\Phi_c} = \int\limits_0^{R_c} n q_\nu e^{2\Phi} dV_p. \tag{8}$$

Combination gives

$$\frac{dT'}{dt} = -\frac{\int\limits_0^{R_c} n q_\nu(r,T) e^{2\Phi} dV_p + 4\pi\sigma R^2 T_S^4 e^{2\Phi_c}}{\int\limits_0^{R_c} n c_v(r,T) dV_p}. \tag{9}$$

Cooling processes

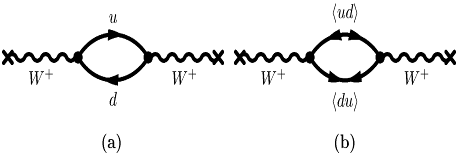
Quark cores of NS emit neutrons via

$$d \longrightarrow u + e + \bar{\nu}_e \quad u + e \longrightarrow d + \nu_e \tag{10}$$

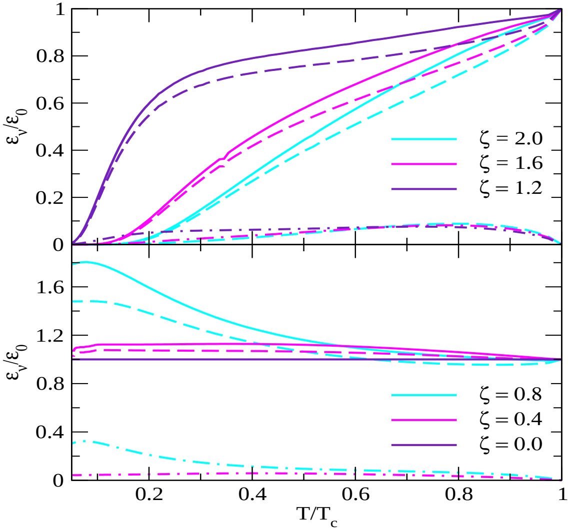
The rate of the process (emissivity) is given through

$$\Pi_{\mu\lambda}(q) = -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(\Gamma_-)_\mu S(p)(\Gamma_+)_\lambda S(p+q)\right]$$

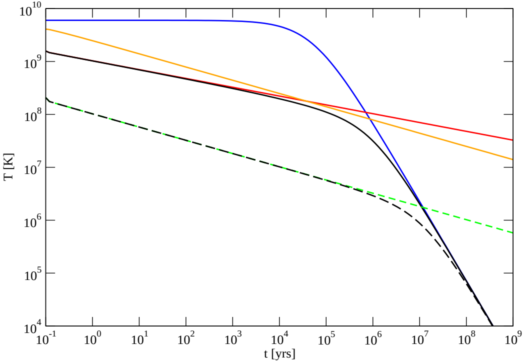
$$\Gamma_\pm(q) = \gamma_\mu(1-\gamma_5) \otimes \tau_\pm$$



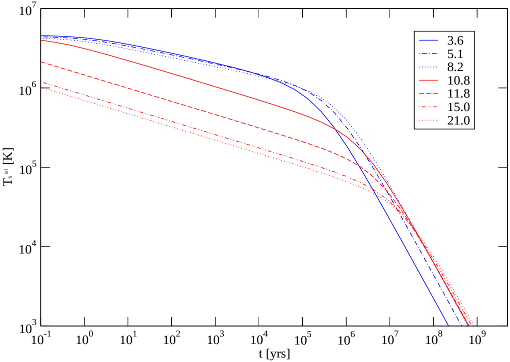
$$S_{f=u,d} = i\delta_{ab}\frac{\Lambda^+(p)}{p_0^2 - \epsilon_p^2}(\not{p} - \mu_f\gamma_0), \quad F(p) = -i\epsilon_{ab3}\epsilon_{fg}\Delta\frac{\Lambda^+(p)}{p_0^2 - \epsilon_p^2}\gamma_5 C$$



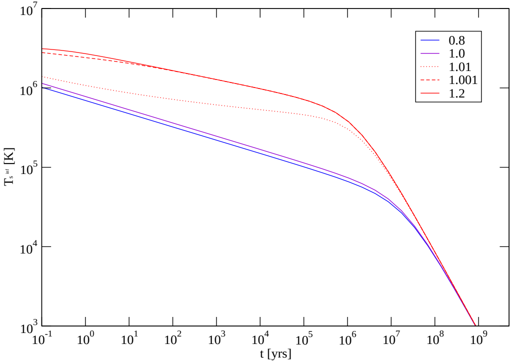
$\zeta = \Delta/\delta\mu$, where $\delta\mu = \mu_d - \mu_u = \mu_e$.



Fast vs slow cooling in standard models with fast cooling agent



Hadonic (blue) hybird (red), central density in units of 10^{14} g cm^{-3} for $f(\zeta) = .8$



Central density 2.1×10^{15} g cm^{-3} various ζ.

Conclusion

- Gravitationally stable hybrid star configurations can be constructed. Hybrid stars are *massive* with $M \sim 1.8 - 2.0 M_\odot$ with radii $R \sim 12$ km.
- The internal structure reveals quark core $M \leq 1. M_\odot$ and radius $R_{\rm core} \sim 7 - 8$ km.
- If the quark core is solid, the gravitational radiation amplitudes are within the range of current upper limits set for the Crab pulsar.
- Cooling of hybrid quark star shows consistency with the current data on surface temperatures of neutron stars. Cooler stars need to have quark cores.