Analytical approximations for the extrapolation of lattice data

Pere Masjuan

University of Vienna

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Work done in collaboration with
G. Ecker and H. Neufeld
[arXiv:hep-ph/1004.3422]

Guideline

- Motivation
- Dissection of the $O(p^6)$ amplitude
- A Viennese recipe
- An example: F_K / F_π
- Conclusions
- Outlook

Motivation

- At low-energies, ChPT is the EFT of SM $\rightarrow$ LECs

  [Gasser,Leutwyler'84'85]

  $O(p^2) \rightarrow 2$
  $O(p^4) \rightarrow 10$
  $O(p^6) \rightarrow 94$

- Lattice calculations involve different quark masses $\rightarrow$ good for LECs
  - LO+NLO $\rightarrow$ O.K.
  - NNLO $\rightarrow$ only recently
    - Involved
    - Mostly available numerically

  [Bernard, Passemar'10]
  [MILC Col. '09]

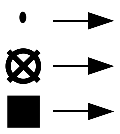
- For comparison with lattice, dependence on masses should be known

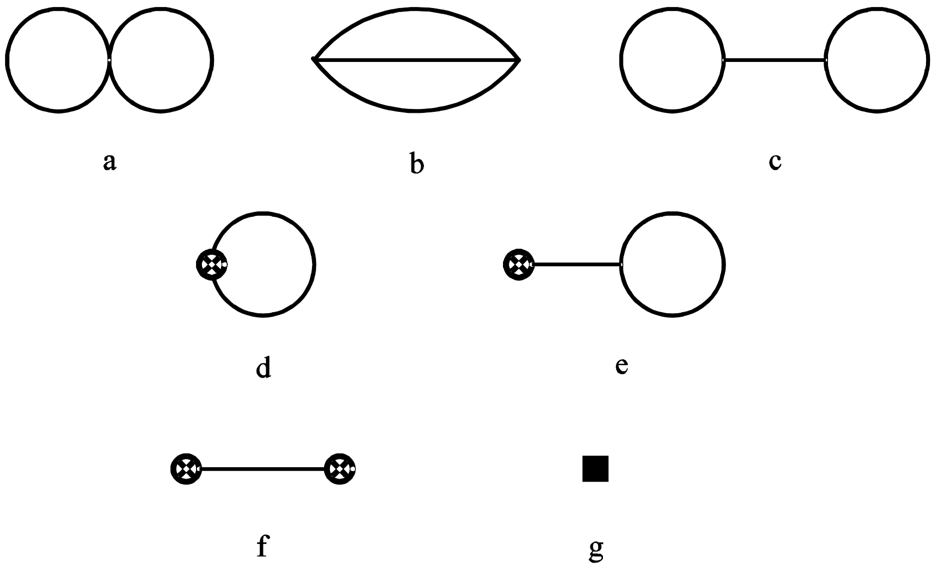
- We propose analytical approximations of chiral SU(3) amplitudes for the extrapolation of lattice data to the physical masses.
- We determine NNLO LECs.

ChPT at O(p^6): Dissection of Z_6

ChPT in meson sector → $Z = Z_2 + Z_4 + Z_6 + \ldots, \quad O(p^2, p^4, p^6, \ldots)$

NNLO functional Z_6 of $O(p^6)$



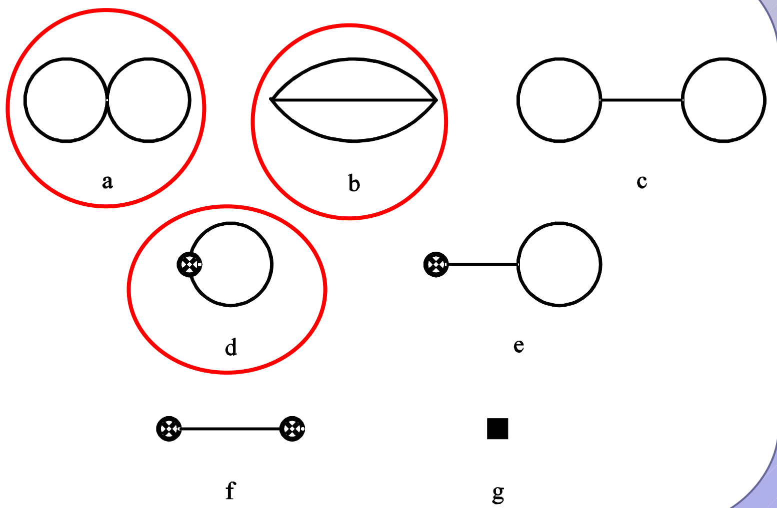


- Tree diagrams of O(p^6) → g
- Irreducible → a,b,d
- Reducible → c, e, f

Dissection of Z_6: irreducible

Diagrams:
- a,b,d → divergent (dim. reg.)
- a+b+d → local diverg (renorm th)

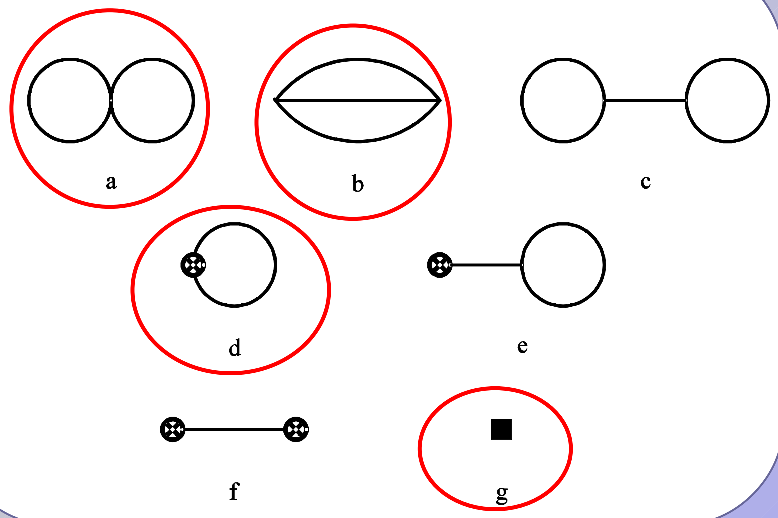
[Bijnens et al ‘00]



Dissection of Z_6: irreducible

Diagrams:
- a,b,d $\longrightarrow$ divergent (dim. reg.)
- a+b+d $\longrightarrow$ local diverg (renorm th)
    - [Bijnens et al '00]
- Chiral sym $\longrightarrow$ div absorbed by "g" $\longrightarrow$ scale μ

$$\left(L = \frac{1}{(4\pi)^2} \ln M^2/\mu^2 \right)$$



$$Z_6^{a+b+d+g} = \int d^4x \left\{ \left[C_a^r(\mu) + \frac{1}{4F_0^2}\left(4\Gamma_a^{(1)}L - \Gamma_a^{(2)}L^2 + 2\Gamma_a^{(L)}(\mu)L\right)\right] O_a(x) + \frac{1}{(4\pi)^2}\left[L_i^r(\mu) - \frac{\Gamma_i}{2}L \right] H_i(x;M) + \frac{1}{(4\pi)^2} K(x;M) \right\}$$

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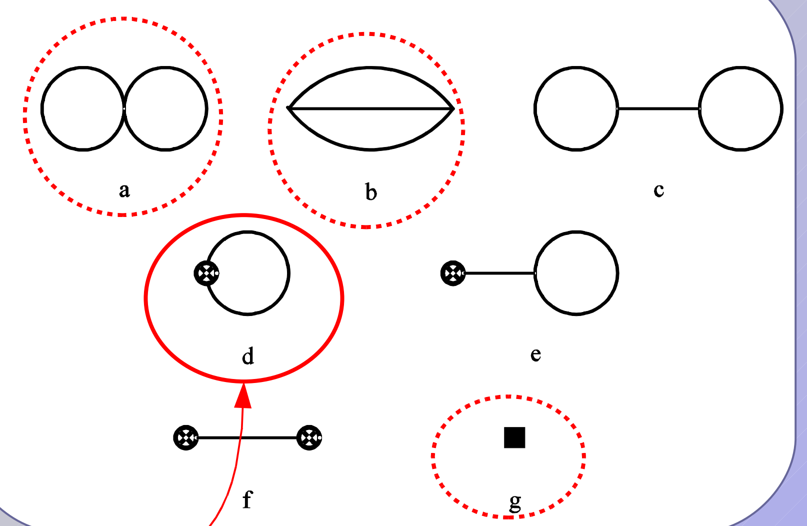

$$Z_6^{a+b+d+g} = \int d^4x \left\{ \left[\left[C_a^r(\mu) + \frac{1}{4F_0^2}\left(4\Gamma_a^{(1)} L - \Gamma_a^{(2)} L^2 + 2\Gamma_a^{(L)}(\mu) L\right)\right] O_a(x) \right.$$

$$\left. + \frac{1}{(4\pi)^2}\left[L_i^r(\mu) - \frac{\Gamma_i}{2} L\right] H_i(x;M) + \frac{1}{(4\pi)^2} K(x;M) \right\}$$

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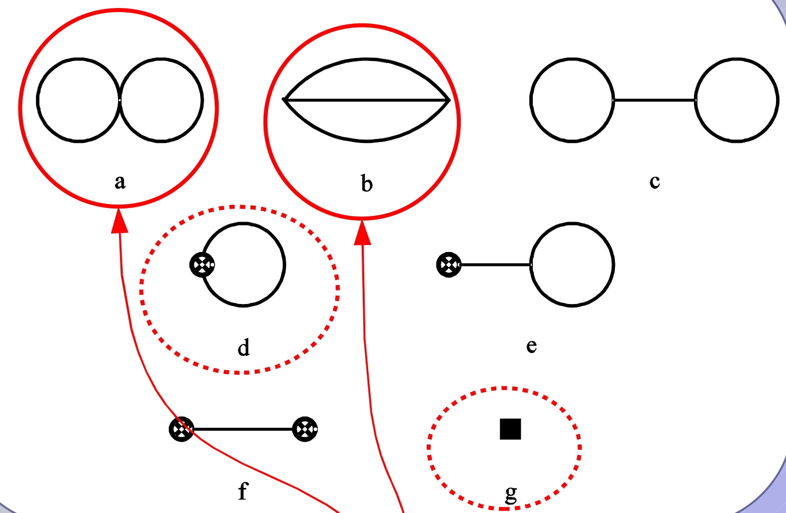

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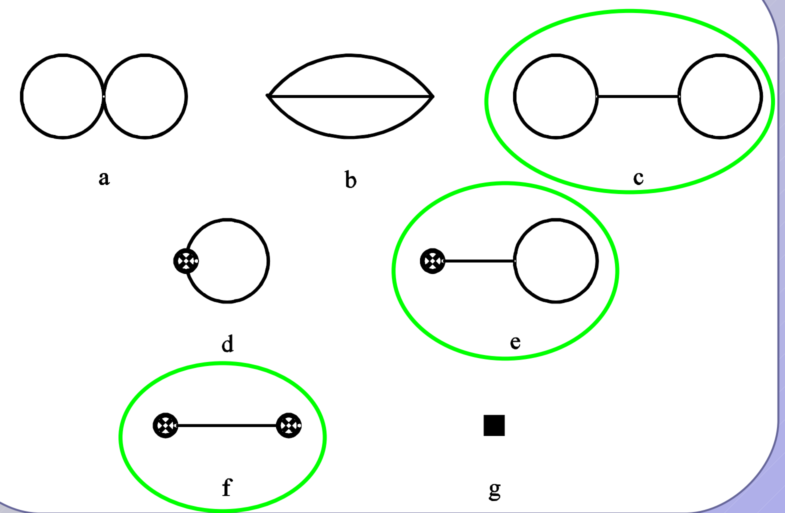
Dissection of Z_6: reducible

Diagrams:
- c+e+f → finite

and scale independent
[Bijnens et al '00]

$$\left(L = \frac{1}{(4\pi)^2} \ln M^2/\mu^2 \right)$$



$$Z_6^{c+e+f} = \int d^4x d^4y \left[\left(L_i^r(\mu) - \frac{\Gamma_i}{2} L \right) P_{i,\alpha}(x) + F_\alpha(x;M) \right] G_{\alpha,\beta}(x,y)$$
$$\left[\left(L_j^r(\mu) - \frac{\Gamma_j}{2} L \right) P_{j,\alpha}(x) + F_\beta(x;M) \right]$$

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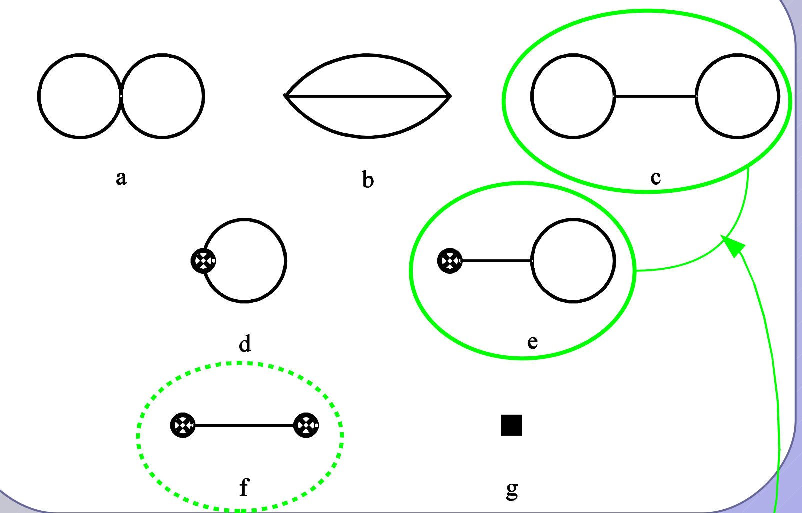

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$$\left[\left(L_j^r(\mu) - \frac{\Gamma_j}{2} L \right) P_{j,\alpha}(x) + F_\beta(x;M) \right]$$

Approximation

$$Z_6 = \underbrace{Z_6^{a+b+d+g} + Z_6^{c+e+f}}_{\substack{\mu,M \\ \text{independent}}} \to Z_6^{app}$$

$$\left.\begin{array}{l} K(x;M) \\ F_\alpha(x;M) \times F_\beta(x;M) \end{array}\right\} \to 0$$

$$Z_6^{app} = \int d^4x \left\{ \left[C_a^r(\mu) + \frac{1}{4F_0^2}\left(4\Gamma_a^{(1)}L - \Gamma_a^{(2)}L^2 + 2\Gamma_a^{(L)}(\mu)L\right)\right] O_a(x) + \frac{1}{(4\pi)^2}\left[L_i^r(\mu) - \frac{\Gamma_i}{2}L \right] H_i(x;M) \right\}$$

$$+ \int d^4x d^4y \left\{ \left(L_i^r(\mu) - \frac{\Gamma_i}{2}L \right) P_{i,\alpha}(x) G_{\alpha,\beta}(x,y) \left(L_j^r(\mu) - \frac{\Gamma_j}{2}L \right) P_{j,\beta}(x) + 2\left(L_i^r(\mu) - \frac{\Gamma_i}{2}L \right) P_{i,\alpha}(x) G_{\alpha,\beta}(x,y) F_\beta(x;M) \right\}$$

Z_6^{app} is M-dependent through chiral logs, $M \approx M_K$

- Scale independent (reliable determination of renormalized LECs)
- Large-N_c behaviour included:
  - leading (C_i, L_j L_k)
  - NLO (L_i × 1-loop)
  - all chiral logs at NNLO

Recipe

The amplitude for a given observable can be determined:

- Calculate tree+one-loop diagrams (d,e,f,g)
- In tree-level $O(p^6)$ (g):

$$C_a^r(\mu) \to C_a^r(\mu) + \frac{1}{4F_0^2}\left(4\Gamma_a^{(1)}L - \Gamma_a^{(2)}L^2 + 2\Gamma_a^{(L)}(\mu)L\right)$$

- Collect L_i and L_iL_j and extract chiral logs
- Replace (in f):

$$L_i^r(\mu)L_j^r(\mu) \to \left(L_i^r(\mu) - \frac{\Gamma_i}{2}L\right)\left(L_j^r(\mu) - \frac{\Gamma_j}{2}L\right)$$

And (in d,e):

$$L_i^r(\mu) \to L_i^r(\mu) - \frac{\Gamma_i}{2}L$$

Application to lattice data: F_K/F_π

Why F_K/F_π ?

- At μ=0.77GeV, genuine two-loop~0.5% [Amoros et al '00][Bernard,Passemar '10]
- BMW collaboration → 13 points (m_π<450MeV) [Dürr et al '10]

Other L_i's from Fit10 [Amoros et al.'01]

- Results:

$$\left.\frac{F_K}{F_\pi}\right|_{\text{BMW}} = 1.192(7)_{stat}(6)_{syst}$$

$$\left.\frac{F_K}{F_\pi}\right|_{\text{fit}} = 1.198 \pm 0.005 \quad \begin{cases} L_5^r = (0.76 \pm 0.09)\cdot 10^{-3} \\ C_{14}^r + C_{15}^r = (0.37 \pm 0.08)\cdot 10^{-3}\,\text{GeV}^{-2} \\ C_{15}^r + 2C_{17}^r = (1.29 \pm 0.16)\cdot 10^{-3}\,\text{GeV}^{-2} \end{cases}$$

(lattice errors)

- Why fit with L_5?
  - At $O(p^4)$ only L_5
  - At $O(p^6)$ L_5^2 is leading in $1/N_c$
  - Ranging $M=M_K(1\pm0.2)$, L_5 and F_K/F_π don't change, C_i's in 2-σ

	L_5^r	$C_{14}^r + C_{15}^r$
$C_{14}^r + 2C_{14}^r$	0.69	-0.87
L_5^r		-0.95

$M_\eta \le M_K(1.2)$

Conclusions

- From Z_6 we propose analytic approx to chiral SU(3) amplitudes
  - User-friendly extrapolation formulas for lattice data
  - Determination of higher-order LECs (specially if lattice sumulations use m_s lighter than the physical case)
- Approx μ independent
- Include all chiral logs+LO+NLO terms in $1/N_c$
- Approach useful when genuine two-loop small (large-N_c counting): F_K/F_π

Outlook (work in progress)

- Further example: Kπ vector fomr factor $f_+(t)$
- Extended approximation (still scale indep) including: