Dissipative processes in superfluid quark matter

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OUTLINE

✦ QCD phase diagram
✦ Color flavor locking
✦ Superfluids
✦ Dissipative processes
✦ R-mode oscillations

QCD phase diagram

- QGP
- Color Superconductor
- Hadron gas
- Nuclear liquid

Temperature ($T$) and chemical potential ($\mu$) axes.
QCD phase diagram

Warning: At high density ab initio calculations using QCD not available
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**QCD Phase Diagram**

![Diagram showing different phase transitions in QCD](image)

**Warning:** At high density, ab initio calculations using QCD are not available.

**Axes:**
- \( T \) (Temperature)
- \( \mu \) (Chemical Potential)

**Phases:**
- **Color Superconductor**
- **QGP**
- **Nuclear liquid**
- **Hadron gas**
- **Compact stars**

**Regions:**
- **Confined**
- **Strong coupling**
- **Weak coupling**

**Transitions:**
- From Hadrons to quarkyonic phase?
- From quarkyonic phase to CSC phase?
- From CSC phase to CFL
Color superconductor

- Confined
- Strong coupling
- Weak coupling

$\mu$
Color superconductor

Confined

Strong coupling

Weak coupling

\( \mu \)
Degenerate system of quarks

Attractive interaction between quarks

\[ 3 \times 3 = \bar{3}_A + 6_S \]

attractive channel
Color superconductor

Confined Strong coupling Weak coupling

* Degenerate system of quarks

* Attractive interaction between quarks

\[ 3 \times 3 = \bar{3}_A + 6_S \]

attractive channel

Using quarks as building blocks, one has color, flavor as well as spin degrees of freedom: the game is complicated

QCD, allows for a zoo of colored phases and one has to single out the one with the smallest free-energy
\( \mu \gg m_s \)  

**CFL phase**  
Alford, Rajagopal, Wilczek hep-ph/9804403

\[ < \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \Delta \epsilon_{I\alpha \beta} \epsilon_{Iij} \]

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2 \]
Xtreme density

$\mu \gg m_s$

CFL phase

$Alford, Rajagopal, Wilczek$ hep-ph/9804403

$< \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \Delta \epsilon_{I \alpha \beta} \epsilon_{I i j}$

$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2$

$\begin{align*}
\langle \psi_L \psi_L \rangle & \quad \langle \psi_R \psi_R \rangle
\end{align*}$
Xtreme density

\[ \mu \gg m_s \]

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\[ <\psi_L\psi_L> \quad <\psi_R\psi_R> \quad SU(3)_L \text{ rotation} \]
**Xtreme density**

\[ \mu \gg m_s \]

CFL phase

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\[
\begin{align*}
&\text{F} \quad \text{C} \quad \text{F} \\
&\langle \psi_L \psi_L \rangle \quad \langle \psi_R \psi_R \rangle \\
&\text{SU}(3)_L \text{ rotation} \\
&\text{SU}(3)_c \text{ rotation}
\end{align*}
\]
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\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2 \]

- Gluons acquire a Meissner mass by Higgs mechanism
- 8 pseudo Nambu-Goldstone bosons
- All quarks are paired; quasiparticles with a gapped excitation spectrum
- No electrons
- 1 Nambu-Goldstone boson → Superfluid
UNVISCID (DRY) FLUID

Continuity equation
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

Euler equation
\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla p}{\rho} - \nabla \phi \]

Vorticity \[ \Omega = \nabla \times \mathbf{v} \]  
Unviscid fluid \[ \Omega = 0 \]

The flow is permanently irrotational \[ \mathbf{v} = \nabla \varphi \]
**Viscous (Wet) Fluid**

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p - \rho \nabla \phi + \eta \nabla^2 \mathbf{v} + \left( \frac{\eta}{3} + \zeta \right) \nabla (\nabla \cdot \mathbf{v})
\]

**Shear viscosity** describes reaction to shear stresses

**Bulk viscosity** describes reaction to compression/rarefaction

Using vorticity
\[
\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times \mathbf{v}) = \frac{\eta}{\rho} \nabla^2 \Omega
\]

Vorticity generated by the shear viscosity
**VISCOUS (WET) FLUID**

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p - \rho \nabla \phi + \eta \nabla^2 \mathbf{v} + \left( \frac{\eta}{3} + \zeta \right) \nabla (\nabla \cdot \mathbf{v}) \]

- **Shear viscosity** describes reaction to shear stresses
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Using vorticity

\[ \frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times \mathbf{v}) = \frac{\eta}{\rho} \nabla^2 \Omega \]

Vorticity generated by the shear viscosity
NON-RELATIVISTIC SUPERFLUIDS

Landau two-fluid theory

\[ \rho = \rho_n + \rho_s \]
\[ \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s \]

Normal component
phonons, rotons...

Superfluid component

\[ \partial_t \rho + \text{div} \mathbf{j} = 0 \]

The two “components” correspond to two different motions of the fluid
NON-RELATIVISTIC SUPERFLUIDS

Landau two-fluid theory

\[ \rho = \rho_n + \rho_s \quad \text{and} \quad \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s \]

Normal component: phonons, rotons...

Superfluid component

\[ \partial_t \rho + \text{div}\mathbf{j} = 0 \]

The two “components” correspond to two different motions of the fluid.

Normal component: viscous fluid

Superfluid component: unviscid fluid

Not completely correct: neglects interactions
dissipative terms

\[ \partial \mu n^\mu = 0 \]
\[ \partial \mu (T^{\mu\nu} + T_d^{\mu\nu}) = 0 \]
\[ u^\mu \partial \mu \phi + \mu + \chi = 0 \]
**RELATIVISTIC HYDRODYNAMICS**

\[ \partial_\mu n^\mu = 0 \]
\[ \partial_\mu (T^{\mu \nu} + T_{d}^{\mu \nu}) = 0 \]
\[ u^\mu \partial_\mu \phi + \mu + \chi = 0 \]

**dissipative terms**

**Close to equilibrium**

\[ \chi = -\zeta_3 \partial_\mu (V^2 w^\mu) - \zeta_4 \partial_\mu u^\mu \]
\[ T_{d}^{\mu \nu} = \kappa (\Delta^{\mu \gamma} u^\nu + \Delta^{\nu \gamma} u^\mu) \left( \partial_\gamma T + T u^\delta \partial_\delta u_\gamma \right) \]
\[ + \eta \Delta^{\mu \gamma} \Delta^{\nu \delta} \left( \partial_\delta u_\gamma + \partial_\gamma u_\delta + \frac{2}{3} g_{\gamma \delta} \partial_\alpha u^\alpha \right) \]
\[ + \Delta^{\mu \nu} (\zeta_1 \partial_\gamma (V^2 w^\gamma) + \zeta_2 \partial_\gamma u_\gamma) \]

\[ w^\mu = - (\partial^\mu \varphi + \mu u^\mu) \]

**where**

\[ \Delta^{\mu \nu} = g^{\mu \nu} - u^\mu u^\nu \]
Bulk viscosity depends on the low energy spectrum of the theory.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Contribution of Phonons</th>
<th>Reference</th>
<th>ArXiv ID</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CFL phase</strong></td>
<td></td>
<td><strong>Manuel and LLanes</strong></td>
<td>0705.3909</td>
</tr>
<tr>
<td>Contribution of phonons</td>
<td></td>
<td><strong>MM and Manuel</strong></td>
<td>0909.4486</td>
</tr>
<tr>
<td>Contribution of kaons</td>
<td></td>
<td><strong>Alford et al.</strong></td>
<td>nucl-th/0701067</td>
</tr>
<tr>
<td><strong>Spin 1 phase</strong></td>
<td></td>
<td><strong>Sa’d et al.</strong></td>
<td>astro-ph/0607643</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Wang et al.</strong></td>
<td>1006.1293</td>
</tr>
<tr>
<td><strong>2SC phase</strong></td>
<td></td>
<td><strong>Alford and Schmitt</strong></td>
<td>nucl-th/0608019</td>
</tr>
</tbody>
</table>

Large amplitude behavior of bulk viscosity: talk by Alford
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

**Effective Lagrangian** (low energy, asymptotic densities) Son hep-ph/0204199

\[ L_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]
Fluid of quasiparticles (phonons) moving on the top of the superfluid background


\[
L_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2
\]

**Scale separation**

\[
\varphi(x) = \bar{\varphi}(x) + \phi(x)
\]
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PHONON EFFECTIVE ACTION

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$$\mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2$$

Scale separation

$$\varphi(x) = \bar{\varphi}(x) + \phi(x)$$

$$S[\varphi] = S[\bar{\varphi}] + \frac{1}{2} \int d^4x \frac{\partial \mathcal{L}_{\text{eff}}}{\partial(\partial_\mu \varphi) \partial(\partial_\nu \varphi)} \bigg|_{\bar{\varphi}} \partial_\mu \phi \partial_\nu \phi + \cdots$$
PHONON EFFECTIVE ACTION

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\[ \mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]

Scale separation

\[ \varphi(x) = \bar{\varphi}(x) + \phi(x) \]

Superfluid

Phonon

Phonon's action

\[ S[\varphi] = S[\bar{\varphi}] + \frac{1}{2} \int d^4 x \frac{\partial \mathcal{L}_{\text{eff}}}{\partial (\partial_\mu \varphi) \partial (\partial_\nu \varphi)} \bigg|_{\bar{\varphi}} \partial_\mu \phi \partial_\nu \phi + \cdots \]

\[ S[\phi] = \frac{1}{2} \int d^4 x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \]
Phonon Effective Action

Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities) Son hep-ph/0204199

\[ \mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]

Scale separation

\[ \varphi(x) = \bar{\varphi}(x) + \phi(x) \]

Superfluid

Phonon

bulk

long-wavelength fluctuations

Phonon's action

\[ S[\phi] = \frac{1}{2} \int d^4 x \sqrt{-g} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \]

Acoustic metric

\[ g_{\mu\nu} = \eta_{\mu\nu} + (c_s^2 - 1) v_\mu v_\nu \]
Phonon contribution

phonon dispersion law  \[ \epsilon_p = c_s p + B p^3 + O(p^5) \]

\[ \partial_t N_{ph} + \text{div}(N_{ph} \mathbf{v}_n) = -\frac{\Gamma_{ph}}{T} \mu_{ph} \]

- \( B > 0 \)  \( \phi \rightarrow \phi\phi \)
- \( B < 0 \)  \( \phi\phi \rightarrow \phi\phi\phi \)
Phonon contribution

phonon dispersion law \[ \epsilon_p = c_s \rho + B \rho^3 + O(p^5) \]

\[ \partial_t N_{ph} + \text{div}(N_{ph} v_n) = -\frac{\Gamma_{ph}}{T} \mu_{ph} \]

\[ B > 0 \quad \phi \rightarrow \phi \phi \]
\[ B < 0 \quad \phi \phi \rightarrow \phi \phi \phi \]

\[ \zeta_1 = -\frac{T}{\Gamma_{ph}} \frac{\partial N_{ph}}{\partial \rho} \left( N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \right) = -\frac{T}{\Gamma_{ph}} I_1 I_2 \]

\[ \zeta_2 = \frac{T}{\Gamma_{ph}} \left( N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{ph}} I_2^2 \]

\[ \zeta_3 = \frac{T}{\Gamma_{ph}} \left( \frac{\partial N_{ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{ph}} I_1^2 \]
Phonon contribution

phonon dispersion law \[ \epsilon_p = c_s \rho + B \rho^3 + O(\rho^5) \]

\[ \partial_t N_{ph} + \text{div}(N_{ph} v_n) = -\frac{\Gamma_{ph}}{T} \mu_{ph} \]

\[ \begin{align*}
\zeta_1 &= -\frac{T}{\Gamma_{ph}} \frac{\partial N_{ph}}{\partial \rho} \left( N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \right) = -\frac{T}{\Gamma_{ph}} I_1 I_2 \\
\zeta_2 &= \frac{T}{\Gamma_{ph}} \left( N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{ph}} I_2^2 \\
\zeta_3 &= \frac{T}{\Gamma_{ph}} \left( \frac{\partial N_{ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{ph}} I_1^2
\end{align*} \]

Notice that \[ \zeta_1^2 = \zeta_2 \zeta_3 \] the system tends toward the state where bulk viscosity does not lead to dissipation

\[ \begin{align*}
B > 0 & \quad \phi \to \phi \phi \\
B < 0 & \quad \phi \phi \to \phi \phi \phi
\end{align*} \]
PHONONS IN CFL

Low temperatures \( T \lesssim 0.01 \text{ MeV} \)

In CFL B<0

Conformal limit
\[
\zeta_1 = \zeta_2 = 0 \quad \zeta_3 \sim \frac{\mu^6}{T \Delta^8}
\]

Conformal breaking due to \( m_s \)
\[
\zeta_1 \sim \frac{m_s^2 \mu^7}{T \Delta^8} \quad \zeta_2 \sim \frac{m_s^4 \mu^8}{T \Delta^8}
\]
Mutual friction

Force between the superfluid component and the normal component mediated by phonon-vortex interaction

\[
\begin{align*}
\rho_s \frac{d\mathbf{v}_s}{dt} &= -\frac{\rho_s}{\rho} \nabla p - \rho_s \nabla \phi - \mathbf{F}^N \\
\rho_n \frac{d\mathbf{v}_n}{dt} &= -\frac{\rho_n}{\rho} \nabla p - \rho_n \nabla \phi + \mathbf{F}^N + \eta \nabla^2 \mathbf{v}_n
\end{align*}
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\end{align*}
\]
Forces acting on a Vortex

- **Magnus force**
  \[ \mathbf{F}^M = \kappa \rho_s (\mathbf{v}_s - \mathbf{v}_L) \times \hat{z} \]

- **Friction force**
  \[ \mathbf{F}^N = D (\mathbf{v}_n - \mathbf{v}_L) + D' \hat{z} \times (\mathbf{v}_n - \mathbf{v}_L) \]

- **Standard hydrodynamic force**

- **Scattering of phonons off vortices**
Forces acting on a Vortex

**Magnus force**

\[ \mathbf{F}^M = \kappa \rho_s (\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{\hat{z}} \]

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**Mutual friction**

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**Scattering of phonons off vortices**
forces acting on a vortex

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**Standard hydrodynamic force**

**Scattering of phonons off vortices**

\[ \frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E} \frac{\cos^2 \theta}{\tan^2 \frac{\theta}{2}} \sin^2 \frac{\pi E}{\Lambda} \]

**Mutual friction**

**Elastic scattering off vortices**
Forces acting on a Vortex

- **Magnum force**
  \[ \mathbf{F}^M = \kappa \rho_s (\mathbf{v}_s - \mathbf{v}_L) \times \hat{z} \]

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- **Mutual friction**

  - Elastic scattering off vortices
    \[ \frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E} \cos^2 \theta \sin^2 \frac{\pi E}{\Lambda} \]
  
  - Inelastic scattering on vortices
    work in progress

  - Scattering of phonons off vortices

  - Standard hydrodynamic force

See the talk by Anglani
R-mode instability

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.
See also Andersson, Kokkotas gr-qc/0010102
Dissipative processes and stars oscillations

**R-mode instability**

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R-mode oscillations difficult to damp in CFL stars


 Emitting gravitational radiation the star quickly spins down
DISsipative processes and stars oscillations

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Emitting gravitational radiation the star quickly spins down

dissipative processes effective for $\nu \lesssim 1 \text{ Hz}$

see however 1005.1163
Dissipative processes and stars oscillations

R-mode instability

Gravitational Radiation

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.
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R-mode oscillations difficult to damp in CFL stars

Emitting gravitational radiation the star quickly spins down

Dissipative processes effective for $\nu \lesssim 1\,\text{Hz}$

see however 1005.1163

For more details see the talk by Sedrakian
Summary

- CFL is a superfluid
- Contribution of phonons to the bulk viscosity coefficients of CFL
- For rotating superfluids one has to include the mutual friction force
- Damping of star oscillations, especially gravitationally unstable r-modes