Dissipative processes in superfluid quark matter

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arXiv:0807.3264

arXiv:0904.3023

arXiv:0909.4486

Collaborators: M.A. Escobedo, C. Manuel, B.A. Sa'd, M.Ruggieri, R.Anglani, G.Colucci

OUTLINE

♦ QCD phase diagram ✦ Color flavor locking ✦ Superfluids ✦ Dissipative processes **R**-mode oscillations

Reviews: hep-ph/0011333, hep-ph/0102047, hep-ph/0202037, 0709.4635









Warning: At high density ab initio calculations using QCD not available





Confined

Strong coupling

 $\xrightarrow{\text{Weak coupling}} \mu$







Using quarks as building blocks, one has color, flavor as well as spin degrees of freedom: the game is complicated

♀ QCD, allows for a zoo of colored phases and one has to single out the one with the smallest free-energy

CFL phase

 $\mu \gg m_s$

Alford, Rajagopal, Wilczek hep-ph/9804403

 $<\psi_{\alpha i}C\gamma_5\psi_{\beta j}>\sim\Delta\epsilon_{I\alpha\beta}\epsilon_{Iij}$

 $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{c+L+R} \times Z_2$

 $\mu \gg m_s$ CF

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FC CF $\langle \psi_L \psi_L \rangle \langle \psi_B \psi_B \rangle$



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 $SU(3)_R$ rotation

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***** Gluons acquire a Meissner mass by Higgs mechanism

淋 All quarks are paired; quasiparticles with a gapped excitation spectrum

***** 8 pseudo Nambu-Goldstone bosons



♯ 1 Nambu-Goldstone boson → Superfluid

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UNVISCID (DRY) FLUID





Vorticity $\Omega = \nabla \times \mathbf{v}$ Unviscid fluid $\Omega = 0$

The flow is permanently irrotational $\mathbf{v} = \nabla \varphi$

VISCOUS (WET) FLUID

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}\right) = -\nabla p - \rho \nabla \phi + \eta \nabla^2 \mathbf{v} + \left(\frac{\eta}{3} + \zeta\right) \nabla (\nabla \cdot \mathbf{v})$$
shear viscosity bulk viscosity

Shear viscosity describes reaction to shear stresses
 Bulk viscosity describes reaction to compression/rarefaction

Using vorticity $\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times \mathbf{v}) = \frac{\eta}{\rho} \nabla^2 \Omega$ Vorticity generated by the shear viscosity

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NON-RELATIVISTIC SUPERFLUIDS

Landau two-fluid theory



The two "components" correspond to two different motions of the fluid

NON-RELATIVISTIC SUPERFLUIDS

Landau two-fluid theory



 $\partial_t \rho + \operatorname{div} \mathbf{j} = 0$

The two "components" correspond to two different motions of the fluid

Normal component: viscous fluid Superfluid component: unviscid fluid

Not completely correct: neglects interactions

RELATIVISTIC HYDRODYAMICS

 $\partial_{\mu}n^{\mu} = 0$ dissipative terms $\partial_{\mu}(T^{\mu\nu} + T^{\mu\nu}_d) = 0$ $u^{\mu}\partial_{\mu}\phi + \mu + \chi = 0$

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$$u^{\mu}\partial_{\mu}\phi + \mu + \chi = 0$$

Close to equilibrium

$$\chi = -\zeta_{3}\partial_{\mu}(V^{2}w^{\mu}) - \zeta_{4}\partial_{\mu}u^{\mu}$$

$$T_{d}^{\mu\nu} = \kappa \left(\Delta^{\mu\gamma}u^{\nu} + \Delta^{\nu\gamma}u^{\mu}\right)\left(\partial_{\gamma}T + Tu^{\delta}\partial_{\delta}u_{\gamma}\right)$$

$$+ \eta \Delta^{\mu\gamma}\Delta^{\nu\delta}\left(\partial_{\delta}u_{\gamma} + \partial_{\gamma}u_{\delta} + \frac{2}{3}g_{\gamma\delta}\partial_{\alpha}u^{\alpha}\right)$$

$$+ \Delta^{\mu\nu}\left(\zeta_{1}\partial_{\gamma}(V^{2}w^{\gamma}) + \zeta_{2}\partial_{\gamma}u^{\gamma}\right)$$

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 $w^{\mu} = -\left(\partial^{\mu}\varphi + \mu u^{\mu}\right)$

where

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$$

BULK VISCOSITY IN COLOR SUPERCONDUCTORS

Bulk viscosity depends on the low energy spectrum of the theory

CFL phase		
Contribution of phonons	Manuel and LLan MM and Manuel	nes 0705.3909 0909.4486
Contribution of kaons	Alford et al. Alford et al.	nucl-th/0701067 0707.2389
Spin 1 phase	Sa'd et al. Wang et al.	astro-ph/0607643 1006.1293
2SC phase	Alford and Schmitt	nucl-th/0608019

Large amplitude behavior of bulk viscosity: talk by Alford

Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities) Son hep-ph/0204199

$$\mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[(\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2$$

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 $\varphi(x) = \bar{\varphi}(x) + \phi(x)$

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 $\varphi(x) = \bar{\varphi}(x) + \phi(x)$ bulk long-wavelength fluctuations

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Fluid of quasiparticles (phonons) moving on the top of the superfluid background



PHONON CONTRIBUTION

phonon dispersion law $\epsilon_p = c_s p + Bp^3 + \mathcal{O}(p^5)$

 $\left[\partial_t \mathcal{N}_{\rm ph} + \operatorname{div}(\mathcal{N}_{\rm ph} \mathbf{v}_{\mathbf{n}}) = -\frac{\Gamma_{\rm ph}}{T} \mu_{\rm ph}\right]$

$$\left\{ \begin{array}{ll} B > 0 & \phi \to \phi \phi \\ B < 0 & \phi \phi \to \phi \phi \phi \end{array} \right.$$

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$$\begin{aligned} \zeta_1 &= -\frac{T}{\Gamma_{\rm ph}} \frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho} \left(\mathcal{N}_{\rm ph} - S \frac{\partial \mathcal{N}_{\rm ph}}{\partial S} - \rho \frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho} \right) = -\frac{T}{\Gamma_{\rm ph}} I_1 I_2 \\ \zeta_2 &= \frac{T}{\Gamma_{\rm ph}} \left(\mathcal{N}_{\rm ph} - S \frac{\partial \mathcal{N}_{\rm ph}}{\partial S} - \rho \frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{\rm ph}} I_2^2 \\ \zeta_3 &= \frac{T}{\Gamma_{\rm ph}} \left(\frac{\partial \mathcal{N}_{\rm ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{\rm ph}} I_1^2 \end{aligned}$$

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Notice that $\zeta_1^2 = \zeta_2 \zeta_3$ the system tends toward the state where bulk viscosity does not lead to dissipation

PHONONS IN CFL

Low temperatures

 $T \lesssim 0.01 {
m MeV}$





Conformal limit





Conformal breaking due to m_s

$$\zeta_1 \sim \frac{m_s^2 \mu^7}{T\Delta^8} \qquad \zeta_2 \sim \frac{m_s^4 \mu^8}{T\Delta^8}$$

MUTUAL FRICTION

Mutual friction

Force between the superfluid component and the normal component mediated by phonon-vortex interaction

$$\rho_s \frac{d\mathbf{v}_s}{dt} = -\frac{\rho_s}{\rho} \nabla p - \rho_s \nabla \phi - \mathbf{F}^N$$
$$\rho_n \frac{d\mathbf{v}_n}{dt} = -\frac{\rho_n}{\rho} \nabla p - \rho_n \nabla \phi + \mathbf{F}^N + \eta \nabla^2 \mathbf{v}_n$$

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 $\bigcirc \text{Magnus force} \qquad \text{Standard hydrodynamic force}$ $\mathbf{F}^{M} = \kappa \rho_{s} (\mathbf{v}_{s} - \mathbf{v}_{L}) \times \hat{\mathbf{z}}$

Scattering of phonons off vortices $\mathbf{F}^{N} = D(\mathbf{v}_{n} - \mathbf{v}_{L}) + D'\hat{\mathbf{z}} \times (\mathbf{v}_{n} - \mathbf{v}_{L})$

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Friction force $\mathbf{F}^{N} = D(\mathbf{v}_{n} - \mathbf{v}_{L}) + D'\hat{\mathbf{z}} \times (\mathbf{v}_{n} - \mathbf{v}_{L})$



R-mode instability



Gravitational

Radiation

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable. See also Andersson, Kokkotas gr-qc/0010102

Lindblom, astro-ph/0101136

R-mode instability



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 \longrightarrow

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CFL

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Emitting gravitational radiation the star quickly spins down

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CFL

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GFL is a superfluid

Contribution of phonons to the bulk viscosity coefficients of CFL

For rotating superfluids one has to include the mutual friction force

Damping of star oscillations, especially gravitationally unstable r-modes

