QCD@WORK, 2010 June 20, Martina Franca

Marco Ruggieri,
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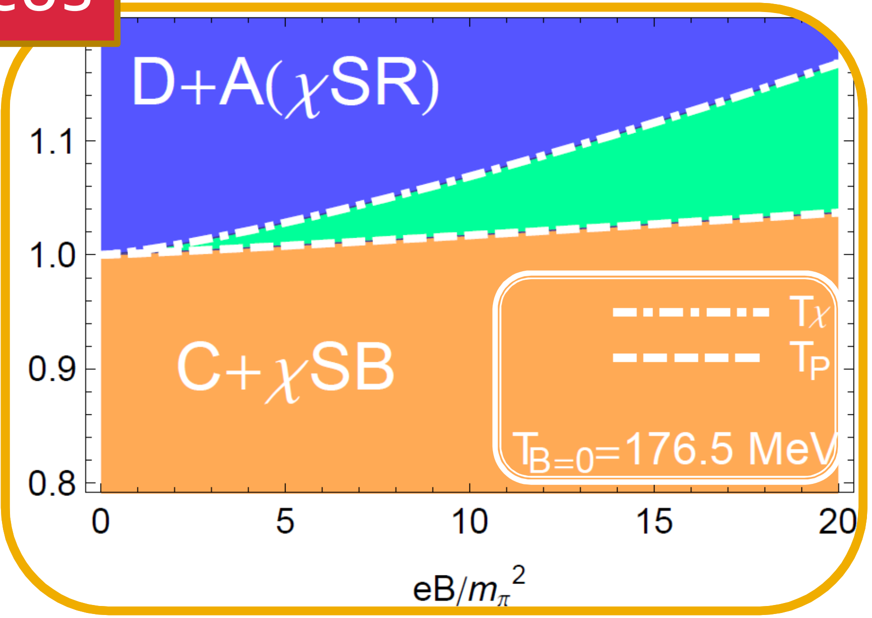
Quark matter in strong magnetic fields

Why the study of QCD in strong magnetic fields

- Understanding confinement and chiral symmetry breaking
- Heavy ion collisions: topological excitations may lead to P and CP odd effects (CME)

Focus



Quantitative determination of the phase diagram, 1

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

Very first **model** calculation of the phase diagram in magnetic field, treating self-consistently
.) dynamical chiral symmetry breaking
.) confinement

Model lagrangian density for two massless flavors:

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu D^\mu + \mu_5\gamma^0\gamma^5\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\boldsymbol{\tau}\psi\right)^2\right]$$

Chiral chemical potential (mimics instanton/sphaleron induced chirality)

Chiral invariant Nambu-Jona Lasinio **4-quark interaction**
(Nambu and Jona Lasinio, **Phys.Rev.122:345-358,1961**.)

Quantitative determination of the phase diagram, 2

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

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Chiral invariant Nambu-Jona Lasinio **4-quark interaction**
(Nambu and Jona Lasinio, **Phys.Rev.122:345-358,1961**.)

Responsible for spontaneous chiral symmetry breaking in the vacuum

Quantitative determination of the phase diagram, 3

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

$$f_\Lambda(p) = \sqrt{\frac{\Lambda^{2N}}{\Lambda^{2N} + |\mathbf{p}|^{2N}}}\,,$$

One loop thermodynamic potential:

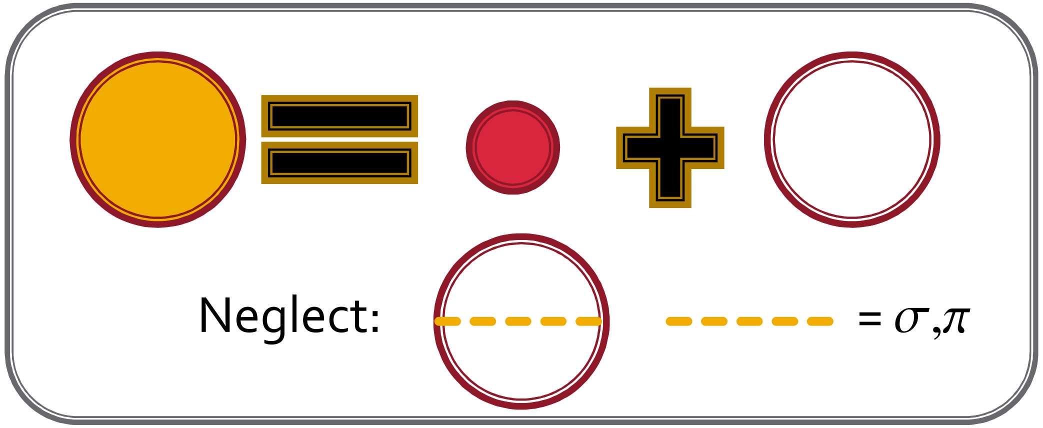
$$\Omega = \mathcal{U} + \frac{\sigma^2}{G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} f_\Lambda^2\, \omega_s(p)$$
$$- 2T \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$
$$\times \ln\left(1 + 3\,\Phi e^{-\beta\omega_s} + 3\,\bar{\Phi} e^{-2\beta\omega_s} + e^{-3\beta\omega_s}\right)\,.$$

$$\omega_s^2 = M^2 + \left[|\mathbf{p}| + s\,\mu_5 \mathrm{sgn}(p_z)\right]^2$$

Quark dispersion law

$$\alpha_{sk} = \begin{cases} \delta_{s,+1} & \text{for} \quad k = 0,\; qB > 0\,, \\ \delta_{s,-1} & \text{for} \quad k = 0,\; qB < 0\,, \\ 1 & \text{for} \quad k \neq 0\,. \end{cases}$$

Landau levels degeneracy



Quantitative determination of the phase diagram, 4

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**
K. Fukushima, **Phys.Lett.B591:277-284,2004**

One loop thermodynamic potential:

$$\Omega = \mathcal{U} + \frac{\sigma^2}{G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} f_\Lambda^2 \omega_s(p)$$
$$- 2T \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$
$$\times \ln\left(1 + 3\,\Phi e^{-\beta\omega_s} + 3\,\bar{\Phi} e^{-2\beta\omega_s} + e^{-3\beta\omega_s}\right) .$$

$$L = \mathcal{P} \exp\left(i \int_0^\beta A_4\, d\tau\right)$$

$$\Phi = (1/N_c)\mathrm{Tr}L$$

$$\mathcal{U}[\Phi, \bar{\Phi}, T] = T^4 \Big\{ -\frac{a(T)}{2} \bar{\Phi}\Phi$$
$$+ b(T) \ln[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2] \Big\}$$

W. Weise *et al*, **Phys.Rev.D75:034007,2007**

P-NJL model, some reference

N. Meisinger and M. C. Ogilvie, Phys. Lett. **B379**, 163 (1996)
K. Fukushima, Phys. Lett. **B591,** 277 (2004)
C. Ratti, M. A. Thaler and W. Weise, Phys.\ Rev.\ D **73**, 014019 (2006).
M. Ruggieri *et al.*, Phys.\ Rev.\ D**77**, 054023 (2008).
M. Ruggieri *et al.*, Phys.\ Rev.\ D**78**, 034034 (2008).
Y. Sakai *et al.*, Phys.\ Rev.\ D**77**, 051901 (2008).
K. Kashiwa *et al.*, Phys.\ Rev.\ D**80**, 117901 (2009).
K. Kashiwa *et al.*, Phys.\ Lett.\ B**662**, 26 (2008).
S. K. Ghosh *et al.*, Phys.\ Rev.\ D**77**, 094024 (2008).
K. Redlich *et al.*, Phys.\ Rev.\ D**75**, 074013 (2007).
E. Megias *et al.*, Phys.\ Rev.\ D**74**, 114014 (2006).

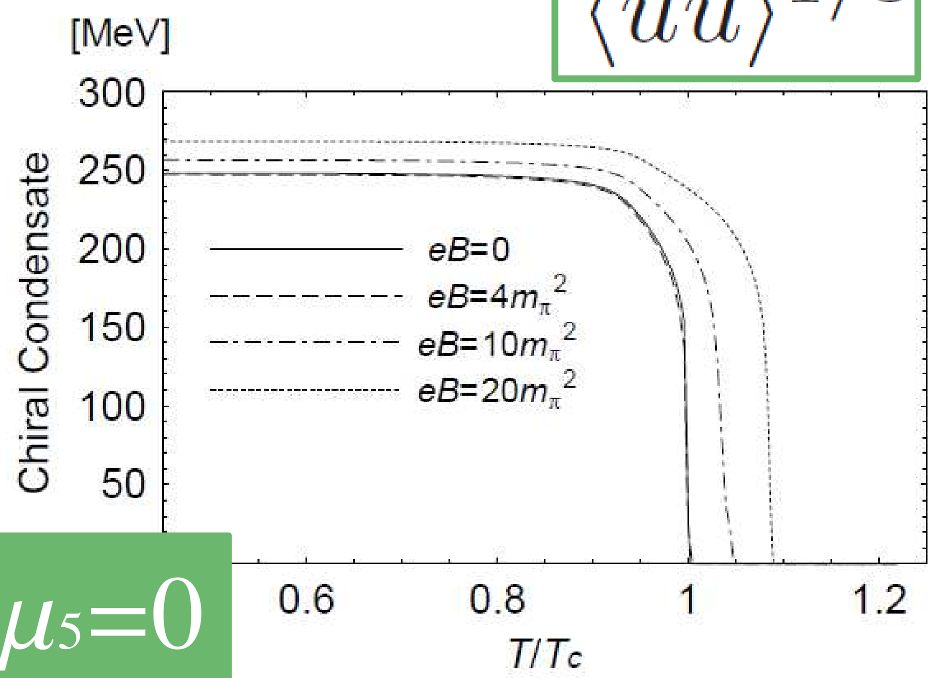
Quantitative determination of the phase diagram, 5.0

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

$$\partial\Omega/\partial\sigma = \partial\Omega/\partial\Phi = 0$$

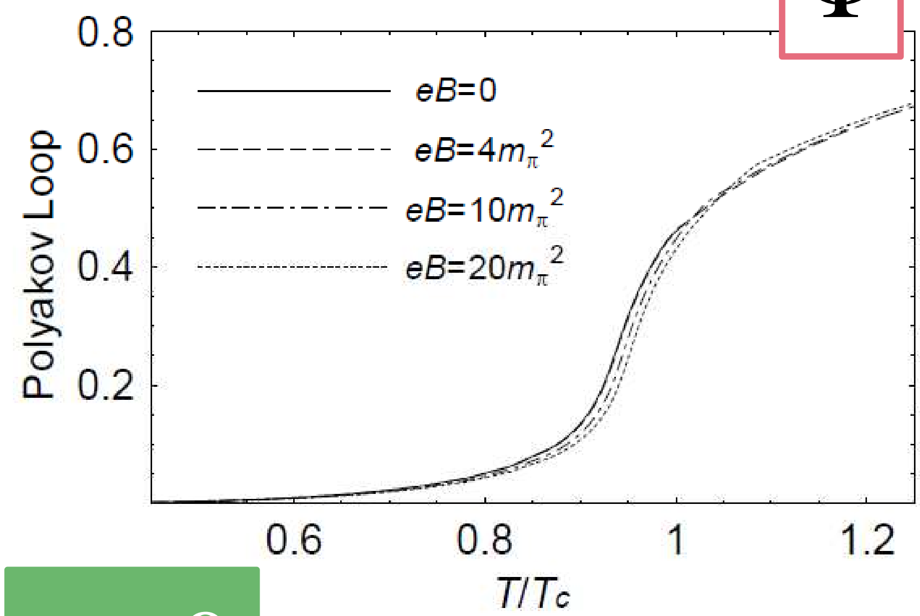
Gap equations
(solved by globally convergent alghoritm with backtrack)

$\langle\bar{u}u\rangle^{1/3}$



$\mu_5=0$

Φ

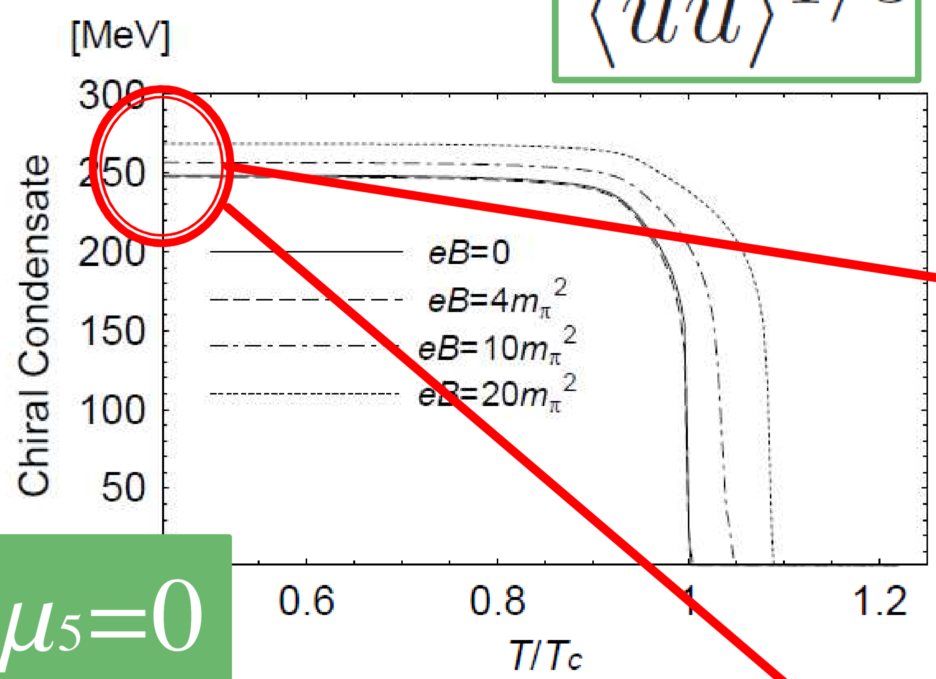


$\mu_5=0$

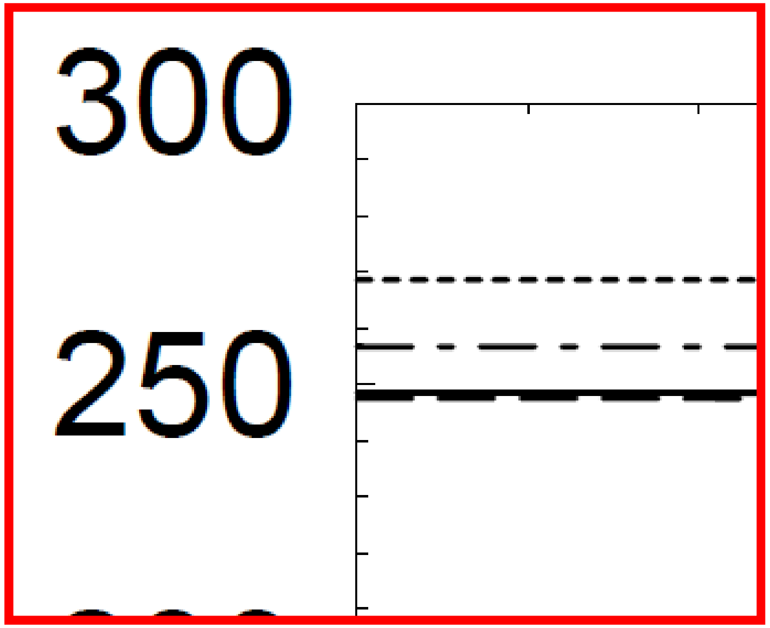
Quantitative determination of the phase diagram, 5.1

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

$\langle \bar{u}u \rangle^{1/3}$



$\mu_5 = 0$



Corresponds to an increasing constituent quark mass:

$$M_q \approx 347 \text{ MeV} \Rightarrow M_q \approx 476 \text{ MeV}$$

eB=0 eB=20

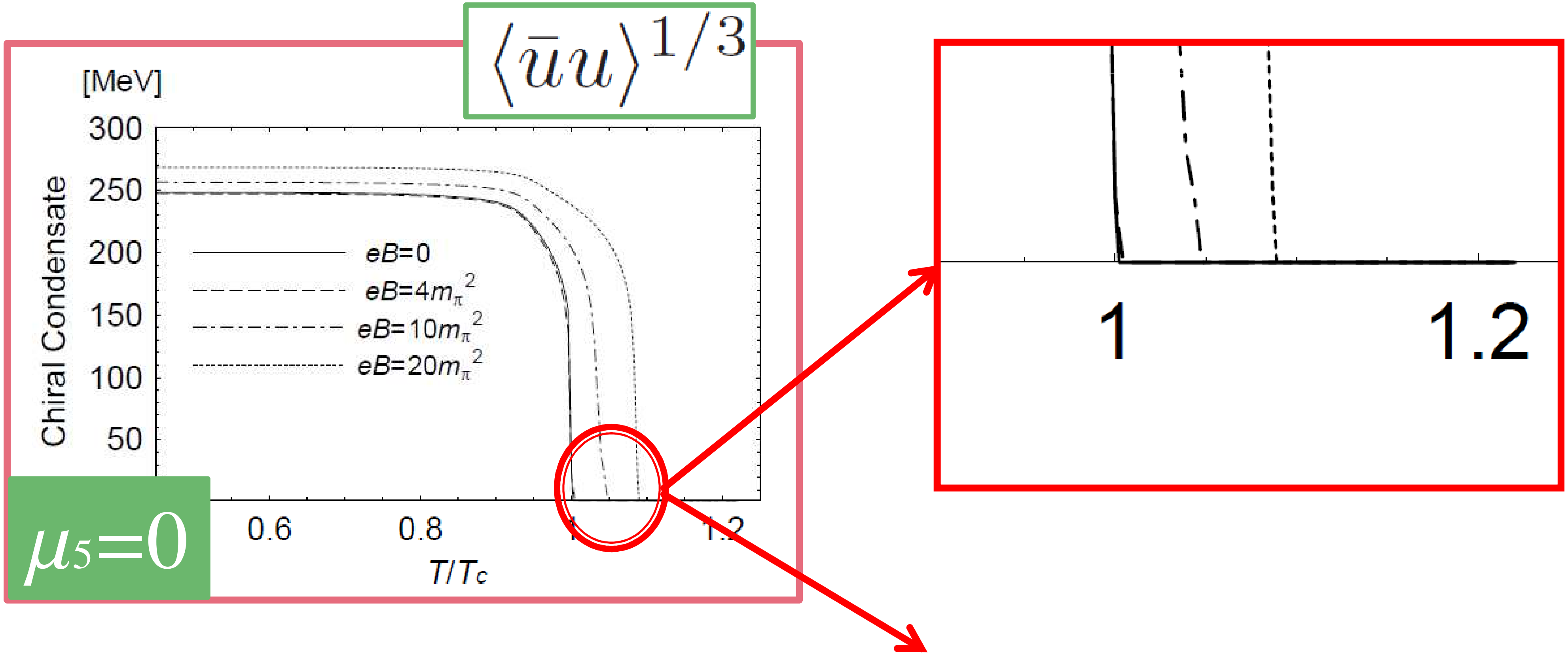
Magnetic catalysis

Magnetic catalysis, some reference

I. A. Shushpanov and A. V. Smilga, Phys. Lett. **B402**, 351 (1997).
D. N. Kabat *et al.*, Phys. Rev. D**66,** 014004 (2002).
T. D. Cohen *et al.*, Phys. Rev. C**76,** 055201 (2007).
V. P. Gusynin *et al.*, Nucl. Phys. B**462**, 249 (1996).
V. A. Miransky and I. A. Shovkovy, Phys. Rev. D**66**, 045006 (2002).
E. S. Fraga and A. J. Mizher, Phys. Rev. D**78**, 025016 (2008).
A. V. Zayakin, JHEP **0807**, 116 (2008).

Quantitative determination of the phase diagram, 5.1.1

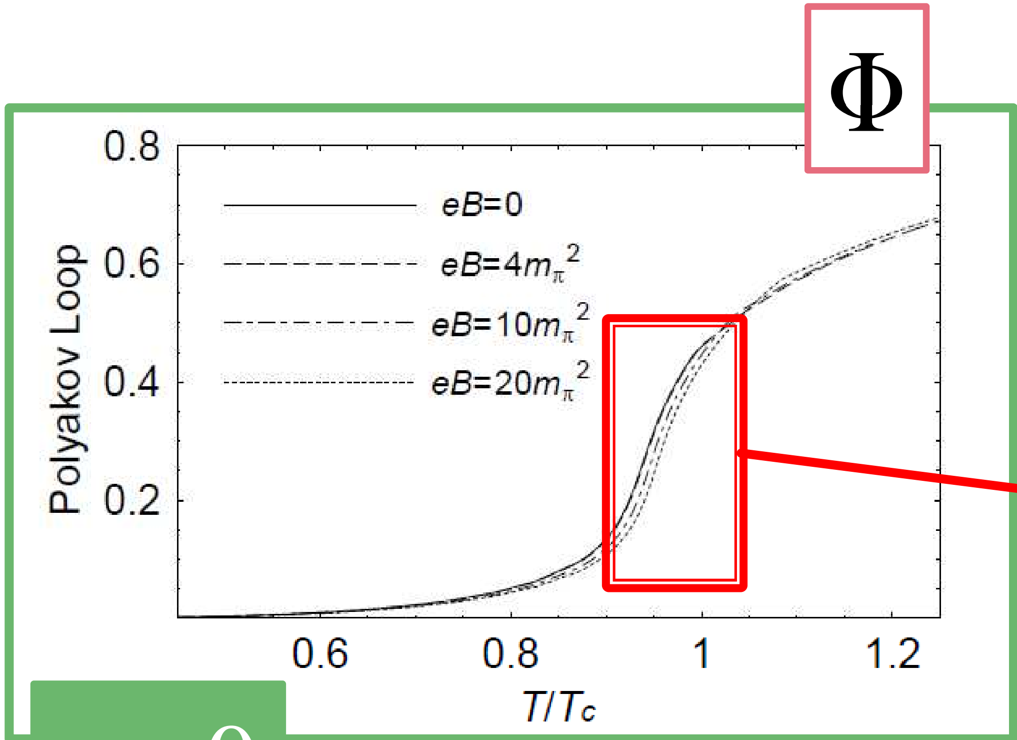
M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**



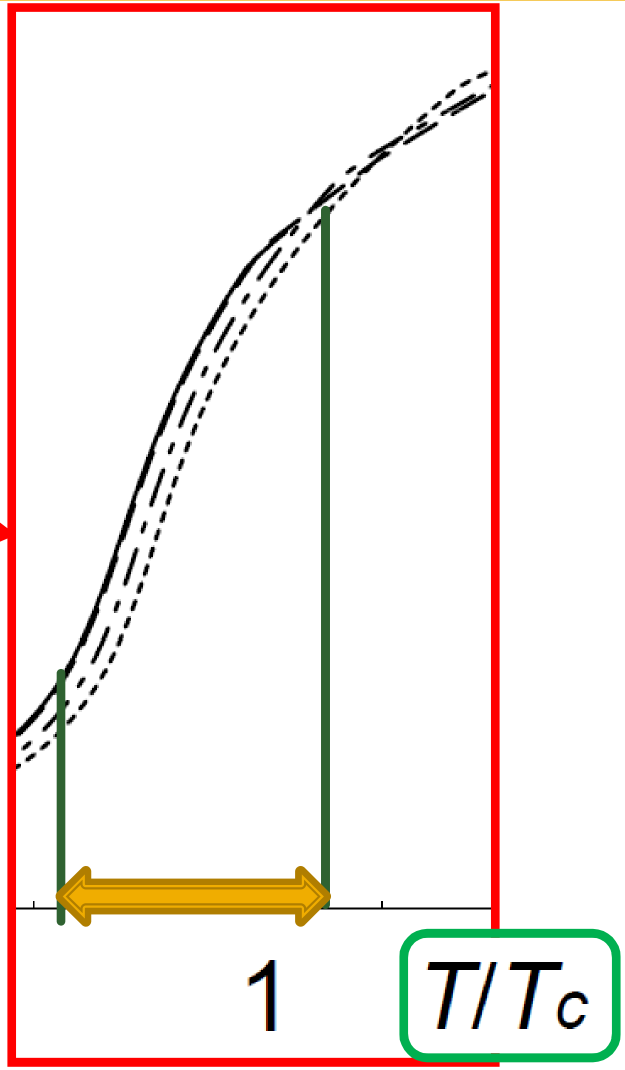
Critical temperature increased

Quantitative determination of the phase diagram, 5.2

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**



$\mu_5=0$

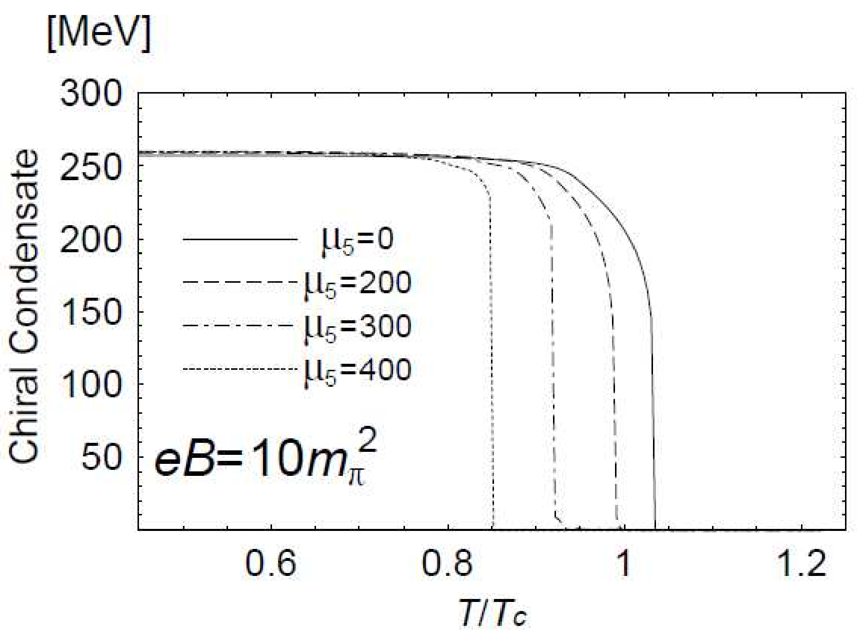


Deconfinement crossover poorly affected by chiral dynamics around Tc

Quantitative determination of the phase diagram, 5.3

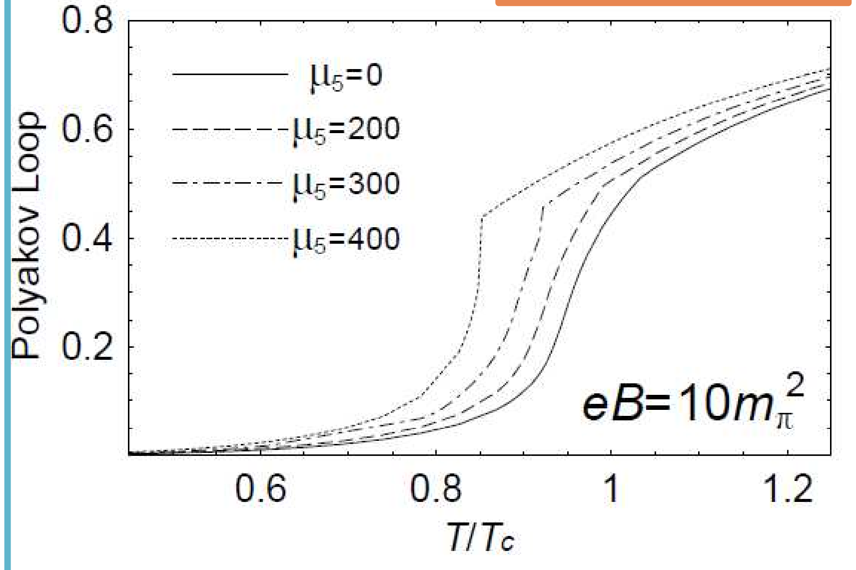
M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

Effect of chiral chemical potential



Chiral condensate

Polyakov loop



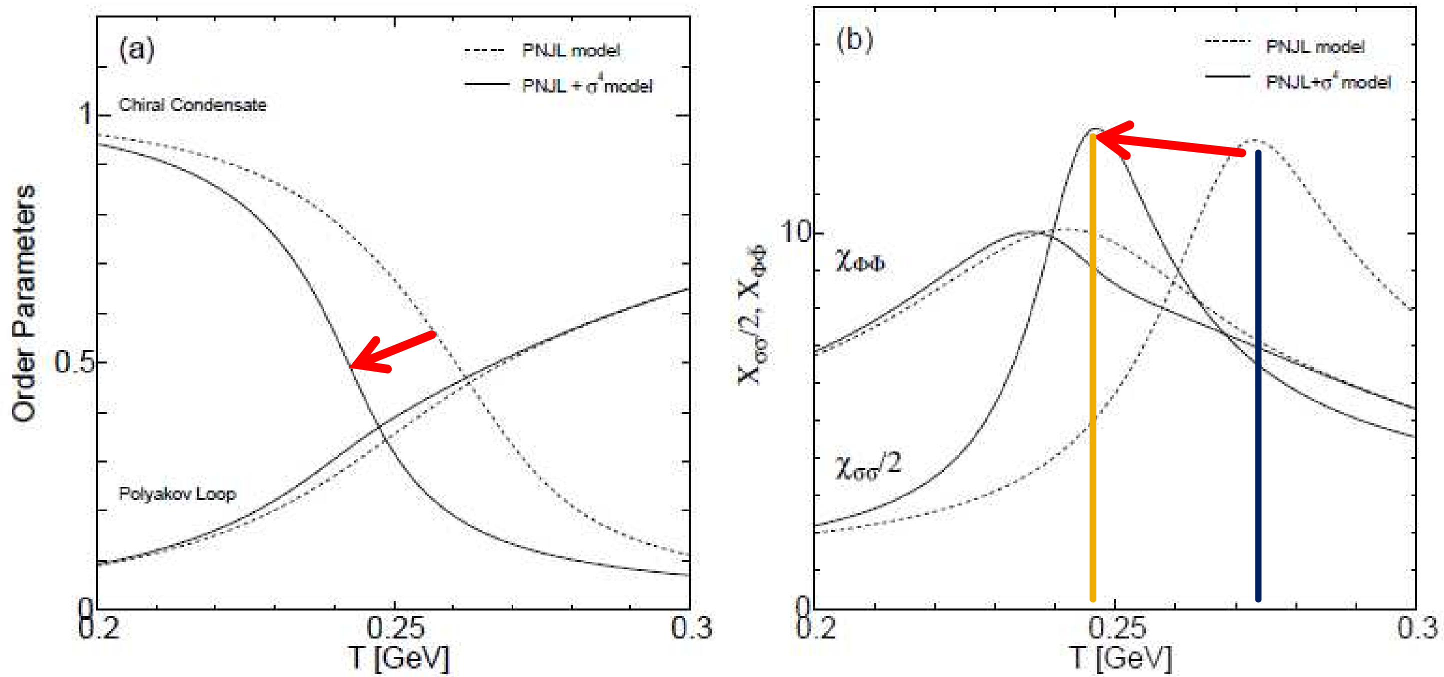
Quantitative determination of the phase diagram, 7

M. Ruggieri and R. Gatto, in preparation

We improve the previous results using **a more realistic model** for in-medium interactions:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - m_0\right)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right] + G_8\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]^2$$

8-quark interaction



Main role: decreases naturally pseudo-critical temperatures

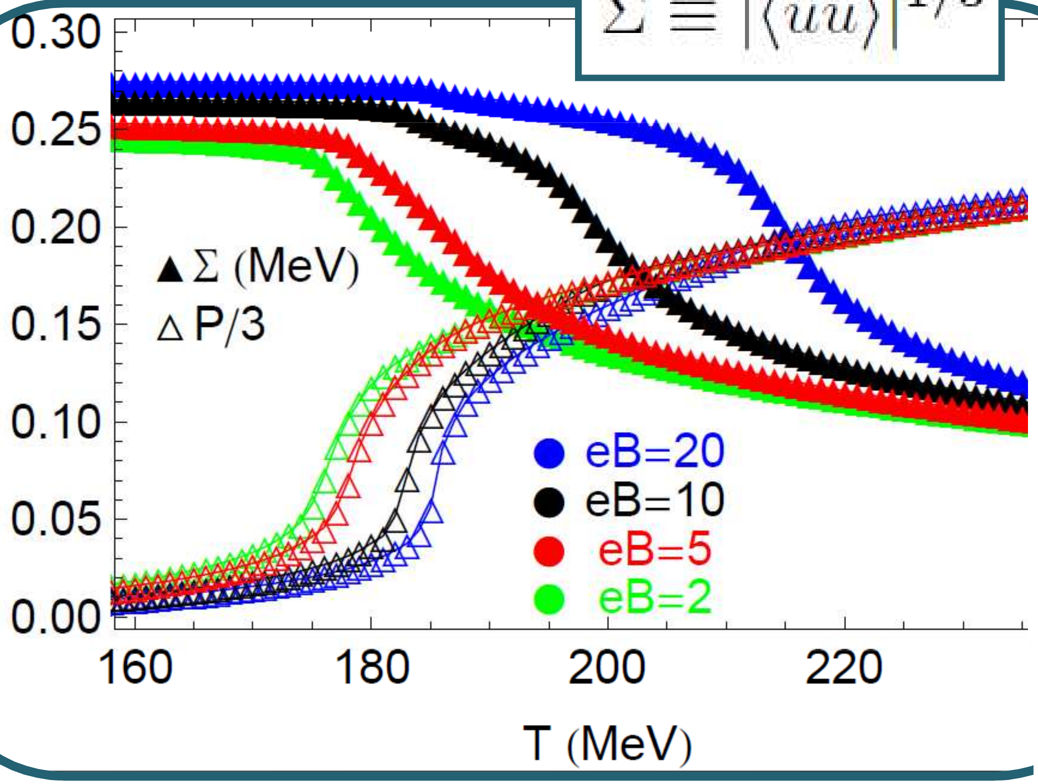
Kashiwa *et al.*, **Phys.Lett.B662:26-32,2008**

8-quark interactions: some reference

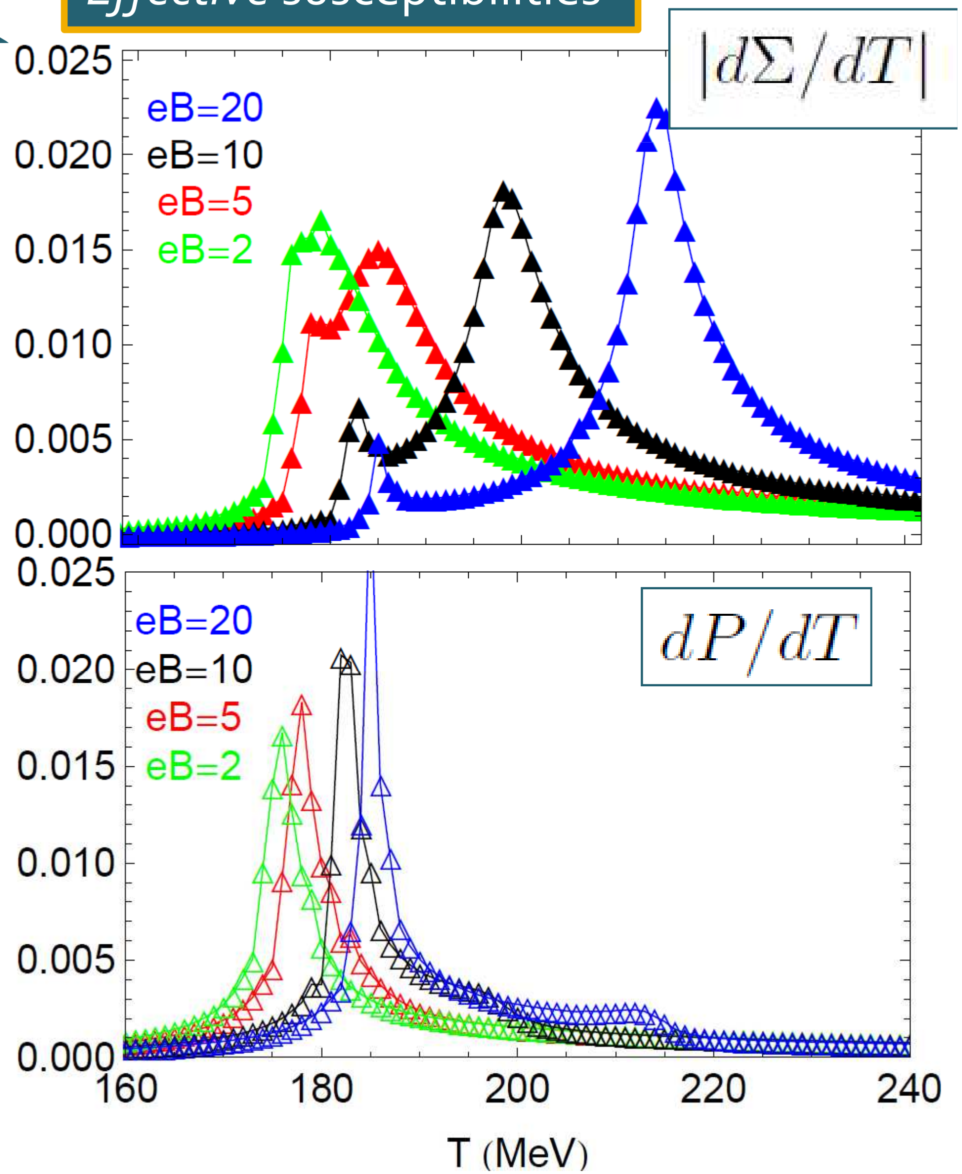
A. A. Osipov et al., Phys. Lett.B**646**, 91 (2007).
K. Kashiwa et al, Phys. Lett. B**647** , 446 (2007).
A. A. Osipov et al., Phys. Lett. B**634**, 48 (2006).
A. A. Osipov et al., Phys. Lett. B**650**, 262 (2007).

Quantitative determination of the phase diagram, 8.1

$\Sigma \equiv |\langle \bar{u}u \rangle|^{1/3}$



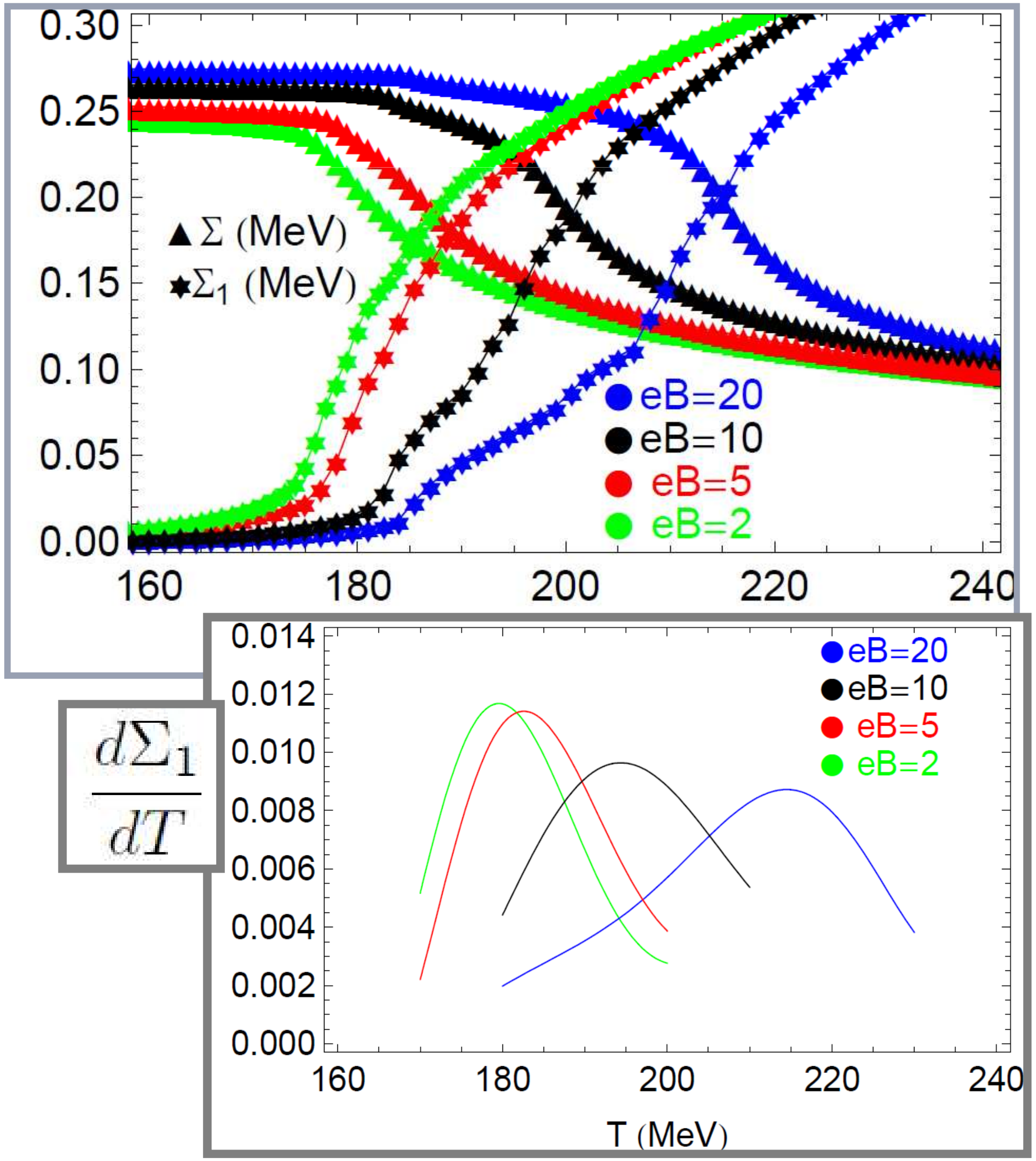
Effective susceptibilities



M. Ruggieri and R. Gatto, in preparation

Quantitative determination of the phase diagram, 8.2

M. Ruggieri and R. Gatto, in preparation



Dressed Polyakov loop

$$\Sigma_1 \equiv -2\pi G \int \frac{d\phi}{2\pi} e^{-i\phi} \langle \bar{q} q \rangle_\phi$$

$$\psi(\vec{x},\beta) = e^{i\varphi}\,\psi(\vec{x},0)$$

Under center symmetry:

$$\Sigma_1 \Rightarrow z\Sigma_1 \ , \quad z \in Z_3$$

In the large **m** limit:

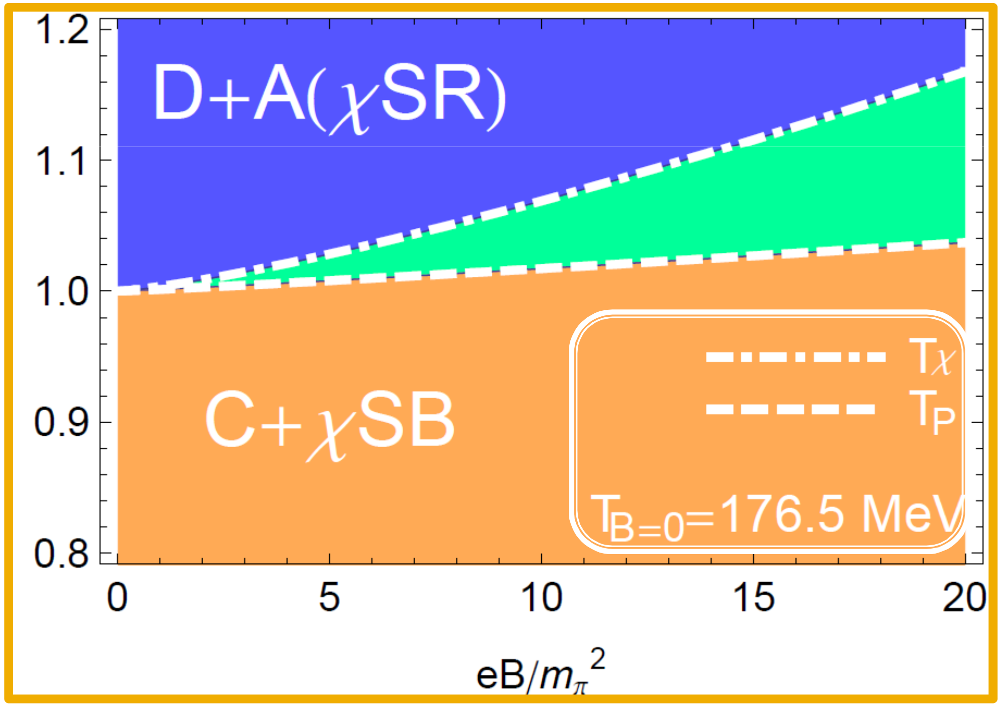
$$\Sigma_1 \propto P$$

Indicator of deconfinement crossover

C. Gattringer *et al.*,
Phys.Rev.D77:094007,2008

Computed phase diagram from Polyakov loop

M. Ruggieri and R. Gatto, in preparation



Best-fit

$$\frac{T_{\chi,P}}{T_0} = 1 + a_{\chi,P}\left(\frac{eB}{T_c^2}\right)^{b_{\chi,P}}$$

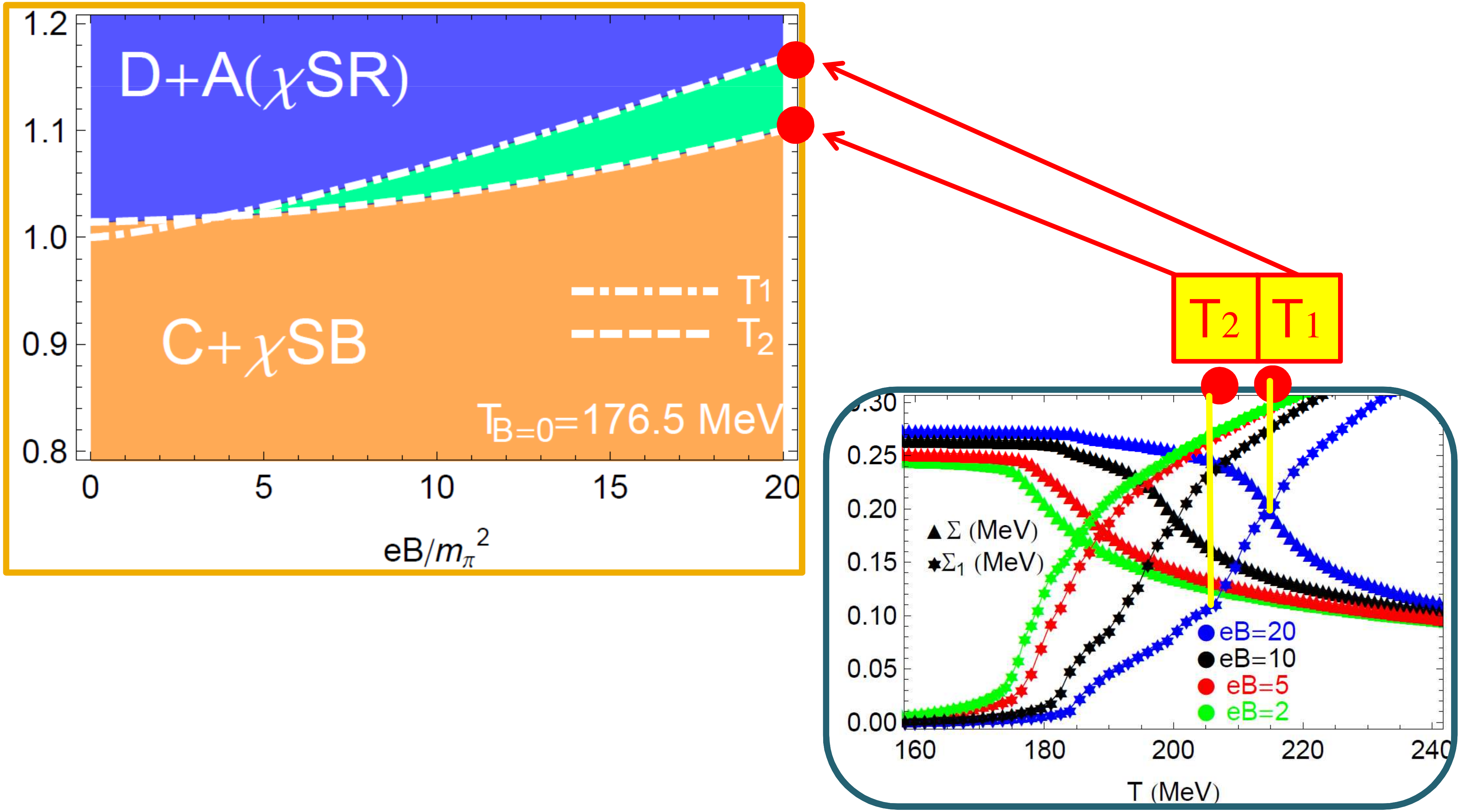
Best-fit parameters

$$a_\chi = 0.13\ , \qquad b_\chi = 1.28$$

$$a_P = 7\times 10^{-3}\ , \qquad b_P = 1.11$$

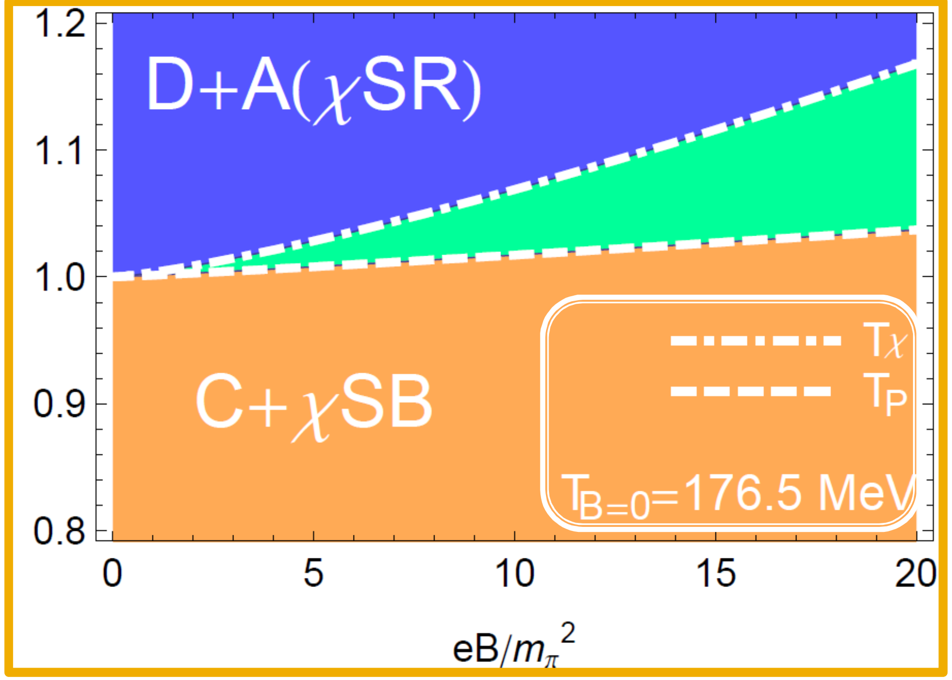
Computed phase diagram from Σ1

M. Ruggieri and R. Gatto, in preparation



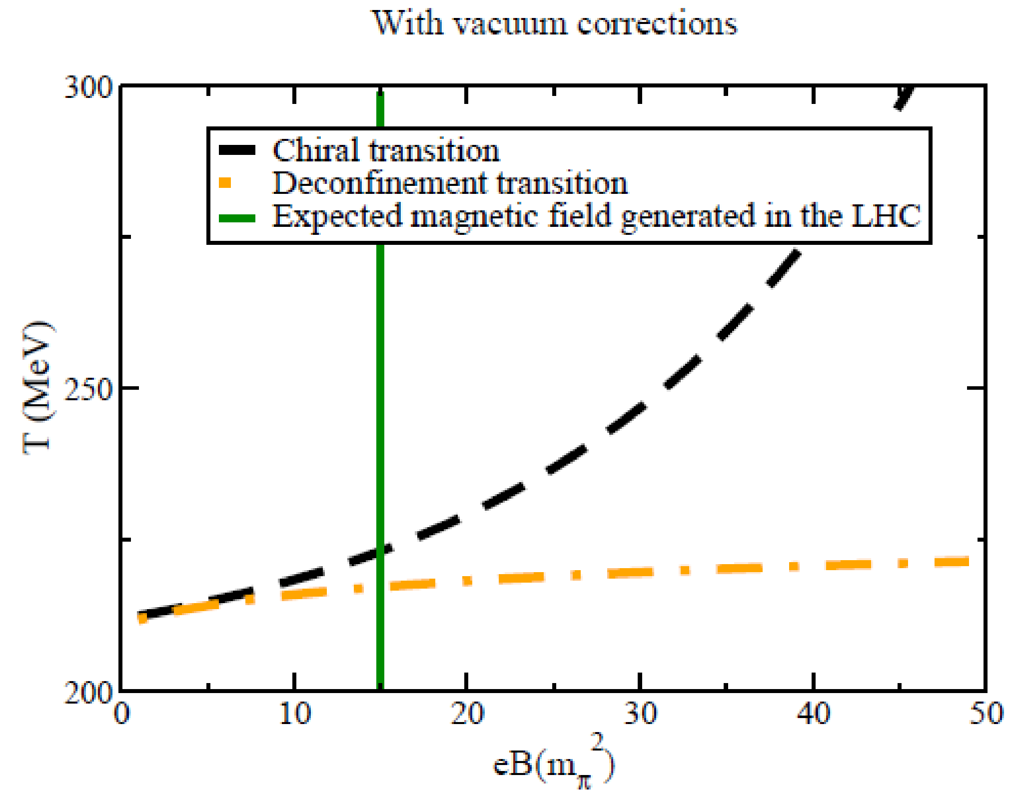
P-NJL and P-QM models comparison

P-NJL



E. Fraga *et al*, **arXiv:1004.2712**

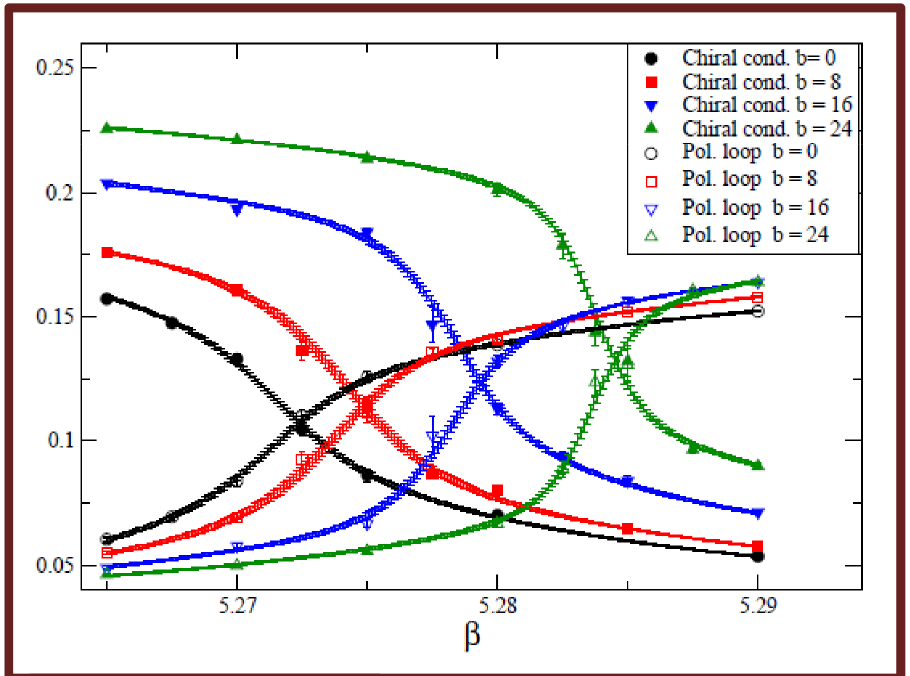
P-QM



Excellent agreement

Quantitative results from Lattice

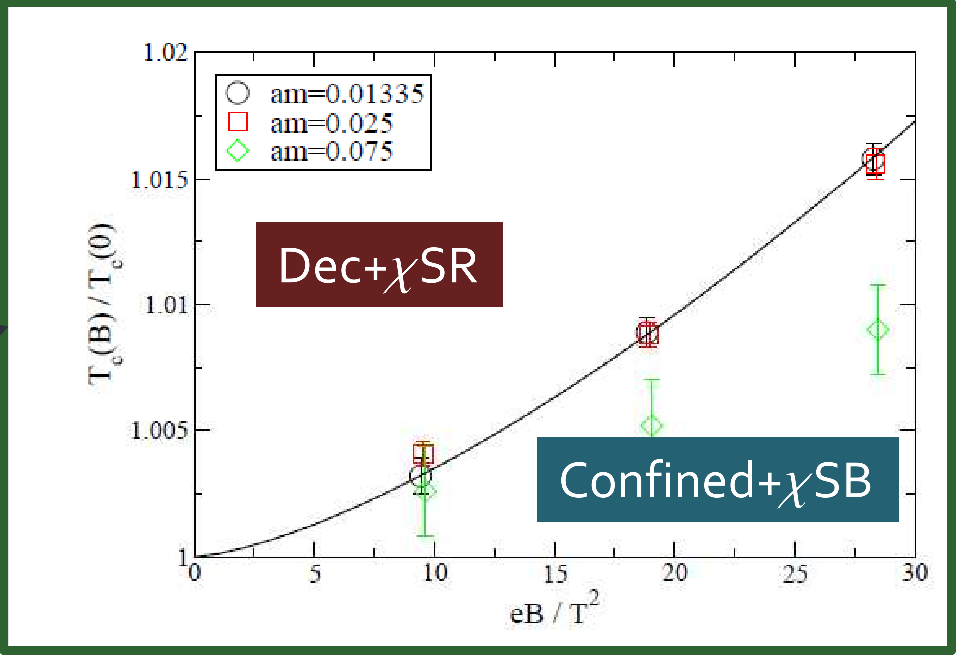
M. d'Elia *et al*, **arXiv:1005.5365**



Condensate
Polyakov loop

Nf=2 staggered fermions
$m\pi$=195 MeV
Lattice size:16x16x16x4

Phase Diagram



Coincidence of the two crossovers

Conclusions, perspectives for the future

- P-NJL model:powerful tool to investigate on the interplay between deconfinement and chiral restoration
- Dressed Polyakov loop opens interesting scenario

- Improve Lattice size
- Get a deeper understanding of the coupling among Polyakov loop and chiral dynamics in external fields

Acknowledgements

For collaboration on the topics discussed in this talk, I acknowledge:
K. Fukushima,
R. Gatto.

Moreover, it is a pleasure to acknowledge the following people for interesting discussions about the topics of this talk:
H. Abuki, L. Campanelli,
E. Cartman, M. d'Elia, S. Nicotri.

The numerical calculations were carried out on Altix3700 BX2 at YITP in Kyoto University.

Appendix

Some backup slide

Quantitative determination of the phase diagram, 1

Determination of the QCD vacuum is a hard task

↓

Lattice QCD

Discretize (euclidean) space-time, and compute QCD partition function via Montecarlo generation of gluon configurations

However:

Lattice QCD is expensive, both in time and in CPU resources.

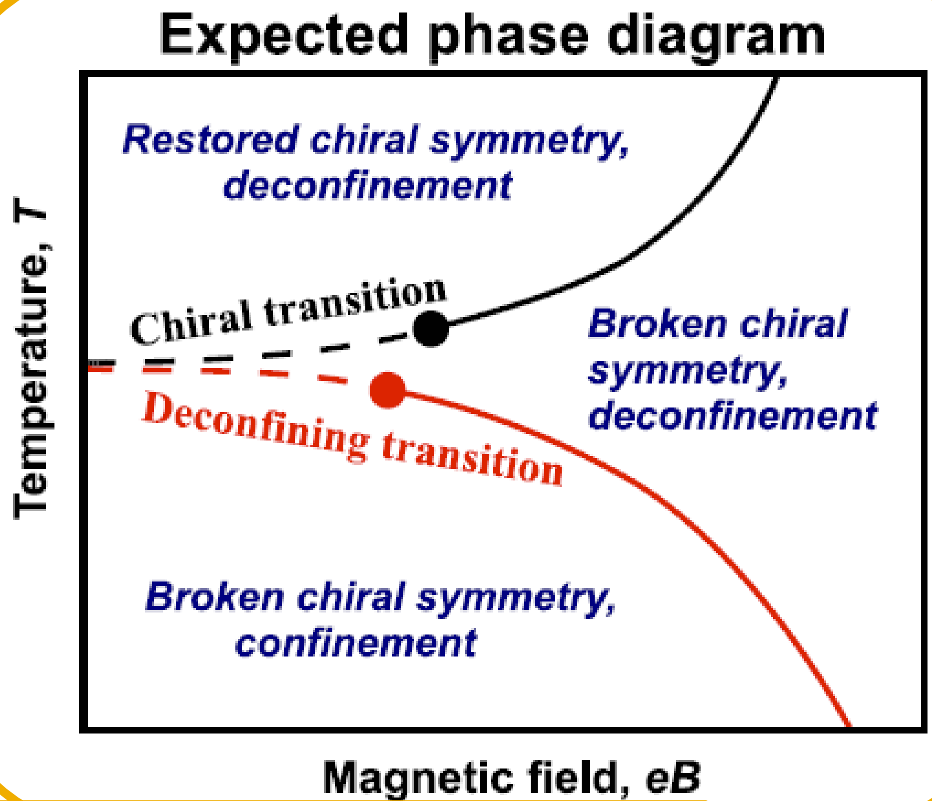
Therefore, if we wish to **speculate** about QCD vacuum and its response to anything:

Either ⟺ Approximate the SDEs of QCD

Or ⟺ Use different models

Qualitative picture of the phase diagram

Fraga *et al*, **arXiv:1004.2712**
Agasian and Fedorov, **Phys.Lett.B663:445-449,2008**.



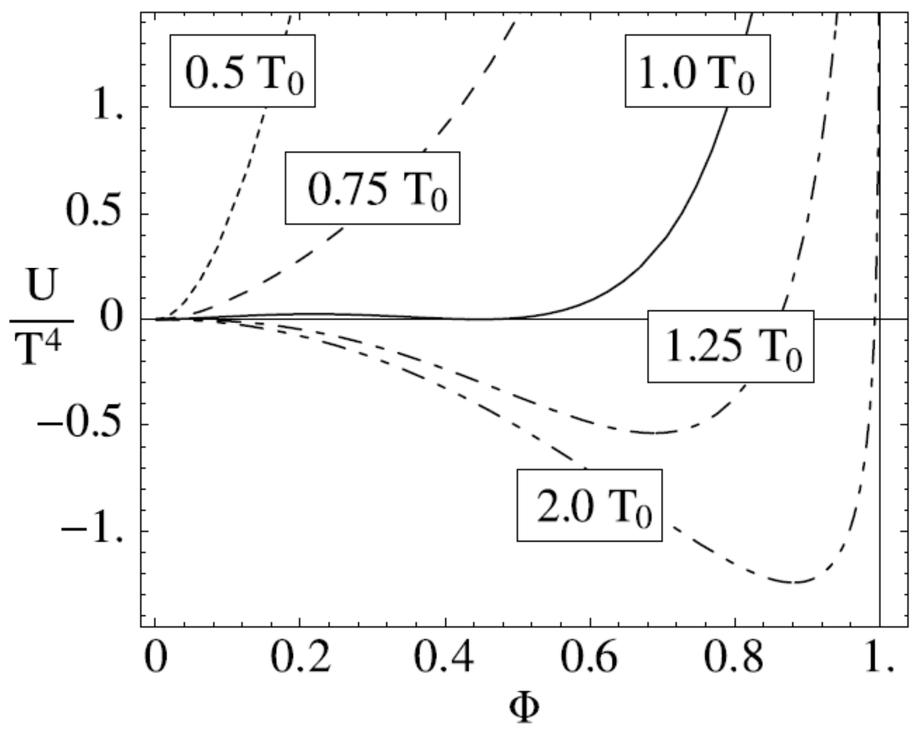
E. Fraga *et al.*, **arXiv:1004.2712**

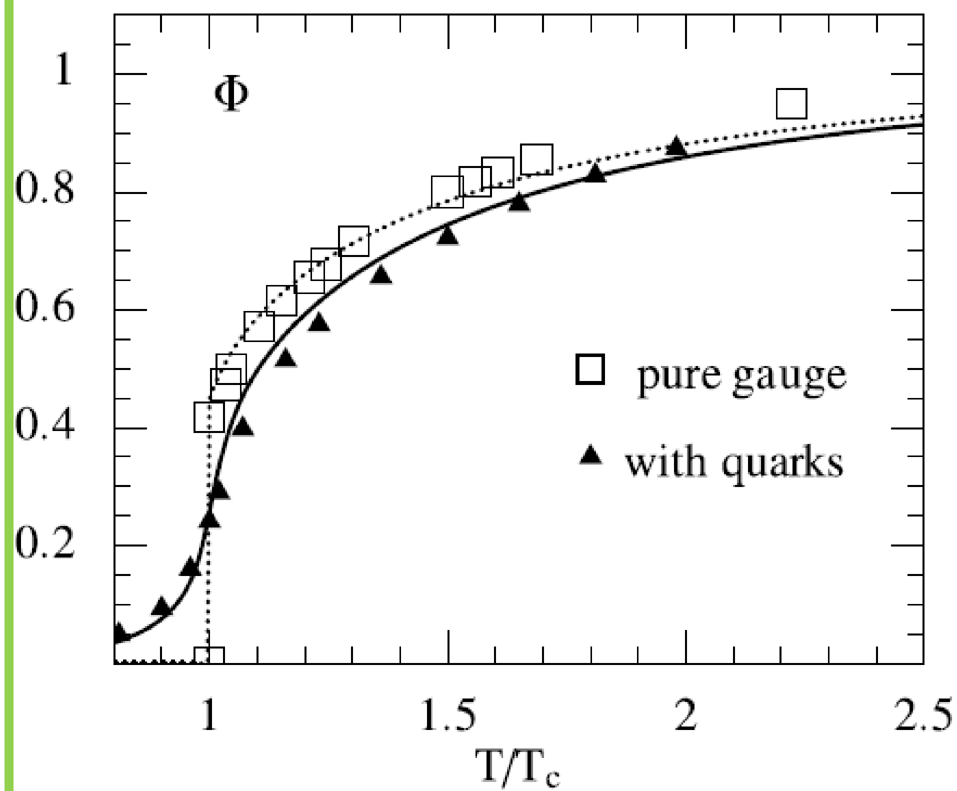
Picture mainly based on:
.) quark-meson models
.) ChPT in magnetic fields
.) phenomenological models of hadrons

Quantitative determination of the phase diagram, 4.1

W. Weise *et al*, **Phys.Rev.D75:034007,2007**

$$\mathcal{U}[\Phi, \bar{\Phi}, T] = T^4 \Big\{ -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln\big[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\big] \Big\}$$





Quantitative determination of the phase diagram, 5

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

Parameter set:

$$\Lambda = 620 \text{ MeV} , \qquad G\Lambda^2 = 2.2$$

Fixed to reproduce some vacuum observables:

$$f_\pi = 92.4 \text{ MeV}$$
$$\langle \bar{u}u \rangle^{1/3} = -245.7 \text{ MeV}$$

Output (NJL sector):

$$T_c \approx 190 \text{ MeV}$$ at B=0

Quantitative determination of the phase diagram, 3

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

One loop thermodynamic potential:

$$\Omega = \mathcal{U} + \frac{\sigma^2}{G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} f_\Lambda^2\, \omega_s(p)$$
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Quark dispersion law

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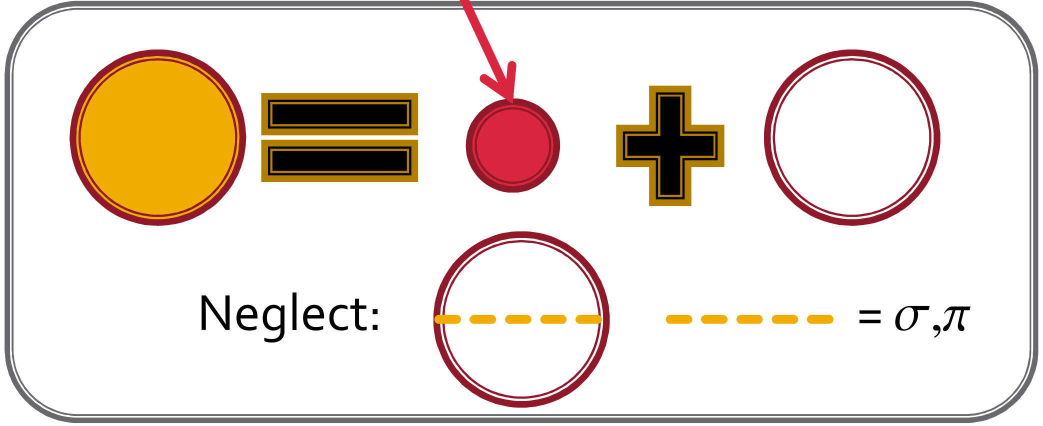
Landau levels degeneracy



Quantitative determination of the phase diagram, 3

M. Ruggieri *et al*, **arXiv:1003.0047 (in press on Phys. Rev. D)**

Tr Log

One loop thermodynamic potential:

$$\Omega = -N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} f_\Lambda^2 \,\omega_s(p) - 2T \sum_{f=u,d} \frac{|q_f B|}{2\pi} \sum_{s,k} \alpha_{sk} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \times \ln\left(1 + 3\,\Phi e^{-\beta\omega_s} + 3\,\bar{\Phi} e^{-2\beta\omega_s} + e^{-3\beta\omega_s}\right) .$$