NJL model study of effects of axial anomaly in quark matter

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Many thanks to:

Gordon Baym, Tetsuo Hatsuda, Naoki Yamamoto

Phys. Rev. D 81, 125010 (2010)

Objective

- How does axial anomaly play a role in quark many-body physics?
  - Does it affect the chiral transition at finite μ?

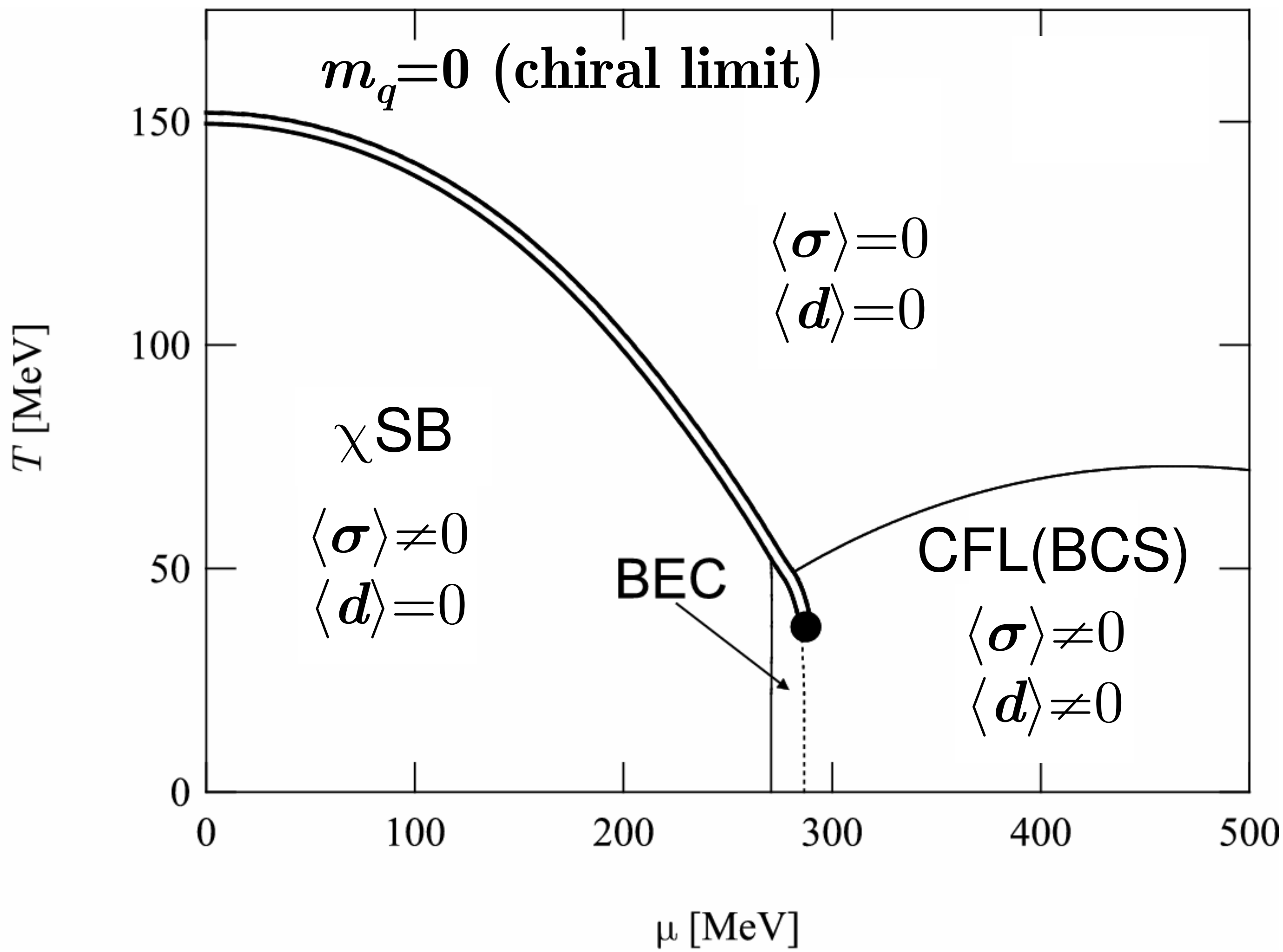
    ⇒ Yes in the Ginzburg-Landau analysis[(1)]
  - Does it modify the diquark phase and how?
- How to investigate?
  - Use an NJL model which allows a microscopic description of quark phases on the basis of quark degree's of freedom[(2)]

(1) Yamamoto, Tachibana, Baym, Hatsuda, PRL97 (06)

(2) HA, G. Baym, T. Hatsuda, N. Yamamoto, PRD81 (10)

What does axial anomaly bring?

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408



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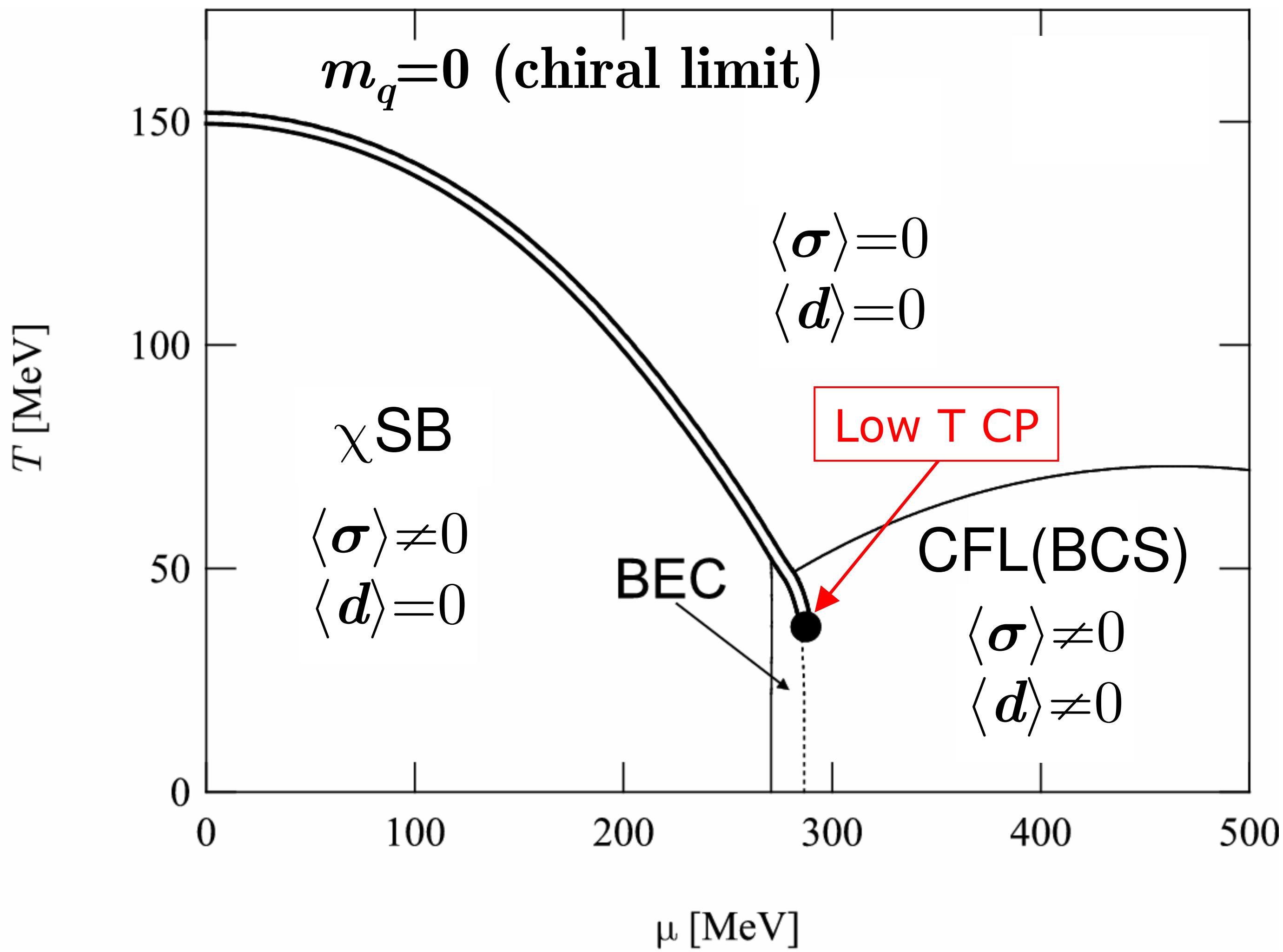

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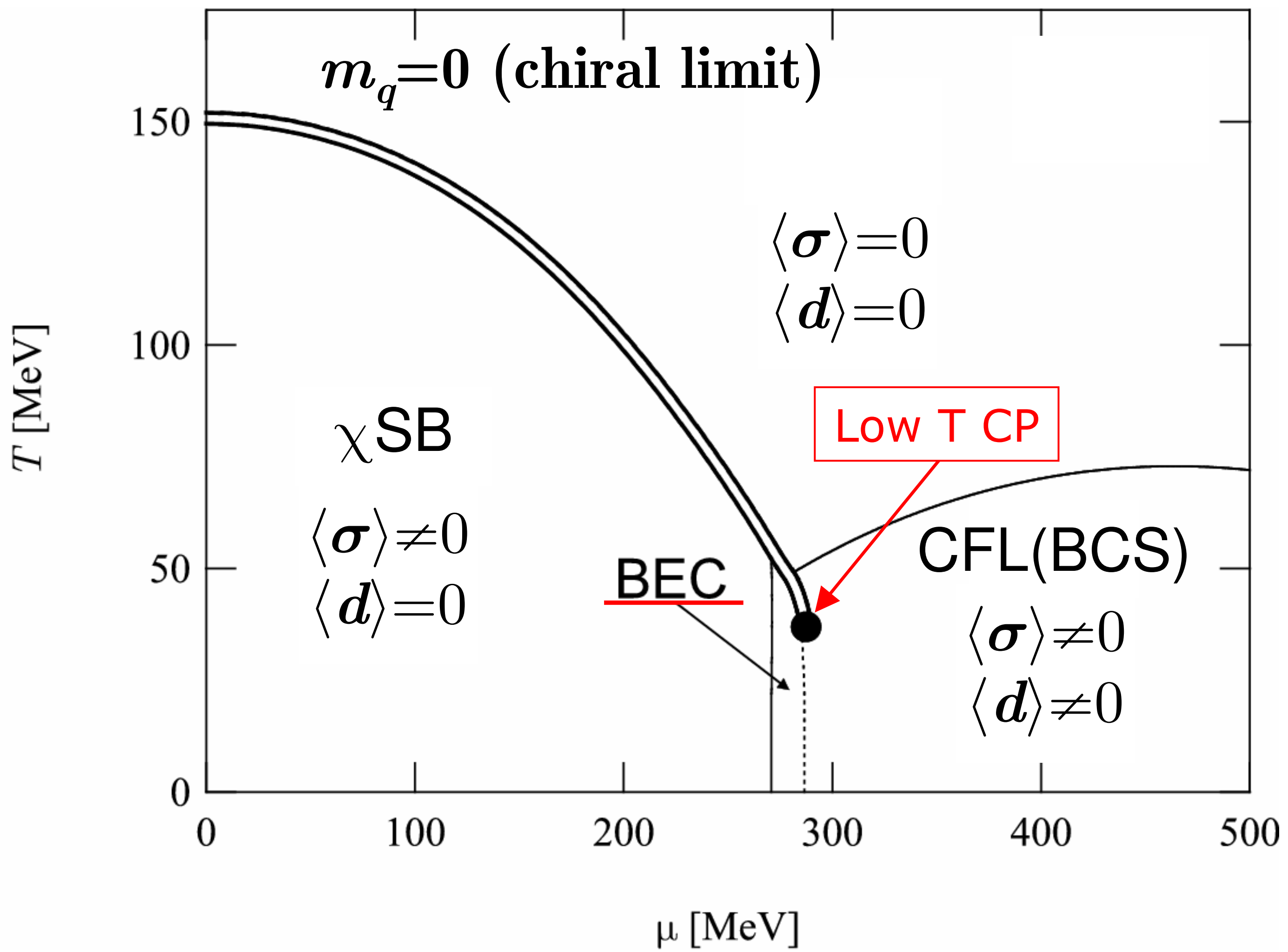
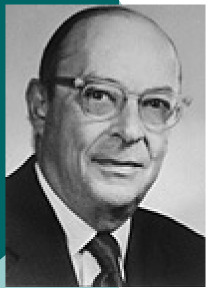
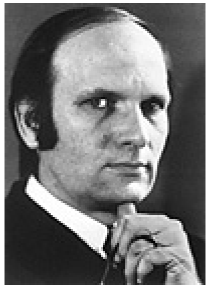


What is the BEC-BCS crossover?



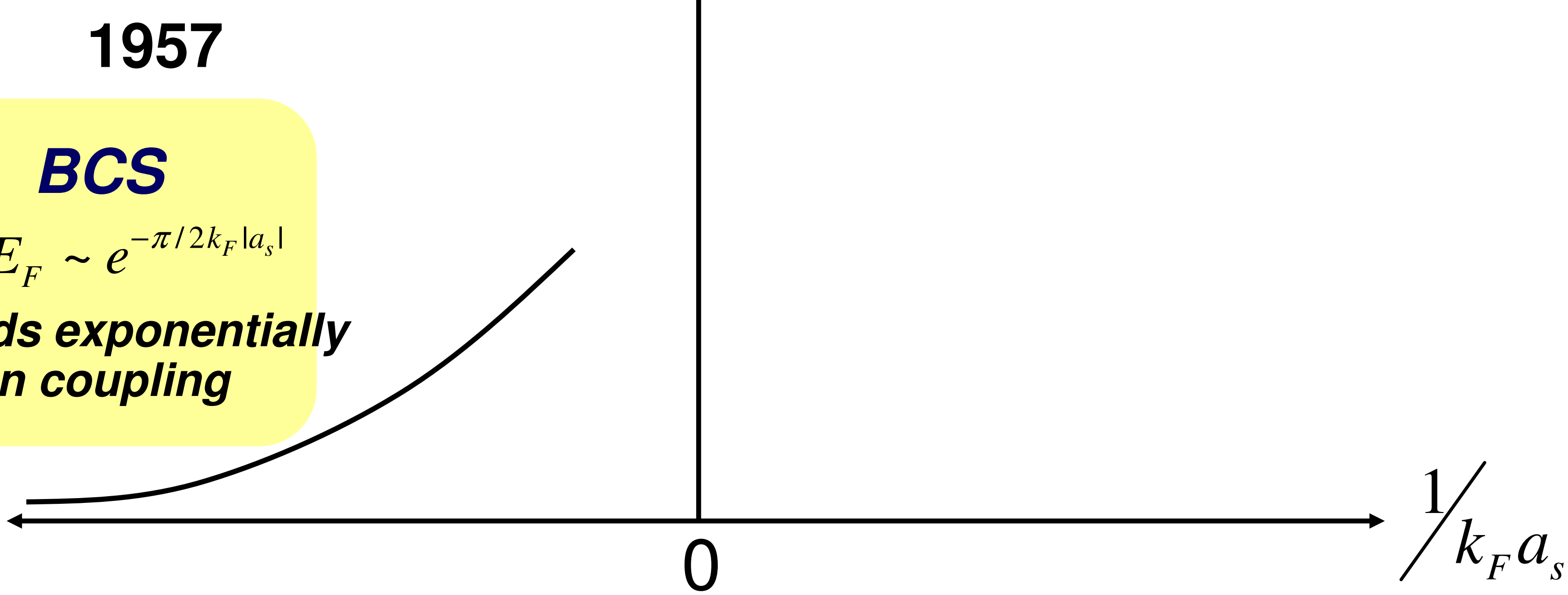


1957

BCS

$T_c / E_F \sim e^{-\pi / 2 k_F |a_s|}$

depends exponentially on coupling



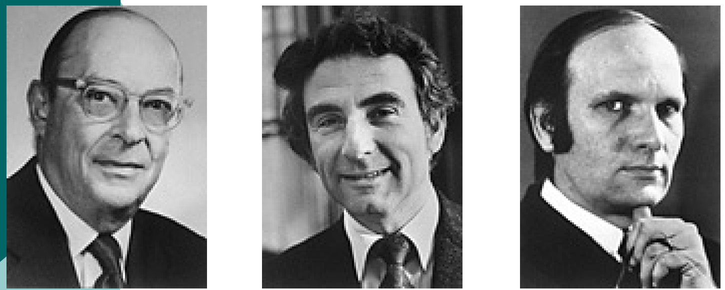
Weak **Strong**

$$k_F \propto n^{1/3}, E_F \propto n^{2/3}$$

What is the BEC-BCS

Naïve application of BCS leads power law blow up

$$T_c^{\text{naive}} / E_F \sim \frac{1}{k_F^2 a_s^2 \log\left(2/k_F^2 a_s^2\right)}$$



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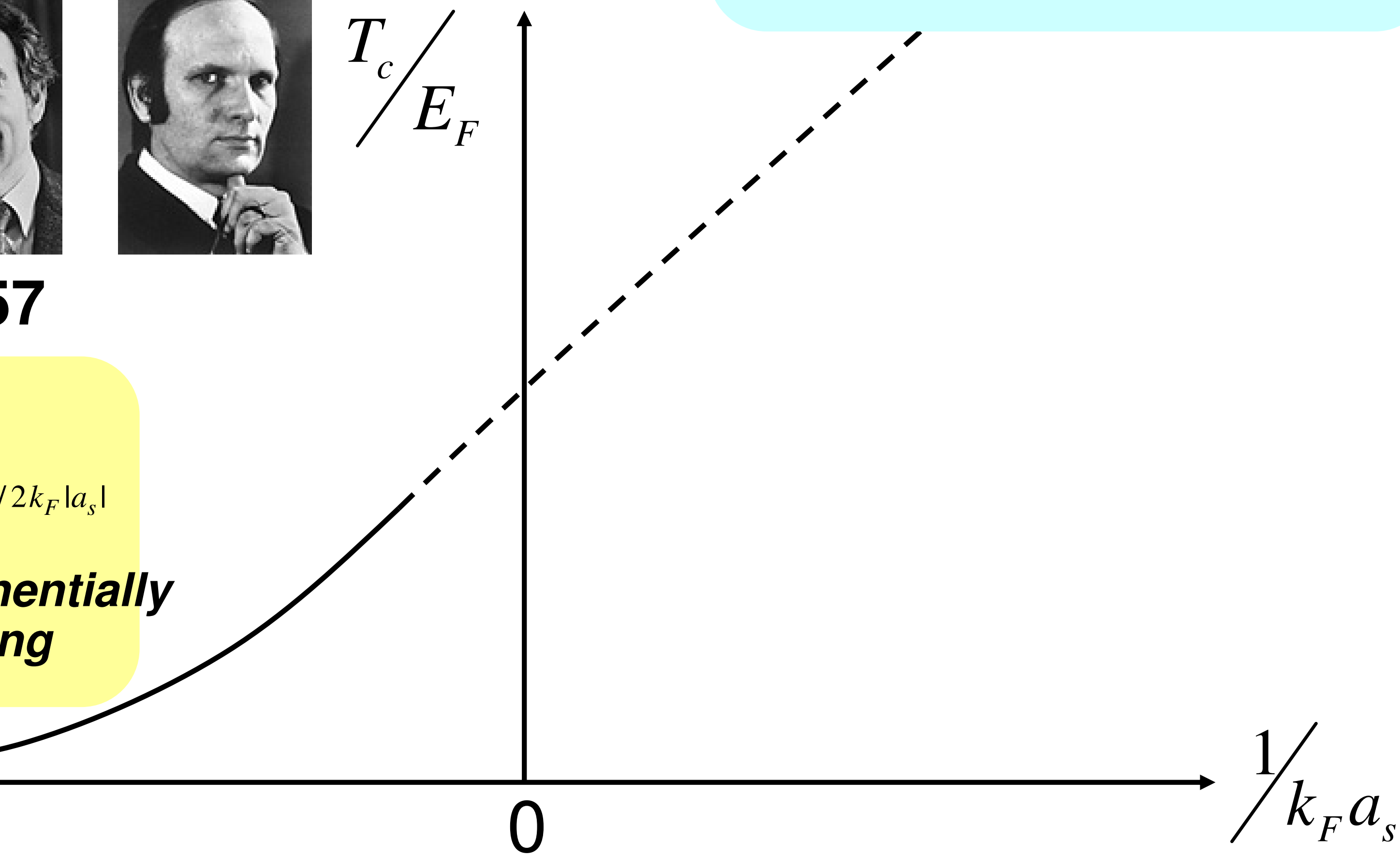

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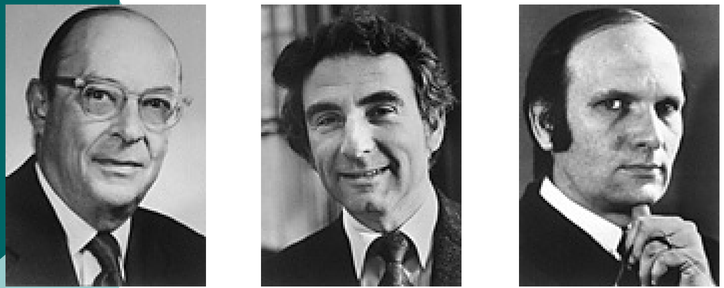

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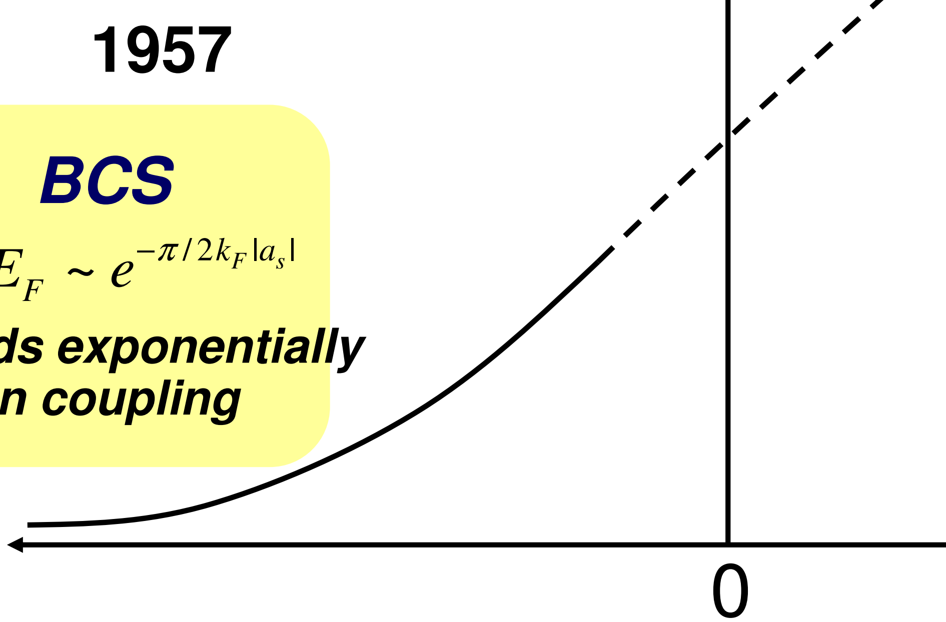
BEC

$T_c / E_F \sim 0.2314..$

"universal value" not depending on coupling



1924

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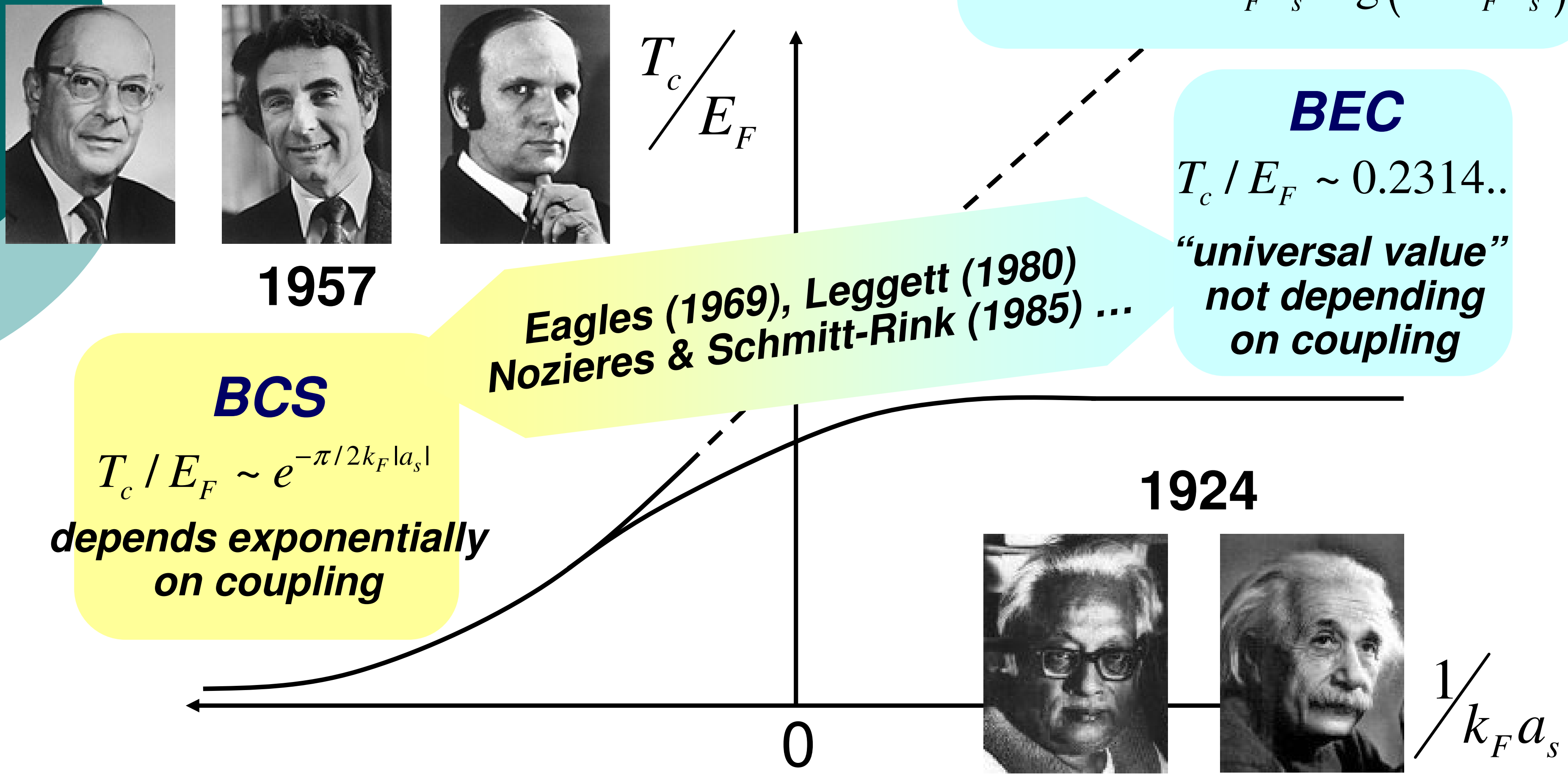


BEC

$T_c / E_F \sim 0.2314..$

"universal value" not depending on coupling

Eagles (1969), Leggett (1980)
Nozieres & Schmitt-Rink (1985) …

BCS

$T_c / E_F \sim e^{-\pi/2k_F|a_s|}$

depends exponentially on coupling

Weak **Strong**

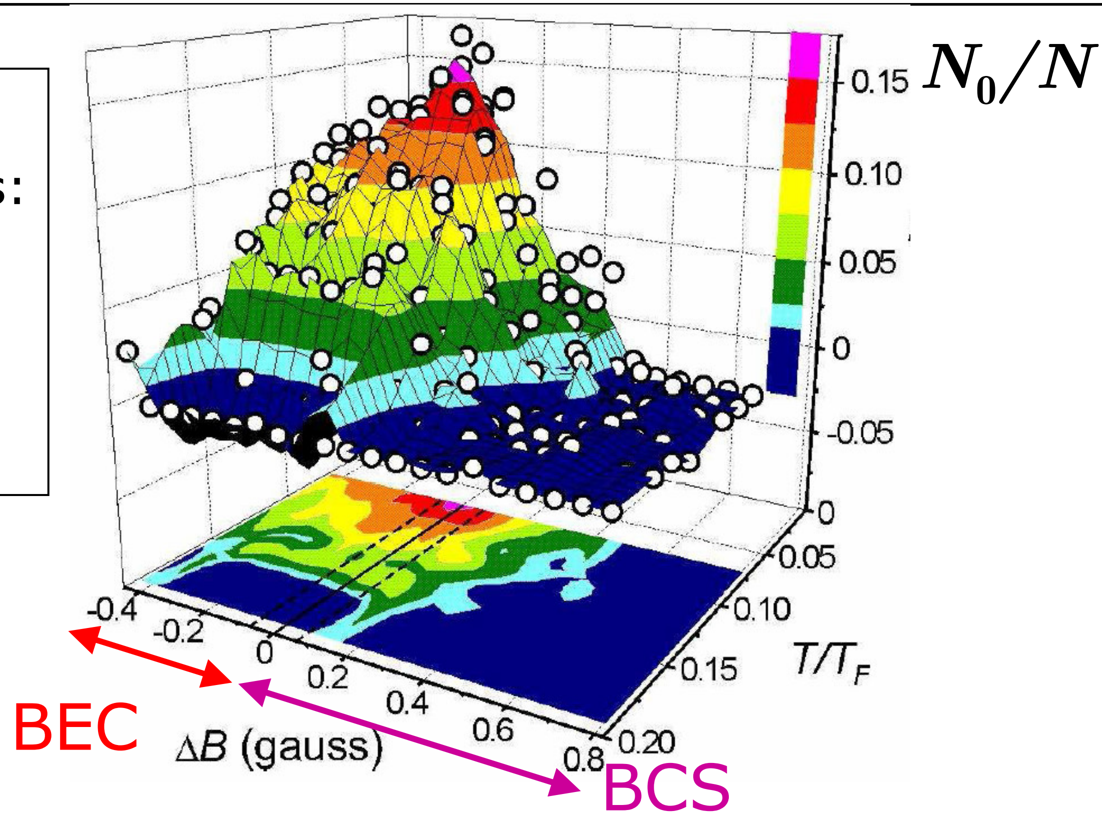
$k_F \propto n^{1/3}, E_F \propto n^{2/3}$

Crossover observed in UCA

K^{40} (JILA)

Relevant scales:

$T_c \sim$ 100nK
$T_F \sim 1\mu$K
$\rho \sim 10^{14}$cm^{-3}
$\lambda \sim$ 100nm



Note: interaction is tunable via Magnetic field!

K40: Regal et al., Nature 424, 47 (03), PRL92 (04): **JILA group**

Li6: Strecker et al., PRL91 (03): **Rice group**; Zwierlen et al., PRL91 (03): **MIT group**; Chin et al., Science 305, 1128 (04): **Austrian group**

In general relativistic systems?

- What is a relativistic matter?
  - $\hbar k_F/mc \sim \lambda_c/d$ (:=relativity parameter)

    $\sim 10^{-9}$ in cold atom systems
    $\sim 10^{-7}$ in ^{3}He
    $\sim 10^{-2}$ nuclear matter (deuteron gas)

    Lombardo, Nozieres, Schuck, Schulze, Sedrakian, PRC64, 064314 (2001)
  - In cold quark matter with CSC

    $\hbar k_F/mc \geqslant 1$ relativistic enough!

    Nishida, Abuki, PRD72 (05): New regime "RBEC"

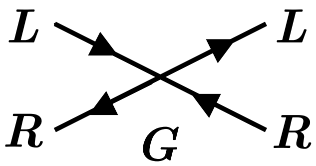
NJL model with axial anomaly (1)

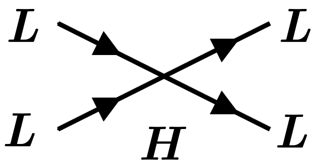
HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

○ Model setup

$$L = \overline{q}(i\not\partial + \mu\gamma_0)q + \boxed{L^{(4)}} + L^{(6)}$$

$$\begin{cases} L_\chi^{(4)} = 8G\ \mathrm{tr}\left(\phi^+\phi\right) \\ L_d^{(4)} = 2H\ \mathrm{tr}\left(d_L^+ d_L + d_R^+ d_R\right) \end{cases}$$





Chiral fields:

$$\phi_{ij} = \left(\overline{q}_{Rj} q_{Li}\right)$$

(u,d,s)

Diquark fields:

$$\left(d_L\right)_i^a = \varepsilon^{abc}\varepsilon_{ijk} q_{Lj}^b C q_{Lk}^c$$

(r,g,b)

Yamamoto, Tachibana, Baym, Hatsuda, PRL97 (06)

NJL model with axial anomaly (2)

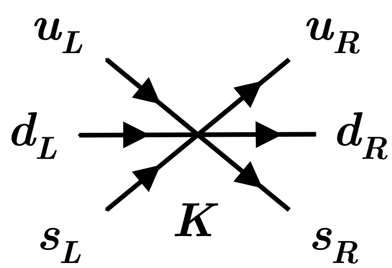
HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- Model setup

$$L = \overline{q}(i\partial\!\!\!/ + \mu\gamma_0)q + L^{(4)} + \boxed{L^{(6)}}$$

- KMT term for $U(1)_A$ breaking

$$\begin{cases} L^{(6)}_{\mathrm{KMT}} = -8K\det(\phi) + \mathrm{h.c.} \\ L^{(6)}_{\mathrm{KMT'}} = K'\,\mathrm{tr}\left(d_R^+ d_L \phi\right) + \mathrm{h.c.} \end{cases}$$

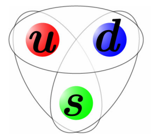


- *K* responsible for η' mass decoupling

$$U(1)_A \to Z_{6A} \quad \left(q_{L(R)} \to (-1)^{\pm n/3} q_{L(R)}\right)$$

NJL model with axial anomaly (3)

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- Favorable mean fields (condensates)
  - Chiral: $\langle \phi_{ij} \rangle = \delta_{ij} (\boldsymbol{\sigma}/2)$ (V-singlet, 0^+)
  - Diquark: $\langle d_{Laj} \rangle = -\langle d_{Raj} \rangle = \delta_{aj} (\boldsymbol{d}/2)$ 

    (for $K' > 0$) (V+C-singlet, 0^+)

Alford, Rajagopal, Wilczek, NPB99

- Spontaneous symmetry breaking in CFL

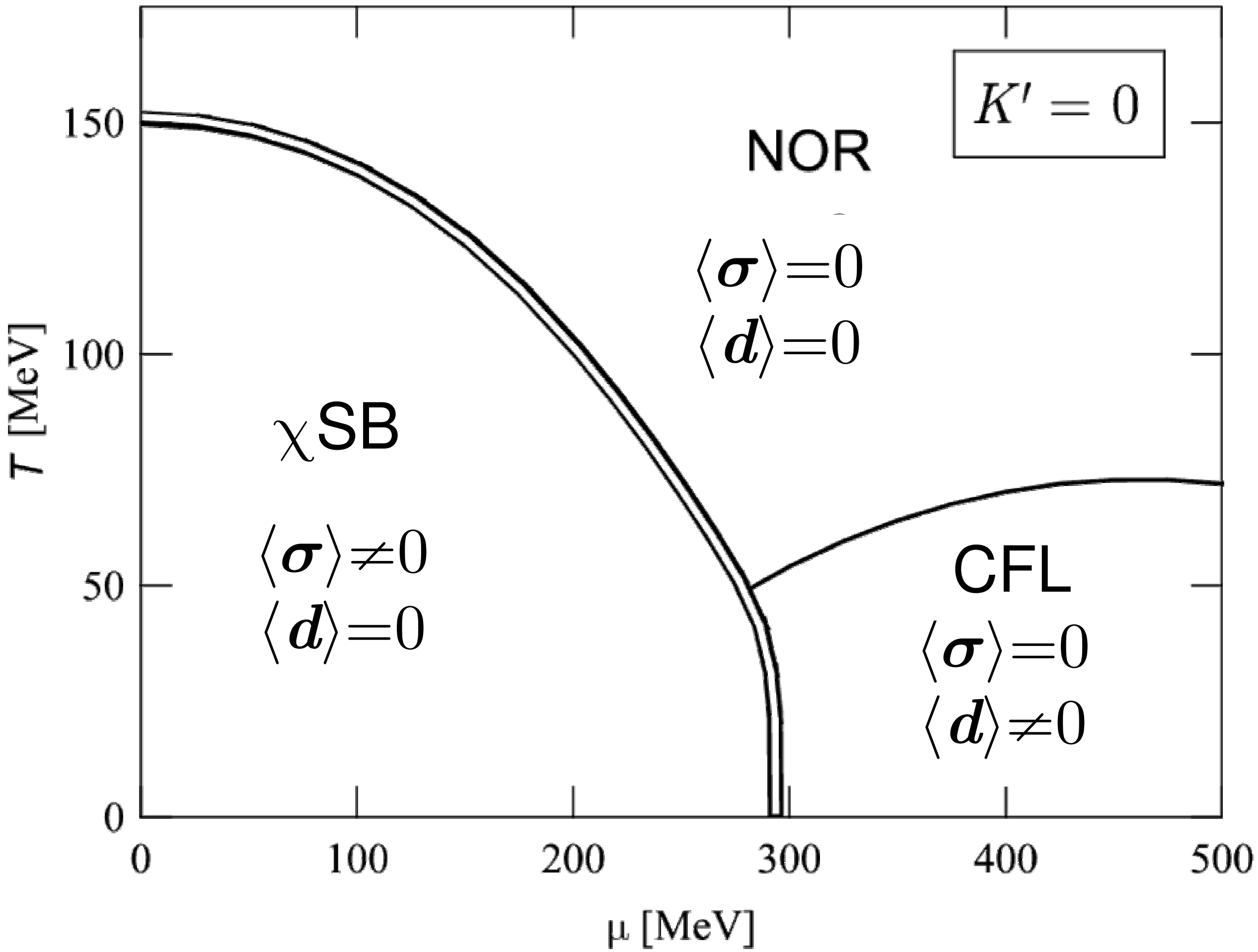
$$SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times Z_{6A}$$

$$\rightarrow SU(3)_{L+R+C} \times Z_2$$

Chiral transition *w/o* d-σ interplay

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- $m=0,\ K'=0$ (no interplay)



typical values for (G,K,H): taken from M. Buballa, PR(05)

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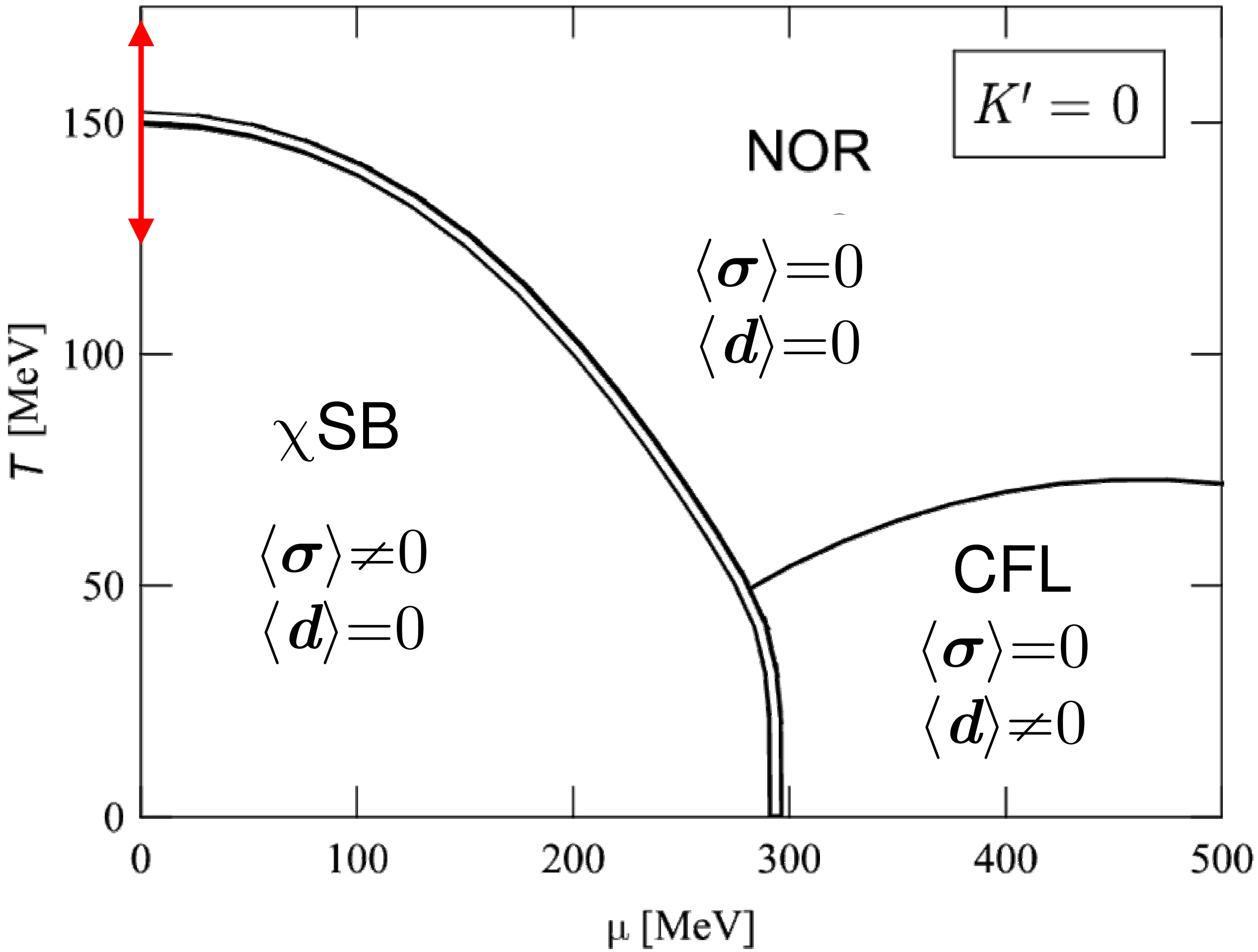

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Chiral transition *w/o d-σ interplay*

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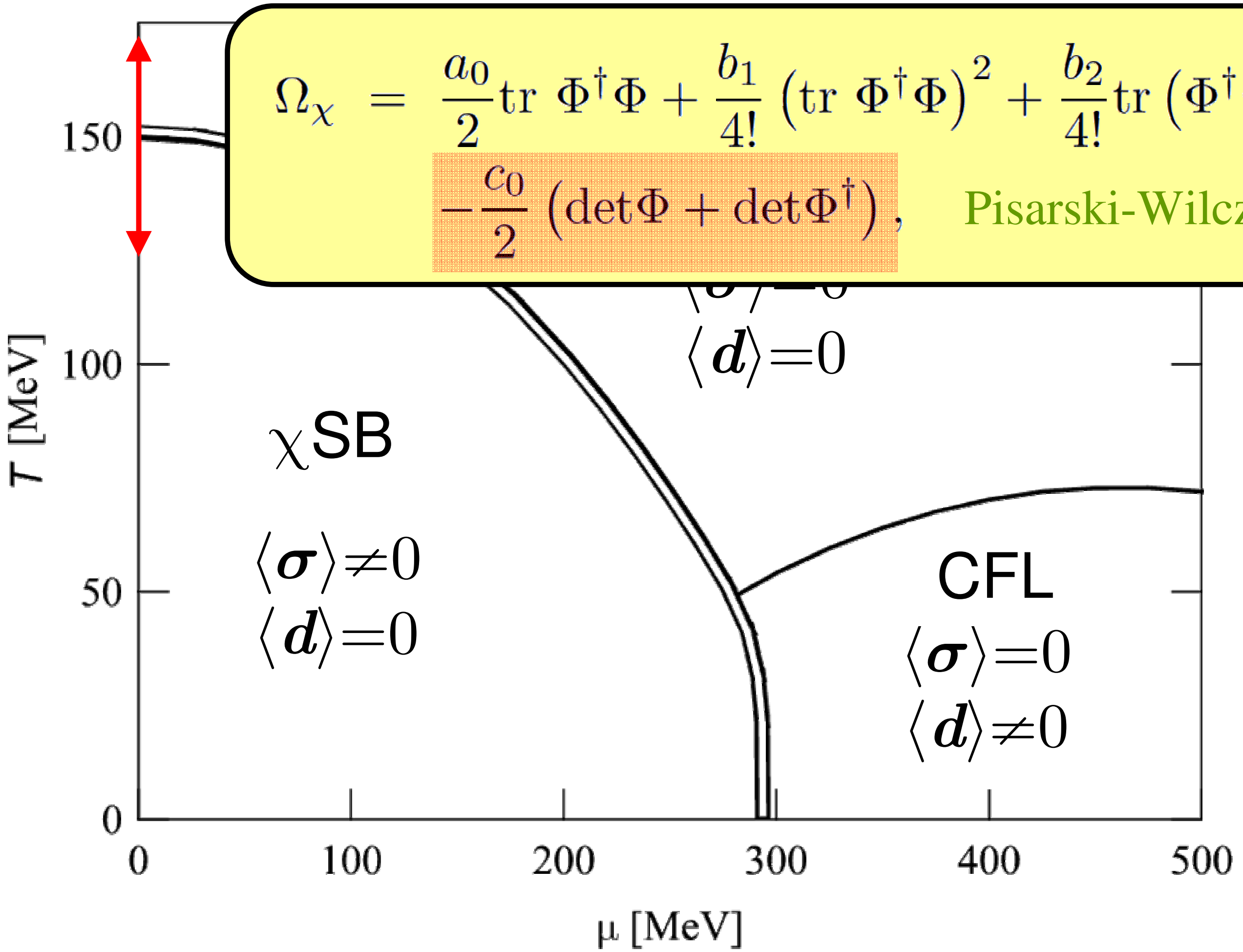
- $m=0,\ K'=0$ (no interplay)

$$\Omega_\chi = \frac{a_0}{2}\mathrm{tr}\ \Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\ \Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2 - \frac{c_0}{2}\left(\det\Phi + \det\Phi^\dagger\right),$$

Pisarski-Wilczek ('84)



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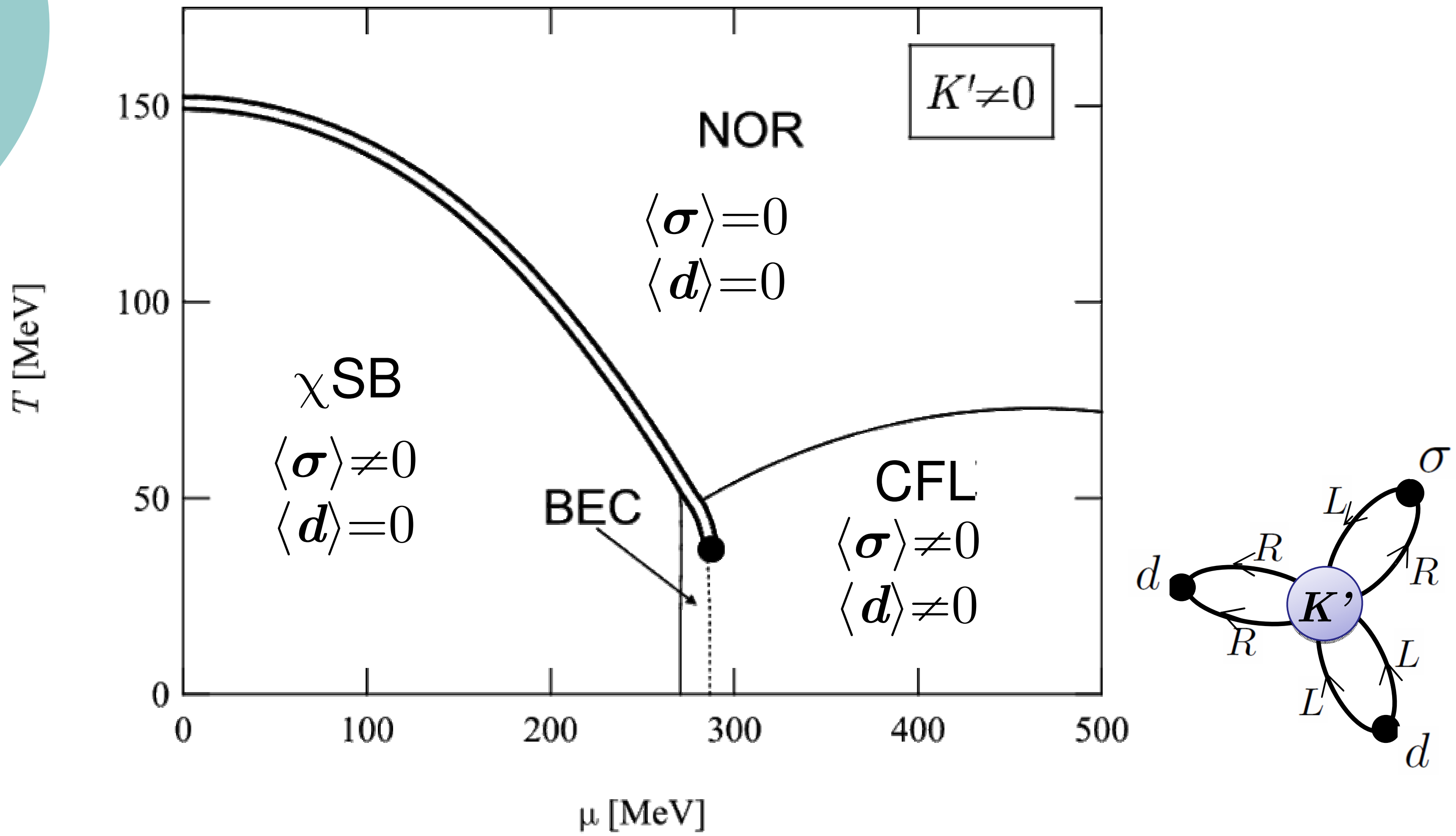
- $m=0$, $K'=4.2K_0$ (with interplay)



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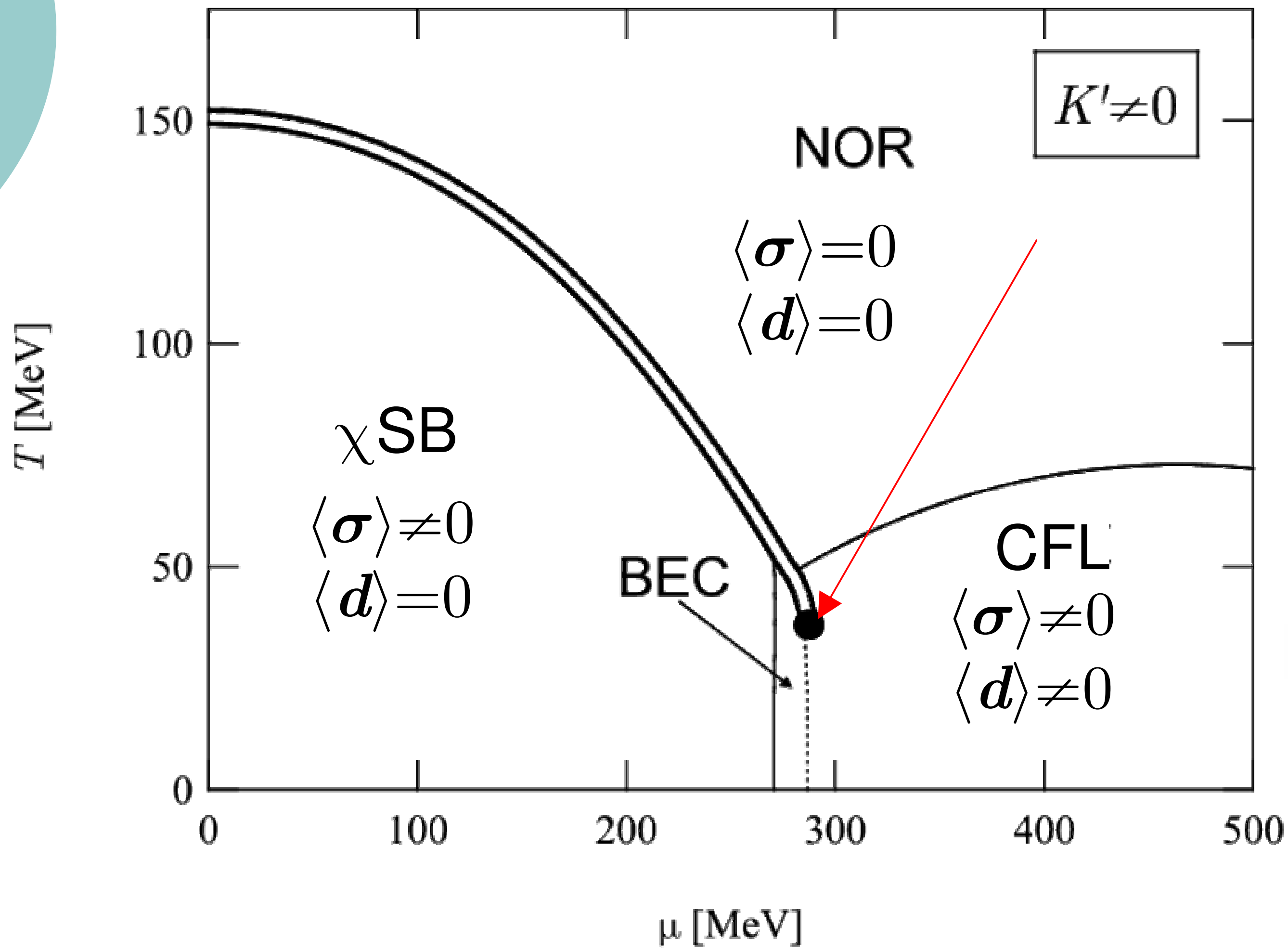
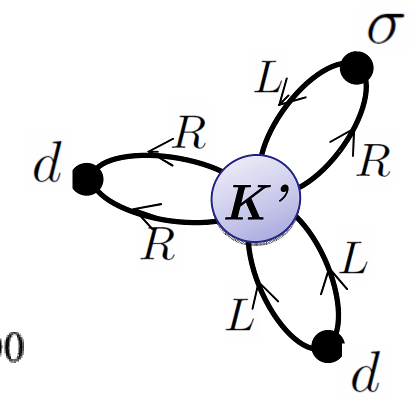




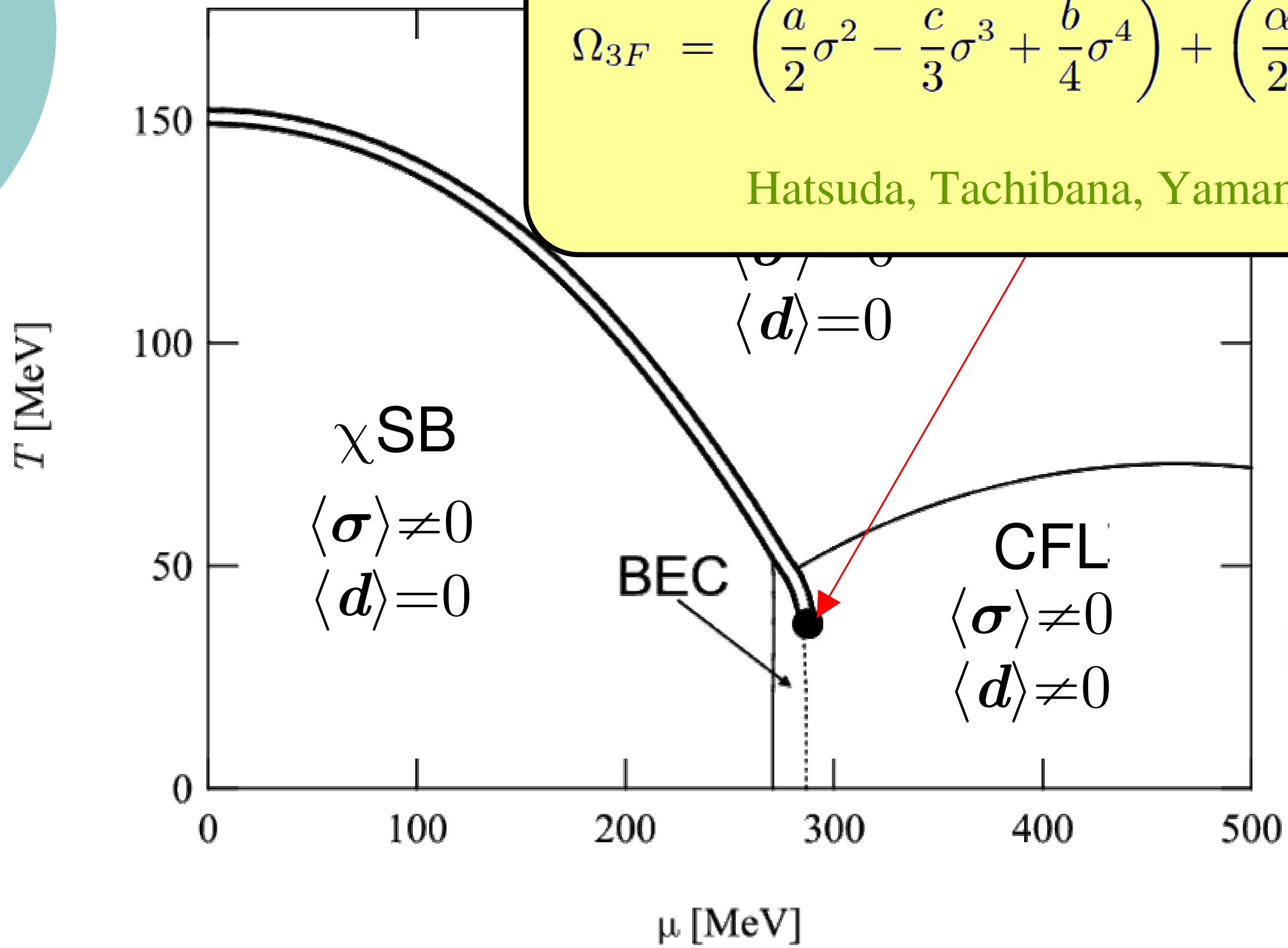
Chiral transition *with d-σ* interplay

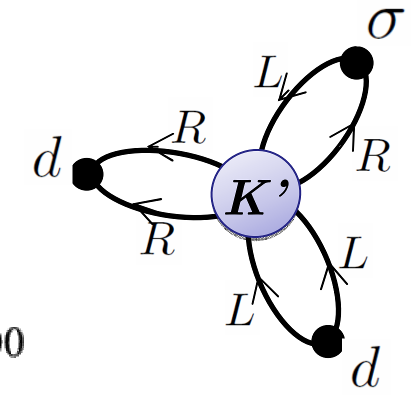
HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- $m=0$, $K'=$ … (with interplay)

$$\Omega_{3F} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2 \sigma$$

Hatsuda, Tachibana, Yamamoto, Baym (06)

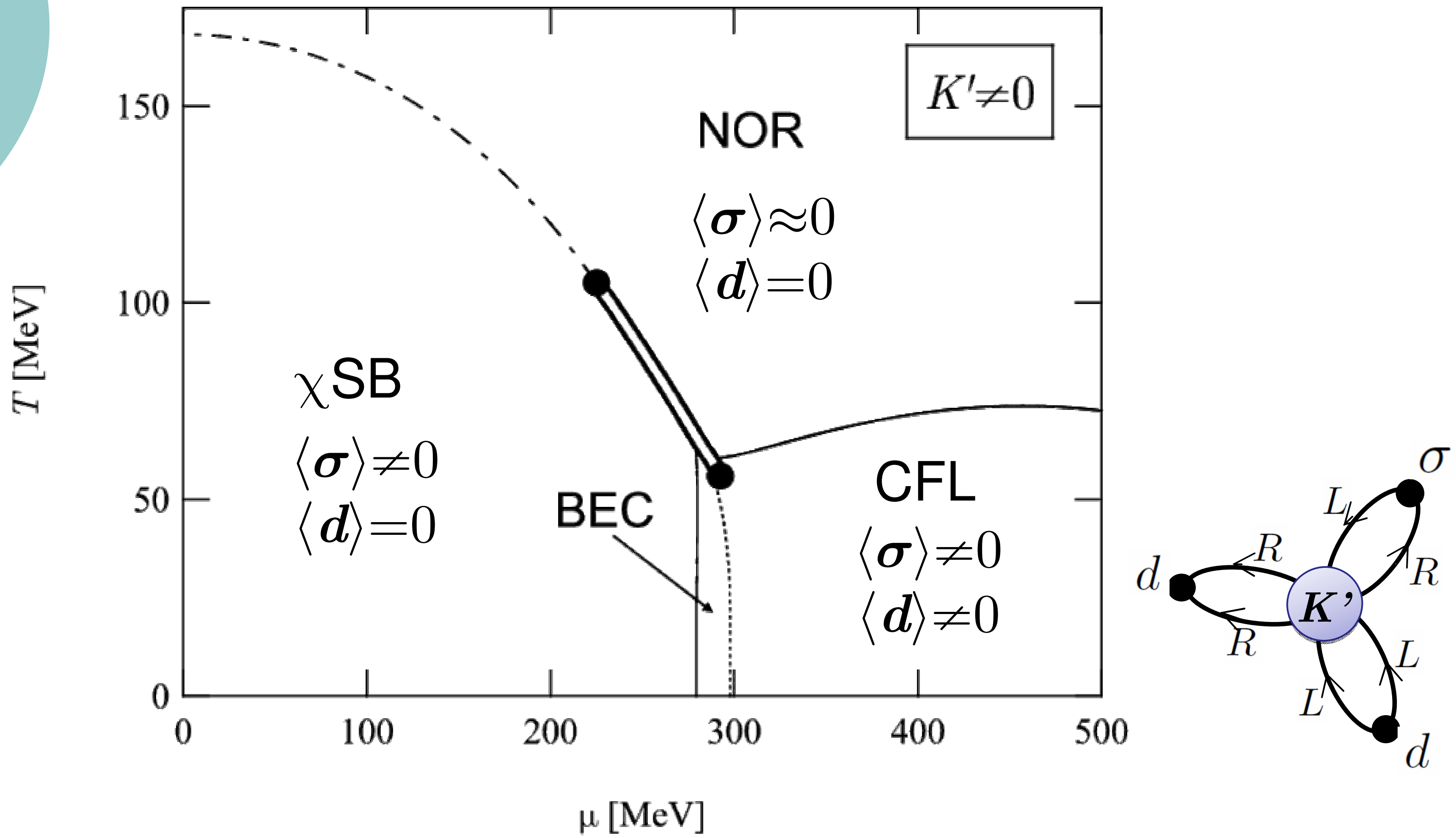




Chiral transition; *current quark mass*

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- $m = 5.5\ \mathrm{MeV}$ (turning on quark mass)



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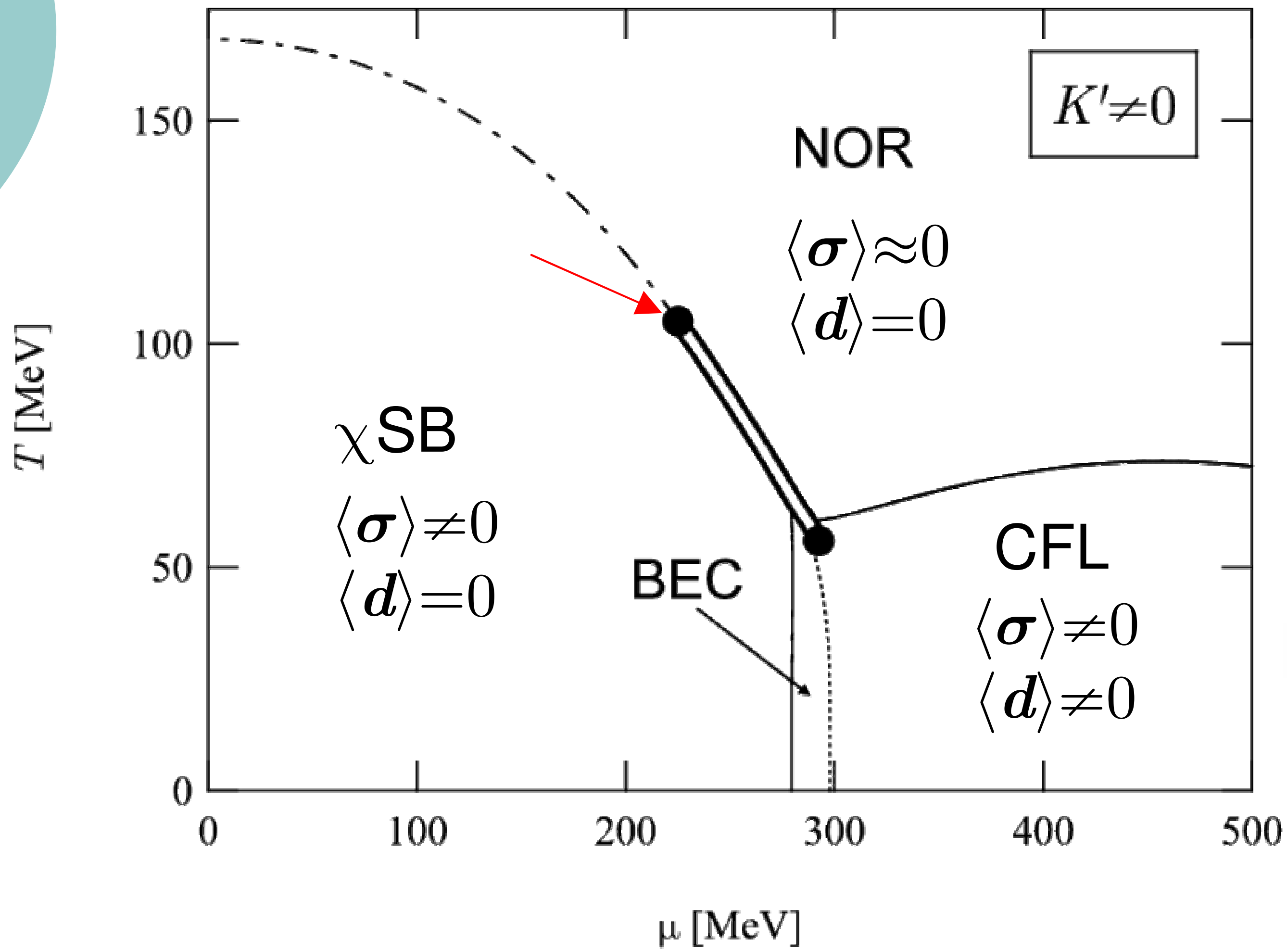

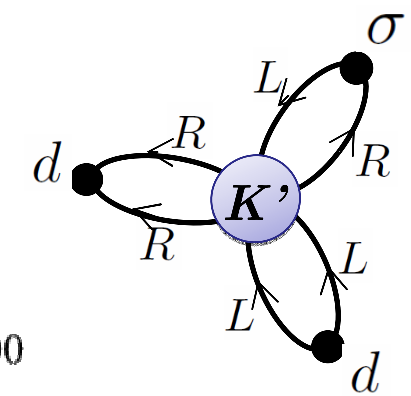

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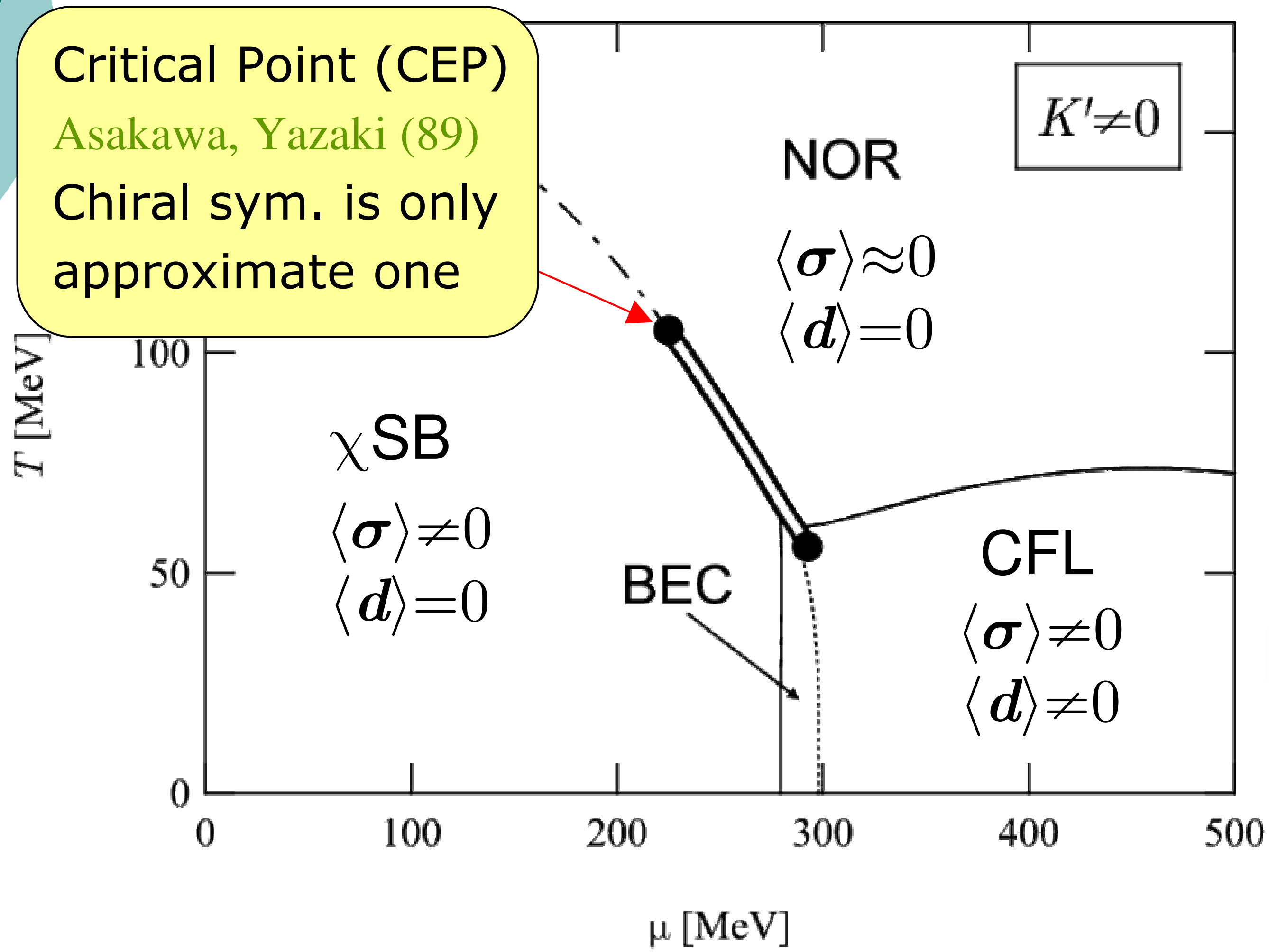
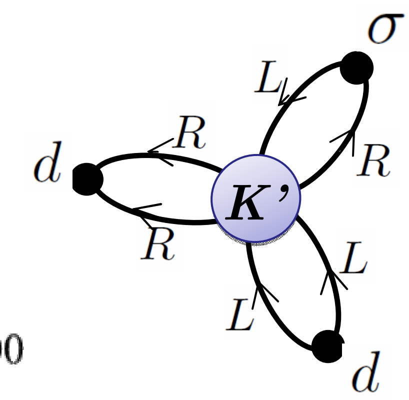
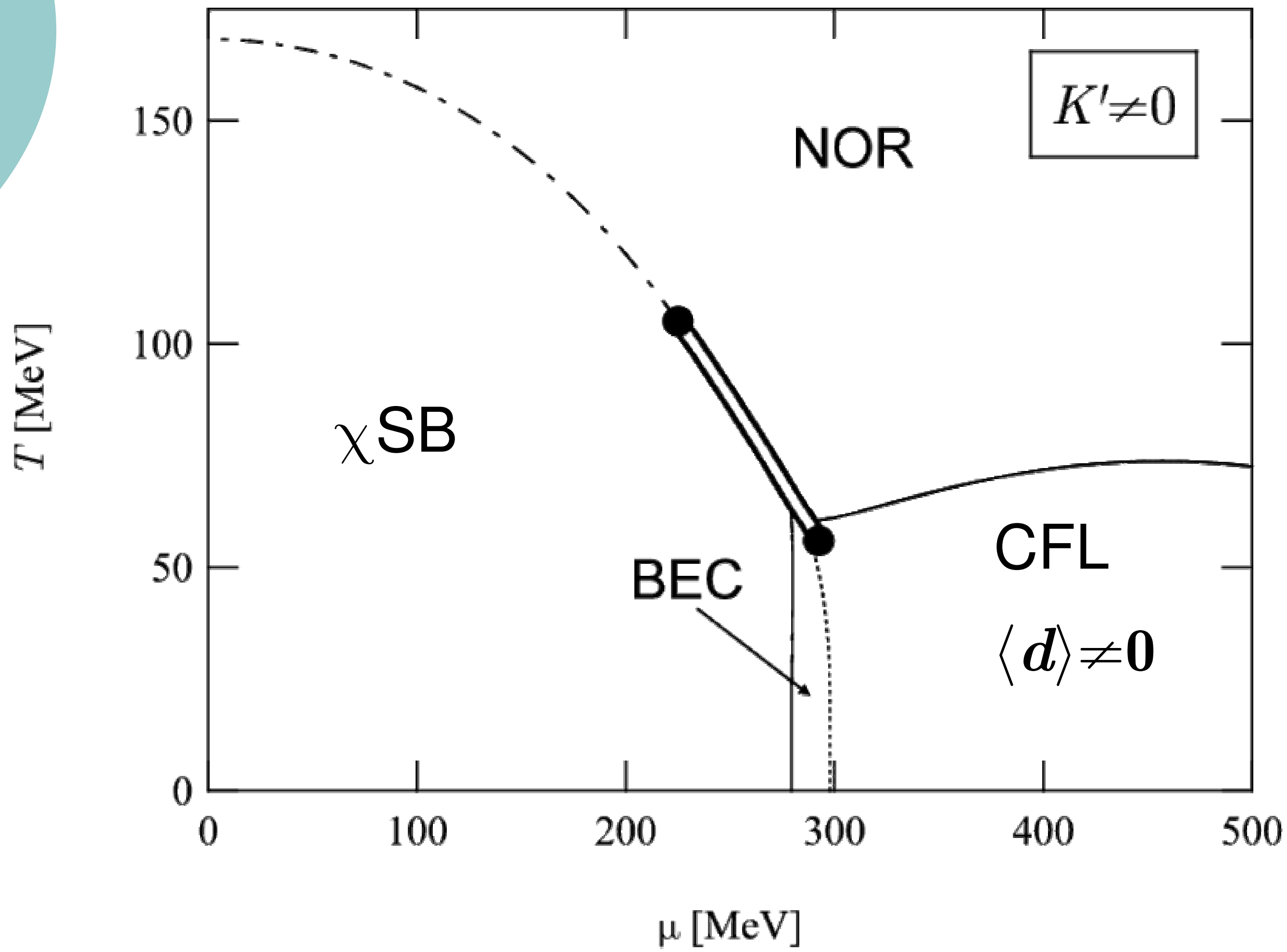




BEC-BCS crossover in the CFL?

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408



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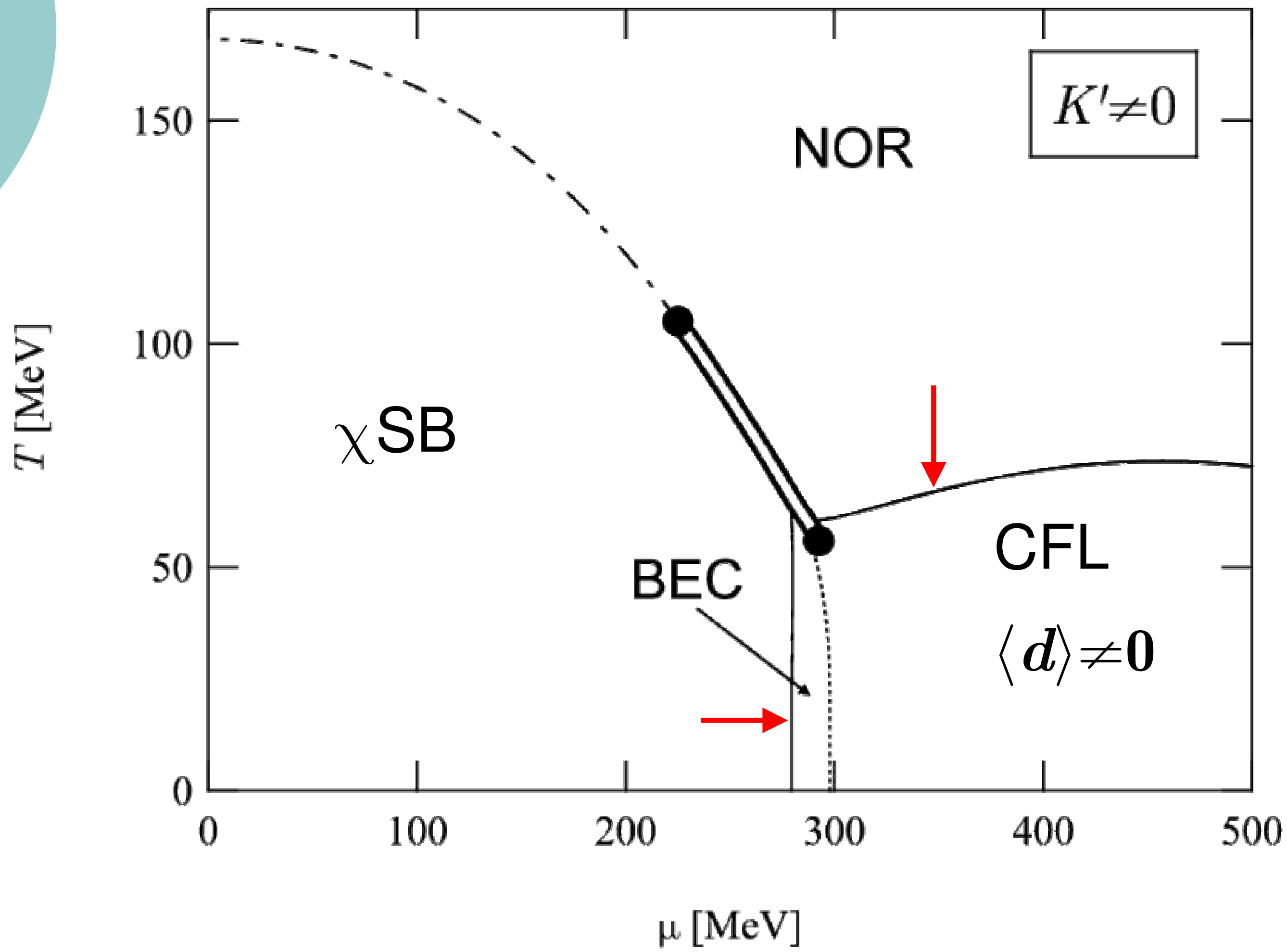

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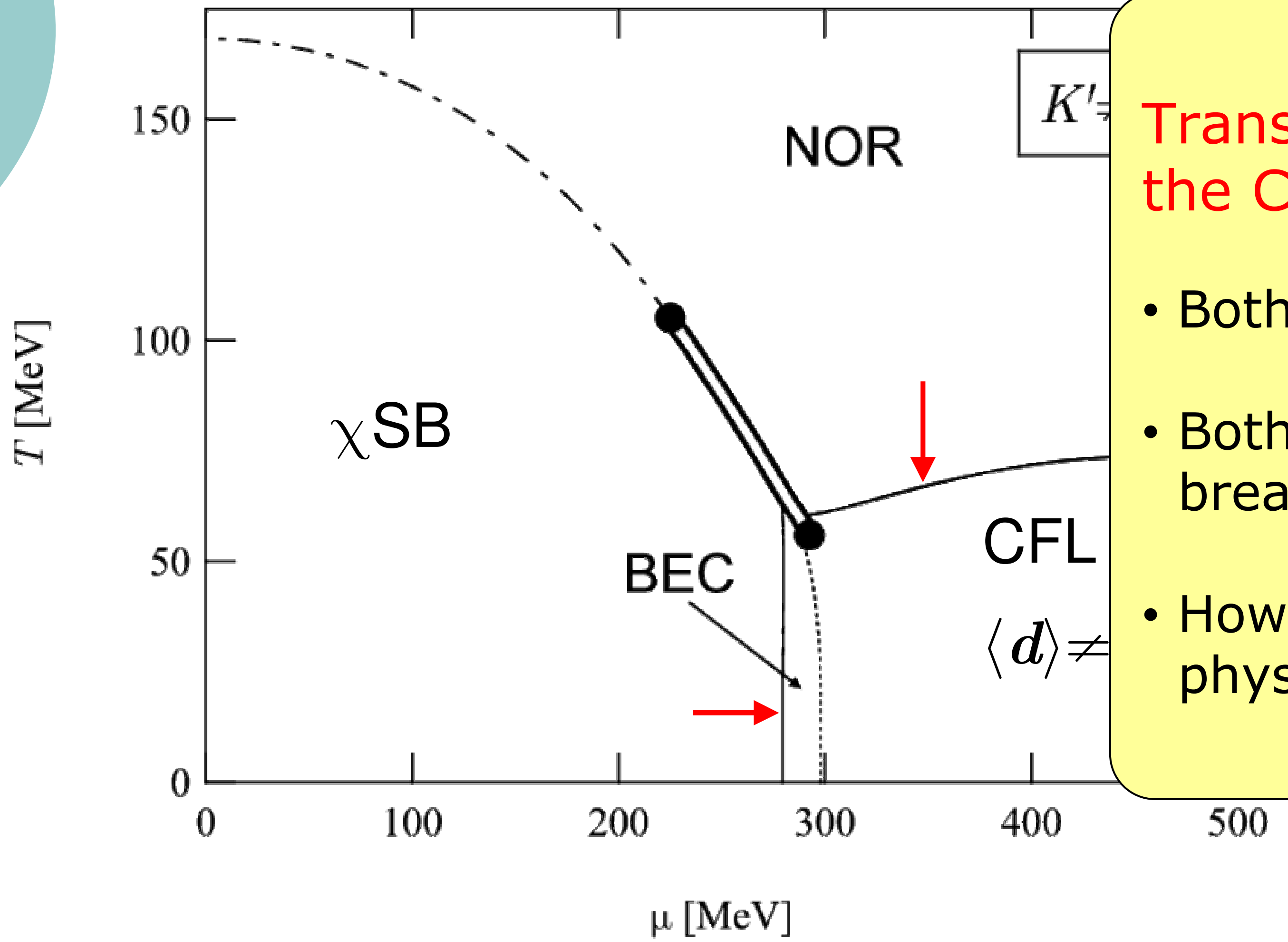

Transitions to the CFL phase:

- Both are 2nd order
- Both singal $U(1)_B$ breaking
- However, different physical nature

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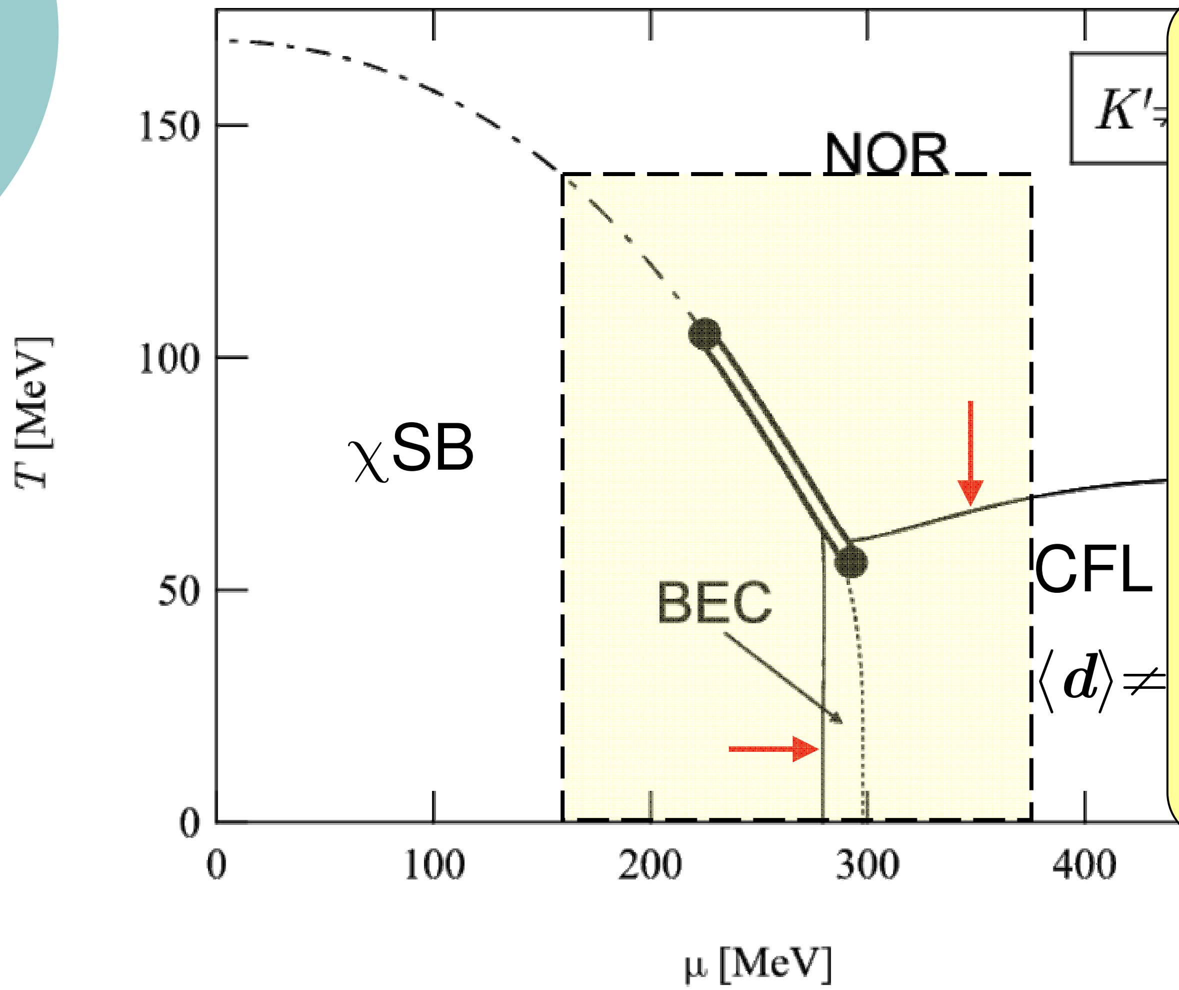

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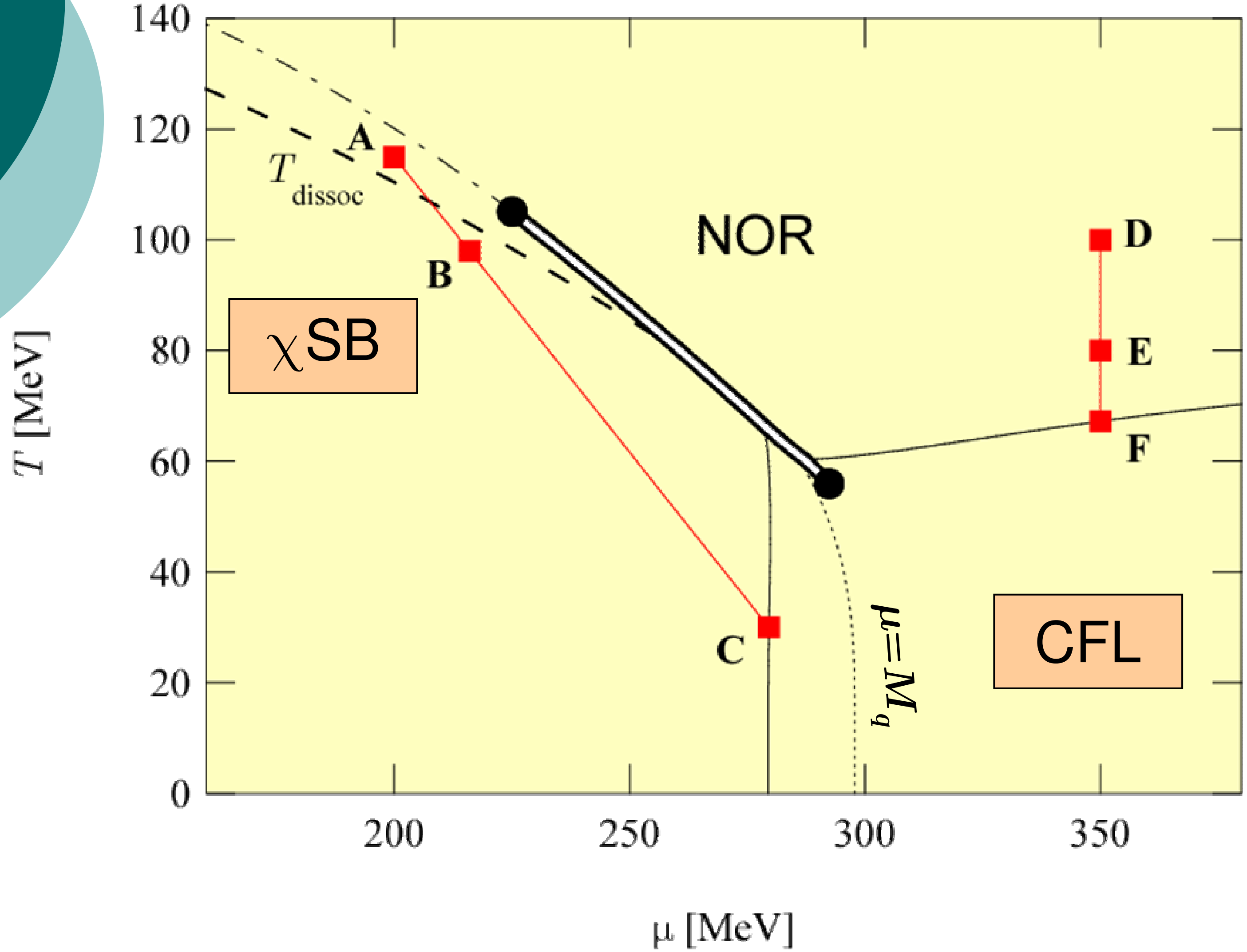
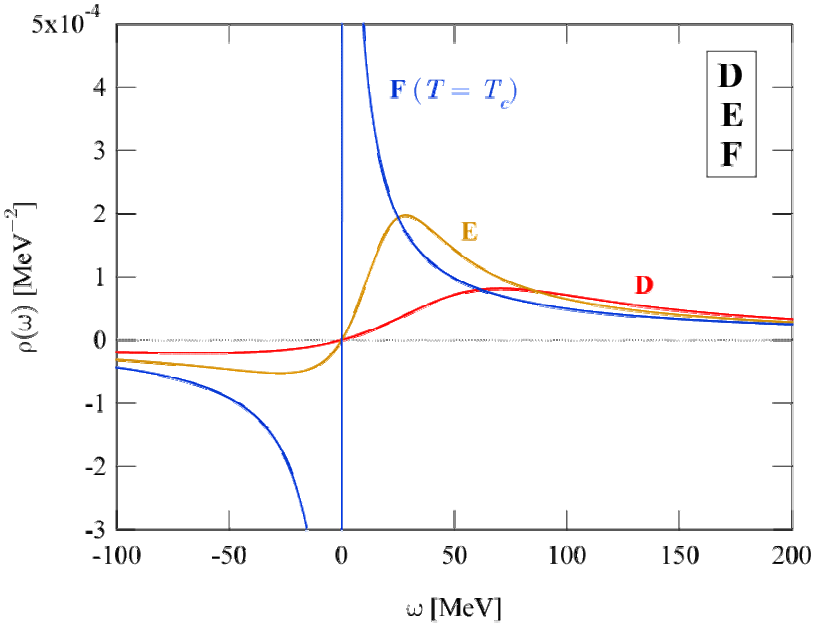


…sover in the CFL?

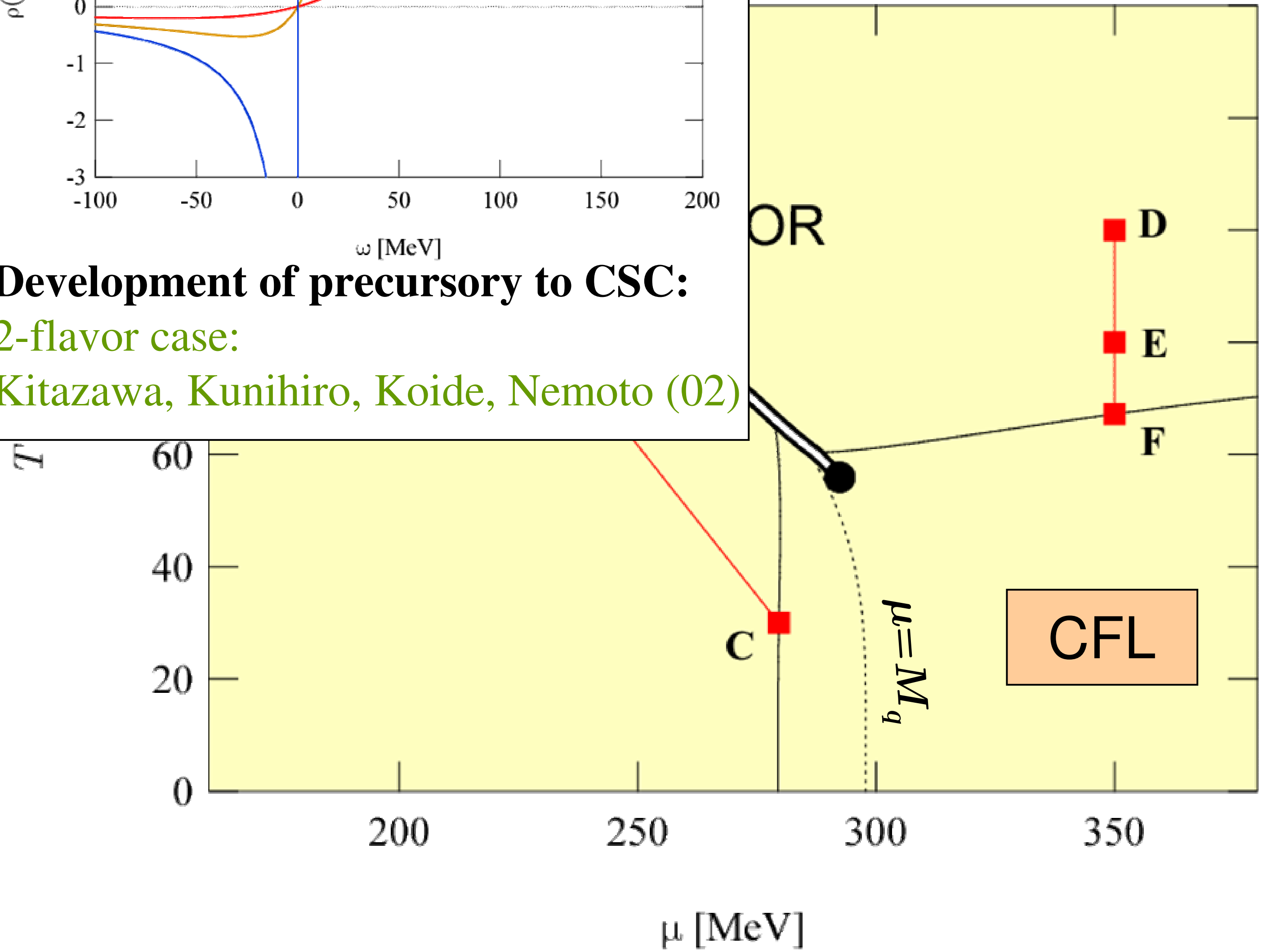
…. Hatsuda, N. Yamamoto, arXiv:1003.0408



Development of precursory to CSC:

2-flavor case:

Kitazawa, Kunihiro, Koide, Nemoto (02)



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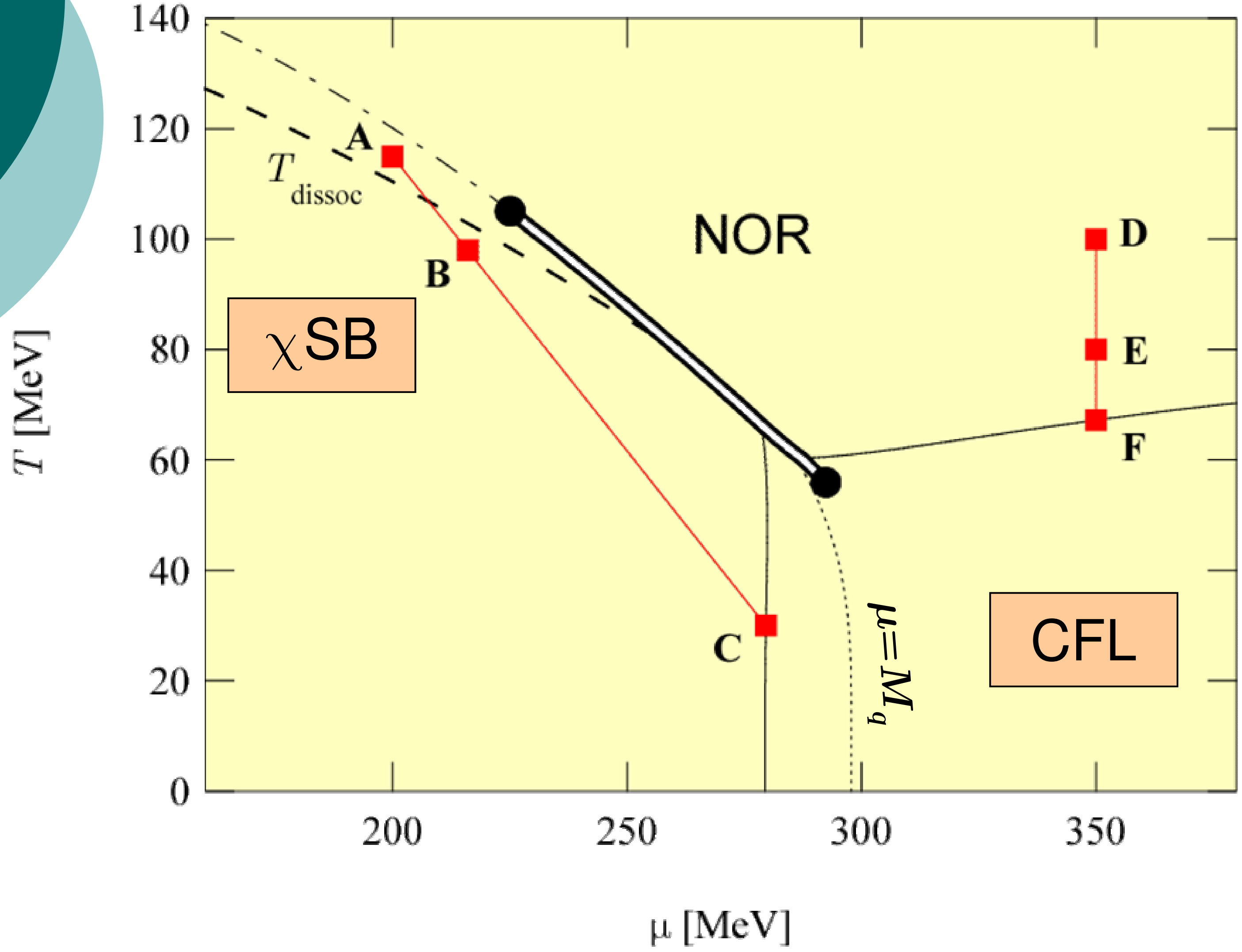

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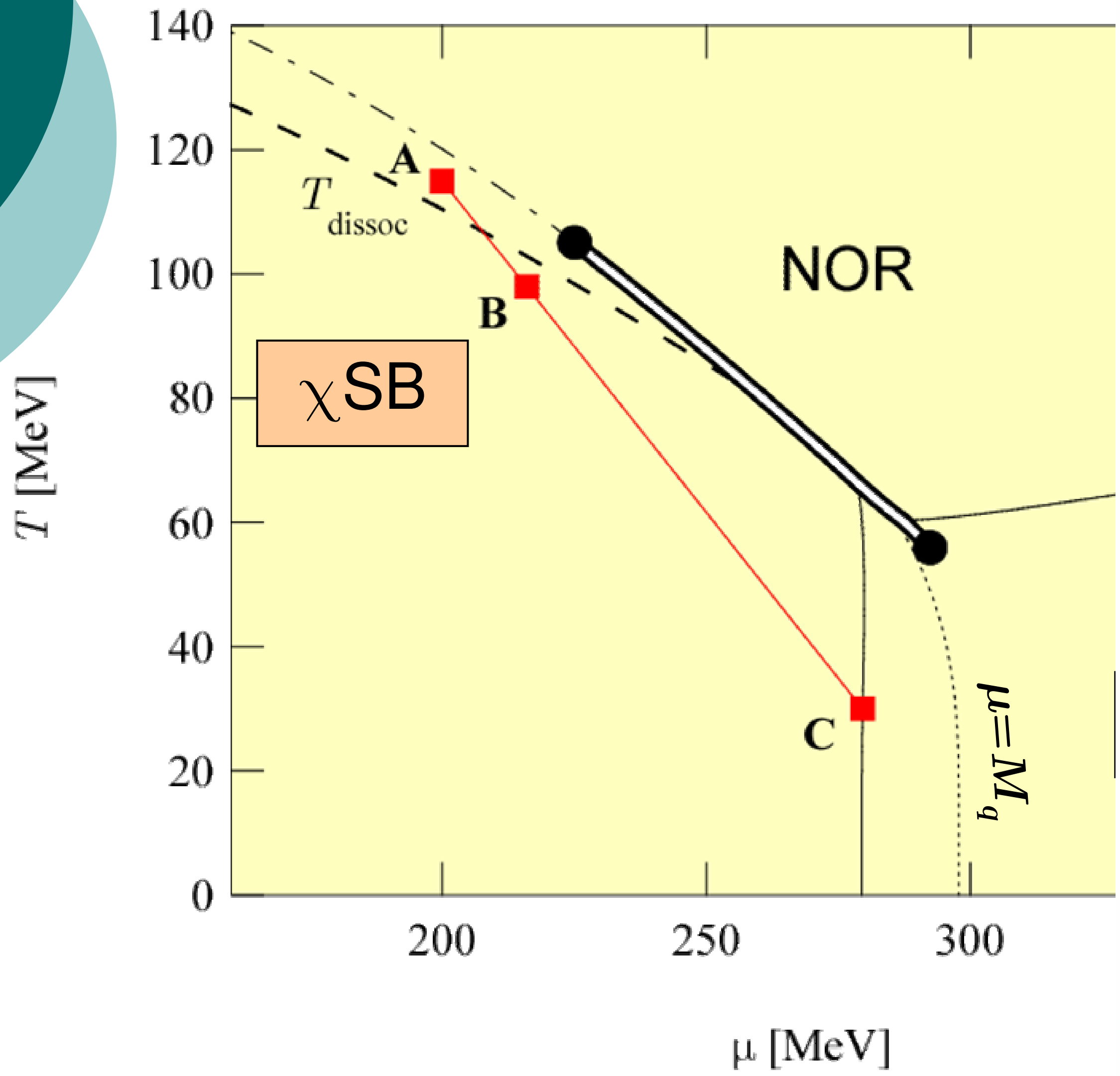

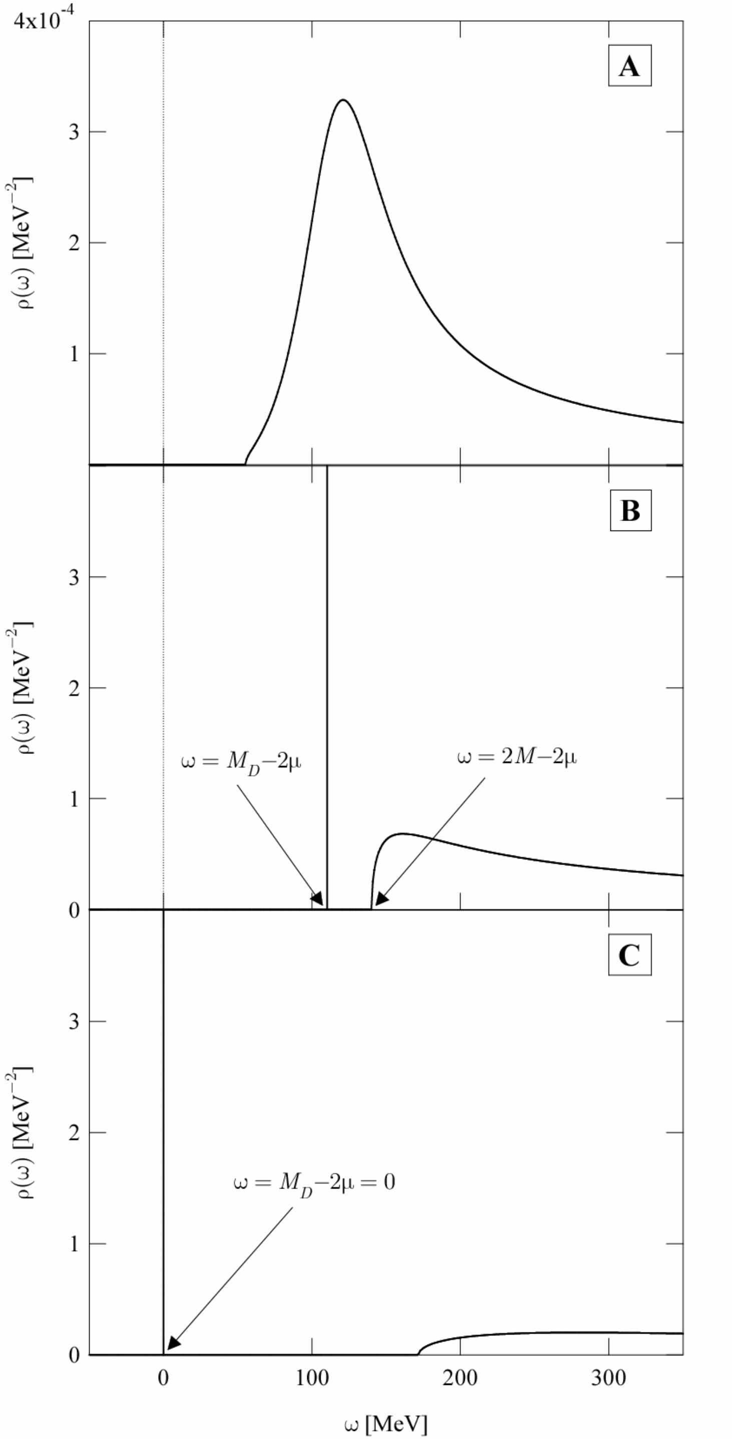

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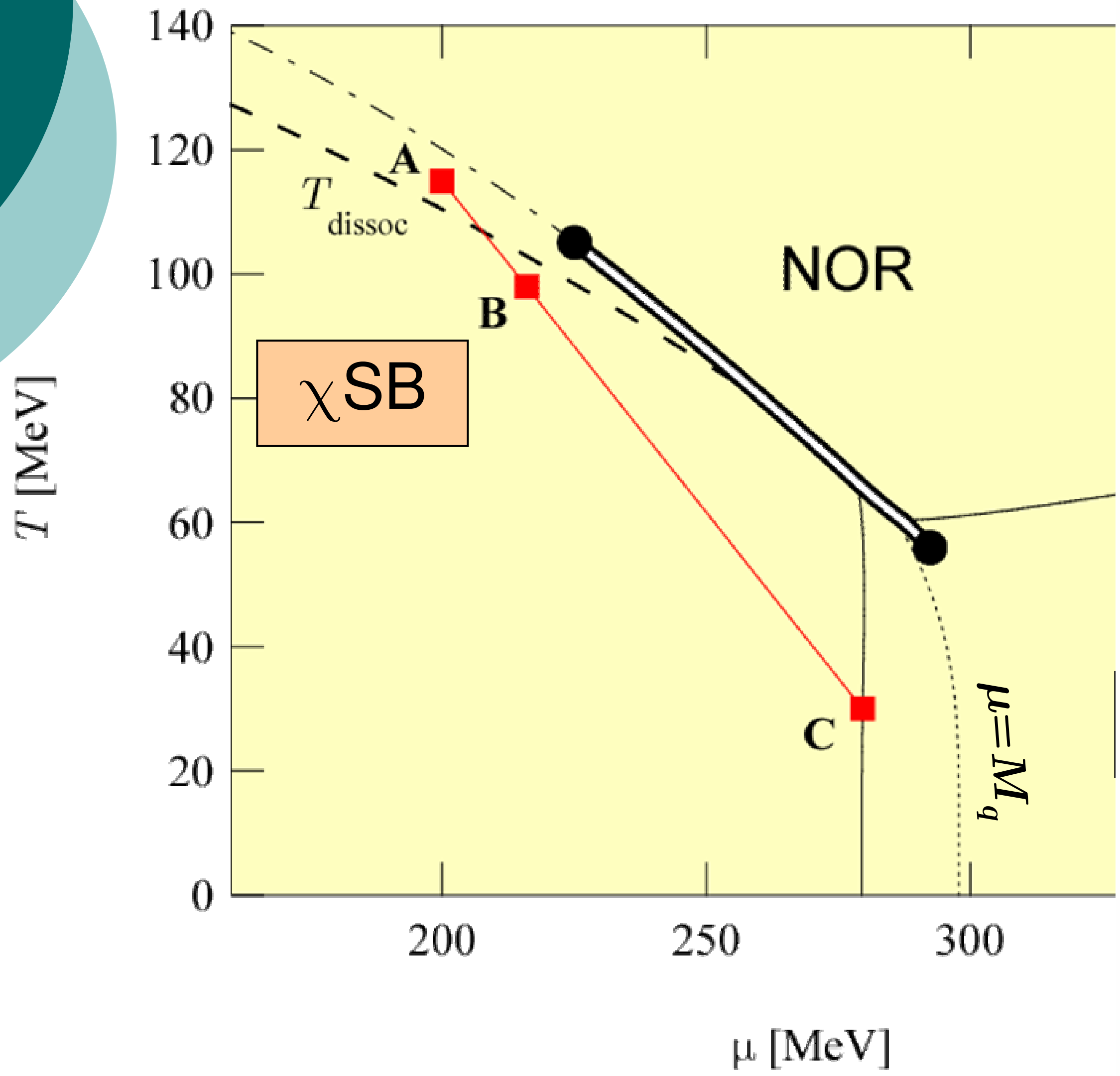
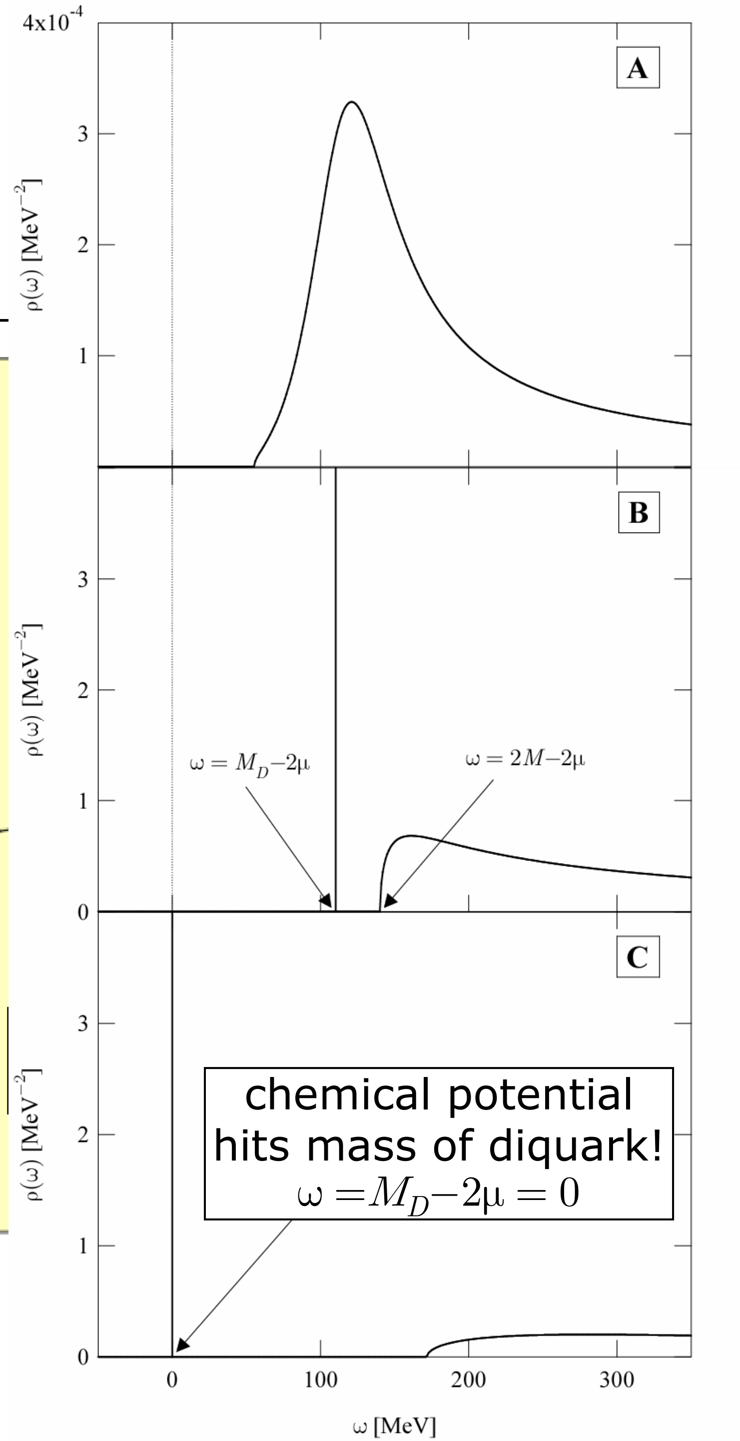


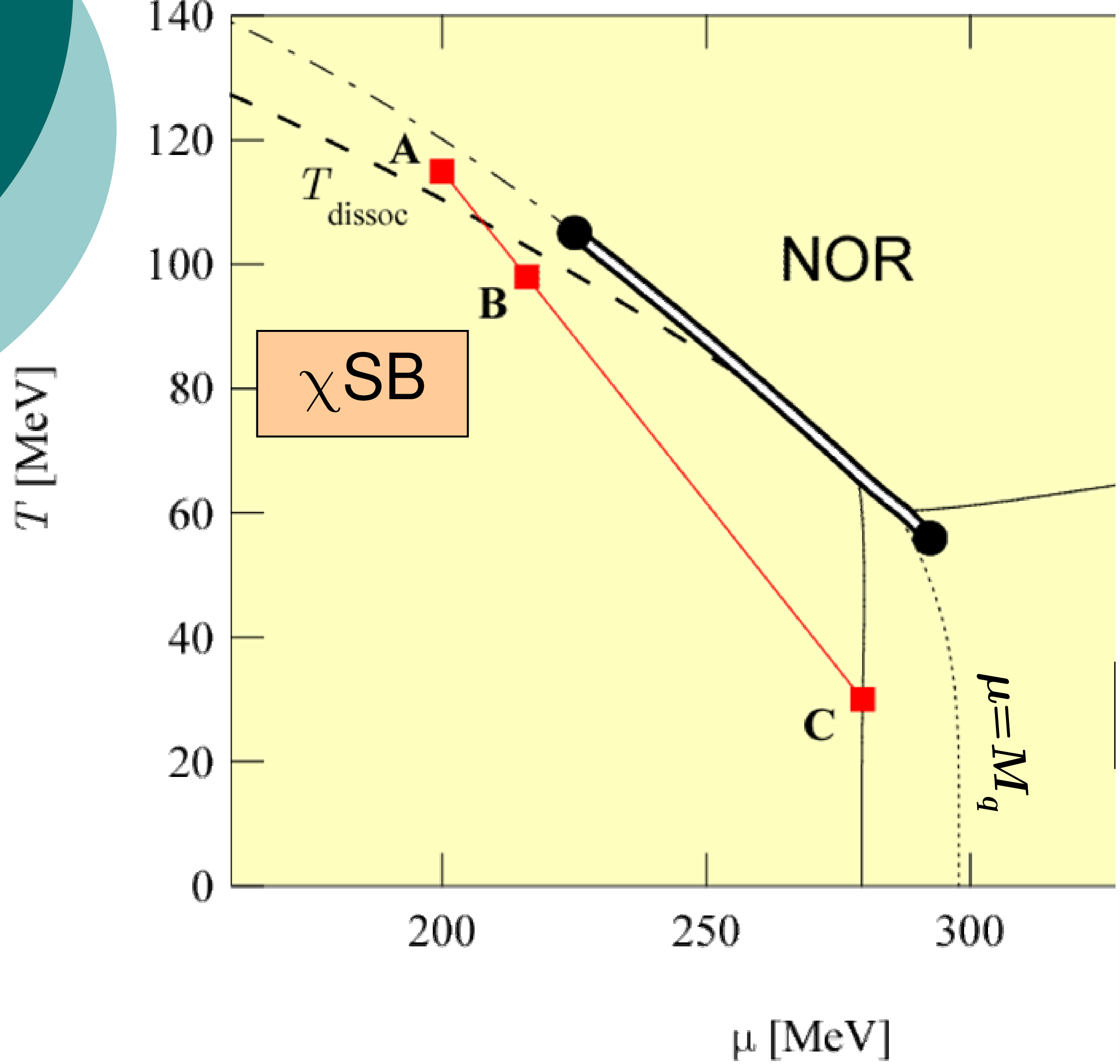


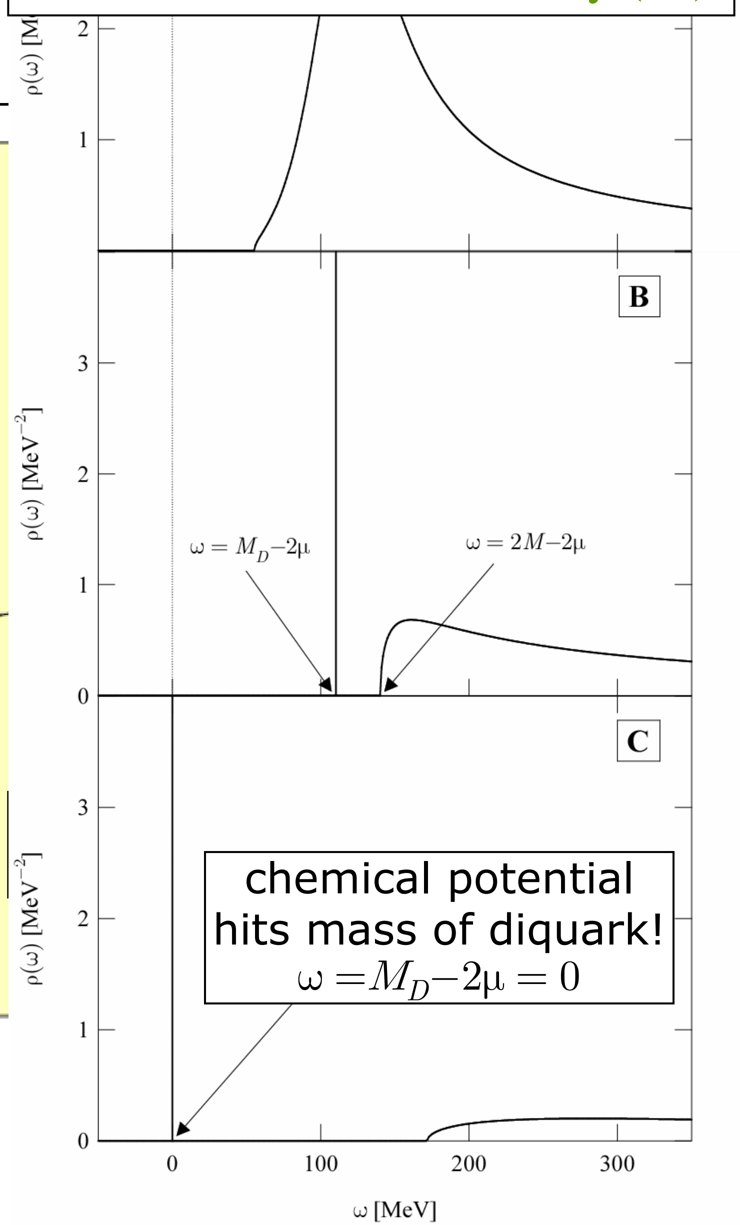
BEC-BCS crossover in

HA, G. Baym, T. Hatsuda, N.

BEC of bound diquarks:

Nishida,Abuki (PRD05, NPA07)
Kitazawa,Rischke,Shovkovy (08)





chemical potential hits mass of diquark!
$\omega = M_D - 2\mu = 0$

BEC-BCS crossover in the CFL; Why?

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- Effect of axial anomaly on (qq) attraction

$$L^{(6)}_{\text{KMT'}} = K'\,\mathrm{tr}\left(d_R^+ d_L \phi + d_R d_L^+ \phi^+\right)$$

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$$L^{(6)}_{\mathrm{KMT'}} = K' \,\mathrm{tr}\left(d_R^+ d_L \phi + d_R d_L^+ \phi^+\right)$$

- Neglecting fluctuation in chiral field

$$L^{(6)}_{\mathrm{KMT'}} \simeq -(1/4)K'\sigma \,\mathrm{tr}\left(\left(d_R^+ - d_L^+\right)\left(d_R - d_L\right)\right)$$
$$+(1/4)K'\sigma \,\mathrm{tr}\left(\left(d_R^+ + d_L^+\right)\left(d_R + d_L\right)\right)$$

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HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

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$$L^{(6)}_{\mathrm{KMT'}} = K'\,\mathrm{tr}\left(d_R^+ d_L \phi + d_R d_L^+ \phi^+\right)$$

- Neglecting fluctuation in chiral field

$$L^{(6)}_{\mathrm{KMT'}} \simeq \boxed{-(1/4)K'\sigma}\,\mathrm{tr}\left(\left(d_R^+ - d_L^+\right)\left(d_R - d_L\right)\right)$$
$$+ (1/4)K'\sigma\,\mathrm{tr}\left(\left(d_R^+ + d_L^+\right)\left(d_R + d_L\right)\right)$$

BEC-BCS crossover in the CFL; Why?

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

- Effect of axial anomaly on (*qq*) attraction

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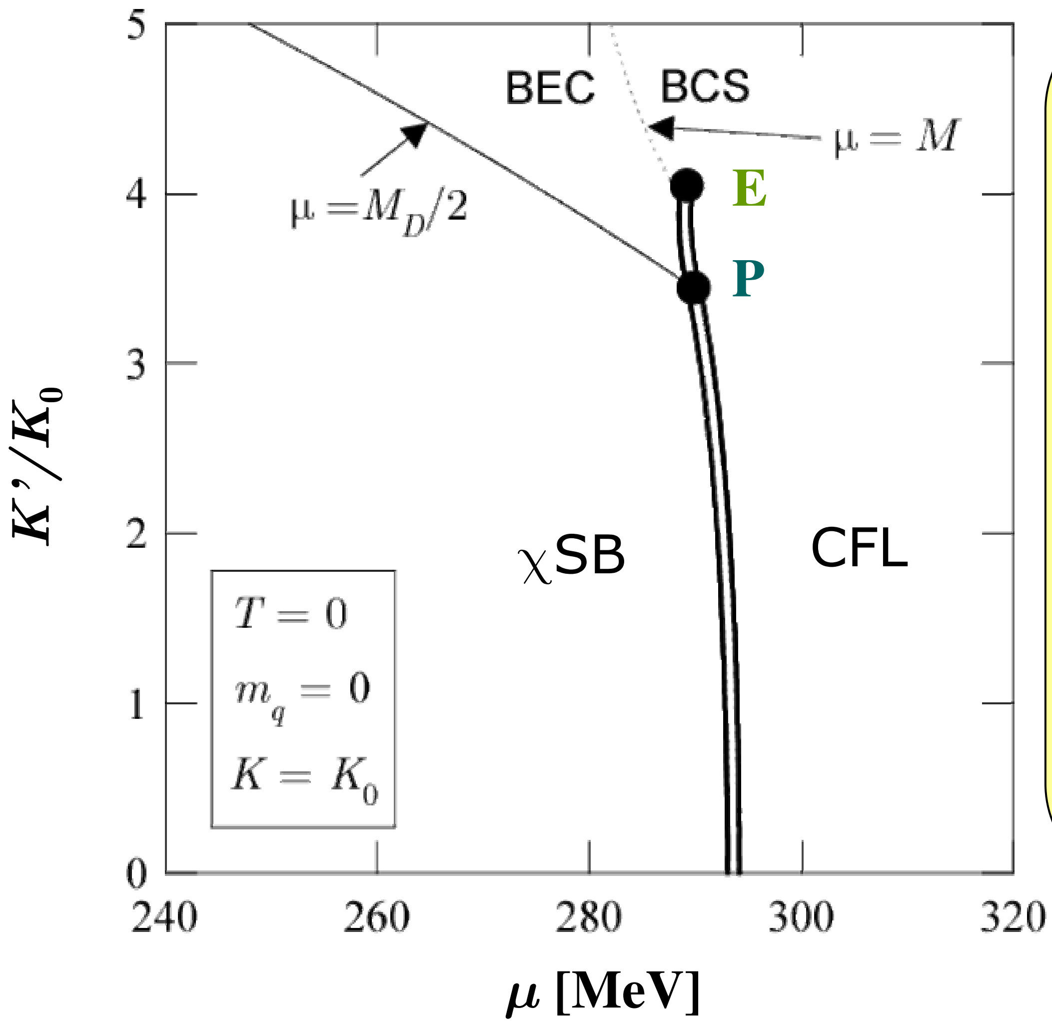
- KMT' term increases (*qq*)-attraction in the positive parity channel

$$H \to H' = H + (1/4)K'|\sigma| > H$$

Effect of K′ term; phase diagram

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408



How robust is the BEC-like CFL?

- K′ increases, then BEC sets in at some critical K′ **(P)**
- This happens before the chiral transition gets crossover **(E)**
- Between **(P)** and **(E)** transition from BEC to BCS is 1st order!

Summary

- Rich phase diagram due to *axial anomaly* and its coupling to chiral/diquark fields

Chiral crossover

- Diquark field contributes as if it is *an external field* for chiral transition
- Low-T critical point indeed appears!

GL prediction: Hatsuda, Tachibana, Yamamoto, Baym (06)

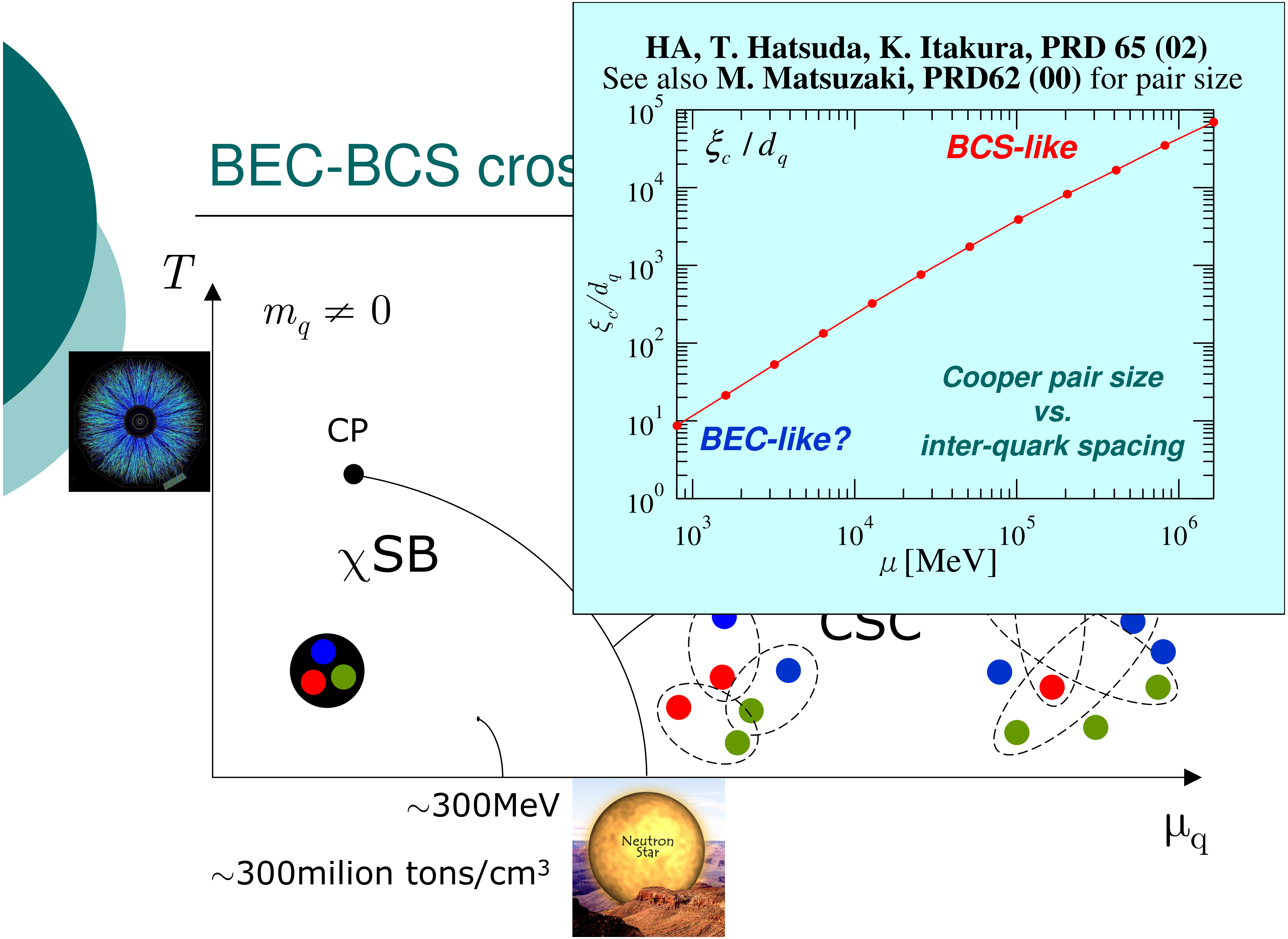
BEC-BCS crossover

- Chiral field behaves as if it is *a mediating field* that increases (qq)-attraction

c.f. Crossover by increasing H: Nishida, Abuki (05,07)
NJL phase diagram: Kitazawa, Rischke, Shovkovy (08)

Backup slides

BEC-BCS cros

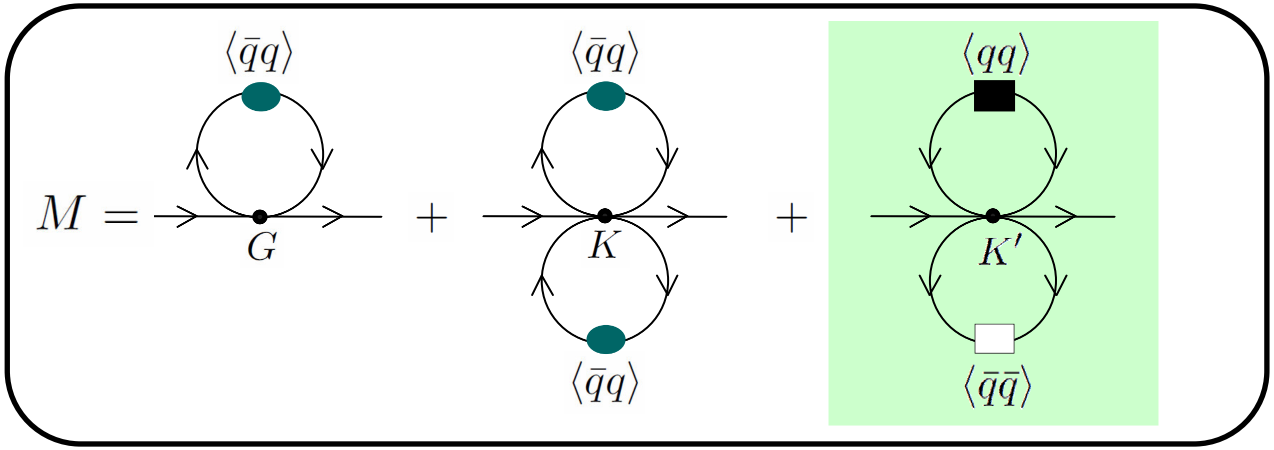


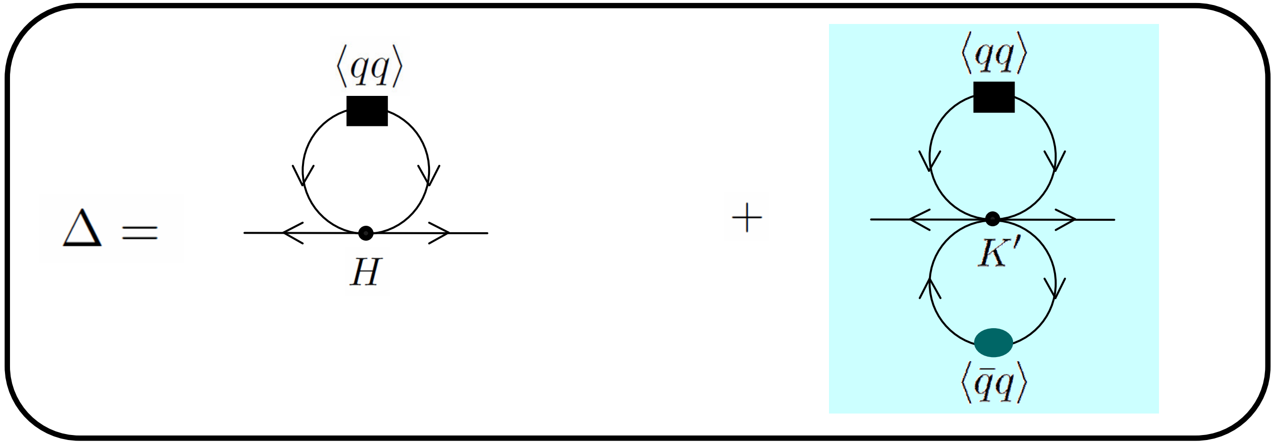
HA, T. Hatsuda, K. Itakura, PRD 65 (02)
See also M. Matsuzaki, PRD62 (00) for pair size
ξ_c / d_q
BCS-like
BEC-like?
Cooper pair size
vs.
inter-quark spacing
μ [MeV]
T
$m_q \neq 0$
CP
χSB
CSC
~300MeV
~300milion tons/cm³
Neutron Star
μ_q

NJL model with axial anomaly (4)

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408

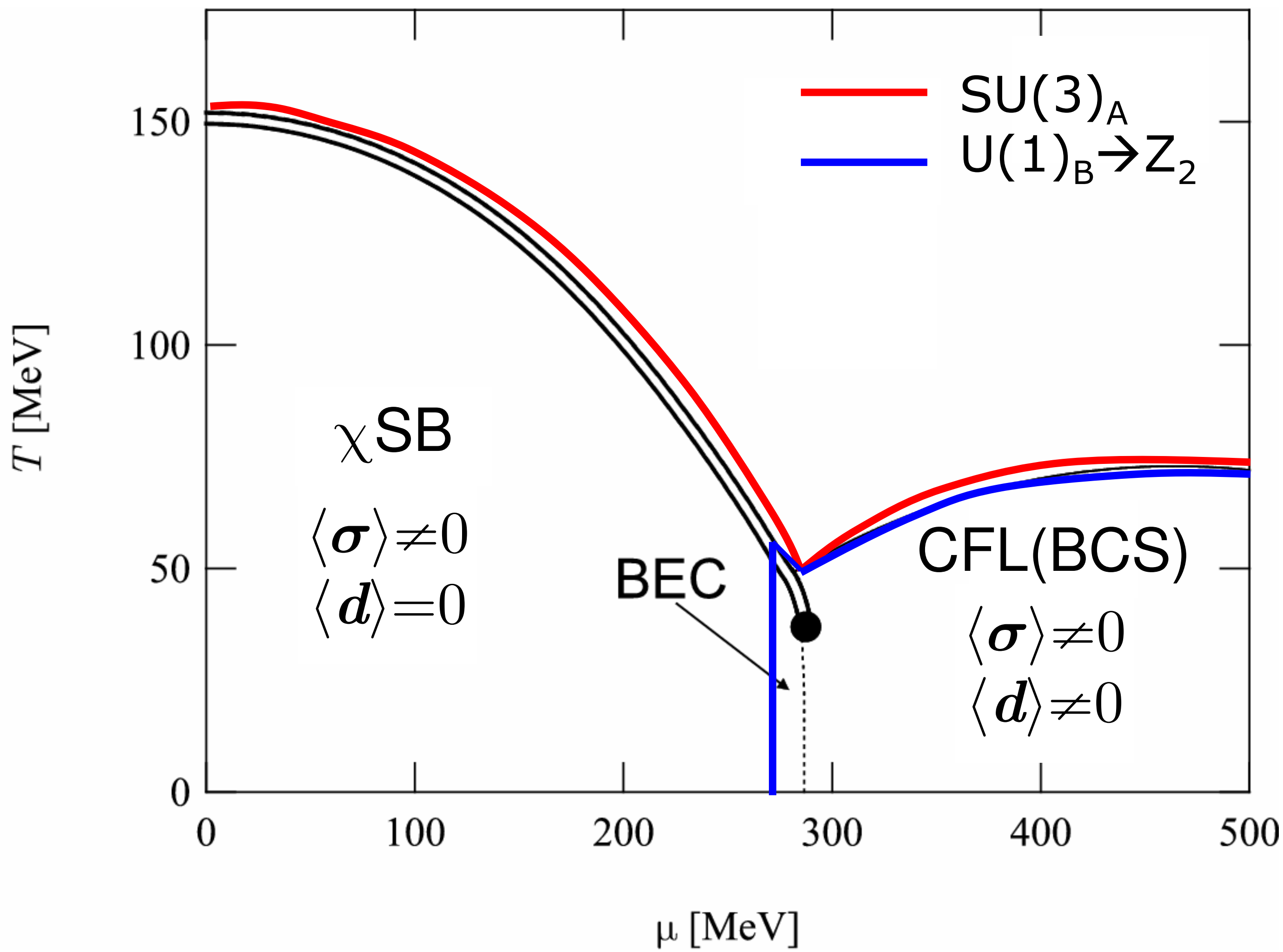
- Dirac/Mojorana mass formations



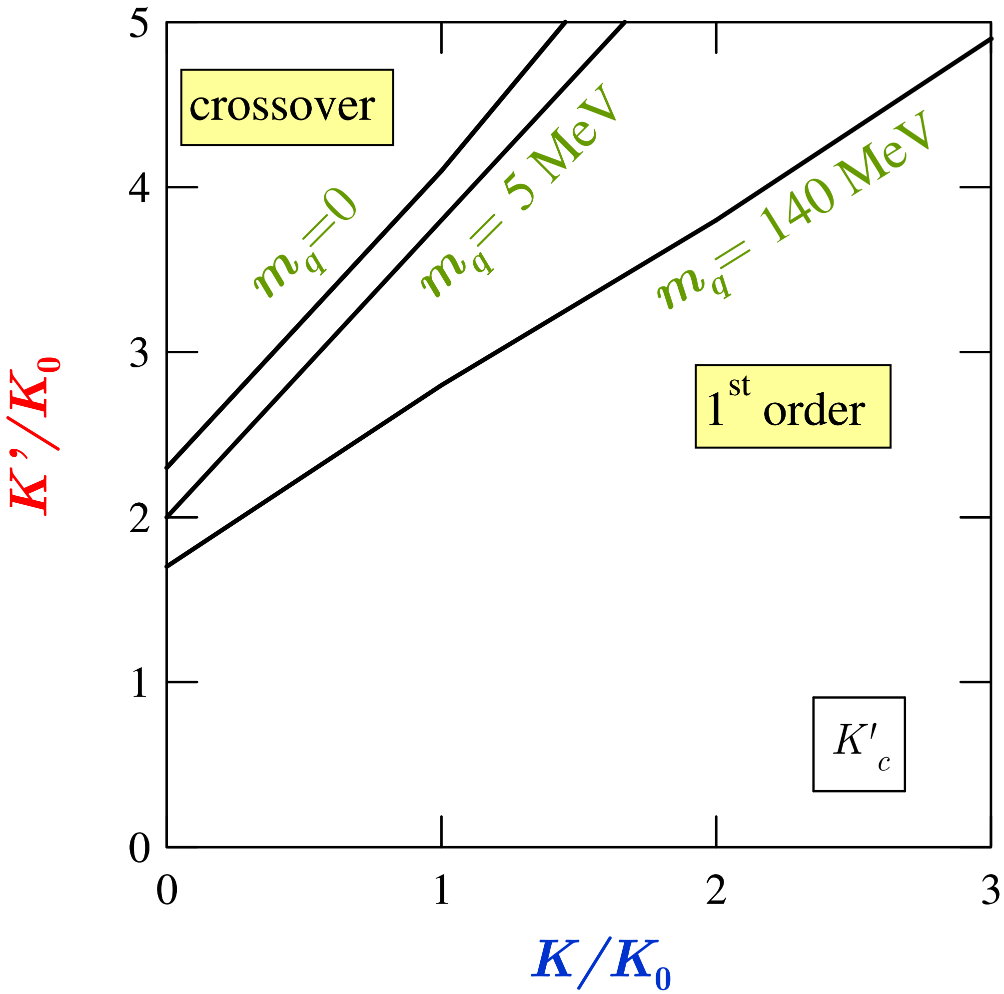


What does axial anomaly bring?

HA, G. Baym, T. Hatsuda, N. Yamamoto, arXiv:1003.0408



Chiral transition at $T = 0$; a role of *quark mass*



Effect of K' :

- weaken the chiral transition

Effect of K :

- strengthen 1^{st} order chiral transition

Effect of m_q :

- helps to weken the chiral transition

Outlook

- General Ginzburg-Landau (GL) analysis
  - What if $m_{u,d} \leqslant m_s$? Neutrality condition?
  - Coupling with the density

Zhang, Kunihiro, (Fukushima) (08, 09)

- Full NJL analysis
  - with flavor asymmetric matter with vector interaction & KMT term & neutrality
  - Polyakov loop? PNJL

c.f. Fukushima, PLB04, Ratti, Thaler, Weise, PRD06
Roessner, Ratti, Weise, PRD07, Abuki, Ruggieri et al., PRD08

  - Nucleon (b-f) formation due to diquark-quark residual interaction

Maeda, Baym, Hatsuda, PRL103 (09)