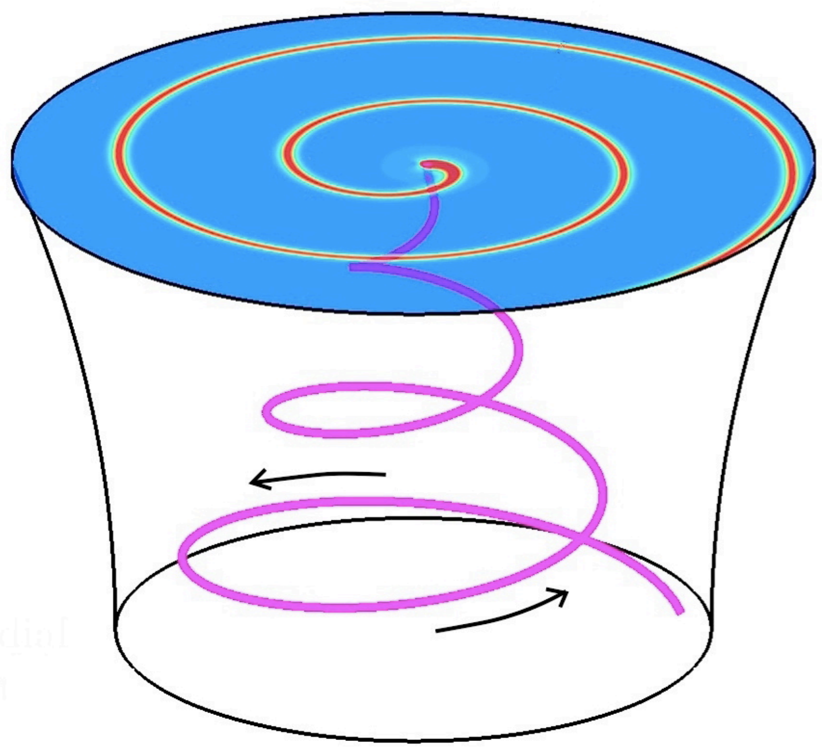
SYNCHROTRON RADIATION IN STRONGLY COUPLED CONFORMAL FIELD THEORIES

Dominik Nickel
Institute of Nuclear Theory, Seattle



C. Athanasiou, P. Chesler, H. Liu, DN, K. Rajagopal

arXiv:1001.3880

UNIVERSITY of WASHINGTON

OUTLINE

- motivation: patterns of radiation
- gauge/gravity duality
- synchrotron radiation
- outlook

RADIATION IN CLASSICAL ELECTRODYNAMICS

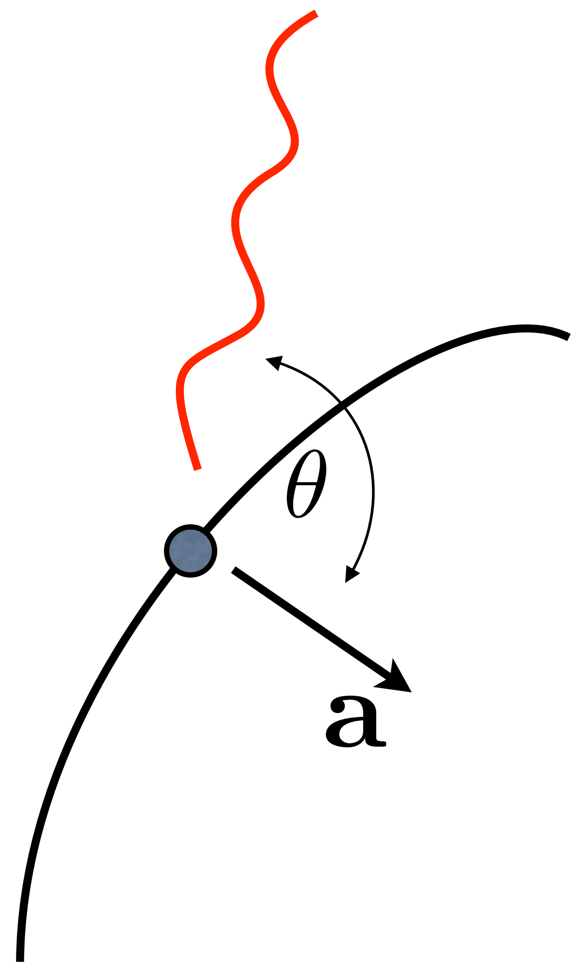
- accelerated charge radiates

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi} a^2 \sin^2\theta$$

➡ total power radiated (energy loss)

$$P = \frac{2}{3}\frac{e^2}{4\pi} a^2$$

➡ free propagation of radiation
(distribution of power)



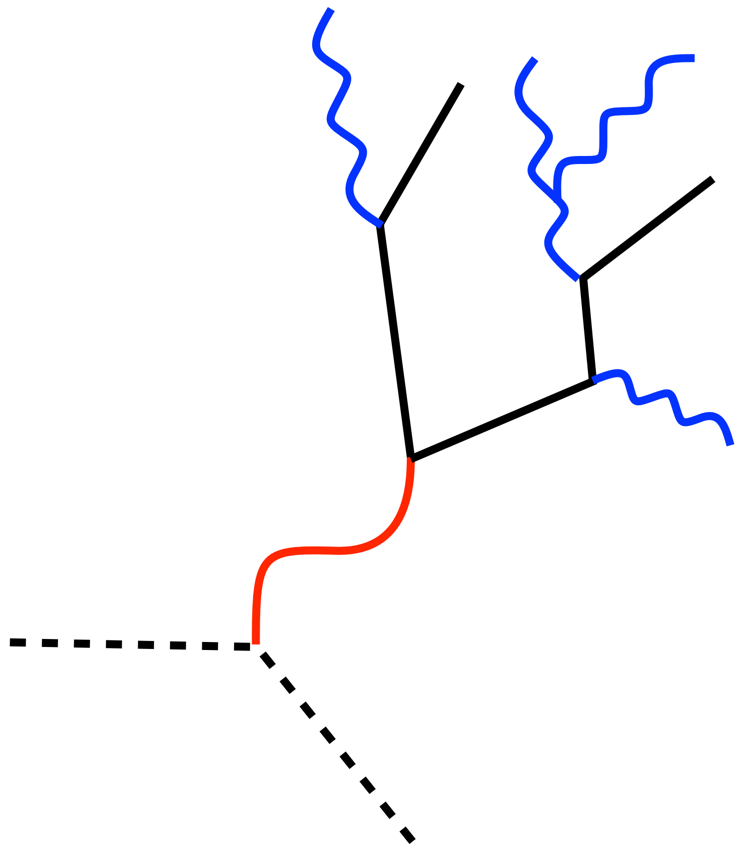
analogy in strongly coupled gauge theories?

RADIATION IN GAUGE THEORIES? – JETS

factorization in jet production

$$e^+e^- \to \gamma^* \to \text{hadrons}$$

- hard production process
- showering
- hadronization



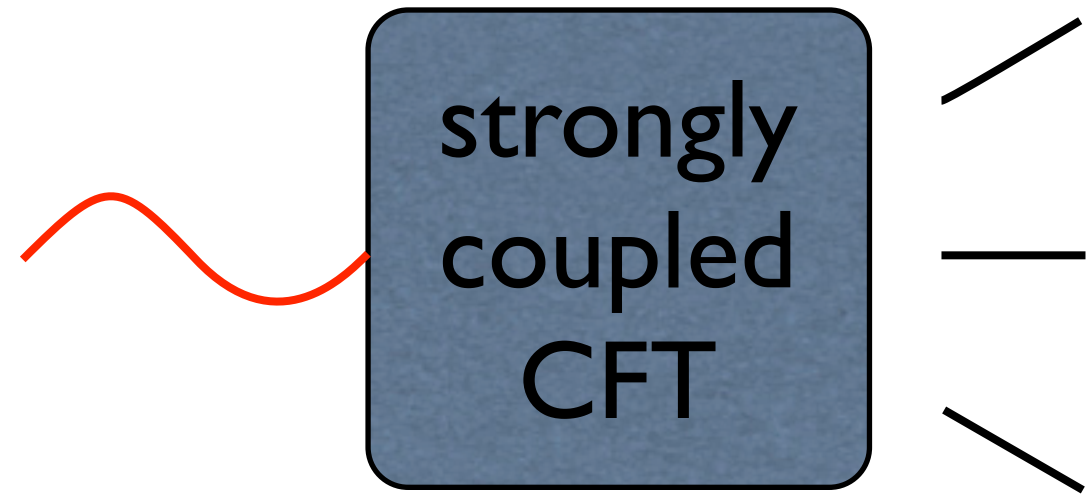
understanding from first principles challenging?

➡ guidance from other approaches?

CONFORMAL COLLIDER PHYSICS

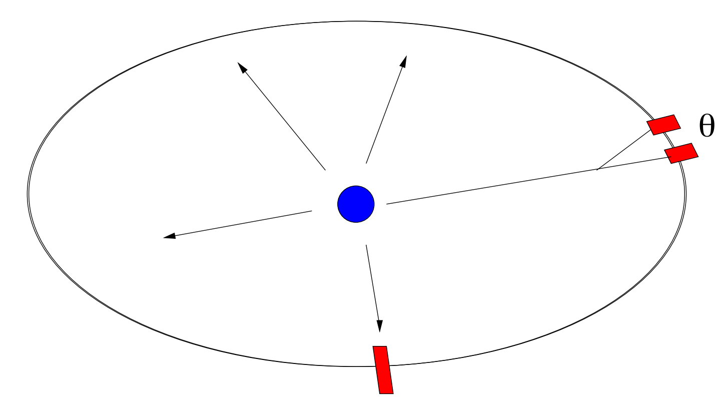
D. Hofman, J. Maldacena [arXiv:0803.1467]

study decay of off-shell bosons in a strongly coupled CFT



quantities of interest:

$$\begin{aligned} \langle \mathcal{E}(\hat{r}) \rangle &= \lim_{r\to\infty} r^2 \int dt \langle \psi(t) | \hat{r} \cdot \mathbf{S} | \psi(t) \rangle \\ \langle \mathcal{E}(\hat{r}) \mathcal{E}(\hat{r}') \rangle &= \dots \end{aligned}$$



key result: all N-point functions isotropic!

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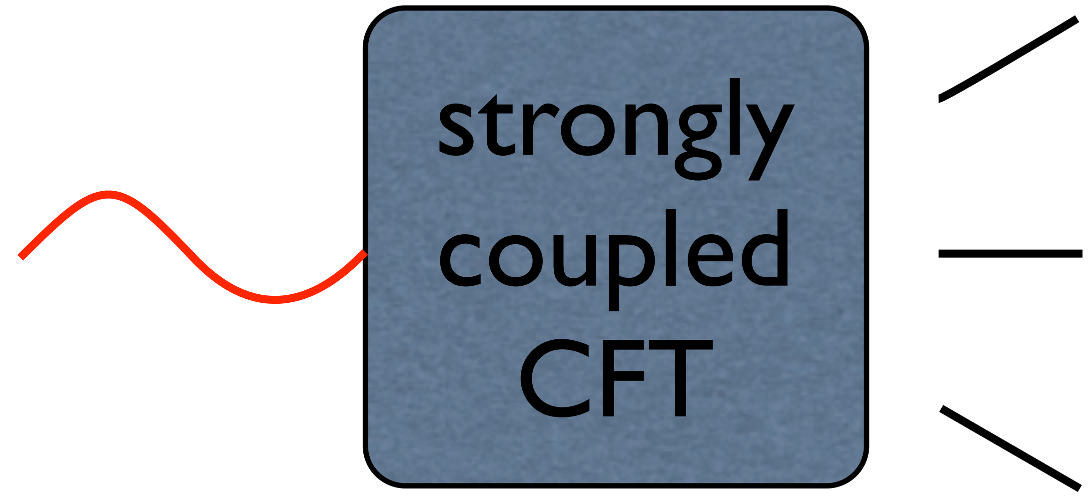

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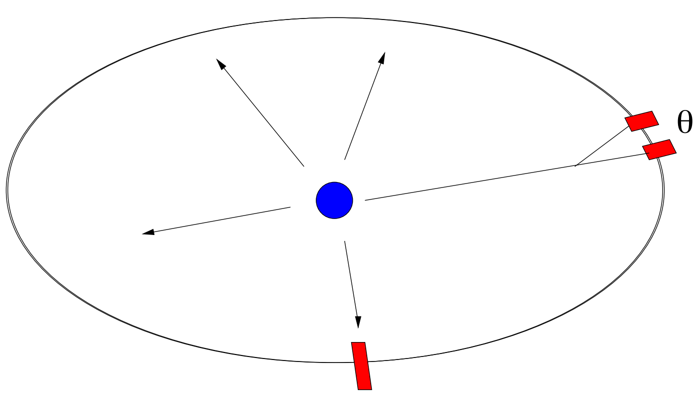


key result: all N-point functions isotropic!

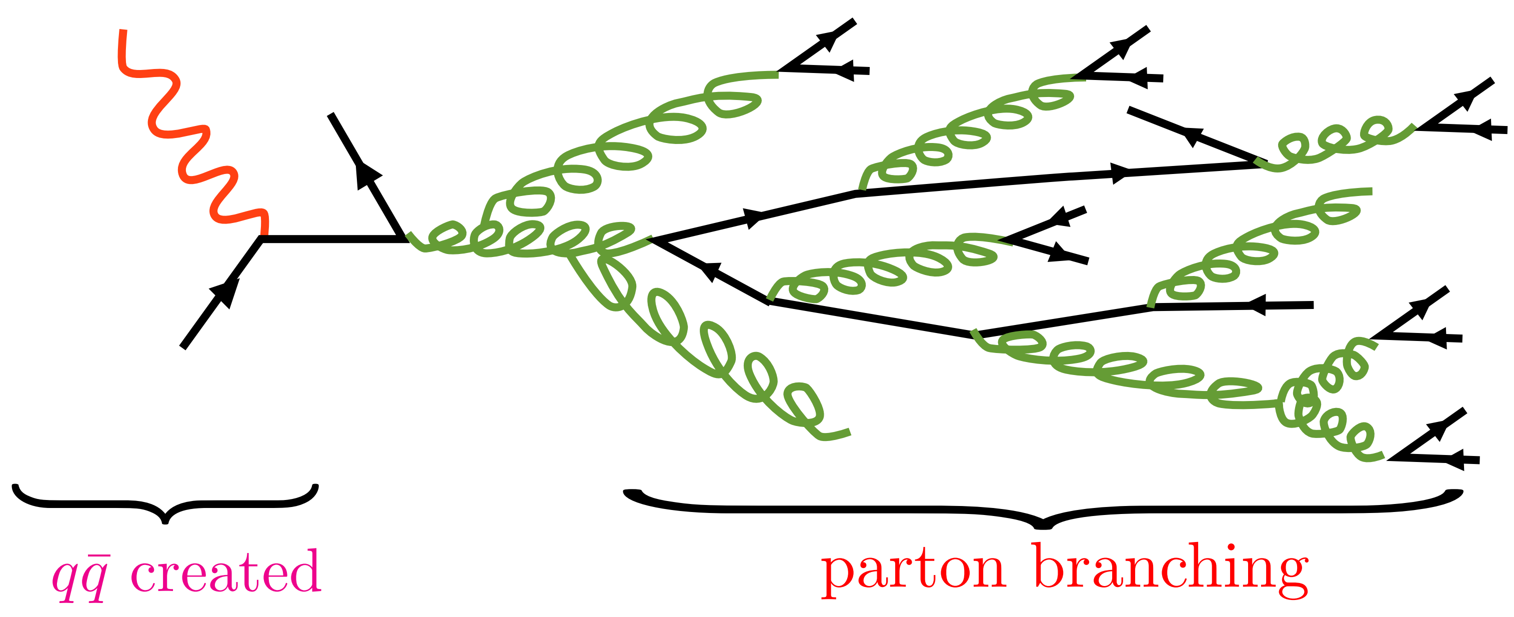
no jets at strongly coupling?

WHERE DOES THE ISOTROPY COME FROM?

- choice of initial conditions?
- propagation through strongly coupled vacuum?

parton branching picture:

Y. Hatta, E. Iancu, A. Mueller [arXiv:0803.2481]



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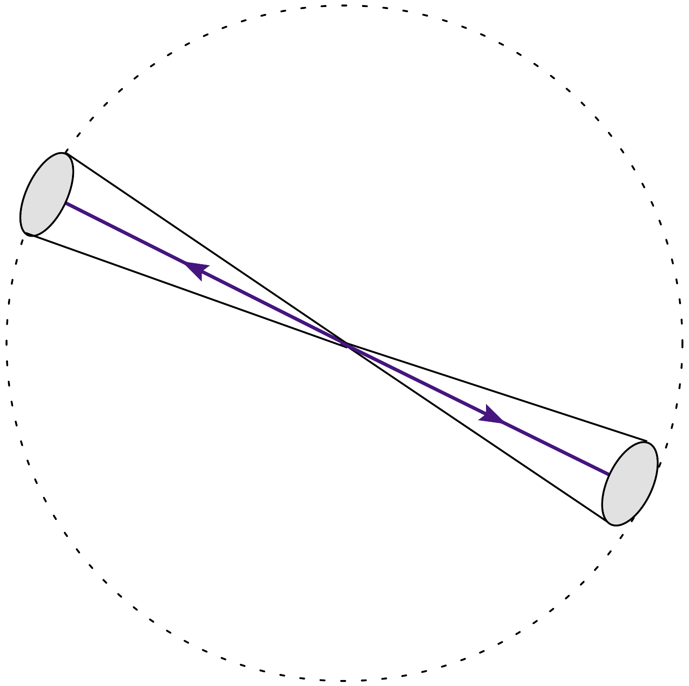
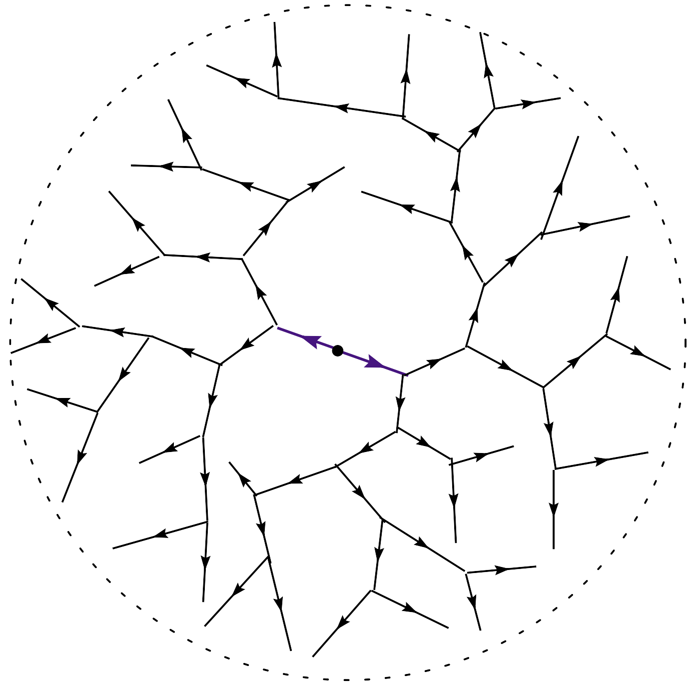


weak coupling



strong coupling

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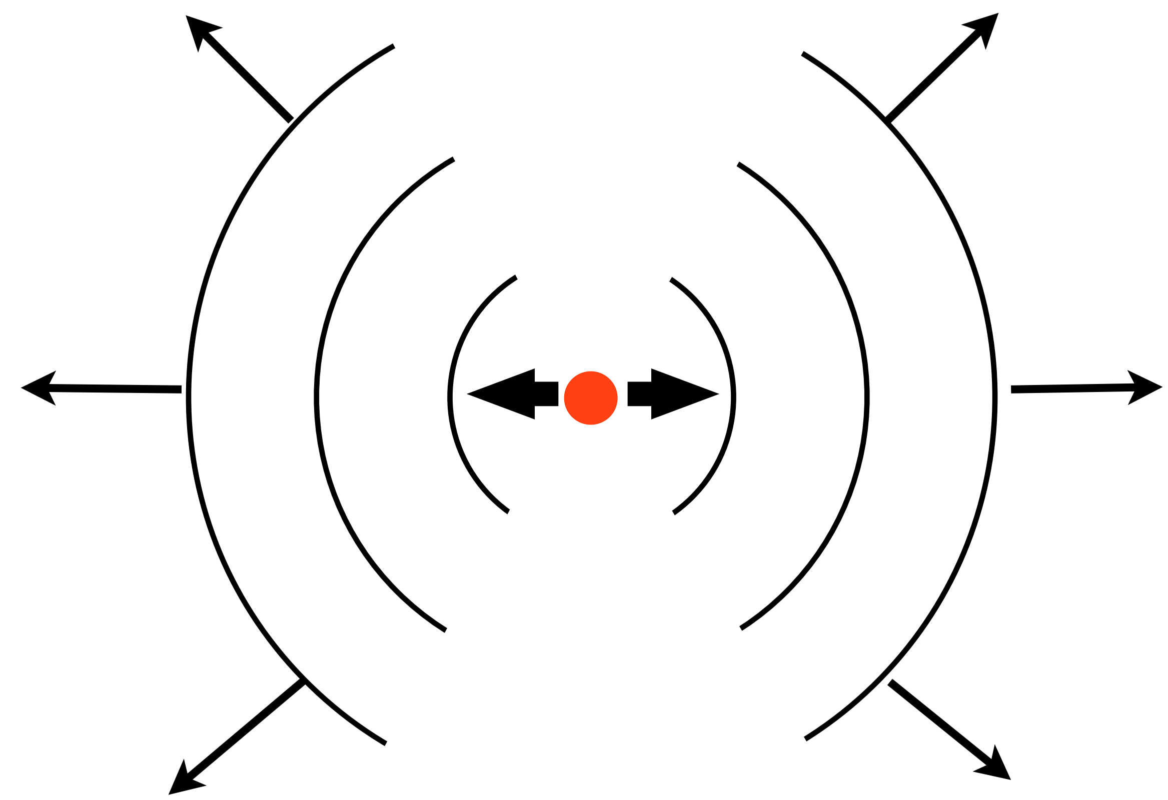
A SIMPLE EXPERIMENT

radiating source in compact domain



- real-time propagation
- power spectrum at infinity
- depletion of short-wavelength modes?

concentrate on theories with classical gravity dual

>exact solution for synchrotron radiation<

ADS/CFT

Maldacena's conjecture:

string theory on aymptotic $AdS_5 \times X^5$	$\longleftrightarrow$	$4d$ (supersymmetric) CFT
classical gravity		large $N_c\,,\,\lambda$

J. Maldacena [hep-th/9711200]

geometrical realization / 'dictionary':

gravity in 5d		4d CFT
gravitational fields	$\longleftrightarrow$	source for stress-energy tensor
open strings		quarks

$$S_{\text{gravity}}[\phi] = S^{\text{boundary}}_{\text{gravity}}[\phi_0] = S_{\text{CFT}}[\phi_0]$$

➡ classical dynamics in 5d captures dynamics of 4d QFT

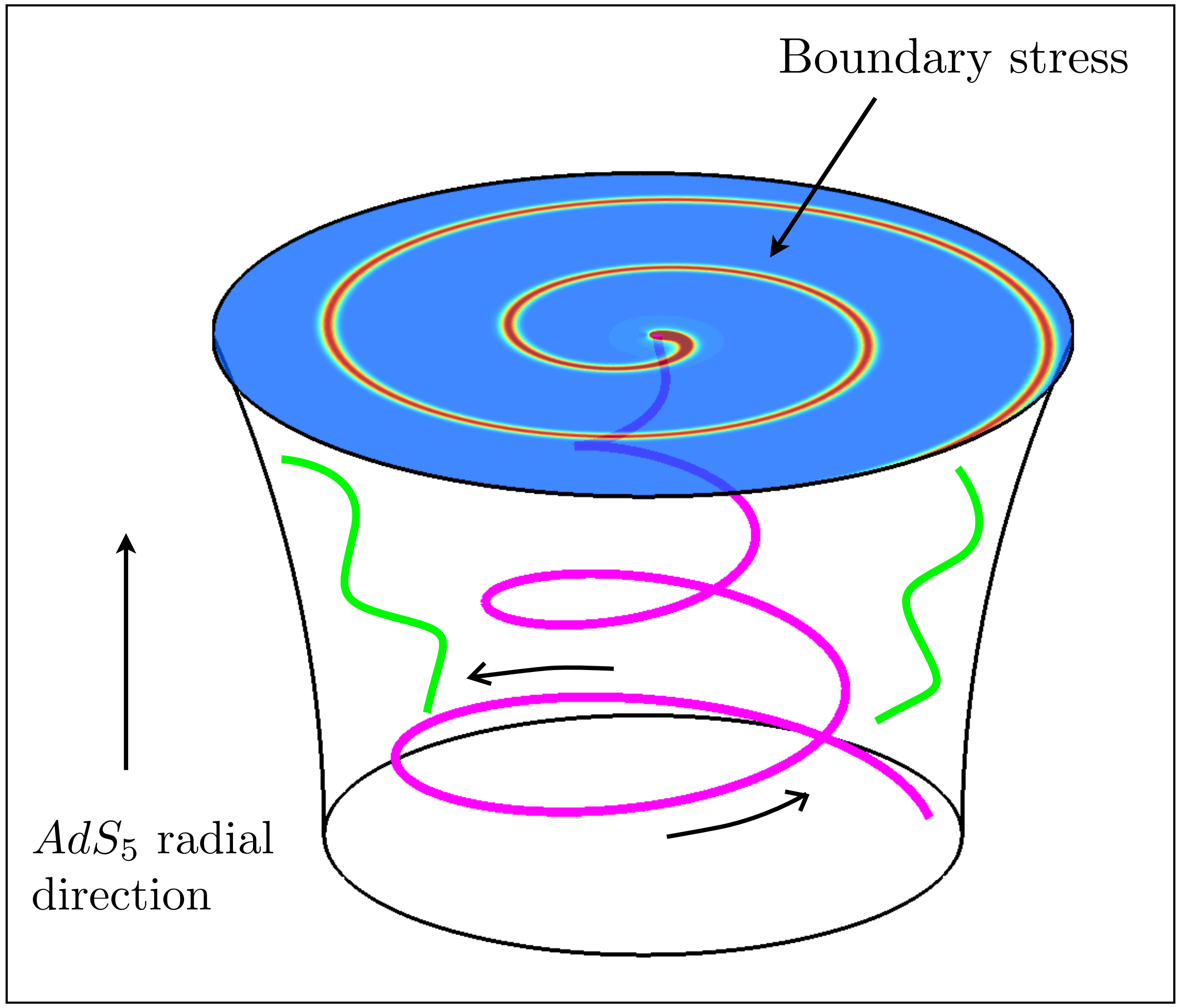
THE GRAVITATIONAL SETUP

AdS background:

$$ds^2 = \frac{L^2}{u^2}\left(-dt^2 + d\mathbf{x}^2 + du^2\right)$$

- ➡ rotating string $X^\mu(\sigma, \tau)$
- ➡ gravitational waves $h_{\mu\nu}(x)$ towards boundary
- ➡ stress-energy tensor/energy density

$$T^{\mu\nu}(x) = \lim_{u\to 0} \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{gravity}}}{\delta h_{\mu\nu}(x)}$$



simplification: $G_{\text{Newton}} \sim 1/N_c^2 \;\longrightarrow$ gravitational response linear

THE ROTATING STRING

- Nambu-Goto action in AdS background:

$$S_{\rm NG} = -T_0 \int dt du \sqrt{-\det(\gamma_{ab})}\,, \quad \gamma_{ab} = G^{\rm AdS}_{\mu\nu} X^\mu X^\nu$$

- parameterization of worldsheet: $X^\mu \quad = \quad (t, R(u), \pi/2, \phi(u) + \omega_0 t, u)^\mu$

- solution (remarkable luck!):

$$R(u) = \sqrt{R_0^2 + \frac{v^2 u^2}{\sqrt{1-v^2}}}\,, \quad \phi(u) = -\frac{u\omega_0}{\sqrt{1-v^2}} + \arctan\left(\frac{u\omega_0}{\sqrt{1-v^2}}\right)$$

- bonus: energy loss from flux on string

$$E = -\pi^u_t \equiv \frac{\delta S_{\rm NG}}{\delta \partial_u X^t} = \frac{\lambda}{2\pi}\left(\frac{R_0 \omega_0^2}{1-v^2}\right)^2$$

A. Mikhailov [hep-th/0305196]

GRAVITATIONAL WAVES AND GAUGE INVARIANTS

- linearized equations of motion for $G_{MN} = G_{MN}^{\rm AdS} + h_{MN}$ from

$$R_{MN} - \frac{1}{2} G_{MN}(R + 2\Lambda) \quad = \quad 8\pi G_{\rm Newton}\, t_{MN}^{\rm rotating\ string}$$

- decomposition into helicities: $h_{MN} \quad = \quad h_a^{\rm tensor} \mathbb{T}^a_{MN} + h_a^{\rm vector} \mathbb{V}^a_{MN} + h_a^{\rm scalar} \mathbb{S}^a_{MN}$

- coordinate invariance/conservation laws: only one gauge invariant per helicity required

- scalar equation

$$\partial_u^2 Z(u) - \frac{5}{u}\,\partial_u Z(u) + \left(-\partial_t^2 + \nabla^2 + \frac{9}{u^2}\right) Z(u) \quad = \quad S(u)$$

- energy density:

$$\boxed{\mathcal{E}(t, \mathbf{r}) \quad = \quad \lim_{u\to 0} \frac{Z(t, \mathbf{r}, u)}{u^3}}$$

ANALYTIC SOLUTION FOR ENERGY DENSITY

- energy density via convolution:

$$\mathcal{E}(t,\mathbf{x}) = \frac{\sqrt{\lambda}}{24\pi^2\gamma^4 r^6 \Xi^6}\Big[-2r^2\Xi^2 + 4r\gamma^2\Xi(t_{\rm ret}-t) + (2\gamma^2-4r^2v^2\gamma^2\omega_0^2\sin^2\theta+3r^2\gamma^4\omega_0^2\Xi^2)(t_{\rm ret}-t)^2$$
$$+7r\gamma^2\omega_0^2\Xi(t_{\rm ret}-t)^3 + 4\gamma^2\omega_0^2(t_{\rm ret}-t)^4 + 8v\gamma^2\omega_0 r(t_{\rm ret}-t)(t_{\rm ret}-t+r\,\Xi)\sin\theta\cos(\varphi-\omega_0 t_{\rm ret})\Big]$$

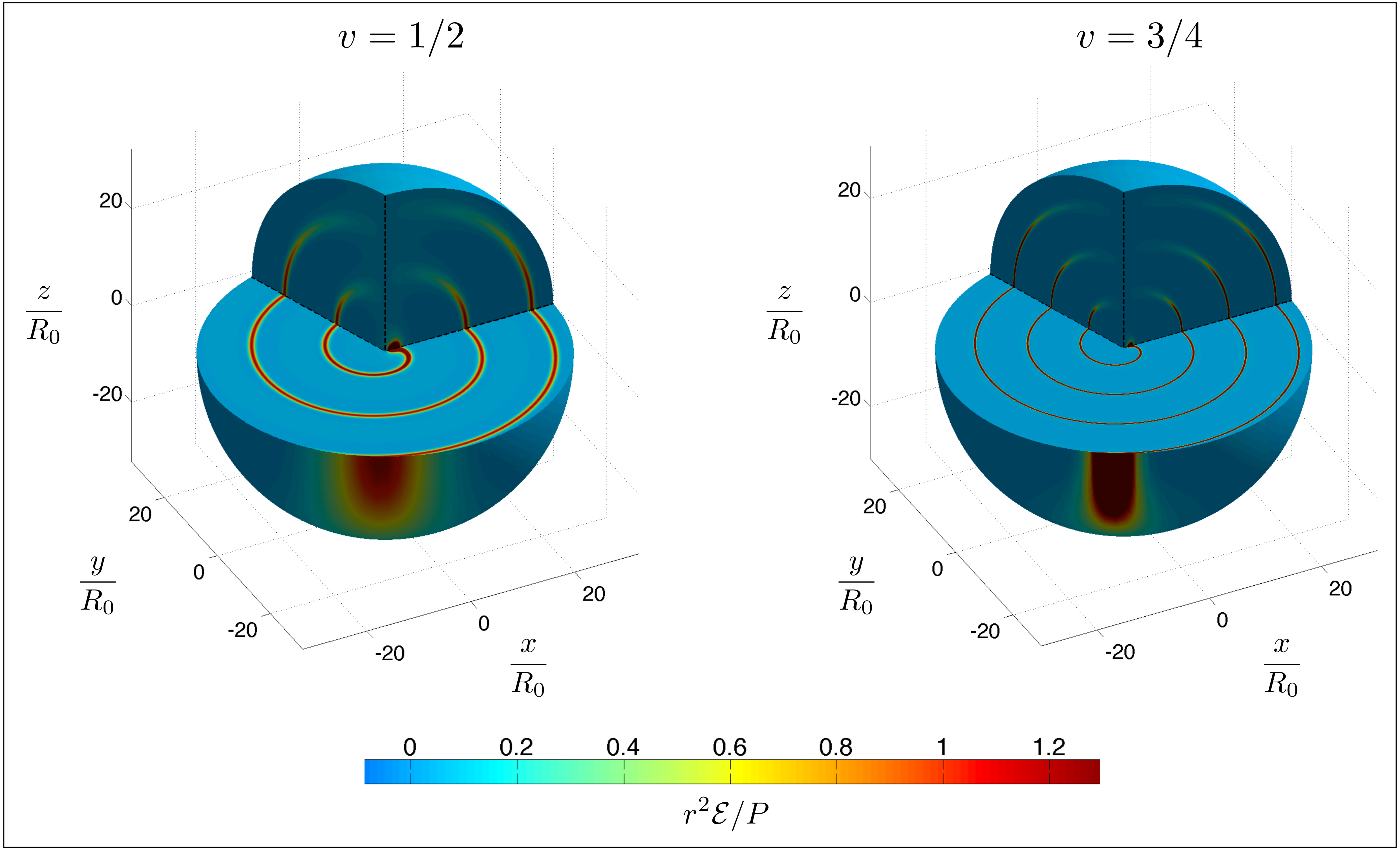
where $t - t_{\rm ret} - |\mathbf{r} - \mathbf{r}_{\rm quark}(t_{\rm ret})| = 0$

$$\Xi \equiv \frac{|\mathbf{r} - \mathbf{r}_{\rm quark}(t_{\rm ret})| - \mathbf{r}\cdot\dot{\mathbf{r}}_{\rm quark}(t_{\rm ret})}{r}$$

$\mathbf{r}_{\rm quark} = \text{quark's trajectory}\,, \quad \mathbf{r} \equiv \{r,\theta,\phi\} = \text{observer}\,, \quad \gamma = 1/\sqrt{1-v^2}$

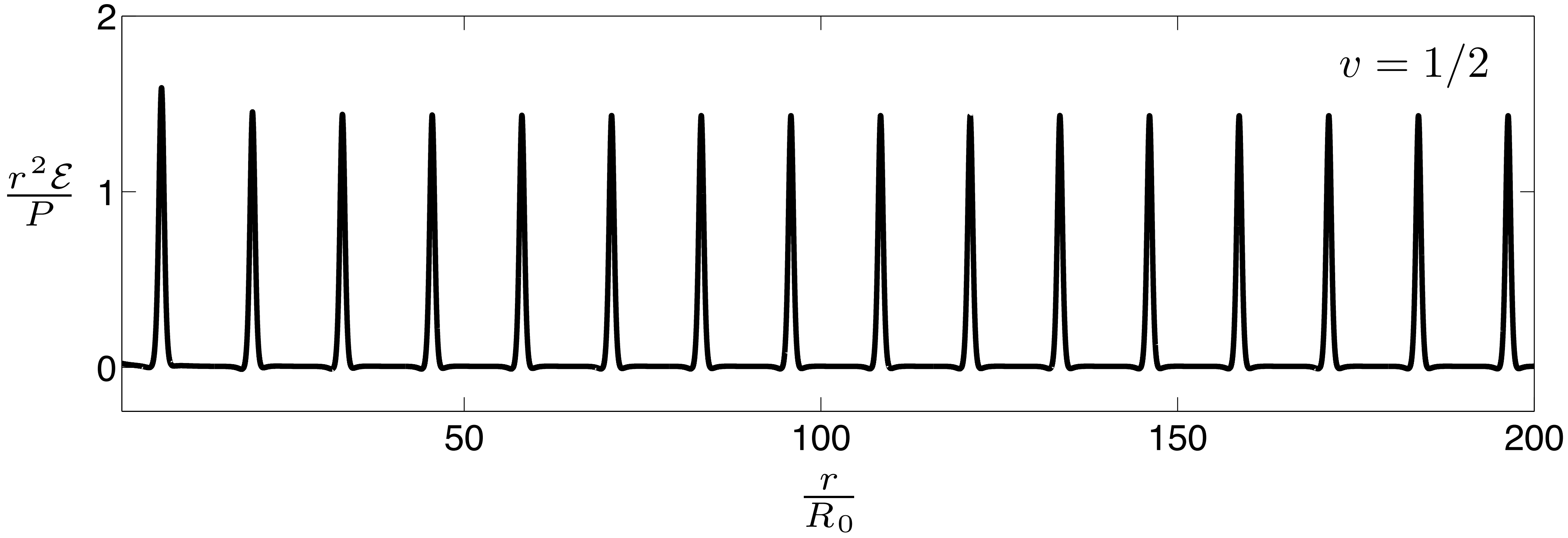
- analytic solution up to $t_{\rm ret}$ (as in electrodynamics)
- similar dependence on Ξ as in electrodynamics

ILLUSTRATION OF $\mathcal{E}(t, \mathbf{r})$

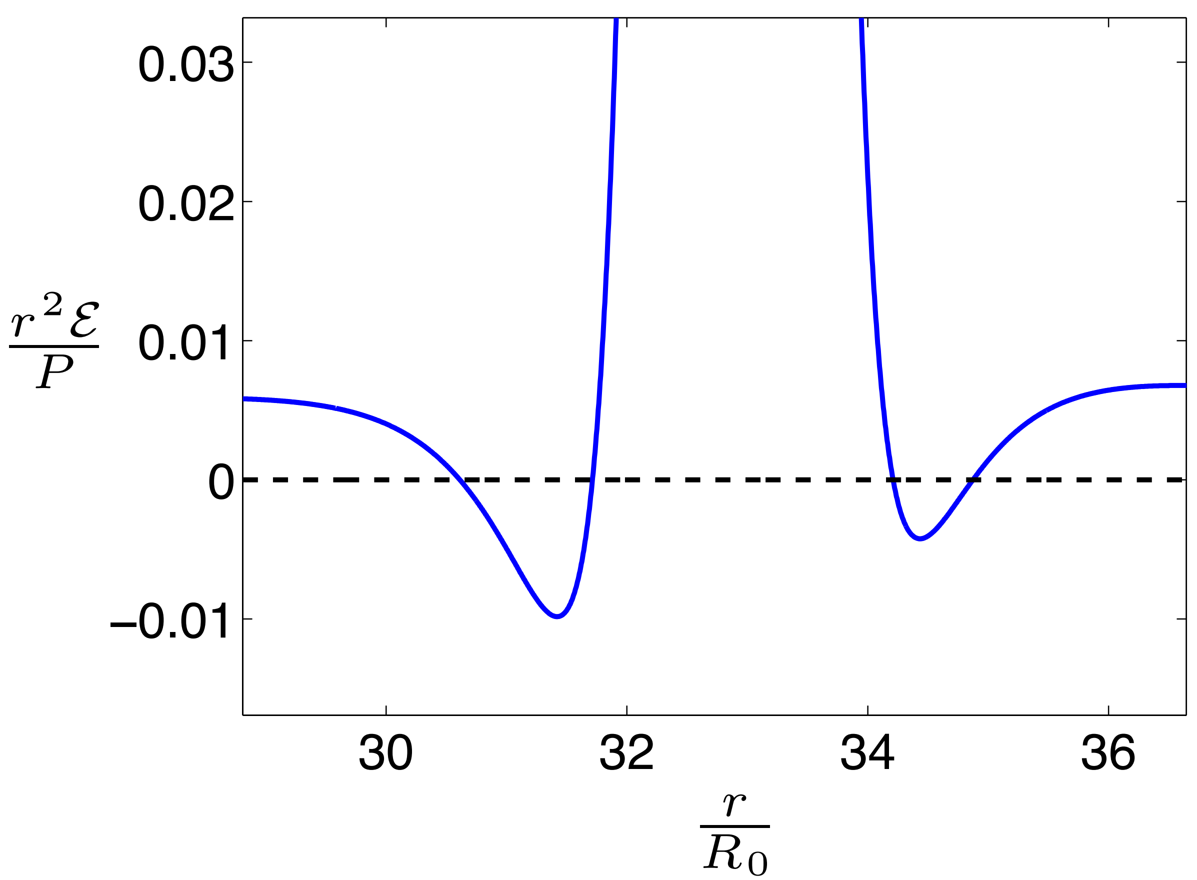


radial thickness: $\sim 1/\gamma^3$, azimuthal thickness: $\sim 1/\gamma$

BEHAVIOR IN FAR ZONE



- radiation does not broaden in far zone!
- similar struture as classical synchrotron radiation
- negative energy density ⟷ quantum effects



GEOMETRICAL INTERPRETATION

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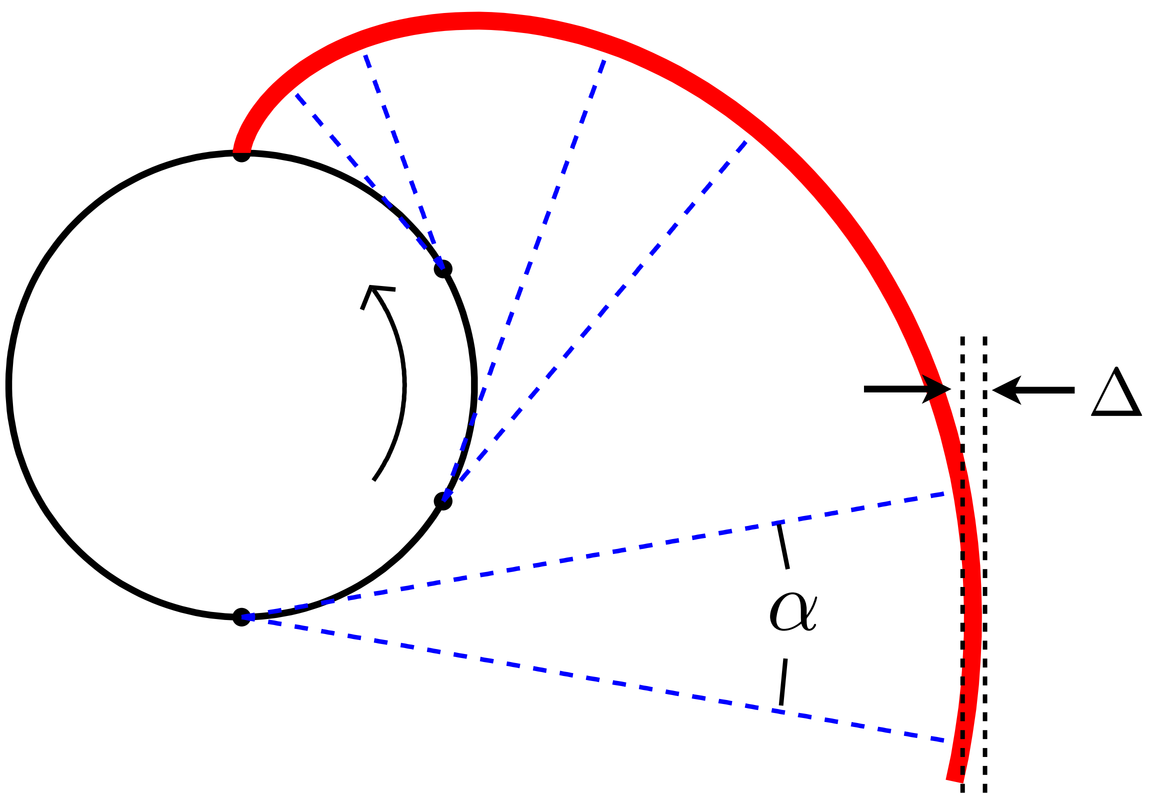
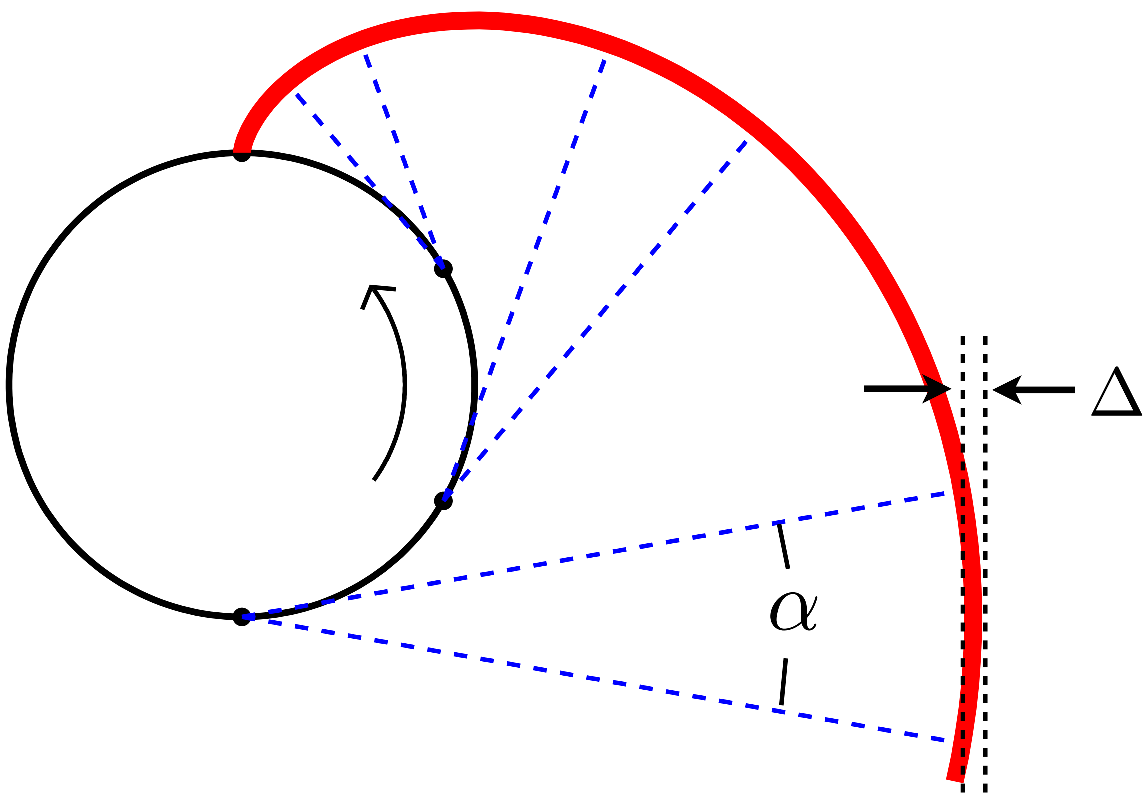
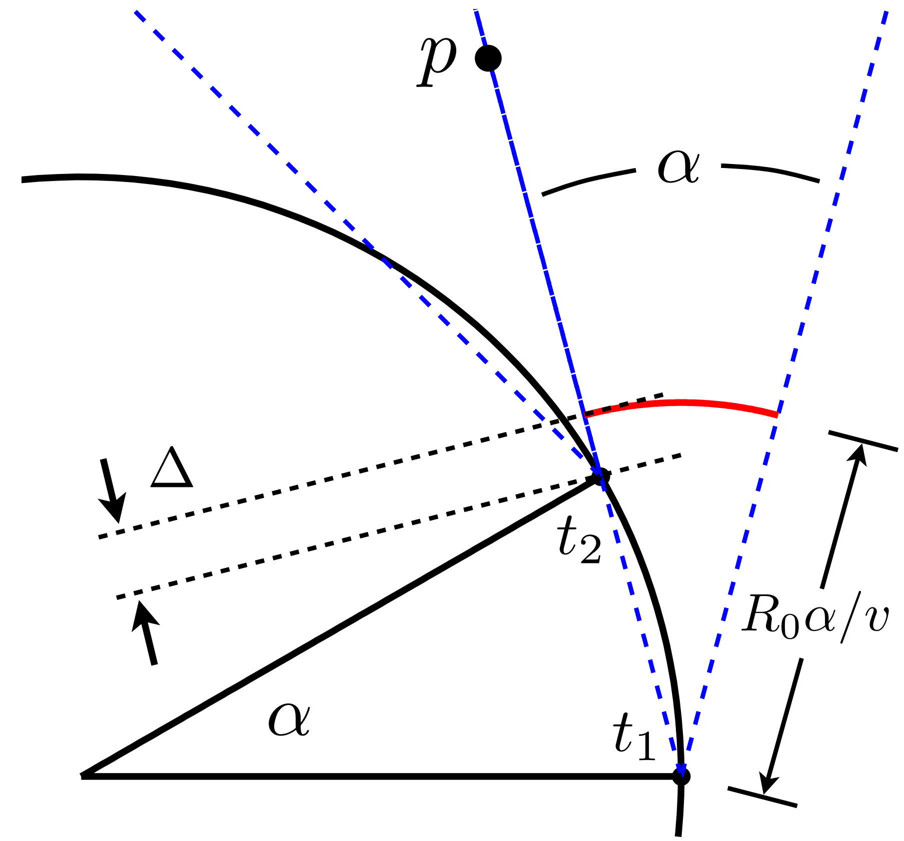


- radiation in fixed relative direction and free without broadening propagation: spiral structure

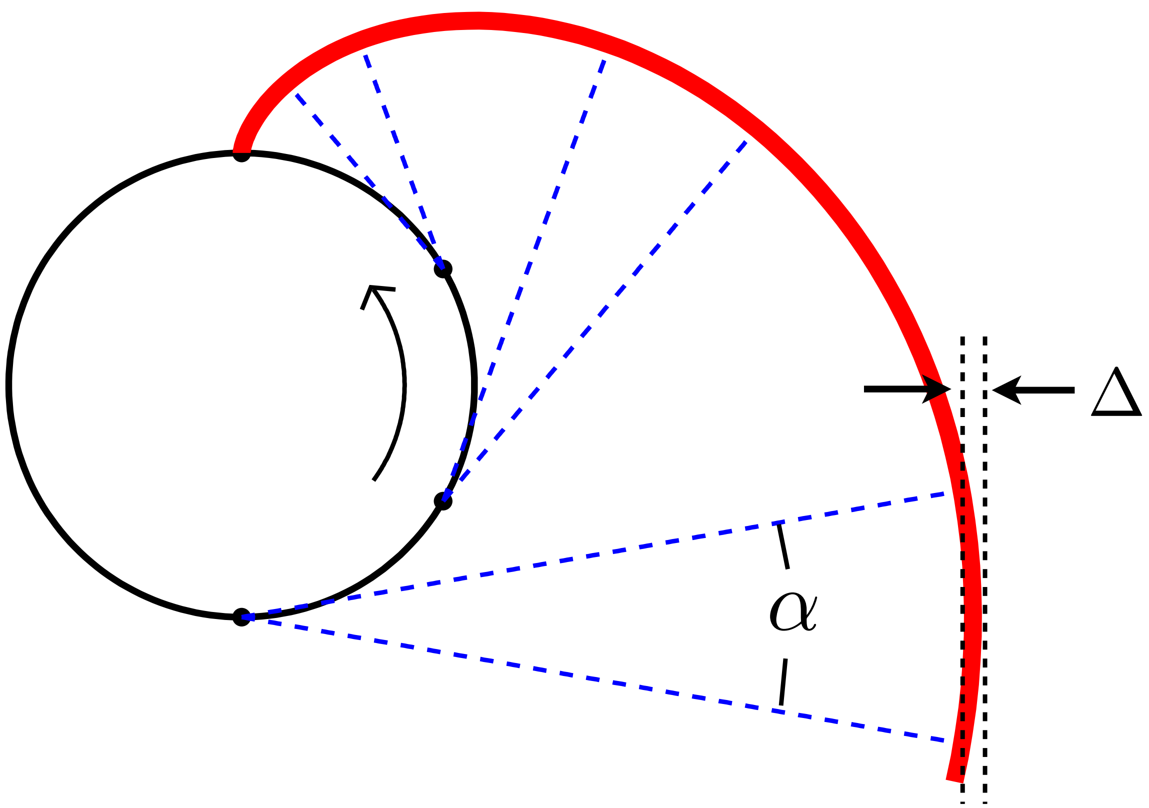
GEOMETRICAL INTERPRETATION

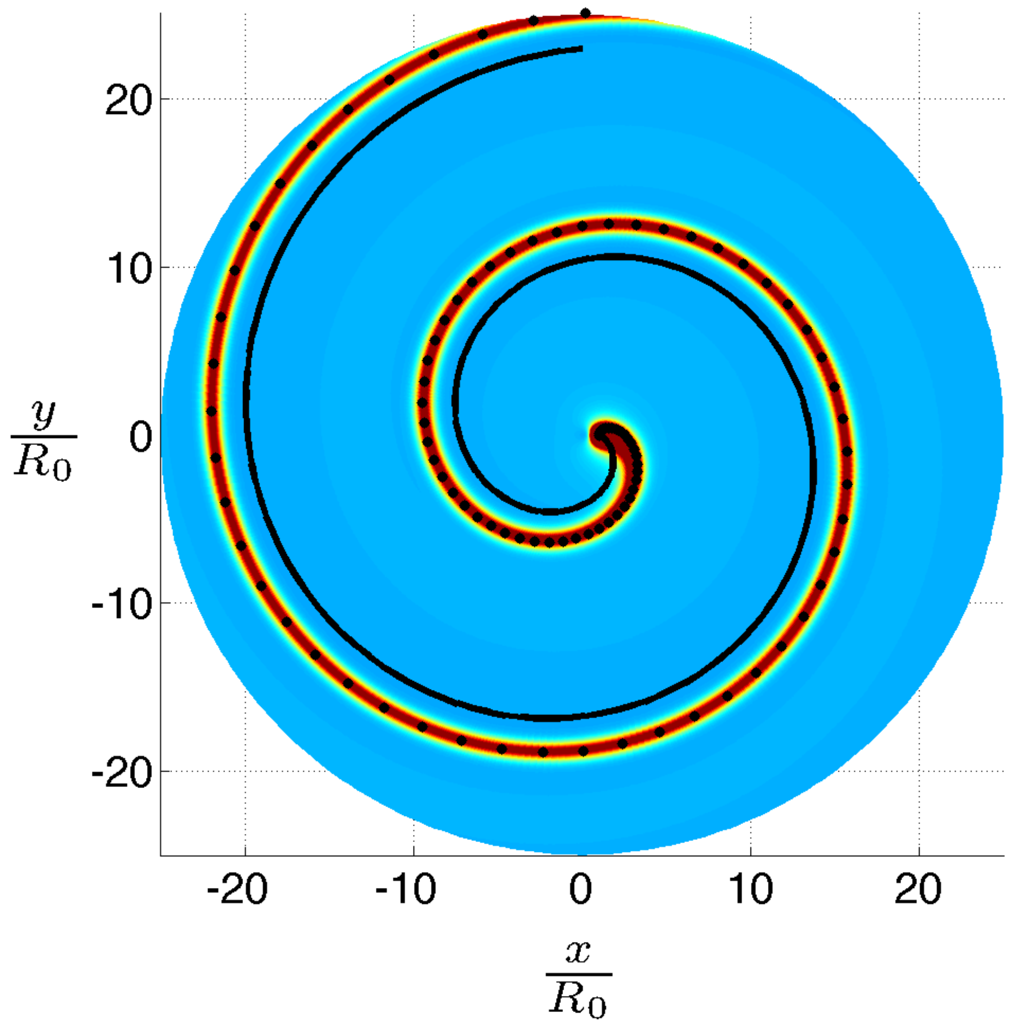




- radiation in fixed relative direction and free without broadening propagation: spiral structure
- radiation in cone with width α : - when radiated in moving direction $\Delta \sim \alpha R_0/v - \alpha R_0 \sim R_0/\gamma^3$
  - otherwise $\Delta \sim \alpha R_0$

GEOMETRICAL INTERPRETATION





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➡ **Lesson:**
- radiation more or less isotropic in rest frame ($\alpha \sim 1/\gamma$)
- free propagation without broadening
- strong similarities to ED/weak coupling regime

COMPARISON TO WEAK COUPLING AND ED

- time-averaged power distributions:

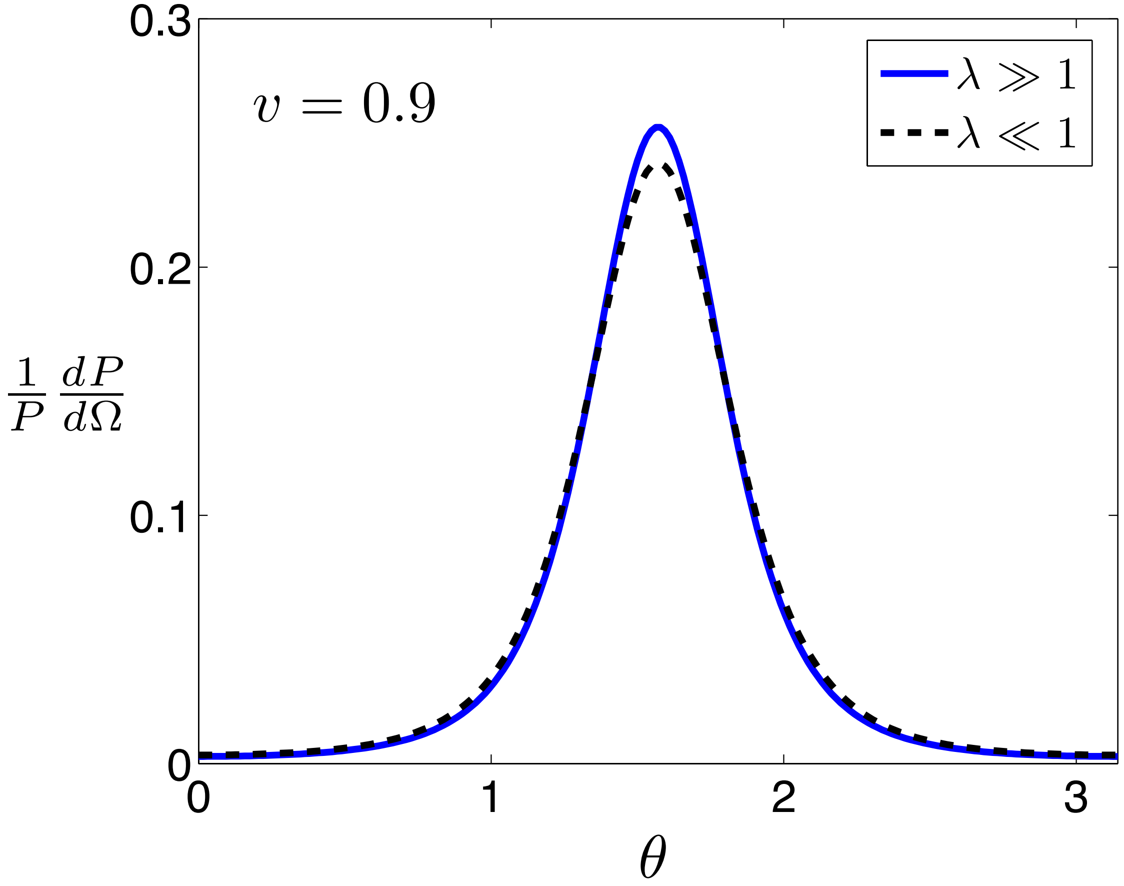
$$\frac{dP}{d\Omega} = \frac{1}{T}\int_t^{t+T} dt' \underbrace{\hat{\mathbf{n}}\cdot\mathbf{S}}_{\to r^2\mathcal{E}}$$

- weak coupling: one vector and one scalar field coupling to $e_{\rm eff}^2 = \lambda/2$

$$\frac{dP^{\lambda\ll 1}}{d\Omega} \propto \frac{dP^{\rm vector}}{d\Omega} + \frac{dP^{\rm scalar}}{d\Omega}$$
$$\frac{dP^{\lambda\gg 1}}{d\Omega} \propto \frac{dP^{\rm vector}}{d\Omega} + 2\frac{dP^{\rm scalar}}{d\Omega}$$

- overall power radiated:

$$P^{\lambda\ll 1} = \frac{5\lambda}{12}\frac{a^2}{2\pi} \quad\longleftrightarrow\quad P^{\lambda\gg 1} = \sqrt{\lambda}\,\frac{a^2}{2\pi}$$



different parametric dependence on coupling, but similar profiles

SUMMARY

- gauge/gravity duality nice tool to study real-time evolution
- synchrotron radiation solved analytically

➡ no isotropization observed

- hope/ideas for modeling jets - also at finite temperature

Thank you for your attention!