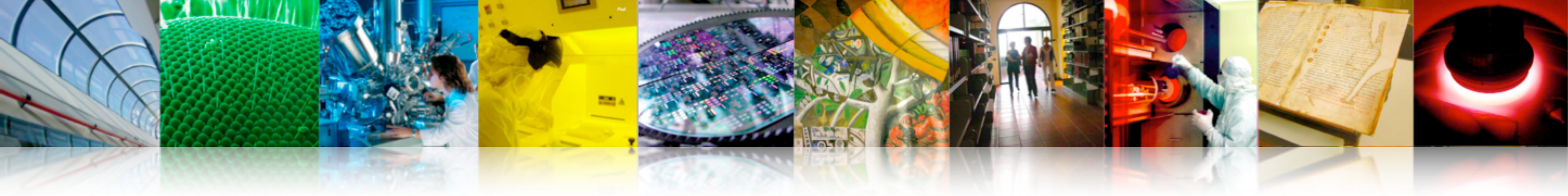


FONDAZIONE
BRUNO KESSLER

ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

IR sector of QCD:
Lattice versus Schwinger-Dyson

Daniele Binosi
binosi@ect.it

with:
Joannis Papavassiliou (University of Valencia, Spain)
Arlene C. Aguilar (University of ABC, Brazil)

QCD@Work 2010
Martina Franca
22|06|10

Motivations

- Gluon and ghost propagators from the lattice

New Schwinger-Dyson equations

- PT-BFM scheme

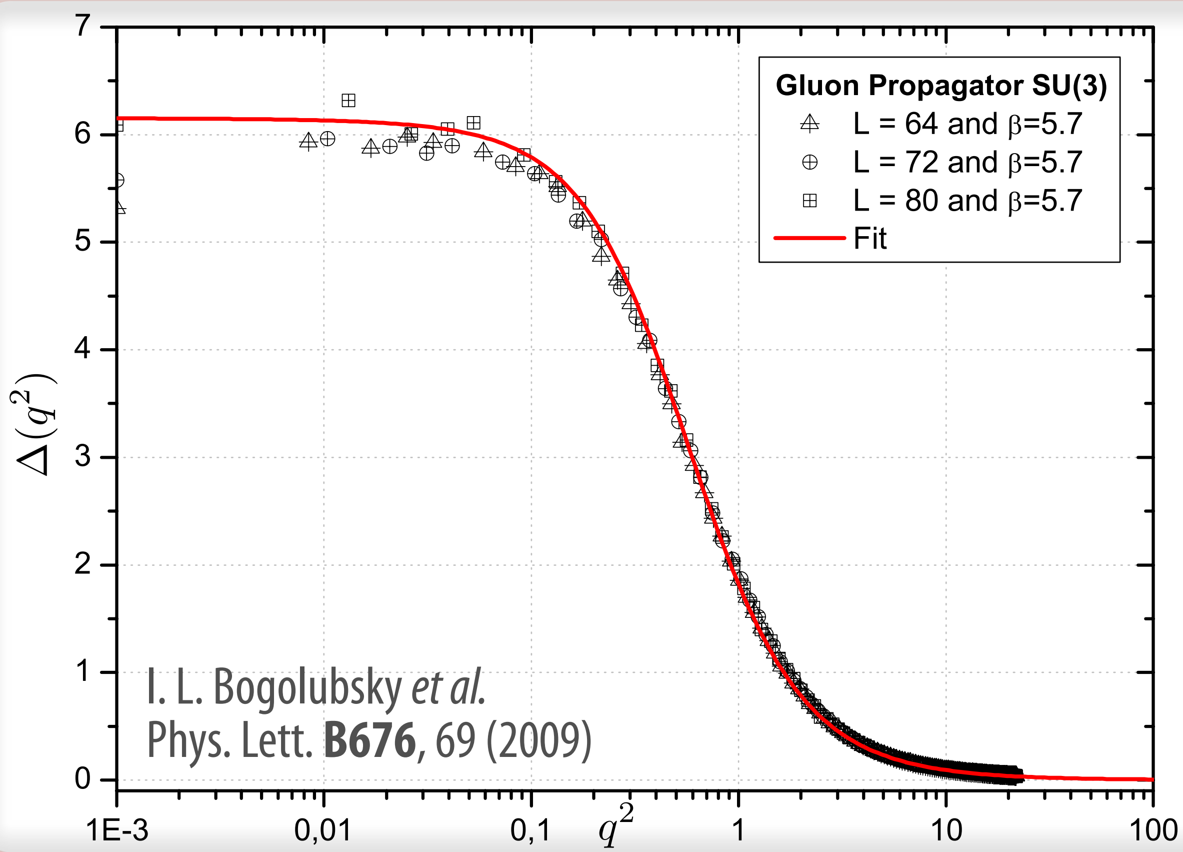
Applications

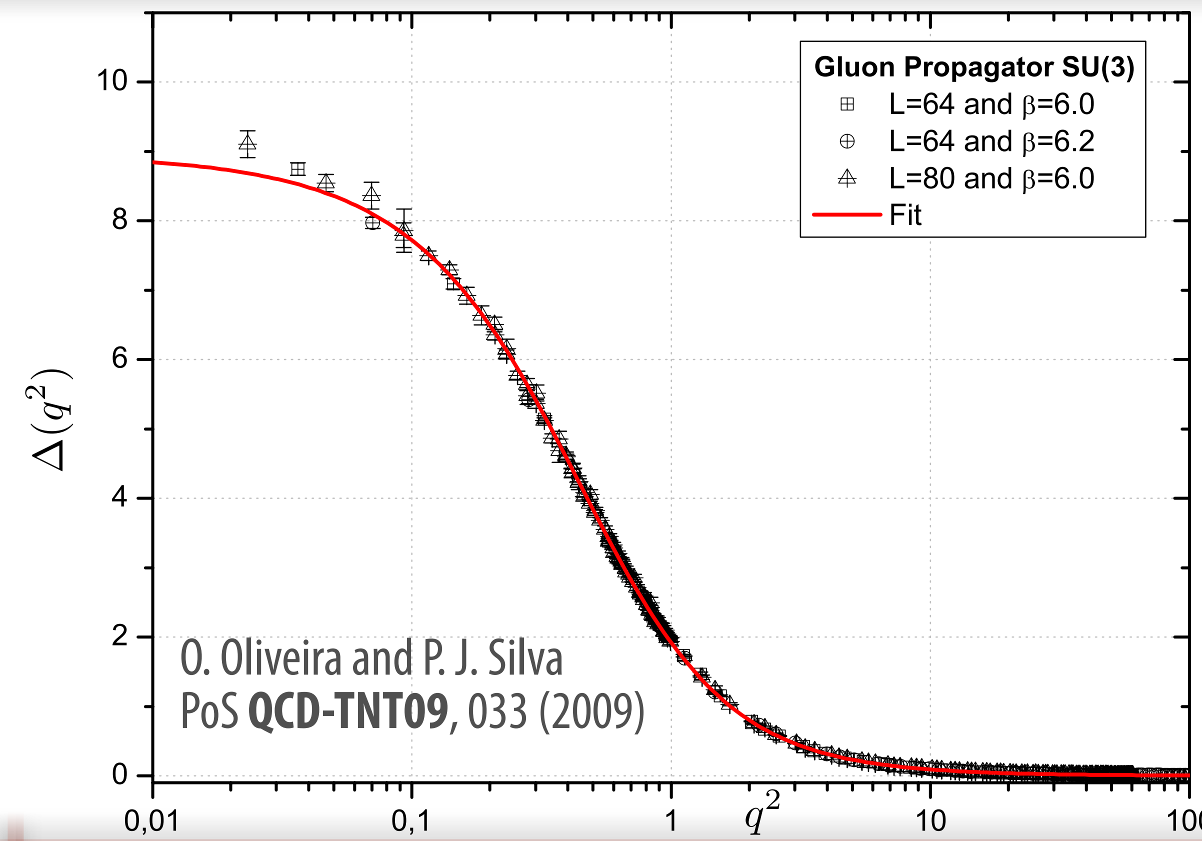
- Gluon and ghost propagators in 4 dimensions
- Gluon and ghost propagators in 3 dimensions
- Kugo-Ojima function
- QCD effective charge

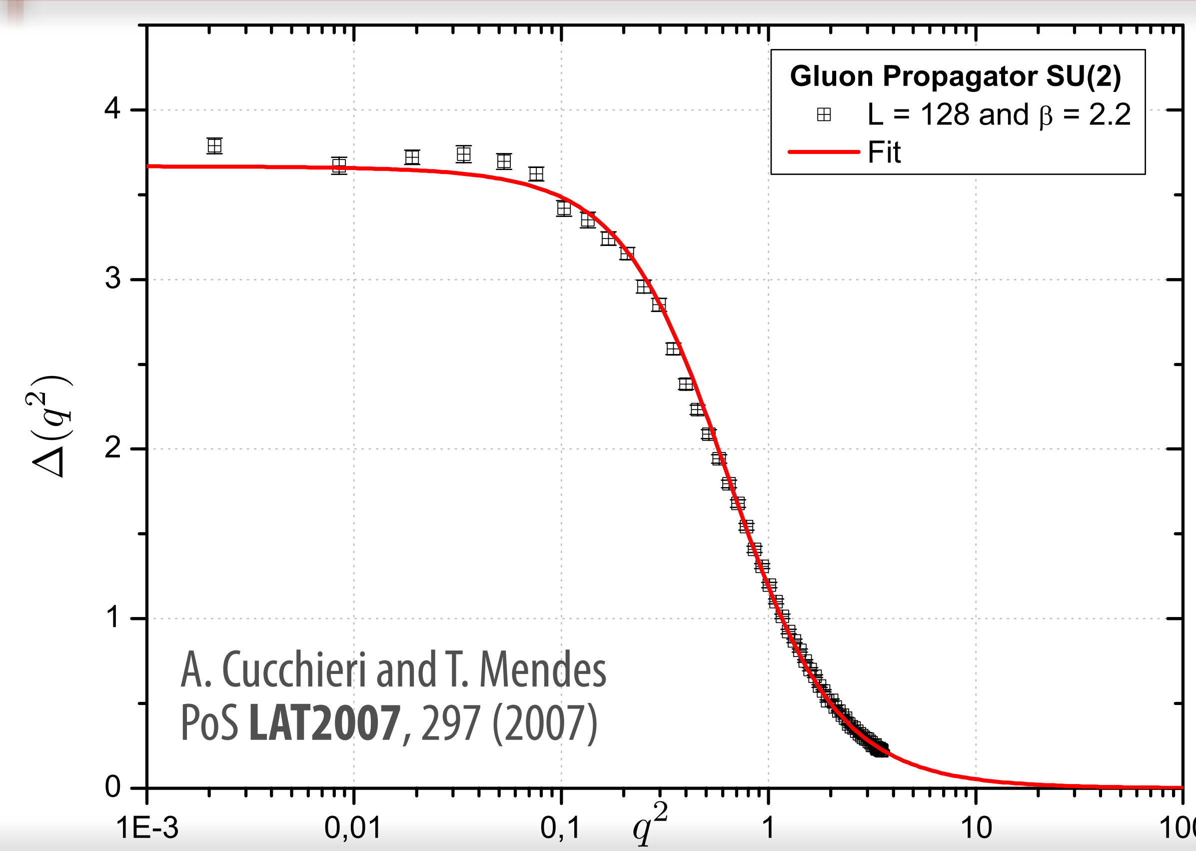
Conclusions and outlook

Motivations

gluon and ghost propagators from lattice QCD

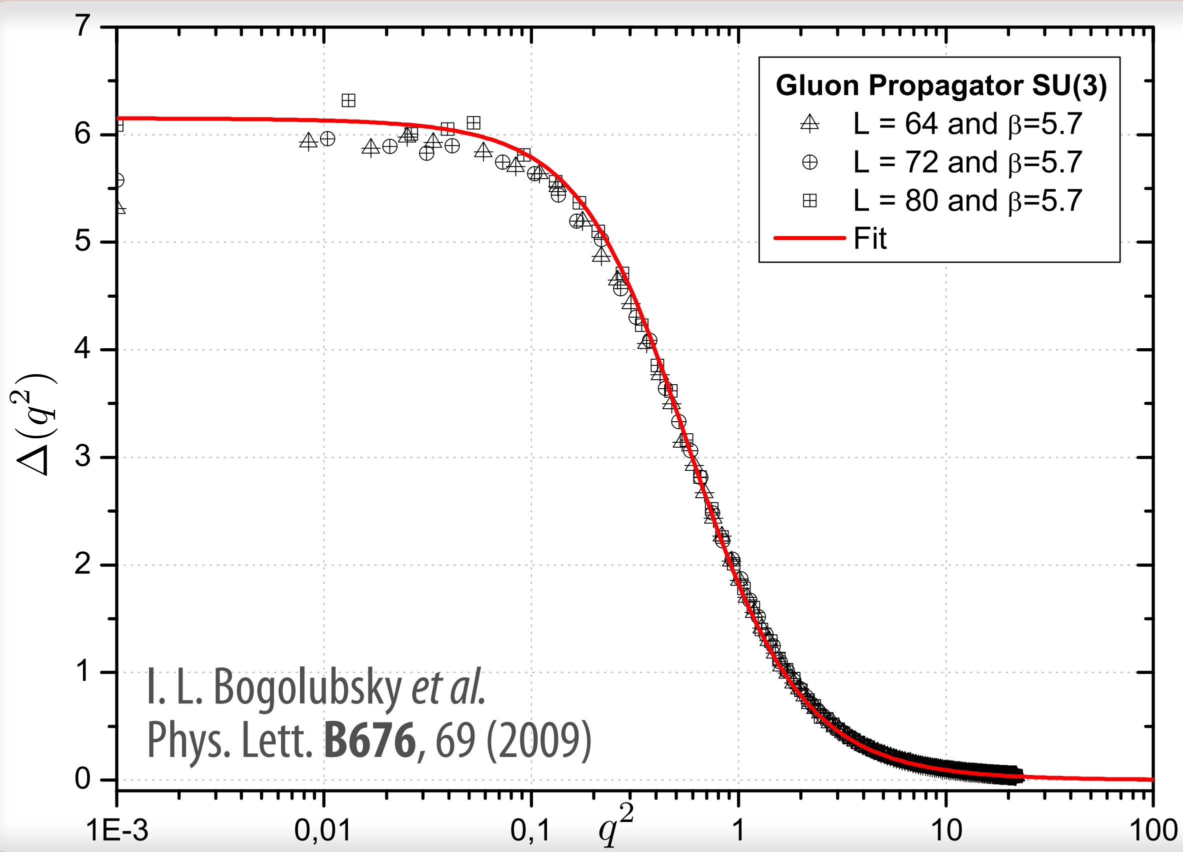


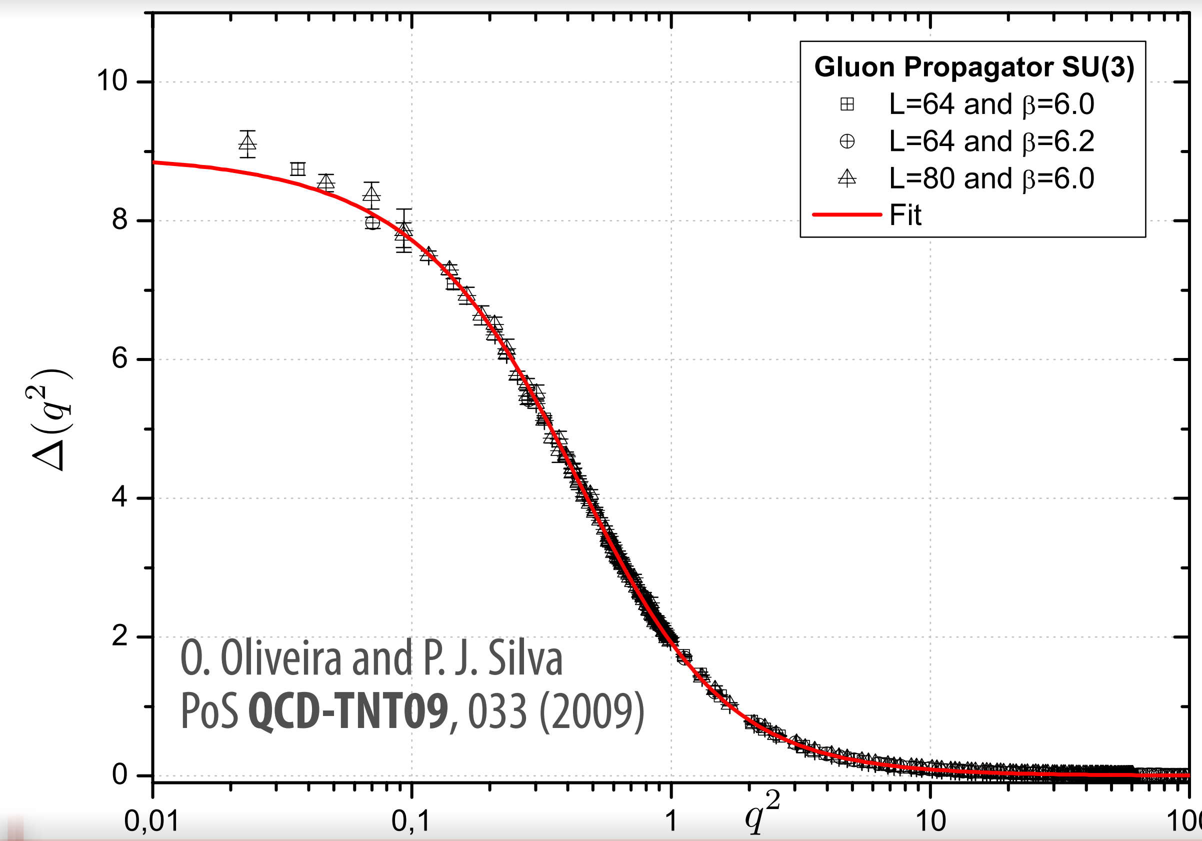


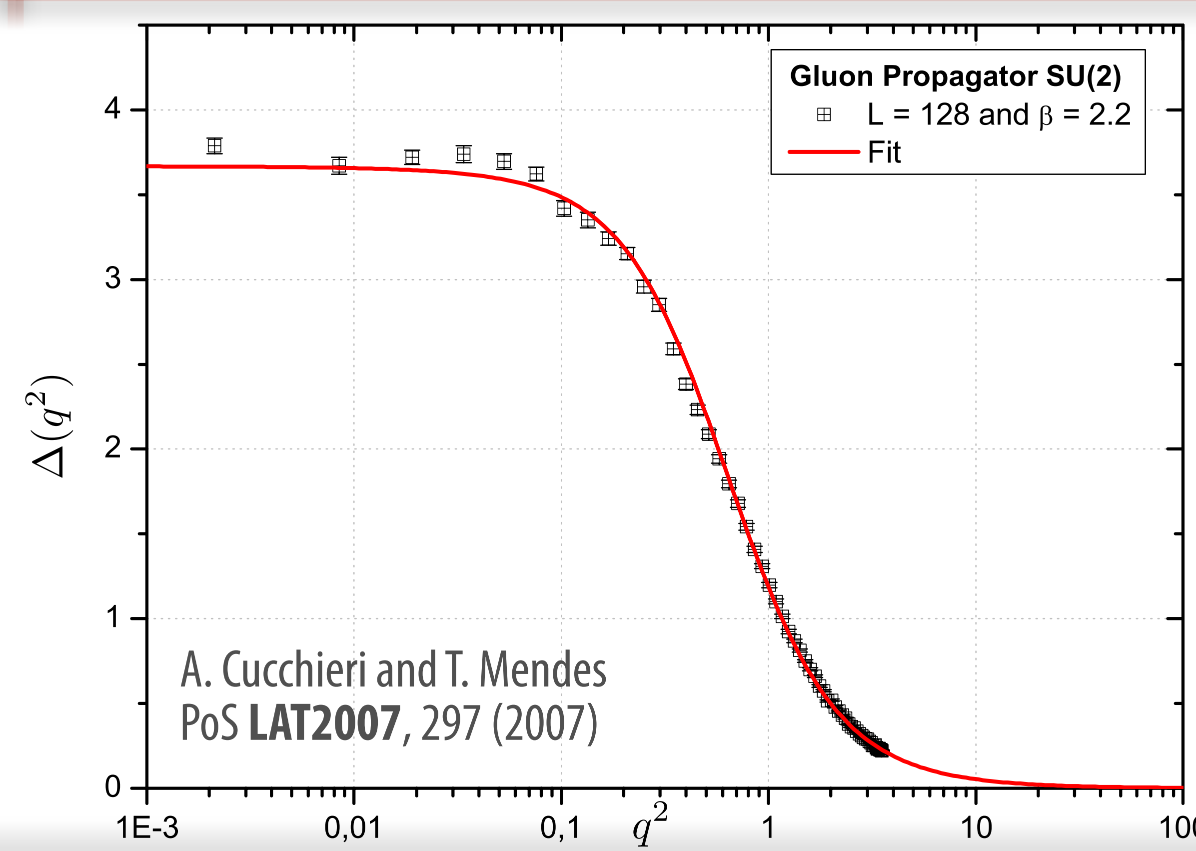


gluon and ghost propagators from lattice QCD

- **Looks like a massive propagator**





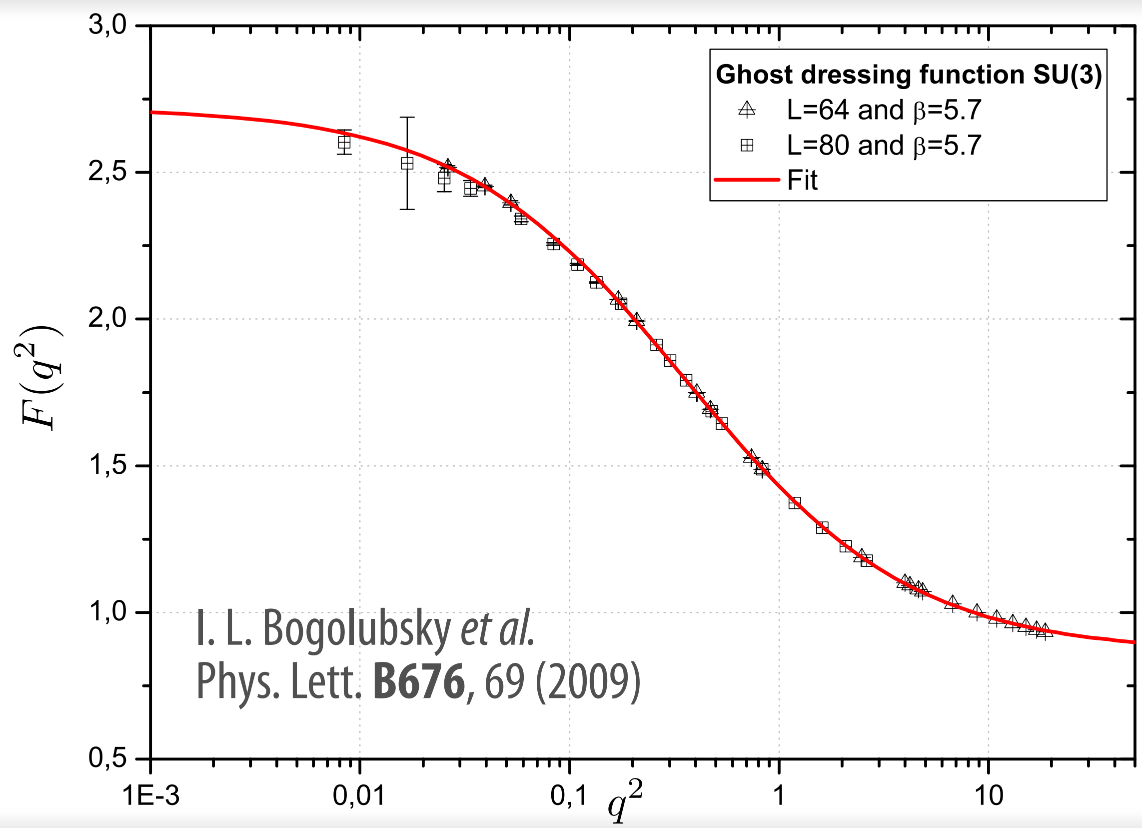


- **Fit by the following function**

$$\Delta^{-1}(q^2) = m^2 + q^2\left[1 + \frac{13C_A g_f^2}{96\pi^2}\ln\left(\frac{q^2+\rho m^2}{\mu^2}\right)\right]$$

Ph. Boucaud *et al.*, Phys. Rev. **D74**, 034505 (2006)

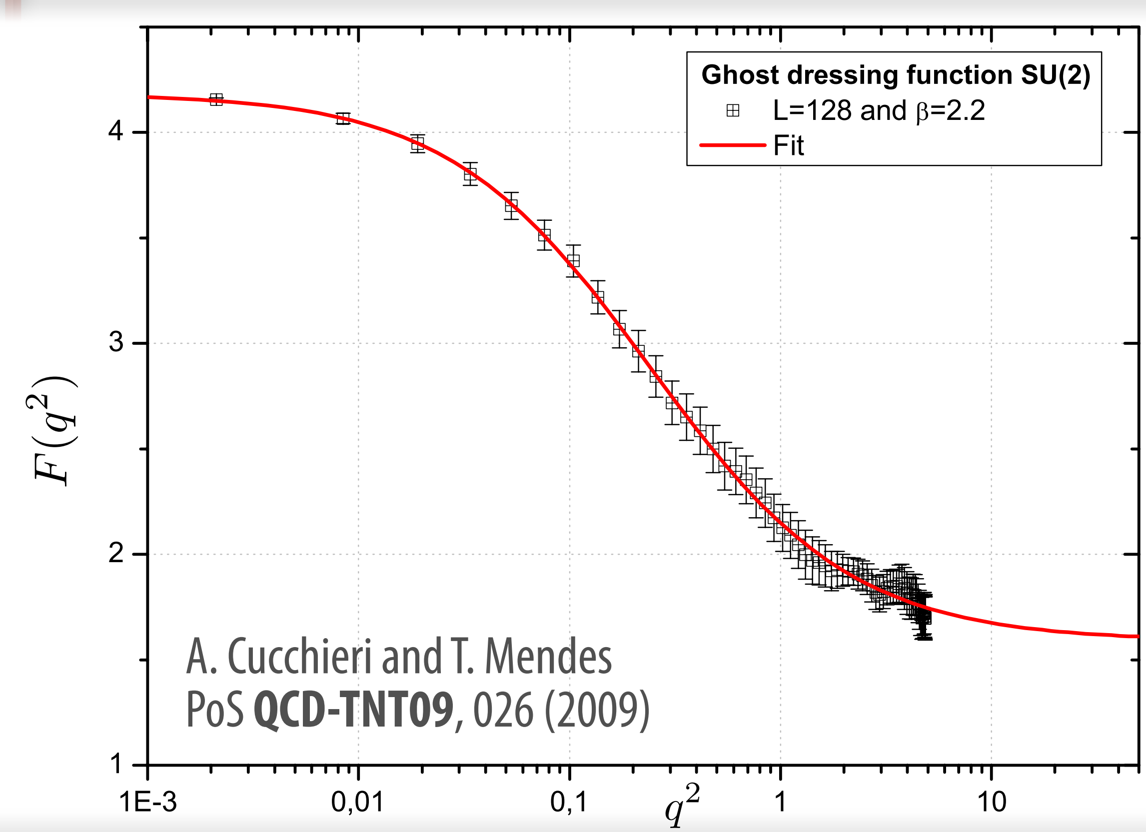
Ghost dressing function $F(q^2)$ saturates ($D=F/q^2$)



- Ghost **propagator** is still **IR divergent** (*i.e.*, there is **no ghost mass**)
- **Ghosts** seem to **play** a **marginal role** in the game (as opposed to ghost-dominance)

R. Alkofer, L. von Smekal, C. Fisher, A. Maas,. . .

- More on this in the talk of J. Rodriguez-Quintero



gluon and **ghost propagators from**
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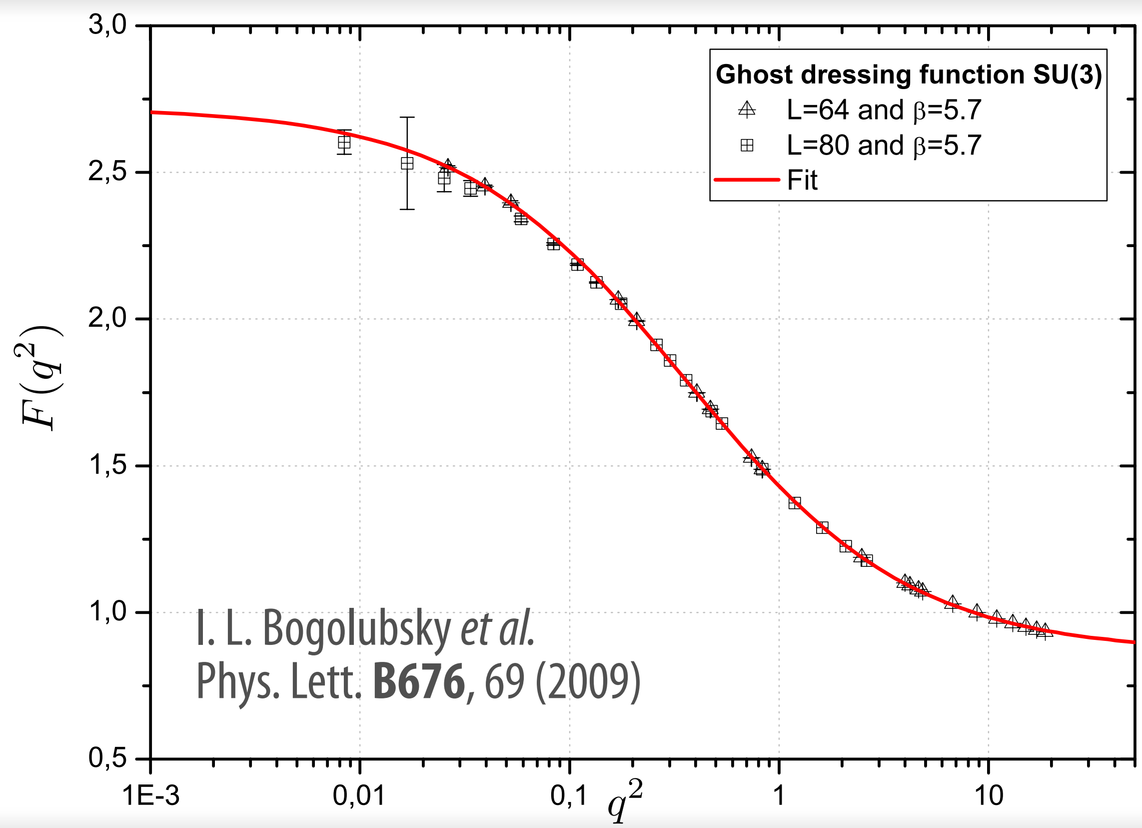

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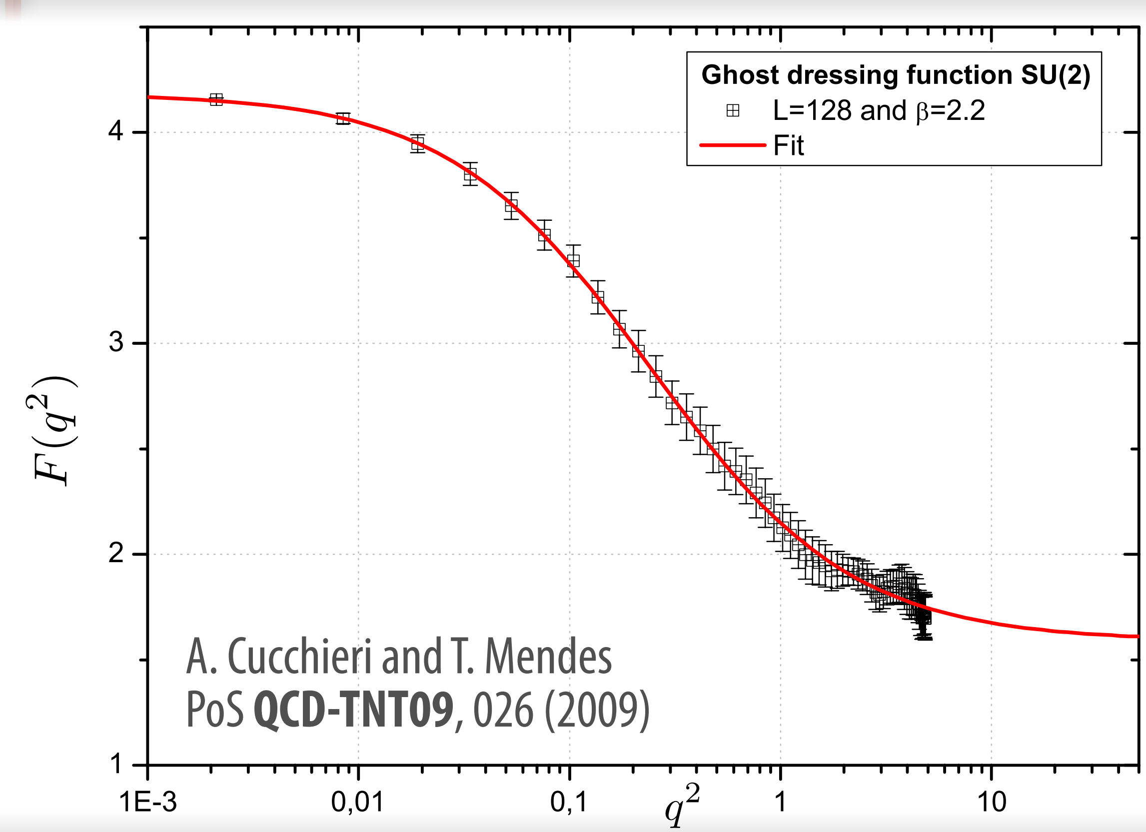


Fit by the following function

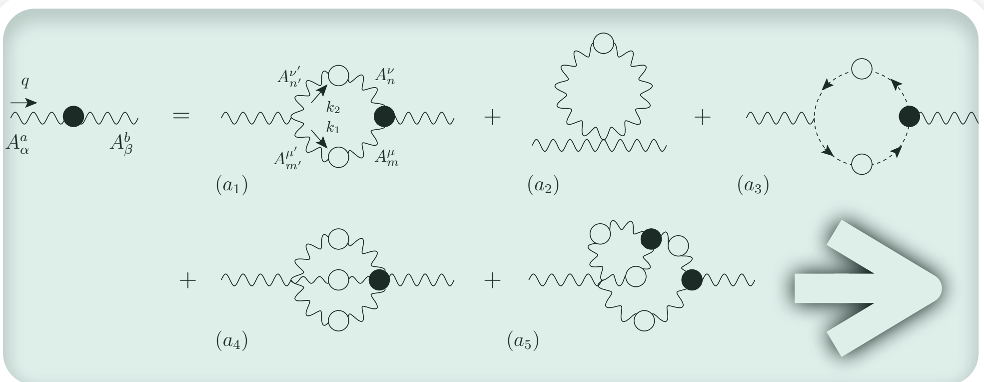
$$F(p^2) = \frac{a_1 - a_2}{1 + (p^2/p_1^2)^\gamma} + a_2$$

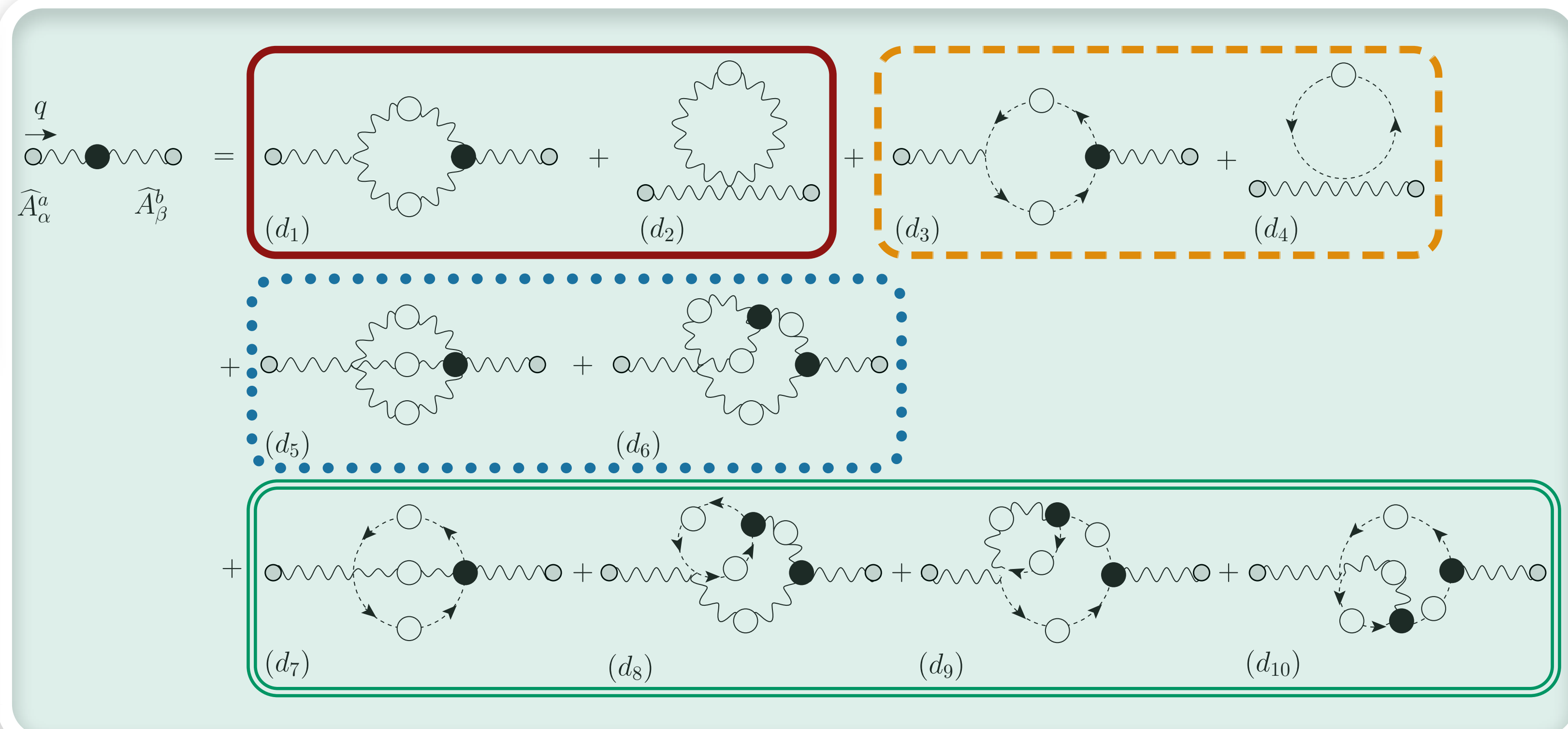
New Schwinger-Dyson eqs

PT-BFM:
new Schwinger-Dyson series

D. B. and J. Papavassiliou, Phys. Rev. **D77**, 061702(R) (2008)
D.B. and J. Papavassiliou, JHEP **11**, 063 (2008)

- Apply the pinch technique to the Schwinger-Dyson eq of the gluon propagator





- Diagrams made out of **new vertices**, but have **inside** the **conventional propagator**
- New vertices corresponds to **BFM vertices**
  - external gluons have been dynamically converted into **background gluons**

- The new Schwinger-Dyson equation has a **special structure**
  - **Subgroups** (one-/two-loop dressed gluon/ghost diagrams) are **individually transverse**

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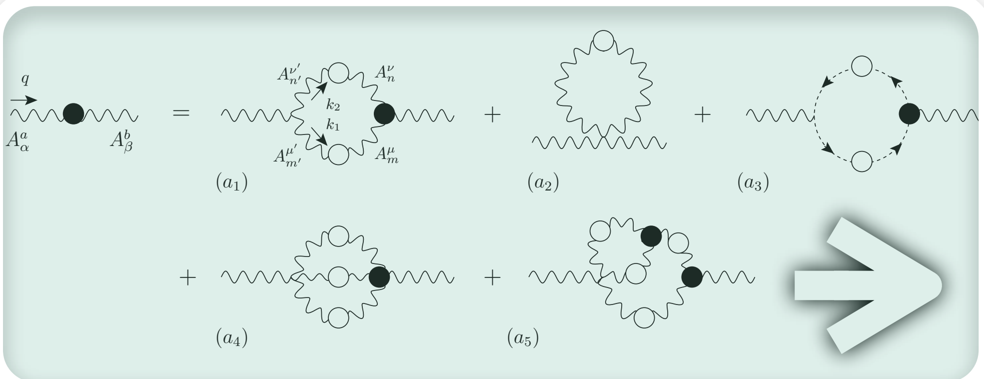

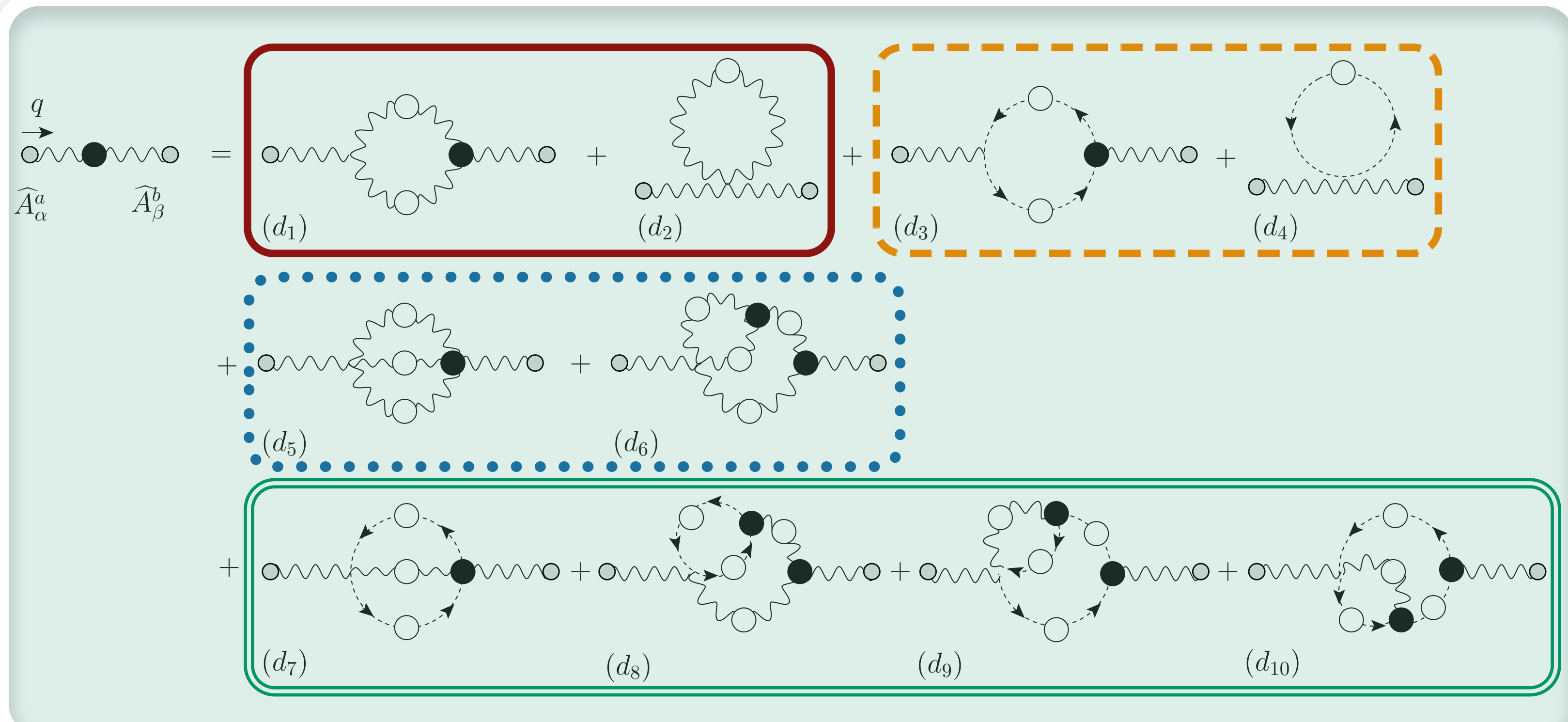

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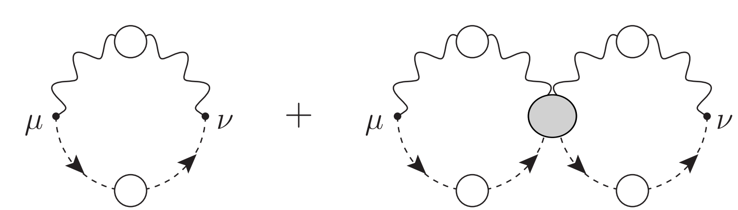
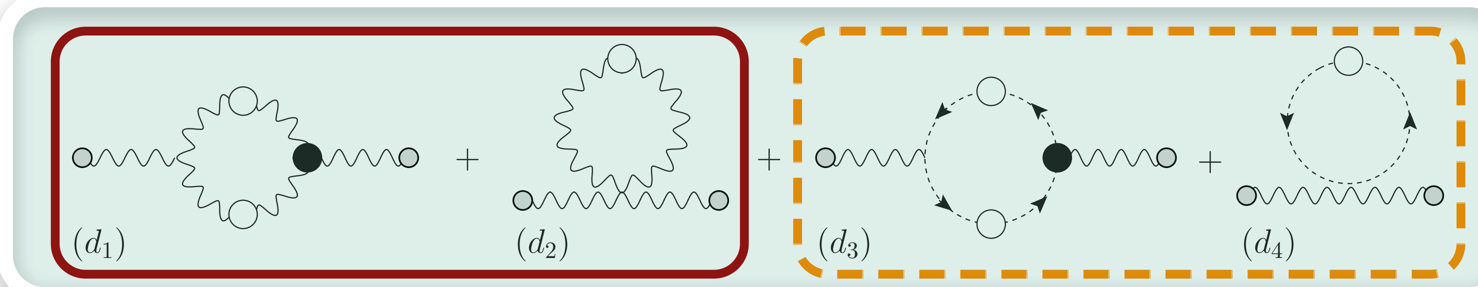
Problem

Not a genuine Schwinger-Dyson equation (**mixes conventional and BFM propagators**)

- Express the **Schwinger-Dyson equation** in terms of a **Background-Quantum identity**
  - $\widehat{\Delta}(q^2) = [1+G(q^2)]^{-2}\Delta(q^2)$
    $\Delta^{-1}(q^2)[1+G(q^2)]^2 P_{\mu\nu}(q) = q^2 P_{\mu\nu}(q) + \sum_{i=1}^{10}(d_i)_{\mu\nu}$
  - $\Lambda_{\mu\nu}(q) =$  $= ig_{\mu\nu}G(q^2)+\cdots$

PT-BFM: new Schwinger-Dyson series



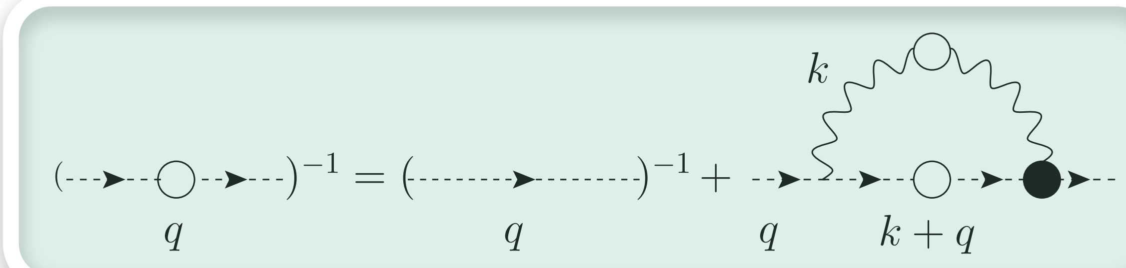
Final equations

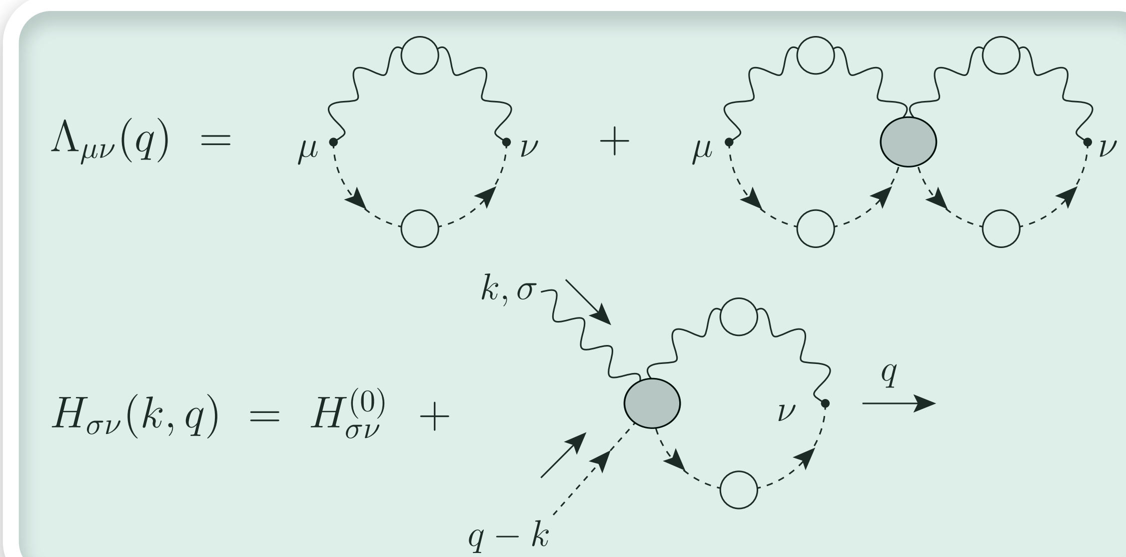
- Consider **one-loop dressed** diagrams **only**
  - **ghost self-energy** equation must be included

$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + \mathrm{i}\sum_{i=1}^{4}(d_i)_{\mu\nu}}{[1+G(q^2)]^2}$$

$$\mathrm{i}D^{-1}(q^2) = q^2 + \mathrm{i}\lambda\int_k \Gamma^{\mu}\Delta_{\mu\nu}\mathbf{\Gamma}^{\nu}(q,k)D(p+k)$$

$$\mathrm{i}\Lambda_{\mu\nu} = \lambda\int_k H^{(0)}_{\mu\rho}D(k+q)\Delta^{\rho\sigma}(k)H_{\sigma\nu}(k,q) = \mathrm{i}g_{\mu\nu}G(q^2) + \mathrm{i}\frac{q_\mu q_\nu}{q^2}L(q^2)$$





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  - contain the **massless poles**
- Keep **normal** ghost vertex at tree-level (consistent with lattice results in d=4)
- One-loop G (tree-level H)

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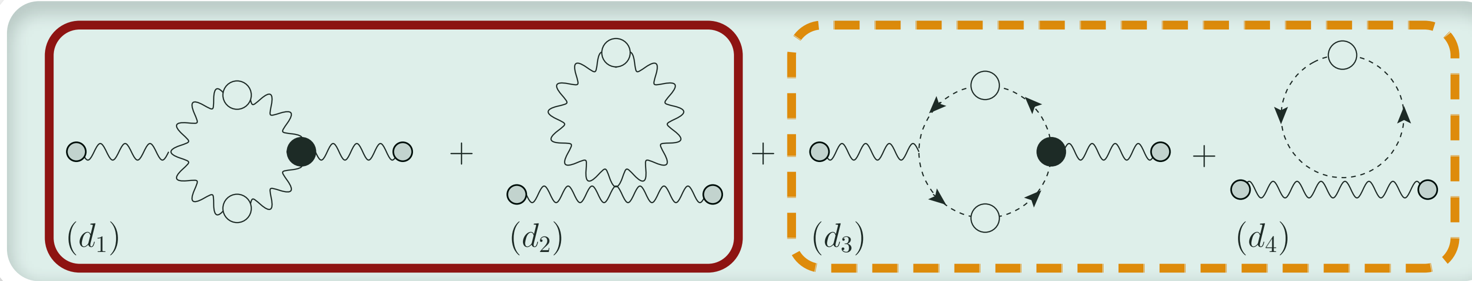

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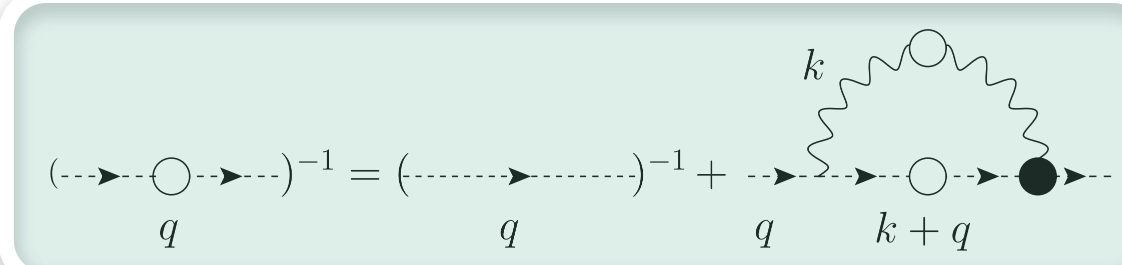

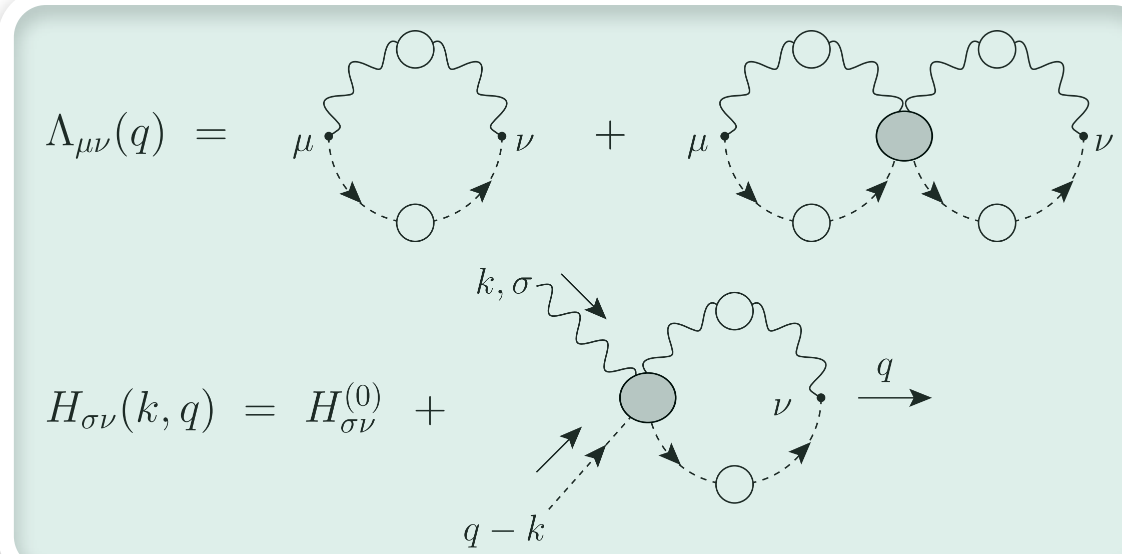

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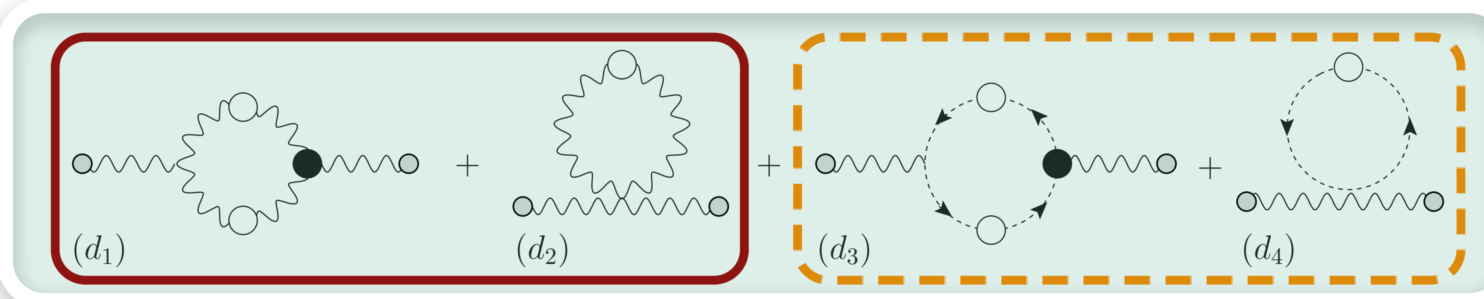
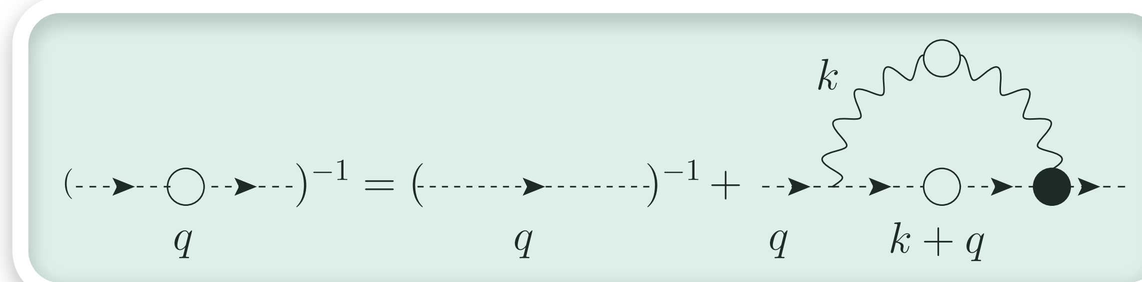
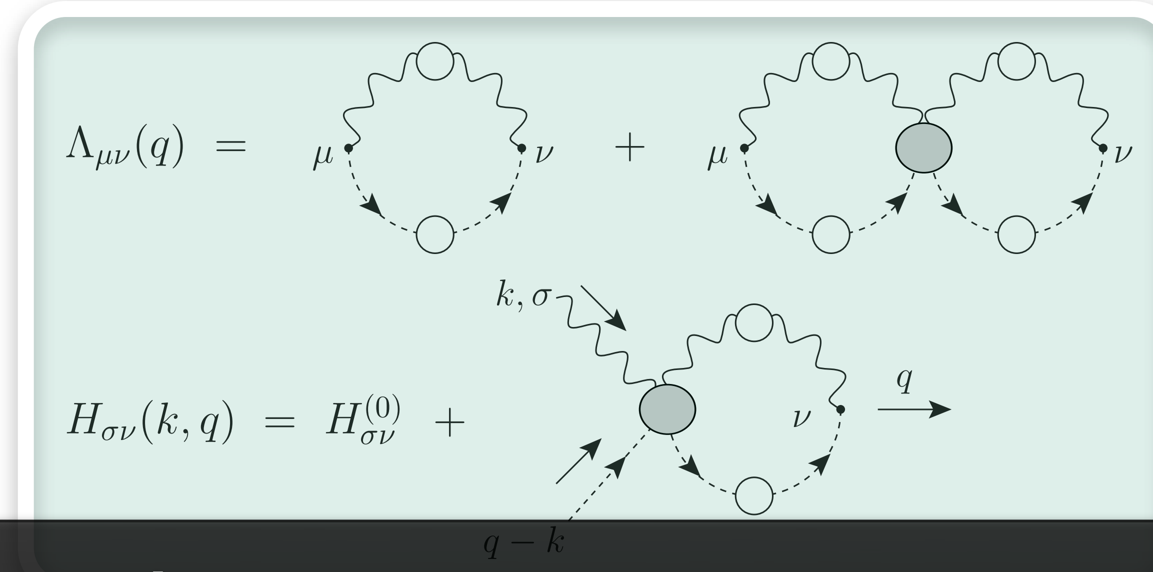






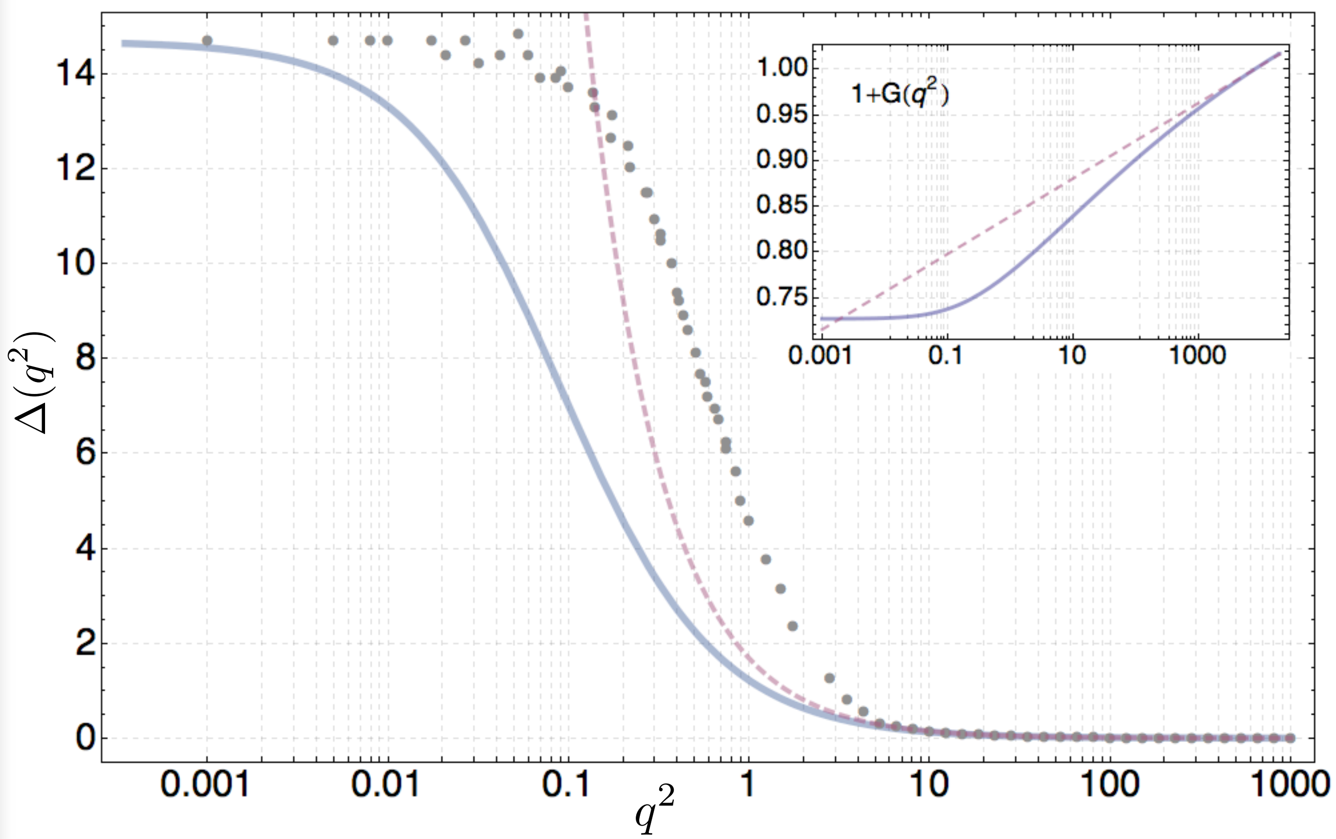
Very flexible framework

- Can **add/remove group** of diagrams **gauge invariantly**
- **Gauge invariance preserved at each step** (transverse projectors can be traced out)
- **Different options for truncation** (always retaining exact gauge invariance)
- Modest price to pay: **additional equation** for $G(q^2)$ (but this function is very worth the effort)
- **PT-BFM** and **conventional vertices** can be **treated independently**
- **Allow the gauge invariant study of ghost contributions**

Applications

application II

gluon and ghost propagators (d=4)



A. C. Aguilar, D. B. and J. Papavassiliou, Phys. Rev. **D78**, 02510 (2008)
SU(3) lattice data: I. L. Bogolubsky *et al.* Phys. Lett. **B676**, 69 (2009)

Solution of the 4d system

- Three unknowns: G, Δ and D (or F)
- IR finiteness condition

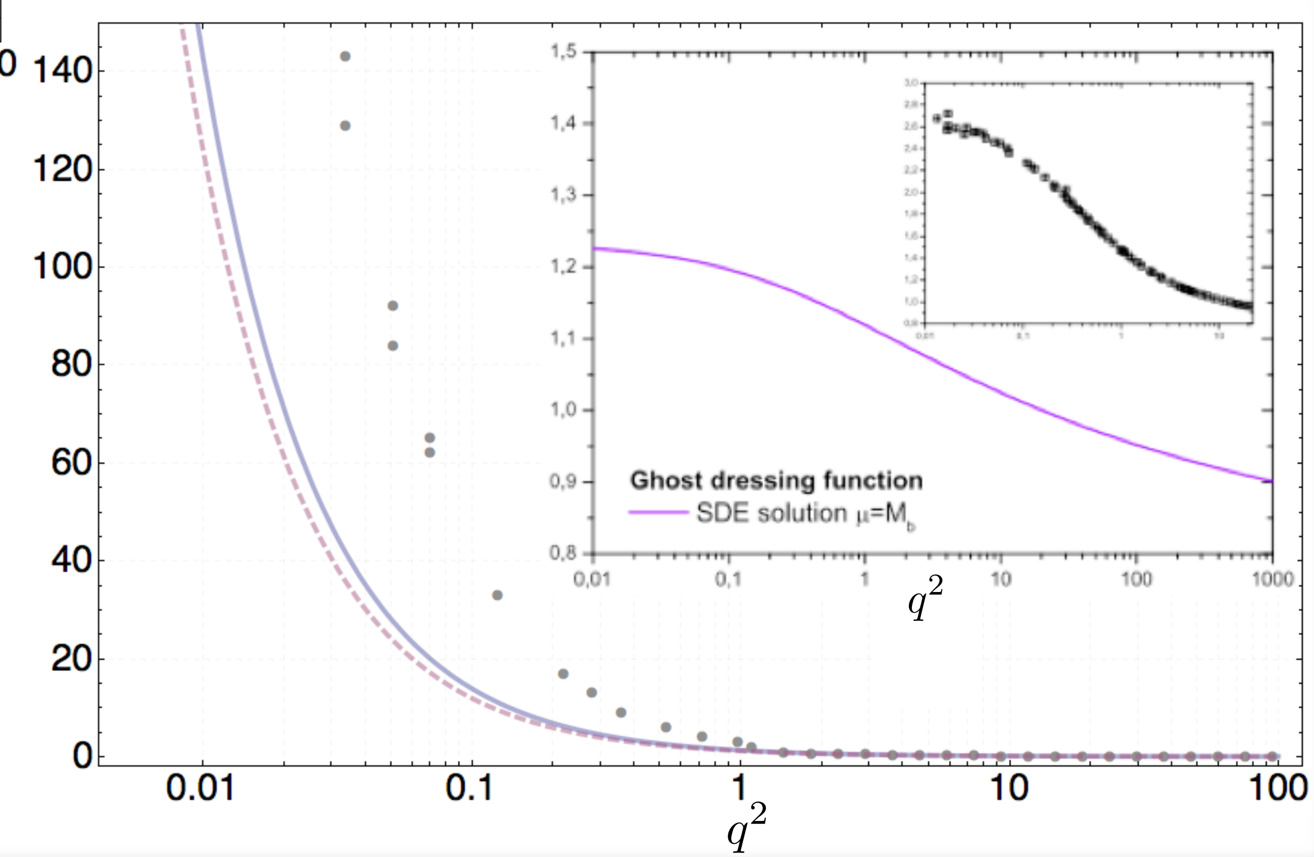
$$\lim_{q^2\to 0} \Delta^{-1}(q^2) = \Delta^{-1}_{\text{lat}}(0)$$

[4d SU(3)] gluon and ghost propagator solutions

- ✓ (almost) **Parameter-free** calculation
- ✓ **Massive** (decoupling) **solutions found**
- ✓ Qualitative **agreement** with lattice simulations
  - ✓ **IR finite gluon propagator**
  - ✓ **IR finite ghost dressing function**

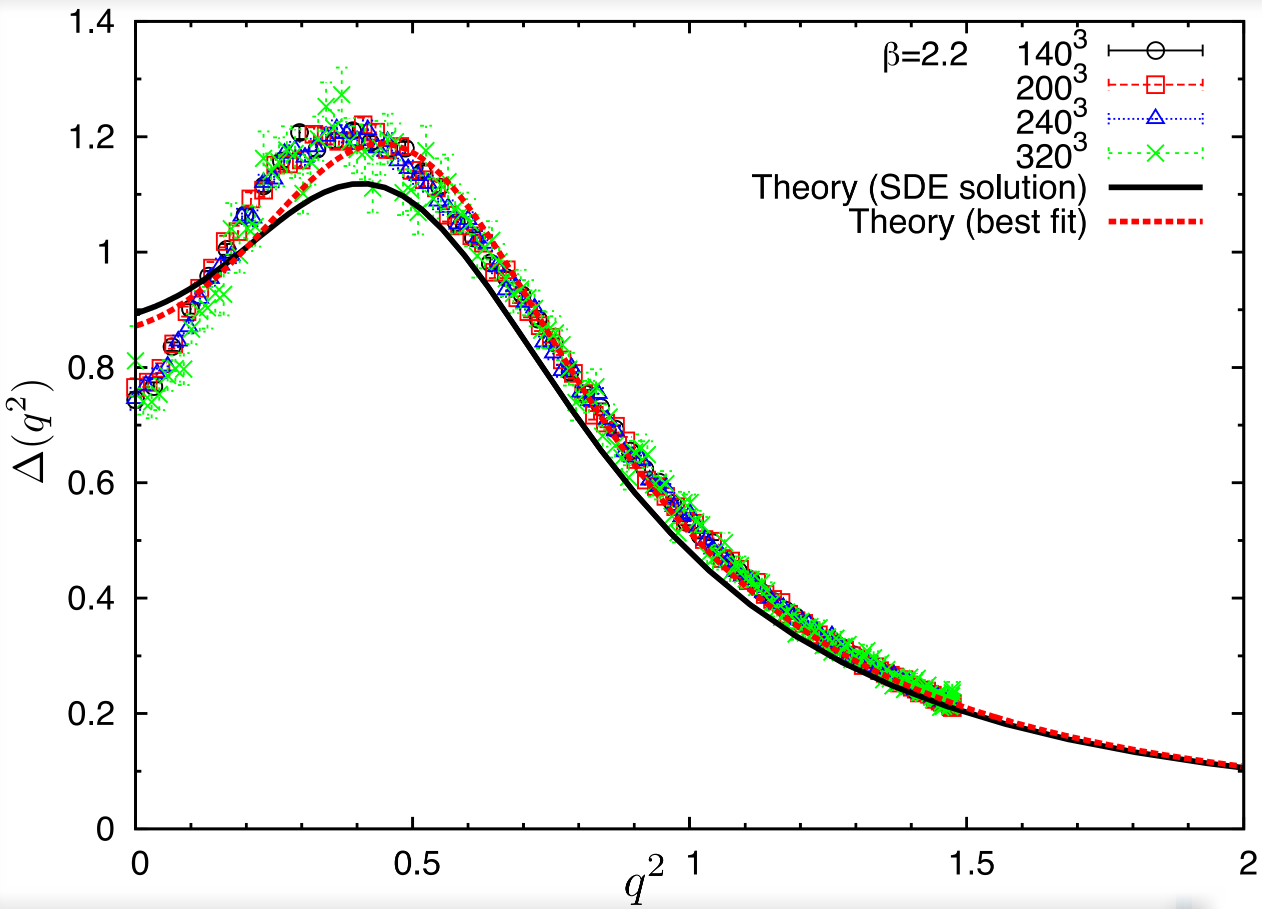
Better agreement could be achieved

- ✳ Mainly going **beyond tree-level** approximations
  - ✳ normal ghost vertex (D)
  - ✳ H auxiliary function (G)



application II

gluon and ghost propagators (*d*=3)



A. C. Aguilar, D. B. and J. Papavassiliou, arXiv:**1004.2011** [hep-th] (2010)
SU(2) lattice data: A. Cucchieri and T. Mendes, PoS **QCD-TNT09**, 026 (2009)

Solution of the 3d system

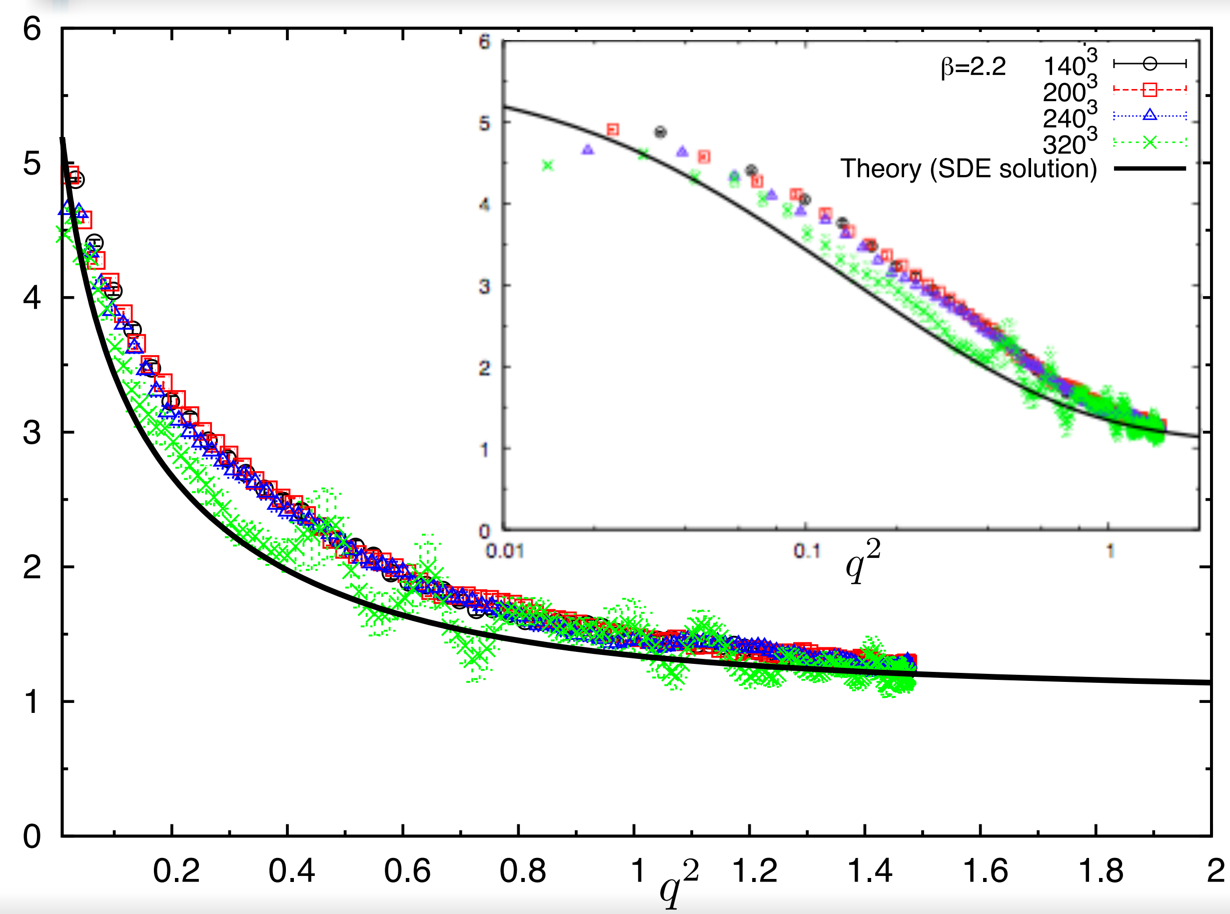
- One-loop approximations for Δ with a tree-level (hard mass)
- Full ghost equation for D (or F)
- More on ghost/gluon propagators in Dudal's and Wschebor's talk

[3d SU(2)] gluon and ghost propagator solutions

- ✓ One-loop (plus tree-level mass) gives good results
- ✓ **Massive** (decoupling) **solution** again present
- ✓ Qualitative **agreement** with lattice simulations
  - ✓ **IR finite gluon propagator**
  - ✓ **IR finite ghost dressing function**

We can do **better**

- ✳ Solve the full system as in the 4d case
  - ✳ expected to be easier than in 4d (integrals are convergent)



application III

G (and L) from the lattice

A. C. Aguilar, D. B. and J. Papavassiliou, JHEP **011**, 066 (2009)

Correct physics encoded in the (truncated) new SDEs

- Use available lattice data to study the behavior of the auxiliary functions G and L, and the QCD effective charge

Dynamical equations

- $G(q^2) = \frac{\lambda}{3}\int_k \left[2 + \frac{(k\cdot q)^2}{k^2q^2}\right]\Delta(k)D(k+q)$
- $L(q^2) = \frac{\lambda}{3}\int_k \left[1 - 4\frac{(k\cdot q)^2}{k^2q^2}\right]\Delta(k)D(k+q)$

- **BRST symmetry** gives (in the **Landau gauge**)
  - $F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$

the **ghost dynamics** is **fully constrained**

Under very general conditions on the gluon and ghost propagators

$$L(0) = 0$$

- In the deep IR limit

$$F^{-1}(0) = 1 + G(0)$$

then **if** $G(0) = -1 \implies F(0) \to \infty \implies$ **ghost dominance**

- **However**
  - Lattice shows a finite F
  - Theoretical arguments suggest $G(0) \simeq -\frac{2}{3}$

K-I. Kondo, Phys. Lett. **B678**, 322 (2009)

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K-I. Kondo, Phys. Lett. **B678**, 322 (2009)

- The condition $G(0) = -1$ looks like the Kugo-Ojima confinement criterion $u = -1$

$$\int d^4x\, \mathrm{e}^{-iq\cdot(x-y)}\langle T\left[\,(\mathcal{D}_\mu c)^m_x\,(\mathcal{D}_\mu \bar{c})^n_y\,\right]\rangle = -\frac{q_\mu q_\nu}{q^2}\delta^{mn} + P_{\mu\nu}(q)\delta^{mn}u(q^2)$$

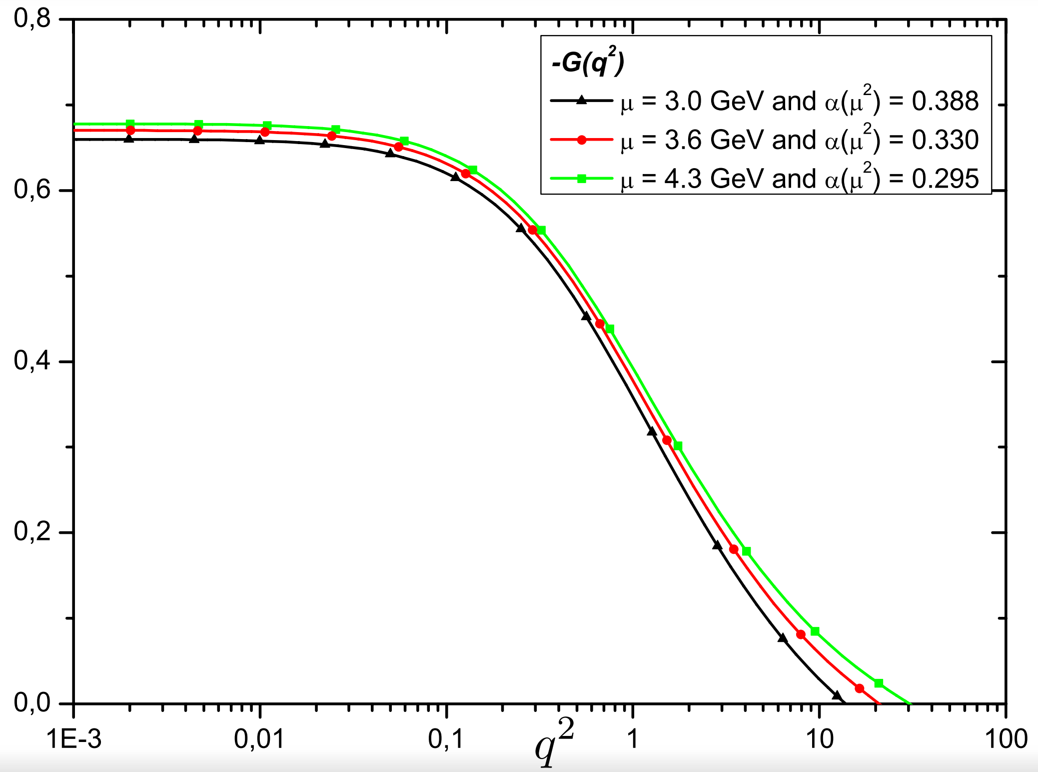
- Indeed one can show that

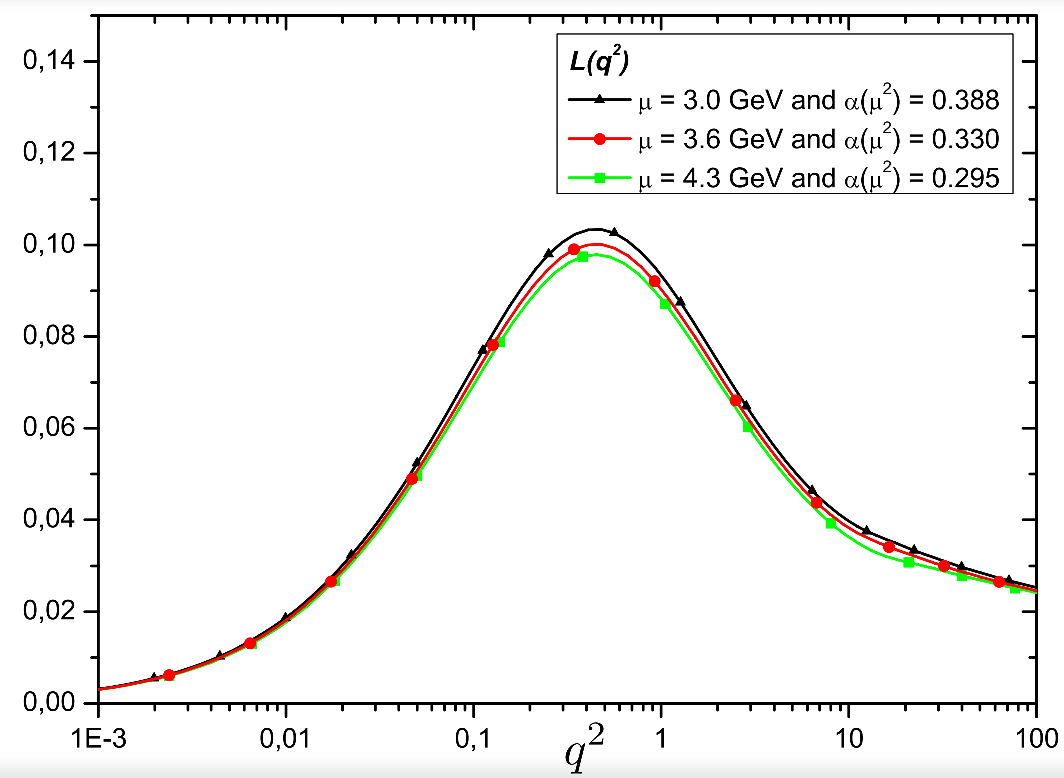
$$G(q^2) \equiv u(q^2)$$

G reveals a great deal about the QCD IR sector

application III
G (and L) from the lattice

- Solve the equation for G and L using the lattice data as input





- Solutions' features
  - In 4d L is subdominant for all momenta
  - Indeed one finds $G(0) \simeq -\frac{2}{3}\ldots$
  - ... but G is μ dependent: $G(0) = G(0;\mu^2)$
  - **No evidence for ghost dominance**

application III

G (and L) from the lattice

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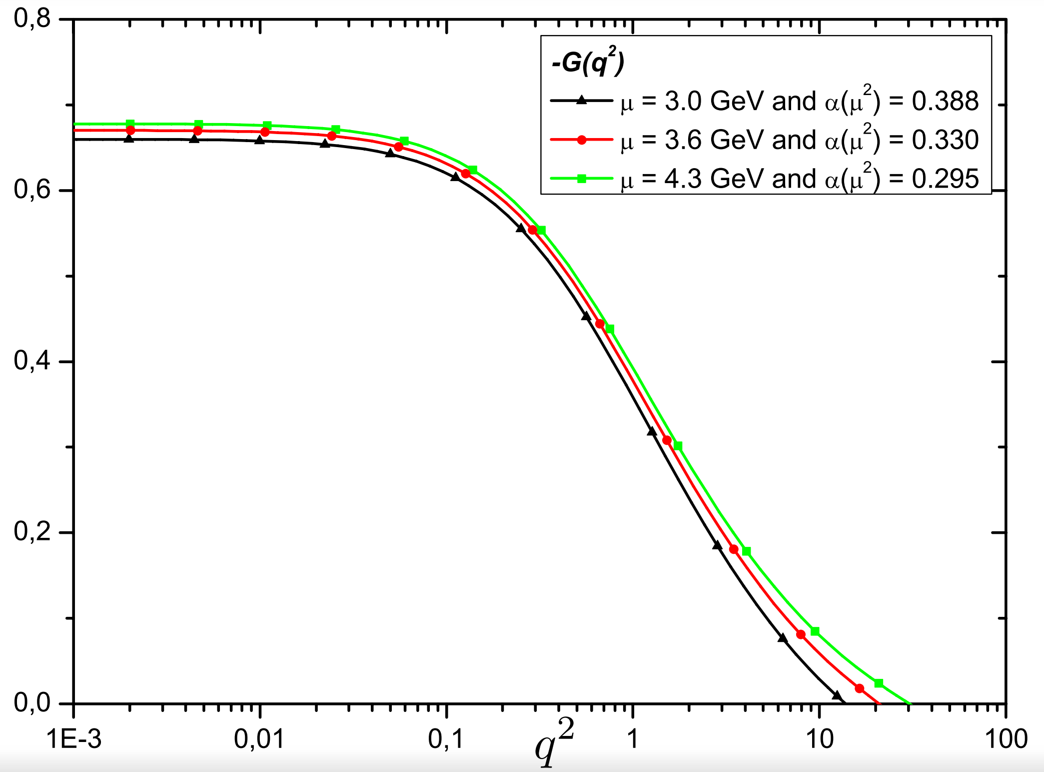
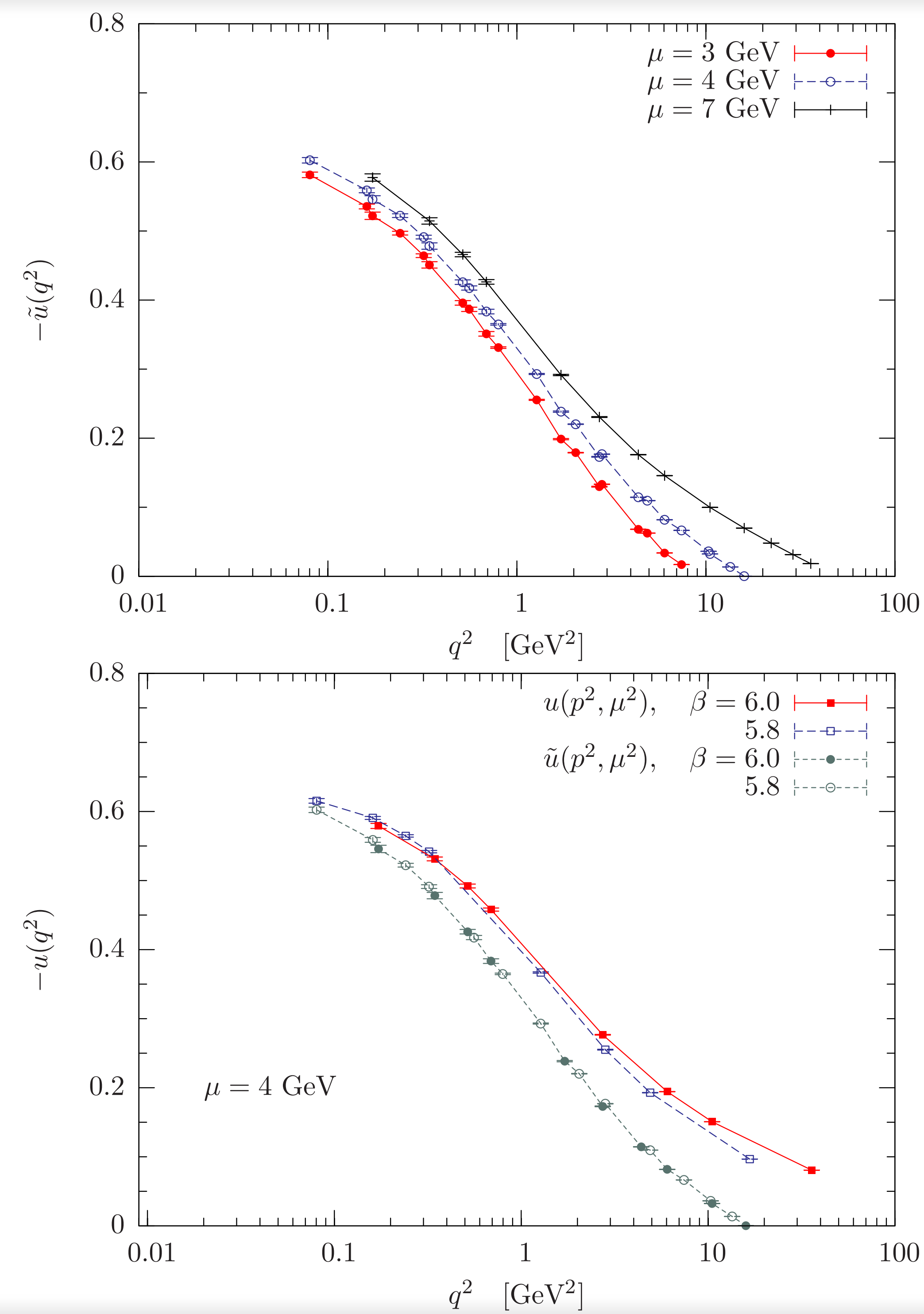


A. Sternbeck, arXiv:hep-lat **0609016** (2006)



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- ... but G is μ dependent: $G(0) = G(0; \mu^2)$
- **No evidence for ghost dominance**
- **Can compare with direct lattice measurement of u in terms of Monte Carlo averages**

G (and L) from the lattice

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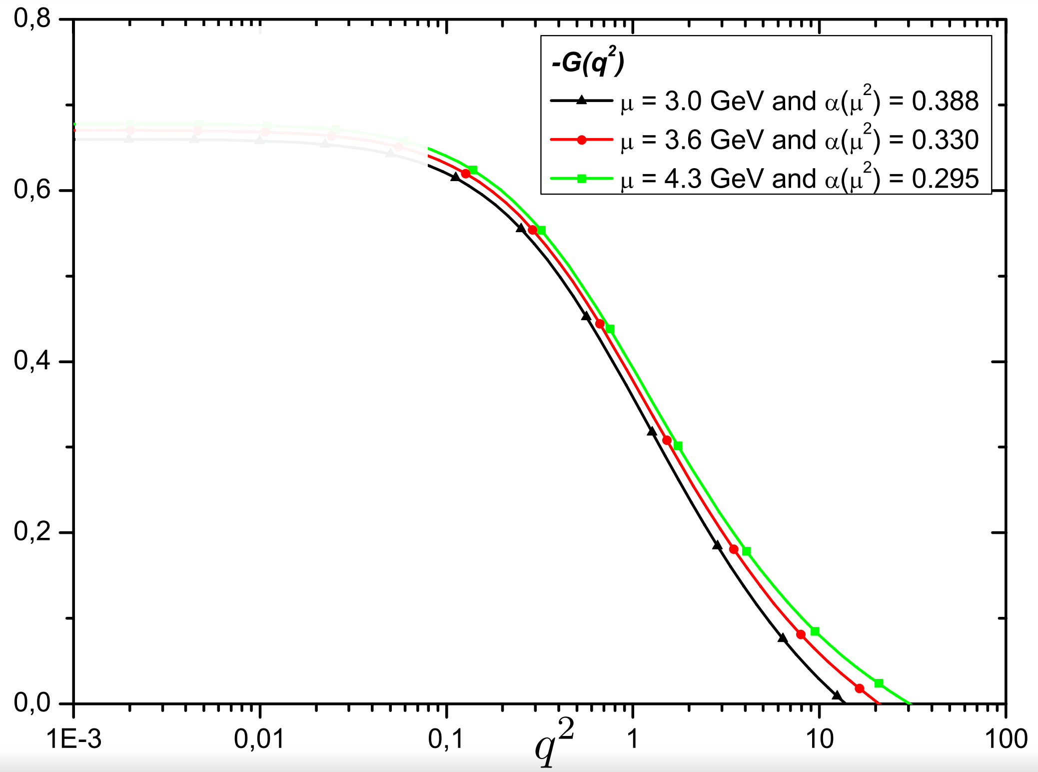
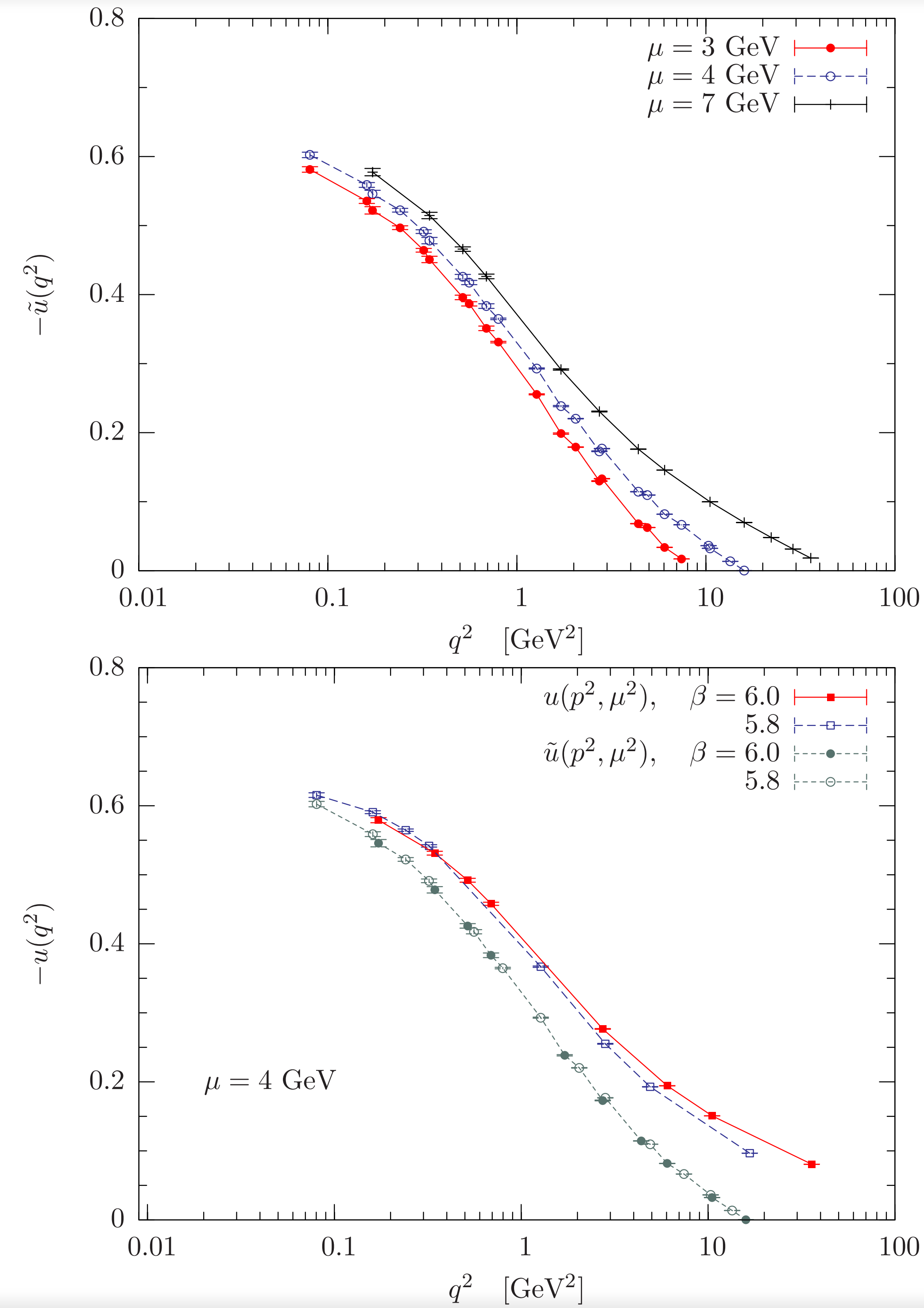




Solutions' features

- In 4d L is subdominant for all momenta
- Indeed one finds $G(0) \simeq -\frac{2}{3} \ldots$
- ... but G is μ dependent: $G(0) = G(0; \mu^2)$
- **No evidence for ghost dominance**
- **Can compare with direct lattice measurement of u in terms of Monte Carlo averages**

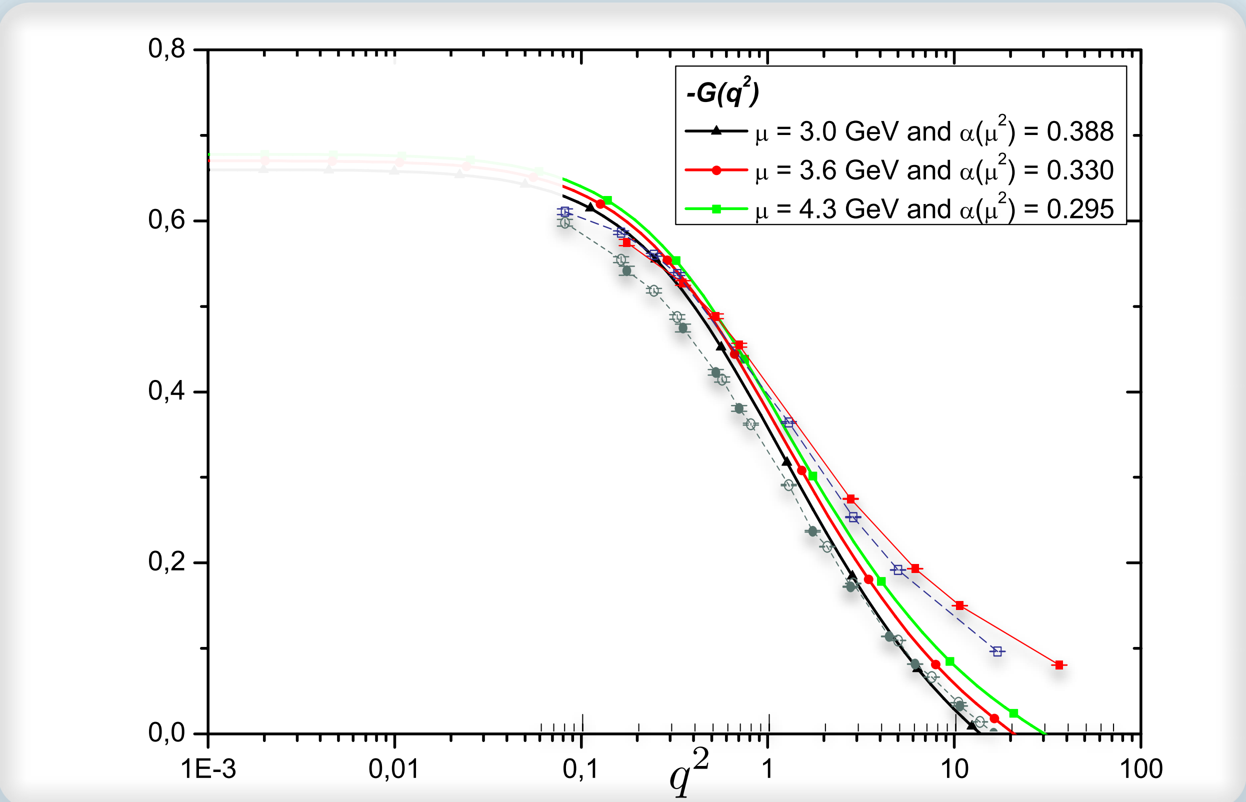
application III

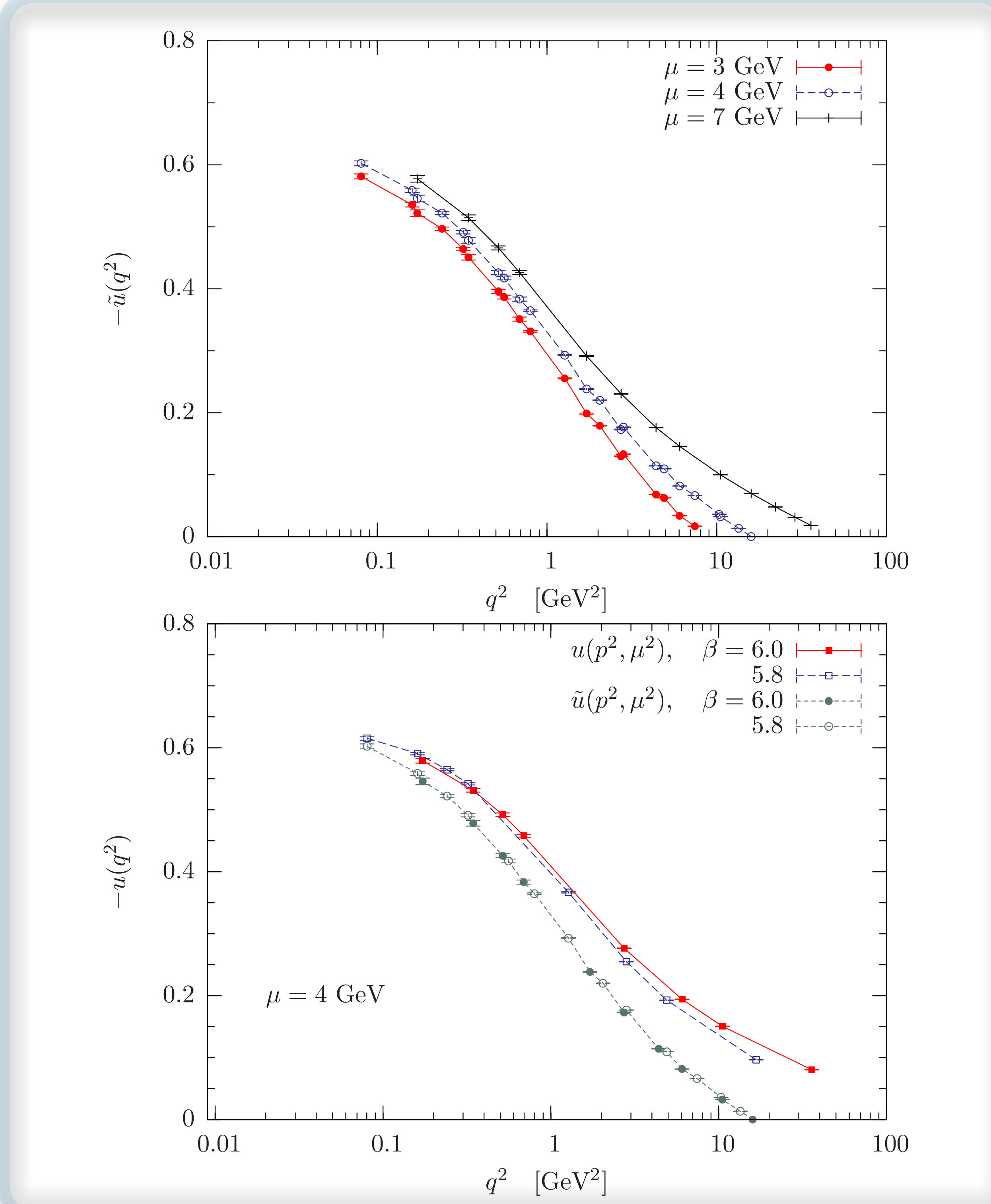
G (and L) from the lattice

LC2010 - Valencia

Solve the equation for G and L

A. Sternbeck, arXiv:hep-lat **0609016** (2006)





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application IV
effective charge

A. C. Aguilar, D. B., J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. **D80**, 085018 (2009)
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- Use the quantum-background identity

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one-loop

$$1+G(q^2;\mu^2) = 1+\frac{9}{4}\frac{C_A g^2}{48\pi^2}\log\frac{q^2}{\mu^2}$$
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$$\alpha(q^2) = [q^2+m^2(q^2)]\frac{\alpha(\mu^2)\Delta(q^2;\mu^2)}{[1+G(q^2;\mu^2)]^2}$$
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R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Phys. Lett. **B611**, 279 (2005)

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The two effective charges coincide in the UV and in the IR

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$$\alpha(q^2) = \left[1 + \frac{L(q^2)}{1+G(q^2)}\right]^2 \alpha_{\rm gh}(q^2)$$

- In the UV both charges reproduce asymptotic freedom
- In the deep IR : $L(0) = 0 \quad \Longrightarrow \quad \alpha(0) \equiv \alpha_{\rm gh}(0)$
- The two charges will only differ in the intermediate momentum region

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effective charge

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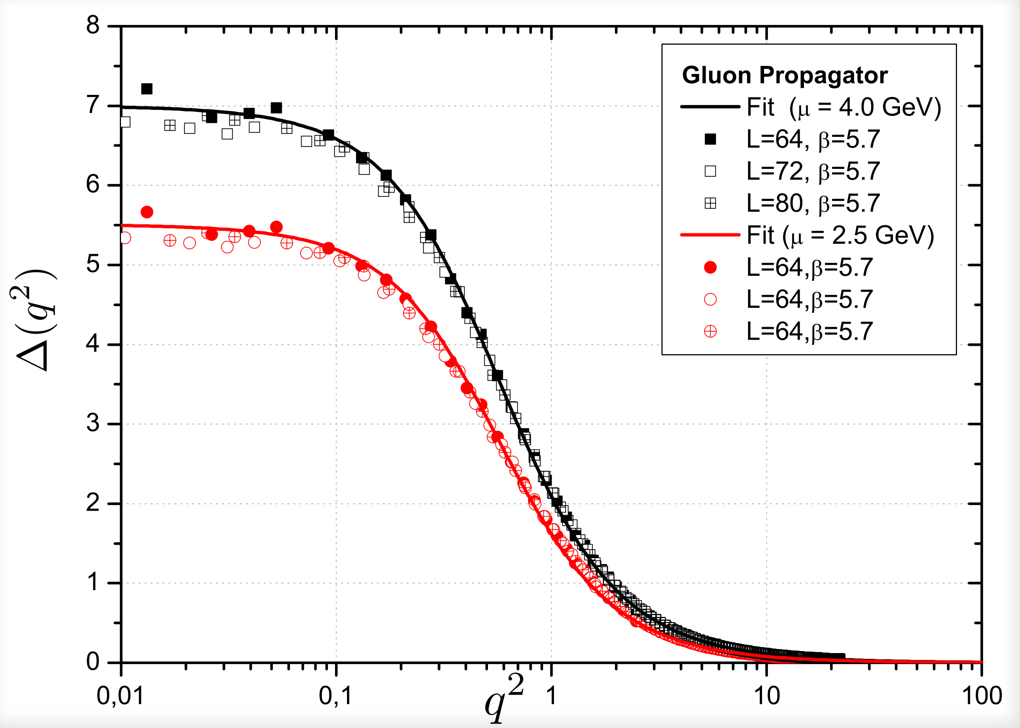

application IV
effective charge

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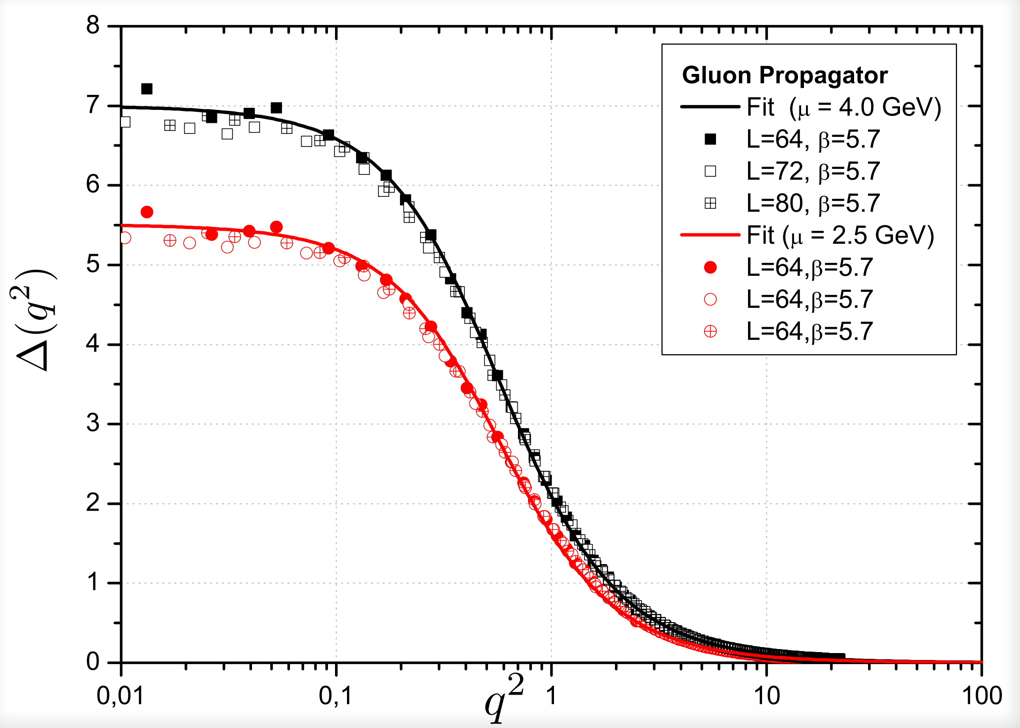

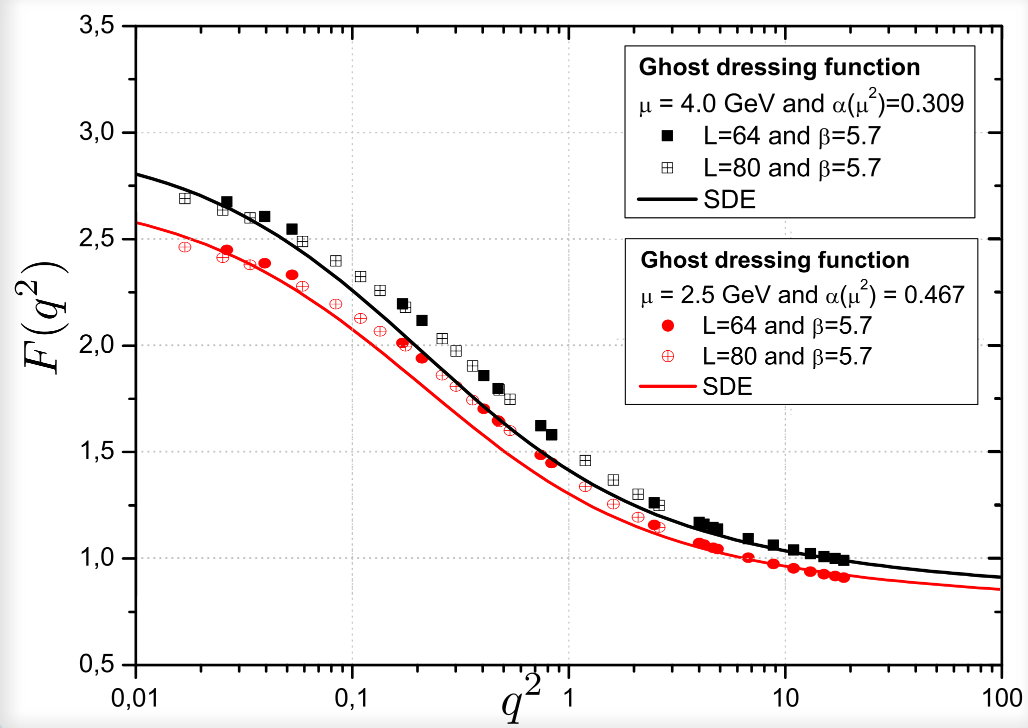

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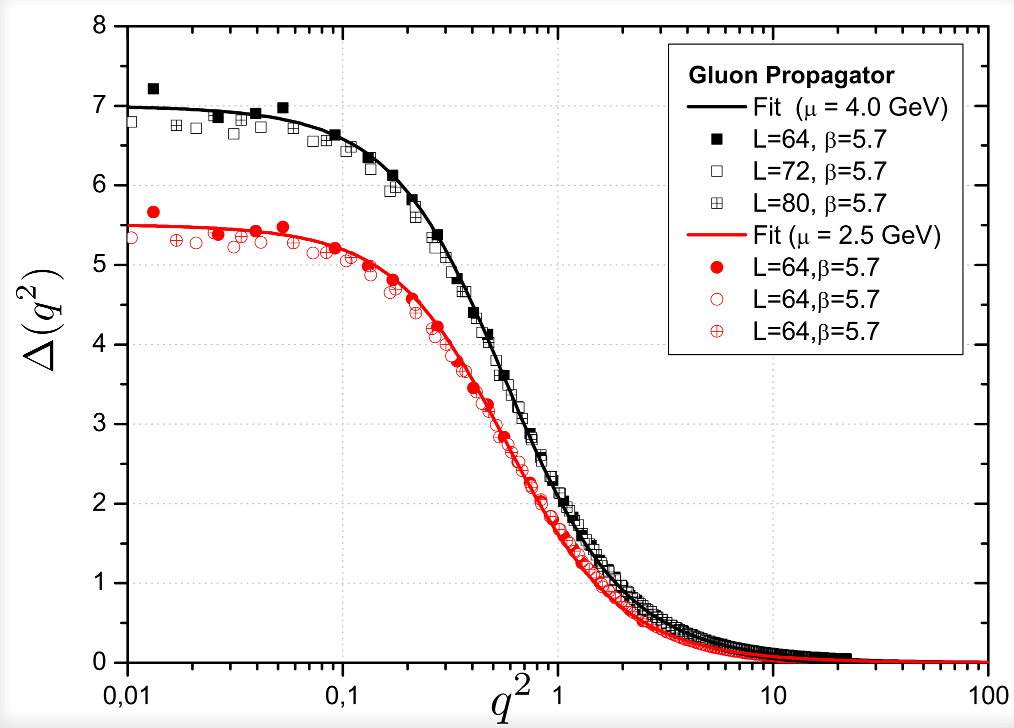

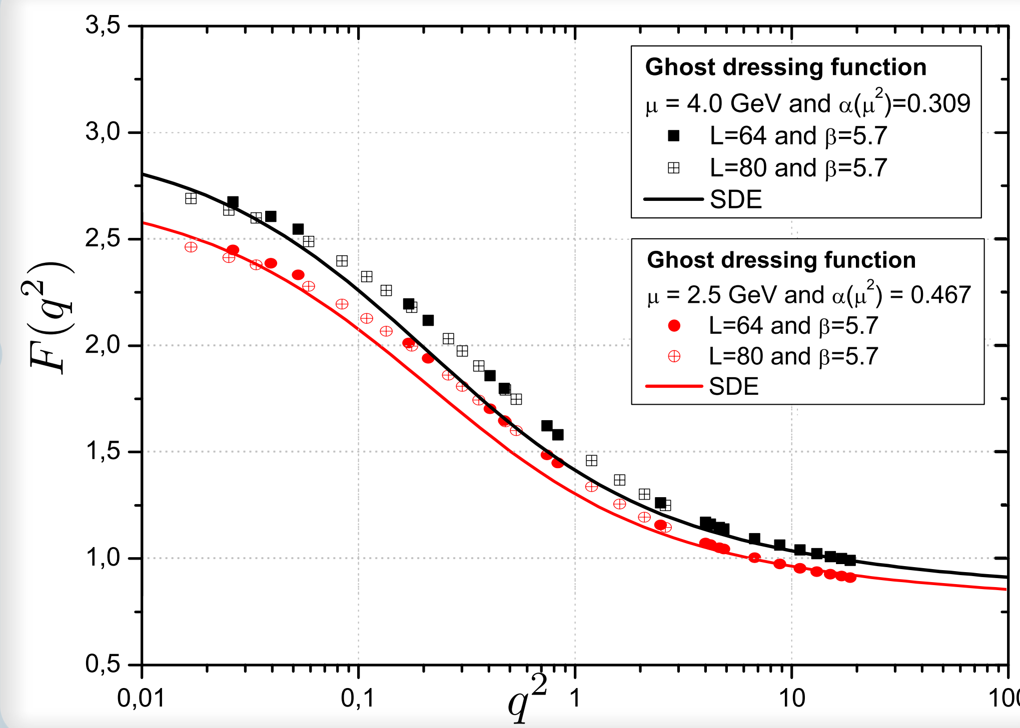

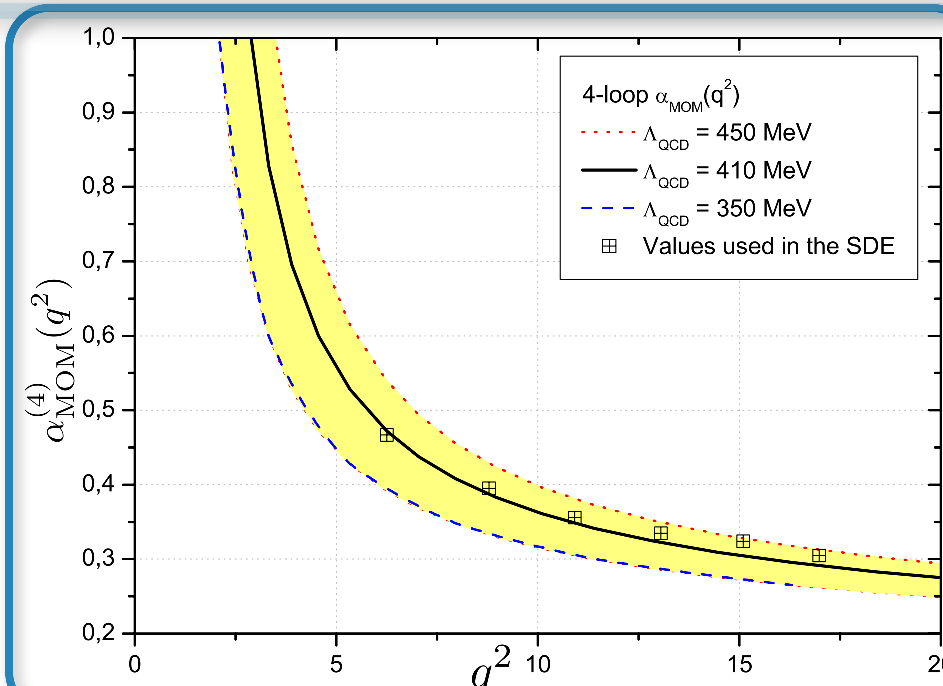

application IV
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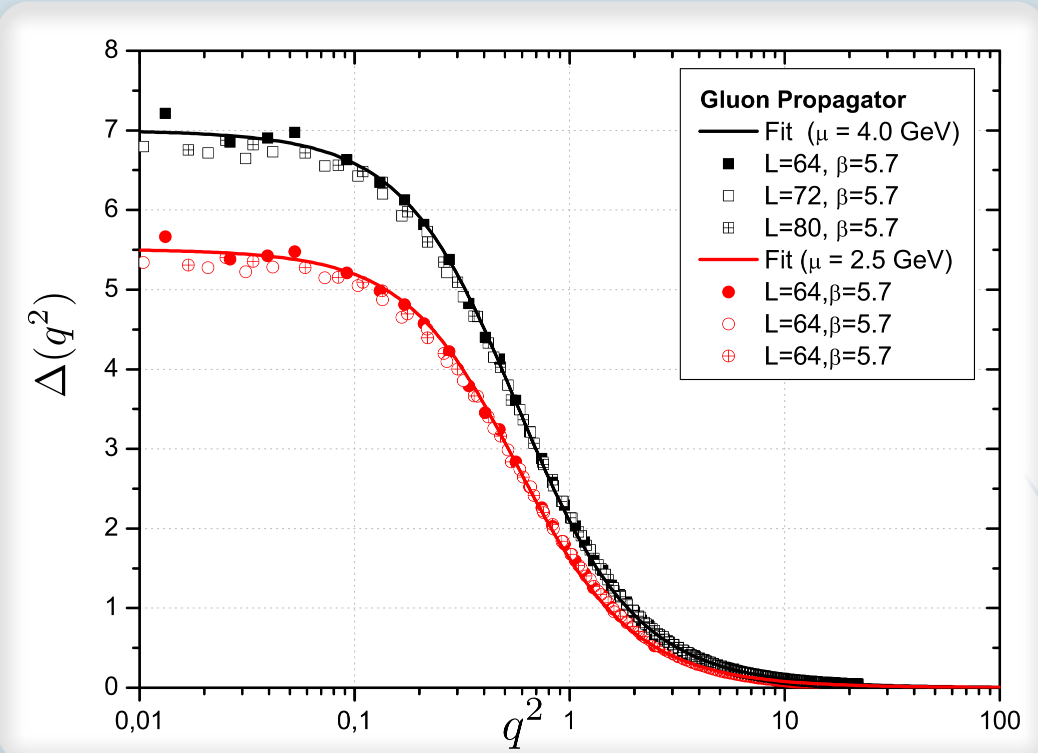

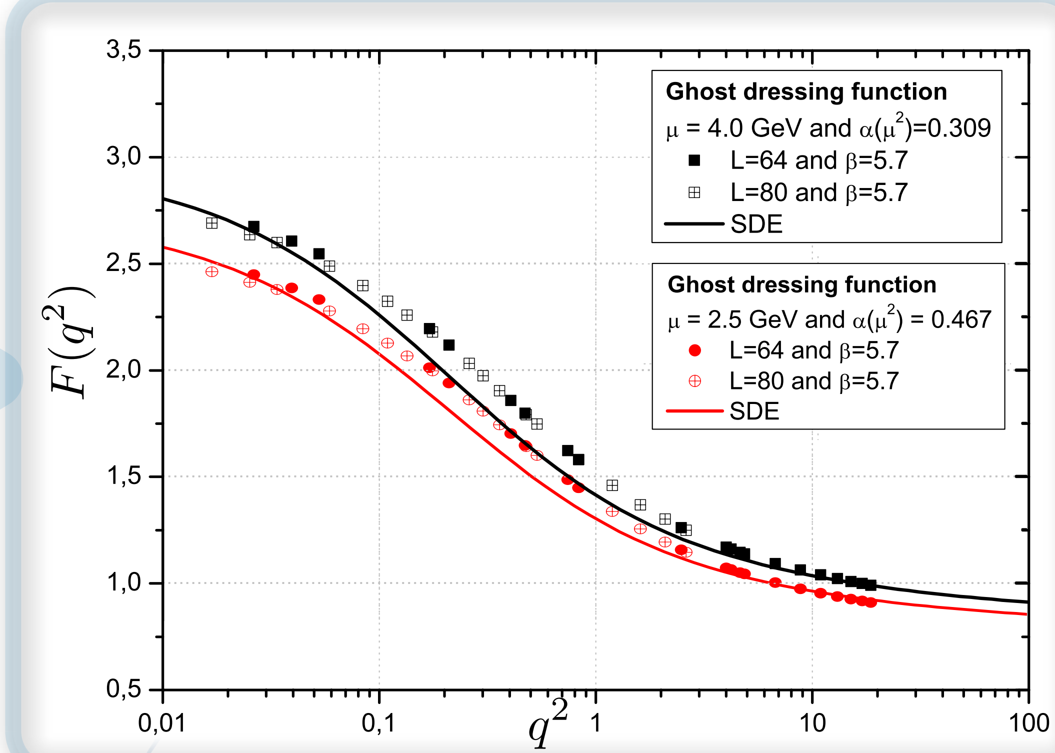

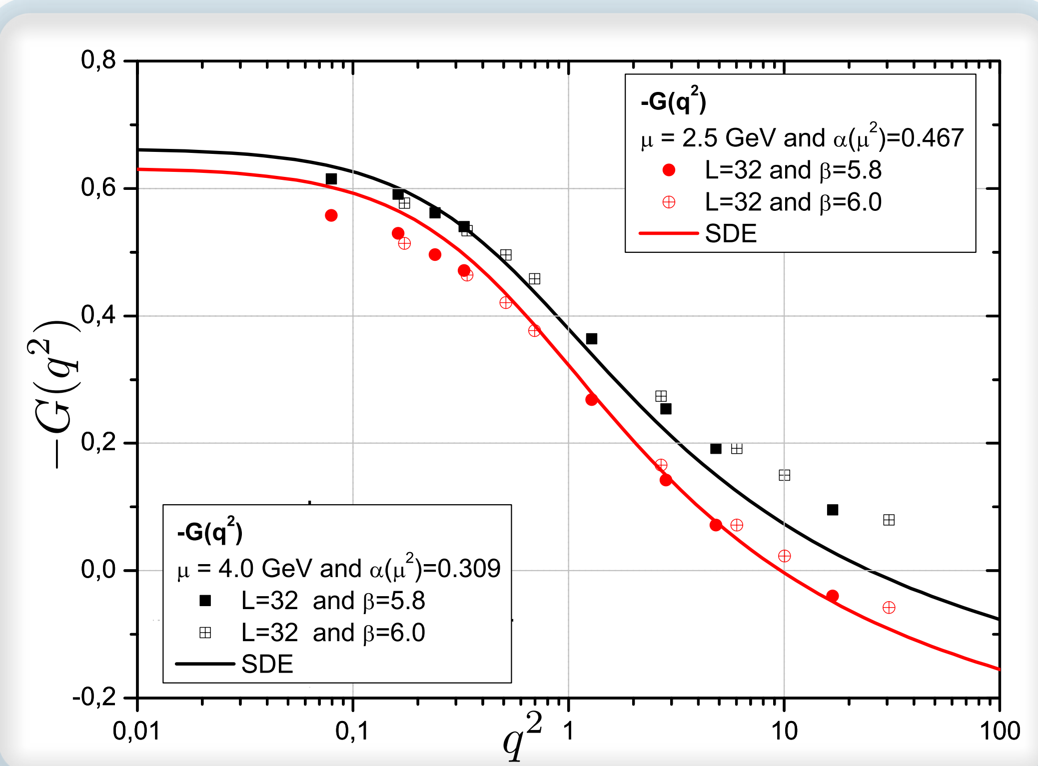

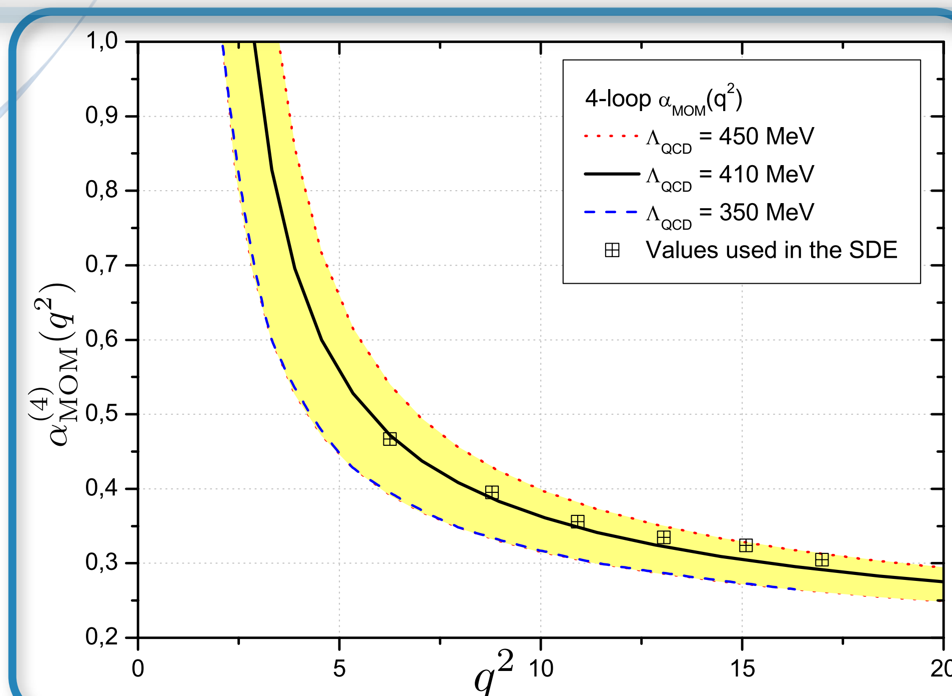

application IV

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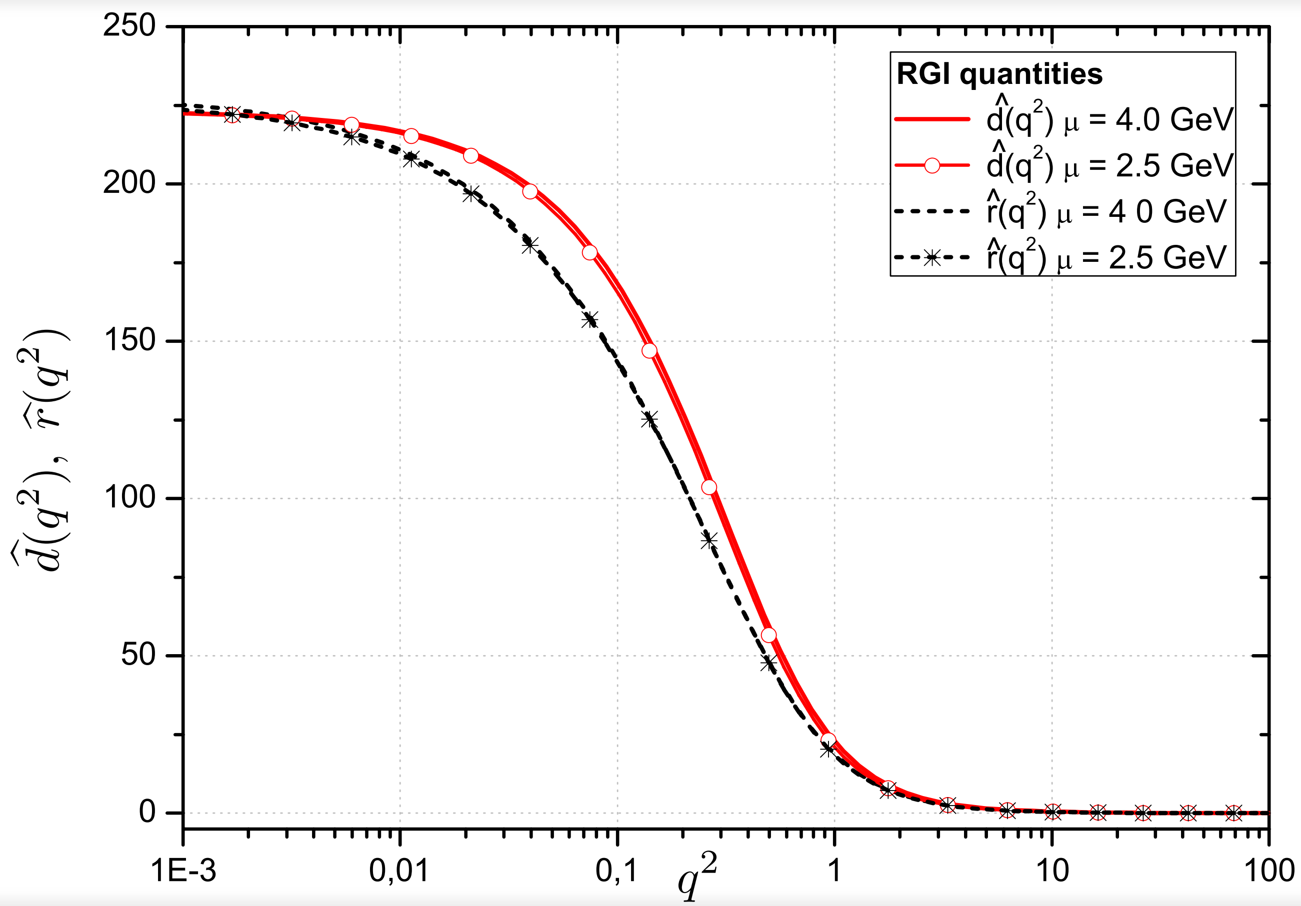

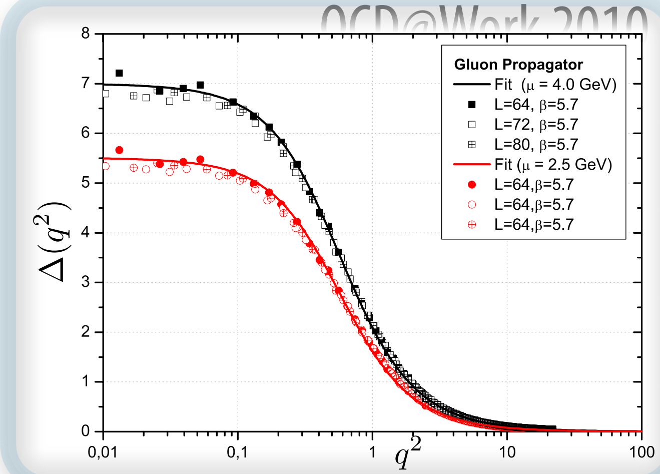

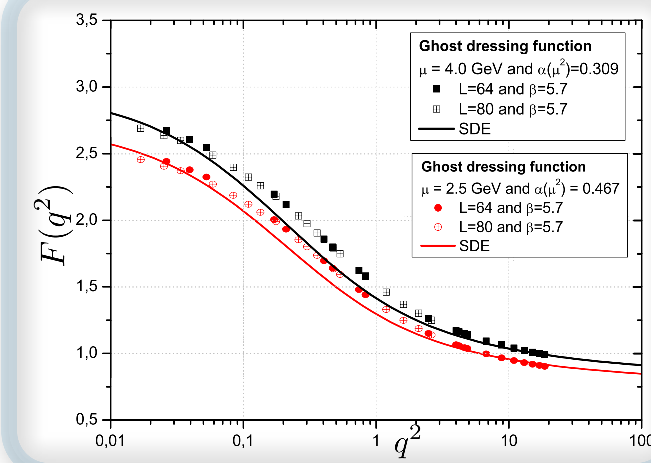

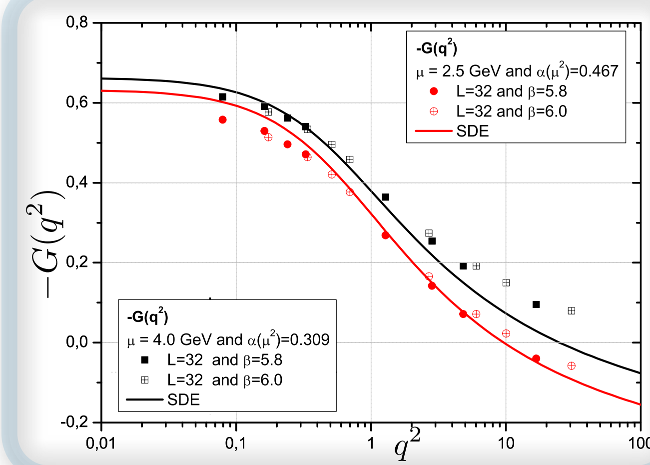

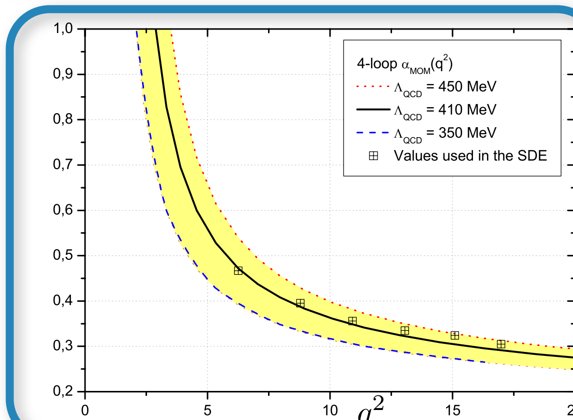

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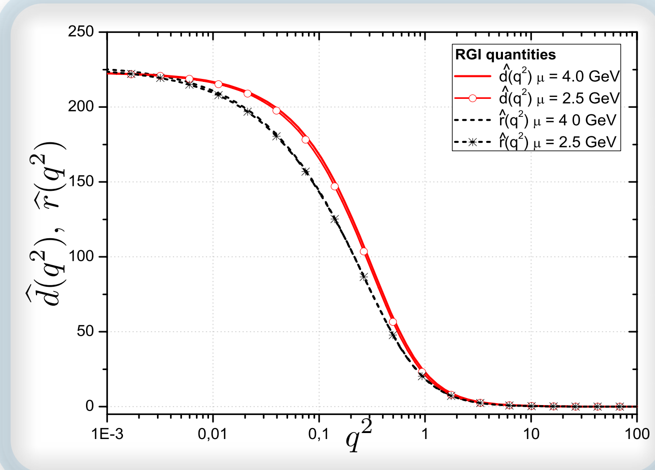
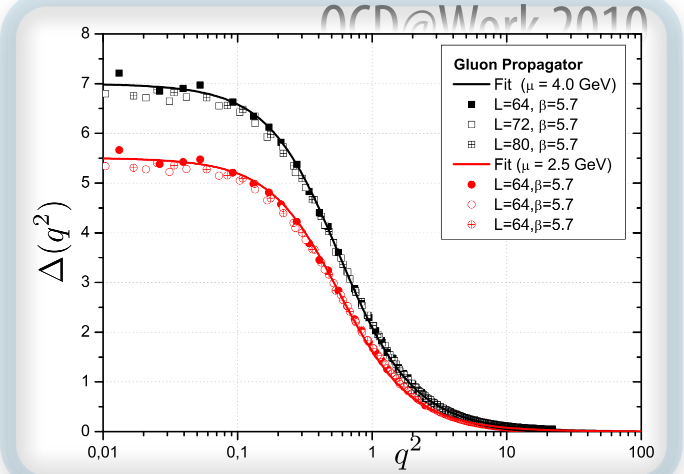
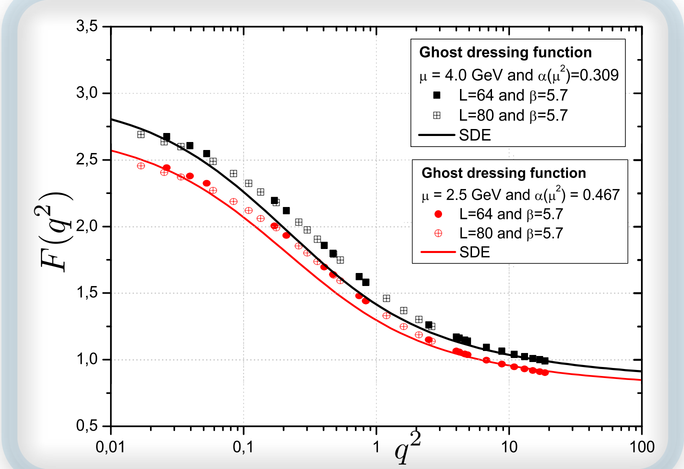
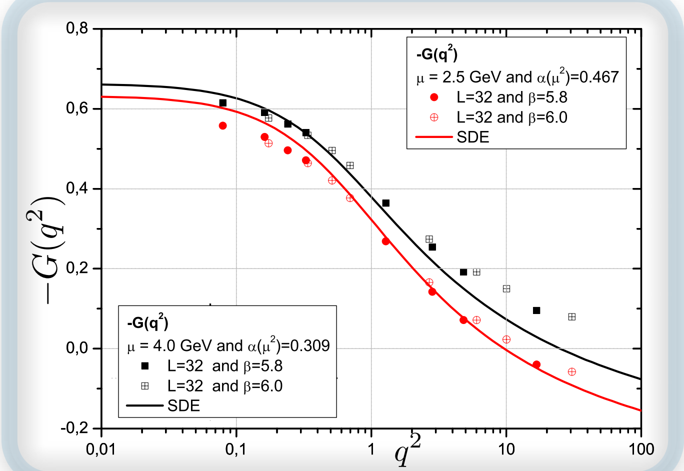
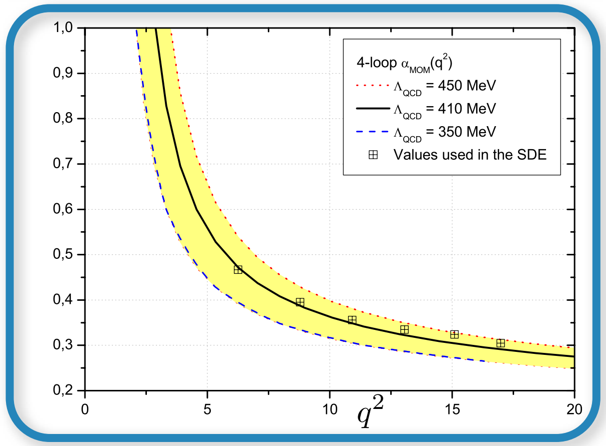
   - M. Lavelle, Phys. Rev. **D44**, 26 (1991)

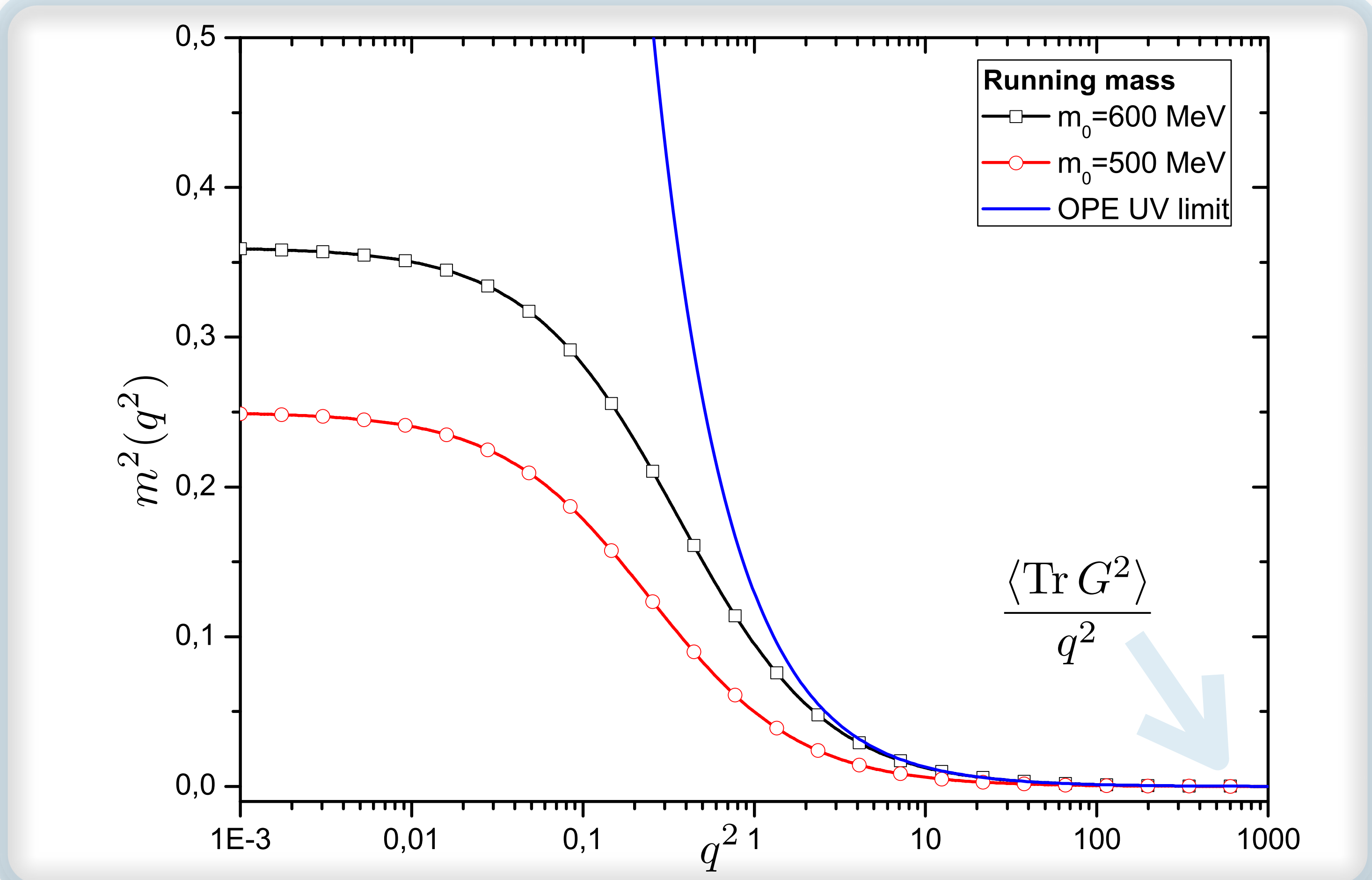












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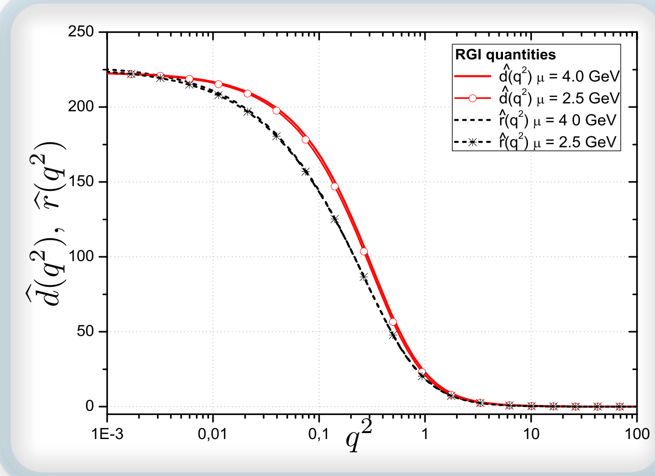
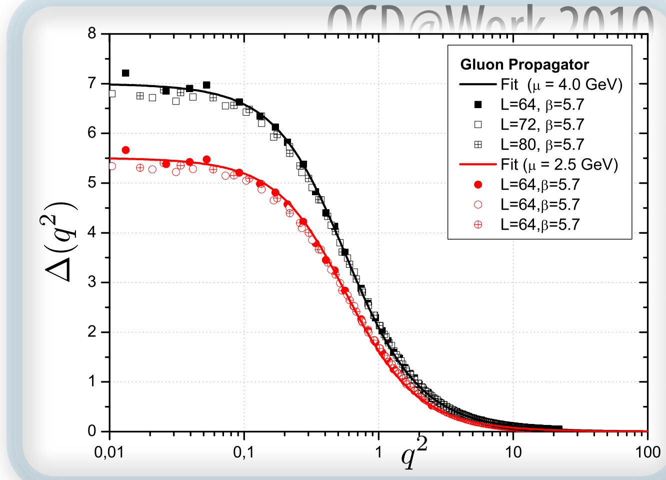
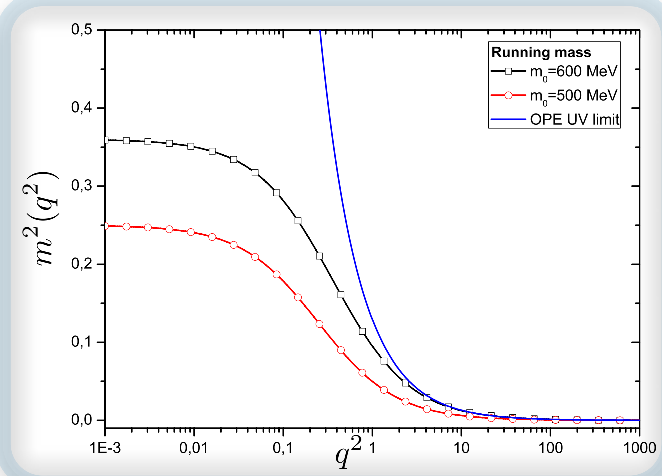
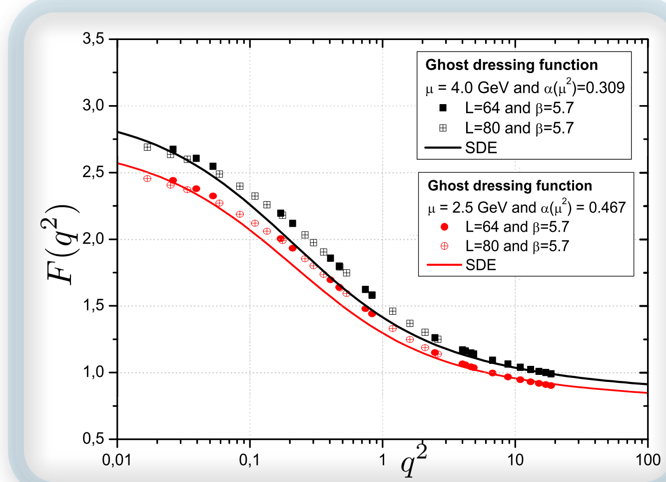
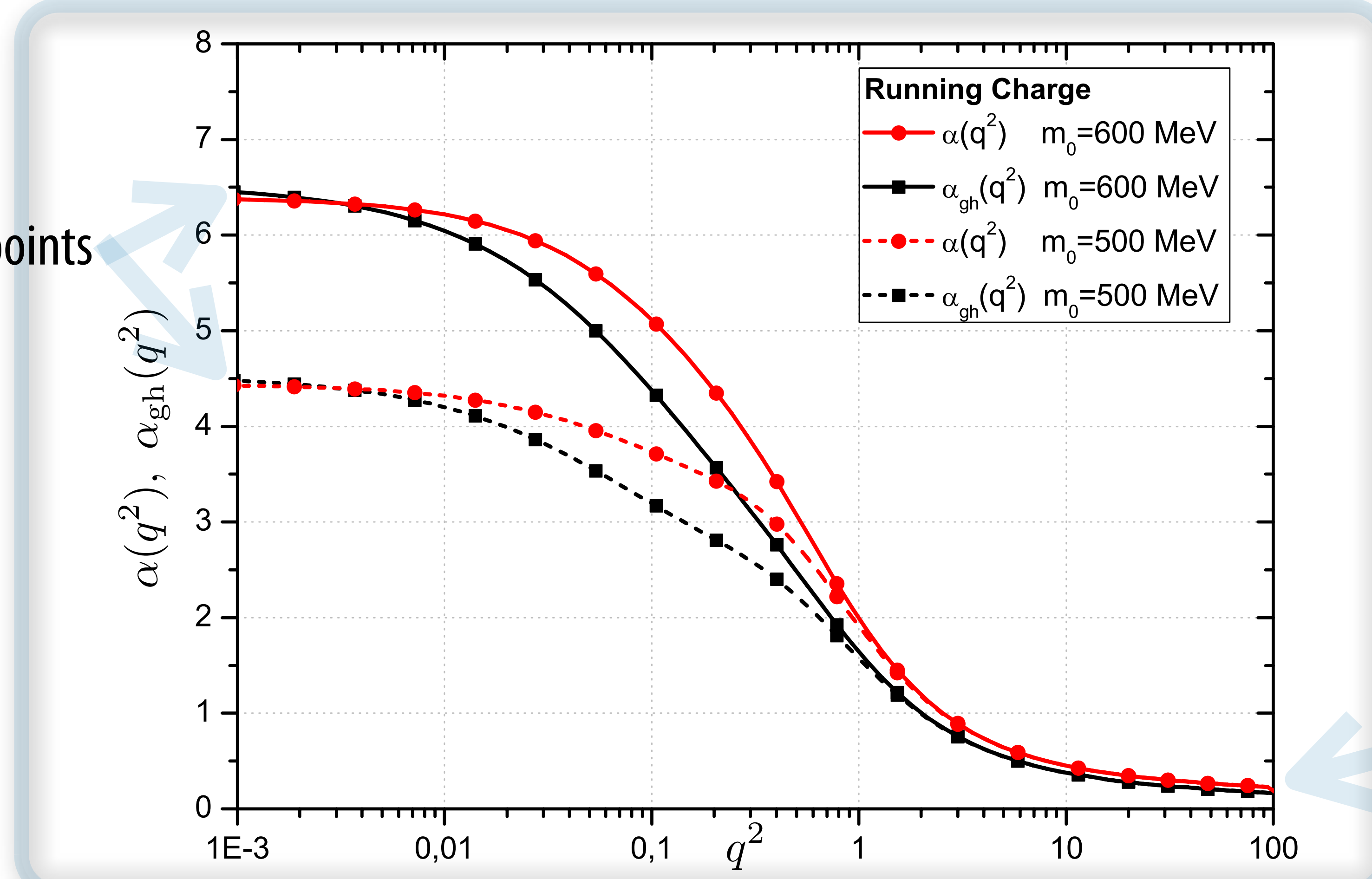




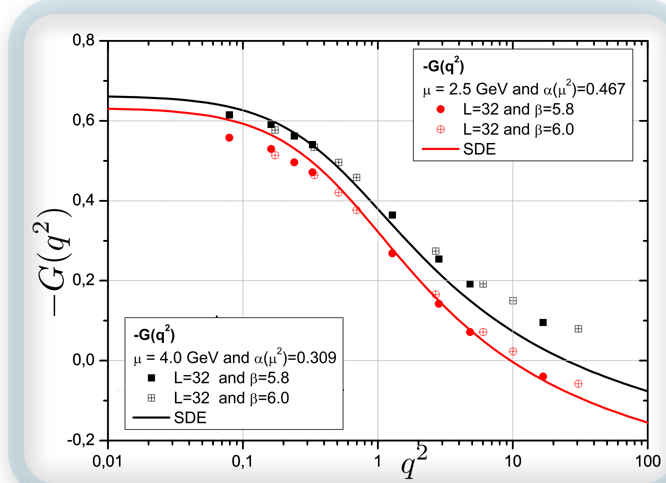


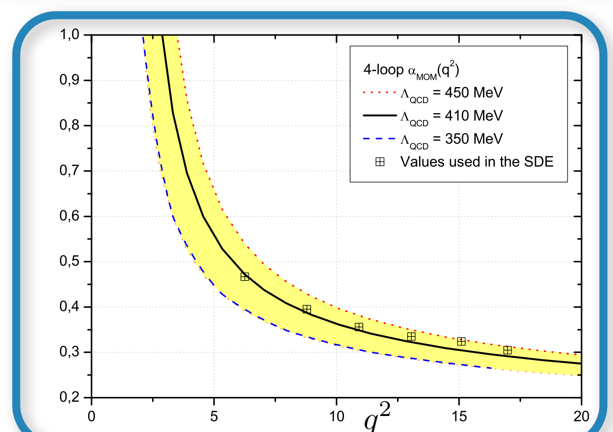




IR fixed points

Asymptotic freedom





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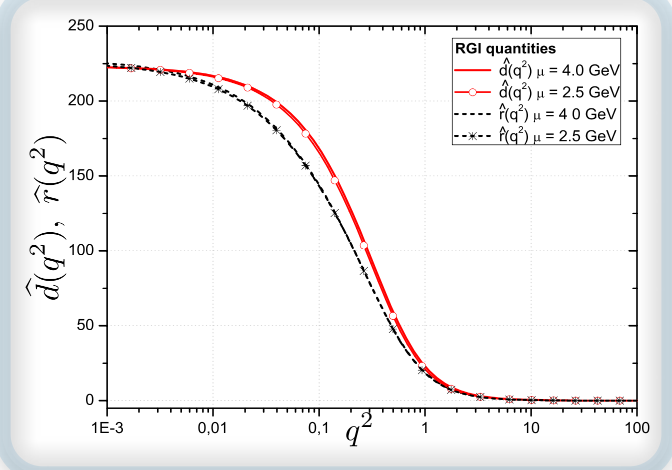
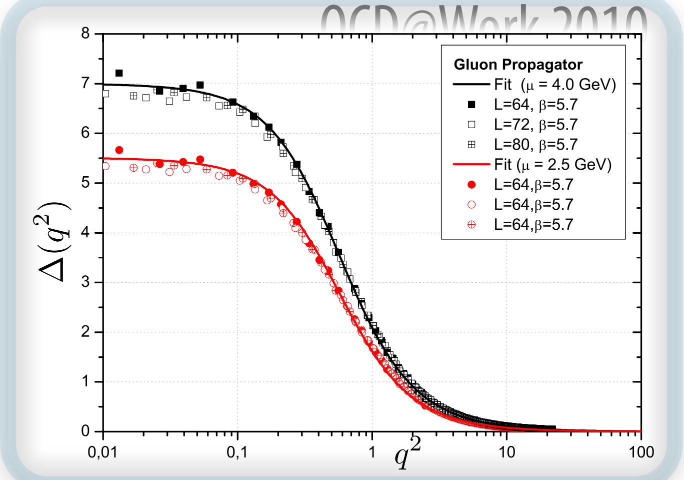
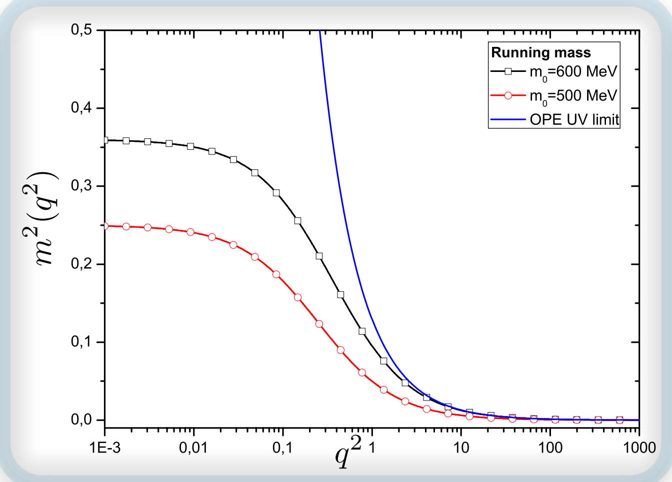
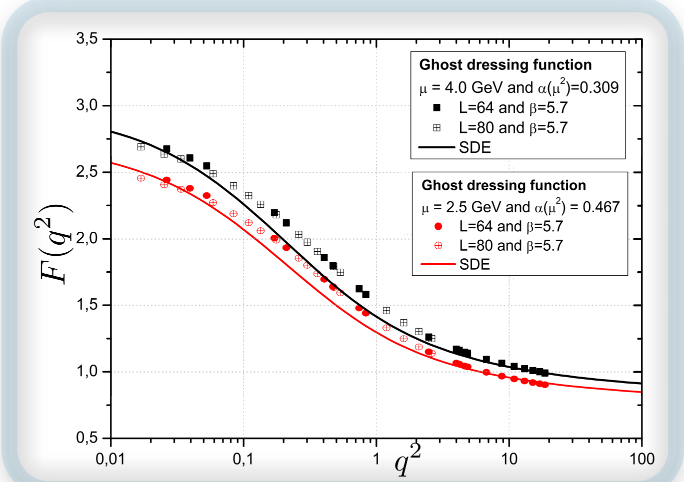
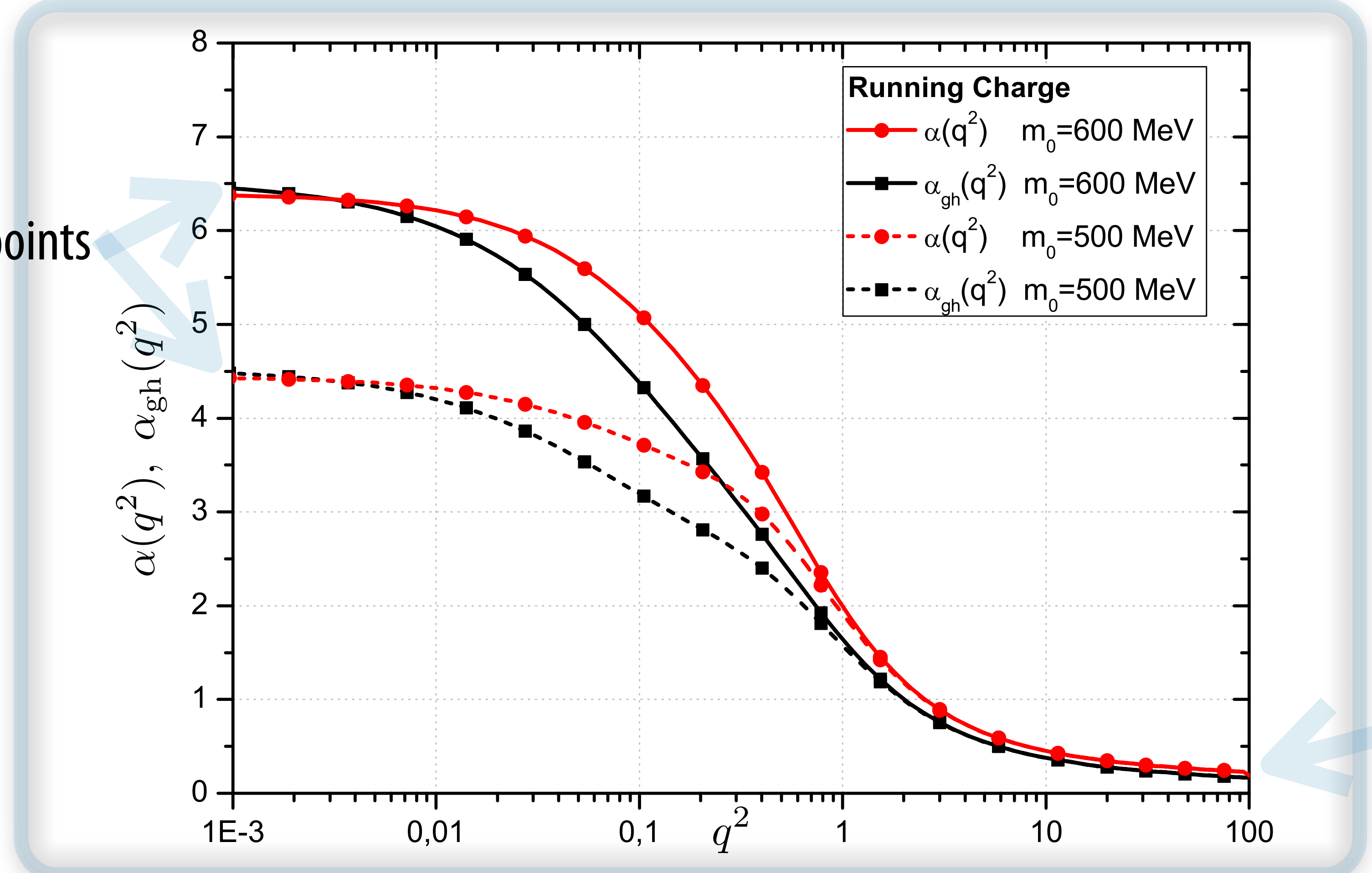




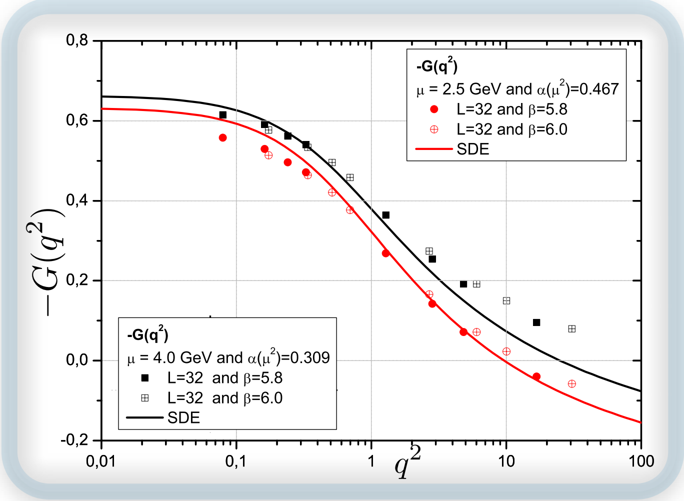


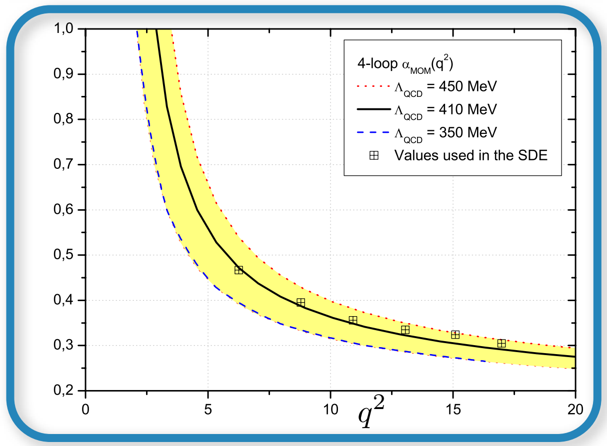


IR fixed points



Asymptotic freedom





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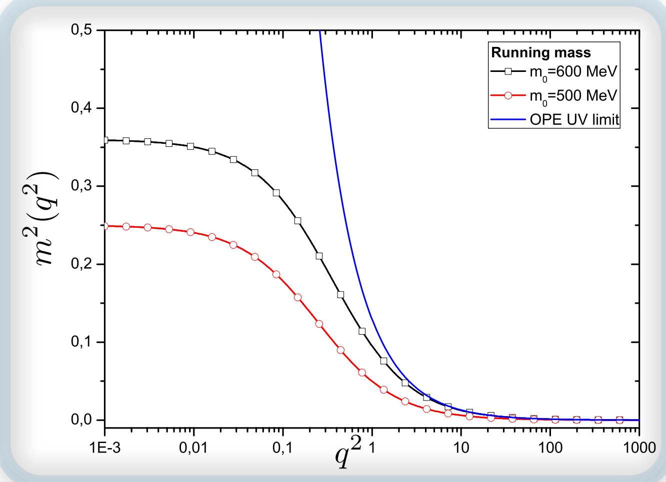
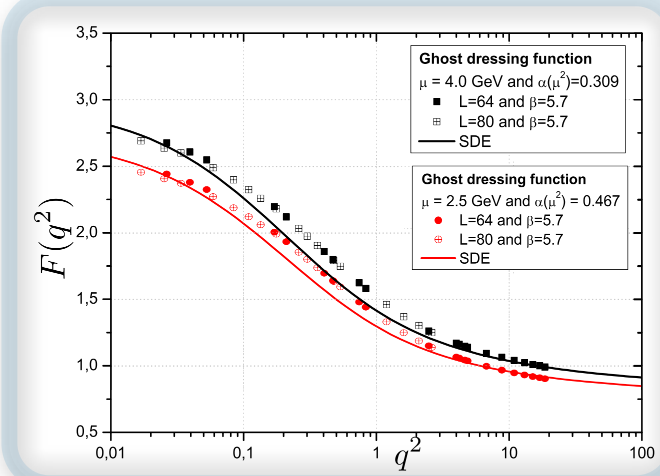
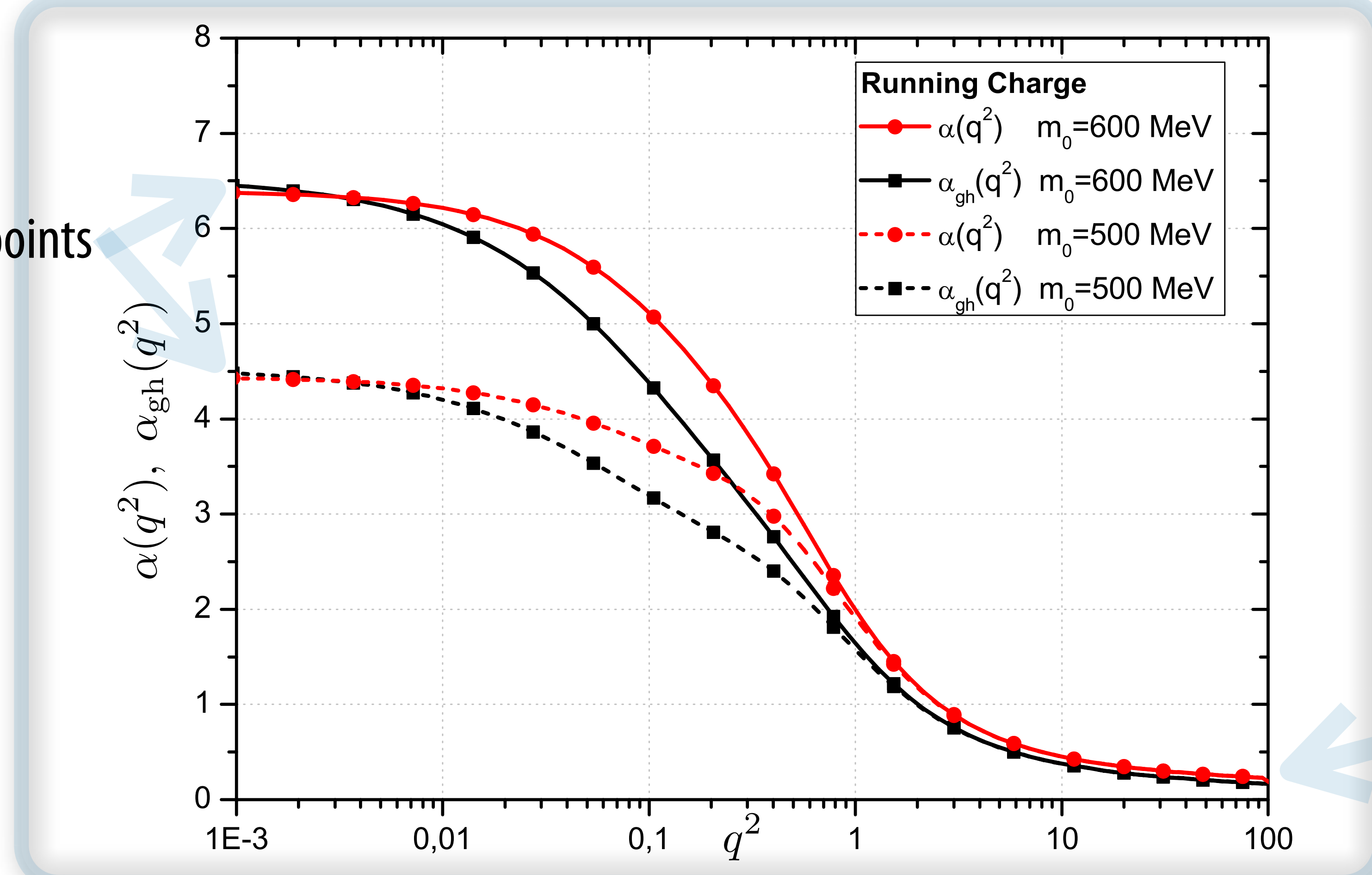
1. Start from the lattice data for $\Delta(q^2)$ and solve the SDE for F and G
2. Construct the two RGI combinations $\widehat{d}(q^2)$ and $\widehat{r}(q^2)$
3. Determine the running mass
4. Construct the effective charge(s)

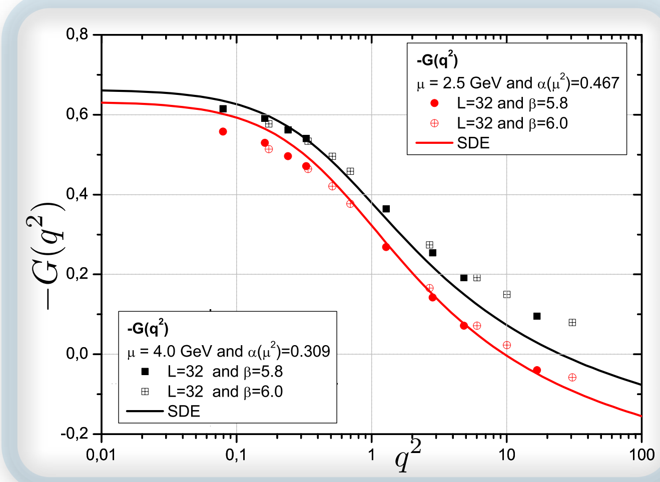
$$\alpha(q^2) = [q^2 + m^2(q^2)]\widehat{d}(q^2)$$
$$\alpha_{\rm gh}(q^2) = [q^2 + m^2(q^2)]\widehat{r}(q^2)$$



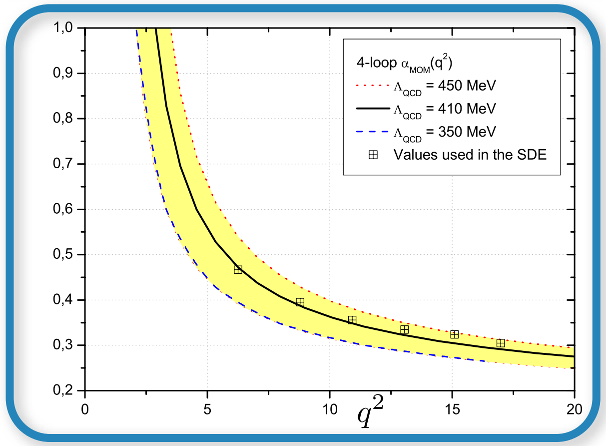


IR fixed points





Asymptotic freedom



Conclusions & Outlook

conclusions
and outlook

- Lattice data in d=3,4 shows clean evidence for
  - **IR saturation of the gluon propagator and the ghost dressing function**

- IR saturation points towards **dynamical gluon mass generation**

Understanding this behavior in the continuum formulation of QCD **is bound to teach us a great deal about QCD non-perturbative dynamics**

- **Schwinger's mechanism** @ work in d=3,4?

A dynamically generated gluon mass
- ✓ **Keeps the theory confined**
- ✓ **Opens up a conformal window**

conclusions and outlook

Gluon and ghost propagators

- Get better agreement with lattice data
- What happens in 2d?

Finite temperature

- PT-BFM beyond $T=0$
- Study the deconfinement/χ restoration temperatures and crossover

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AdS/QCD

- Can we gather information about (quenched) QCD Green's functions?
- Identify high energy (small x) observables sensible to the IR dynamics

Lattice

- Detailed study of G in 4d, 3d (and 2d?)
- Calculate in gauges other than Landau — A. Cucchieri, T. Mendes and E. M. S. Santos , Phys. Rev. Lett. **103**, 141602 (2009)
- Implement the background field method — R. F. Dashen and D. J. Gross, Phys. Rev. **D23**, 2340 (1981)

thank you

ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

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