Heavy-light mesons in the Static Limit of QCD

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Why Static Limit?

simplified framework to solving the non-perturbative dynamics of light degrees of freedom - due to heavy quark symmetry

- ✘ $m_b \to \infty$

$$\underbrace{[B(0^-), B^*(1^-)]}_{j_\ell^P = \frac{1}{2}^- (L=0)}, \quad \underbrace{[B_0^*(0^+), B_1'(1^+)]}_{j_\ell^P = \frac{1}{2}^+ (L=1)}, \quad \underbrace{[B_1(1^+), B_2^*(2^+)]}_{j_\ell^P = \frac{3}{2}^+ (L=1)}, \quad \ldots$$

- ✘ $m_c \to \infty$

$$B \to D$$

most interesting phenomenology involves lowest mesons: leptonic, semileptonic (H/H, H/L) and radiative decays

$\bar{B}^0_{d,s} - B^0_{d,s}$*-mixing amplitude (BSM)*

Heavy Quark Effective Theory

$m_{c,b,t} > \Lambda_{\text{QCD}}$

$$\mathcal{L}_Q = \bar{Q}(x)\left[i\not{D} - m_Q\right]Q(x)$$

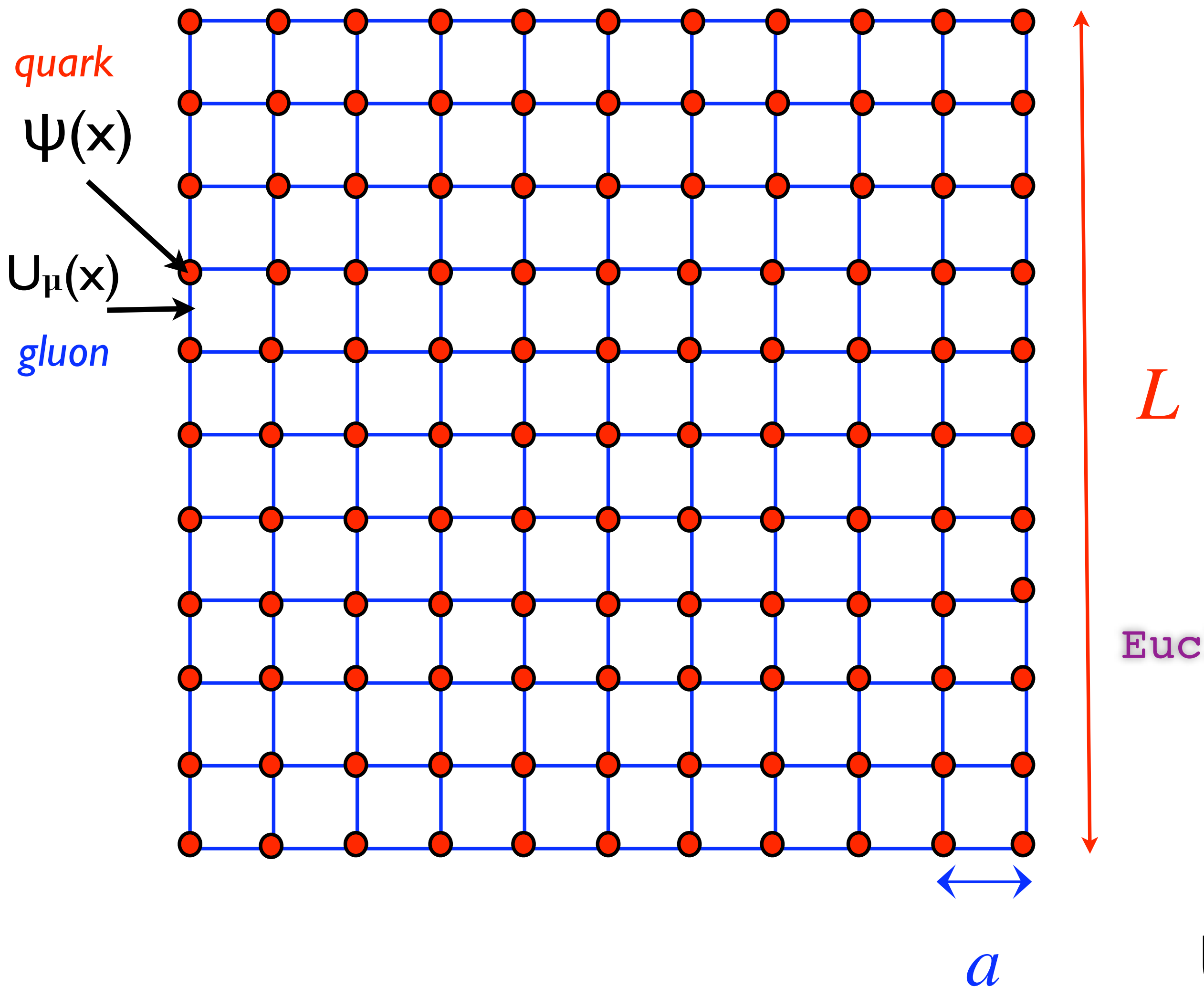
$Q(x) = e^{im_Q vx}\left[h_v^{(-)}(x) + H_v^{(-)}(x)\right]$

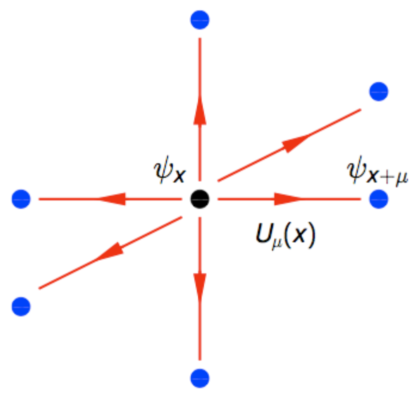
$$\mathcal{L}_Q = \bar{h}_v\, i\, v\cdot D\, h_v \;-\; \bar{H}_v\,(i\, v\cdot D + 2m_Q)\, H_v + \bar{h}_v\, i\, \not{D}_\perp\, H_v$$

$$\begin{aligned}\mathcal{L}_{\text{HQET}} &= \bar{h}_v\, i\, v\cdot D h_v + \bar{h}_v\, i\, \not{D}_\perp \frac{1}{(i\, v\cdot D + 2m_Q - i\epsilon)} i\not{D}_\perp\, h_v \\ &= \bar{h}_v\, i\, v\cdot D h_v + \mathcal{O}(1/m_Q)\end{aligned}$$

in $\quad v^\mu = (1,0,0,0) \quad \mathcal{L}_{\text{HQET}} = \bar{h}\, i\, D_0\, h$

Lattice QCD





IR cutoff

Euclidean space-time

UV cutoff

Latticized HQET...

$$S^E_{\rm HQET} = \int d^4x \, (\, h^\dagger \, D_0 \, h + \tilde{h} \, D_0 \, \tilde{h}^\dagger \,)$$

$$S^{\rm latt}_{\rm HQET} = a^3 \sum_n \{ \, h^\dagger(n) \, [\, h(n) - U_0 \, (n - \hat{0}\,) \, h(n - \hat{0}) \,] + \tilde{h}(n) \, [U_0(n) \, \tilde{h}^\dagger \, (n + \hat{0}) - \tilde{h}^\dagger(n) \,] \}$$

$$S(x,y) = -iW^{x^0}_{y^0} \delta(\vec{x} - \vec{y}) \left\{ \left[\theta(x^0 - y^0) e^{-im_Q(x^0 - y^0)} \left(\frac{1+\gamma^0}{2} \right) \right] + \left[\begin{array}{ccc} x^0 & \leftrightarrow & y^0 \\ \frac{1+\gamma^0}{2} & \leftrightarrow & \frac{1-\gamma^0}{2} \end{array} \right] \right\} + \mathcal{O}(1/m_Q)$$

$$W^{x^0}_{y^0} \equiv W^x_y = \exp \left(ig \int_{y^o}^{x^0} dz^0 A^a_0(\vec{x}, z^0) t^a \right)$$

$$W^x_y = U_0(\vec{x}, x^0 - a) U_0(\vec{x}, x^0 - 2a) \cdots U_0(\vec{x}, y^0) \big|_{\vec{x} = \vec{y}}$$

! $$\exp \left(- \int_{\alpha_s(m)}^{\alpha_s(\mu)} \frac{\gamma_O(\alpha)}{\beta(\alpha)} \frac{d\alpha}{\alpha} \right) O(\mu) = C_\Gamma(m) \times \exp \left(- \int_{\alpha_s(m)}^{\alpha_s(\bar{\mu})} \frac{\widetilde{\gamma}_O(\alpha)}{\beta(\alpha)} \frac{d\alpha}{\alpha} \right) \widetilde{O}(\bar{\mu}) + \mathcal{O}\,(1/m)$$

HMChPT

$m_{c,b,t} > \Lambda_{\mathrm{QCD}}$

$m_{u,d,s} < \Lambda_{\mathrm{QCD}}$

$SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$

$$H_a = \frac{1+\not{v}}{2}(P_a^{\mu}\gamma_\mu - P_a\gamma_5)$$

$$\mathcal{L}^{(1)} = -\mathrm{Tr}(\bar{H}_a iv\cdot D_{ab}H_b) + g\,\mathrm{Tr}(\bar{H}_a H_b \gamma_\mu \mathcal{A}^{\mu}_{ba}\gamma_5)$$

$$D^{\mu}_{ab}H_b = \partial^{\mu}H_a - H_b\mathcal{V}^{\mu}_{ba}$$

$$\xi = \exp(i\phi/f)$$

$$\mathcal{V}_\mu = \frac{1}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi)$$

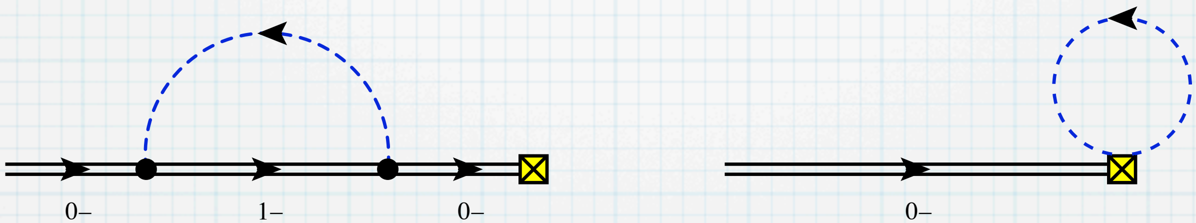
$$\phi = \sum_i P^i t^i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$\mathcal{A}_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$$

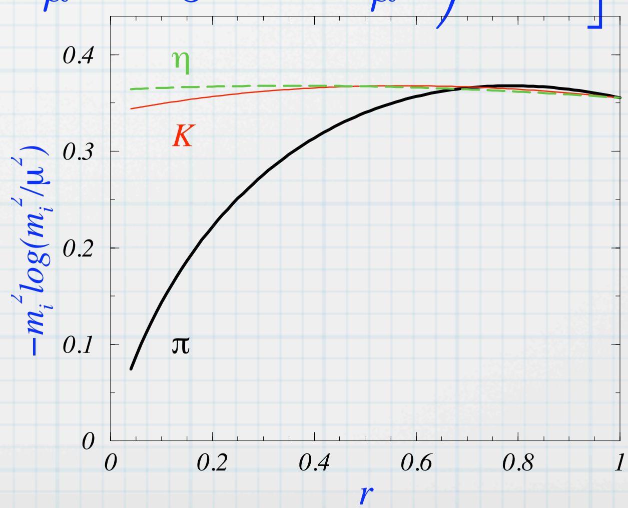
HMChPT

$f_B\sqrt{m_B} \to \phi_q$

$$(V-A)^\mu_q = \frac{i\phi_q}{2}\left\{\mathrm{Tr}\left[(\xi\overline{H})_q\gamma^\mu(1-\gamma_5)\right] + \text{ct.}\right\}$$



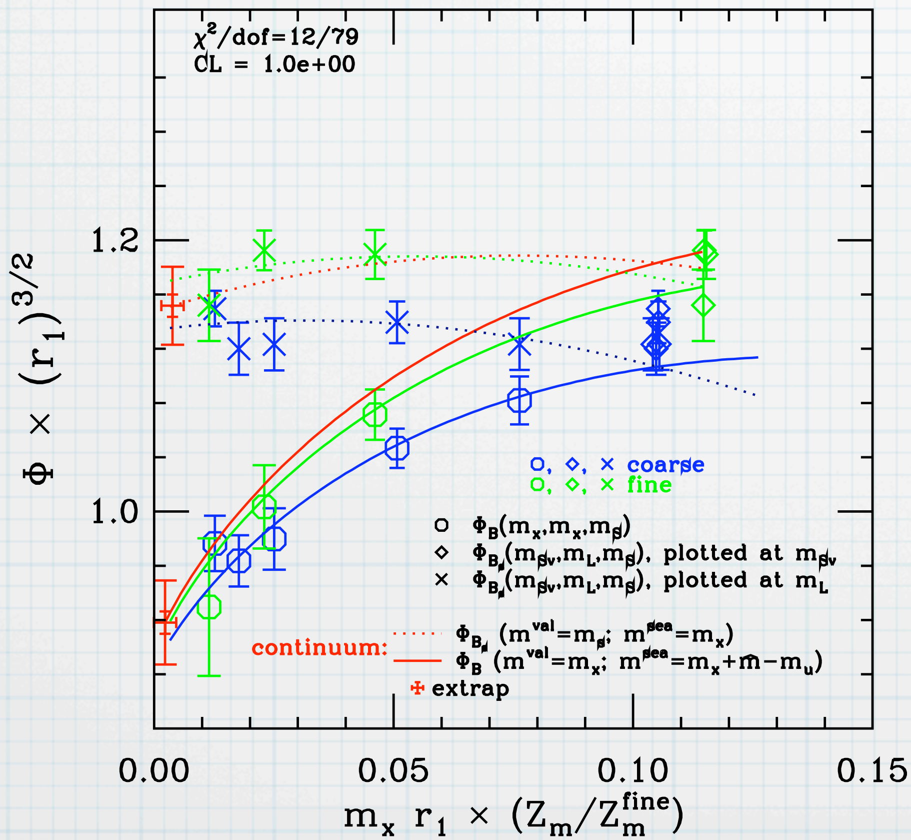
$$\phi_d = \phi_0\left[1 - \frac{1+3g^2}{4(4\pi f)^2}\left(3m_\pi^2\log\frac{m_\pi^2}{\mu^2} + 2m_K^2\log\frac{m_K^2}{\mu^2} + \frac{1}{3}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right) + \text{ct.}\right]$$



Extrapolate lattice data

$f_B\sqrt{m_B} \rightarrow \phi_q$

$$\phi_d = \phi_0 \left[1 - \frac{1+3g^2}{4(4\pi f)^2}\left(3m_\pi^2 \log\frac{m_\pi^2}{\mu^2} + 2m_K^2 \log\frac{m_K^2}{\mu^2} + \frac{1}{3}m_\eta^2 \log\frac{m_\eta^2}{\mu^2}\right) + \text{ct.}\right]$$



* *finite volume effects can be accounted for by ChPT tools (at NLO completely independent of counterterms!)*

* *HMStagChPT Aubin, Bernard, 2006*

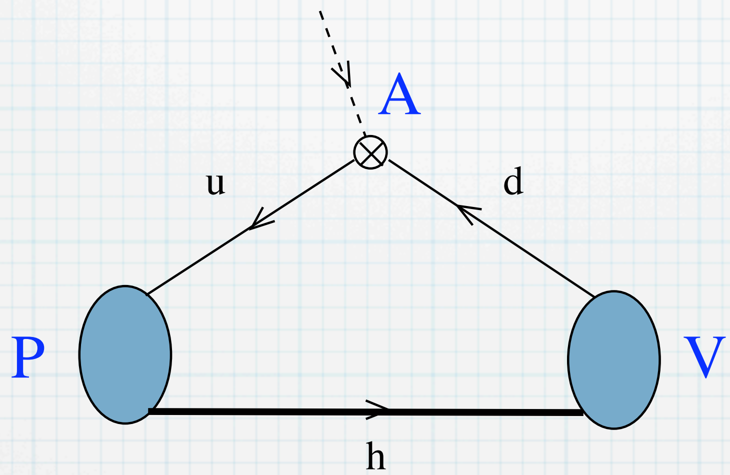
* *MILC & Fermilab-lattice, 2009*

Need coupling - g

@ $|\vec{q}| = 0$

$$\langle B|\vec{A}|B^*(\varepsilon)\rangle = g\ \vec{\varepsilon}_\lambda$$

$$A_\mu = \bar{u}\gamma_\mu\gamma_5 d$$



$$C_2(t) = \langle\sum_{\vec{x}} P(x)P^\dagger(0)\rangle_U \overset{\text{HQS}}{=} \frac{1}{3}\langle\sum_{i,\vec{x}} V_i(x)V_i^\dagger(0)\rangle_U = \langle\sum_{\vec{x}} \mathrm{Tr}\left[\frac{1+\gamma_0}{2}W_x^0\gamma_5\mathcal{S}_{u,d}(0,x)\gamma_5\right]\rangle_U$$

$$C_3(t_y,t_x) = \langle\sum_{i,\vec{x},\vec{y}} V_i(y)A_i(x)P^\dagger(0)\rangle_U = \langle\sum_{\vec{x},\vec{y}} \mathrm{Tr}\left[\frac{1+\gamma_0}{2}W_0^y\gamma_i\mathcal{S}_u(y,x)\gamma_i\gamma_5\mathcal{S}_d(x,0)\gamma_5\right]\rangle_U$$

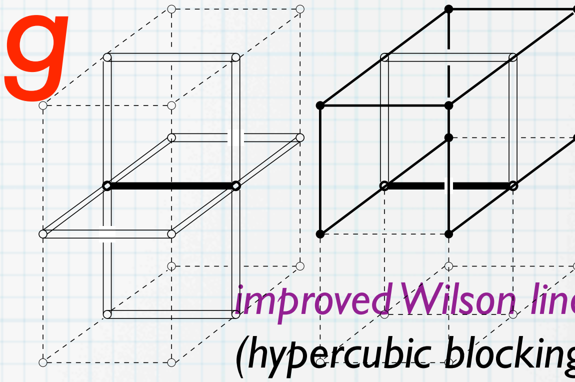
$\bar{h}\gamma_i q$

$(\bar{h}\gamma_5 q)^\dagger$

Need coupling - g

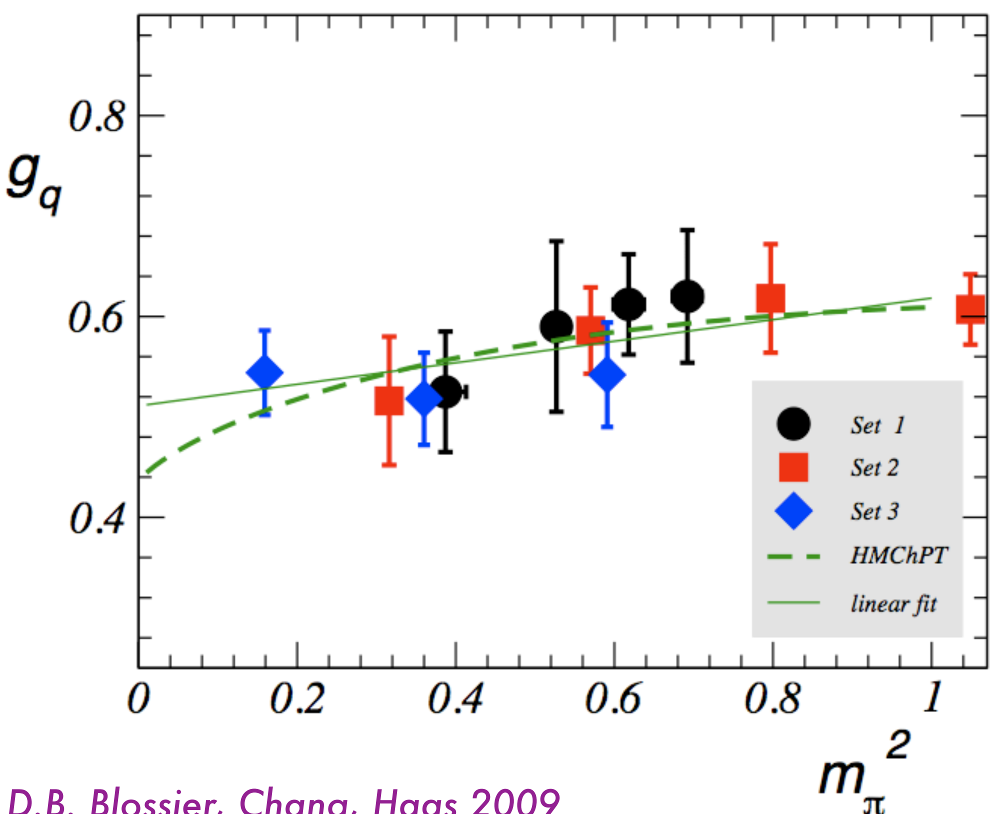
$$C_3(t_x) \simeq \sum_n \mathcal{Z}_n^2 e^{-\mathcal{E}_n^q\ (t_x - t_{\rm fix})} \langle B_n | A_i | B_n^* \rangle \varepsilon_i^{(n)}$$

$$C_2(t_x) \simeq \sum_n \mathcal{Z}_n^2 e^{-\mathcal{E}_n^q\ t_x}$$



improved Wilson line

(hypercubic blocking)



$$g^q = g_{\rm lin}\left(1 + c_{\rm lin} m_\pi^2\right)$$

$$g_{\rm lin} = 0.51 \pm 0.04$$

$$c_{\rm lin} = (0.21 \pm 0.12)\ {\rm GeV}^{-1}$$

$$g_q = g_0\left[1 - \frac{4g_0^2}{(4\pi f)^2} m_\pi^2 \log(m_\pi^2) + c_0 m_\pi^2\right]$$

$$g_0 = 0.44 \pm 0.03$$

$$c_0 = (0.40 \pm 0.12)\ {\rm GeV}^{-1}$$

D.B. Blossier, Chang, Haas 2009

We must extend HMChPT !

Nearest orbital excitations $[j_\ell^P = (1/2)^+]$ not far from the ground states $[j_\ell^P = (1/2)^-]$:

- BaBar, Belle and CLEO $\to$ scalar D_{0s}^* and axial D_{1s}:

$$\Delta_{S_s} \equiv m_{D_{0s}^*} - m_{D_s} = m_{D_{1s}} - m_{D_s^*} = 350 \text{ MeV}$$

- Belle and Focus $\Delta_{S_q} \approx 430(30)$ MeV
- Lattice: situation similar in the static limit $(m_Q \to \infty) \Rightarrow$ also B's **N.B.!**

$$\Delta_{S_{s,q}} < \Lambda_\chi, m_\eta, m_K$$

- No kaon or eta, unless we include scalars in the loops!
+ Not big a problem: SU(2) is cool too

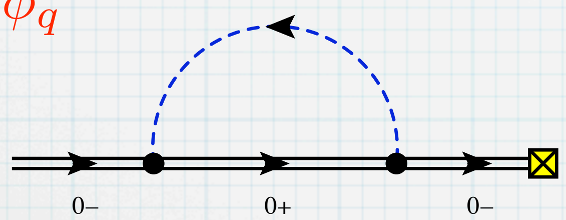
Extended HMChPT

$$\mathcal{L}^{(1)}_{\text{HMChPT}} = \mathcal{L}^{(1)}_{\frac{1}{2}^-} + \mathcal{L}^{(1)}_{\frac{1}{2}^+} + \mathcal{L}^{(1)}_{\text{mix}} + \mathcal{L}^{(1)}_{\text{ct.}}$$

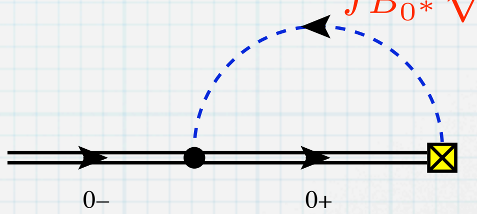
$$\mathcal{L}_{\frac{1}{2}^+} = -\text{Tr}\left[S_b(iv\cdot D_{ba} + \Delta_S)\overline{S}_a\right] + \tilde{g}\ \text{Tr}\left[S_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\bar{S}_a\right]$$

$$\mathcal{L}_{\text{mix}} = h\ \text{Tr}\left[S_b\gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\overline{H}_a\right] + \text{h.c.}$$

$f_B\sqrt{m_B} \to \phi_q$



$f_{B_{0^*}}\sqrt{m_{B_{0^*}}} \to \widetilde{\phi}_q$



$$\phi_q = \phi_0\left\{1 + \frac{t^i_{qa}t^{i\dagger}_{aq}}{2(4\pi f)^2}\left[3g^2\lim_{x\to 0}\frac{d}{dx}[xJ_1(m_i^2,x)] - I_1(m_i^2) - h^2\left(J_1(m_i^2,\Delta_S) + J_2(m_i^2,\Delta_S)\right.\right.\right.$$
$$\left.\left.\left. + \Delta_S\frac{d}{d\Delta_S}\left(J_1(m_i^2,\Delta_S) + J_2(m_i^2,\Delta_S)\right)\right) - 2h\frac{\widetilde{\phi}_0}{\phi_0}\left(I_1(m_i^2) + I_2(m_i^2,\Delta_S)\right)\right]\right\}$$

Extended HMChPT

In the limit $m_\pi/\Delta_S \to 0$ all LO corr's due to $1/2^+$ states are analytic in m_π

$$\phi_q = \phi_0 \left[1 - \frac{1+3g^2}{2(4\pi f)^2} \frac{3}{2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_f(\mu) m_\pi^2 \right]$$

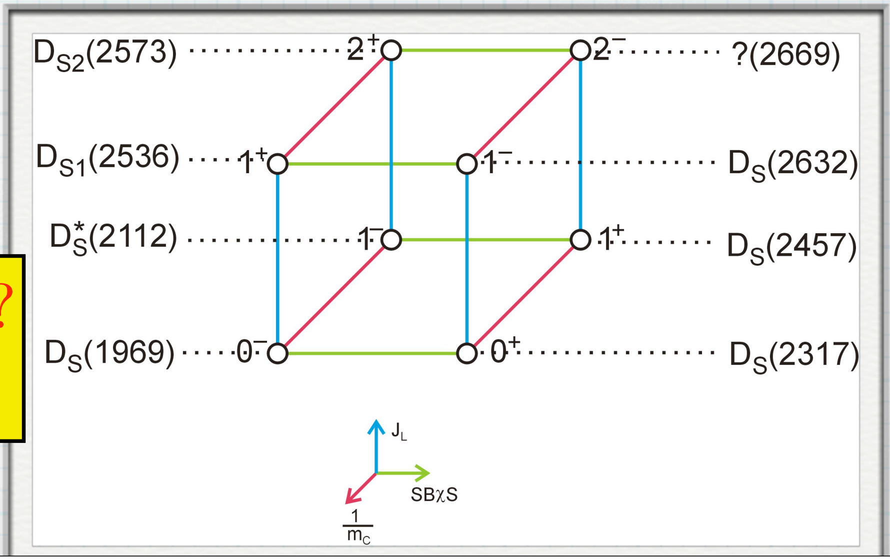
$$\widetilde{\phi}_q = \widetilde{\phi}_0 \left[1 - \frac{1+3\widetilde{g}^2}{2(4\pi f)^2} \frac{3}{2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + \widetilde{c}_f(\mu) m_\pi^2 \right]$$

Chiral doubler model

Nowak, Zahed, Rho, 2003;
Bardeen, Eichten, Hill, 2003

What are the values of h, $\widetilde{g}$?

Is $\widetilde{g} \approx g$ and h very small?



Axial charge in detail

$$C_{\gamma_i\gamma_5}(-t_{s1},t_{s2};\vec{r}) = \sum_{\vec{x},\vec{y}} \left\langle \bar{h}(x)\gamma_5 q(x)\ \bar{q}(z+\vec{r})\gamma_i\gamma_5 q(z+\vec{r})\ \ \left(\bar{h}(y)\gamma_i q(y)\right)^\dagger \right\rangle_{z-\text{fixed}}$$

$$R_{\gamma_i\gamma_5}(\vec{r}) = \frac{C_{\gamma_i\gamma_5}(-t_{s1},t_{s2};\vec{r})}{\left(\mathcal{Z}_q^S\right)^2 \times \exp[-\mathcal{E}_q\,(t_{s2}-t_{s1})]} \xrightarrow{|t_{s1}|,t_{s2}\gg 0} \langle B_q|\bar{q}\gamma_i\gamma_5 q(\vec{r})|B_q^*\rangle \equiv f_{\gamma_i\gamma_5}(r)$$

$$\widetilde{C}_{\gamma_i\gamma_5}(-t_{s1},t_{s2};r) = \sum_{\vec{x},\vec{y}} \left\langle \bar{h}(x)\mathbb{1}q(x)\ \bar{q}(z+\vec{r})\gamma_i\gamma_5 q(z+\vec{r})\ \ \left(\bar{h}(y)\gamma_i\gamma_5 q(y)\right)^\dagger \right\rangle_{z-\text{fixed}}$$

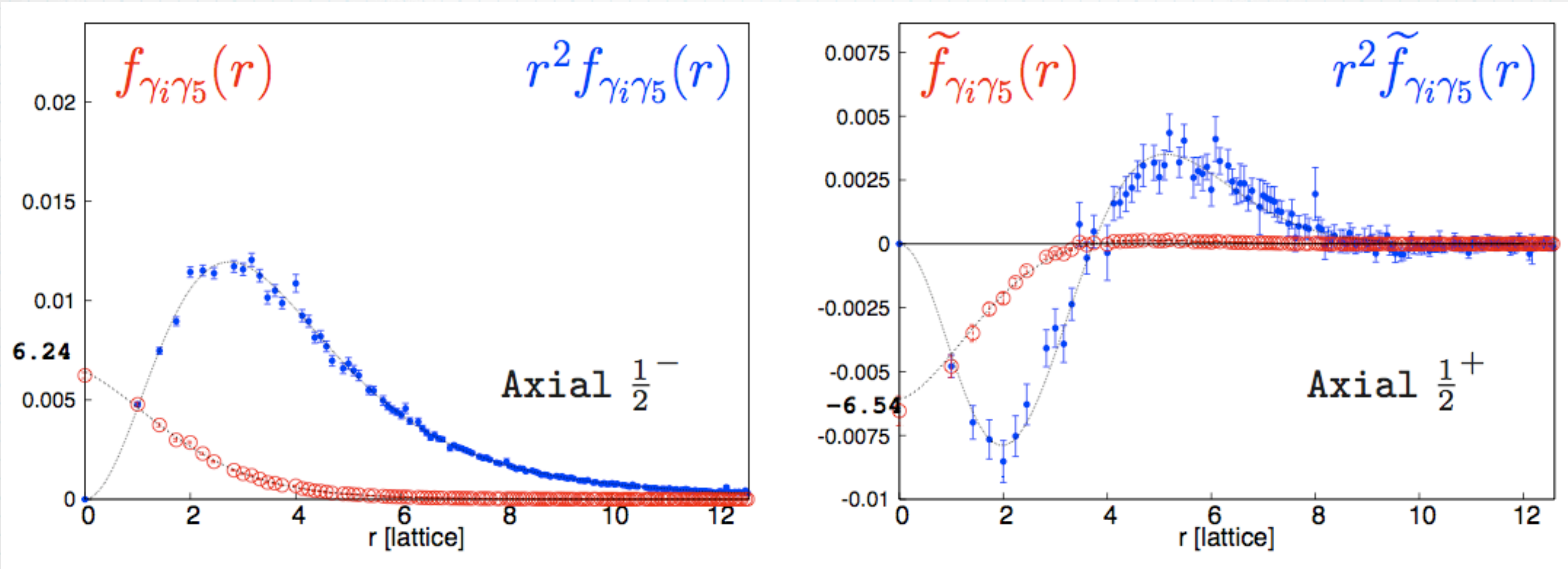
$$\widetilde{R}_{\gamma_i\gamma_5}(r) = \frac{\widetilde{C}_{\gamma_i\gamma_5}(-t_{s1},t_{s2};\vec{r})}{(\widetilde{\mathcal{Z}}_q^S)^2 \times \exp[-\widetilde{\mathcal{E}}_q\,(t_{s2}-t_{s1})]} \xrightarrow{|t_{s1}|,t_{s2}\gg 0} \langle B_{0q}^*|\bar{q}\gamma_i\gamma_5 q(r)|B_{1q}\rangle \equiv \widetilde{f}_{\gamma_i\gamma_5}(r)$$

Nontrivial check!!!
for `large' lattice (> 2 fm)

$$\sum_{r_x,r_y,r_z} f_\Gamma(r_x,r_y,r_z) = 4\pi\int_0^\infty r^2 f_\Gamma(r)dr$$

✔

Axial charge in detail



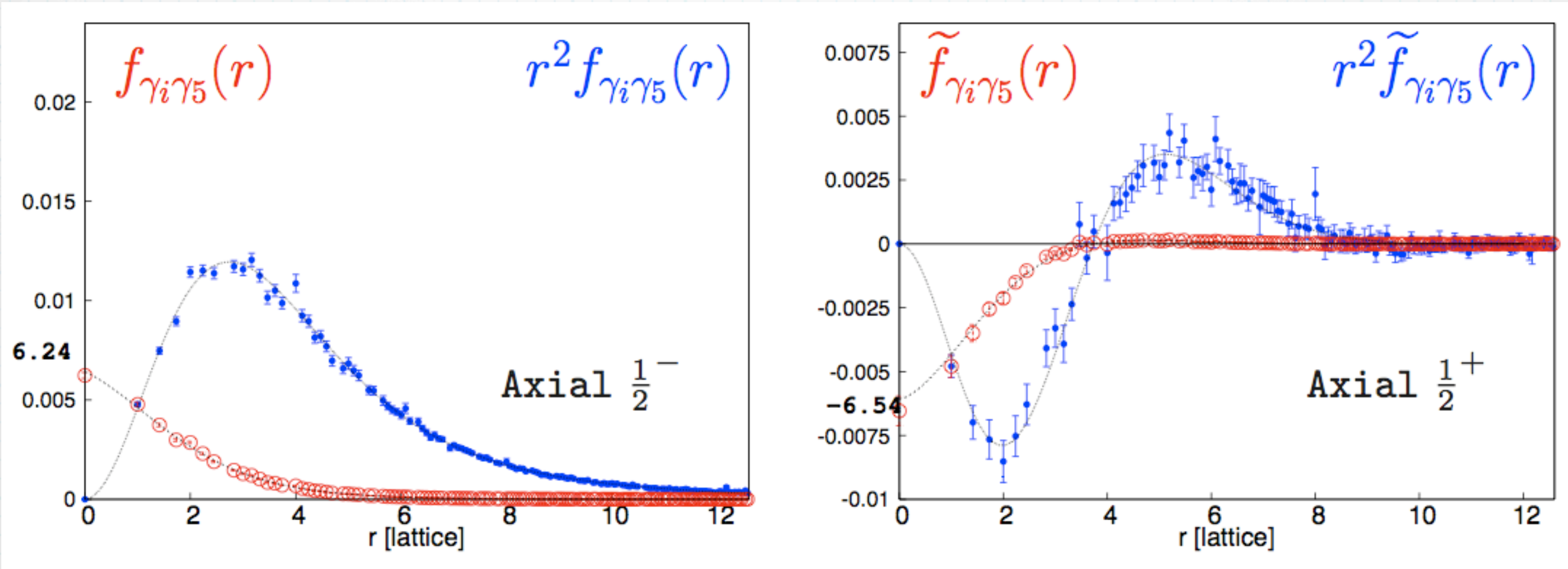
above:

light quark near the physical strange quark mass

Nf=2 dynamical Wilson/Clover quarks & Iwasaki gauge action, 24^3 X 48, a=0.1 fm

D.B. Chang, LeYaouanc 2009
Koponen 2007

Axial charge in detail



chiral extrapolation

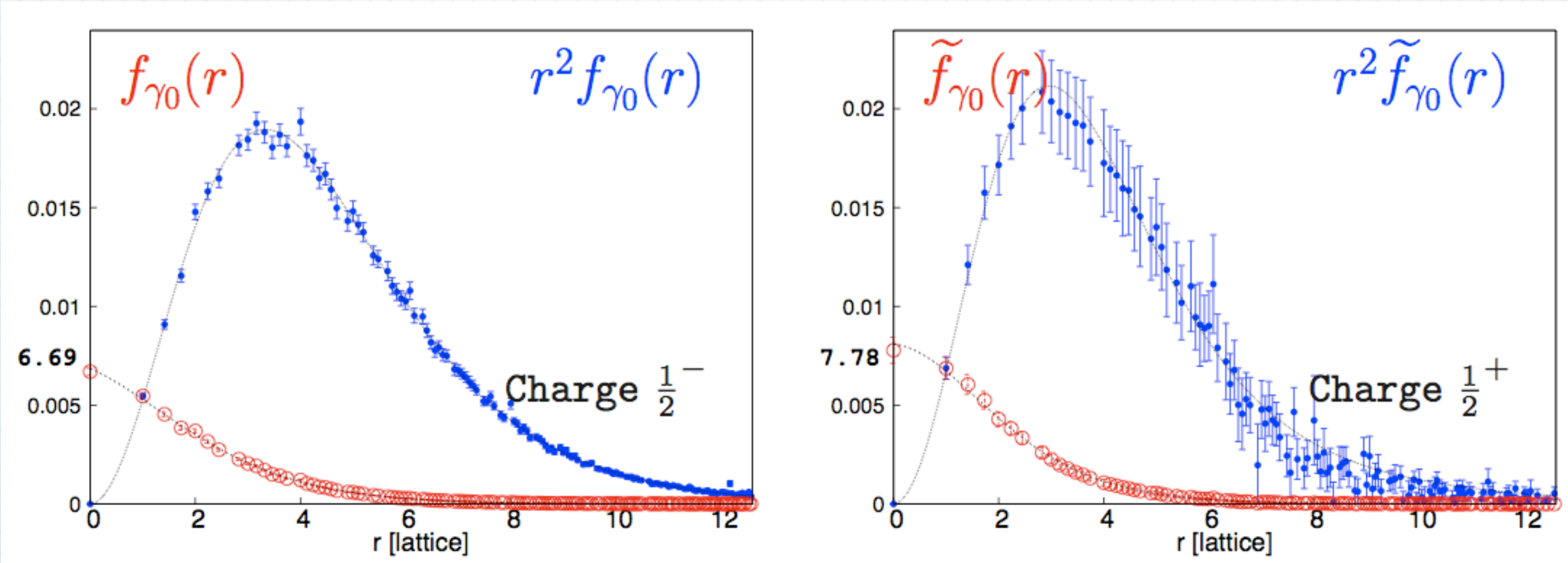
$$g_{\text{lin}} = 0.526(22) \qquad \tilde{g}_{\text{lin}} = -0.157(48)$$

$$g_0^{\text{HMChPT}} = 0.444(18) \qquad \tilde{g}_0^{\text{HMChPT}} = -0.14(6)$$

This is where you can test quark models: fix parameters with lowest lying masses and splittings - and check for both axial charge radial distributions, point by point!

$$\tilde{g} \ll g$$

Vector charge (fixed by VWI)

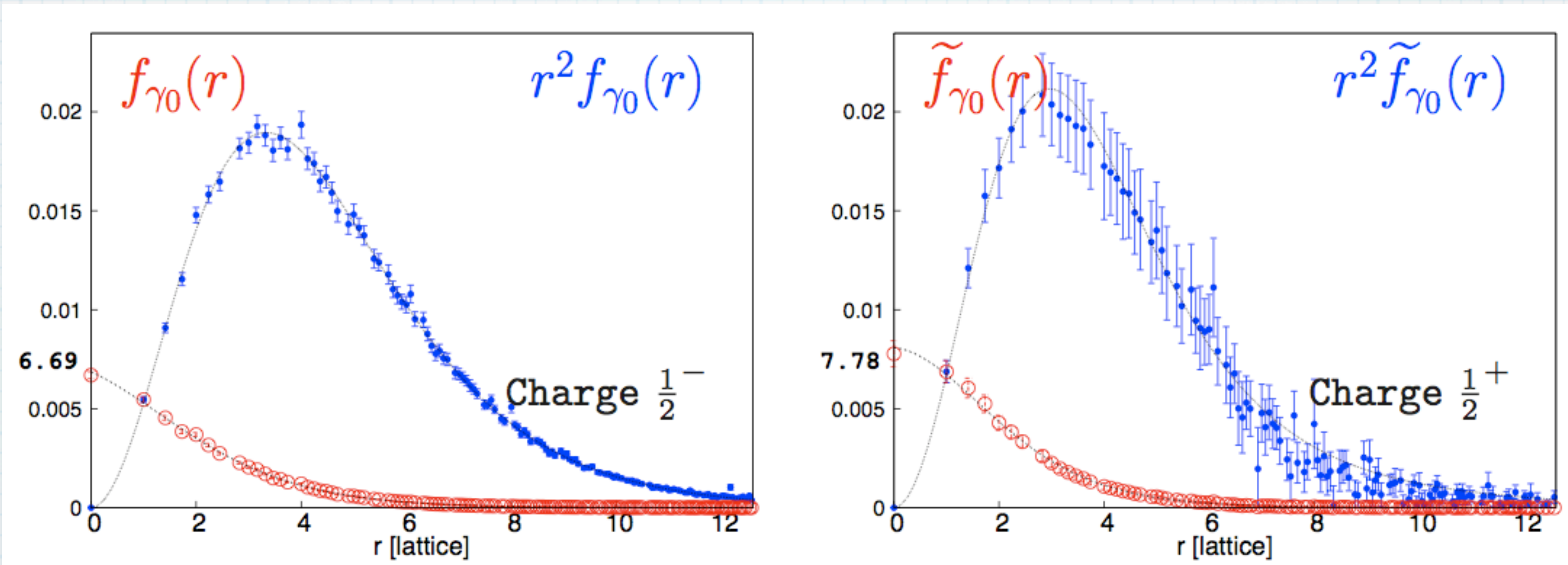


✖ $\langle P(p')|J_\mu^{\rm em}|P(p)\rangle = F(q^2)\,(p+p')\,, \quad \langle r^2\rangle = 6\left.\frac{dF(q^2)}{dq^2}\right|_{q^2=0}$

$$\langle r^2\rangle_{\frac{1}{2}^-} = \frac{\int_0^\infty r^4 f_{\gamma 0}(r)\;dr}{\int_0^\infty r^2 f_{\gamma 0}(r)\;dr}$$

$$\langle r^2\rangle^{\rm lin.}_{\frac{1}{2}^-} = 0.334(3)\ {\rm fm}^2\,, \qquad \langle r^2\rangle^{\rm lin.}_{\frac{1}{2}^+} = 0.280(16)\ {\rm fm}^2$$

Vector charge (fixed by VWI)



$$\langle P(p')|J_\mu^{\rm em}|P(p)\rangle = F(q^2)\,(p+p')\ , \quad \langle r^2\rangle = 6\left.\frac{dF(q^2)}{dq^2}\right|_{q^2=0}$$

CRSR (Pirjol, Uraltsev 1999)

$$\langle r^2\rangle^{\rm lin.}_{\frac{1}{2}^-} \approx \langle r^2\rangle^{\rm VMD}_{\frac{1}{2}^-} = 6/m_\rho^2 = 0.393\ {\rm fm}^2$$

$$\left.\frac{dF(q^2)}{dq^2}\right|_{q^2=0} = \frac{1}{8\alpha_{\rm em} e_q^2}\sum_{\rm exc}(2J+1)\frac{\Gamma(B_{\rm exc}\to B\gamma)}{|\vec{k}_\gamma|^3}$$

$$\Gamma(B^{*\pm}\to B^\pm\gamma) \le \frac{16}{81}\alpha_{\rm em}\langle r^2\rangle|\vec{k}_\gamma|^3 = 1.16(1)\ {\rm keV}$$

How to compute h-coupling ?

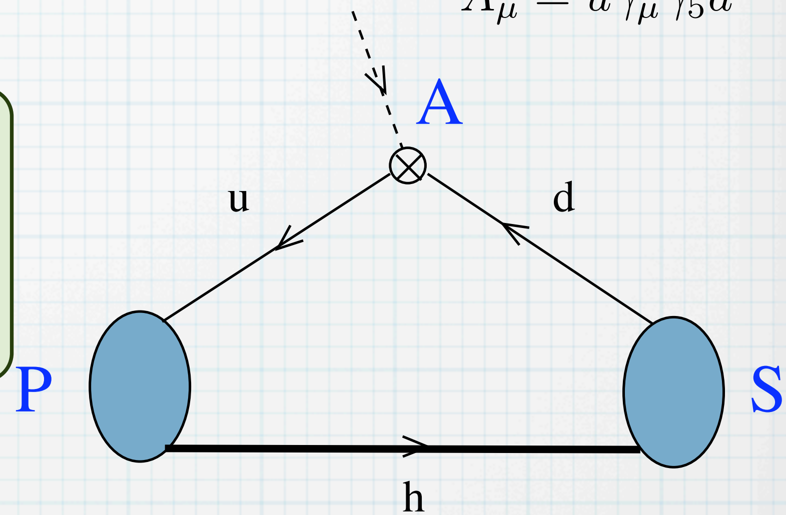
`Easy' part in two steps

$$\langle B(p')|A_\mu(0)|B_0^*(p)\rangle = (p+p')_\mu A_+(q^2) + q_\mu A_-(q^2)$$

$$\lim_{q^2\to 0}\langle B(p')|q^\mu A_\mu(0)|B_0^*(p)\rangle = (m_{B_0^*}^2 - m_B^2)A_+(0)$$

$$\frac{f_\pi m_\pi^2}{m_\pi^2 - q^2}\langle \pi^\pm(q)B(p')|B_0^*(p)\rangle = \langle B(p')|q^\mu A_\mu(0)|B_0^*(p)\rangle$$

$A_\mu = \bar{u}\gamma_\mu\gamma_5 d$



$\Rightarrow h = A_+(0)$

✘

$$\langle B|A_0(0)|B_0^*\rangle^{\rm HQET} = \frac{m_B + m_{B_0^*}}{2\sqrt{m_B m_{B_0^*}}}A_+(\Delta_S^2) + \frac{\Delta_S}{2\sqrt{m_B m_{B_0^*}}}A_-(\Delta_S^2) \xrightarrow{m_b\to\infty} A_+(\Delta_S^2)$$

q^2 non-zero in the static limit !
cannot give momentum
to a static source

How to compute h-coupling ?

$$C_{PAS}(t_y,t_x) = \langle\sum_{\vec{x},\vec{y}} P(y)A_0(0)S^\dagger(x)\rangle$$

$$C_{SAP}(t_y,t_x) = \langle\sum_{\vec{x},\vec{y}} S(y)A_0(0)P^\dagger(x)\rangle$$

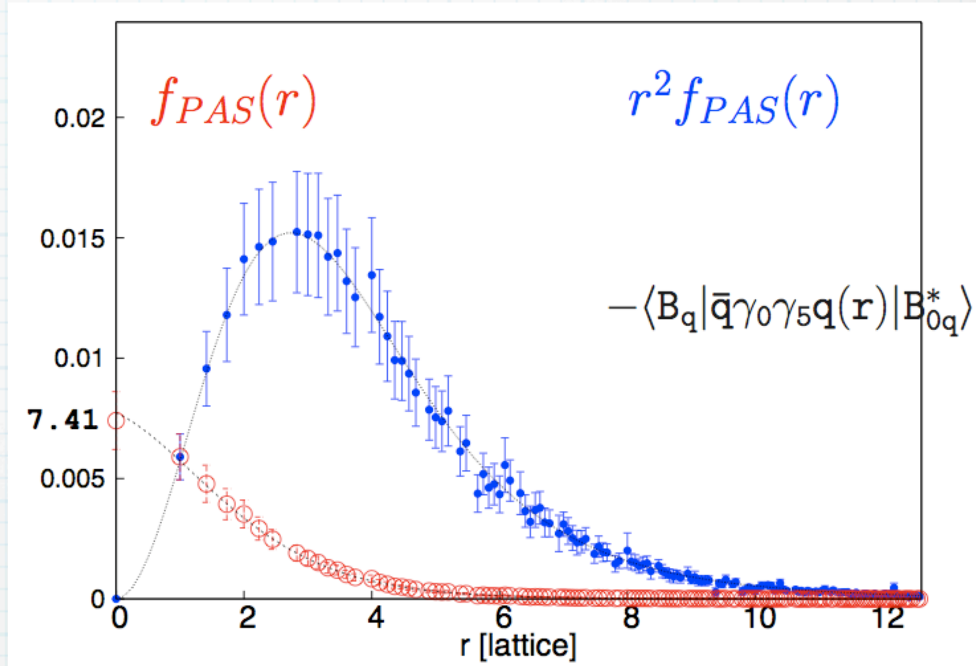
$$C_{PVP}(t_y,t_x) = \langle\sum_{\vec{x},\vec{y}} P(y)V_0(0)P^\dagger(x)\rangle$$

$$C_{SVS}(t_y,t_x) = \langle\sum_{\vec{x},\vec{y}} S(y)V_0(0)S^\dagger(x)\rangle$$

$$R_h(t) = -\frac{Z_A^2}{Z_V^2}\frac{C_{SAP}(t_y,t_x)\, C_{PAS}(t_y,t_x)}{C_{PVP}(t_y,t_x)\, C_{SVS}(t_y,t_x)}$$

$$\to -\frac{\langle B_0^*|A_0(0)|B\rangle\,\langle B|A_0(0)|B_0^*\rangle}{\langle B|V_0(0)|B\rangle\,\langle B_0^*|V_0(0)|B_0^*\rangle}$$

$$= [A_+(\Delta_S^2)]^2$$

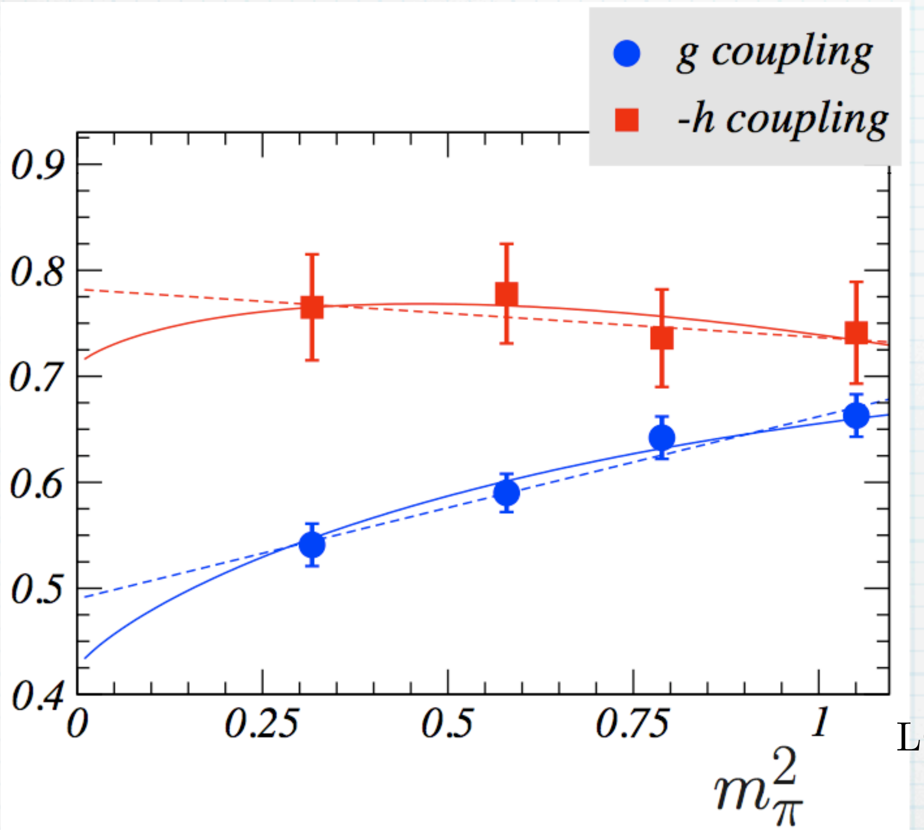


$$A_+(\Delta^2) = 4\pi\int_0^\infty dr\; r^2 f_{PAS}(r)$$

$$A_+(\Delta_S^2 - q_z^2) = \int d\vec{r} e^{-i\vec{q}\vec{r}} f_{PAS}(r) = A_+(\Delta^2)\left[1 - \frac{q_z^2}{6}\langle r^2\rangle_{PAS} + \frac{q_z^4}{120}\langle r^4\rangle_{PAS} + \ldots\right]$$

$$\Rightarrow A_+(0) = A_+(\Delta_q^2)\left(1 - \frac{\Delta_q^2}{6}\langle r^2\rangle_{PAS}^q + \frac{\Delta_q^4}{120}\langle r^4\rangle_{PAS}^q + \ldots\right)$$

h-coupling (1st latt.estimate ever!)



$$h = h_0 \left[1 - \frac{3}{4} \frac{3g_0^2 + 3\tilde{g}_0^2 + 2g_0\tilde{g}_0}{(4\pi f)^2} m_\pi^2 \log m_\pi^2 + c_h m_\pi^2 \right]$$

$$g = g_0 \left[1 - \frac{4g_0^2}{(4\pi f)^2} m_\pi^2 \log m_\pi^2 + c_g m_\pi^2 \right]$$

$$h_0 = -0.78(5)(^{+6}_{-0})$$

sic!

LARGE VALUE

LCSR: $h_0 = -0.60 \pm 0.13$ Colangelo, DeFazio 1998

✘ *if LCSR fail getting g right - why it works for h ?*

h-coupling (1st latt.estimate ever!)

$h_0 = -0.78(5)(^{+6}_{-0})$ $g_0 = 0.52(2)(^{+0}_{-8})$ $\widetilde{g}_0 = -0.16(5)(^{+2}_{-0})$

3 consequences:

❶ $$\Gamma(B_0^{*0} \to B^+\pi^-) = \frac{(m^2_{B_0^*} - m^2_B)^2}{8\pi f_\pi^2 m^3_{B_0^*}} m_B h^2 |\vec{q}_\pi| \Rightarrow \Gamma(B_0^{*\pm}) = 0.45 \left(^{+0.07}_{-0.12}\right) \text{ GeV}$$

❷ *HMChPT reliable only for extremely low pion masses. In lattice chiral extrapolations the contributions coming from scalar heavy-light states must be included in the expressions*

❸ *Models with chiral doubling in heavy-light mesons are be viable.*
Soft pion coupling in L=1 doublet is much smaller than in the L=0 doublet

Instead of conclusions

HMChPT:

✔ useful in guiding the chiral extrapolation of quantities computed on the lattice: Nf=3 less viable than Nf=2 [@ present level of accuracy!]

✔ inclusion of first excitations is necessary (formulas for decay constants and B-parameters in and beyond SM are available; NOT for the form factors!)

✔ dominant couplings in the lagrangian computed here (strategy to compute h devised!)

Instead of conclusions

Internal structure of heavy-light mesons:

- ✔ peculiar shapes of radial distributions of axial charge: very different in the lowest doublet from that of the first excitations
- ✔ radial distribution of vector/axial charge and matter for both lowest and excited doublet of mesons computed: test quark models (Dirac Model with scalar potential works very well -- refine the choice of parameters; the models of type Bakamjan-Thomas works well too)
- ✔ Can use models to study physics of higher excitations