Hadronic Matrix Elements with Heavy Quarks: The Charm-Loop Effect in $B \to K^{(*)}\ell^+\ell^-$

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[A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, to appear soon]

Introduction

- $B \to K^{(*)}\ell^+\ell^-$ via FCNC $b \to s\ell^+\ell^-$
- a benchmark channel for B-physics,
  distributions in $q^2 = (p_{\ell^+} + p_{\ell^-})^2$:
  the first measurements from BABAR,Belle,CDF,
  future: more precise LHCb data expected
- zero in the forward-backward asymmetry in $B \to K^*\ell^+\ell^-$
  an important indicator of New Physics
- Standard Model:

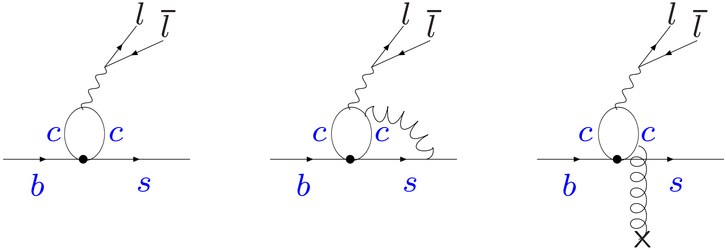
$$A(B \to K^{(*)}\ell^+\ell^-) = \langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B\rangle\,,$$

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\,,$$

- dominant contributions from $(\bar{s}b)(\bar{\ell}\ell)$ ($O_{9,10}$) operators,
  hadronic matrix elements reduced to $B \to K^{(*)}$ form factors
  available from the lattice QCD or QCD sum rules

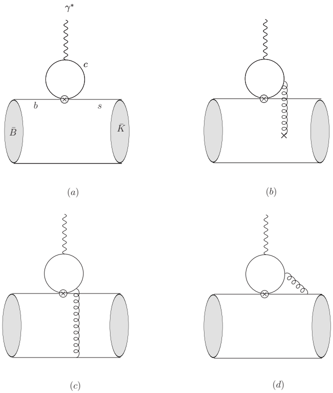
Charm-loops and other complications

- a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$ “mimicking FCNC”
- Charm-loop effect:



- similar effects:
  u, d, s, c, b-quark loops (quark-penguin operators O_{3-6}),
  u-loops from $O^u_{1,2}$ (CKM suppressed in $b \to s$),
  weak annihilation.
- In SM $A(B \to K^* \ell^+ \ell^-)$ includes new hadronic matrix elements, not simply form factors

Charm-loop in $B \to K^{(*)}\ell^+\ell^-$



- factorizable c-quark loop
$C_9 \to C_9 + (C_1 + 3C_2)g(m_c^2, q^2)$

- perturbative gluons $\to$ (nonfactorizable) corrections being factorized in $O(\alpha_s)$ and added to C_9
[M. Beneke,T. Feldmann,D. Seidel (2001)]

- how important are the soft gluons (low-virtuality, nonvanishing momenta) emitted from the c-quark loop ?

Charm loop turns charmonium

- at $q^2 \to m_{J/\psi}, \dots$ an on-shell hadronic state: $B \to J/\psi K \otimes J/\psi \to \ell^+\ell^-$
- other ψ-levels (charmonia with $J^P = 1^-$), open-charm states $B \to \bar{D}DK \to K\ell^+\ell^-$, exotic charmonia?, ($\bar{c}c$ states with the masses up to $m_B - m_K^{(*)}$)
- $B \to \psi K$: the naive factorization fails, hinting at large nonfactorizable contribuitons
- to avoid huge backgr., the charmonium resonances are subtracted from the q^2-distribuiton data in $B \to K^{(*)}\ell^+\ell^-$
- the effect of intermediate/virtual $\bar{c}c$ states remains at $q^2 \ll m^2_{J/\psi}$ (nonperturbative at $q^2 \sim 4m_c^2$)
- Can we use the $\{$ loop $\oplus$ corrections $\}$ ansatz?

Isolating the charm-loop in the decay amplitude

- the contribuiton of $O_{1,2}$ and e.m. interaction:

$$A(B \to K^{(*)}\ell^+\ell^-)^{(O_{1,2})} = -(4\pi\alpha_{em}Q_c)\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{\bar{\ell}\gamma^\mu\ell}{q^2}\mathcal{H}_\mu^{(B\to K^{(*)})},$$

- the relevant hadronic matrix element:

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q) = i\int d^4x e^{iq\cdot x}\langle K^{(*)}(p)|T\Big\{\bar{c}(x)\gamma_\mu c(x),$$
$$\Big[C_1O_1(0)+C_2O_2(0)\Big]\Big\}|B(p+q)\rangle,$$

- at small $q^2 \ll 4m_c^2$ use the operator-product expansion (OPE) for the T-product:

$$\mathcal{C}_\mu^a(q) = \int d^4x e^{iq\cdot x}T\Big\{\bar{c}(x)\gamma_\mu c(x),\bar{c}_L(0)\Gamma^a c_L(0)\Big\},$$

Expansion near the light-cone

- the dominant region in this T-product: $\langle x^2 \rangle \sim 1/(2m_c - \sqrt{q^2})^2$
- at $q^2 \ll 4m_c^2$: T- product of $\bar{c}c$-operators can be expanded near the light-cone $x^2 \sim 0$, schematically,

$$T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\gamma_\rho c_L(0)\} = C_0^{\mu\rho}(x^2, m_c^2) + \text{two-gluon term} + ...$$

$$T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\gamma_\rho \frac{\lambda^a}{2} c_L(0)\} = \int\limits_0^1 du C_1^{\mu\rho\alpha\beta}(x^2, m_c^2, u) G^a_{\alpha\beta}(ux) + ...$$

- after x-integration and taking hadronic matrix element: $O(C_1)/O(C_0) \sim O(C_{n+1})/O(C_n) \sim \Lambda^2_{QCD}/(4m_c^2 - q^2)$,
- but ! no local expansion possible in each term of LC OPE: $O(C_1) \sim \sum_{k=0}^{\infty} (q\Lambda_{QCD})^k/(4m_c^2 - q^2)^{k+1}$,

  $q \sim m_b/2$ and $m_b\Lambda_{QCD} \sim m_c^2$.

The resulting effective operators

- LO reduced to simple $\bar{c}c$ -loop,
  no difference between local and LC,

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho})\frac{9}{32\pi^2}\, g(m_c^2, q^2)\bar{s}_L\gamma^\rho b_L\,.$$

- gluon emission: use c-quark propagator near the light-cone in the external gluon field [I. Balitsky, V. Braun (1999)]
- define LC kinematics ($n_\pm$) in the rest-frame of B,
  $q \simeq (m_b/2)n_+$
- one-gluon emission yields a new nonlocal operator:

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q, m_c, \omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L\ ,$$

The local OPE limit

- $\omega \to 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\widetilde{\mathcal{O}}^{(0)}_{\mu}(q) = I_{\mu\rho\alpha\beta}(q)\bar{s}_L\gamma^{\rho}\widetilde{G}_{\alpha\beta}b_L \ ,$$

$$I_{\mu\rho\alpha\beta}(q, m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha} g_{\rho\beta}) \times \frac{1}{16\pi^2}\int_0^1 dt \, \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained
in $B \to X_s\gamma$ in *[M.Voloshin (1997)]*
in $B \to K^*\gamma$ *[A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]*

- the neccesity of resummation was discussed before
[Z. Ligeti, L. Randall and M.B. Wise,(1997);
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
J. W. Chen, G. Rupak and M. J. Savage,(1997);
G. Buchalla, G. Isidori and S.J. Rey (1997)]

Hadronic matrix elements for the charm-loop effect

- the LO: factorized $\bar{c}c$ loop

$$\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{fact} = \left(\frac{C_1}{3}+C_2\right)\langle K^{(*)}(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle\,,$$

- reduced to $B \to K^{(*)}$ form factors, nothing new
- The gluon emission yields:

$$\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle\,.$$

- new hadronic matrix element

$$\langle K^{(*)}(p)|\bar{s}_L\gamma^\rho\delta[\omega-\frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L|B(p+q)\rangle\,,$$

Charm-loop effect in $B \to K\ell^+\ell^-$

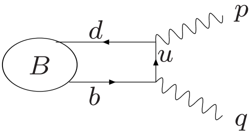
- The invariant amplitude:

$$\left[\mathcal{H}_\mu^{(B\to K)}(p,q)\right]_{fact.+nonfact.} = \left[(p\cdot q)q_\mu - q^2 p_\mu\right]$$
$$\times\left[\left(\frac{C_1}{3}+C_2\right)A(q^2) + 2C_1\widetilde{A}(q^2)\right]$$

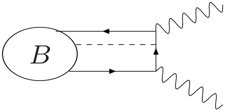
- the factorizable part $A(q^2) = \frac{9}{32\pi^2}\, g(m_c^2, q^2) f_{BK}^+(q^2)$
- Wilson coefficients enhance the nonfact. part $C_1/3 + C_2 \ll C_1$
- need nonperturbative QCD methods to calculate the form factor $f_{BK}^+(q^2)$ and the nonfactorizable amplitude $\widetilde{A}(q^2)$
- use one and the same LCSR approach

LCSR with B-meson distribution amplitudes

[A.K., T. Mannel, N.Offen,2005]



(a)



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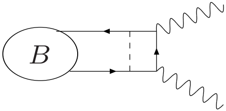


(c)

- a similar approach: LCSR for $B \to \pi$ in SCET

  [F.De Fazio, Th. Feldmann and T. Hurth, (2005)]

B-meson DA's

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

- defined in HQET; key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV
  [V.Braun, D.Ivanov, G.Korchemsky,2004]
- all $B \to \pi, K^{(*)}, \rho$ form factors calculated
- so far only tree-level calculations, 2,3-particle DA's
- model for 3-particle DA's obtained from 2-point QCD sum rules in HQET *[A.K., T.Mannel, N.Offen (2007)]*

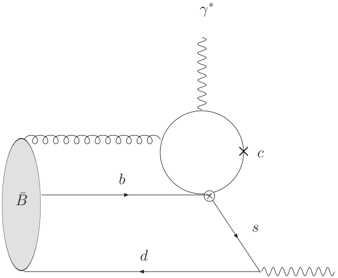
Form factors from LCSR with B-meson DA's

form factor	this work	LCSR with light-meson DA's
		[P.Ball and R.Zwicky] ([Duplancic et al])
$f^+_{B\pi}(0)$	0.25±0.05	0.258±0.031, $(0.26^{+0.04}_{-0.03})$
$f^+_{BK}(0)$	0.31±0.04	0.301±0.041±0.008
$f^T_{B\pi}(0)$	0.21±0.04	0.253±0.028
$f^T_{BK}(0)$	0.27±0.04	0.321±0.037±0.009
$V^{B\rho}(0)$	0.32±0.10	0.323±0.029
$V^{BK^*}(0)$	0.39±0.11	0.411±0.033±0.031
$A_1^{B\rho}(0)$	0.24±0.08	0.242±0.024
$A_1^{BK^*}(0)$	0.30±0.08	0.292±0.028±0.023
$A_2^{B\rho}(0)$	0.21±0.09	0.221±0.023
$A_2^{BK^*}(0)$	0.26±0.08	0.259±0.027±0.022
$T_1^{B\rho}(0)$	0.28±0.09	0.267±0.021
$T_1^{BK^*}(0)$	0.33±0.10	0.333±0.028±0.024

LCSR for the soft-gluon hadronic matrix element

- the correlation function:

$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i\int d^4y e^{ip\cdot y}$
$\langle 0|T\{j^K_\nu(y)\widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle\,,$



- hadronic dispersion relation in the kaon channel

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{if_K p_\nu}{m_K^2 - p^2}[(p\cdot q)q_\mu - q^2 p_\mu]\tilde{A}(q^2) + \int_{s_h}^{\infty} ds\, \frac{\tilde{\rho}_{\mu\nu}(s,q^2)}{s-p^2}$$

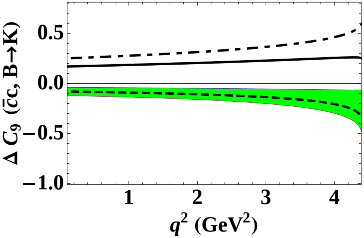
Charm-loop effect in $B \to K\ell^+\ell^-$ in terms of ΔC_9

- the effective coefficient $C_9(\mu = m_b) \simeq 4.4$
  a process-dependent correction to be added:

$$\Delta C_9^{(\bar{c}c, B\to K)}(q^2) = \frac{32\pi^2}{3}\frac{\mathcal{H}^{(B\to K)}(q^2)}{f^+_{BK}(q^2)}$$

$$= (C_1 + 3C_2)\, g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3}\frac{\tilde{A}(q^2)}{f^+_{BK}(q^2)}$$

$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}$,
$(\mu = m_b = 4.2\text{GeV})$



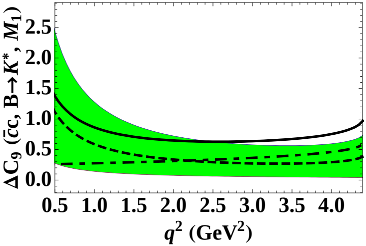
Charm-loop effect for $B \to K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \to K^*$ form factors $V^{BK*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) - 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)},$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\text{GeV}^2) = 0.7^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\text{GeV}^2) = 0.8^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\text{GeV}^2) = 1.1^{+1.1}_{-0.7}$



Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

$$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}]_{1,2}\,,$$

$$\left[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2}\,,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

$$[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2}\,,$$
$$[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2}\,. \qquad (1)$$

[P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]

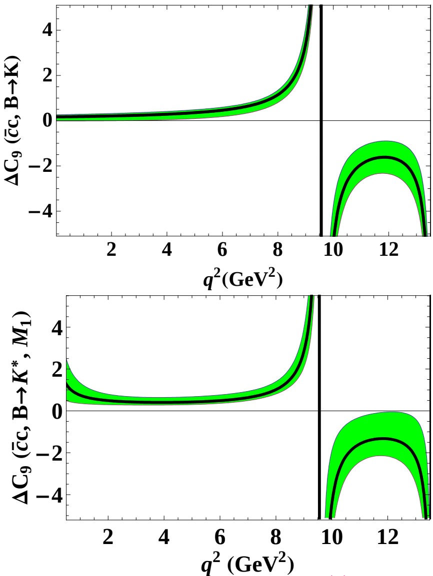
Accessing large q^2 with dispersion relation

- analyticity of the hadronic matrix element in q^2, dispersion relation:

$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2\Big[\sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - im_\psi\Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)}\Big]$$

- absolute values of the residues $|f_\psi A_{B\psi K}|$ from exp. data
- the integral over $\rho(s)$ fitted as an effective pole
  no attempt to use semi-local duality
- complex phases neglected, destructive intererence preferable, small integral over $\rho(s)$
- previous uses of dispersion relation: only factorizable part, positive residues, k-factors to accomodate data
  [F. Krüger, L. Sehgal (1997); A. Ali, P. Ball, L. T. Handoko, G. Hiller (2000)]

Charm-loop effect at large q^2

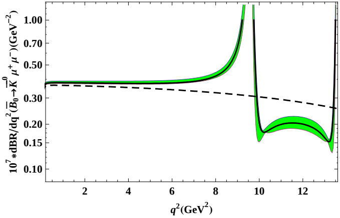


solid- central input,
green-shaded - uncertainties

- the dispersion relation ansatz coinsides with OPE result at $q^2 < 4.0$ GeV^2 and is valid up to $s = 4m_D^2$ (at $q^2 < m^2_{J/\psi}$ largely independent of higher-states ansatz)

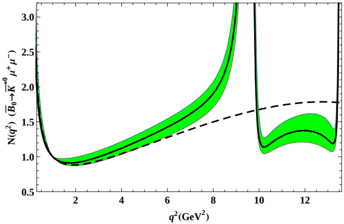
Influence on the observables for $B \to K\ell^+\ell^-$

- adding $\delta C_9(q^2)$ to the decay amplitude
- differential distribution in q^2 with (solid) and without (dashed) charn-loop effect



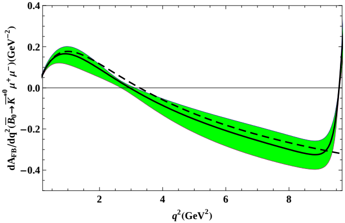
Observables for $B \to K^* \ell^+ \ell^-$

- differential distribution in q^2 with (solid) and without (dashed) charm-loop effect



- forward-backward asymmetry : $q_0^2 = 2.9^{+0.2}_{-0.3} \mathrm{GeV}^2$

  $\sim$ 10% larger without nonfactorizable correction



Summary

- contribution of four-quark operators with c-quarks in $B \to K^{(*)}\ell^+\ell^-$ is calculated at $q^2 \ll 4m_c^2$ using light-cone OPE
- soft-gluon emission - a nonlocal operator, effective resummmation of local operators, $\sim 1/(4m_c^2 - q^2)$-suppression
- LCSR with B meson DAs used to calculate the emerging hadronic matrix element

  (other methods, especially lattice QCD are welcome)
- charm loop yields an important correction to C_9 , especially for $B \to K^*\ell^+\ell^-$
- accuracy can be improved by including $O(\alpha_s)$ effects and CKM/Wilson coeff. suppressed loop corrections
- analytical continuation using dispersion relation and data on $B \to \psi K$ allows to access $q^2 \leq 4m_D^2$
- plans to extend this analysis to $B \to \rho(\pi)\ell^+\ell^-$