TWO-PHOTON AND TWO-GLUON DECAYS OF 0^{++} AND 2^{++} P-WAVE HEAVY QUARKONIUM STATES

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1 Introduction

• I would like to dedicate this talk to the memory of Professor Giuseppe Nardulli who, with great kindness and generosity has initiated the long collaboration I have with members of the Physics Department and INFN Bari.

• Recent CLEO measurements of $\chi_{c0,2}$ two-photon width has renewed interest in P-wave Heavy Quarkonium decays.

• Two-photon and two-gluon decay rates of S wave quarkonium state $\eta_{c,b}$ can be obtained from the wave function at the origin extracted from leptonic decay rates of the 1^{-} J/ψ and Υ states using Heavy quark spin symmetry (HQSS).

• No similar prediction for P wave χ_c and χ_b state.

• For P wave heavy quarkonium, the decay rates depends on the derivative of the wave function which can only obtained from a potential model in non relativistic bound state calculation.

• The derivative of the wave function at the origin is obtained in non relativistic bound state calculation from potential models.

• Need to express the P wave two-photon and two-gluon decay amplitudes in terms of a heavy quark field local operator extracted from some known physical processes or computed by QCD sum rules [Novikov, Okun, Shifman, Vainshtein, and Zakharov (1978); Reinders, Rubinstein , and Yazaki (1982); Dudek, Edwards and Richards (2006); Colangelo and Khodjamirian (2001); Colangelo, De Fazio and Pham (2002), Dudek, Edwards and Richards (2006)] or lattice simulations [Dudek, Edwards and Richards (2006); Chiu, Hsieh, and Ogawa (TWQCD Collaboration)(2007)] .

• This work :To derive an effective Lagrangian for the process $c\bar{c} \to \gamma\gamma$ by expanding the charm-quark propagator of the $c\bar{c} \to \gamma\gamma$ annihilation amplitudes in powers of q^2/m_c^2 , with $q = p_c - p_{\bar{c}}$, and neglecting terms of $\mathcal{O}(q^2/m_c^2)$ terms.

• The two-photon decay amplitude for the P-wave heavy quarkonium $\chi_{c0,2}$ given by the matrix element of the heavy quark energy-momentum tensor which can be expressed in terms of the heavy quark scalar density $\bar{c}c$.

• The matrix elements of the heavy quark vector, axial vector, energy momentum tensor and scalar density operators, like the full decay amplitude [Barbieri, Gatto, and Kogerler (1976)], be computed in terms of the derivative of the spatial wave function at the origin using relativistic spin projection operators [Kuhn, Kaplan and Safiani, 1979; Kuhn, Guberina, Peccei and Ruckl, (1979)].

• Obtain the corresponding decay constants in terms of the derivative at the origin of the spatial wave function of the P state.

• Express the decay amplitude in terms of the decay constants of local operators

• Our approach: Express directly the decay amplitude in terms of the corresponding decay constants for these local heavy quark field operator, as done in short distance operator expansion technique for non-leptonic weak interactions.

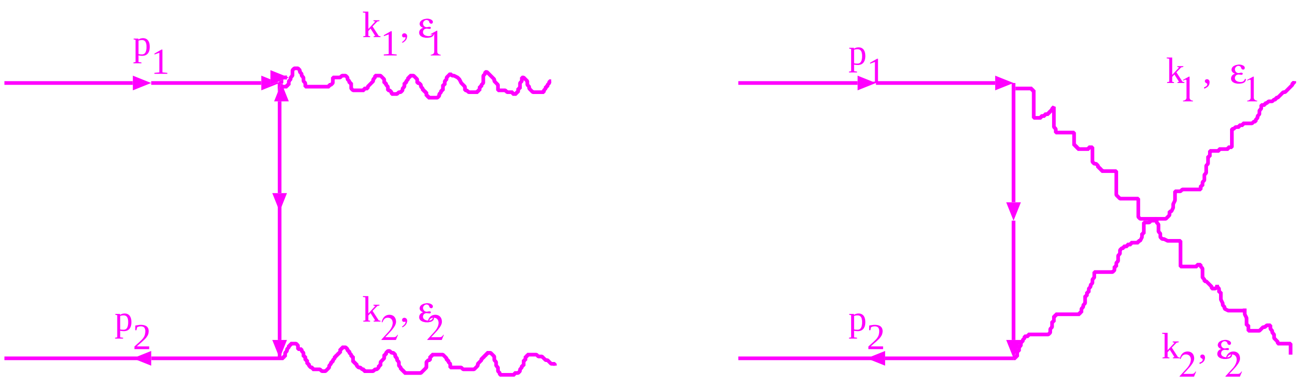

Figure 1: Diagrams for $Q\bar{Q}$ annihilation to two photons.

2 Matrix elements of Local operators

• Matrix elements of local operators in a fermion-antifermion system with a given spin and angular momentum is given by:

$$\mathcal{A} = \int \frac{d^4 q}{(2\pi)^4} \, \mathrm{Tr}\, \mathcal{O}(q)\chi(P,q) \tag{1}$$

P is the total 4-momentum of the quarkonium system, q is the relative 4-momentum between the quark and anti-quark and $\chi(P,q)$ is the Bethe-Salpeter wave function. This expression is that of Kuhn, Kaplan and Safiani; Guberina, Kuhn, Peccei and Ruckl (1979).

• For a quarkonium system in a fixed total, orbital and spin angular momentum $\chi(P,q)$ is given by ($\mathbf{q}$ is the relative 3-momentum of the bound state)

$$\chi(P,q;J,J_z,L,S) = \sum_{M,S_z} 2\pi\delta(q^0 - \frac{\mathbf{q}^2}{2m})\psi_{LM}(\mathbf{q}) < LM; SS_z|JJ_z >$$

$$\times\sqrt{\frac{3}{m}}\sum_{s,\bar{s}} u(P/2+q,s)\bar{v}(P/2-q,\bar{s}) < \frac{1}{2}s;\frac{1}{2}\bar{s}|SS_z>$$

$$= \sum_{M,S_z} 2\,\pi\,\delta(q^0 - \frac{\mathbf{q}^2}{2\,m})\psi_{LM}(\mathbf{q})\mathcal{P}_{SS_z}(P,q) < LM; SS_z|JJ_z> \tag{2}$$

The spin projection operators $\mathcal{P}_{SS_z}(P,q)$ are

$$\mathcal{P}_{0,0}(P,q) = \sqrt{\frac{3}{8m^3}}[-(\not{P}/2+\not{q})+m]\gamma_5\,[(\not{P}/2-\not{q})+m]$$

$$\mathcal{P}_{1,S_z}(P,q) = \sqrt{\frac{3}{8m^3}}[-(\not{P}/2+\not{q})+m]\not{\epsilon}(S_z)\,[(\not{P}/2-\not{q})+m] \tag{3}$$

3 Effective Lagrangian for $\chi_{c0,2} \to \gamma\gamma$ and $\chi_{b0,2} \to \gamma\gamma$

- From the P-wave part of the $c\bar{c} \to \gamma\gamma, gg$ and $b\bar{b} \to \gamma\gamma, gg$ amplitudes:

$$\begin{aligned} \mathcal{L}_{\rm eff}(Q\bar{Q} \to \gamma\gamma) &= -ic_1 A_{\mu\nu}\bar{Q}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu Q \\ c_1 &= -e^2 Q^2_{c,b}[(k_1 - k_2)^2/4 - m_Q^2]^{-2} \end{aligned} \tag{4}$$

With the matrix element of $\theta_{Q\mu\nu} = \bar{Q}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu Q$ between the vacuum and $\chi_{c0,2}$ or $\chi_{b0,2}$ given by $(Q^2 = M^2)$:

$$\begin{aligned} < 0|\theta_{Q\mu\nu}|\chi_0 > &= T_0 M^2(-g_{\mu\nu} + Q_\mu Q_\nu/M^2), \\ < 0|\theta_{Q\mu\nu}|\chi_2 > &= -T_2 M^2 \epsilon_{\mu\nu}. \end{aligned} \tag{5}$$

- The two-photon decay amplitudes are then easily obtained:

$$\mathcal{M}(\chi_0 \to \gamma\gamma) = -e^2 Q^2_{c,b} \frac{T_0 A_0}{[M^2/4 + m_Q^2]^2} \tag{6}$$

$$\mathcal{M}(\chi_2 \to \gamma\gamma) = -e^2 Q_{c,b}^2 \frac{T_2 A_2}{[M^2/4 + m_Q^2]^2} \tag{7}$$

where

$$A_0 = (\frac{3}{2}) M^2 (M^2 \epsilon_1 \cdot \epsilon_2 - 2\epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1) \tag{8}$$

$$A_2 = M^2 \epsilon_{\mu\nu} [M^2 \epsilon_{1\mu} \epsilon_{2\nu} - 2(\epsilon_1 \cdot k_2 \epsilon_{2\mu} k_{1\nu} + \epsilon_2 \cdot k_1 \epsilon_{1\mu} k_{2\nu} + \epsilon_1 \cdot \epsilon_2 k_{1\mu} k_{2\nu})] \tag{9}$$

• $T_2 = \sqrt{3} T_0$ from HQSS.

• For QCD sum rules calculation or lattice simulation, it is simpler to compute the trace of the energy-momentum tensor $\theta_{Q\mu\mu}$ to obtain the decay amplitudes.

$$\theta_{Q\mu\mu} = 2m_Q \bar{Q} Q \tag{10}$$

and

$$\bar{v}(p_2) T_{\mu\mu} u(p_1) = 2m_Q \bar{v}(p_2) u(p_1) \tag{11}$$

• The problem of computing the two-photon or two-gluon decays of $\chi_{c0,2}$ and $\chi_{b0,2}$ states is reduced to computing the decays constants $f_{\chi_{c0}}$ or

$f_{\chi_{b0}}$ defined as:

$$< 0|\bar{Q}Q|\chi_0 >= m_{\chi_0} f_{\chi_0} \tag{12}$$

• T_0 is given directly in terms of f_{χ_0} without using the derivative at the origin of the P-wave saptial wave function.

$$T_0 = \frac{f_{\chi_0}}{3} \tag{13}$$

• The decay rates of $\chi_{c0,2}$, $\chi_{b0,2}$ states can now be obtained in terms of the decay constant f_{χ_0}.

$$\Gamma_{\gamma\gamma}(\chi_{c0}) = \frac{4\pi Q_c^4 \alpha_{em}^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}} + b)^4} \left[1 + B_0(\alpha_s/\pi)\right], \tag{14}$$

$$\Gamma_{\gamma\gamma}(\chi_{c2}) = \left(\frac{4}{15}\right) \frac{4\pi Q_c^4 \alpha_{em}^2 M_{\chi_{c2}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c2}} + b)^4} \left[1 + B_2(\alpha_s/\pi)\right] \tag{15}$$

where $B_0 = \pi^2/3 - 28/9$ and $B_2 = -16/3$ are NLO QCD radiative corrections.

- This expression is similar to that for η_c:

$$\Gamma_{\gamma\gamma}(\eta_c) = \frac{4\pi Q_c^4 \alpha_{em}^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c}+b)^2}\left[1 - \frac{\alpha_s}{\pi}\frac{(20-\pi^2)}{3}\right] \tag{16}$$

- The two-gluon decay rates:

$$\Gamma_{gg}(\chi_{c0}) = \left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}}+b)^4}[1 + C_0(\alpha_s/\pi)], \tag{17}$$

$$\Gamma_{gg}(\chi_{c2}) = \left(\frac{4}{15}\right)\left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2 M_{\chi_{c2}}^3 f_{\chi 0}^2}{(M_{\chi_{c2}}+b)^4}[1 + C_2(\alpha_s/\pi)] \tag{18}$$

where $C_0 = 8.77$ and $C_2 = -4.827$ are NLO QCD radiative corrections.

- The expression for $\Gamma_{gg}(\eta_c)$ is similar:

$$\Gamma_{gg}(\eta_c) = \left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c}+b)^2}\left[1 + 4.8\frac{\alpha_s}{\pi}\right] \tag{19}$$

• The decay constants in terms of the wave function:

$$f_{\eta_c} = \sqrt{\frac{3}{32\,\pi\,m_Q^3}}\,\mathcal{R}_0(0)\,(4\,m_Q)\ , \tag{20}$$

$$f_{\chi_0} = 12\sqrt{\frac{3}{(8\pi m_Q)}}\left(\frac{\mathcal{R}_1'(0)}{M}\right) \tag{21}$$

$$\tag{22}$$

• In terms of the wave functions, $f_{\chi_{c0}} = 6\left(\frac{\mathcal{R}_1'(0)}{\mathcal{R}_0(0)M}\right) f_{\eta_c}$ and becomes comparable to f_{η_c}.

• Thus by comparing the expression for χ_{c0} and η_c we could already have some estimate for the χ_{c0} two-photon and two-gluon decay rates. For $f_{\chi_{c0}}$ of $O(f_{\eta_c})$, one would expect $\Gamma_{\gamma\gamma}(\chi_{c0})$ to be in the range of a few keV.

• The decay constants from QCD sum rules : $f_{\chi_{c0}} = m_{\chi_0}\, g$:

$$f_{\chi_{c0}} = 357\,\mathrm{MeV}, \mathrm{Novikov} \quad \mathrm{et\ al}$$
$$= 510\,\mathrm{MeV}, \mathrm{Colangelo} \quad \mathrm{et\ al} \qquad (23)$$

• As shown in Table 1, the predicted two-photon width of χ_{c0} from the sum rules value for $f_{\chi_{c0}}$ is however almost twice the CLEO value, but possibly with large theoretical uncertainties in sum rules calculation for $f_{\chi_{c0}}$, as to be expected.

• The measured ratio $\Gamma_{\gamma\gamma}(\chi_{c2})/\Gamma_{\gamma\gamma}(\chi_{c0})$ is then $\approx 0.24 \pm 0.09$, somewhat bigger than the predicted value of about 0.14 as shown in Table 1.

Reference	$\Gamma_{\gamma\gamma}(\chi_{c0})$(keV)	$\Gamma_{\gamma\gamma}(\chi_{c2})$(keV)	$R = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})}$
Barbieri	3.5	0.93	0.27
Godfrey	1.29	0.46	0.36
Barnes	1.56	0.56	0.36
Bodwin	6.70 ± 2.80	0.82 ± 0.23	$0.12^{+0.15}_{-0.06}$
Gupta	6.38	0.57	0.09
Münz	1.39 ± 0.16	0.44 ± 0.14	$0.32^{+0.16}_{-0.12}$
Huang	3.72 ± 1.10	0.49 ± 0.16	$0.13^{+0.11}_{-0.06}$
Ebert	2.90	0.50	0.17
Schuler	2.50	0.28	0.11
Crater	$3.34 - 3.96$	$0.43 - 0.74$	$0.13 - 0.19$
Wang	3.78	–	–
Laverty	$1.99 - 2.10$	$0.30 - 0.73$	$0.14 - 0.37$
This work	5.00	0.70	0.14
Exp(CLEO)	$2.53 \pm 0.37 \pm 0.26$	$0.60 \pm 0.06 \pm 0.06$	$0.24 \pm 0.04 \pm 0.03$
Exp(Average)	$2.31 \pm 0.10 \pm 0.12$	$0.51 \pm 0.02 \pm 0.02$	$0.20 \pm 0.01 \pm 0.02$

Table 1: Potential model predictions for $\chi_{c0,2}$ two-photon widths compared with this work.

- CLEO measurements give : $(2.53 \pm 0.37 \pm 0.26)\,\text{keV}$ and $(0.60 \pm 0.06 \pm 0.06)\,\text{keV}$ for the two-photon width of χ_{c0} and χ_{c2} respectively.

- Ratio of the decay constants:

$$f_{\chi_{c0}} = 6 \left(\frac{\mathcal{R}_1'(0)}{\mathcal{R}_0(0) M} \right) f_{\eta_c} \tag{24}$$

- Two-photon branching ratio for η_c and $\eta_{c'}$ is given by

$$\mathcal{B}(\eta_c, \eta_{c'} \to \gamma\gamma) = \frac{9}{2}\, Q_c^4\, \frac{\alpha_{em}^2}{\alpha_s^2} \left(1 - 8.2\, \frac{\alpha_s}{\pi} \right) \tag{25}$$

with α_s evaluated at the appropriate scale.

- This relation gives, $\Gamma(\eta_{c'} \to \gamma\gamma) = (4.1 \pm 2.3)\,\text{keV}$.

- Extraction of $\Gamma_{\gamma\gamma}(\eta_c')$

- Using the Babar measurement of the ratio:

$$R(\eta_c(2S)K^+/\eta_c K^+) = \frac{\mathcal{B}(B^+ \to \eta_c(2S)K^+) \times \mathcal{B}(\eta_c(2S) \to K\bar{K}\pi)}{\mathcal{B}(B^+ \to \eta_c K^+) \times \mathcal{B}(\eta_c \to K\bar{K}\pi)} = 0.096^{+0.020}_{-0.019}(\mathrm{stat}) \pm 0.025(\mathrm{syst}) \quad (26)$$

and the Belle measurement:
$\mathcal{B}(B^+ \to \eta_c K^+) \times \mathcal{B}(\eta_c \to K\bar{K}\pi) = (6.88 \pm 0.77^{+0.55}_{-0.66}) \times 10^{-5}$,
BABAR Collaboration gives (quoted by CLEO) :

$$\mathcal{B}(\eta_c' \to K_S K\pi) = (1.9 \pm 0.4(\mathrm{stat}) \pm 1.1(\mathrm{syst}))\% \quad [\mathrm{BABAR}] \quad (27)$$

- which is rather small comopared with:

$$\mathcal{B}(\eta_c \to K_S K\pi) = (7.0 \pm 1.2)\% \quad [\mathrm{PDG}] \quad (28)$$

- With the Babar result for $\mathcal{B}(\eta_c' \to K_S K\pi)$ and the CLEO measurement:

$$R(\eta_c'/\eta_c) = \frac{\Gamma_{\gamma\gamma}(\eta_c') \times \mathcal{B}(\eta_c' \to K_S K\pi)}{\Gamma_{\gamma\gamma}(\eta_c) \times \mathcal{B}(\eta_c \to K_S K\pi)} = 0.18 \pm 0.05 \pm 0.02 \quad (29)$$

- CLEO Collaboration obtains:

$$\Gamma(\eta_{c'} \to \gamma\gamma) = (4.8 \pm 3.7)\,\text{keV} \tag{30}$$

in agreement with the predicted value $\Gamma(\eta_{c'} \to \gamma\gamma) = (4.1 \pm 2.3)\,\text{keV}$

- while the Belle result

$$R(\eta_c' K/\eta_c K) = \frac{\mathcal{B}(B^0 \to \eta_c' K^0) \times \mathcal{B}(\eta_c' \to K_S K^+ \pi^-)}{\mathcal{B}(B^0 \to \eta_c K^0) \times \mathcal{B}(\eta_c \to K_S K^+ \pi^-)} = 0.38 \pm 0.12 \pm 0.05 \tag{31}$$

leads to

$$\Gamma_{\gamma\gamma}(\eta_c') = 1.3 \pm 0.6\,\text{keV} \tag{32}$$

- Similarly, for $\chi_{c0,2}$:

$$\mathcal{B}(\chi_{c0}, \chi_{c'0} \to \gamma\gamma) = \frac{9}{2}\, Q_c^4\, \frac{\alpha_{em}^2}{\alpha_s^2} \left(1 + (B_0 - C_0)\frac{\alpha_s}{\pi}\right) \tag{33}$$

$$\mathcal{B}(\chi_{c2}, \chi_{c'2} \to \gamma\gamma) = \frac{6}{5}\, Q_c^4\, \frac{\alpha_{em}^2}{\alpha_s^2} \left(1 + (B_2 - C_2)\frac{\alpha_s}{\pi}\right) \tag{34}$$

with $B_0 = \pi^2/3 - 28/9, B_2 = -16/3, C_0 = 8.77, C_2 = -4.827$

• For $\alpha_s = 0.26$, $\mathcal{B}(\eta_c \to \gamma\gamma) = 3.6 \times 10^{-4}$ to be compared with the measured value of $(2.8 \pm 0.9) \times 10^{-4}$(PDG), but this prediction is rather sensitive to α_s: with $\alpha_s = 0.28$, one would get $\mathcal{B}(\eta_c \to \gamma\gamma) = 2.95 \times 10^{-4}$, in better agreement with experiment.

• The predicted and measured BR: with $\alpha_s = 0.28$:

$$\eta_c : 2.90 \times 10^{-4}, (2.4^{+1.1}_{-0.9}) \times 10^{-4} \tag{35}$$

$$\chi_{c0} : 3.45 \times 10^{-4}, (2.35 \pm 0.23) \times 10^{-4} \tag{36}$$

$$\chi_{c2} : 4.45 \times 10^{-4}, (2.43 \pm 0.18) \times 10^{-4},$$

• For η_c and χ_{c0} QCD radiative corrections are important in bringing the predictions close to experiments.

• For χ_{c2} , QCD radiative corrections give a too large ratio of two-photon to two-gluon decay rates. Need $\alpha_s = 0.36$ to get the measured $\mathcal{B}(\chi_{c2} \to \gamma\gamma)$.

• The $Z(3930)$ state with $M = (3928 \pm 5 \pm 2)\,\mathrm{MeV}$ found by Belle Collaboration : $\Gamma_{\gamma\gamma}(\chi'_{c2}) \times \mathcal{B}(D\bar{D}) = (0.18 \pm 0.05 \pm 0.03)\,\mathrm{keV}$

• Babar Collaboration : $\Gamma_{\gamma\gamma}(\chi'_{c2}) \times \mathcal{B}(D\bar{D}) = (0.24 \pm 0.05 \pm 0.04)\,\mathrm{keV}$

• $\mathcal{B}(D\bar{D}) \approx 0.70 - 1$ [Colangelo; Swanson; Li, Chao]:
$\Gamma_{\gamma\gamma}(\chi'_{c2}) = (0.18 - 0.24 \pm 0.05 \pm 0.03)\,\mathrm{keV}$

• $f_{\chi'_{c0}} \simeq 195 - 225\,\mathrm{MeV}$. This implies $\Gamma_{gg}(\chi'_{c0})$ in the range $5 - 10\,\mathrm{MeV}$.

• For $\chi_{b0,2}$ potential model calculation gives the two-photon width about 1/10 of that for η_b , which implies $f_{\chi_{b0}} = f_{\eta_b}/3$, smaller than Cornell potential value $f_{\chi_{b0}} = 0.46\, f_{\eta_b}$.

4 Conclusion

• Effective Lagrangian approach and HQSS can be used to compute quarkonium decays into lepton and photon with relativistic kinematic.

• Babar measurement of $\mathcal{B}(\eta_c' \to K_S K \pi)$ allows an extraction $\Gamma_{\gamma\gamma}(\eta_c') = (4.8 \pm 3.7)\,\mathrm{keV}$ in good agreement with the predicted value of $(4.1 \pm 2.3)\,\mathrm{keV}$.

• For χ_{c0} and χ_{b0}, existing sum rules calculation for $f_{\chi_{c0}}$ however produces a two-photon width about $5\,\mathrm{keV}$, somewhat bigger than the CLEO measurement.

• It remains to be seen whether a better determination of $f_{\chi_{c0}}$ could bring the $\chi_{c0,2}$ two-photon decay rates closer to experiments or higher order QCD radiative corrections and relativistic corrections are needed to explain the data.

• The measured η_c and $\chi_{c0,2}$ two-photon branching ratios require $\alpha_s \approx 0.28$, far bigger than the value of $\alpha_s \approx 0.19$ from $J/\psi \to ggg$ [Kwong, Mackenzie, Rosner(1988); Voloshin (2008)].

• Relativistic corrections should be small for $\chi_{b0,2}$. Two-photon and two-gluon decays could be used to test QCD predictions and determination of α_s at the $\chi_{b0,2}$ mass.

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