PREDICTIONS FOR HEAVY QUARKONIUM AT WEAK COUPLING

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Motivation:

- 1st principle computation of heavy quarkonium properties from QCD
- Determination of Standard Model parameters: m_Q, α_s, ...

We have an effective field theory, Potential Non-Relativistic QCD, which describes the heavy quarkonium dynamics in the weak and strong coupling situation. $m \gg mv \gg mv^2$

$$\left.\begin{array}{l} \left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_s^{(0)}(r)\right)\Phi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with other low} \\ \quad\text{energy degrees of freedom} \end{array}\right\} \text{potential NRQCD} \qquad E \sim mv^2$$

In the weak coupling regime the starting point is $V_s^{(0)} = -C_f \frac{\alpha_s(\mu)}{r}$.

Golden Mode: Bottomonium ground state (?)

In the strong coupling regime case

$$V_s^{(0)}(\mathbf{r}) = \lim_{T\to\infty} \frac{i}{T} \log \langle W_\Box \rangle \quad \text{Wilson, Susskind}$$

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Phenomenological analysis:

- Which states belong to the weak/strong coupling regime?
  To which extent the static potential can be described with perturbation theory
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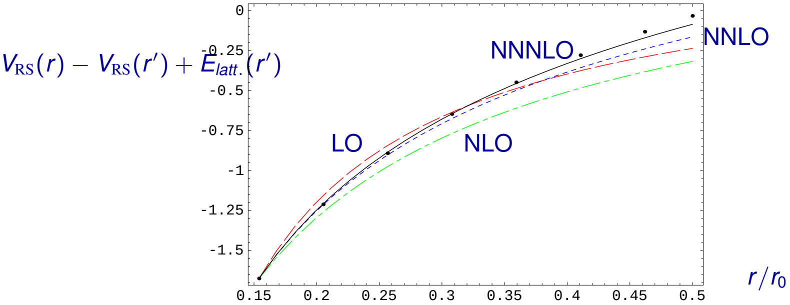
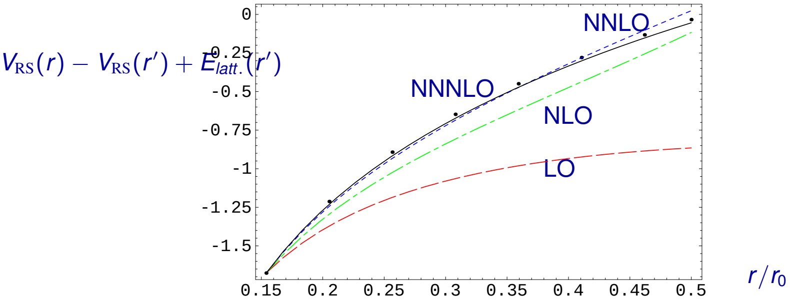

$V_{RS}(r) - V_{RS}(r') + E_{latt.}(r')$
NNLO
NNNLO
LO
NLO
r/r_0
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5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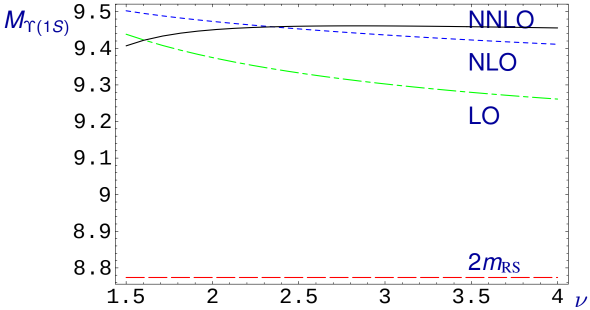
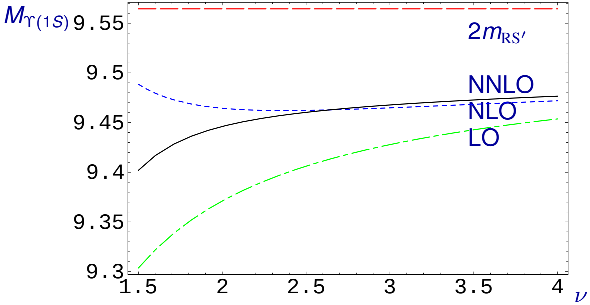

$M_{\Upsilon(1S)}$
9.5
9.4
9.3
9.2
9.1
9
8.9
8.8
NNLO
NLO
LO
$2m_{RS}$
1.5
2
2.5
3
3.5
4
ν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9.5
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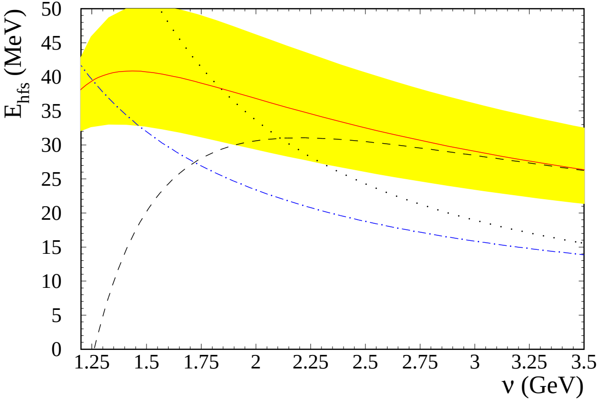
Kniehl, Penin, Smirnov, Steinhauser, Pineda;
Penin, Smirnov, Steinhauser, Pineda

$$\delta E \sim m\alpha^4 + m\alpha^5 \ln\alpha + m\alpha^6 \ln^2\alpha + \cdots$$
$$+ \quad m\alpha^5 + m\alpha^6 \ln\alpha + m\alpha^7 \ln^2\alpha + m\alpha^8 \ln^3\alpha + \cdots$$



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Relation of the vacuum polarization and $\Gamma(V \to e^+e^-)$

$$J^\mu = \bar{Q}\gamma^\mu Q = c_1 \psi^\dagger \boldsymbol{\sigma}\chi + \cdots , \qquad c_1 = 1 + a_1\alpha_s + a_2\alpha_s^2 + \cdots$$

$$(q_\mu q_\nu - g_{\mu\nu})\Pi(q^2) = i\int d^4x e^{iqx}\langle \mathrm{vac}|J_\mu(x)J_\nu(0)|\mathrm{vac}\rangle$$

$$\Pi(q^2) \sim c_1^2 \langle \mathbf{r}=\mathbf{0}|\frac{1}{E-H}|\mathbf{r}=\mathbf{0}\rangle$$

$$G(0,0,E) = \sum_{m=0}^{\infty}\frac{|\phi_m(0)|^2}{E_m - E + i\epsilon} + \frac{1}{\pi}\int_0^\infty dE' \frac{|\phi_{E'}(0)|^2}{E' - E + i\epsilon}$$

$$\Gamma(V \to e^+e^-) \sim \frac{1}{m^2}c_v^2|\phi_n^v(\mathbf{0})|^2 \qquad \Gamma(P \to \gamma\gamma) \sim \frac{1}{m^2}c_s^2|\phi_n^s(\mathbf{0})|^2$$

$$\left|\phi_n^{v/s}(\mathbf{0})\right|^2 = \left|\phi_n^C(\mathbf{0})\right|^2\left(1 + \delta\phi_n^{v/s}\right) = \mathop{\mathrm{Res}}_{E=E_n} G(\mathbf{0},\mathbf{0};E)\,,$$

where the Coulomb wave function is given by

$$\left|\phi_n^C(\mathbf{0})\right|^2 = \frac{1}{\pi}\left(\frac{m_Q C_F\alpha_s}{2n}\right)^3 .$$

Note that $\left|\phi_n^{v/s}(\mathbf{0})\right|^2$ are SCHEME and SCALE dependent.

Inclusive electromagnetic decays: bottomonium

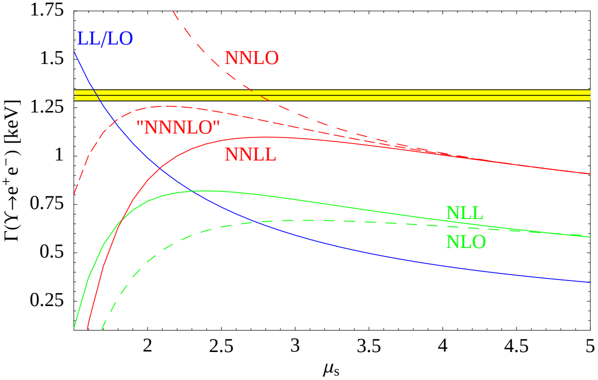


Figure: Prediction for the $\Upsilon(1S)$ decay rate to e^+e^-. We work in the RS' scheme. Pineda, Signer

The effect of the resummation of logarithms is important if compared with just keeping the single logarithm.

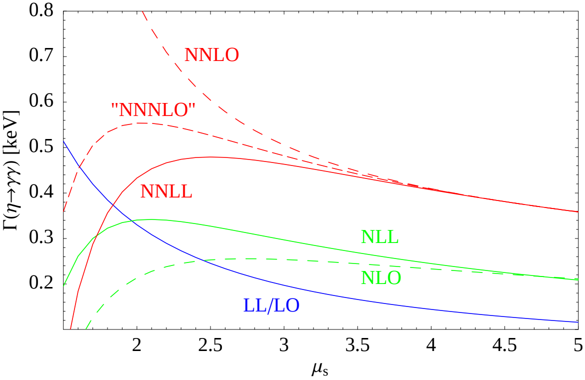


Figure: Prediction for the $\eta_b(1S)$ decay rate to two photons. We work in the RS' scheme.

Decay Ratio at NNLL

Penin, Smirnov, Steinhauser, Pineda

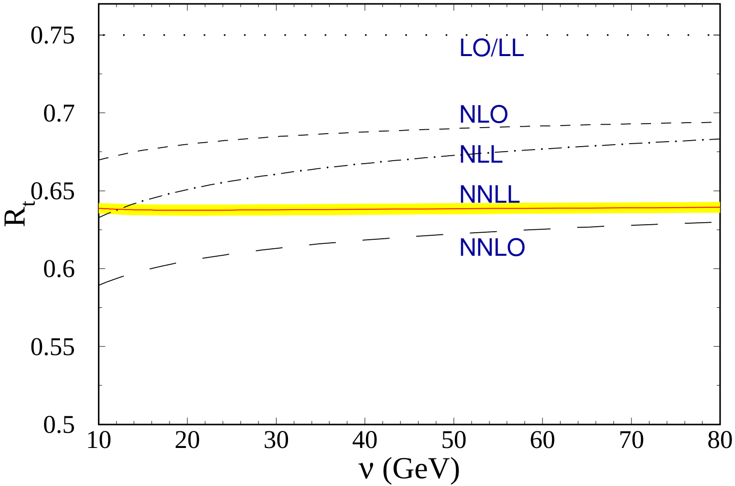
$$\frac{\Gamma(V_Q(nS) \to e^+e^-)}{\Gamma(P_Q(nS) \to \gamma\gamma)} \sim 1 + \alpha \ln \alpha + \alpha^2 \ln^2 \alpha + \cdots$$
$$+\alpha + \alpha^2 \ln \alpha + \alpha^3 \ln^2 \alpha + \cdots$$
$$+\alpha^2 + \alpha^3 \ln \alpha + \alpha^4 \ln^2 \alpha + \cdots$$

$$\left|\phi_n^{v/s}(\mathbf{0})\right|^2 = \left|\phi_n^C(\mathbf{0})\right|^2 \left(1 + \delta\phi_n^{v/s}\right) = \underset{E=E_n}{\mathrm{Res}} G(\mathbf{0}, \mathbf{0}; E) \,,$$

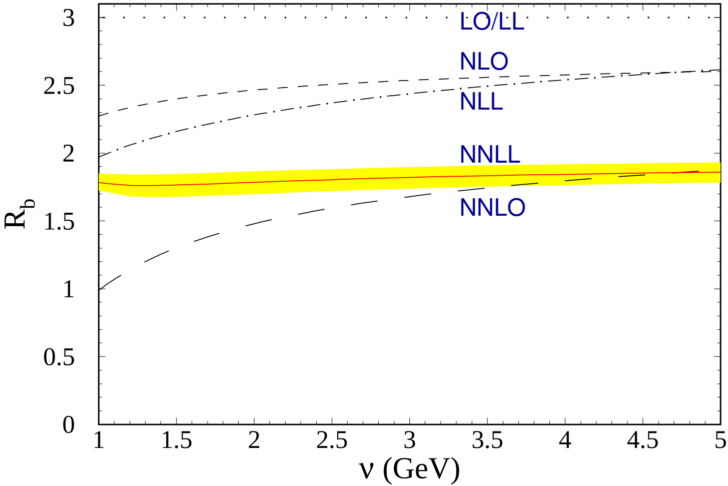
Note that $\delta\phi_n^{v/s}$ are DIVERGENT: SCHEME and SCALE dependent.

$$\frac{\Gamma(V \to e^+e^-)}{\Gamma(P \to \gamma\gamma)} \sim \frac{c_v}{c_s}(\mu)\left(1 + \delta\phi_n^v(\mu) - \delta\phi_n^s(\mu)\right)$$
$$\sim 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(v^2) + \cdots$$

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.659 \pm 0.089(\mathrm{th.})^{+0.019}_{-0.018}(\delta\alpha_\mathrm{s}) \pm 0.015(\mathrm{exp.}) \ \mathrm{KeV} \,,$$



The spin ratio as the function of the renormalization scale ν for the (would be) toponium ground state. The yellow band reflects the errors due to $\alpha_s(M_Z)$.



The spin ratio as the function of the renormalization scale ν for the bottomonium ground state. The yellow band reflects the errors due to $\alpha_s(M_Z)$.

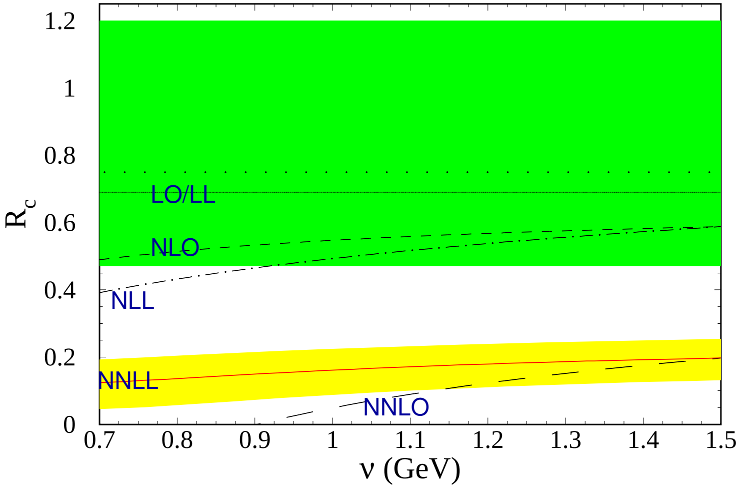


Figure: The decay ratio. The horizontal band represents the experimental error of the ratio. The NNLL band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$.

Proposal: Reorganization of perturbation theory (Kiyo, Pineda, Signer)

Improved perturbation theory "acceleration of perturbation theory"

$$H^{(0)} = \frac{\mathbf{p}^2}{m} + V^{(0)} \longrightarrow E_n^{(0)}, \quad \phi_n^{(0)}(\mathbf{r})$$

Keep the static potential exactly

$$V_s^{(0)} = -C_F \frac{\alpha_s(1/r)}{r} \left(1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \frac{\alpha_s^2(1/r)}{16\pi^2} + \cdots \right)$$

Relativistic corrections:

$$\Delta H = \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \cdots$$

$$\Gamma(V \to e^+e^-) \sim \frac{1}{m^2} c_v^2 |\phi_n^v(\mathbf{0})|^2 + \cdots \qquad \Gamma(P \to \gamma\gamma) \sim \frac{1}{m^2} c_s^2 |\phi_n^s(\mathbf{0})|^2 + \cdots$$

$$E_{HF} \sim c_{HF} |\phi_n^v(\mathbf{0})|^2 + \cdots$$

$$\left|\phi_n^{v/s}(\mathbf{0})\right|^2 = \left|\phi_n^{(0)}(\mathbf{0})\right|^2 \left(1+\delta\phi_n^{v/s}\right) = \mathop{\mathrm{Res}}_{E=E_n} G(\mathbf{0},\mathbf{0};E)\,,$$

HF and decays sensitive to the behavior of the wave function at the origin:

- $$\left|\phi_n^{C}(\mathbf{0})\right|^2 \longrightarrow \left|\phi_n^{(0)}(\mathbf{0})\right|^2$$
- $\mathcal{O}(v^2)$ relativistic corrections beyond the Coulomb approximation.

$$\frac{\Gamma(V\to e^+e^-)}{\Gamma(P\to\gamma\gamma)} \sim \frac{c_v}{c_s}(\mu)\left(1+\delta\phi_n^v(\mu)-\delta\phi_n^s(\mu)\right)$$

$$\sim 1+\mathcal{O}(\alpha_s)+\mathcal{O}(\alpha_s^2)+\mathcal{O}(v^2)+\cdots$$

Use of "Improved" potential $\to$ numerical analysis (relevant for future analysis with nonperturbative potentials!!)
Regularization in position space $\to$ change to $\overline{\mathrm{MS}}$ (known at one loop)
RG plays an important role.

TOP

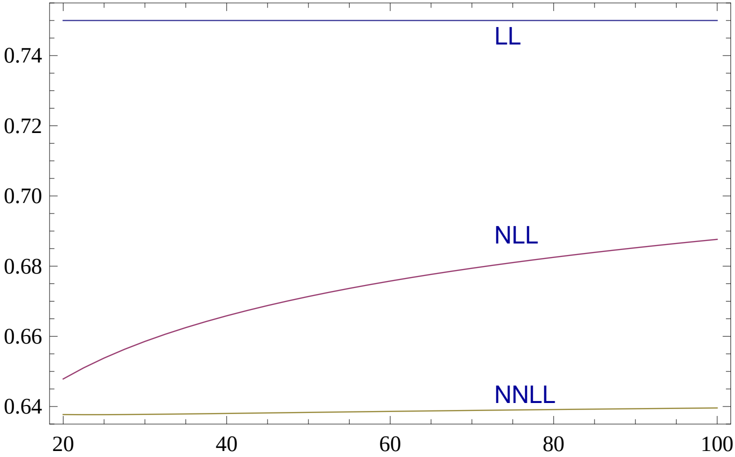


Figure: Decay ratio.

TOP

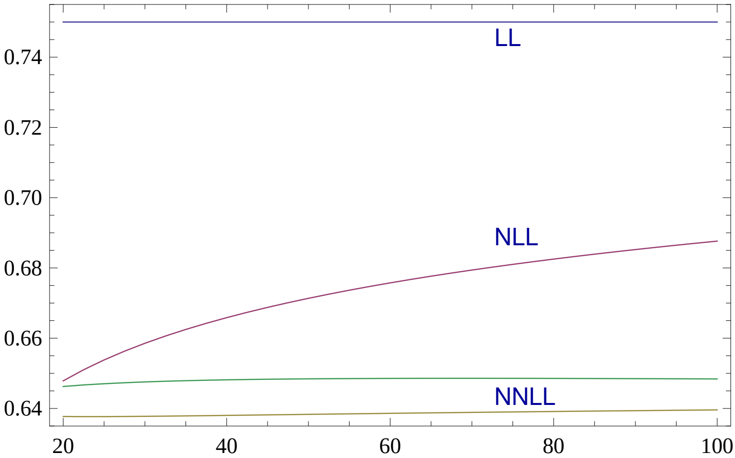


Figure: Decay ratio. $\mathcal{O}(\alpha_s)$ in the static potential.

Sizable but small correction: $\sim$ 1.3%

TOP

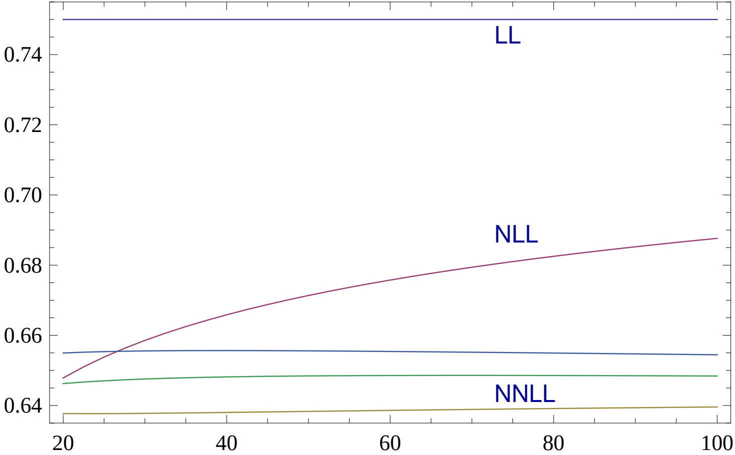


Figure: Decay ratio. $\mathcal{O}(\alpha_s^2)$ in the static potential.

Small correction: $\lesssim 1$ %

TOP

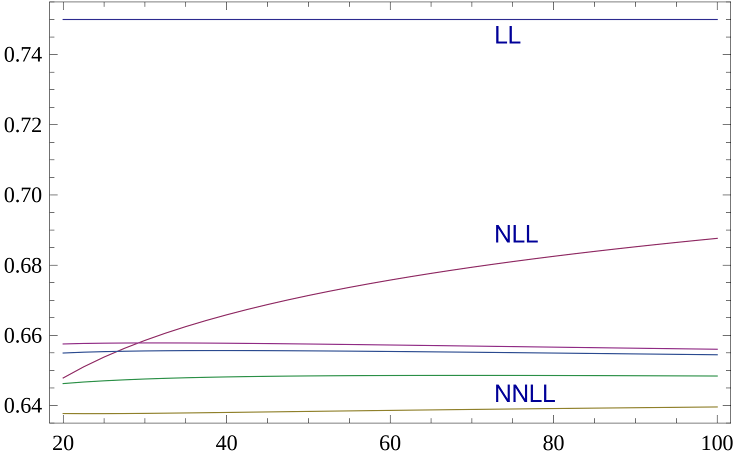


Figure: Decay ratio. $\mathcal{O}(\alpha_s^3)$ in the static potential.

Very small correction. Convergent series

TOP

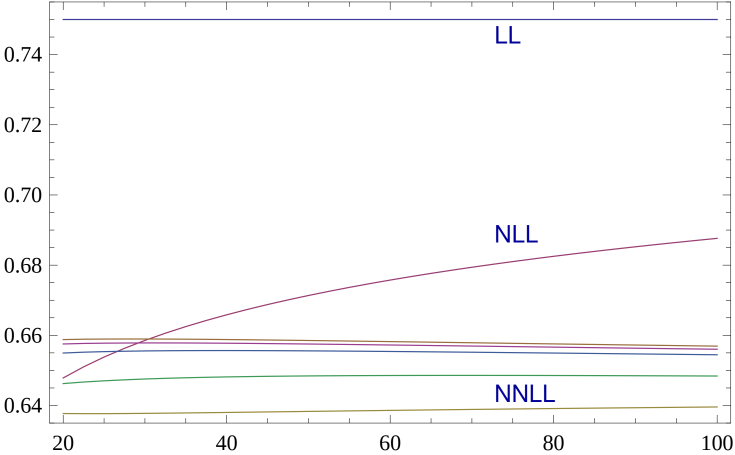


Figure: Decay ratio. $\mathcal{O}(\alpha_s^4)$ in the static potential.

Very small correction. Convergent series.

BOTTOM

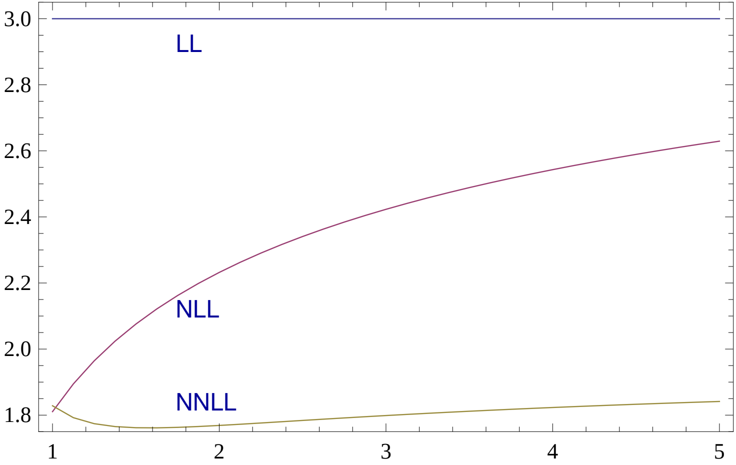


Figure: Decay ratio.

Magnitude of the splitting larger than for top.

BOTTOM

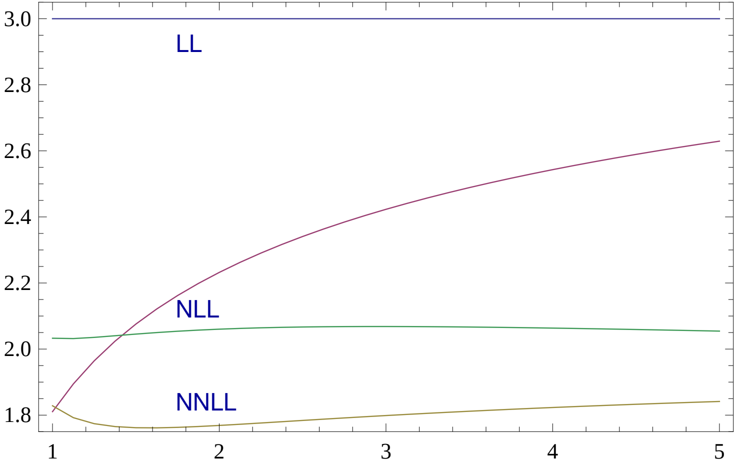


Figure: Decay ratio. $\mathcal{O}(\alpha_s)$ in the static potential.

Sizable but small correction: $\sim$ 10%

BOTTOM

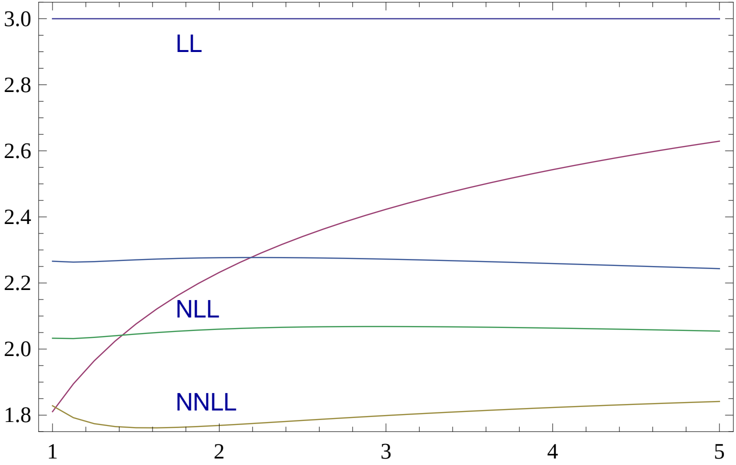


Figure: Decay ratio. $\mathcal{O}(\alpha_s^2)$ in the static potential.

Sizable but small correction: $\sim$ 6.7%

BOTTOM

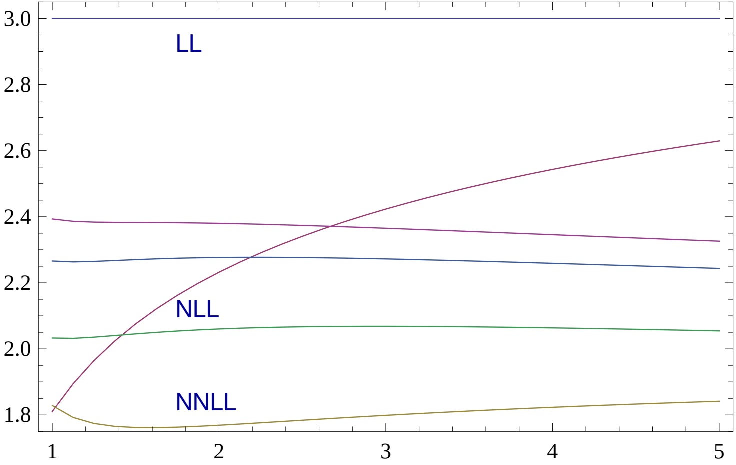


Figure: Decay ratio. $\mathcal{O}(\alpha_s^3)$ in the static potential.

Sizable but convergent correction: $\sim$ 3.3%

BOTTOM

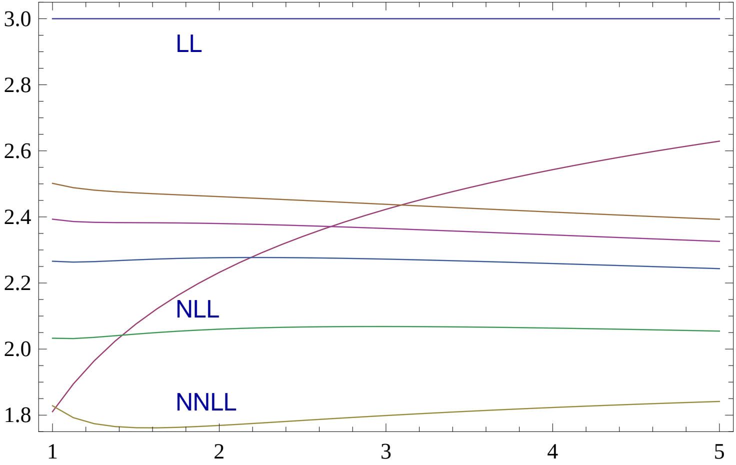


Figure: Decay ratio. $\mathcal{O}(\alpha_s^4)$ in the static potential.

Small correction but not very much convergent. $\mathcal{O}(\alpha_s^4)$ incomplete.

BOTTOM

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \,\mathrm{keV},$$

CHARM

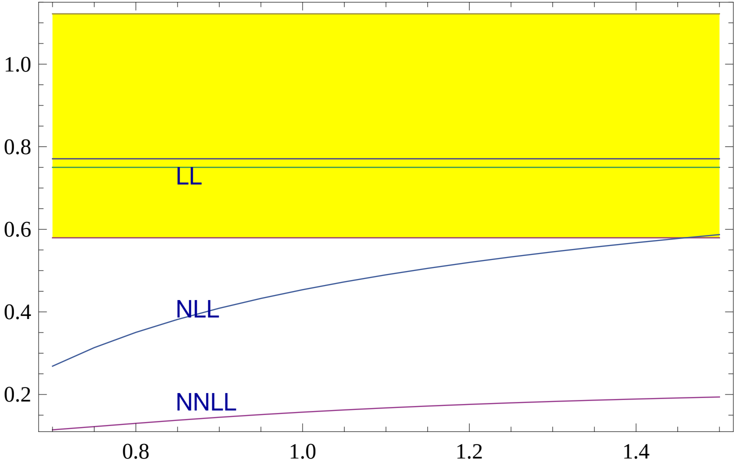


Figure: Decay ratio.

Magnitude of the corrections are large.

CHARM

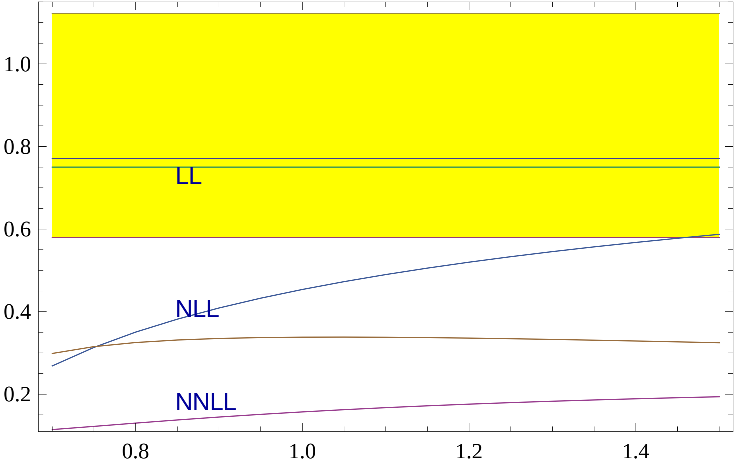


Figure: Decay ratio. $\mathcal{O}(\alpha_s)$ in the static potential.

Sizable correction: $\sim$ 27%

CHARM

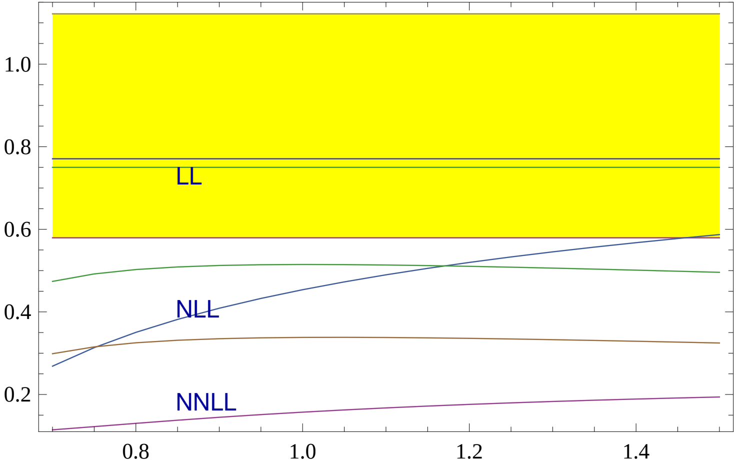


Figure: Decay ratio. $\mathcal{O}(\alpha_s^2)$ in the static potential.

Sizable correction: $\sim$20%

CHARM

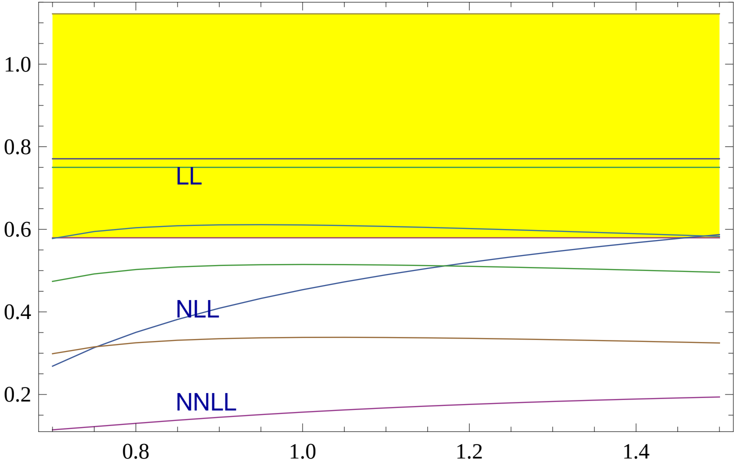


Figure: Decay ratio. $\mathcal{O}(\alpha_s^3)$ in the static potential.

Sizable but convergent: $\sim$13.3%

CHARM

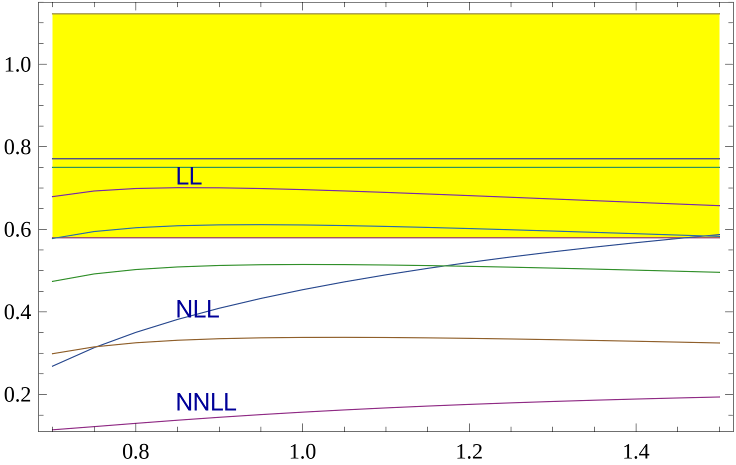


Figure: Decay ratio. $\mathcal{O}(\alpha_s^4)$ in the static potential.

Not very convergent: $\sim$ 10 %. $\mathcal{O}(\alpha_s^4)$ incomplete.
Pattern to bring closer agreement with experiment.

CONCLUSIONS

pNRQCD: Effective field theory from QCD that describes Heavy Quarkonium.

- Weak coupling regime.
- Strong coupling regime.

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- Weak coupling regime

Problems with hyperfine splitting and decays?
Reorganization of perturbative series may lead to a faster convergence. Sizable effect.
Applied to decay ratio. Updated prediction for

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV},$$

Future:
Heavy Quarkonium hyperfine splitting, Decays, sum rules, Spectrum, ...
Application with nonperturbative potential (charm?). Assign errors to (unquenched) lattice potentials.
Wave function at the origin: relation with lattice/experimental. Determination of NRQCD matrix element (scheme and scale dependent).
Other considerations:
The hard and RG contribution are not included in non-relativistic lattice determinations of heavy quarkonium properties. Those effects are sizable.
We need not to do perturbation theory/matching in the lattice. Our approach is analytic as far as the divergences is concerned.

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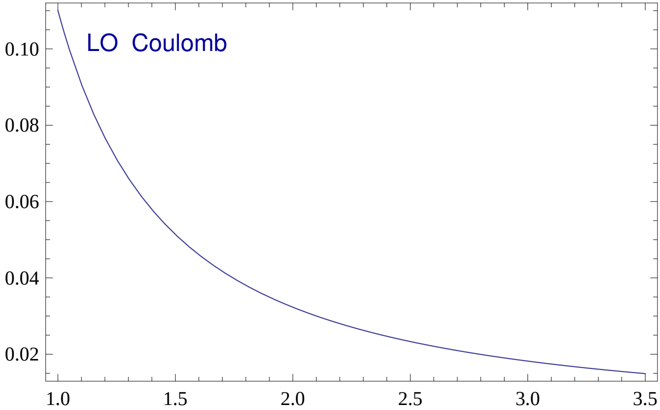
Bottomonium hyperfine splitting? VERY PRELIMINARY

$$E_{HF} \sim c_{HF}|\phi_n^v(\mathbf{0})|^2 + \cdots$$



LO Coulomb: $c_{HF} \sim \alpha_s$ $\quad |\phi_n^v(\mathbf{0})|^2 = |\phi_n^C(\mathbf{0})|^2 \sim m\alpha_s^3$

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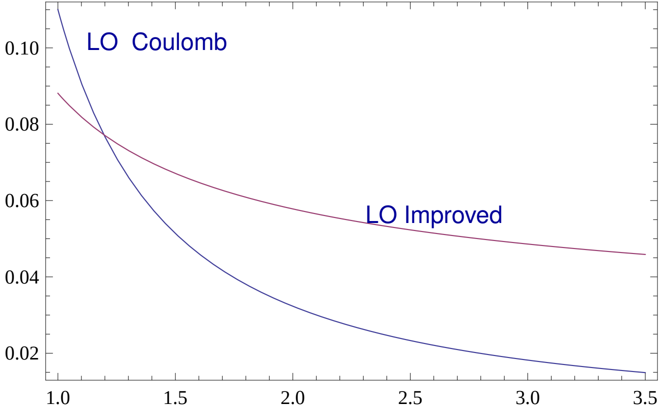

LO Improved: $c_{HF} \sim \alpha_s$ $|\phi_n^v(\mathbf{0})|^2 \sim 0.35$
Result roughly equivalent to non-relativistic lattice.

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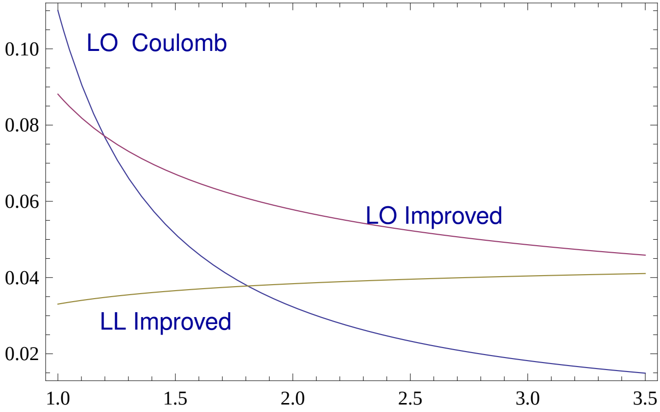

LI Improved: $c_{HF} \sim \alpha_s + \alpha_s^2 \ln + \cdots$ $\quad |\phi_n^v(\mathbf{0})|^2 \sim 0.35$
This gap is not included in non-relativistic lattice.

$$\widehat{G}(E_n) \equiv \sum_m{}' \frac{|\psi_m^{(0)}(0)|^2}{E_m^{(0)} - E_n^{(0)}} = \lim_{E \to E_n^{(0)}} \left(G(E) - \frac{|\psi_n^{(0)}(0)|^2}{E_n^{(0)} - E} \right).$$

The prime indicates that the sum does not include the state n and

$$G(E) = G(0,0;E) \equiv \lim_{r \to 0} G(r,r;E) = \lim_{r \to 0} \langle \mathbf{r} | \frac{1}{H^{(0)} - E - i0} | \mathbf{r} \rangle$$

$$\begin{aligned} G^{(r)}(E) &= \frac{m_r}{2\pi} \left[A^{(r)}(r_0;\mu) + B^{(r)}_{V_s^{(0)}}(E;\mu) \right], \\ A^{(r)}(r_0;\mu) &= \frac{u_0(r_0)}{r_0} = \frac{1}{r_0} - 2 m_r C_F \alpha_s \ln\left(\mu\, e^{\gamma_E} r_0\right) + \mathcal{O}(\alpha_s^2), \end{aligned}$$

$$\widehat{G}^{(r)}(E_n) = \frac{m_r}{2\pi} \left[A^{(r)}(r_0;\mu) + \widehat{B}^{(r)}_{V_s^{(0)}}(E_n;\mu) \right].$$

$$\delta\rho_n^{\overline{\mathrm{MS}}}(\mu) = -\frac{8 m_r C_F}{3 m_1 m_2} D^{(2)}_{S^2,s}(\mu) \left(\widehat{B}^{(r)}_{V_s^{(0)}}(E_n^{(0)};\mu) + \frac{1}{3} m_r C_F\, \alpha_s + \mathcal{O}(\alpha_s^2) \right).$$